PREFACE for Volume I

This volume contains the answers to the Focus on Concepts questions and the complete solutions to the problems for chapters 1 through 17. These chapters include all of mechanics (including fluids), thermal physics and wave motion. The solutions for chapters 18 through 32 (electricity and magnetism, light and optics, and modern physics) are contained in Volume 2.

Each chapter is organized so that the answers to the Focus on Concepts questions appear first, followed by the solutions to the problems.

An electronic version of this manual is available on the Instructor’s Companion Website. The files are available in three formats: Microsoft Word 97-2003, Microsoft Word 2007, and PDF files of each individual solution.

The icon SSM at the beginning of some of the problems indicates that the solution is also available in the Student Solutions Manual.

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1. (d) The resultant vector \( \mathbf{R} \) is drawn from the tail of the first vector to the head of the last vector.

2. (c) Note from the drawing that the magnitude \( R \) of the resultant vector \( \mathbf{R} \) is equal to the shortest distance between the tail of \( \mathbf{A} \) and the head of \( \mathbf{B} \). Thus, \( R \) is less than the magnitude (length) of \( \mathbf{A} \) plus the magnitude of \( \mathbf{B} \).

3. (a) The triangle in the drawing is a right triangle. The lengths \( A \) and \( B \) of the two sides are known, so the Pythagorean theorem can be used to determine the length \( R \) of the hypotenuse.

4. (b) The angle is found by using the inverse tangent function, \( \theta = \tan^{-1}\left(\frac{4.0 \text{ km}}{3.0 \text{ km}}\right) = 53^\circ \).

5. (b) In this drawing the vector \( -\mathbf{C} \) is reversed relative to \( \mathbf{C} \), while vectors \( \mathbf{A} \) and \( \mathbf{B} \) are not reversed.

6. (c) In this drawing the vectors \( -\mathbf{B} \) and \( -\mathbf{C} \) are reversed relative to \( \mathbf{B} \) and \( \mathbf{C} \), while vector \( \mathbf{A} \) is not reversed.

7. (e) These vectors form a closed four-sided polygon, with the head of the fourth vector exactly meeting the tail of the first vector. Thus, the resultant vector is zero.

8. (c) When the two vector components \( A_x \) and \( A_y \) are added by the tail-to-head method, the sum equals the vector \( \mathbf{A} \). Therefore, these vector components are the correct ones.

9. (b) The three vectors form a right triangle, so the magnitude of \( \mathbf{A} \) is given by the Pythagorean theorem as \( A = \sqrt{A_x^2 + A_y^2} \). If \( A_x \) and \( A_y \) double in size, then the magnitude of \( \mathbf{A} \) doubles:

\[
\left(2A_x\right)^2 + \left(2A_y\right)^2 = 4A_x^2 + 4A_y^2 = 2\sqrt{A_x^2 + A_y^2} = 2A.
\]

10. (a) The angle \( \theta \) is determined by the inverse tangent function, \( \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \). If \( A_x \) and \( A_y \) both become twice as large, the ratio does not change, and \( \theta \) remains the same.

11. (b) The displacement vector \( \mathbf{A} \) points in the \( -y \) direction. Therefore, it has no scalar component along the \( x \) axis (\( A_x = 0 \text{ m} \)) and its scalar component along the \( y \) axis is negative.
12. (e) The scalar components are given by \( A_x' = -(450 \text{ m}) \sin 35.0^\circ = -258 \text{ m} \) and 
\[ A_y' = -(450 \text{ m}) \cos 35.0^\circ = -369 \text{ m}. \]

13. (d) The distance (magnitude) traveled by each runner is the same, but the directions are different. Therefore, the two displacement vectors are not equal.

14. (c) \( A_x \) and \( B_x \) point in opposite directions, and \( A_y \) and \( B_y \) point in the same direction.

15. (d)

16. \( A_y = 3.4 \text{ m}, B_y = 3.4 \text{ m} \)

17. \( R_x = 0 \text{ m}, R_y = 6.8 \text{ m} \)

18. \( R = 7.9 \text{ m}, \theta = 21 \text{ degrees} \)
CHAPTER 1

INTRODUCTION AND
MATHEMATICAL CONCEPTS

PROBLEMS

1. **REASONING** We use the fact that 1 m = 3.28 ft to form the following conversion factor:
   \[
   \frac{1 \text{ m}}{3.28 \text{ ft}} = 1.
   \]

   **SOLUTION** To convert \( \text{ft}^2 \) into \( \text{m}^2 \), we apply the conversion factor twice:
   \[
   \text{Area} = \left(1330 \text{ ft}^2\right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right) = 124 \text{ m}^2
   \]

2. **REASONING**
   a. To convert the speed from miles per hour (mi/h) to kilometers per hour (km/h), we need to convert miles to kilometers. This conversion is achieved by using the relation 1.609 km = 1 mi (see the page facing the inside of the front cover of the text).
   b. To convert the speed from miles per hour (mi/h) to meters per second (m/s), we must convert miles to meters and hours to seconds. This is accomplished by using the conversions 1 mi = 1609 m and 1 h = 3600 s.

   **SOLUTION**
   a. Multiplying the speed of 34.0 mi/h by a factor of unity, \( \frac{1.609 \text{ km}}{1 \text{ mi}} = 1 \), we find the speed of the bicyclists is
   \[
   \text{Speed} = \left(34.0 \frac{\text{mi}}{\text{h}}\right)(1) = \left(34.0 \frac{\text{mi}}{\text{h}}\right) \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) = 54.7 \frac{\text{km}}{\text{h}}
   \]
   b. Multiplying the speed of 34.0 mi/h by two factors of unity, \( \frac{1609 \text{ m}}{1 \text{ mi}} = 1 \) and \( \frac{1 \text{ h}}{3600 \text{ s}} = 1 \), the speed of the bicyclists is
   \[
   \text{Speed} = \left(34.0 \frac{\text{mi}}{\text{h}}\right)(1)(1) = \left(34.0 \frac{\text{mi}}{\text{h}}\right) \left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 15.2 \frac{\text{m}}{\text{s}}
   \]

3. **SSM REASONING** We use the facts that 1 mi = 5280 ft, 1 m = 3.281 ft, and 1 yd = 3 ft. With these facts we construct three conversion factors: \( \frac{5280 \text{ ft}}{1 \text{ mi}} = 1 \), \( \frac{1 \text{ m}}{3.281 \text{ ft}} = 1 \), and \( \frac{1 \text{ yd}}{3 \text{ ft}} = 1 \).
4. **REASONING** The word “per” indicates a ratio, so “0.35 mm per day” means 0.35 mm/d, which is to be expressed as a rate in ft/century. These units differ from the given units in both length and time dimensions, so both must be converted. For length, 1 m = 10^3 mm, and 1 ft = 0.3048 m. For time, 1 year = 365.24 days, and 1 century = 100 years. Multiplying the resulting growth rate by one century gives an estimate of the total length of hair a long-lived adult could grow over his lifetime.

**SOLUTION** Multiply the given growth rate by the length and time conversion factors, making sure units cancel properly:

\[
\text{Growth rate} = \left(0.35 \text{ mm/day}\right) \left(\frac{1 \text{ m}}{10^3 \text{ mm}}\right) \left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right) \left(\frac{365.24 \text{ d}}{1 \text{ yr}}\right) \left(\frac{100 \text{ yr}}{1 \text{ century}}\right) = 42 \text{ ft/century}
\]

5. **REASONING** In order to calculate \(d\), the units of \(a\) and \(b\) must be, respectively, cubed and squared along with their numerical values, then combined algebraically with each other and the units of \(c\). Ignoring the values and working first with the units alone, we have

\[
d = \frac{a^3}{cb^2} \rightarrow \frac{(\text{m})^3}{(\text{m/s})(\text{s})^2} = \frac{\text{m}^3}{\text{m}/\text{s} \cdot \text{s}^2} = \frac{\text{m}^2}{\text{s}}
\]

Therefore, the units of \(d\) are \(\text{m}^2/\text{s}\).

**SOLUTION** With the units known, the numerical value may be calculated:

\[
d = \frac{(9.7)^3}{(69)(4.2)^2} \text{ m}^2/\text{s} = 0.75 \text{ m}^2/\text{s}
\]
**SOLUTION** Since \( v = \frac{1}{3} z x t^2 \), it follows that \( z = \frac{3v}{x t^2} \). We know the following dimensions: 
\( v = [L]/[T] \), \( x = [L] \), and \( t = [T] \). Since the factor 3 is dimensionless, \( z \) has the dimensions of 
\[
\frac{v}{x t^2} = \frac{[L]/[T]}{[L][T]^2} = \frac{1}{[T]^3}
\]

7. **SSM REASONING** This problem involves using unit conversions to determine the number of magnums in one jeroboam. The necessary relationships are 
\[
\begin{align*}
1.0 \text{ magnum} &= 1.5 \text{ liters} \\
1.0 \text{ jeroboam} &= 0.792 \text{ U. S. gallons} \\
1.00 \text{ U. S. gallon} &= 3.785 \times 10^{-3} \text{ m}^3 = 3.785 \text{ liters}
\end{align*}
\]
These relationships may be used to construct the appropriate conversion factors. 

**SOLUTION** By multiplying one jeroboam by the appropriate conversion factors we can determine the number of magnums in a jeroboam as shown below: 
\[
1.0 \text{ jeroboam} \left( \frac{0.792 \text{ gallons}}{1.0 \text{ jeroboam}} \right) \left( \frac{3.785 \text{ liters}}{1.0 \text{ gallon}} \right) \left( \frac{1.0 \text{ magnum}}{1.5 \text{ liters}} \right) = 2.0 \text{ magnums}
\]

8. **REASONING** By multiplying the quantity \( 1.78 \times 10^{-3} \text{ kg/}(s \cdot m) \) by the appropriate conversions factors, we can convert the quantity to units of poise (P). These conversion factors are obtainable from the following relationships between the various units: 
\[
\begin{align*}
1 \text{ kg} &= 1.00 \times 10^3 \text{ g} \\
1 \text{ m} &= 1.00 \times 10^2 \text{ cm} \\
1 \text{ P} &= 1 \text{ g/}(s \cdot cm)
\end{align*}
\]

**SOLUTION** The conversion from the unit \( \text{ kg/}(s \cdot m) \) to the unit P proceeds as follows: 
\[
\left( 1.78 \times 10^{-3} \frac{\text{kg}}{s \cdot m} \right) \left( \frac{1.00 \times 10^3 \text{ g}}{1 \text{ kg}} \right) \left( \frac{1 \text{ m}^2}{1.00 \times 10^2 \text{ cm}^2} \right) \left( \frac{1 \text{ P}}{1 \text{ g/}(s \cdot cm)} \right) = 1.78 \times 10^{-2} \text{ P}
\]
9. **REASONING** Multiplying an equation by a factor of 1 does not alter the equation; this is the basis of our solution. We will use factors of 1 in the following forms:

\[
\frac{1 \text{ gal}}{128 \text{ oz}} = 1, \text{ since } 1 \text{ gal} = 128 \text{ oz}
\]

\[
\frac{3.785 \times 10^{-3} \text{ m}^3}{1 \text{ gal}} = 1, \text{ since } 3.785 \times 10^{-3} \text{ m}^3 = 1 \text{ gal}
\]

\[
\frac{1 \text{ mL}}{10^{-6} \text{ m}^3} = 1, \text{ since } 1 \text{ mL} = 10^{-6} \text{ m}^3
\]

**SOLUTION** The starting point for our solution is the fact that

\[
\text{Volume} = 1 \text{ oz}
\]

Multiplying this equation on the right by factors of 1 does not alter the equation, so it follows that

\[
\text{Volume} = (1 \text{ oz})(1)(1)(1) = \left( 1 \text{ oz} \right) \left( \frac{1 \text{ gal}}{128 \text{ oz}} \right) \left( \frac{3.785 \times 10^{-3} \text{ m}^3}{1 \text{ gal}} \right) \left( \frac{1 \text{ mL}}{10^{-6} \text{ m}^3} \right) = 29.6 \text{ mL}
\]

Note that all the units on the right, except one, are eliminated algebraically, leaving only the desired units of milliliters (mL).

10. **REASONING** To convert from gallons to cubic meters, use the equivalence 1 U.S. gal = 3.785×10⁻³ m³. To find the thickness of the painted layer, we use the fact that the paint’s volume is the same, whether in the can or painted on the wall. The layer of paint on the wall can be thought of as a very thin “box” with a volume given by the product of the surface area (the “box top”) and the thickness of the layer. Therefore, its thickness is the ratio of the volume to the painted surface area: Thickness = Volume/Area. That is, the larger the area it’s spread over, the thinner the layer of paint.

**SOLUTION**

a. The conversion is

\[
\left( 0.67 \text{ U.S. gallons} \right) \left( \frac{3.785 \times 10^{-3} \text{ m}^3}{\text{ U.S. gallons}} \right) = 2.5 \times 10^{-3} \text{ m}^3
\]

b. The thickness is the volume found in (a) divided by the area,

\[
\text{Thickness} = \frac{\text{Volume}}{\text{Area}} = \frac{2.5 \times 10^{-3} \text{ m}^3}{13 \text{ m}^2} = 1.9 \times 10^{-4} \text{ m}
\]
11. **SSM REASONING** The dimension of the spring constant \(k\) can be determined by first solving the equation \(T = 2\pi\sqrt{m/k}\) for \(k\) in terms of the time \(T\) and the mass \(m\). Then, the dimensions of \(T\) and \(m\) can be substituted into this expression to yield the dimension of \(k\).

**SOLUTION** Algebraically solving the expression above for \(k\) gives \(k = 4\pi^2 m/T^2\). The term \(4\pi^2\) is a numerical factor that does not have a dimension, so it can be ignored in this analysis. Since the dimension for mass is \([M]\) and that for time is \([T]\), the dimension of \(k\) is

\[
\text{Dimension of } k = \left[\frac{M}{[T]^2}\right]
\]

12. **REASONING AND SOLUTION** The following figure (not drawn to scale) shows the geometry of the situation, when the observer is a distance \(r\) from the base of the arch.

The angle \(\theta\) is related to \(r\) and \(h\) by \(\tan \theta = h/r\).

Solving for \(r\), we find

\[
r = \frac{h}{\tan \theta} = \frac{192 \text{ m}}{\tan 2.0^\circ} = 5.5 \times 10^3 \text{ m} = 5.5 \text{ km}
\]

13. **SSM REASONING** The shortest distance between the two towns is along the line that joins them. This distance, \(h\), is the hypotenuse of a right triangle whose other sides are \(h_o = 35.0 \text{ km}\) and \(h_a = 72.0 \text{ km}\), as shown in the figure below.

**SOLUTION** The angle \(\theta\) is given by \(\tan \theta = h_o / h_a\) so that

\[
\theta = \tan^{-1} \left(\frac{35.0 \text{ km}}{72.0 \text{ km}}\right) = 25.9^\circ \text{ S of W}
\]

We can then use the Pythagorean theorem to find \(h\).

\[
h = \sqrt{h_o^2 + h_a^2} = \sqrt{(35.0 \text{ km})^2 + (72.0 \text{ km})^2} = 80.1 \text{ km}
\]
14. **REASONING** The drawing shows a schematic representation of the hill. We know that the hill rises 12.0 m vertically for every 100.0 m of distance in the horizontal direction, so that \( h_o = 12.0 \) m and \( h_a = 100.0 \) m. Moreover, according to Equation 1.3, the tangent function is \( \tan \theta = \frac{h_o}{h_a} \). Thus, we can use the inverse tangent function to determine the angle \( \theta \).

**SOLUTION** With the aid of the inverse tangent function (see Equation 1.6) we find that

\[
\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{12.0 \text{ m}}{100.0 \text{ m}}\right) = 6.84^\circ
\]

15. **REASONING** Using the Pythagorean theorem (Equation 1.7), we find that the relation between the length \( D \) of the diagonal of the square (which is also the diameter of the circle) and the length \( L \) of one side of the square is \( D = \sqrt{L^2 + L^2} = \sqrt{2}L \).

**SOLUTION** Using the above relation, we have

\[
D = \sqrt{2}L \quad \text{or} \quad L = \frac{D}{\sqrt{2}} = \frac{0.35 \text{ m}}{\sqrt{2}} = 0.25 \text{ m}
\]

16. **REASONING** In both parts of the drawing the line of sight, the horizontal dashed line, and the vertical form a right triangle. The angles \( \theta_a = 35.0^\circ \) and \( \theta_b = 38.0^\circ \) at which the person’s line of sight rises above the horizontal are known, as is the horizontal distance \( d = 85.0 \) m from the building. The unknown vertical sides of the right triangles correspond, respectively, to the heights \( H_a \) and \( H_b \) of the bottom and top of the antenna relative to the person’s eyes. The antenna’s height \( H \) is the difference between \( H_b \) and \( H_a \): \( H = H_b - H_a \).

The horizontal side \( d \) of the triangle is adjacent to the angles \( \theta_a \) and \( \theta_b \), while the vertical sides \( H_a \) and \( H_b \) are opposite these angles. Thus, in either triangle, the angle \( \theta \) is related to the horizontal and vertical sides by Equation 1.3

\[
\tan \theta = \frac{h_o}{h_a}
\]

1. \( \tan \theta_a = \frac{H_a}{d} \)  
2. \( \tan \theta_b = \frac{H_b}{d} \)
SOLUTION Solving Equations (1) and (2) for the heights of the bottom and top of the antenna relative to the person’s eyes, we find that

\[ H_a = d \tan \theta_a \quad \text{and} \quad H_b = d \tan \theta_b \]

The height of the antenna is the difference between these two values:

\[ H = H_b - H_a = d \tan \theta_b - d \tan \theta_a = d \left( \tan \theta_b - \tan \theta_a \right) \]

\[ H = (85.0 \text{ m}) \left( \tan 38.0^\circ - \tan 35.0^\circ \right) = 6.9 \text{ m} \]

17. REASONING The drawing shows the heights of the two balloonists and the horizontal distance \( x \) between them. Also shown in dashed lines is a right triangle, one angle of which is 13.3°. Note that the side adjacent to the 13.3° angle is the horizontal distance \( x \), while the side opposite the angle is the distance between the two heights, 61.0 m – 48.2 m. Since we know the angle and the length of one side of the right triangle, we can use trigonometry to find the length of the other side.

SOLUTION The definition of the tangent function, Equation 1.3, can be used to find the horizontal distance \( x \), since the angle and the length of the opposite side are known:

\[ \tan 13.3^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side (}= x)} \]

Solving for \( x \) gives

\[ x = \frac{\text{length of opposite side}}{\tan 13.3^\circ} = \frac{61.0 \text{ m} - 48.2 \text{ m}}{\tan 13.3^\circ} = 54.1 \text{ m} \]
18. **REASONING** As given in Appendix E, the law of cosines is

\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]

where \( c \) is the side opposite angle \( \gamma \), and \( a \) and \( b \) are the other two sides. Solving for \( \gamma \), we have

\[ \gamma = \cos^{-1}\left( \frac{a^2 + b^2 - c^2}{2ab} \right) \]

**SOLUTION** For \( c = 95 \) cm, \( a = 150 \) cm, and \( b = 190 \) cm

\[ \gamma = \cos^{-1}\left( \frac{150^2 + 190^2 - 95^2}{2 \times 150 \times 190} \right) = \cos^{-1}\left( \frac{(150 \text{ cm})^2 + (190 \text{ cm})^2 - (95 \text{ cm})^2}{2 \times 150 \text{ cm} \times 190 \text{ cm}} \right) = 30^\circ \]

Thus, the angle opposite the side of length 95 cm is \( 30^\circ \).

Similarly, for \( c = 150 \) cm, \( a = 95 \) cm, and \( b = 190 \) cm, we find that the angle opposite the side of length 150 cm is \( 51^\circ \).

Finally, for \( c = 190 \) cm, \( a = 150 \) cm, and \( b = 95 \) cm, we find that the angle opposite the side of length 190 cm is \( 99^\circ \).

As a check on these calculations, we note that \( 30^\circ + 51^\circ + 99^\circ = 180^\circ \), which must be the case for the sum of the three angles in a triangle.

19. **REASONING** Note from the drawing that the shaded right triangle contains the angle \( \theta \), the side opposite the angle (length = 0.281 nm), and the side adjacent to the angle (length = \( L \)). If the length \( L \) can be determined, we can use trigonometry to find \( \theta \). The bottom face of the cube is a square whose diagonal has a length \( L \). This length can be found from the Pythagorean theorem, since the lengths of the two sides of the square are known.

**SOLUTION** The angle can be obtained from the inverse tangent function, Equation 1.6, as \( \theta = \tan^{-1}\left[ \left( \frac{0.281 \text{ nm}}{L} \right) \right] \). Since \( L \) is the length of the hypotenuse of a right triangle whose sides have lengths of 0.281 nm, its value can be determined from the Pythagorean theorem:
Thus, the angle is

\[ \theta = \tan^{-1}\left( \frac{0.281 \text{ nm}}{L} \right) = \tan^{-1}\left( \frac{0.281 \text{ nm}}{0.397 \text{ nm}} \right) = 35.3^\circ \]

20. **REASONING**

   a. The drawing shows the person standing on the earth and looking at the horizon. Notice the right triangle, the sides of which are \( R \), the radius of the earth, and \( d \), the distance from the person’s eyes to the horizon. The length of the hypotenuse is \( R + h \), where \( h \) is the height of the person’s eyes above the water. Since we know the lengths of two sides of the triangle, the Pythagorean theorem can be employed to find the length of the third side.

   b. To convert the distance from meters to miles, we use the relation 1609 m = 1 mi (see the page facing the inside of the front cover of the text).

**SOLUTION**

   a. The Pythagorean theorem (Equation 1.7) states that the square of the hypotenuse is equal to the sum of the squares of the sides, or \( (R + h)^2 = d^2 + R^2 \). Solving this equation for \( d \) yields

\[ d = \sqrt{(R + h)^2 - R^2} = \sqrt{R^2 + 2Rh + h^2 - R^2} \]

\[ = \sqrt{2Rh + h^2} = \sqrt{2(6.38 \times 10^6 \text{ m})(1.6 \text{ m}) + (1.6 \text{ m})^2} = 4500 \text{ m} \]

   b. Multiplying the distance of 4500 m by a factor of unity, \( (1 \text{ mi})/(1609 \text{ m}) = 1 \), the distance (in miles) from the person’s eyes to the horizon is

\[ d = (4500 \text{ m})(1) = (4500 \text{ m})(\frac{1 \text{ mi}}{1609 \text{ m}}) = 2.8 \text{ mi} \]
21. **REASONING** The drawing at the right shows the location of each deer A, B, and C. From the problem statement it follows that
\[ b = 62 \text{ m} \]
\[ c = 95 \text{ m} \]
\[ \gamma = 180^\circ - 51^\circ - 77^\circ = 52^\circ \]
Applying the law of cosines (given in Appendix E) to the geometry in the figure, we have
\[ a^2 - 2ab \cos \gamma + (b^2 - c^2) = 0 \]
which is an expression that is quadratic in \( a \). It can be simplified to \( Aa^2 + Ba + C = 0 \), with
\[ A = 1 \]
\[ B = -2b \cos \gamma = -2(62 \text{ m}) \cos 52^\circ = -76 \text{ m} \]
\[ C = (b^2 - c^2) = (62 \text{ m})^2 - (95 \text{ m})^2 = -5181 \text{ m}^2 \]
This quadratic equation can be solved for the desired quantity \( a \).

**SOLUTION** Suppressing units, we obtain from the quadratic formula
\[ a = \frac{-(76) \pm \sqrt{(-76)^2 - 4(1)(-5181)}}{2(1)} = 1.2 \times 10^2 \text{ m} \text{ and } -43 \text{ m} \]
Discarding the negative root, which has no physical significance, we conclude that the distance between deer A and C is \( 1.2 \times 10^2 \text{ m} \).

22. **REASONING** The trapeze cord is \( L = 8.0 \text{ m} \) long, so that the trapeze is initially \( h_1 = L \cos 41^\circ \) meters below the support. At the instant he releases the trapeze, it is \( h_2 = L \cos \theta \) meters below the support. The difference in the heights is
\[ d = h_2 - h_1 = 0.75 \text{ m} \]
Given that the trapeze is released at a lower elevation than the platform, we expect to find \( \theta < 41^\circ \).

**SOLUTION** Putting the above relationships together, we have
Chapter 1  Problems

23. **SSM REASONING**

a. Since the two force vectors \( \mathbf{A} \) and \( \mathbf{B} \) have directions due west and due north, they are perpendicular. Therefore, the resultant vector \( \mathbf{F} = \mathbf{A} + \mathbf{B} \) has a magnitude given by the Pythagorean theorem: 
\[
F^2 = A^2 + B^2
\]
Knowing the magnitudes of \( A \) and \( B \), we can calculate the magnitude of \( F \). The direction of the resultant can be obtained using trigonometry.

b. For the vector \( \mathbf{F}' = \mathbf{A} - \mathbf{B} \) we note that the subtraction can be regarded as an addition in the following sense: \( \mathbf{F}' = \mathbf{A} + (-\mathbf{B}) \). The vector \(-\mathbf{B}\) points due south, opposite the vector \( \mathbf{B} \), so the two vectors are once again perpendicular and the magnitude of \( \mathbf{F}' \) again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

**SOLUTION**  a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have
\[
F^2 = A^2 + B^2
\]
\[
F = \sqrt{A^2 + B^2}
\]
\[
F = \sqrt{(445 \text{ N})^2 + (325 \text{ N})^2}
\]
\[
= 551 \text{ N}
\]
Using trigonometry, we can see that the direction of the resultant is
\[
\tan \theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{325 \text{ N}}{445 \text{ N}} \right) = 36.1^\circ \text{ north of west}
\]

b. Referring to the drawing and following the same procedure as in part a, we find
\[
F'^2 = A^2 + (-B)^2 \quad \text{or} \quad F' = \sqrt{A^2 + (-B)^2} = \sqrt{(445 \text{ N})^2 + (-325 \text{ N})^2} = 551 \text{ N}
\]
\[
\tan \theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{325 \text{ N}}{445 \text{ N}} \right) = 36.1^\circ \text{ south of west}
\]

\[
d = h_2 - h_1 = L \cos \theta - L \cos 41^\circ \quad \text{or} \quad d + L \cos 41^\circ = L \cos \theta
\]
\[
\cos \theta = \frac{d}{L} + \cos 41^\circ
\]
\[
\theta = \cos^{-1} \left( \frac{d}{L} + \cos 41^\circ \right) = \cos^{-1} \left( \frac{0.75 \text{ m}}{8.0 \text{ m}} + \cos 41^\circ \right) = 32^\circ
\]
24. **REASONING** Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west, so there are two answers. We use “N” to denote the units of the forces, which are specified in newtons.

**SOLUTION** If the second force points **due east**, both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: \( F_1 + F_2 = F_R \). Therefore,

\[
F_2 = F_R - F_1 = 400 \text{ N} - 200 \text{ N} = 200 \text{ N}
\]

If the second force points **due west**, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: \( F_2 - F_1 = F_R \). Therefore,

\[
F_2 = F_R + F_1 = 400 \text{ N} + 200 \text{ N} = 600 \text{ N}
\]

25. **SSM REASONING** For convenience, we can assign due east to be the positive direction and due west to be the negative direction. Since all the vectors point along the same east-west line, the vectors can be added just like the usual algebraic addition of positive and negative scalars. We will carry out the addition for all of the possible choices for the two vectors and identify the resultants with the smallest and largest magnitudes.

**SOLUTION** There are six possible choices for the two vectors, leading to the following resultant vectors:

\[
\begin{align*}
F_1 + F_2 &= 50.0 \text{ newtons} + 10.0 \text{ newtons} = 60.0 \text{ newtons}, \text{ due east} \\
F_1 + F_3 &= 50.0 \text{ newtons} - 40.0 \text{ newtons} = 10.0 \text{ newtons}, \text{ due east} \\
F_1 + F_4 &= 50.0 \text{ newtons} - 30.0 \text{ newtons} = 20.0 \text{ newtons}, \text{ due east} \\
F_2 + F_3 &= 10.0 \text{ newtons} - 40.0 \text{ newtons} = -30.0 \text{ newtons}, \text{ due west} \\
F_2 + F_4 &= 10.0 \text{ newtons} - 30.0 \text{ newtons} = -20.0 \text{ newtons}, \text{ due west} \\
F_3 + F_4 &= -40.0 \text{ newtons} - 30.0 \text{ newtons} = -70.0 \text{ newtons}, \text{ due west}
\end{align*}
\]

The resultant vector with the smallest magnitude is \( F_1 + F_3 = 10.0 \text{ newtons}, \text{ due east} \).

The resultant vector with the largest magnitude is \( F_3 + F_4 = 70.0 \text{ newtons}, \text{ due west} \).
26. **REASONING** The Pythagorean theorem (Equation 1.7) can be used to find the magnitude of the resultant vector, and trigonometry can be employed to determine its direction.

a. Arranging the vectors in tail-to-head fashion, we can see that the vector \( \mathbf{A} \) gives the resultant a westerly direction and vector \( \mathbf{B} \) gives the resultant a southerly direction. Therefore, the resultant \( \mathbf{A} + \mathbf{B} \) points south of west.

b. Arranging the vectors in tail-to-head fashion, we can see that the vector \( \mathbf{A} \) gives the resultant a westerly direction and vector \( -\mathbf{B} \) gives the resultant a northerly direction. Therefore, the resultant \( \mathbf{A} + (-\mathbf{B}) \) points north of west.

**SOLUTION** Using the Pythagorean theorem and trigonometry, we obtain the following results:

a. Magnitude of \( \mathbf{A} + \mathbf{B} = \sqrt{(63 \text{ units})^2 + (63 \text{ units})^2} = 89 \text{ units} \)

\[ \theta = \tan^{-1} \left( \frac{63 \text{ units}}{63 \text{ units}} \right) = 45^\circ \text{ south of west} \]

b. Magnitude of \( \mathbf{A} - \mathbf{B} = \sqrt{(63 \text{ units})^2 + (63 \text{ units})^2} = 89 \text{ units} \)

\[ \theta = \tan^{-1} \left( \frac{63 \text{ units}}{63 \text{ units}} \right) = 45^\circ \text{ north of west} \]

27. **REASONING** At the turning point, the distance to the campground is labeled \( d \) in the drawing. Note that \( d \) is the length of the hypotenuse of a right triangle. Since we know the lengths of the other two sides of the triangle, the Pythagorean theorem can be used to find \( d \). The direction that cyclist #2 must head during the last part of the trip is given by the angle \( \theta \). It can be determined by using the inverse tangent function.

**SOLUTION**

a. The two sides of the triangle have lengths of 1080 m and 520 m (1950 m \(- 1430 \text{ m} = 520 \text{ m} \)). The length \( d \) of the hypotenuse can be determined from the Pythagorean theorem, Equation (1.7), as

\[ d = \sqrt{(1080 \text{ m})^2 + (520 \text{ m})^2} = 1200 \text{ m} \]
INTRODUCTION AND MATHEMATICAL CONCEPTS

b. Since the lengths of the sides opposite and adjacent to the angle \( \theta \) are known, the inverse tangent function (Equation 1.6) can be used to find \( \theta \).

\[
\theta = \tan^{-1} \left( \frac{520 \text{ m}}{1080 \text{ m}} \right) = 26^\circ \text{ south of east}
\]

28. **REASONING** The triple jump consists of a double jump in one direction, followed by a perpendicular single jump, which we can represent with displacement vectors \( \mathbf{J} \) and \( \mathbf{K} \) (see the drawing). These two perpendicular vectors form a right triangle with their resultant \( \mathbf{D} = \mathbf{J} + \mathbf{K} \), which is the displacement of the colored checker. In order to find the magnitude \( D \) of the displacement, we first need to find the magnitudes \( J \) and \( K \) of the double jump and the single jump. As the three sides of a right triangle, \( J, K \), and \( D \) (the hypotenuse) are related to one another by the Pythagorean theorem (Equation 1.7) The double jump moves the colored checker a straight-line distance equal to the length of four square’s diagonals \( d \), and the single jump moves a length equal to two square’s diagonals. Therefore,

\[
J = 4d \quad \text{and} \quad K = 2d \quad (1)
\]

Let the length of a square’s side be \( s \). Any two adjacent sides of a square form a right triangle with the square’s diagonal (see the drawing). The Pythagorean theorem gives the diagonal length \( d \) in terms of the side length \( s \):

\[
d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2} \quad (2)
\]

**SOLUTION** First, we apply the Pythagorean theorem to the right triangle formed by the three displacement vectors, using Equations (1) for \( J \) and \( K \):

\[
D = \sqrt{J^2 + K^2} = \sqrt{(4d)^2 + (2d)^2} = \sqrt{16d^2 + 4d^2} = \sqrt{20d^2} = d\sqrt{20} \quad (3)
\]

Substituting Equation (2) into Equation (3) gives

\[
D = d\sqrt{20} = \left( s\sqrt{2} \right) \sqrt{20} = s\sqrt{40} = (4.0 \text{ cm})\sqrt{40} = 25 \text{ cm}
\]
29. **REASONING** Both \( P \) and \( Q \) and the vector sums \( K \) and \( M \) can be drawn with correct magnitudes and directions by counting grid squares. To add vectors, place them tail-to-head and draw the resultant vector from the tail of the first vector to the head of the last. The vector \( 2P \) is equivalent to \( P + P \), and \(-Q\) is a vector that has the same magnitude as \( Q \), except it is directed in the opposite direction.

The vector \( M \) runs 11 squares horizontally and 3 squares vertically, and the vector \( K \) runs 4 squares horizontally and 9 squares vertically. These distances can be converted from grid squares to centimeters with the grid scale: 1 square = 4.00 cm. Once the distances are calculated in centimeters, the Pythagorean theorem (Equation 1.7) will give the magnitudes of the vectors.

**SOLUTION**

a. The vector \( M = P + Q \) runs 11 squares horizontally and 3 squares vertically, and these distances are equivalent to, respectively, \( \left(4.00 \frac{\text{cm}}{\text{square}}\right)(11 \text{ squares}) = 44.0 \text{ cm} \) and \( \left(4.00 \frac{\text{cm}}{\text{square}}\right)(3 \text{ squares}) = 12.0 \text{ cm} \). Thus, the magnitude of \( M \) is

\[
M = \sqrt{(44.0 \text{ cm})^2 + (12.0 \text{ cm})^2} = 45.6 \text{ cm}
\]

b. Similarly, the lengths of the horizontal and vertical distances of \( K = 2P - Q \) are 4 horizontal squares and 9 vertical squares, or 16.0 cm and 36.0 cm, respectively. The magnitude of \( K \) is then

\[
K = \sqrt{(16.0 \text{ cm})^2 + (36.0 \text{ cm})^2} = 39.4 \text{ cm}
\]

30. **REASONING**

a. and b. The following drawing shows the two vectors \( A \) and \( B \), as well as the resultant vector \( A + B \). The three vectors form a right triangle, of which two of the sides are known. We can employ the Pythagorean theorem, Equation 1.7, to find the length of the third side. The angle \( \theta \) in the drawing can be determined by using the inverse cosine function, Equation 1.5, since the side adjacent to \( \theta \) and the length of the hypotenuse are known.
c. and d. The following drawing shows the vectors $\mathbf{A}$ and $-\mathbf{B}$, as well as the resultant vector $\mathbf{A} - \mathbf{B}$. The three vectors form a right triangle, which is identical to the previous drawing, except for orientation. Thus, the lengths of the hypotenuses and the angles are equal.

**SOLUTION**

a. Let $\mathbf{R} = \mathbf{A} + \mathbf{B}$. The Pythagorean theorem (Equation 1.7) states that the square of the hypotenuse is equal to the sum of the squares of the sides, so that $R^2 = A^2 + B^2$. Solving for $B$ yields

$$B = \sqrt{R^2 - A^2} = \sqrt{(15.0 \text{ units})^2 - (12.3 \text{ units})^2} = 8.6 \text{ units}$$

b. The angle $\theta$ can be found from the inverse cosine function, Equation 1.5:

$$\theta = \cos^{-1}\left(\frac{12.3 \text{ units}}{15.0 \text{ units}}\right) = 34.9^\circ \text{ north of west}$$

c. Except for orientation, the triangles in the two drawings are the same. Thus, the value for $B$ is the same as that determined in part (a) above: $B = 8.6 \text{ units}$

d. The angle $\theta$ is the same as that found in part (a), except the resultant vector points south of west, rather than north of west: $\theta = 34.9^\circ \text{ south of west}$

---

31. **SSM REASONING AND SOLUTION** The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The following figure shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure.
32. **REASONING**  

a. Since the two displacement vectors \( \mathbf{A} \) and \( \mathbf{B} \) have directions due south and due east, they are perpendicular. Therefore, the resultant vector \( \mathbf{R} = \mathbf{A} + \mathbf{B} \) has a magnitude given by the Pythagorean theorem: \( R^2 = A^2 + B^2 \). Knowing the magnitudes of \( \mathbf{R} \) and \( \mathbf{A} \), we can calculate the magnitude of \( \mathbf{B} \). The direction of the resultant can be obtained using trigonometry.

b. For the vector \( \mathbf{R'} = \mathbf{A} - \mathbf{B} \) we note that the subtraction can be regarded as an addition in the following sense: \( \mathbf{R'} = \mathbf{A} + (-\mathbf{B}) \). The vector \(-\mathbf{B}\) points due west, opposite the vector \( \mathbf{B} \), so the two vectors are once again perpendicular and the magnitude of \( \mathbf{R'} \) again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

**SOLUTION**  

a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have  
\[
R^2 = A^2 + B^2 \quad \text{or} \quad B = \sqrt{R^2 - A^2}
\]  
\[
B = \sqrt{(3.75 \text{ km})^2 - (2.50 \text{ km})^2} = 2.8 \text{ km}
\]  
Using trigonometry, we can see that the direction of the resultant is  
\[
\tan \theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8 \text{ km}}{2.50 \text{ km}}\right) = 48^\circ \text{ east of south}
\]

b. Referring to the drawing and following the same procedure as in part a, we find  
\[
R'^2 = A^2 + (-B)^2 \quad \text{or} \quad B = \sqrt{R'^2 - A^2} = \sqrt{(3.75 \text{ km})^2 - (2.50 \text{ km})^2} = 2.8 \text{ km}
\]  
\[
\tan \theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8 \text{ km}}{2.50 \text{ km}}\right) = 48^\circ \text{ west of south}
\]
33. **REASONING AND SOLUTION** The following figure is a scale diagram of the forces drawn tail-to-head. The scale factor is shown in the figure. The head of $F_3$ touches the tail of $F_1$, because the resultant of the three forces is zero.

a. From the figure, $F_3$ must have a magnitude of $78 \text{ N}$ if the resultant force acting on the ball is zero.

b. Measurement with a protractor indicates that the angle $\theta = 34^\circ$.

![Diagram of forces](image)

34. **REASONING** The magnitude of the $x$-component of the force vector is the product of the magnitude of the force times the cosine of the angle between the vector and the $x$ axis. Since the $x$-component points in the $+x$ direction, it is positive. Likewise, the magnitude of the $y$ component of the force vector is the product of the magnitude of the force times the sine of the angle between the vector and the $x$ axis. Since the vector points $36.0^\circ$ below the positive $x$ axis, the $y$ component of the vector points in the $-y$ direction; thus, a minus sign must be assigned to the $y$-component to indicate this direction.

**SOLUTION** The $x$ and $y$ scalar components are

a. $F_x = (575 \text{ newtons}) \cos 36.0^\circ = 465 \text{ newtons}$

b. $F_y = -(575 \text{ newtons}) \sin 36.0^\circ = -338 \text{ newtons}$

35. **SSM REASONING AND SOLUTION** In order to determine which vector has the largest $x$ and $y$ components, we calculate the magnitude of the $x$ and $y$ components explicitly and compare them. In the calculations, the symbol $u$ denotes the units of the vectors.

$A_x = (100.0 \text{ u}) \cos 90.0^\circ = 0.00 \text{ u}$  
$A_y = (100.0 \text{ u}) \sin 90.0^\circ = 1.00 \times 10^2 \text{ u}$

$B_x = (200.0 \text{ u}) \cos 60.0^\circ = 1.00 \times 10^2 \text{ u}$  
$B_y = (200.0 \text{ u}) \sin 60.0^\circ = 173 \text{ u}$

$C_x = (150.0 \text{ u}) \cos 0.00^\circ = 150.0 \text{ u}$  
$C_y = (150.0 \text{ u}) \sin 0.00^\circ = 0.00 \text{ u}$

a. **C** has the largest $x$ component.

b. **B** has the largest $y$ component.
36. **REASONING** The triangle in the drawing is a right triangle. We know one of its angles is 30.0°, and the length of the hypotenuse is 8.6 m. Therefore, the sine and cosine functions can be used to find the magnitudes of $A_x$ and $A_y$. The directions of these vectors can be found by examining the diagram.

**SOLUTION**

a. The magnitude $A_x$ of the displacement vector $A_x$ is related to the length of the hypotenuse and the 30.0° angle by the sine function (Equation 1.1). The drawing shows that the direction of $A_x$ is due east.

$$A_x = A \sin 30.0^\circ = (8.6 \text{ m}) \sin 30.0^\circ = 4.3 \text{ m, due east}$$

b. In a similar manner, the magnitude $A_y$ of $A_y$ can be found by using the cosine function (Equation 1.2). Its direction is due south.

$$A_y = A \cos 30.0^\circ = (8.6 \text{ m}) \cos 30.0^\circ = 7.4 \text{ m, due south}$$

37. **REASONING** Using trigonometry, we can determine the angle $\theta$ from the relation $\tan \theta = A_y / A_x$.

**SOLUTION**

a. $\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \tan^{-1} \left( \frac{12 \text{ m}}{12 \text{ m}} \right) = 45^\circ$

b. $\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \tan^{-1} \left( \frac{12 \text{ m}}{17 \text{ m}} \right) = 35^\circ$

c. $\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \tan^{-1} \left( \frac{17 \text{ m}}{12 \text{ m}} \right) = 55^\circ$

38. **REASONING** The drawing assumes that the horizontal direction along the ground is the $x$ direction and shows the plane’s velocity vector $\mathbf{v}$, along with its horizontal component $v_x$ and vertical component $v_y$. These components, together with the velocity vector, form a right triangle, as indicated. Based on this right triangle, we
can use the cosine function to determine the horizontal velocity component.

**SOLUTION** According to Equation 1.2, the cosine of an angle is the side of the right triangle adjacent to the angle divided by the hypotenuse. Thus, for the $34^\circ$ angle in the drawing we have

$$\cos 34^\circ = \frac{v_x}{v} \quad \text{or} \quad v_x = v \cos 34^\circ = (180 \text{ m/s}) \cos 34^\circ = 150 \text{ m/s}$$

39. **SSM REASONING** The $x$ and $y$ components of $r$ are mutually perpendicular; therefore, the magnitude of $r$ can be found using the Pythagorean theorem. The direction of $r$ can be found using the definition of the tangent function.

**SOLUTION** According to the Pythagorean theorem, we have

$$r = \sqrt{x^2 + y^2} = \sqrt{(-125 \text{ m})^2 + (-184 \text{ m})^2} = 222 \text{ m}$$

The angle $\theta$ is

$$\theta = \tan^{-1} \left( \frac{184 \text{ m}}{125 \text{ m}} \right) = 55.8^\circ$$

40. **REASONING AND SOLUTION**

a. From the Pythagorean theorem, we have

$$F = \sqrt{(150 \text{ N})^2 + (130 \text{ N})^2} = 2 \times 10^2 \text{ N}$$

b. The angle $\theta$ is given by

$$\theta = \tan^{-1} \left( \frac{130 \text{ N}}{150 \text{ N}} \right) = 41^\circ$$

41. **REASONING** Two vectors that are equal must have the same magnitude and direction. Equivalently, they must have identical $x$ components and identical $y$ components. We will begin by examining the given information with respect to these criteria, in order to see if there are obvious reasons why some of the vectors could not be equal. Then we will compare our choices for the equal vectors by calculating the magnitude and direction and the scalar components, as needed.

**SOLUTION** Vectors $A$ and $B$ cannot possibly be equal, because they have different $x$ scalar components of $A_x = 80.0 \text{ m}$ and $B_x = 60.0 \text{ m}$. Furthermore, vectors $B$ and $C$ cannot possibly be equal, because they have different magnitudes of $B = 75.0 \text{ m}$ and $C = 100.0 \text{ m}$. 
Therefore, we conclude that vectors \( \mathbf{A} \) and \( \mathbf{C} \) are the equal vectors. To verify that this is indeed the case we have two choices. We can either calculate the magnitude and direction of \( \mathbf{A} \) (and compare it to the given magnitude and direction of \( \mathbf{C} \)) or determine the scalar components of \( \mathbf{C} \) (and compare them to the given components of \( \mathbf{A} \)). Either choice will do, although both are shown below.

The magnitude and direction of \( \mathbf{A} \) are

\[
A = \sqrt{A_x^2 + A_y^2} = \sqrt{(80.0 \text{ m})^2 + (60.0 \text{ m})^2} = 100.0 \text{ m}
\]

\[
\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \tan^{-1} \left( \frac{60.0 \text{ m}}{80.0 \text{ m}} \right) = 36.9^\circ
\]

These results are identical to those given for \( \mathbf{C} \).

The scalar components of \( \mathbf{C} \) are

\[
C_x = C \cos 36.9^\circ = (100.0 \text{ m}) \cos 36.9^\circ = 80.0 \text{ m}
\]

\[
C_y = C \sin 36.9^\circ = (100.0 \text{ m}) \sin 36.9^\circ = 60.0 \text{ m}
\]

These results are identical to the components given for \( \mathbf{A} \).

---

42. **REASONING** Because both boats travel at 101 km per hour, each one ends up (0.500 h)(101 km/h) = 50.5 km from the dock after a half-hour. They travel along straight paths, so this is the magnitude of both displacement vectors: \( \mathbf{B} = \mathbf{G} = 50.5 \text{ km} \). Since the displacement vector \( \mathbf{G} \) makes an angle of 37° south of due west, its direction can also be expressed as 90° – 37° = 53° west of south. With these angles and the magnitudes of both vectors in hand, we can consider the westward and southward components of each vector.

**SOLUTION**

a. First we calculate the magnitudes of the westward component of the displacement of each boat and then subtract them to find the difference:

\[
\text{Magnitude of } B_{\text{west}} = B \cos 25.0^\circ = (50.5 \text{ km}) \cos 25.0^\circ = 45.8 \text{ km}
\]

\[
\text{Magnitude of } G_{\text{west}} = G \sin 53.0^\circ = (50.5 \text{ km}) \sin 53.0^\circ = 40.3 \text{ km}
\]
The blue boat travels farther by the following amount:

$$45.8 \text{ km} - 40.3 \text{ km} = 5.5 \text{ km}$$

b. Similarly, we find for the magnitudes of the southward components that

$$\text{Magnitude of } B_{\text{south}} = B \sin 25.0^\circ = (50.5 \text{ km}) \sin 25.0^\circ = 21.3 \text{ km}$$

$$\text{Magnitude of } G_{\text{south}} = G \cos 53.0^\circ = (50.5 \text{ km}) \cos 53.0^\circ = 30.4 \text{ km}$$

The green boat travels farther by the following amount:

$$30.4 \text{ km} - 21.3 \text{ km} = 9.1 \text{ km}$$

43. **REASONING**  The force $\mathbf{F}$ and its two components form a right triangle. The hypotenuse is 82.3 newtons, and the side parallel to the $+x$ axis is $F_x = 74.6$ newtons. Therefore, we can use the trigonometric cosine and sine functions to determine the angle of $\mathbf{F}$ relative to the $+x$ axis and the component $F_y$ of $\mathbf{F}$ along the $+y$ axis.

**SOLUTION**

a. The direction of $\mathbf{F}$ relative to the $+x$ axis is specified by the angle $\theta$ as

$$\theta = \cos^{-1} \left( \frac{74.6 \text{ newtons}}{82.3 \text{ newtons}} \right) = 25.0^\circ$$

(1.5)

b. The component of $\mathbf{F}$ along the $+y$ axis is

$$F_y = F \sin 25.0^\circ = (82.3 \text{ newtons}) \sin 25.0^\circ = 34.8 \text{ newtons}$$

(1.4)

44. **REASONING AND SOLUTION**  The force $\mathbf{F}$ can be first resolved into two components; the $z$ component $F_z$ and the projection onto the $x$-$y$ plane, $F_p$ as shown on the left in the following figure. According to this figure,

$$F_p = F \sin 54.0^\circ = (475 \text{ N}) \sin 54.0^\circ = 384 \text{ N}$$

The projection onto the $x$-$y$ plane, $F_p$, can then be resolved into $x$ and $y$ components.
a. From the figure on the right,

\[ F_x = F_p \cos 33.0^\circ = (384 \text{ N}) \cos 33.0^\circ = 322 \text{ N} \]

b. Also from the figure on the right,

\[ F_y = F_p \sin 33.0^\circ = (384 \text{ N}) \sin 33.0^\circ = 209 \text{ N} \]

c. From the figure on the left,

\[ F_z = F \cos 54.0^\circ = (475 \text{ N}) \cos 54.0^\circ = 279 \text{ N} \]

45. **REASONING** The individual displacements of the golf ball, A, B, and C are shown in the figure. Their resultant, \( R \), is the displacement that would have been needed to "hole the ball" on the very first putt. We will use the component method to find \( R \).

\[ \begin{align*}
\text{N} & \quad \text{R} \quad \text{C} \\
\text{E} & \quad \text{A} \quad \text{B}
\end{align*} \]

**SOLUTION** The components of each displacement vector are given in the table below.

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x ) Components</th>
<th>( y ) Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(5.0 m) \cos 0^\circ = 5.0 m</td>
<td>(5.0 m) \sin 0^\circ = 0</td>
</tr>
<tr>
<td>B</td>
<td>(2.1 m) \cos 20.0^\circ = 2.0 m</td>
<td>(2.1 m) \sin 20.0^\circ = 0.72 m</td>
</tr>
<tr>
<td>C</td>
<td>(0.50 m) \cos 90.0^\circ = 0</td>
<td>(0.50 m) \sin 90.0^\circ = 0.50 m</td>
</tr>
<tr>
<td>( R = A + B + C )</td>
<td>7.0 m</td>
<td>1.22 m</td>
</tr>
</tbody>
</table>
The resultant vector \( \mathbf{R} \) has magnitude

\[
R = \sqrt{(7.0 \text{ m})^2 + (1.22 \text{ m})^2} = 7.1 \text{ m}
\]

and the angle \( \theta \) is

\[
\theta = \tan^{-1} \left( \frac{1.22 \text{ m}}{7.0 \text{ m}} \right) = 9.9^\circ
\]

Thus, the required direction is \( 9.9^\circ \) north of east.

---

46. **REASONING** To apply the component method for vector addition, we must first determine the \( x \) and \( y \) components of each vector. The algebraic sum of the three \( x \) components gives the \( x \) component of the resultant. The algebraic sum of the three \( y \) components gives the \( y \) component of the resultant. Knowing the \( x \) and \( y \) components of the resultant will allow us to use the Pythagorean theorem to determine the magnitude of the resultant. Finally, the directional angle of the resultant will be obtained using the trigonometric sine function.

**SOLUTION** Referring to the drawing in the text, we see that the \( x \) and \( y \) components of the vectors are

\[
\begin{align*}
A_x &= -A \cos 20.0^\circ = -(5.00 \text{ m}) \cos 20.0^\circ = -4.70 \text{ m} \\
B_x &= B \cos 60.0^\circ = (5.00 \text{ m}) \cos 60.0^\circ = 2.50 \text{ m} \\
C_x &= 0.00 \text{ m} \\
R_x &= -4.70 \text{ m} + 2.50 \text{ m} + 0.00 \text{ m} = -2.20 \text{ m}
\end{align*}
\]

\[
\begin{align*}
A_y &= A \sin 20.0^\circ = (5.00 \text{ m}) \sin 20.0^\circ = 1.71 \text{ m} \\
B_y &= B \sin 60.0^\circ = (5.00 \text{ m}) \sin 60.0^\circ = 4.33 \text{ m} \\
C_y &= -4.00 \text{ m} \\
R_y &= 1.71 \text{ m} + 4.33 \text{ m} - 4.00 \text{ m} = 2.04 \text{ m}
\end{align*}
\]

Note that the value for \( A_x \) is negative because this component points in the \(-x\) direction and that the value for \( C_x \) is zero because the vector \( \mathbf{C} \) points along the \(-y\) axis. Note also that the value for \( C_y \) is negative because the vector \( \mathbf{C} \) points along the \(-y\) axis.

The \( x \) component \( R_x \) of the resultant vector, being negative, points in the \(-x\) direction. The \( y \) component \( R_y \) of the resultant vector, being positive, points in the \(+y\) direction. The drawing shows these two components and the resultant vector. Since the components are perpendicular, the magnitude \( R \) of the resultant can be obtained using the Pythagorean theorem.
\[ R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20 \text{ m})^2 + (2.04 \text{ m})^2} = 3.00 \text{ m} \]

Referring to the drawing, we can see that \( \theta = R_x / R \), so that the directional angle \( \theta \) is

\[ \theta = \sin^{-1} \left( \frac{R_y}{R} \right) = \sin^{-1} \left( \frac{2.04 \text{ m}}{3.00 \text{ m}} \right) = 42.8^\circ \]

Thus, the resultant vector points in a direction of 42.8° above the negative \( x \) axis.

47. **REASONING** Using the component method for vector addition, we will find the \( x \) component of the resultant force vector by adding the \( x \) components of the individual vectors. Then we will find the \( y \) component of the resultant vector by adding the \( y \) components of the individual vectors. Once the \( x \) and \( y \) components of the resultant are known, we will use the Pythagorean theorem to find the magnitude of the resultant and trigonometry to find its direction. We will take east as the +\( x \) direction and north as the +\( y \) direction.

**SOLUTION** The \( x \) component of the resultant force \( \mathbf{F} \) is

\[ F_x = (2240 \text{ N}) \cos 34.0^\circ + (3160 \text{ N}) \cos 90.0^\circ = (2240 \text{ N}) \cos 34.0^\circ \]

The \( y \) component of the resultant force \( \mathbf{F} \) is

\[ F_y = -(2240 \text{ N}) \sin 34.0^\circ + (-3160 \text{ N}) \]

Using the Pythagorean theorem, we find that the magnitude of the resultant force is

\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2240 \text{ N}) \cos 34.0^\circ}^2 + -(2240 \text{ N}) \sin 34.0^\circ - 3160 \text{ N}^2 = 4790 \text{ N} \]

Using trigonometry, we find that the direction of the resultant force is

\[ \theta = \tan^{-1} \left( \frac{(2240 \text{ N}) \sin 34.0^\circ + 3160 \text{ N}}{(2240 \text{ N}) \cos 34.0^\circ} \right) = 67.2^\circ \text{ south of east} \]

48. **REASONING** The resultant force in part \( a \) is \( \mathbf{F}_A \), because that is the only force applied. The resultant force \( \mathbf{R} \) in part \( b \), where the two additional forces with identical magnitudes \( (F_B = F_C = F) \) are applied, is the sum of all three vectors: \( \mathbf{R} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C \). Because the magnitude \( R \) of the resultant force \( \mathbf{R} \) is \( k \) times larger than the magnitude \( F_A \) of \( \mathbf{F}_A \), we have
To solve the problem, we need to find an expression for $R$ in terms of $F_A$ and $F$. Let the $+x$ direction be in the direction of $F_A$ and the $+y$ direction be upward (see the following drawing), and consider the resultant of the two additional force vectors in part $b$. Because $F_B$ and $F_C$ are directed symmetrically about the $x$ axis, and have the same magnitude $F$, their $y$ components are equal and opposite. Therefore, they cancel out of the vector sum $R$, leaving only the $x$ components of $F_B$ and $F_C$. Now, since $F_A$ also has only an $x$ component, the resultant $R$ can be written as the sum of $F_A$ and the vector $x$ components of $F_B$ and $F_C$: $R = F_A + F_{Bx} + F_{Cx}$. Because these vectors all point in the same direction, we can write down a first expression for the magnitude of the resultant:

$$R = F_A + F_{Bx} + F_{Cx}$$

(2)

The $x$ components of the vectors $F_B$ and $F_C$ are adjacent to the $20.0^\circ$-angles, and so are related to the common magnitude $F$ of both vectors by Equation 1.2 \(\cos \theta = \frac{h_a}{h}\), with $h_a = F_{Bx}$ and $h = F$:

$$F_{Bx} = F \cos \theta \quad \text{and} \quad F_{Cx} = F \cos \theta$$

(3)

**SOLUTION** First, we use Equations (3) to replace $F_{Bx}$ and $F_{Cx}$ in Equation (2):

$$R = F_A + F \cos \theta + F \cos \theta = F_A + 2F \cos \theta$$

(4)

Then, we combine Equation (4) with Equation (1) \((R = kF_A)\) to eliminate $R$, and solve for the desired ratio $F/F_A$:
\[ F_A + 2F \cos \theta = kF_A \quad \text{or} \quad 2F \cos \theta = kF_A - F_A \quad \text{or} \quad 2F \cos \theta = (k-1)F_A \]

\[ \frac{F}{F_A} = \frac{k-1}{2 \cos \theta} = \frac{2.00-1}{2 \cos 20.0^\circ} = 0.532 \]

49. **REASONING**  Using the component method, we find the components of the resultant \( \mathbf{R} \) that are due east and due north. The magnitude and direction of the resultant \( \mathbf{R} \) can be determined from its components, the Pythagorean theorem, and the tangent function.

**SOLUTION**  The first four rows of the table below give the components of the vectors \( \mathbf{A}, \mathbf{B}, \mathbf{C}, \) and \( \mathbf{D} \). Note that east and north have been taken as the positive directions; hence vectors pointing due west and due south will appear with a negative sign.

<table>
<thead>
<tr>
<th>Vector</th>
<th>East/West Component</th>
<th>North/South Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{A} )</td>
<td>+ 2.00 km</td>
<td>0</td>
</tr>
<tr>
<td>( \mathbf{B} )</td>
<td>0</td>
<td>+ 3.75 km</td>
</tr>
<tr>
<td>( \mathbf{C} )</td>
<td>- 2.50 km</td>
<td>0</td>
</tr>
<tr>
<td>( \mathbf{D} )</td>
<td>0</td>
<td>- 3.00 km</td>
</tr>
</tbody>
</table>

\[ \mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} \]

\[ R = \sqrt{(-0.50 \text{ km})^2 + (+0.75 \text{ km})^2} = 0.90 \text{ km} \]

The angle \( \theta \) that \( \mathbf{R} \) makes with the direction due west is

\[ \theta = \tan^{-1}\left(\frac{0.75 \text{ km}}{-0.50 \text{ km}}\right) = 56^\circ \text{ north of west} \]

50. **REASONING**  We will use the scalar \( x \) and \( y \) components of the resultant vector to obtain its magnitude and direction. To obtain the \( x \) component of the resultant we will add together the \( x \) components of each of the vectors. To obtain the \( y \) component of the resultant we will add together the \( y \) components of each of the vectors.
**SOLUTION**

The $x$ and $y$ components of the resultant vector $\mathbf{R}$ are $R_x$ and $R_y$, respectively. In terms of these components, the magnitude $R$ and the directional angle $\theta$ (with respect to the $x$ axis) for the resultant are

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{and} \quad \theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) \quad (1)$$

The following table summarizes the components of the individual vectors shown in the drawing:

<table>
<thead>
<tr>
<th>Vector</th>
<th>$x$ component</th>
<th>$y$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$A_x = -(16.0 \text{ m}) \cos 20.0^\circ = -15.0 \text{ m}$</td>
<td>$A_y = (16.0 \text{ m}) \sin 20.0^\circ = 5.47 \text{ m}$</td>
</tr>
<tr>
<td>B</td>
<td>$B_x = 0 \text{ m}$</td>
<td>$B_y = 11.0 \text{ m}$</td>
</tr>
<tr>
<td>C</td>
<td>$C_x = -(12.0 \text{ m}) \cos 35.0^\circ = -9.83 \text{ m}$</td>
<td>$C_y = -(12.0 \text{ m}) \sin 35.0^\circ = -6.88 \text{ m}$</td>
</tr>
<tr>
<td>D</td>
<td>$D_x = (26.0 \text{ m}) \cos 50.0^\circ = 16.7 \text{ m}$</td>
<td>$D_y = -(26.0 \text{ m}) \sin 50.0^\circ = -19.9 \text{ m}$</td>
</tr>
<tr>
<td>R</td>
<td>$R_x = -15.0 \text{ m} + 0 \text{ m} - 9.83 \text{ m} + 16.7 \text{ m}$</td>
<td>$R_y = 5.47 \text{ m} + 11.0 \text{ m} - 6.88 \text{ m} - 19.9 \text{ m}$</td>
</tr>
<tr>
<td></td>
<td>$= -8.1 \text{ m}$</td>
<td>$= -10.3 \text{ m}$</td>
</tr>
</tbody>
</table>

Note that the component $B_x$ is zero, because $\mathbf{B}$ points along the $y$ axis. Note also that the components $C_x$ and $C_y$ are both negative, since $\mathbf{C}$ points between the $-x$ and $-y$ axes. Finally, note that the component $D_y$ is negative since $\mathbf{D}$ points below the $+x$ axis.

Using equation (1), we find that

$$R = \sqrt{(-8.1 \text{ m})^2 + (-10.3 \text{ m})^2} = 13 \text{ m}$$

$$\theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) = \tan^{-1} \left( \frac{-10.3 \text{ m}}{-8.1 \text{ m}} \right) = 52^\circ$$

Since both $R_x$ and $R_y$ are negative, the resultant points between the $-x$ and $-y$ axes.
51. **REASONING**  If we let the directions due east and due north be the positive directions, then the desired displacement \( \mathbf{A} \) has components

\[
A_E = (4.8 \text{ km}) \cos 42^\circ = 3.57 \text{ km}
\]
\[
A_N = (4.8 \text{ km}) \sin 42^\circ = 3.21 \text{ km}
\]

while the actual displacement \( \mathbf{B} \) has components

\[
B_E = (2.4 \text{ km}) \cos 22^\circ = 2.23 \text{ km}
\]
\[
B_N = (2.4 \text{ km}) \sin 22^\circ = 0.90 \text{ km}
\]

Therefore, to reach the research station, the research team must go

\[
3.57 \text{ km} - 2.23 \text{ km} = 1.34 \text{ km eastward}
\]

and

\[
3.21 \text{ km} - 0.90 \text{ km} = 2.31 \text{ km northward}
\]

**SOLUTION**

a. From the Pythagorean theorem, we find that the magnitude of the displacement vector required to bring the team to the research station is

\[
R = \sqrt{(1.34 \text{ km})^2 + (2.31 \text{ km})^2} = 2.7 \text{ km}
\]

b. The angle \( \theta \) is given by

\[
\theta = \tan^{-1} \left( \frac{2.31 \text{ km}}{1.34 \text{ km}} \right) = 6.0 \times 10^1 \text{ degrees, north of east}
\]

52. **REASONING**  Let \( \mathbf{A} \) be the vector from base camp to the first team, \( \mathbf{B} \) the vector from base camp to the second team, and \( \mathbf{C} \) the vector from the first team’s position to the second team’s position. \( \mathbf{C} \) is the vector whose magnitude and direction are given by the first team’s GPS unit. Since you can get from the base camp to the second team’s position either by traveling along vector \( \mathbf{B} \) alone, or by traveling first along \( \mathbf{A} \) and then along \( \mathbf{C} \), we know that \( \mathbf{B} \) is the vector sum of the other two: \( \mathbf{B} = \mathbf{A} + \mathbf{C} \).
The reading on the first team’s GPS unit is then \( \mathbf{C} = \mathbf{B} - \mathbf{A} \). The components of \( \mathbf{C} \) are found from the components of \( \mathbf{A} \) and \( \mathbf{B} \): \( C_x = B_x - A_x \), \( C_y = B_y - A_y \). Once we have these components, we can calculate the magnitude and direction of \( \mathbf{C} \), as shown on the GPS readout. Because the first team is northwest of camp, and the second team is northeast, we expect the vector \( \mathbf{C} \) to be directed east and either north or south.

**SOLUTION** Let east serve as the positive \( x \) direction and north as the positive \( y \) direction. We then calculate the components of \( \mathbf{C} \), noting that \( \mathbf{B} \)’s components are both positive, and \( \mathbf{A} \)’s \( x \)-component is negative:

\[
C_x = (29 \text{ km}) \sin 35° - (-38 \text{ km}) \cos 19° = 53 \text{ km}
\]

\[
C_y = (29 \text{ km}) \cos 35° - (38 \text{ km}) \sin 19° = 11 \text{ km}
\]

The second team is therefore 53 km east of the first team (since \( C_x \) is positive), and 11 km north (since \( C_y \) is positive). The straight-line distance between the teams can be calculated with the Pythagorean theorem (Equation 1.7):

\[
C = \sqrt{C_x^2 + C_y^2} = \sqrt{(53 \text{ km})^2 + (11 \text{ km})^2} = 54 \text{ km}
\]

The direction of the vector \( \mathbf{C} \) is to be measured relative to due east, so we apply the inverse tangent function (Equation 1.6) to get the angle \( \theta \):

\[
\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{11 \text{ km}}{53 \text{ km}}\right) = 12°
\]

53. **SSM REASONING** Since the finish line is coincident with the starting line, the net displacement of the sailboat is zero. Hence the sum of the components of the displacement vectors of the individual legs must be zero. In the drawing in the text, the directions to the right and upward are taken as positive.

**SOLUTION** In the horizontal direction \( R_h = A_h + B_h + C_h + D_h = 0 \)

\[
R_h = (3.20 \text{ km}) \cos 40.0° - (5.10 \text{ km}) \cos 35.0° - (4.80 \text{ km}) \cos 23.0° + D \cos \theta = 0
\]

\[
D \cos \theta = 6.14 \text{ km}
\]
In the vertical direction  \( R_v = A_v + B_v + C_v + D_v = 0. \)

\[ R_v = (3.20 \text{ km}) \sin 40.0^\circ + (5.10 \text{ km}) \sin 35.0^\circ - (4.80 \text{ km}) \sin 23.0^\circ - D \sin \theta = 0. \]

Dividing (2) by (1) gives

\[ \tan \theta = \frac{(3.11 \text{ km})}{(6.14 \text{ km})} \quad \text{or} \quad \theta = 26.9^\circ \]

Solving (1) gives

\[ D = \frac{(6.14 \text{ km})}{\cos 26.9^\circ} = 6.88 \text{ km} \]

54. **REASONING**  The following table shows the components of the individual displacements and the components of the resultant. The directions due east and due north are taken as the positive directions.

<table>
<thead>
<tr>
<th>Displacement</th>
<th>( \text{East/West Component} )</th>
<th>( \text{North/South Component} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(-27.0 \text{ cm})</td>
<td>(0)</td>
</tr>
<tr>
<td>(2)</td>
<td>(-23.0 \text{ cm}) \cos 35.0^\circ = -18.84 cm</td>
<td>(-23.0 \text{ cm}) \sin 35.0^\circ = -13.19 cm</td>
</tr>
<tr>
<td>(3)</td>
<td>(28.0 \text{ cm}) \cos 55.0^\circ = 16.06 cm</td>
<td>(-28.0 \text{ cm}) \sin 55.0^\circ = -22.94 cm</td>
</tr>
<tr>
<td>(4)</td>
<td>(35.0 \text{ cm}) \cos 63.0^\circ = 15.89 cm</td>
<td>(35.0 \text{ cm}) \sin 63.0^\circ = 31.19 cm</td>
</tr>
</tbody>
</table>

**RESULTANT**

\( -13.89 \text{ cm} \quad -4.94 \text{ cm} \)

**SOLUTION**

a. From the Pythagorean theorem, we find that the magnitude of the resultant displacement vector is

\[ R = \sqrt{(13.89 \text{ cm})^2 + (4.94 \text{ cm})^2} = 14.7 \text{ cm} \]

b. The angle \( \theta \) is given by

\[ \theta = \tan^{-1} \left( \frac{4.94 \text{ cm}}{13.89 \text{ cm}} \right) = 19.6^\circ, \text{ south of west} \]
55. **REASONING** The drawing shows the vectors A, B, and C. Since these vectors add to give a resultant that is zero, we can write that \( A + B + C = 0 \). This addition will be carried out by the component method. This means that the \( x \)-component of this equation must be zero \( (A_x + B_x + C_x = 0) \) and the \( y \)-component must be zero \( (A_y + B_y + C_y = 0) \). These two equations will allow us to find the magnitudes of B and C.

**SOLUTION** The \( x \)- and \( y \)-components of A, B, and C are given in the table below. The plus and minus signs indicate whether the components point along the positive or negative axes.

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x ) Component</th>
<th>( y ) Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-(145 \text{ units}) \cos 35.0^\circ = -119 \text{ units})</td>
<td>(+ (145 \text{ units}) \sin 35.0^\circ = +83.2 \text{ units})</td>
</tr>
<tr>
<td>B</td>
<td>(+ B \sin 65.0^\circ = +B (0.906))</td>
<td>(+ B \cos 65.0^\circ = +B (0.423))</td>
</tr>
<tr>
<td>C</td>
<td>(- C \sin 15.0^\circ = -C (0.259))</td>
<td>(- C \cos 15.0^\circ = -C (0.966))</td>
</tr>
<tr>
<td>A + B + C</td>
<td>(-119 \text{ units} + B (0.906) - C (0.259))</td>
<td>(+83.2 \text{ units} + B (0.423) - C (0.966))</td>
</tr>
</tbody>
</table>

Setting the separate \( x \)- and \( y \)-components of \( A + B + C \) equal to zero gives

- **\( x \)-component**
  \((-119 \text{ units}) + B (0.906) - C (0.259) = 0\)

- **\( y \)-component**
  \((+83.2 \text{ units}) + B (0.423) - C (0.966) = 0\)

Solving these two equations simultaneously, we find that

- a. \( B = \boxed{178 \text{ units}} \)
- b. \( C = \boxed{164 \text{ units}} \)

56. **REASONING** We know that the three displacement vectors have a resultant of zero, so that \( A + B + C = 0 \). This means that the sum of the \( x \) components of the vectors and the sum of the \( y \) components of the vectors are separately equal to zero. From these two equations we will be able to determine the magnitudes of vectors B and C. The directions east and north are, respectively, the \(+x\) and \(+y\) directions.

**SOLUTION** Setting the sum of the \( x \) components of the vectors and the sum of the \( y \) components of the vectors separately equal to zero, we have
\[
\frac{(1550 \text{ m}) \cos 25.0^\circ + B \sin 41.0^\circ + (-C \cos 35.0^\circ)}{A_x} = 0 \\
\frac{(1550 \text{ m}) \sin 25.0^\circ + (-B \cos 41.0^\circ) + C \sin 35.0^\circ}{A_y} = 0 \\
\]

These two equations contain two unknown variables, \(B\) and \(C\). They can be solved simultaneously to show that
a. \(B = \boxed{5550 \text{ m}}\)  
   b. \(C = \boxed{6160 \text{ m}}\)

57. REASONING The shortest distance between the tree and the termite mound is equal to the magnitude of the chimpanzee's displacement \(r\).

SOLUTION
a. From the Pythagorean theorem, we have
\[
r = \sqrt{(51 \text{ m})^2 + (39 \text{ m})^2} = \boxed{64 \text{ m}}
\]

b. The angle \(\theta\) is given by
\[
\theta = \tan^{-1} \left( \frac{39 \text{ m}}{51 \text{ m}} \right) = 37^\circ \text{ south of east}
\]

58. REASONING When the monkey has climbed as far up the pole as it can, its leash is taut, making a straight line from the stake to the monkey, that is, \(L = 3.40 \text{ m}\) long. The leash is the hypotenuse of a right triangle, and the other sides are a line drawn from the stake to the base of the pole \((d = 3.00 \text{ m})\), and a line from the base of the pole to the monkey \((height = h)\).

SOLUTION These three lengths are related by the Pythagorean theorem (Equation 1.7):
\[
h^2 + d^2 = L^2 \quad \text{or} \quad h^2 = L^2 - d^2
\]
\[
h = \sqrt{L^2 - d^2} = \sqrt{(3.40 \text{ m})^2 - (3.00 \text{ m})^2} = \boxed{1.6 \text{ m}}
\]
59. **REASONING** The ostrich’s velocity vector \( \mathbf{v} \) and the desired components are shown in the figure at the right. The components of the velocity in the directions due west and due north are \( v_W \) and \( v_N \), respectively. The sine and cosine functions can be used to find the components.

**SOLUTION**

a. According to the definition of the sine function, we have for the vectors in the figure

\[
\sin \theta = \frac{v_N}{v} \quad \text{or} \quad v_N = v \sin \theta = (17.0 \text{ m/s}) \sin 68^\circ = 15.8 \text{ m/s}
\]

b. Similarly,

\[
\cos \theta = \frac{v_W}{v} \quad \text{or} \quad v_W = v \cos \theta = (17.0 \text{ m/s}) \cos 68.0^\circ = 6.37 \text{ m/s}
\]

60. **REASONING** In the expression for the volume flow rate, the dimensions on the left side of the equals sign are \([L]^3/[T]\). If the expression is to be valid, the dimensions on the right side of the equals sign must also be \([L]^3/[T]\). Thus, the dimensions for the various symbols on the right must combine algebraically to yield \([L]^3/[T]\). We will substitute the dimensions for each symbol in the expression and treat the dimensions of \([M]\), \([L]\), and \([T]\) as algebraic variables, solving the resulting equation for the value of the exponent \(n\).

**SOLUTION** We begin by noting that the symbol \( \pi \) and the number 8 have no dimensions. It follows, then, that

\[
Q = \frac{\pi R^n(P_2 - P_1)}{8 \eta L} \quad \text{or} \quad \frac{[L]^3}{[T]} = \frac{[L]^n}{[M][T]^2} = \frac{[L]^n}{[L][T]} = \frac{[L]^n}{[L][T]}
\]

Thus, we find that \( n = 4 \).
61. **REASONING AND SOLUTION** The east and north components are, respectively

a. \[ A_e = A \cos \theta = (155 \text{ km}) \cos 18.0^\circ = 147 \text{ km} \]

b. \[ A_n = A \sin \theta = (155 \text{ km}) \sin 18.0^\circ = 47.9 \text{ km} \]

62. **REASONING** According to the component method for vector addition, the \( x \) component of the resultant vector is the sum of the \( x \) component of \( A \) and the \( x \) component of \( B \). Similarly, the \( y \) component of the resultant vector is the sum of the \( y \) component of \( A \) and the \( y \) component of \( B \). The magnitude \( R \) of the resultant can be obtained from the \( x \) and \( y \) components of the resultant by using the Pythagorean theorem. The directional angle \( \theta \) can be obtained using trigonometry.

**SOLUTION** We find the following results:

\[
R_x = \left( 244 \text{ km}\right) \cos 30.0^\circ + \left( -175 \text{ km} \right) = 36 \text{ km}
\]

\[
R_y = \left( 244 \text{ km}\right) \sin 30.0^\circ + \left( 0 \text{ km} \right) = 122 \text{ km}
\]

\[
R = \sqrt{R_x^2 + R_y^2} = \sqrt{(36 \text{ km})^2 + (122 \text{ km})^2} = 127 \text{ km}
\]

\[
\theta = \tan^{-1}\left( \frac{R_y}{R_x} \right) = \tan^{-1}\left( \frac{122 \text{ km}}{36 \text{ km}} \right) = 74^\circ
\]

63. **SSM REASONING** The performer walks out on the wire a distance \( d \), and the vertical distance to the net is \( h \). Since these two distances are perpendicular, the magnitude of the displacement is given by the Pythagorean theorem as \( s = \sqrt{d^2 + h^2} \). Values for \( s \) and \( h \) are given, so we can solve this expression for the distance \( d \). The angle that the performer’s displacement makes below the horizontal can be found using trigonometry.

**SOLUTION**

a. Using the Pythagorean theorem, we find that
s = \sqrt{d^2 + h^2}\quad \text{or} \quad d = \sqrt{s^2 - h^2} = \sqrt{(26.7 \text{ ft})^2 - (25.0 \text{ ft})^2} = 9.4 \text{ ft}

b. The angle \( \theta \) that the performer’s displacement makes below the horizontal is given by

\[ \tan \theta = \frac{h}{d}\quad \text{or} \quad \theta = \tan^{-1} \left( \frac{h}{d} \right) = \tan^{-1} \left( \frac{25.0 \text{ ft}}{9.4 \text{ ft}} \right) = 69^\circ \]

64. **REASONING** The force vector \( \mathbf{F} \) points at an angle of \( \theta \) above the +x axis. Therefore, its \( x \) and \( y \) components are given by \( F_x = F \cos \theta \) and \( F_y = F \sin \theta \).

**SOLUTION**

a. The magnitude of the vector can be obtained from the \( y \) component as follows:

\[ F_y = F \sin \theta \quad \text{or} \quad F = \frac{F_y}{\sin \theta} = \frac{290 \text{ N}}{\sin 52^\circ} = 370 \text{ N} \]

b. Now that the magnitude of the vector is known, the \( x \) component of the vector can be calculated as follows:

\[ F_x = F \cos \theta = (370 \text{ N}) \cos 52^\circ = +230 \text{ N} \]

65. **SSM REASONING AND SOLUTION** We take due north to be the direction of the +y axis. Vectors \( \mathbf{A} \) and \( \mathbf{B} \) are the components of the resultant, \( \mathbf{C} \). The angle that \( \mathbf{C} \) makes with the \( x \) axis is then \( \theta = \tan^{-1}(B/A) \). The symbol \( \text{u} \) denotes the units of the vectors.

a. Solving for \( B \) gives

\[ B = A \tan \theta = (6.00 \text{ u}) \tan 60.0^\circ = 10.4 \text{ u} \]

b. The magnitude of \( \mathbf{C} \) is

\[ C = \sqrt{A^2 + B^2} = \sqrt{(6.00 \text{ u})^2 + (10.4 \text{ u})^2} = 12.0 \text{ u} \]

66. **REASONING** We are given that the vector sum of the three forces is zero, so \( \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0 \text{ N} \). Since \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) are known, \( \mathbf{F}_3 \) can be found from the relation \( \mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2) \). We will use the \( x \)- and \( y \)-components of this equation to find the magnitude and direction of \( \mathbf{F}_3 \).

**SOLUTION** The \( x \)- and \( y \)-components of the equation \( \mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2) \) are:
\[ x\text{-component} \quad F_{3x} = -(F_{1x} + F_{2x}) \quad (1) \]

\[ y\text{-component} \quad F_{3y} = -(F_{1y} + F_{2y}) \quad (2) \]

The table below gives the \( x \)- and \( y \)-components of \( F_1 \) and \( F_2 \):

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x ) component</th>
<th>( y ) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>( F_{1x} = -(21.0 , \text{N} ) sin 30.0° = -10.5 , \text{N} )</td>
<td>( F_{1y} = -(21.0 , \text{N} ) cos 30.0° = +18.2 , \text{N} )</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>( F_{2x} = +15.0 , \text{N} )</td>
<td>( F_{2y} = 0 , \text{N} )</td>
</tr>
</tbody>
</table>

Substituting the values for \( F_{1x} \) and \( F_{2x} \) into Equation (1) gives

\[ F_{3x} = -(F_{1x} + F_{2x}) = -(10.5 \, \text{N} + 15.2 \, \text{N}) = -4.5 \, \text{N} \]

Substituting \( F_{1y} \) and \( F_{2y} \) into Equation (2) gives

\[ F_{3y} = -(F_{1y} + F_{2y}) = -(18.2 \, \text{N} + 0 \, \text{N}) = -18.2 \, \text{N} \]

The magnitude of \( F_3 \) can now be obtained by employing the Pythagorean theorem:

\[ F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-4.5 \, \text{N})^2 + (-18.2 \, \text{N})^2} = 18.7 \, \text{N} \]

The angle \( \theta \) that \( F_3 \) makes with respect to the \(-x\) axis can be determined from the inverse tangent function (Equation 1.6),

\[ \theta = \tan^{-1} \left( \frac{F_{3y}}{F_{3x}} \right) = \tan^{-1} \left( \frac{-18.2 \, \text{N}}{-4.5 \, \text{N}} \right) = 76^\circ \]

67. **REASONING AND SOLUTION**  The following figures are scale diagrams of the forces drawn tail-to-head. The scale factor is shown in the figure.

a. From the figure on the left, we see that \[ F_A - F_B = 142 \, \text{N}, \; \theta = 67^\circ \text{ south of east} \].

b. Similarly, from the figure on the right, \[ F_B - F_A = 142 \, \text{N}, \; \theta = 67^\circ \text{ north of west} \].
68. **REASONING** There are two right triangles in the drawing. Each contains the common side that is shown as a dashed line and is labeled $D$, which is the distance between the buildings. The hypotenuse of each triangle is one of the lines of sight to the top and base of the taller building. The remaining (vertical) sides of the triangles are labeled $H_1$ and $H_2$. Since the height of the taller building is $H_1 + H_2$ and the height of the shorter building is $H_1$, the ratio that we seek is $(H_1 + H_2)/H_1$. We will use the tangent function to express $H_1$ in terms of the $52^\circ$ angle and to express $H_2$ in terms of the $21^\circ$ angle. The unknown distance $D$ will be eliminated algebraically when the ratio $(H_1 + H_2)/H_1$ is calculated.

**SOLUTION** The ratio of the building heights is

\[
\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1}
\]

Using the tangent function, we have that
\[
\tan 52^\circ = \frac{H_1}{D} \quad \text{or} \quad H_1 = D \tan 52^\circ
\]

\[
\tan 21^\circ = \frac{H_2}{D} \quad \text{or} \quad H_2 = D \tan 21^\circ
\]

Substituting these results into the expression for the ratio of the heights gives

\[
\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1} = \frac{D \tan 52^\circ + D \tan 21^\circ}{D \tan 52^\circ}
\]

\[
= 1 + \frac{\tan 21^\circ}{\tan 52^\circ} = 1.30
\]

Since 1.30 is less than 1.50, your friend is wrong.

69. **REASONING AND SOLUTION** If \( \mathbf{D} \) is the unknown vector, then \( \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} = 0 \) requires that

\[
\mathbf{D}_E = -(\mathbf{A}_E + \mathbf{B}_E + \mathbf{C}_E) \quad \text{or} \quad \mathbf{D}_E = (113 \, \text{u}) \cos 60.0^\circ - (222 \, \text{u}) \cos 35.0^\circ -(177 \, \text{u}) \cos 23.0^\circ = -288 \, \text{units}
\]

The minus sign indicates that \( \mathbf{D}_E \) has a direction of due west.

Also, \( \mathbf{D}_N = -(\mathbf{A}_N + \mathbf{B}_N + \mathbf{C}_N) \) or

\[
\mathbf{D}_N = (113 \, \text{u}) \sin 60.0^\circ + (222 \, \text{u}) \sin 35.0^\circ - (177 \, \text{u}) \sin 23.0^\circ = 156 \, \text{units}
\]
CHAPTER 2 | KINEMATICS IN ONE DIMENSION

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (b) Displacement, being a vector, conveys information about magnitude and direction. Distance conveys no information about direction and, hence, is not a vector.

2. (c) Since each runner starts at the same place and ends at the same place, the three displacement vectors are equal.

3. (c) The average speed is the distance of 16.0 km divided by the elapsed time of 2.0 h. The average velocity is the displacement of 0 km divided by the elapsed time. The displacement is 0 km, because the jogger begins and ends at the same place.

4. (a) Since the bicycle covers the same number of meters per second everywhere on the track, its speed is constant.

5. (e) The average velocity is the displacement (2.0 km due north) divided by the elapsed time (0.50 h), and the direction of the velocity is the same as the direction of the displacement.

6. (c) The average acceleration is the change in velocity (final velocity minus initial velocity) divided by the elapsed time. The change in velocity has a magnitude of 15.0 km/h. Since the change in velocity points due east, the direction of the average acceleration is also due east.

7. (d) This is always the situation when an object at rest begins to move.

8. (b) If neither the magnitude nor the direction of the velocity changes, then the velocity is constant, and the change in velocity is zero. Since the average acceleration is the change in velocity divided by the elapsed time, the average acceleration is also zero.

9. (a) The runners are always moving after the race starts and, therefore, have a non-zero average speed. The average velocity is the displacement divided by the elapsed time, and the displacement is zero, since the race starts and finishes at the same place. The average acceleration is the change in the velocity divided by the elapsed time, and the velocity changes, since the contestants start at rest and finish while running.

10. (c) The equations of kinematics can be used only when the acceleration remains constant and cannot be used when it changes from moment to moment.

11. (a) Velocity, not speed, appears as one of the variables in the equations of kinematics. Velocity is a vector. The magnitude of the instantaneous velocity is the speed.
12. (b) According to one of the equation of kinematics \( v^2 = v_0^2 + 2ax \), with \( v_0 = 0 \) m/s, the displacement is proportional to the square of the velocity.

13. (d) According to one of the equation of kinematics \( x = v_0t + \frac{1}{2}at^2 \), with \( v_0 = 0 \) m/s, the displacement is proportional to the acceleration.

14. (b) For a single object each equation of kinematics contains four variables, one of which is the unknown variable.

15. (e) An equation of kinematics \( v = v_0 + at \) gives the answer directly, since the initial velocity, the final velocity, and the time are known.

16. (c) An equation of kinematics \( x = \frac{1}{2}(v_0 + v)t \) gives the answer directly, since the initial velocity, the final velocity, and the time are known.

17. (e) An equation of kinematics \( v^2 = v_0^2 + 2ax \) gives the answer directly, since the initial velocity, the final velocity, and the acceleration are known.

18. (d) This statement is false. Near the earth’s surface the acceleration due to gravity has the approximate magnitude of 9.80 m/s² and always points downward, toward the center of the earth.

19. (b) Free-fall is the motion that occurs while the acceleration is solely the acceleration due to gravity. While the rocket is picking up speed in the upward direction, the acceleration is not just due to gravity, but is due to the combined effect of gravity and the engines. In fact, the effect of the engines is greater than the effect of gravity. Only when the engines shut down does the free-fall motion begin.

20. (c) According to an equation of kinematics \( v^2 = v_0^2 + 2ax \), with \( v_0 = 0 \) m/s, the launch speed \( v_0 \) is proportional to the square root of the maximum height.

21. (a) An equation of kinematics \( v = v_0 + at \) gives the answer directly.

22. (d) The acceleration due to gravity points downward, in the same direction as the initial velocity of the stone thrown from the top of the cliff. Therefore, this stone picks up speed as it approaches the nest. In contrast, the acceleration due to gravity points opposite to the initial velocity of the stone thrown from the ground, so that this stone loses speed as it approaches the nest. The result is that, on average, the stone thrown from the top of the cliff travels faster than the stone thrown from the ground and hits the nest first.

23. 1.13 s
24. (a) The slope of the line in a position versus time graph gives the velocity of the motion. The slope for part A is positive. For part B the slope is negative. For part C the slope is positive.

25. (b) The slope of the line in a position versus time graph gives the velocity of the motion. Section A has the smallest slope and section B the largest slope.

26. (c) The slope of the line in a position versus time graph gives the velocity of the motion. Here the slope is positive at all times, but it decreases as time increases from left to right in the graph. This means that the positive velocity is decreasing as time increases, which is a condition of deceleration.
CHAPTER 2

KINEMATICS IN ONE DIMENSION

PROBLEMS

1. **REASONING** The distance traveled by the Space Shuttle is equal to its speed multiplied by the time. The number of football fields is equal to this distance divided by the length \( L \) of one football field.

**SOLUTION**  The number of football fields is

\[
\text{Number} = \frac{x}{L} = \frac{vt}{L} = \frac{(7.6 \times 10^3 \text{ m/s})(110 \times 10^{-3} \text{s})}{91.4 \text{ m}} = 9.1
\]

2. **REASONING** The displacement is a vector that points from an object’s initial position to its final position. If the final position is greater than the initial position, the displacement is positive. On the other hand, if the final position is less than the initial position, the displacement is negative.  (a) The final position is greater than the initial position, so the displacement will be positive.  (b) The final position is less than the initial position, so the displacement will be negative.  (c) The final position is greater than the initial position, so the displacement will be positive.

**SOLUTION**  The displacement is defined as 
\[\text{Displacement} = x - x_0,\]  where \( x \) is the final position and \( x_0 \) is the initial position. The displacements for the three cases are:

(a) Displacement = 6.0 m – 2.0 m = +4.0 m
(b) Displacement = 2.0 m – 6.0 m = -4.0 m
(c) Displacement = 7.0 m – (-3.0 m) = +10.0 m

3. **SSM REASONING** The average speed is the distance traveled divided by the elapsed time (Equation 2.1). Since the average speed and distance are known, we can use this relation to find the time.

**SOLUTION**  The time it takes for the continents to drift apart by 1500 m is

\[
\text{Elapsed time} = \frac{\text{Distance}}{\text{Average speed}} = \frac{1500 \text{ m}}{\left(\frac{3 \text{ cm}}{\text{yr}}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)} = 5 \times 10^4 \text{ yr}
\]
4. **REASONING** Since the average speed of the impulse is equal to the distance it travels divided by the elapsed time (see Equation 2.1), the elapsed time is just the distance divided by the average speed.

**SOLUTION**
The time it takes for the impulse to travel from the foot to the brain is

\[
\text{Time} = \frac{\text{Distance}}{\text{Average speed}} = \frac{1.8 \text{ m}}{1.1 \times 10^2 \text{ m/s}} = 1.6 \times 10^{-2} \text{ s}
\]  

(2.1)

5. **REASONING** According to Equation 2.2 \( \bar{v} = \frac{x - x_0}{t - t_0} \), the average velocity \( \bar{v} \) is equal to the displacement \( x - x_0 \) divided by the elapsed time \( t - t_0 \), and the direction of the average velocity is the same as that of the displacement. The displacement is equal to the difference between the final and initial positions.

**SOLUTION**
Equation 2.2 gives the average velocity as

\[
\bar{v} = \frac{x - x_0}{t - t_0}
\]

Therefore, the average velocities for the three cases are:

(a) Average velocity = \( (6.0 \text{ m} - 2.0 \text{ m})/(0.50 \text{ s}) = +8.0 \text{ m/s} \)

(b) Average velocity = \( (2.0 \text{ m} - 6.0 \text{ m})/(0.50 \text{ s}) = -8.0 \text{ m/s} \)

(c) Average velocity = \( [7.0 \text{ m} - (-3.0 \text{ m})]/(0.50 \text{ s}) = +2.0 \times 10^1 \text{ m/s} \)

The algebraic sign of the answer conveys the direction in each case.

6. **REASONING** Distance and displacement are different physical quantities. Distance is a scalar, and displacement is a vector. Distance and the magnitude of the displacement, however, are both measured in units of length.

**SOLUTION**
a. The distance traveled is equal to three-fourths of the circumference of the circular lake. The circumference of a circle is \( 2\pi r \), where \( r \) is the radius of the circle. Thus, the distance \( d \) that the couple travels is

\[
d = \frac{3}{4} (2\pi r) = \frac{3}{4} [2\pi (1.50 \text{ km})] = 7.07 \text{ km}
\]
b. The couple’s displacement is the hypotenuse of a right triangle with sides equal to the radius of the circle (see the drawing). The magnitude $R$ of the displacement can be obtained with the aid of the Pythagorean theorem:

$$ R = \sqrt{r^2 + r^2} = \sqrt{2(1.50 \text{ km})^2} = 2.12 \text{ km} $$

The angle $\theta$ that the displacement makes with due east is

$$ \theta = \tan^{-1}\left(\frac{r}{r}\right) = \tan^{-1}(1) = 45.0^\circ \text{ north of east} $$

7. **REASONING AND SOLUTION** In 12 minutes the sloth travels a distance of

$$ x_s = v_s t = (0.037 \text{ m/s})(12 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 27 \text{ m} $$

while the tortoise travels a distance of

$$ x_t = v_t t = (0.076 \text{ m/s})(12 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 55 \text{ m} $$

The tortoise goes farther than the sloth by an amount that equals $55 \text{ m} - 27 \text{ m} = 28 \text{ m}$

8. **REASONING** The younger (and faster) runner should start the race after the older runner, the delay being the difference between the time required for the older runner to complete the race and that for the younger runner. The time for each runner to complete the race is equal to the distance of the race divided by the average speed of that runner (see Equation 2.1).

**SOLUTION** The difference between the times for the two runners to complete the race is $t_{50} - t_{18}$, where

$$ t_{50} = \frac{\text{Distance}}{(\text{Average Speed})_{50-yr-old}} \quad \text{and} \quad t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{18-yr-old}} $$

(2.1)

The difference between these two times (which is how much later the younger runner should start) is
\[ t_{50} - t_{18} = \frac{\text{Distance}}{(\text{Average Speed})_{50-yr-old}} - \frac{\text{Distance}}{(\text{Average Speed})_{18-yr-old}} \]

\[ = \frac{10.0 \times 10^3 \text{ m}}{4.27 \text{ m/s}} - \frac{10.0 \times 10^3 \text{ m}}{4.39 \text{ m/s}} = 64 \text{ s} \]

9. **REASONING** In order for the bear to catch the tourist over the distance \( d \), the bear must reach the car at the same time as the tourist. During the time \( t \) that it takes for the tourist to reach the car, the bear must travel a total distance of \( d + 26 \text{ m} \). From Equation 2.1,

\[ v_{\text{tourist}} = \frac{d}{t} \quad (1) \quad \text{and} \quad v_{\text{bear}} = \frac{d + 26 \text{ m}}{t} \quad (2) \]

Equations (1) and (2) can be solved simultaneously to find \( d \).

**SOLUTION** Solving Equation (1) for \( t \) and substituting into Equation (2), we find

\[ v_{\text{bear}} = \frac{d + 26 \text{ m}}{d / v_{\text{tourist}}} = \frac{(d + 26 \text{ m})v_{\text{tourist}}}{d} \]

\[ v_{\text{bear}} = \left(1 + \frac{26 \text{ m}}{d}\right)v_{\text{tourist}} \]

Solving for \( d \) yields:

\[ d = \frac{26 \text{ m}}{\frac{v_{\text{bear}}}{v_{\text{tourist}}} - 1} = \frac{26 \text{ m}}{6.0 \text{ m/s} / 4.0 \text{ m/s} - 1} = 52 \text{ m} \]

10. **REASONING AND SOLUTION** Let west be the positive direction. The average velocity of the backpacker is

\[ v = \frac{x_w + x_e}{t_w + t_e} \quad \text{where} \quad t_w = \frac{x_w}{v_w} \quad \text{and} \quad t_e = \frac{x_e}{v_e} \]

Combining these equations and solving for \( x_e \) (suppressing the units) gives

\[ x_e = \frac{-\left(1 - v/v_w\right)x_w}{\left(1 - v/v_e\right)} = \frac{-1 - (1.34 \text{ m/s} / 2.68 \text{ m/s}) \times 6.44 \text{ km}}{1 - (1.34 \text{ m/s} / 0.447 \text{ m/s})} = -0.81 \text{ km} \]
The distance traveled is the magnitude of $x_e$, or $0.81 \text{ km}$.

11. SSM REASONING AND SOLUTION

a. The total displacement traveled by the bicyclist for the entire trip is equal to the sum of the displacements traveled during each part of the trip. The displacement traveled during each part of the trip is given by Equation 2.2: $\Delta x = \Delta x = v \Delta t$. Therefore,

$$\Delta x_1 = (7.2 \text{ m/s})(22 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 9500 \text{ m}$$

$$\Delta x_2 = (5.1 \text{ m/s})(36 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 11000 \text{ m}$$

$$\Delta x_3 = (13 \text{ m/s})(8.0 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 6200 \text{ m}$$

The total displacement traveled by the bicyclist during the entire trip is then

$$\Delta x = 9500 \text{ m} + 11000 \text{ m} + 6200 \text{ m} = 2.67 \times 10^4 \text{ m}$$

b. The average velocity can be found from Equation 2.2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2.67 \times 10^4 \text{ m}}{(22 \text{ min} + 36 \text{ min} + 8.0 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right)} = 6.74 \text{ m/s, due north}$$

12. REASONING The definition of average velocity is given by Equation 2.2 as

Average velocity = Displacement/(Elapsed time).

The displacement in this expression is the total displacement, which is the sum of the displacements for each part of the trip. Displacement is a vector quantity, and we must be careful to account for the fact that the displacement in the first part of the trip is north, while the displacement in the second part is south.

SOLUTION According to Equation 2.2, the displacement for each part of the trip is the average velocity for that part times the corresponding elapsed time. Designating north as the positive direction, we find for the total displacement that

$$\text{Displacement} = (27 \text{ m/s})t_{\text{North}} + (-17 \text{ m/s})t_{\text{South}}$$

where $t_{\text{North}}$ and $t_{\text{South}}$ denote, respectively, the times for each part of the trip. Note that the minus sign indicates a direction due south. Noting that the total elapsed time is
we can use Equation 2.2 to find the average velocity for the entire trip as follows:

\[
\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}} = \frac{(27 \text{ m/s})t_{\text{North}} + (-17 \text{ m/s})t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}}
\]

\[
= (27 \text{ m/s})\left(\frac{t_{\text{North}}}{t_{\text{North}} + t_{\text{South}}}\right) + (-17 \text{ m/s})\left(\frac{t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}}\right)
\]

But \(\frac{t_{\text{North}}}{t_{\text{North}} + t_{\text{South}}} = \frac{3}{4}\) and \(\frac{t_{\text{South}}}{t_{\text{North}} + t_{\text{South}}} = \frac{1}{4}\). Therefore, we have that

\[
\text{Average velocity} = (27 \text{ m/s})\left(\frac{3}{4}\right) + (-17 \text{ m/s})\left(\frac{1}{4}\right) = +16 \text{ m/s}
\]

The plus sign indicates that the average velocity for the entire trip points north.

13. **REASONING AND SOLUTION** The upper edge of the wall will disappear after the train has traveled the distance \(d\) in the figure below.

The distance \(d\) is equal to the length of the window plus the base of the 12° right triangle of height 0.90 m.

The base of the triangle is given by

\[
b = \frac{0.90 \text{ m}}{\tan 12^\circ} = 4.2 \text{ m}
\]

Thus, \(d = 2.0 \text{ m} + 4.2 \text{ m} = 6.2 \text{ m}\).

The time required for the train to travel 6.2 m is, from the definition of average speed,

\[
t = \frac{x}{v} = \frac{6.2 \text{ m}}{3.0 \text{ m/s}} = 2.1 \text{ s}
\]
14. **REASONING AND SOLUTION** Since \( v = v_0 + at \), the acceleration is given by \( a = \frac{(v - v_0)}{t} \). Since the direction of travel is in the negative direction throughout the problem, all velocities will be negative.

\[
a = \frac{(-29.0 \text{ m/s}) - (-27.0 \text{ m/s})}{5.0 \text{ s}} = -0.40 \text{ m/s}^2
\]

Since the acceleration is negative, it is in the same direction as the velocity and the car is speeding up.

\[
a = \frac{(-23.0 \text{ m/s}) - (-27.0 \text{ m/s})}{5.0 \text{ s}} = +0.80 \text{ m/s}^2
\]

Since the acceleration is positive, it is in the opposite direction to the velocity and the car is slowing down or decelerating.

15. **REASONING** The average acceleration \( \bar{a} \) is defined by Equation 2.4 \( \bar{a} = \frac{v - v_0}{t - t_0} \) as the change in velocity \( (v - v_0) \) divided by the elapsed time \( (t - t_0) \). The change in velocity is equal to the final velocity minus the initial velocity. Therefore, the change in velocity, and hence the acceleration, is positive if the final velocity is greater than the initial velocity. The acceleration is negative if the final velocity is less than the initial velocity. The acceleration is zero if the final and initial velocities are the same.

**SOLUTION** Equation 2.4 gives the average acceleration as

\[
\bar{a} = \frac{v - v_0}{t - t_0}
\]

a. The initial and final velocities are both +82 m/s, since the velocity is constant. The average acceleration is

\[
\bar{a} = \frac{82 \text{ m/s} - 82 \text{ m/s}}{t - t_0} = 0 \text{ m/s}^2
\]

b. The initial velocity is +82 m/s, and the final velocity is -82 m/s. The average acceleration is

\[
\bar{a} = \frac{-82 \text{ m/s} - 82 \text{ m/s}}{12 \text{ s}} = -14 \text{ m/s}^2
\]
16. **REASONING** Although the planet follows a curved, two-dimensional path through space, this causes no difficulty here because the initial and final velocities for this period are in opposite directions. Thus, the problem is effectively a problem in one dimension only.

Equation 2.4 \( \bar{a} = \frac{\Delta v}{\Delta t} \) relates the change \( \Delta v \) in the planet’s velocity to its average acceleration and the elapsed time \( \Delta t = 2.16 \) years. It will be convenient to convert the elapsed time to seconds before calculating the average acceleration.

**SOLUTION**

a. The net change in the planet’s velocity is the final minus the initial velocity:

\[
\Delta v = v - v_0 = -18.5 \text{ km/s} - 20.9 \text{ km/s} = -39.4 \text{ km/s}
\]

\[
\Delta v = \left( -39.4 \text{ km/s} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) = -3.94 \times 10^4 \text{ m/s}
\]

b. Although the planet’s velocity changes by a large amount, the change occurs over a long time interval, so the average acceleration is likely to be small. Expressed in seconds, the interval is

\[
\Delta t = 2.16 \text{ yr} \left( \frac{365 \text{ d}}{1 \text{ yr}} \right) \left( \frac{24 \text{ h}}{1 \text{ d}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 6.81 \times 10^7 \text{ s}
\]

Then the average acceleration is

\[
\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-3.94 \times 10^4 \text{ m/s}}{6.81 \times 10^7 \text{ s}} = -5.79 \times 10^{-4} \text{ m/s}^2
\]

17. **REASONING** Since the velocity and acceleration of the motorcycle point in the same direction, their numerical values will have the same algebraic sign. For convenience, we will choose them to be positive. The velocity, acceleration, and the time are related by Equation 2.4: \( v = v_0 + at \).

**SOLUTION**

a. Solving Equation 2.4 for \( t \) we have

\[
t = \frac{v - v_0}{a} = \frac{(+31 \text{ m/s}) - (+21 \text{ m/s})}{+2.5 \text{ m/s}^2} = 4.0 \text{ s}
\]

b. Similarly,

\[
t = \frac{v - v_0}{a} = \frac{(+61 \text{ m/s}) - (+51 \text{ m/s})}{+2.5 \text{ m/s}^2} = 4.0 \text{ s}
\]
18. **REASONING** We can use the definition of average acceleration $\bar{a} = (v - v_0)/(t - t_0)$ (Equation 2.4) to find the sprinter’s final velocity $v$ at the end of the acceleration phase, because her initial velocity ($v_0 = 0$ m/s, since she starts from rest), her average acceleration $\bar{a}$, and the time interval $t - t_0$ are known.

**SOLUTION**

a. Since the sprinter has a constant acceleration, it is also equal to her average acceleration, so $\bar{a} = +2.3$ m/s$^2$. Her velocity at the end of the 1.2-s period is

$$v = v_0 + \bar{a}(t - t_0) = (0 \text{ m/s}) + (+2.3 \text{ m/s}^2)(1.2 \text{ s}) = +2.8 \text{ m/s}$$

b. Since her acceleration is zero during the remainder of the race, her velocity remains constant at $+2.8 \text{ m/s}$.

19. **REASONING** When the velocity and acceleration vectors are in the same direction, the speed of the object increases in time. When the velocity and acceleration vectors are in opposite directions, the speed of the object decreases in time. (a) The initial velocity and acceleration are in the same direction, so the speed is increasing. (b) The initial velocity and acceleration are in opposite directions, so the speed is decreasing. (c) The initial velocity and acceleration are in opposite directions, so the speed is decreasing. (d) The initial velocity and acceleration are in the same direction, so the speed is increasing.

**SOLUTION** The final velocity $v$ is related to the initial velocity $v_0$, the acceleration $a$, and the elapsed time $t$ through Equation 2.4 ($v = v_0 + at$). The final velocities and speeds for the four moving objects are:

a. $v = 12 \text{ m/s} + (3.0 \text{ m/s}^2)(2.0 \text{ s}) = 18 \text{ m/s}$. The final speed is $18 \text{ m/s}$.

b. $v = 12 \text{ m/s} + (-3.0 \text{ m/s}^2)(2.0 \text{ s}) = 6.0 \text{ m/s}$. The final speed is $6.0 \text{ m/s}$.

c. $v = -12 \text{ m/s} + (3.0 \text{ m/s}^2)(2.0 \text{ s}) = -6.0 \text{ m/s}$. The final speed is $6.0 \text{ m/s}$.

d. $v = -12 \text{ m/s} + (-3.0 \text{ m/s}^2)(2.0 \text{ s}) = -18 \text{ m/s}$. The final speed is $18 \text{ m/s}$.

20. **REASONING** The fact that the emu is slowing down tells us that the acceleration and the velocity have opposite directions. Furthermore, since the acceleration remains the same in both parts of the motion, we can determine its value from the first part of the motion and then use it in the second part to determine the bird’s final velocity at the end of the total 6.0-s time interval.

**SOLUTION**
a. The initial velocity of the emu is directed due north. Since the bird is slowing down, its acceleration must point in the opposite direction, or due south.

b. We assume that due north is the positive direction. With the data given for the first part of the motion, Equation 2.4 shows that the average acceleration is

\[
\bar{a} = \frac{v - v_0}{t - t_0} = \frac{(10.6 \text{ m/s}) - (13.0 \text{ m/s})}{4.0 \text{ s} - 0 \text{ s}} = -0.60 \text{ m/s}^2
\]

The negative value for the acceleration indicates that it indeed points due south, which is the negative direction. Solving Equation 2.4 for the final velocity gives

\[
v = v_0 + \bar{a}(t - t_0) = +10.6 \text{ m/s} + (-0.60 \text{ m/s}^2)(2.0 \text{ s} - 0 \text{ s}) = 9.4 \text{ m/s}
\]

Since this answer is positive, the bird’s velocity after an additional 2.0 s is in the positive direction and is [9.4 m/s, due north].

---

21. **REASONING AND SOLUTION**  
The magnitude of the car's acceleration can be found from Equation 2.4 \((v = v_0 + at)\) as

\[
a = \frac{v - v_0}{t} = \frac{26.8 \text{ m/s} - 0 \text{ m/s}}{3.275 \text{ s}} = 8.18 \text{ m/s}^2
\]

---

22. **REASONING**  
According to Equation 2.4, the average acceleration of the car for the first twelve seconds after the engine cuts out is

\[
\bar{a}_1 = \frac{v_{1f} - v_{10}}{\Delta t_1} \quad \text{(1)}
\]

and the average acceleration of the car during the next six seconds is

\[
\bar{a}_2 = \frac{v_{2f} - v_{20}}{\Delta t_2} = \frac{v_{2f} - v_{1f}}{\Delta t_2} \quad \text{(2)}
\]

The velocity \(v_{1f}\) of the car at the end of the initial twelve-second interval can be found by solving Equations (1) and (2) simultaneously.
SOLUTION  Dividing Equation (1) by Equation (2), we have

\[
\frac{a_1}{a_2} = \frac{(v_{1f} - v_{10})/\Delta t_1}{(v_{2f} - v_{1f})/\Delta t_2} = \frac{(v_{1f} - v_{10})\Delta t_2}{(v_{2f} - v_{1f})\Delta t_1}
\]

Solving for \(v_{1f}\), we obtain

\[
v_{1f} = \frac{a_1\Delta t_1 v_{2f} + a_2\Delta t_2 v_{10}}{a_1\Delta t_1 + a_2\Delta t_2} = \frac{(a_1/a_2)\Delta t_1 v_{2f} + \Delta t_2 v_{10}}{(a_1/a_2)\Delta t_1 + \Delta t_2}
\]

\[
v_{1f} = \frac{1.50(12.0 \text{ s})(+28.0 \text{ m/s}) + (6.0 \text{ s})(+36.0 \text{ m/s})}{1.50(12.0 \text{ s}) + 6.0 \text{ s}} = +30.0 \text{ m/s}
\]

23. REASONING AND SOLUTION  Both motorcycles have the same velocity \(v\) at the end of the four second interval. Now

\[
v = v_{0A} + a_A t
\]

for motorcycle A and

\[
v = v_{0B} + a_B t
\]

for motorcycle B. Subtraction of these equations and rearrangement gives

\[
v_{0A} - v_{0B} = (4.0 \text{ m/s}^2 - 2.0 \text{ m/s}^2)(4 \text{ s}) = +8.0 \text{ m/s}
\]

The positive result indicates that motorcycle A was initially traveling faster.

24. REASONING AND SOLUTION  The average acceleration of the basketball player is

\[
\bar{a} = \frac{v}{t}, \text{ so}
\]

\[
x = \frac{1}{2} \bar{a}t^2 = \frac{1}{2} \left( \frac{6.0 \text{ m/s}}{1.5 \text{ s}} \right)(1.5 \text{ s})^2 = 4.5 \text{ m}
\]

25. SSM REASONING AND SOLUTION

a. The magnitude of the acceleration can be found from Equation 2.4 \((v = v_0 + at)\) as

\[
a = \frac{v - v_0}{t} = \frac{3.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}} = 1.5 \text{ m/s}^2
\]

b. Similarly the magnitude of the acceleration of the car is
\[ a = \frac{v - v_0}{t} = \frac{41.0 \text{ m/s} - 38.0 \text{ m/s}}{2.0 \text{ s}} = 1.5 \text{ m/s}^2 \]

c. Assuming that the acceleration is constant, the displacement covered by the car can be found from Equation 2.9 \((v^2 = v_0^2 + 2ax)\):

\[ x = \frac{v^2 - v_0^2}{2a} = \frac{(41.0 \text{ m/s})^2 - (38.0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 79 \text{ m} \]

Similarly, the displacement traveled by the jogger is

\[ x = \frac{v^2 - v_0^2}{2a} = \frac{(3.0 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 3.0 \text{ m} \]

Therefore, the car travels \(79 \text{ m} - 3.0 \text{ m} = 76 \text{ m}\) further than the jogger.

26. **REASONING**  The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given.

**SOLUTION**  
a. The time \(\Delta t\) that it takes for the VW Beetle to change its velocity by an amount \(\Delta v = v - v_0\) is (and noting that 0.4470 m/s = 1 mi/h)

\[ \Delta t = \frac{v - v_0}{a} = \frac{60.0 \text{ mi/h}}{2.35 \text{ m/s}^2} \left( \frac{0.4470 \text{ m/s}}{1 \text{ mi/h}} \right) - 0 \text{ m/s} = 11.4 \text{ s} \]

b. From Equation 2.4, the acceleration (in m/s^2) of the dragster is

\[ a = \frac{v - v_0}{t - t_0} = \frac{60.0 \text{ mi/h}}{0.600 \text{ s} - 0 \text{ s}} \left( \frac{0.4470 \text{ m/s}}{1 \text{ mi/h}} \right) - 0 \text{ m/s} = 44.7 \text{ m/s}^2 \]

27. **REASONING**  We know the initial and final velocities of the blood, as well as its displacement. Therefore, Equation 2.9 \((v^2 = v_0^2 + 2ax)\) can be used to find the acceleration
of the blood. The time it takes for the blood to reach its final velocity can be found by using Equation 2.7
\[ t = \frac{x}{\frac{1}{2}(v_0 + v)} \].

**SOLUTION**
a. The acceleration of the blood is
\[ a = \frac{v^2 - v_0^2}{2x} = \frac{(26 \text{ cm/s})^2 - (0 \text{ cm/s})^2}{2(2.0 \text{ cm})} = 1.7 \times 10^2 \text{ cm/s}^2 \]

b. The time it takes for the blood, starting from 0 cm/s, to reach a final velocity of +26 cm/s is
\[ t = \frac{x}{\frac{1}{2}(v_0 + v)} = \frac{2.0 \text{ cm}}{\frac{1}{2}(0 \text{ cm/s} + 26 \text{ cm/s})} = 0.15 \text{ s} \]

### 28. REASONING AND SOLUTION
a. From Equation 2.4, the definition of average acceleration, the magnitude of the average acceleration of the skier is
\[ \bar{a} = \frac{v - v_0}{t - t_0} = \frac{8.0 \text{ m/s} - 0 \text{ m/s}}{5.0 \text{ s}} = 1.6 \text{ m/s}^2 \]

b. With \( x \) representing the displacement traveled along the slope, Equation 2.7 gives:
\[ x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(8.0 \text{ m/s} + 0 \text{ m/s})(5.0 \text{ s}) = 2.0 \times 10^1 \text{ m} \]

### 29. SSM REASONING AND SOLUTION
The average acceleration of the plane can be found by solving Equation 2.9 \( v^2 = v_0^2 + 2ax \) for \( a \). Taking the direction of motion as positive, we have
\[ a = \frac{v^2 - v_0^2}{2x} = \frac{(+6.1 \text{ m/s})^2 - (+69 \text{ m/s})^2}{2(+750 \text{ m})} = -3.1 \text{ m/s}^2 \]

The minus sign indicates that the direction of the acceleration is opposite to the direction of motion, and the plane is slowing down.

### 30. REASONING
At a constant velocity the time required for Secretariat to run the final mile is given by Equation 2.2 as the displacement (+1609 m) divided by the velocity. The actual time required for Secretariat to run the final mile can be determined from Equation 2.8,
since the initial velocity, the acceleration, and the displacement are given. It is the
difference between these two results for the time that we seek.

**SOLUTION** According to Equation 2.2, with the assumption that the initial time is $t_0 = 0 \text{ s}$, the run time at a constant velocity is

$$\Delta t = t - t_0 = t = \frac{\Delta x}{v} = \frac{+1609 \text{ m}}{+16.58 \text{ m/s}} = 97.04 \text{ s}$$

Solving Equation 2.8 \(x = v_0t + \frac{1}{2}at^2\) for the time shows that

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-x)}}{2\left(\frac{1}{2}a\right)}$$

$$= \frac{-16.58 \text{ m/s} \pm \sqrt{(+16.58 \text{ m/s})^2 - 4\left(\frac{1}{2}\right)(+0.0105 \text{ m/s}^2)(-1609 \text{ m})}}{2\left(\frac{1}{2}\right)(+0.0105 \text{ m/s}^2)} = 94.2 \text{ s}$$

We have ignored the negative root as being unphysical. The acceleration allowed Secretariat
to run the last mile in a time that was faster by

$$97.04 \text{ s} - 94.2 \text{ s} = 2.8 \text{ s}$$

### 31. **REASONING**
The cart has an initial velocity of $v_0 = +5.0 \text{ m/s}$, so initially it is moving to the right, which is the positive direction. It eventually reaches a point where the displacement is $x = +12.5 \text{ m}$, and it begins to move to the left. This must mean that the cart comes to a momentary halt at this point (final velocity is $v = 0 \text{ m/s}$), before beginning to move to the left. In other words, the cart is decelerating, and its acceleration must point opposite to the velocity, or to the left. Thus, the acceleration is negative. Since the initial velocity, the final velocity, and the displacement are known, Equation 2.9 \(v^2 = v_0^2 + 2ax\)
can be used to determine the acceleration.

**SOLUTION** Solving Equation 2.9 for the acceleration $a$ shows that

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(0 \text{ m/s})^2 - (+5.0 \text{ m/s})^2}{2(+12.5 \text{ m})} = \frac{-1.0 \text{ m/s}^2}{2}$$

### 32. **REASONING** At time $t$ both rockets return to their starting points and have a displacement of zero. This occurs, because each rocket is decelerating during the first half of its journey.
However, rocket A has a smaller initial velocity than rocket B. Therefore, in order for rocket B to decelerate and return to its point of origin in the same time as rocket A, rocket B must have a deceleration with a greater magnitude than that for rocket A. Since we know that the displacement of each rocket is zero at time $t$, since both initial velocities are given, and since we seek information about the acceleration, we begin our solution with Equation 2.8, for it contains just these variables.

**SOLUTION** Applying Equation 2.8 to each rocket gives

$$
x_A = v_{0A}t + \frac{1}{2}a_At^2$$
$$x_B = v_{0B}t + \frac{1}{2}a_Bt^2$$

$$0 = v_{0A} + \frac{1}{2}a_At$$
$$0 = v_{0B} + \frac{1}{2}a_Bt$$

$$t = \frac{-2v_{0A}}{a_A}$$
$$t = \frac{-2v_{0B}}{a_B}$$

The time for each rocket is the same, so that we can equate the two expressions for $t$, with the result that

$$\frac{-2v_{0A}}{a_A} = \frac{-2v_{0B}}{a_B}$$

or

$$\frac{v_{0A}}{a_A} = \frac{v_{0B}}{a_B}$$

Solving for $a_B$ gives

$$a_B = \frac{a_A}{v_{0A}}v_{0B} = \frac{-15 \text{ m/s}^2}{5800 \text{ m/s}}(8600 \text{ m/s}) = -22 \text{ m/s}^2$$

As expected, the magnitude of the acceleration for rocket B is greater than that for rocket A.

33. **REASONING** The stopping distance is the sum of two parts. First, there is the distance the car travels at 20.0 m/s before the brakes are applied. According to Equation 2.2, this distance is the magnitude of the displacement and is the magnitude of the velocity times the time. Second, there is the distance the car travels while it decelerates as the brakes are applied. This distance is given by Equation 2.9, since the initial velocity, the acceleration, and the final velocity (0 m/s when the car comes to a stop) are given.

**SOLUTION** With the assumption that the initial position of the car is $x_0 = 0$ m, Equation 2.2 gives the first contribution to the stopping distance as

$$\Delta x_1 = x_1 = vt_1 = (20.0 \text{ m/s})(0.530 \text{ s})$$
Solving Equation 2.9 \( (v^2 = v_0^2 + 2ax) \) for \( x \) shows that the second part of the stopping distance is
\[
x_2 = \frac{v^2 - v_0^2}{2a} = \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(-7.00 \text{ m/s}^2)}
\]

Here, the acceleration is assigned a negative value, because we have assumed that the car is traveling in the positive direction, and it is decelerating. Since it is decelerating, its acceleration points opposite to its velocity. The stopping distance, then, is
\[
x_{\text{Stopping}} = x_1 + x_2 = (20.0 \text{ m/s})(0.530 \text{ s}) + \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(-7.00 \text{ m/s}^2)} = 39.2 \text{ m}
\]

34. **REASONING** The entering car maintains a constant acceleration of \( a_1 = 6.0 \text{ m/s}^2 \) from the time it starts from rest in the pit area until it catches the other car, but it is convenient to separate its motion into two intervals. During the first interval, lasting \( t_1 = 4.0 \text{ s} \), it accelerates from rest to the velocity \( v_{01} \) with which it enters the main speedway. This velocity is found from Equation 2.4 \( (v = v_0 + at) \), with \( v_0 = 0 \text{ m/s}, a = a_1, t = t_1, \) and \( v = v_{01} \):
\[
v_{10} = a_1 t_1 \tag{1}
\]

The second interval begins when the entering car enters the main speedway with velocity \( v_{01} \), and ends when it catches up with the other car, which travels with a constant velocity \( v_{02} = 70.0 \text{ m/s} \). Since both cars begin and end the interval side-by-side, they both undergo the same displacement \( x \) during this interval. The displacement of each car is given by Equation 2.8 \( (x = v_0 t + \frac{1}{2}at^2) \). For the accelerating car, \( v_0 = v_{10}, a = a_1 \), so
\[
x = v_{10} t + \frac{1}{2} a_1 t^2 \tag{2}
\]

For the other car, \( v_0 = v_{02} \) and \( a = 0 \text{ m/s}^2 \), and so Equation 2.8 yields
\[
x = v_{20} t \tag{3}
\]

**SOLUTION** The displacement during the second interval is not required, so equating the right hand sides of Equations (2) and (3) eliminates \( x \), leaving an equation that may be solved for the elapsed time \( t \), which is now the only unknown quantity:
Substituting Equation (1) for $v_{10}$ into Equation (4), we find that

$$t = \frac{2(v_{20} - a_t t)}{a_1} = \frac{2}{a_1} \left[ 70.0 \text{ m/s} - (6.0 \text{ m/s}^2)(4.0 \text{ s}) \right] = 15 \text{ s}$$

35. **REASONING** The drawing shows the two knights, initially separated by the displacement $d$, traveling toward each other. At any moment, Sir George’s displacement is $x_G$ and that of Sir Alfred is $x_A$. When they meet, their displacements are the same, so $x_G = x_A$.

According to Equation 2.8, Sir George's displacement as a function of time is

$$x_G = v_{0,G} t + \frac{1}{2} a_G t^2 = (0 \text{ m/s}) t + \frac{1}{2} a_G t^2 = \frac{1}{2} a_G t^2$$

where we have used the fact that Sir George starts from rest ($v_{0,G} = 0 \text{ m/s}$).

Since Sir Alfred starts from rest at $x = d$ at $t = 0$ s, we can write his displacement as (again, employing Equation 2.8)

$$x_A = d + v_{0,A} t + \frac{1}{2} a_A t^2 = d + (0 \text{ m/s}) t + \frac{1}{2} a_A t^2 = d + \frac{1}{2} a_A t^2$$
Solving Equation 1 for $t^2 \left( t^2 = 2x_G / a_G \right)$ and substituting this expression into Equation 2 yields

$$x_A = d + \frac{1}{2} a_A \left( \frac{2x_G}{a_G} \right) = d + a_A \left( \frac{x_G}{a_G} \right)$$  \hspace{1cm} (3)

Noting that $x_A = x_G$ when the two riders collide, we see that Equation 3 becomes

$$x_G = d + a_A \left( \frac{x_G}{a_G} \right)$$

Solving this equation for $x_G$ gives $x_G = \frac{d}{1 - \frac{a_A}{a_G}}$.

**SOLUTION** Sir George’s acceleration is positive ($a_G = +0.300 \text{ m/s}^2$) since he starts from rest and moves to the right (the positive direction). Sir Alfred’s acceleration is negative ($a_A = -0.200 \text{ m/s}^2$) since he starts from rest and moves to the left (the negative direction). The displacement of Sir George is, then,

$$x_G = \frac{d}{1 - \frac{a_A}{a_G}} = \frac{88.0 \text{ m}}{1 - \left(\frac{-0.200 \text{ m/s}^2}{+0.300 \text{ m/s}^2}\right)} = 52.8 \text{ m}$$

36. **REASONING** The players collide when they have the same $x$ coordinate relative to a common origin. For convenience, we will place the origin at the starting point of the first player. From Equation 2.8, the $x$ coordinate of each player is given by

$$x_1 = v_{01}t_1 + \frac{1}{2} a_{11}t_1^2 = \frac{1}{2} a_{11}t_1^2 \hspace{1cm} (1)$$

$$x_2 = d + v_{02}t_2 + \frac{1}{2} a_{22}t_2^2 = d + \frac{1}{2} a_{22}t_2^2 \hspace{1cm} (2)$$

where $d = +48 \text{ m}$ is the initial position of the second player. When $x_1 = x_2$, the players collide at time $t = t_1 = t_2$.

**SOLUTION**

a. Equating Equations (1) and (2) when $t = t_1 = t_2$, we have

$$\frac{1}{2} a_{11}t^2 = d + \frac{1}{2} a_{22}t^2$$
We note that $a_1 = +0.50 \text{ m/s}^2$, while $a_2 = -0.30 \text{ m/s}^2$, since the first player accelerates in the $+x$ direction and the second player in the $-x$ direction. Solving for $t$, we have

$$t = \frac{2d}{\sqrt{a_1 - a_2}} = \frac{2(48 \text{ m})}{\sqrt{(0.50 \text{ m/s}^2) - (-0.30 \text{ m/s}^2)}} = 11 \text{ s}$$

b. From Equation (1),

$$x_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} (0.50 \text{ m/s}^2)(11 \text{ s})^2 = 3.0 \times 10^4 \text{ m}$$

37. **REASONING** At a constant velocity the time required for the first car to travel to the next exit is given by Equation 2.2 as the magnitude of the displacement ($2.5 \times 10^3 \text{ m}$) divided by the magnitude of the velocity. This is also the travel time for the second car to reach the next exit. The acceleration for the second car can be determined from Equation 2.8, since the initial velocity, the displacement, and the time are known. This equation applies, because the acceleration is constant.

**SOLUTION** According to Equation 2.2, with the assumption that the initial time is $t_0 = 0 \text{ s}$, the time for the first car to reach the next exit at a constant velocity is

$$\Delta t = t - t_0 = t = \frac{\Delta x}{v} = \frac{2.5 \times 10^3 \text{ m}}{33 \text{ m/s}} = 76 \text{ s}$$

Remembering that the initial velocity $v_0$ of the second car is zero, we can solve Equation 2.8

$$\left(x = v_0 t + \frac{1}{2} at^2 = \frac{1}{2} at^2\right)$$

for the acceleration to show that

$$a = \frac{2x}{t^2} = \frac{2(2.5 \times 10^3 \text{ m})}{(76 \text{ s})^2} = 0.87 \text{ m/s}^2$$

Since the second car’s speed is increasing, this acceleration must be in the same direction as the velocity.

38. **REASONING** Let the total distance between the first and third sign be equal to $2d$. Then, the time $t_A$ is given by

$$t_A = \frac{d}{v_{35}} + \frac{d}{v_{55}} = \frac{d(v_{35} + v_{55})}{v_{55}v_{35}} \quad (1)$$
Equation 2.7 \[ x = \frac{1}{2}(v_0 + v)t \] can be written as \[ t = 2x/(v_0 + v) \], so that

\[
t_B = \frac{2d}{v_{35} + v_{35}} + \frac{2d}{v_{35} + v_{25}} = \frac{2d[(v_{35} + v_{25}) + (v_{55} + v_{55})]}{(v_{35} + v_{35})(v_{35} + v_{25})}
\]  \hspace{1cm} (2)

**SOLUTION** Dividing Equation (2) by Equation (1) and suppressing units for convenience, we obtain

\[
t_B = \frac{2v_{35}v_{35}[(v_{35} + v_{25}) + (v_{55} + v_{55})]}{(v_{35} + v_{35})^2(v_{35} + v_{25})} = \frac{2(55)(35)[(35 + 25) + (55 + 55)]}{(55 + 35)^2(35 + 25)} = 1.2
\]

39. **REASONING** Because the car is traveling in the +x direction and decelerating, its acceleration is negative: \( a = -2.70 \text{ m/s}^2 \). The final velocity for the interval is given \( v = +4.50 \text{ m/s} \), as well as the elapsed time \( t = 3.00 \text{ s} \). Both the car’s displacement \( x \) and its initial velocity \( v_0 \) at the instant braking begins are unknown.

Compare the list of known kinematic quantities \( (v, a, t) \) to the equations of kinematics for constant acceleration: \( v = v_0 + at \) \hspace{1cm} (Equation 2.4), \( x = \frac{1}{2}(v_0 + v)t \) \hspace{1cm} (Equation 2.7), \( x = v_0t + \frac{1}{2}at^2 \) \hspace{1cm} (Equation 2.8), and \( v^2 = v_0^2 + 2ax \) \hspace{1cm} (Equation 2.9). None of these four equations contains all three known quantities and the desired displacement \( x \), and each of them contains the initial velocity \( v_0 \). Since the initial velocity is neither known nor requested, we can combine two kinematic equations to eliminate it, leaving an equation in which \( x \) is the only unknown quantity.

**SOLUTION** For the first step, solve Equation 2.4 \( (v = v_0 + at) \) for \( v_0 \):

\[
v_0 = v - at
\]  \hspace{1cm} (1)

Substituting the expression for \( v_0 \) in Equation (1) into Equation 2.8 \( (x = v_0t + \frac{1}{2}at^2) \) yields an expression for the car’s displacement solely in terms of the known quantities \( v, a, \) and \( t \):

\[
x = (v - at)t + \frac{1}{2}at^2 = vt - at^2 + \frac{1}{2}at^2
\]

\[
x = vt - \frac{1}{2}at^2
\]  \hspace{1cm} (2)

Substitute the known values of \( v, a, \) and \( t \) into Equation (2):
\[
x = (+4.50 \text{ m/s})(3.00 \text{ s}) - \frac{1}{2}(-2.70 \text{ m/s}^2)(3.00 \text{ s})^2 = +25.7 \text{ m}
\]

Note: Equation (2) can also be obtained by combining Equation (1) with Equation 2.7 \[
[ x = \frac{1}{2}(v_0 + v)t ]\]
, or, with more effort, by combining Equation (1) with Equation 2.9 \[
(v^2 = v_0^2 + 2ax)
\].

40. **REASONING AND SOLUTION** As the plane decelerates through the intersection, it covers a total distance equal to the length of the plane plus the width of the intersection, so

\[
x = 59.7 \text{ m} + 25.0 \text{ m} = 84.7 \text{ m}
\]

The speed of the plane as it enters the intersection can be found from Equation 2.9. Solving Equation 2.9 for \( v_0 \) gives

\[
v_0 = \sqrt{v^2 - 2ax} = \sqrt{(45.0 \text{ m})^2 - 2(-5.70 \text{ m/s}^2)(84.7 \text{ m})} = 54.7 \text{ m/s}
\]

The time required to traverse the intersection can then be found from Equation 2.4. Solving Equation 2.4 for \( t \) gives

\[
t = \frac{v - v_0}{a} = \frac{45.0 \text{ m/s} - 54.7 \text{ m/s}}{-5.70 \text{ m/s}^2} = 1.7 \text{ s}
\]

41. **SSM REASONING** As the train passes through the crossing, its motion is described by Equations 2.4 \((v = v_0 + at)\) and 2.7 \[
[ x = \frac{1}{2}(v + v_0)t ]\]
, which can be rearranged to give

\[
v - v_0 = at \quad \text{and} \quad v + v_0 = \frac{2x}{t}
\]

These can be solved simultaneously to obtain the speed \( v \) when the train reaches the end of the crossing. Once \( v \) is known, Equation 2.4 can be used to find the time required for the train to reach a speed of 32 m/s.

**SOLUTION** Adding the above equations and solving for \( v \), we obtain

\[
v = \frac{1}{2}\left(at + \frac{2x}{t}\right) = \frac{1}{2}\left[(1.6 \text{ m/s}^2)(2.4 \text{ s}) + \frac{2(20.0 \text{ m})}{2.4 \text{ s}}\right] = 1.0 \times 10^1 \text{ m/s}
\]
The motion from the end of the crossing until the locomotive reaches a speed of 32 m/s requires a time
\[ t = \frac{v - v_0}{a} = \frac{32 \text{ m/s} - 1.0 \times 10^1 \text{ m/s}}{1.6 \text{ m/s}^2} = 14 \text{ s} \]

42. **REASONING**  Since the car is moving with a constant velocity, the displacement of the car in a time \( t \) can be found from Equation 2.8 with \( a = 0 \text{ m/s}^2 \) and \( v_0 \) equal to the velocity of the car: \( x_{\text{car}} = v_{\text{car}} t \). Since the train starts from rest with a constant acceleration, the displacement of the train in a time \( t \) is given by Equation 2.8 with \( v_0 = 0 \text{ m/s} \):

\[ x_{\text{train}} = \frac{1}{2} a_{\text{train}} t^2 \]

At a time \( t_1 \), when the car just reaches the front of the train, \( x_{\text{car}} = L_{\text{train}} + x_{\text{train}} \), where \( L_{\text{train}} \) is the length of the train. Thus, at time \( t_1 \),

\[ v_{\text{car}} t_1 = L_{\text{train}} + \frac{1}{2} a_{\text{train}} t_1^2 \quad (1) \]

At a time \( t_2 \), when the car is again at the rear of the train, \( x_{\text{car}} = x_{\text{train}} \). Thus, at time \( t_2 \)

\[ v_{\text{car}} t_2 = \frac{1}{2} a_{\text{train}} t_2^2 \quad (2) \]

Equations (1) and (2) can be solved simultaneously for the speed of the car \( v_{\text{car}} \) and the acceleration of the train \( a_{\text{train}} \).

**SOLUTION**

a. Solving Equation (2) for \( a_{\text{train}} \) we have

\[ a_{\text{train}} = \frac{2v_{\text{car}}}{t_2} \quad (3) \]

Substituting this expression for \( a_{\text{train}} \) into Equation (1) and solving for \( v_{\text{car}} \), we have

\[ v_{\text{car}} = \frac{L_{\text{train}}}{t_1 \left(1 - \frac{t_1}{t_2}\right)} = \frac{92 \text{ m}}{\left(1 - \frac{14 \text{ s}}{28 \text{ s}}\right)} = 13 \text{ m/s} \]

b. Direct substitution into Equation (3) gives the acceleration of the train:
43. **REASONING AND SOLUTION** When air resistance is neglected, free fall conditions are applicable. The final speed can be found from Equation 2.9;

\[ v^2 = v_0^2 + 2ay \]

where \( v_0 \) is zero since the stunt man falls from rest. If the origin is chosen at the top of the hotel and the upward direction is positive, then the displacement is \( y = -99.4 \text{ m} \). Solving for \( v \), we have

\[ v = -\sqrt{2ay} = -\sqrt{2(-9.80 \text{ m/s}^2)(-99.4 \text{ m})} = -44.1 \text{ m/s} \]

The speed at impact is the magnitude of this result or \( 44.1 \text{ m/s} \).

44. **REASONING** Because there is no effect due to air resistance, the rock is in free fall from its launch until it hits the ground, so that the acceleration of the rock is always \(-9.8 \text{ m/s}^2\), assuming upward to be the positive direction. In (a), we will consider the interval beginning at launch and ending 2.0 s later. In (b), we will consider the interval beginning at launch and ending 5.0 s later. Since the displacement isn’t required, Equation 2.4 \((v = v_0 + at)\) suffices to solve both parts of the problem. The stone slows down as it rises, so we expect the speed in (a) to be larger than 15 m/s. The speed in (b) could be smaller than 15 m/s (the rock does not reach its maximum height) or larger than 15 m/s (the rock reaches its maximum height and falls back down below its height at the 2.0-s point).

**SOLUTION**

a. For the interval from launch to \( t = 2.0 \text{ s} \), the final velocity is \( v = 15 \text{ m/s} \), the acceleration is \( a = -9.8 \text{ m/s}^2 \), and the initial velocity is to be found. Solving Equation 2.4 \((v = v_0 + at)\) for \( v_0 \) gives

\[ v_0 = v - at = 15 \text{ m/s} - (-9.8 \text{ m/s}^2)(2.0 \text{ s}) = 35 \text{ m/s} \]

Therefore, at launch,

\[ \text{Speed} = 35 \text{ m/s} \]

b. Now we consider the interval from launch to \( t = 5.0 \text{ s} \). The initial velocity is that found in part (a), \( v_0 = 35 \text{ m/s} \). The final velocity is
\[ v = v_0 + at = 35 \text{ m/s} + (-9.8 \text{ m/s}^2)(5.0 \text{ s}) = -14 \text{ m/s} \]  

(2.4)

Instantaneous speed is the magnitude of the instantaneous velocity, so we drop the minus sign and find that

\[
\text{Speed} = 14 \text{ m/s}
\]

45. **REASONING AND SOLUTION** In a time \( t \) the card will undergo a vertical displacement \( y \) given by

\[ y = \frac{1}{2} at^2 \]

where \( a = -9.80 \text{ m/s}^2 \). When \( t = 60.0 \text{ ms} = 6.0 \times 10^{-2} \text{ s} \), the displacement of the card is 0.018 m, and the distance is the magnitude of this value or \[ d_1 = 0.018 \text{ m} \].

Similarly, when \( t = 120 \text{ ms} \), \[ d_2 = 0.071 \text{ m} \], and when \( t = 180 \text{ ms} \), \[ d_3 = 0.16 \text{ m} \].

46. **REASONING**

Assuming that air resistance can be neglected, the acceleration is the same for both the upward and downward parts, namely \(-9.80 \text{ m/s}^2 \) (upward is the positive direction). Moreover, the displacement is \( y = 0 \text{ m} \), since the final and initial positions of the ball are the same. The time is given as \( t = 8.0 \text{ s} \). Therefore, we may use Equation 2.8 \( \left( y = v_0 t + \frac{1}{2} at^2 \right) \) to find the initial velocity \( v_0 \) of the ball.

**SOLUTION** Solving Equation 2.8 \( \left( y = v_0 t + \frac{1}{2} at^2 \right) \) for the initial velocity \( v_0 \) gives

\[
v_0 = \frac{y - \frac{1}{2} at^2}{t} = \frac{0 \text{ m} - \frac{1}{2}(-9.80 \text{ m/s}^2)(8.0 \text{ s})^2}{8.0 \text{ s}} = 39 \text{ m/s}
\]

47. **REASONING AND SOLUTION** The figure at the right shows the paths taken by the pellets fired from gun A and gun B. The two paths differ by the extra distance covered by the pellet from gun A as it rises and falls back to the edge of the cliff. When it falls back to the edge of the cliff, the pellet from gun A will have the same speed as the pellet fired from gun B, as Conceptual Example 15 discusses. Therefore, the flight time of pellet A will be greater than that of B by the amount of time that it takes for pellet A to cover the extra distance.
The time required for pellet A to return to the cliff edge after being fired can be found from Equation 2.4: \( v = v_0 + at \). If "up" is taken as the positive direction then \( v_0 = +30.0 \, \text{m/s} \) and \( v = -30.0 \, \text{m/s} \). Solving Equation 2.4 for \( t \) gives

\[
t = \frac{v - v_0}{a} = \frac{(-30.0 \, \text{m/s}) - (+30.0 \, \text{m/s})}{-9.80 \, \text{m/s}^2} = 6.12 \, \text{s}
\]

Notice that this result is independent of the height of the cliff.

48. **REASONING** The initial velocity and the elapsed time are given in the problem. Since the rock returns to the same place from which it was thrown, its displacement is zero \((y = 0 \, \text{m})\). Using this information, we can employ Equation 2.8 \((y = v_0 t + \frac{1}{2}a t^2)\) to determine the acceleration \( a \) due to gravity.

**SOLUTION** Solving Equation 2.8 for the acceleration yields

\[
a = \frac{2(y - v_0 t)}{t^2} = \frac{2[0 \, \text{m} - (+15 \, \text{m/s})(20.0 \, \text{s})]}{(20.0 \, \text{s})^2} = -1.5 \, \text{m/s}^2
\]

49. **SSM REASONING** The initial velocity of the compass is \(+2.50 \, \text{m/s}\). The initial position of the compass is \(3.00 \, \text{m}\) and its final position is \(0 \, \text{m}\) when it strikes the ground. The displacement of the compass is the final position minus the initial position, or \(y = -3.00 \, \text{m}\). As the compass falls to the ground, its acceleration is the acceleration due to gravity, \(a = -9.80 \, \text{m/s}^2\). Equation 2.8 \((y = v_0 t + \frac{1}{2}a t^2)\) can be used to find how much time elapses before the compass hits the ground.

**SOLUTION** Starting with Equation 2.8, we use the quadratic equation to find the elapsed time.

\[
t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(\frac{1}{2}a)(-y)}}{2(\frac{1}{2}a)} = \frac{-2.50 \, \text{m/s} \pm \sqrt{(2.50 \, \text{m/s})^2 - 4(-4.90 \, \text{m/s}^2)(-3.00 \, \text{m})}}{2(-4.90 \, \text{m/s}^2)}
\]

There are two solutions to this quadratic equation, \(t_1 = 1.08 \, \text{s}\) and \(t_2 = -0.568 \, \text{s}\). The second solution, being a negative time, is discarded.
50. **REASONING** The initial speed of the ball can be determined from Equation 2.9 \( v^2 = v_0^2 + 2ay \). Once the initial speed of the ball is known, Equation 2.9 can be used a second time to determine the height above the launch point when the speed of the ball has decreased to one half of its initial value.

**SOLUTION** When the ball has reached its maximum height, its velocity is zero. If we take upward as the positive direction, we have from Equation 2.9 that

\[
v_0 = \sqrt{v^2 - 2ay} = \sqrt{(0 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(16 \text{ m})} = 18 \text{ m/s}
\]

When the speed of the ball has decreased to one half of its initial value, \( v = \frac{1}{2}v_0 \), and Equation 2.9 gives

\[
y = \frac{v^2 - v_0^2}{2a} = \frac{(\frac{1}{2}v_0)^2 - v_0^2}{2a} = \frac{v_0^2}{2a}\left(\frac{1}{4} - 1\right) = \frac{(18 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)}\left(\frac{1}{4} - 1\right) = 12 \text{ m}
\]

51. **REASONING AND SOLUTION**

a. \( v^2 = v_0^2 + 2ay \)

\[
v = \pm \sqrt{(1.8 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-3.0 \text{ m})} = \pm 7.9 \text{ m/s}
\]

The minus is chosen, since the diver is now moving down. Hence, \( v = -7.9 \text{ m/s} \).

b. The diver's velocity is zero at his highest point. The position of the diver relative to the board is

\[
y = -\frac{v_0^2}{2a} = -\frac{(1.8 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 0.17 \text{ m}
\]

The position above the water is 3.0 m + 0.17 m = 3.2 m.

52. **REASONING** Equation 2.9 \( v^2 = v_0^2 + 2ay \) can be used to determine the maximum height above the launch point, where the final speed is \( v = 0 \text{ m/s} \). However, we will need to know the initial speed \( v_0 \), which can be determined via Equation 2.9 and the fact that \( v = \frac{1}{2}v_0 \) when \( y = 4.00 \text{ m} \) (assuming upward to be the positive direction).

**SOLUTION** When the ball has reached its maximum height, we have \( v = 0 \text{ m/s} \) and \( y = y_{\text{max}} \), so that Equation 2.9 becomes
\[ v^2 = v_0^2 + 2ay \quad \text{or} \quad (0 \text{ m/s})^2 = v_0^2 + 2ay_{\text{max}} \quad \text{or} \quad y_{\text{max}} = \frac{-v_0^2}{2a} \quad (1) \]

Using Equation 2.9 and the fact that \( v = \frac{1}{2}v_0 \) when \( y = 4.00 \text{ m} \) (assuming upward to be the positive direction), we find that
\[ v^2 = v_0^2 + 2ay \quad \text{or} \quad \left(\frac{1}{2}v_0\right)^2 = v_0^2 + 2a(4.00 \text{ m}) \quad \text{or} \quad v_0^2 = \frac{2a(4.00 \text{ m})}{-(3/4)} \quad (2) \]

Substituting Equation (2) into Equation (1) gives
\[ y_{\text{max}} = \frac{-v_0^2}{2a} = \frac{-2a(4.00 \text{ m})}{2a} = \frac{5.33 \text{ m}}{} \]

53. [SSM] REASONING AND SOLUTION Since the balloon is released from rest, its initial velocity is zero. The time required to fall through a vertical displacement \( y \) can be found from Equation 2.8 \( (y = v_0t + \frac{1}{2}at^2) \) with \( v_0 = 0 \text{ m/s} \). Assuming upward to be the positive direction, we find
\[ t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-6.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 1.1 \text{ s} \]

54. REASONING Equation 2.9 \( (v^2 = v_0^2 + 2ay) \) can be used to find out how far above the cliff’s edge the pellet would have gone if the gun had been fired straight upward, provided that we can determine the initial speed imparted to the pellet by the gun. This initial speed can be found by applying Equation 2.9 to the downward motion of the pellet described in the problem statement.

SOLUTION If we assume that upward is the positive direction, the initial speed of the pellet is, from Equation 2.9,
\[ v_0 = \sqrt{v^2 - 2ay} = \sqrt{(-27 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(-15 \text{ m})} = 20.9 \text{ m/s} \]

Equation 2.9 can again be used to find the maximum height of the pellet if it were fired straight up. At its maximum height, \( v = 0 \text{ m/s} \), and Equation 2.9 gives
\[ y = \frac{-v_0^2}{2a} = \frac{-(20.9 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 22 \text{ m} \]

55. REASONING The displacement \( y \) of the diver is equal to her average velocity \( \bar{v} \) multiplied by the time \( t \), or \( y = \bar{v}t \). Since the diver has a constant acceleration (the acceleration due to gravity), her average velocity is equal to \( \bar{v} = \frac{1}{2}(v_0 + v) \), where \( v_0 \) and \( v \)
are, respectively, the initial and final velocities. Thus, according to Equation 2.7, the displacement of the diver is

\[ y = \frac{1}{2} (v_0 + v) t \]  

(2.7)

The final velocity and the time in this expression are known, but the initial velocity is not. To determine her velocity at the beginning of the 1.20-s period (her initial velocity), we turn to her acceleration. The acceleration is defined by Equation 2.4 as the change in her velocity, \( v - v_0 \), divided by the elapsed time \( t \):  

\[ a = \frac{v - v_0}{t} \]

Solving this equation for the initial velocity \( v_0 \) yields

\[ v_0 = v - at \]

Substituting this relation for \( v_0 \) into Equation 2.7, we obtain

\[ y = \frac{1}{2} (v_0 + v) t = \frac{1}{2} (v - at + v) t = vt - \frac{1}{2} at^2 \]

**SOLUTION**  The diver’s acceleration is that due to gravity, or \( a = -9.80 \text{ m/s}^2 \). The acceleration is negative because it points downward, and this direction is the negative direction. The displacement of the diver during the last 1.20 s of the dive is

\[ y = vt - \frac{1}{2} at^2 = (-10.1 \text{ m/s})(1.20 \text{ s}) - \frac{1}{2} (-9.80 \text{ m/s}^2)(1.20 \text{ s})^2 = -5.06 \text{ m} \]

The displacement of the diver is negative because she is moving downward.

---

56. **REASONING**  The ball is initially in free fall, then collides with the pavement and rebounds, which puts it into free fall again, until caught by the boy. We don’t have enough information to analyze its collision with the pavement, but we’re only asked to calculate the time it spends in the air, undergoing free-fall motion. The motion can be conveniently divided into three intervals: from release \( (h_1 = 9.50 \text{ m}) \) to impact, from impact to the second highest point \( (h_2 = 5.70 \text{ m}) \), and from the second highest point to \( h_3 = 1.20 \text{ m} \) above the pavement. For each of the intervals, the acceleration is that due to gravity. For the first and last interval, the ball’s initial velocity is zero, so the time to fall a given distance can be found from Equation 2.8 \( (y = v_0 t + \frac{1}{2} at^2) \).

The second interval begins at the pavement and ends at \( h_2 \), so the initial velocity isn’t zero. However, the symmetry of free-fall motion is such that it takes the ball as much time to rise from the ground to a maximum height \( h_2 \) as it would take for a ball dropped from \( h_2 \) to fall to the pavement, so we can again use Equation 2.8 to find the duration of the second interval.

**SOLUTION**  Taking upward as the positive direction, we have \( a = -9.80 \text{ m/s}^2 \) for the
acceleration in each of the three intervals. Furthermore, the initial velocity for each of the
intervals is \( v_0 = 0 \) m/s. Remember, we are using symmetry to treat the second interval as if
the ball were dropped from rest at a height of 5.70 m and fell to the pavement. Using
Equation 2.8 \( y = v_0 t + \frac{1}{2} a t^2 \), with \( v_0 = 0 \) m/s, we can solve for the time to find that

\[
t = \sqrt{\frac{2y}{a}}
\]

Applying this result to each interval gives the total time as

\[
t_{total} = \sqrt{\frac{2(-9.50 \text{ m})}{-9.80 \text{ m/s}^2}} + \sqrt{\frac{2(-5.70 \text{ m})}{-9.80 \text{ m/s}^2}} + \sqrt{\frac{2(-5.70 \text{ m} - 1.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 3.43 \text{ s}
\]

Note that the displacement \( y \) for each interval is negative, because upward has been
designated as the positive direction.

57. **REASONING** To calculate the speed of the raft, it is necessary to determine the distance it
travels and the time interval over which the motion occurs. The speed is the distance divided
by the time, according to Equation 2.1. The distance is 7.00 m – 4.00 m = 3.00 m. The time is the time it takes for the stone to fall, which can be
obtained from Equation 2.8 \( y = v_0 t + \frac{1}{2} a t^2 \), since the displacement \( y \), the initial velocity
\( v_0 \), and the acceleration \( a \) are known.

**SOLUTION** During the time \( t \) that it takes the stone to fall, the raft travels a distance of
7.00 m – 4.00 m = 3.00 m, and according to Equation 2.1, its speed is

\[
\text{speed} = \frac{3.00 \text{ m}}{t}
\]

The stone falls downward for a distance of 75.0 m, so its displacement is
\( y = -75.0 \) m, where the downward direction is taken to be the negative direction. Equation
2.8 can be used to find the time of fall. Setting \( v_0 = 0 \) m/s, and solving Equation 2.8 for the
time \( t \), we have

\[
t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-75.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 3.91 \text{ s}
\]

Therefore, the speed of the raft is
\[
\text{speed} = \frac{3.00 \text{ m}}{3.91 \text{ s}} = 0.767 \text{ m/s}
\]

58. **REASONING**

The stone that is thrown upward loses speed on the way up. The stone that is thrown downward gains speed on the way down. The stones cross paths below the point that corresponds to half the height of the cliff. To see why, consider where they would cross paths if they each maintained their initial speed as they moved. Then, they would cross paths exactly at the halfway point. However, the stone traveling upward begins immediately to lose speed, while the stone traveling downward immediately gains speed. Thus, the upward moving stone travels more slowly than the downward moving stone. Consequently, the stone thrown downward has traveled farther when it reaches the crossing point than the stone thrown upward.

The initial velocity \(v_0\) is known for both stones, as is the acceleration \(a\) due to gravity. In addition, we know that at the crossing point the stones are at the same place at the same time \(t\). Furthermore, the position of each stone is specified by its displacement \(y\) from its starting point. The equation of kinematics that relates the variables \(v_0\), \(a\), \(t\) and \(y\) is Equation 2.8 \((y = v_0t + \frac{1}{2}at^2)\), and we will use it in our solution. In using this equation, we will assume upward to be the positive direction.

**SOLUTION** Applying Equation 2.8 to each stone, we have

\[
\begin{align*}
\text{Upward moving stone: } & y_{up} = v_0^{up} t + \frac{1}{2}at^2 \\
\text{Downward moving stone: } & y_{down} = v_0^{down} t + \frac{1}{2}at^2
\end{align*}
\]

In these expressions \(t\) is the time it takes for either stone to reach the crossing point, and \(a\) is the acceleration due to gravity. Note that \(y_{up}\) is the displacement of the upward moving stone above the base of the cliff, \(y_{down}\) is the displacement of the downward moving stone below the top of the cliff, and \(H\) is the displacement of the cliff-top above the base of the cliff, as the drawing shows. The distances above and below the crossing point must add to equal the height of the cliff, so we have

\[
y_{up} - y_{down} = H
\]

where the minus sign appears because the displacement \(y_{down}\) points in the negative direction. Substituting the two expressions for \(y_{up}\) and \(y_{down}\) into this equation gives

\[
v_0^{up} t + \frac{1}{2}at^2 - \left(v_0^{down} t + \frac{1}{2}at^2\right) = H
\]
This equation can be solved for \( t \) to show that the travel time to the crossing point is

\[
 t = \frac{H}{v_0^{\text{up}} - v_0^{\text{down}}}
\]

Substituting this result into the expression from Equation 2.8 for \( y_{\text{up}} \) gives

\[
y_{\text{up}} = v_0^{\text{up}} t + \frac{1}{2} a t^2 = v_0^{\text{up}} \left( \frac{H}{v_0^{\text{up}} - v_0^{\text{down}}} \right) + \frac{1}{2} a \left( \frac{H}{v_0^{\text{up}} - v_0^{\text{down}}} \right)^2
\]

\[
= (9.00 \text{ m/s}) \left[ \frac{6.00 \text{ m}}{9.00 \text{ m/s} - (-9.00 \text{ m/s})} \right] + \frac{1}{2} \left( -9.80 \text{ m/s}^2 \right) \left[ \frac{6.00 \text{ m}}{9.00 \text{ m/s} - (-9.00 \text{ m/s})} \right]^2
\]

\[
= 2.46 \text{ m}
\]

Thus, the crossing is located a distance of 2.46 m above the base of the cliff, which is below the halfway point of 3.00 m, as expected.

59. [SSM] **REASONING AND SOLUTION**

a. We can use Equation 2.9 to obtain the speed acquired as she falls through the distance \( H \). Taking downward as the positive direction, we find

\[
v^2 = v_0^2 + 2ay = (0 \text{ m/s})^2 + 2aH \quad \text{or} \quad v = \sqrt{2aH}
\]

To acquire a speed of twice this value or \( 2\sqrt{2aH} \), she must fall an additional distance \( H' \). According to Equation 2.9 \( (v^2 = v_0^2 + 2ay) \), we have

\[
\left( 2\sqrt{2aH} \right)^2 = \left( \sqrt{2aH} \right)^2 + 2aH' \quad \text{or} \quad 4(2aH) = 2aH + 2aH'
\]

The acceleration due to gravity \( a \) can be eliminated algebraically from this result, giving

\[
4H = H + H' \quad \text{or} \quad H' = 3H
\]

b. In the previous calculation the acceleration due to gravity was eliminated algebraically. Thus, a value other than 9.80 m/s\(^2\) would not have affected the answer to part (a).
60. **REASONING** When the arrows reach their maximum heights, they come instantaneously to a halt, and the final speed of each arrow is zero. Using this fact, we will be able to determine the time it takes for each arrow to reach its maximum height. Knowing this time for the second arrow will allow us to determine its initial speed at launch.

**SOLUTION** The time required for the first arrow to reach its maximum height can be determined from Equation 2.4 \((v = v_0 + at)\). Taking upward as the positive direction, we have

\[
t = \frac{v - v_0}{a} = \frac{0 \text{ m/s} - 25.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 2.55 \text{ s}
\]

Note that the second arrow is shot 1.20 s after the first arrow. Therefore, since both arrows reach their maximum height at the same time, the second arrow reaches its maximum height

\[2.55 \text{ s} - 1.20 \text{ s} = 1.35 \text{ s}\]

after being fired. The initial speed of the second arrow can then be found from Equation 2.4:

\[v_0 = v - at = 0 \text{ m/s} - (-9.80 \text{ m/s}^2)(1.35 \text{ s}) = 13.2 \text{ m/s}\]

61. **SSM REASONING** Once the man sees the block, the man must get out of the way in the time it takes for the block to fall through an additional 12.0 m. The velocity of the block at the instant that the man looks up can be determined from Equation 2.9. Once the velocity is known at that instant, Equation 2.8 can be used to find the time required for the block to fall through the additional distance.

**SOLUTION** When the man first notices the block, it is 14.0 m above the ground and its displacement from the starting point is \(y = 14.0 \text{ m} - 53.0 \text{ m}\). Its velocity is given by Equation 2.9 \((v^2 = v_0^2 + 2ay)\). Since the block is moving down, its velocity has a negative value,

\[v = -\sqrt{v_0^2 + 2ay} = -\sqrt{(0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(14.0 \text{ m} - 53.0 \text{ m})} = -27.7 \text{ m/s}\]

The block then falls the additional 12.0 m to the level of the man's head in a time \(t\) which satisfies Equation 2.8:

\[y = v_0 t + \frac{1}{2} at^2\]

where \(y = -12.0 \text{ m}\) and \(v_0 = -27.7 \text{ m/s}\). Thus, \(t\) is the solution to the quadratic equation

\[4.90t^2 + 27.7t - 12.0 = 0\]

where the units have been suppressed for brevity. From the quadratic formula, we obtain
\[
t = \frac{-27.7 \pm \sqrt{(27.7)^2 - 4(4.90)(-12.0)}}{2(4.90)} = 0.40 \text{ s or } -6.1 \text{ s}
\]

The negative solution can be rejected as nonphysical, and the time it takes for the block to reach the level of the man is 0.40 s.

**62. REASONING** Once its fuel is gone, the rocket is in free fall, so its motion consists of two intervals of constant but different acceleration. We will take upward as the positive direction. From launch to engine burn-out, the acceleration is \(a_1 = +86.0 \text{ m/s}^2\), and the rocket’s displacement is \(y_1\). Its velocity at the end of the burn, \(v_1\), is also the initial velocity for the second portion of its flight: engine burn-out to maximum altitude. During this second portion, the rocket slows down with the acceleration of gravity \(a_2 = -9.80 \text{ m/s}^2\) and undergoes an additional displacement of \(y_2\) in reaching its maximum height. Its maximum altitude is the sum of these two vertical displacements: \(h = y_1 + y_2\).

**SOLUTION** First we consider the time period \(t_1 = 1.70 \text{ s}\) from the ignition of the engine until the fuel is gone. The rocket accelerates from \(v_0 = 0 \text{ m/s}\) to \(v = v_1\), rising a displacement \(y_1\), as given by Equation 2.8 \(\left( y = v_0 t + \frac{1}{2} a_1 t_1^2 \right)\):

\[
y_1 = v_0 t_1 + \frac{1}{2} a_1 t_1^2 = (0 \text{ m/s}) t_1 + \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_1 t_1^2
\]

Equation 2.4 \((v = v_0 + at)\) gives its velocity \(v_1\) at the instant the fuel runs out:

\[
v_1 = v_0 + a_1 t_1 = 0 \text{ m/s} + a_1 t_1 = a_1 t_1
\]

From that moment onward, the second part of the rocket’s motion is free fall \((a_2 = -9.80 \text{ m/s}^2)\). It takes a time \(t_2\) for the rocket’s velocity to decrease from \(v_0 = v_1\) to \(v_2 = 0 \text{ m/s}\) at its maximum altitude. We solve Equation 2.9 \((v^2 = v_0^2 + 2ay)\) to find its upward displacement \(y_2\) during this time:

\[
(0 \text{ m/s})^2 = v_1^2 + 2a_2 y_2 \quad \text{or} \quad y_2 = \frac{-v_1^2}{2a_2}
\]

Substituting for \(v_1\) from Equation (2), we find for \(y_2\) that
Using Equations (1) and (3), we find that the rocket’s maximum altitude, relative to the ground, is

\[ h = y_1 + y_2 = \frac{1}{2} a t_1^2 - \frac{(a t_1)^2}{2a_2} = \frac{1}{2} a t_1^2 \left(1 - \frac{a}{a_2}\right) \]

Using the values given, we find that

\[ h = \frac{1}{2} \left(86.0 \, \text{m/s}^2\right) (1.70 \, \text{s})^2 \left(1 - \frac{86.0 \, \text{m/s}^2}{-9.80 \, \text{m/s}^2}\right) = 1210 \, \text{m} \]

63. **REASONING** To find the initial velocity \( v_{0,2} \) of the second stone, we will employ Equation 2.8, \( y = v_{0,2} t_2 + \frac{1}{2} a t_2^2 \). In this expression \( t_2 \) is the time that the second stone is in the air, and it is equal to the time \( t_1 \) that the first stone is in the air minus the time \( t_{3,20} \) it takes for the first stone to fall 3.20 m:

\[ t_2 = t_1 - t_{3,20} \]

We can find \( t_1 \) and \( t_{3,20} \) by applying Equation 2.8 to the first stone.

**SOLUTION** To find the initial velocity \( v_{0,2} \) of the second stone, we employ Equation 2.8, \( y = v_{0,2} t_2 + \frac{1}{2} a t_2^2 \). Solving this equation for \( v_{0,2} \) yields

\[ v_{0,2} = \frac{y - \frac{1}{2} a t_2^2}{t_2} \]

The time \( t_1 \) for the first stone to strike the ground can be obtained from Equation 2.8, \( y = v_{0,1} t_1 + \frac{1}{2} a t_1^2 \). Noting that \( v_{0,1} = 0 \, \text{m/s} \) since the stone is dropped from rest and solving this equation for \( t_1 \), we have

\[ t_1 = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-15.0 \, \text{m})}{-9.80 \, \text{m/s}^2}} = 1.75 \, \text{s} \quad (1) \]

Note that the stone is falling down, so its displacement is negative \( (y = -15.0 \, \text{m}) \). Also, its acceleration \( a \) is that due to gravity, so \( a = -9.80 \, \text{m/s}^2 \).
The time \( t_{3.20} \) for the first stone to fall 3.20 m can also be obtained from Equation 1:

\[
t_{3.20} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-3.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.808 \text{ s}
\]

The time \( t_2 \) that the second stone is in the air is

\[
t_2 = t_1 - t_{3.20} = 1.75 \text{ s} - 0.808 \text{ s} = 0.94 \text{ s}
\]

The initial velocity of the second stone is

\[
v_{0,2} = \frac{y - \frac{1}{2} at_2^2}{t_2} = \frac{(-15.0 \text{ m}) - \frac{1}{2} (-9.80 \text{ m/s}^2)(0.94 \text{ s})^2}{0.94 \text{ s}} = -11 \text{ m/s}
\]

64. **REASONING** We assume that downward is the positive direction. The tile falls from rest, so its initial velocity \( v_0 \) is zero. The tile falls through a displacement \( y \) in going from the roof top to the top of the window. It is a value for \( y \) that we seek, and it can be obtained from \( v_{\text{window}}^2 = v_0^2 + 2ay \) (Equation 2.9). In this expression \( v_0 = 0 \text{ m/s} \), and \( a \) is the acceleration due to gravity. The velocity \( v_{\text{window}} \) at the top of the window is not given, but it can be obtained from the time of 0.20 s that it takes the tile to pass the window.

**SOLUTION** Solving Equation 2.9 for \( y \) and using the fact that \( v_0 = 0 \text{ m/s} \) gives

\[
y = \frac{v_{\text{window}}^2 - v_0^2}{2a} = \frac{v_{\text{window}}^2}{2a} \quad (1)
\]

The tile travels an additional displacement \( y_{\text{window}} = 1.6 \text{ m} \) in traversing the window in a time \( t = 0.20 \text{ s} \). These data can be used in \( v_{\text{window}} = v_{\text{window}} + \frac{1}{2} at^2 \) (Equation 2.8) to find the velocity \( v_{\text{window}} \) at the top of the window. Solving Equation 2.8 for \( v_{\text{window}} \) gives

\[
v_{\text{window}} = \frac{2y_{\text{window}} - at^2}{2t} = \frac{2(1.6 \text{ m}) - (9.80 \text{ m/s}^2)(0.20 \text{ s})^2}{2(0.20 \text{ s})} = 7.0 \text{ m/s}
\]

Using this value for \( v_{\text{window}} \) in Equation (1), we obtain

\[
y = \frac{v_{\text{window}}^2}{2a} = \frac{(7.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 2.5 \text{ m}
\]
65. **REASONING** The slope of a straight-line segment in a position-versus-time graph is the average velocity. The algebraic sign of the average velocity, therefore, corresponds to the sign of the slope.

**SOLUTION**

a. The slope, and hence the average velocity, is *positive* for segments A and C, *negative* for segment B, and *zero* for segment D.

b. In the given position-versus-time graph, we find the slopes of the four straight-line segments to be

\[
\begin{align*}
    v_A &= \frac{1.25 \text{ km} - 0 \text{ km}}{0.20 \text{ h} - 0 \text{ h}} = 6.3 \text{ km/h} \\
    v_B &= \frac{0.50 \text{ km} - 1.25 \text{ km}}{0.40 \text{ h} - 0.20 \text{ h}} = -3.8 \text{ km/h} \\
    v_C &= \frac{0.75 \text{ km} - 0.50 \text{ km}}{0.80 \text{ h} - 0.40 \text{ h}} = 0.63 \text{ km/h} \\
    v_D &= \frac{0.75 \text{ km} - 0.75 \text{ km}}{1.00 \text{ h} - 0.80 \text{ h}} = 0 \text{ km/h}
\end{align*}
\]

66. **REASONING** On a position-versus-time graph, the velocity is the slope. Since the object’s velocity is constant and it moves in the +x direction, the graph will be a straight line with a positive slope, beginning at \(x = -16 \text{ m}\) when \(t = 0 \text{ s}\). At \(t = 18 \text{ s}\), its position should be \(x = -16 \text{ m} + 48 \text{ m} = 32 \text{ m}\). Once the graph is constructed, the object’s velocity is found by calculating the slope of the graph: \(v = \frac{\Delta x}{\Delta t}\).

**SOLUTION** The position-versus-time graph for the motion is as follows:
The object’s displacement is +48 m, and the elapsed time is 18 s, so its velocity is

\[ v = \frac{\Delta x}{\Delta t} = \frac{+48 \text{ m}}{18 \text{ s}} = +2.7 \text{ m/s} \]

67. **REASONING AND SOLUTION** The average acceleration for each segment is the slope of that segment.

\[ a_A = \frac{40 \text{ m/s} - 0 \text{ m/s}}{21 \text{ s} - 0 \text{ s}} = 1.9 \text{ m/s}^2 \]

\[ a_B = \frac{40 \text{ m/s} - 40 \text{ m/s}}{48 \text{ s} - 21 \text{ s}} = 0 \text{ m/s}^2 \]

\[ a_C = \frac{80 \text{ m/s} - 40 \text{ m/s}}{60 \text{ s} - 48 \text{ s}} = 3.3 \text{ m/s}^2 \]

68. **REASONING** The average velocity for each segment is the slope of the line for that segment.

**SOLUTION** Taking the direction of motion as positive, we have from the graph for segments A, B, and C,

\[ v_A = \frac{10.0 \text{ km} - 40.0 \text{ km}}{1.5 \text{ h} - 0.0 \text{ h}} = -2.0 \times 10^1 \text{ km/h} \]

\[ v_B = \frac{20.0 \text{ km} - 10.0 \text{ km}}{2.5 \text{ h} - 1.5 \text{ h}} = 1.0 \times 10^1 \text{ km/h} \]

\[ v_C = \frac{40.0 \text{ km} - 20.0 \text{ km}}{3.0 \text{ h} - 2.5 \text{ h}} = 40 \text{ km/h} \]

69. **REASONING** The slope of the position-time graph is the velocity of the bus. Each of the three segments of the graph is a straight line, so the bus has a different constant velocity for each part of the trip: \( v_A \), \( v_B \), and \( v_C \). The slope of each segment may be calculated from Equation 2.2 \( \left( v = \frac{\Delta x}{\Delta t} \right) \), where \( \Delta x \) is the difference between the final and initial positions of the bus and \( \Delta t \) is the elapsed time during each segment. The average acceleration of the bus is the change in its velocity divided by the elapsed time, as in Equation 2.4 \( \left( \bar{a} = \frac{v - v_0}{\Delta t} \right) \).

The trip lasts from \( t = 0 \text{ h} \) (the initial instant on the graph) to \( t = 3.5 \text{ h} \) (the final instant on the graph), so the total elapsed time is \( \Delta t = 3.5 \text{ h} \). The initial velocity of the bus is its
velocity at \( t = 0 \), which is its constant velocity for segment \( A \): \( v_0 = v_A \). Similarly, the velocity of the bus at the last instant of segment \( C \) is its final velocity for the trip: \( v = v_C \).

**SOLUTION** In using Equation 2.2 \( v = \frac{\Delta x}{\Delta t} \) to calculate the slopes of segments \( A \) and \( C \), any displacement \( \Delta x \) within a segment may be chosen, so long as the corresponding elapsed time \( \Delta t \) is used in the calculation. If the full displacements for each segment are chosen, then

\[
v_A = \frac{\Delta x_A}{\Delta t_A} = \frac{24 \text{ km} - 0 \text{ km}}{1.0 \text{ h} - 0 \text{ h}} = 24 \text{ km/h}
\]

\[
v_C = \frac{\Delta x_C}{\Delta t_C} = \frac{27 \text{ km} - 33 \text{ km}}{3.5 \text{ h} - 2.2 \text{ h}} = -5 \text{ km/h}
\]

Apply these results to Equation 2.4:

\[
\bar{a} = \frac{v - v_0}{\Delta t} = \frac{(-5 \text{ km/h}) - (24 \text{ km/h})}{3.5 \text{ h}} = -8.3 \text{ km/h}^2
\]

70. **REASONING** The runner is at the position \( x = 0 \text{ m} \) when time \( t = 0 \text{ s} \); the finish line is 100 m away. During each ten-second segment, the runner has a constant velocity and runs half the remaining distance to the finish line. The following table shows the first four segments of the motion:

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Change in Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 \text{ s} \rightarrow t = 10.0 \text{ s} )</td>
<td>( x = 0 \text{ m} \rightarrow x = 50.0 \text{ m} )</td>
</tr>
<tr>
<td>( t = 10.0 \text{ s} \rightarrow t = 20.0 \text{ s} )</td>
<td>( x = 50.0 \text{ m} \rightarrow x = 50.0 \text{ m} + 25.0 \text{ m} = 75.0 \text{ m} )</td>
</tr>
<tr>
<td>( t = 20.0 \text{ s} \rightarrow t = 30.0 \text{ s} )</td>
<td>( x = 75.0 \text{ m} \rightarrow x = 75.0 \text{ m} + 12.5 \text{ m} = 87.5 \text{ m} )</td>
</tr>
<tr>
<td>( t = 30.0 \text{ s} \rightarrow t = 40.0 \text{ s} )</td>
<td>( x = 87.5 \text{ m} \rightarrow x = 87.5 \text{ m} + 6.25 \text{ m} = 93.8 \text{ m} )</td>
</tr>
</tbody>
</table>

This data can be used to construct the position-time graph. Since the runner has a constant velocity during each ten-second segment, we can find the velocity during each segment from the slope of the position-time graph for that segment.

**SOLUTION**

a. The following figure shows the position-time graph for the first forty seconds.
b. The slope of each segment of the position-time graph is calculated as follows:

\[
\begin{align*}
[0.00 \text{ s to } 10.0 \text{ s}] \quad v &= \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ m} - 0.00 \text{ m}}{10.0 \text{ s} - 0 \text{ s}} = 5.00 \text{ m/s} \\
[10.0 \text{ s to } 20.0 \text{ s}] \quad v &= \frac{\Delta x}{\Delta t} = \frac{75.0 \text{ m} - 50.0 \text{ m}}{20.0 \text{ s} - 10.0 \text{ s}} = 2.50 \text{ m/s} \\
[20.0 \text{ s to } 30.0 \text{ s}] \quad v &= \frac{\Delta x}{\Delta t} = \frac{87.5 \text{ m} - 75.0 \text{ m}}{30.0 \text{ s} - 20.0 \text{ s}} = 1.25 \text{ m/s} \\
[30.0 \text{ s to } 40.0 \text{ s}] \quad v &= \frac{\Delta x}{\Delta t} = \frac{93.8 \text{ m} - 87.5 \text{ m}}{40.0 \text{ s} - 30.0 \text{ s}} = 0.625 \text{ m/s}
\end{align*}
\]

Therefore, the velocity-time graph is:
71. **REASONING**  The two runners start one hundred meters apart and run toward each other. Each runs ten meters during the first second and, during each second thereafter, each runner runs ninety percent of the distance he ran in the previous second. While the velocity of each runner changes from second to second, it remains constant during any one second.

**SOLUTION** The following table shows the distance covered during each second for one of the runners, and the position at the end of each second (assuming that he begins at the origin) for the first eight seconds.

<table>
<thead>
<tr>
<th>Time $t$ (s)</th>
<th>Distance covered (m)</th>
<th>Position $x$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>10.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.00</td>
<td>9.00</td>
<td>10.00</td>
</tr>
<tr>
<td>2.00</td>
<td>8.10</td>
<td>19.00</td>
</tr>
<tr>
<td>3.00</td>
<td>7.29</td>
<td>27.10</td>
</tr>
<tr>
<td>4.00</td>
<td>6.56</td>
<td>34.39</td>
</tr>
<tr>
<td>5.00</td>
<td>5.90</td>
<td>40.95</td>
</tr>
<tr>
<td>6.00</td>
<td>5.31</td>
<td>46.85</td>
</tr>
<tr>
<td>7.00</td>
<td>4.78</td>
<td>52.16</td>
</tr>
<tr>
<td>8.00</td>
<td></td>
<td>56.94</td>
</tr>
</tbody>
</table>

The following graph is the position-time graph constructed from the data in the table above.

a. Since the two runners are running toward each other in exactly the same way, they will meet halfway between their respective starting points. That is, they will meet at $x = 50.0$ m. According to the graph, therefore, this position corresponds to a time of $\boxed{6.6}$ s.
b. Since the runners collide during the seventh second, the speed at the instant of collision can be found by taking the slope of the position-time graph for the seventh second. The speed of either runner in the interval from \( t = 6.00 \, \text{s} \) to \( t = 7.00 \, \text{s} \) is

\[
v = \frac{\Delta x}{\Delta t} = \frac{52.16 \, \text{m} - 46.85 \, \text{m}}{7.00 \, \text{s} - 6.00 \, \text{s}} = 5.3 \, \text{m/s}
\]

Therefore, at the moment of collision, the speed of either runner is 5.3 m/s.

72. **REASONING** The average acceleration \( \left( \bar{a} \right) \) is defined by Equation 2.4 \( \left( \bar{a} = \frac{v - v_0}{t - t_0} \right) \) as the change in velocity \( \left( v - v_0 \right) \) divided by the elapsed time \( \left( t - t_0 \right) \). The change in velocity is equal to the final velocity minus the initial velocity. Therefore, the change in velocity, and hence the acceleration, is positive if the final velocity is greater than the initial velocity. The acceleration is negative if the final velocity is less than the initial velocity. (a) The final velocity is greater than the initial velocity, so the acceleration will be positive. (b) The final velocity is less than the initial velocity, so the acceleration will be negative. (c) The final velocity is greater than the initial velocity (–3.0 m/s is greater than –6.0 m/s), so the acceleration will be positive. (d) The final velocity is less than the initial velocity, so the acceleration will be negative.

**SOLUTION** Equation 2.4 gives the average acceleration as

\[
\bar{a} = \frac{v - v_0}{t - t_0}
\]

Therefore, the average accelerations for the four cases are:

(a) \( \bar{a} = (5.0 \, \text{m/s} - 2.0 \, \text{m/s})/(2.0 \, \text{s}) = +1.5 \, \text{m/s}^2 \)
(b) \( \bar{a} = (2.0 \, \text{m/s} - 5.0 \, \text{m/s})/(2.0 \, \text{s}) = -1.5 \, \text{m/s}^2 \)
(c) \( \bar{a} = [-3.0 \, \text{m/s} - (-6.0 \, \text{m/s})]/(2.0 \, \text{s}) = +1.5 \, \text{m/s}^2 \)
(d) \( \bar{a} = (-4.0 \, \text{m/s} - 4.0 \, \text{m/s})/(2.0 \, \text{s}) = -4.0 \, \text{m/s}^2 \)

73. **SSM REASONING AND SOLUTION**

a. Once the pebble has left the slingshot, it is subject only to the acceleration due to gravity. Since the downward direction is negative, the acceleration of the pebble is \(-9.80 \, \text{m/s}^2\). The pebble is not decelerating. Since its velocity and acceleration both point downward, the magnitude of the pebble’s velocity is increasing, not decreasing.
b. The displacement $y$ traveled by the pebble as a function of the time $t$ can be found from Equation 2.8. Using Equation 2.8, we have

$$y = v_0t + \frac{1}{2}at^2 = (-9.0 \text{ m/s})(0.50 \text{ s}) + \frac{1}{2} \left[ (-9.80 \text{ m/s}^2)(0.50 \text{ s})^2 \right] = -5.7 \text{ m}$$

Thus, after 0.50 s, the pebble is $5.7 \text{ m}$ beneath the cliff-top.

74. **REASONING** In a race against el-Guerrouj, Bannister would run a distance given by his average speed times the time duration of the race (see Equation 2.1). The time duration of the race would be el-Guerrouj’s winning time of 3:43.13 (223.13 s). The difference between Bannister’s distance and the length of the race is el-Guerrouj’s winning margin.

**SOLUTION** From the table of conversion factors on the page facing the front cover, we find that one mile corresponds to 1609 m. According to Equation 2.1, Bannister’s average speed is

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}} = \frac{1609 \text{ m}}{239.4 \text{ s}}$$

Had he run against el-Guerrouj at this average speed for the 223.13-s duration of the race, he would have traveled a distance of

$$\text{Distance} = \text{Average speed} \times \text{Time} = \left( \frac{1609 \text{ m}}{239.4 \text{ s}} \right)(223.13 \text{ s})$$

while el-Guerrouj traveled 1609 m. Thus, el-Guerrouj would have won by a distance of

$$1609 \text{ m} - \left( \frac{1609 \text{ m}}{239.4 \text{ s}} \right)(223.13 \text{ s}) = 109 \text{ m}$$

75. **REASONING** Since the belt is moving with constant velocity, the displacement $(x_0 = 0 \text{ m})$ covered by the belt in a time $t_{belt}$ is giving by Equation 2.2 (with $x_0$ assumed to be zero) as

$$x = v_{belt}t_{belt} \quad (1)$$

Since Clifford moves with constant acceleration, the displacement covered by Clifford in a time $t_{Cliff}$ is, from Equation 2.8,

$$x = v_{0Cliff}t_{Cliff} + \frac{1}{2}at_{Cliff}^2 = \frac{1}{2}at_{Cliff}^2 \quad (2)$$
The speed $v_{belt}$ with which the belt of the ramp is moving can be found by eliminating $x$ between Equations (1) and (2).

**SOLUTION** Equating the right hand sides of Equations (1) and (2), and noting that $t_{Cliff} = \frac{1}{4} t_{belt}$, we have

$$v_{belt}^ 2 = \frac{1}{2} a \left( \frac{1}{4} t_{belt} \right)^2$$

$$v_{belt} = \frac{1}{32} a t_{belt} = \frac{1}{32} (0.37 \text{ m/s}^2)(64 \text{ s}) = 0.74 \text{ m/s}$$

76. **REASONING** The minimum time that a player must wait before touching the basketball is the time required for the ball to reach its maximum height. The initial and final velocities are known, as well as the acceleration due to gravity, so Equation 2.4 ($v = v_0 + at$) can be used to find the time.

**SOLUTION** Solving Equation 2.4 for the time yields

$$t = \frac{v - v_0}{a} = \frac{0 \text{ m/s} - 4.6 \text{ m/s}}{-9.8 \text{ m/s}^2} = 0.47 \text{ s}$$

77. **REASONING** Average speed is the ratio of distance to elapsed time (Equation 2.1), so the elapsed time is distance divided by average speed. Both the average speed and the distance are given in SI base units, so the elapsed time will come out in seconds, which can then be converted to minutes (1 min = 60 s).

**SOLUTION** First, calculate the elapsed time $\Delta t$ in seconds:

$$\Delta t = \frac{\text{Distance}}{\text{Average speed}} = \frac{1.5 \text{ m}}{1.1 \times 10^{-2} \text{ m/s}} = 140 \text{ s}$$

(2.1)

Converting the elapsed time from seconds to minutes, we find that

$$\Delta t = \left(140 \frac{s}{60 \frac{s}{min}}\right) = 2.3 \text{ min}$$

78. **REASONING** The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given. Since the acceleration of the spacecraft is constant, it is equal to the average acceleration.
SOLUTION

a. The time $\Delta t$ that it takes for the spacecraft to change its velocity by an amount $\Delta v = +2700 \text{ m/s}$ is

$$\Delta t = \frac{\Delta v}{a} = \frac{+2700 \text{ m/s}}{+9.0 \text{ m/s/day}} = 3.0 \times 10^2 \text{ days}$$

b. Since 24 hr = 1 day and 3600 s = 1 hr, the acceleration of the spacecraft (in m/s$^2$) is

$$a = \frac{\Delta v}{t} = \frac{+9.0 \text{ m/s}}{(1 \text{ day}) \left( \frac{24 \text{ hr}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right)} = +1.04 \times 10^{-4} \text{ m/s}^2$$

79. REASONING The cheetah and its prey run the same distance. The prey runs at a constant velocity, so that its distance is the magnitude of its displacement, which is given by Equation 2.2 as the product of velocity and time. The distance for the cheetah can be expressed using Equation 2.8, since the cheetah’s initial velocity (zero, since it starts from rest) and the time are given, and we wish to determine the acceleration. The two expressions for the distance can be equated and solved for the acceleration.

SOLUTION We begin by using Equation 2.2 and assuming that the initial position of the prey is $x_0 = 0 \text{ m}$. The distance run by the prey is

$$\Delta x = x - x_0 = x = v_{\text{Prey}} t$$

The distance run by the cheetah is given by Equation 2.8 as

$$x = v_{0, \text{Cheetah}} t + \frac{1}{2} a_{\text{Cheetah}} t^2$$

Equating the two expressions for $x$ and using the fact that $v_{0, \text{Cheetah}} = 0 \text{ m/s}$, we find that

$$v_{\text{Prey}} t = \frac{1}{2} a_{\text{Cheetah}} t^2$$

Solving for the acceleration gives

$$a_{\text{Cheetah}} = \frac{2v_{\text{Prey}}}{t} = \frac{2(+9.0 \text{ m/s})}{3.0 \text{ s}} = +6.0 \text{ m/s}^2$$
80. **REASONING AND SOLUTION**  The distance covered by the cab driver during the two phases of the trip must satisfy the relation

\[ x_1 + x_2 = 2.00 \text{ km} \]  

where \( x_1 \) and \( x_2 \) are the displacements of the acceleration and deceleration phases of the trip, respectively. The quantities \( x_1 \) and \( x_2 \) can be calculated from Equation 2.9 (\( v^2 = v_0^2 + 2ax \)):

\[
x_1 = \frac{v_1^2 - (0 \text{ m/s})^2}{2a_1} = \frac{v_1^2}{2a_1} \quad \text{and} \quad x_2 = \frac{(0 \text{ m/s})^2 - v_{02}^2}{2a_2} = -\frac{v_{02}^2}{2a_2}
\]

with \( v_{02} = v_1 \) and \( a_2 = -3a_1 \). Thus,

\[
\frac{x_1}{x_2} = \frac{v_1^2/(2a_1)}{-v_1^2/(-6a_1)} = 3
\]

so that

\[ x_1 = 3x_2 \]  

(2)

Combining (1) and (2), we have,

\[ 3x_2 + x_2 = 2.00 \text{ km} \]

Therefore, \( x_2 = 0.50 \text{ km} \), and from Equation (1), \( x_1 = 1.50 \text{ km} \). Thus, the length of the acceleration phase of the trip is \( x_1 = 1.50 \text{ km} \), while the length of the deceleration phase is \( x_2 = 0.50 \text{ km} \).

81. **SSM REASONING**  Since the woman runs for a known distance at a known constant speed, we can find the time it takes for her to reach the water from Equation 2.1. We can then use Equation 2.1 to determine the total distance traveled by the dog in this time.

**SOLUTION**  The time required for the woman to reach the water is

\[
\text{Elapsed time} = \frac{d_{\text{woman}}}{v_{\text{woman}}} = \left( \frac{4.0 \text{ km}}{2.5 \text{ m/s}} \right) \left( \frac{1000 \text{ m}}{1.0 \text{ km}} \right) = 1600 \text{ s}
\]

In 1600 s, the dog travels a total distance of

\[
d_{\text{dog}} = v_{\text{dog}}t = (4.5 \text{ m/s})(1600 \text{ s}) = 7.2 \times 10^3 \text{ m}
\]
82. **REASONING** When the second-place cyclist catches the leader, the displacement \( x_{2\text{nd}} \) of the second-place cyclist is 10.0 m greater than the displacement \( x_{\text{leader}} \) of the leader, so \( x_{2\text{nd}} = x_{\text{leader}} + 10.0 \) m. The initial velocity and acceleration of the second-place cyclist are known \((v_0 = +9.50 \text{ m/s}, a = +1.20 \text{ m/s}^2)\), as well as those of the leader \((v_0 = +11.10 \text{ m/s}, \ a = 0.00 \text{ m/s}^2)\). Note that the leader has zero acceleration, since his velocity is constant. Equation 2.8 may be used to provide a relationship between these variables and the displacement \( x \).

**SOLUTION** Substituting Equation 2.8 into each side of the relation \( x_{2\text{nd}} = x_{\text{leader}} + 10.0 \) m, we have that

\[
\frac{v_0 t + \frac{1}{2}a t^2}{x_{2\text{nd}}} = \frac{v_0 t + \frac{1}{2}a t^2}{x_{\text{leader}}} + 10.0 \text{ m}
\]

\[
(9.50 \text{ m/s})t + \frac{1}{2}(1.20 \text{ m/s}^2)t^2 = (11.10 \text{ m/s})t + \frac{1}{2}(0.00 \text{ m/s}^2)t^2 + 10.0 \text{ m}
\]

Rearranging the terms of this equation so it is in quadratic form, we have

\[
\frac{1}{2}(1.20 \text{ m/s}^2)t^2 - (1.60 \text{ m/s})t - 10.0 \text{ m} = 0
\]

This equation can be solved using the quadratic formula, with the result that \( t = 5.63 \text{ s} \).

83. **REASONING** The time \( t_{\text{trip}} \) to make the entire trip is equal to the time \( t_{\text{cart}} \) that the golfer rides in the golf cart plus the time \( t_{\text{walk}} \) that she walks; \( t_{\text{trip}} = t_{\text{cart}} + t_{\text{walk}} \). Therefore, the time that she walks is

\[
t_{\text{walk}} = t_{\text{trip}} - t_{\text{cart}}
\]

The average speed \( \bar{v}_{\text{trip}} \) for the entire trip is equal to the total distance, \( x_{\text{cart}} + x_{\text{walk}} \), she travels divided by the time to make the entire trip (see Equation 2.1);

\[
\bar{v}_{\text{trip}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{t_{\text{trip}}}
\]

Solving this equation for \( t_{\text{trip}} \) and substituting the resulting expression into Equation 1 yields

\[
t_{\text{walk}} = \frac{x_{\text{cart}} + x_{\text{walk}}}{\bar{v}_{\text{trip}}} - t_{\text{cart}}
\]
The distance traveled by the cart is \( x_{\text{cart}} = \overline{v}_{\text{cart}} t_{\text{cart}} \), and the distance walked by the golfer is \( x_{\text{walk}} = \overline{v}_{\text{walk}} t_{\text{walk}} \). Substituting these expressions for \( x_{\text{cart}} \) and \( x_{\text{walk}} \) into Equation 2 gives

\[
t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} + \overline{v}_{\text{walk}} t_{\text{walk}}}{\overline{v}_{\text{trip}}} - t_{\text{cart}}
\]

The unknown variable \( t_{\text{walk}} \) appears on both sides of this equation. Algebraically solving for this variable gives

\[
t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} - \overline{v}_{\text{trip}} t_{\text{cart}}}{\overline{v}_{\text{trip}} - \overline{v}_{\text{walk}}}
\]

**SOLUTION** The time that the golfer spends walking is

\[
t_{\text{walk}} = \frac{\overline{v}_{\text{cart}} t_{\text{cart}} - \overline{v}_{\text{trip}} t_{\text{cart}}}{\overline{v}_{\text{trip}} - \overline{v}_{\text{walk}}} = \frac{(3.10 \text{ m/s})(28.0 \text{ s}) - (1.80 \text{ m/s})(28.0 \text{ s})}{(1.80 \text{ m/s}) - (1.30 \text{ m/s})} = 73 \text{ s}
\]

84. **REASONING** The definition of average velocity is given by Equation 2.2 as the displacement divided by the elapsed time. When the velocity is constant, as it is for car A, the average velocity is the same as the constant velocity. We note that, since both displacement and time are the same for each car, this equation gives the same value for car B’s average velocity and car A’s constant velocity.

Since the acceleration of car B is constant, we know that its average velocity is given by Equation 2.6 as \( \overline{v}_B = \frac{1}{2} \left( v_B + v_{B0} \right) \), where \( v_B \) is the final velocity and \( v_{B0} = 0 \text{ m/s} \) is the initial velocity (car B starts from rest). Thus, we can use Equation 2.6 to find the final velocity.

Car B’s constant acceleration can be calculated from Equation 2.4 \( (v_B = v_{B0} + a_B t) \), which is one of the equations of kinematics and gives the acceleration as \( [a_B = (v_B - v_{B0})/t] \). Since car B starts from rest, we know that \( v_{B0} = 0 \text{ m/s} \). Furthermore, \( t \) is given. Therefore, calculation of the acceleration \( a_B \) requires that we use the value calculated for the final velocity \( v_B \).

**SOLUTION**

a. According to Equation 2.2, the velocity of car A is the displacement \( L \) divided by the time \( t \). Thus, we obtain

\[
v_A = \frac{L}{t} = \frac{460 \text{ m}}{210 \text{ s}} = 2.2 \text{ m/s}
\]
b. The average velocity of car B is given by Equation 2.6 as
\[ \bar{v}_B = \frac{1}{2} (v_B + v_{B0}) \]
where \( v_B \) is the final velocity and \( v_{B0} \) is the initial velocity. Solving for the final velocity and using the fact that car B starts from rest \( (v_{B0} = 0 \text{ m/s}) \) gives

\[ v_B = 2\bar{v}_B - v_{B0} = 2\bar{v}_B \tag{1} \]

As discussed in the REASONING, the average velocity of car B is equal to the constant velocity of car A. Substituting this result into Equation (1), we find that

\[ v_B = 2\bar{v}_B = 2v_A = 2(2.2 \text{ m/s}) = 4.4 \text{ m/s} \]

\[ a_B = \frac{v_B - v_{B0}}{t} = \frac{4.4 \text{ m/s} - 0 \text{ m/s}}{210 \text{ s}} = 0.021 \text{ m/s}^2 \]

85. REASONING We choose due north as the positive direction. Our solution is based on the fact that when the police car catches up, both cars will have the same displacement, relative to the point where the speeder passed the police car. The displacement of the speeder can be obtained from the definition of average velocity given in Equation 2.2, since the speeder is moving at a constant velocity. During the 0.800-s reaction time of the policeman, the police car is also moving at a constant velocity. Once the police car begins to accelerate, its displacement can be expressed as in Equation 2.8 \( (x = v_0t + \frac{1}{2}at^2) \), because the initial velocity \( v_0 \) and the acceleration \( a \) are known and it is the time \( t \) that we seek. We will set the displacements of the speeder and the police car equal and solve the resulting equation for the time \( t \).

SOLUTION Let \( t \) equal the time during the accelerated motion of the police car. Relative to the point where he passed the police car, the speeder then travels a time of \( t + 0.800 \text{ s} \) before the police car catches up. During this time, according to the definition of average velocity given in Equation 2.2, his displacement is

\[ x_{\text{Speeder}} = v_{\text{Speeder}} (t + 0.800 \text{ s}) = (42.0 \text{ m/s})(t + 0.800 \text{ s}) \]

The displacement of the police car consists of two contributions, the part due to the constant-velocity motion during the reaction time and the part due to the accelerated motion. Using Equation 2.2 for the contribution from the constant-velocity motion and Equation 2.9 for the contribution from the accelerated motion, we obtain
\[ x_{\text{Police car}} = v_{0, \text{Police car}} (0.800 \text{ s}) + \frac{v_{0, \text{Police car}} t}{2} + \frac{1}{2} at^2 \]

Constant velocity motion, 
Equation 2.2

\[ = (18.0 \text{ m/s}) (0.800 \text{ s}) + (18.0 \text{ m/s}) t + \frac{1}{2} (5.00 \text{ m/s}^2) t^2 \]

Setting the two displacements equal we obtain

\[ \frac{(42.0 \text{ m/s})(t + 0.800 \text{ s})}{\text{Displacement of speeder}} = \frac{(18.0 \text{ m/s})(0.800 \text{ s}) + (18.0 \text{ m/s}) t + \frac{1}{2} (5.00 \text{ m/s}^2) t^2}{\text{Displacement of police car}} \]

Rearranging and combining terms gives this result in the standard form of a quadratic equation:

\[ \left( 2.50 \text{ m/s}^2 \right) t^2 - (24.0 \text{ m/s}) t - 19.2 \text{ m} = 0 \]

Solving for \( t \) shows that

\[ t = \frac{-(-24.0 \text{ m/s}) \pm \sqrt{(-24.0 \text{ m/s})^2 - 4 \left( 2.50 \text{ m/s}^2 \right)(-19.2 \text{ m})}}{2 \left( 2.50 \text{ m/s}^2 \right)} = 10.3 \text{ s} \]

We have ignored the negative root, because it leads to a negative value for the time, which is unphysical. The total time for the police car to catch up, including the reaction time, is

\[ 0.800 \text{ s} + 10.3 \text{ s} = \boxed{11.1 \text{ s}} \]

86. **REASONING AND SOLUTION** We measure the positions of the balloon and the pellet relative to the ground and assume up to be positive. The balloon has no acceleration, since it travels at a constant velocity \( v_B \), so its displacement in time \( t \) is \( v_B t \). Its position above the ground, therefore, is

\[ y_B = H_0 + v_B t \]

where \( H_0 = 12 \text{ m} \). The pellet moves under the influence of gravity \((a = -9.80 \text{ m/s}^2)\), so its position above the ground is given by Equation 2.8 as

\[ y_p = v_0 t + \frac{1}{2} at^2 \]

But \( y_p = y_B \) at time \( t \), so that

\[ v_0 t + \frac{1}{2} at^2 = H_0 + v_B t \]
Rearranging this result and suppressing the units gives

\[ \frac{1}{2} at^2 + (v_0 - v_B)t - H_0 = \frac{1}{2} (-9.80)t^2 + (30.0 - 7.0)t - 12.0 = 0 \]

\[ 4.90t^2 - 23.0t + 12.0 = 0 \]

\[ t = \frac{23.0 \pm \sqrt{23.0^2 - 4(4.90)(12.0)}}{2(4.90)} = 4.09 \text{ s or 0.602 s} \]

Substituting each of these values in the expression for \( y_B \) gives

\[ y_B = 12.0 \text{ m} + (7.0 \text{ m/s})(4.09 \text{ s}) = 41 \text{ m} \]

\[ y_B = 12.0 \text{ m} + (7.0 \text{ m/s})(0.602 \text{ s}) = 16 \text{ m} \]

87. SSM REASONING Since 1 mile = 1609 m, a quarter-mile race is \( L = 402 \text{ m} \) long. If a car crosses the finish line before reaching its maximum speed, then there is only one interval of constant acceleration to consider. We will first determine whether this is true by calculating the car’s displacement \( x_1 \) while accelerating from rest to top speed from Equation 2.9 \( v^2 = v_0^2 + 2ax \), with \( v_0 = 0 \text{ m/s} \) and \( v = v_{\text{max}} \):

\[ v_{\text{max}}^2 = (0 \text{ m/s})^2 + 2ax_1 \quad \text{or} \quad x_1 = \frac{v_{\text{max}}^2}{2a} \tag{1} \]

If \( x_1 > L \), then the car crosses the finish line before reaching top speed, and the total time for its race is found from Equation 2.8 \( x = v_0t + \frac{1}{2}at^2 \), with \( x = L \) and \( v_0 = 0 \text{ m/s} \):

\[ L = (0 \text{ m/s})t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \text{or} \quad t = \sqrt{\frac{2L}{a}} \tag{2} \]

On the other hand, if a car reaches its maximum speed before crossing the finish line, the race divides into two intervals, each with a different constant acceleration. The displacement \( x_1 \) is found as given in Equation (1), but the time \( t_1 \) to reach the maximum speed is most easily found from Equation 2.4 \( v = v_0 + at \), with \( v_0 = 0 \text{ m/s} \) and \( v = v_{\text{max}} \):

\[ v_{\text{max}} = 0 \text{ m/s} + at_1 \quad \text{or} \quad t_1 = \frac{v_{\text{max}}}{a} \tag{3} \]

The time \( t_2 \) that elapses during the rest of the race is found by solving Equation 2.8 \( x = v_0t + \frac{1}{2}at^2 \). Let \( x_2 = L - x_1 \) represent the displacement for this part of the race. With
the aid of Equation (1), this becomes \( x_2 = L - \frac{v_{\text{max}}^2}{2a} \). Then, since the car is at its maximum speed, the acceleration is \( a = 0 \text{ m/s}^2 \), and the displacement is

\[
x_2 = v_{\text{max}} t_2 + \frac{1}{2}\left(0 \text{ m/s}^2\right)t_2^2 = v_{\text{max}}^2 t_2 \quad \text{or} \quad t_2 = \frac{x_2}{v_{\text{max}}} = \frac{L - \frac{v_{\text{max}}^2}{2a}}{v_{\text{max}}} = \frac{L}{v_{\text{max}}} - \frac{v_{\text{max}}}{2a} \quad (4)
\]

Using this expression for \( t_2 \) and Equation (3) for \( t_1 \) gives the total time for a two-part race:

\[
t = t_1 + t_2 = \frac{v_{\text{max}}}{a} + \left(\frac{L}{v_{\text{max}}} - \frac{v_{\text{max}}}{2a}\right) = \frac{L}{v_{\text{max}}} + \frac{v_{\text{max}}}{2a} \quad (5)
\]

**SOLUTION** First, we use Equation (1) to determine whether either car finishes the race while accelerating:

**Car A**

\[
x_1 = \frac{v_{\text{max}}^2}{2a} = \frac{(106 \text{ m/s})^2}{2 \left(11.0 \text{ m/s}^2\right)} = 511 \text{ m}
\]

**Car B**

\[
x_1 = \frac{v_{\text{max}}^2}{2a} = \frac{(92.4 \text{ m/s})^2}{2 \left(11.6 \text{ m/s}^2\right)} = 368 \text{ m}
\]

Therefore, car A finishes the race before reaching its maximum speed, but car B has \( 402 \text{ m} - 368 \text{ m} = 34 \text{ m} \) to travel at its maximum speed. Equation (2) gives the time for car A to reach the finish line as

**Car A**

\[
t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2(402 \text{ m})}{11.0 \text{ m/s}^2}} = 8.55 \text{ s}
\]

Equation (5) gives the time for car B to reach the finish line as

**Car B**

\[
t = \frac{L}{v_{\text{max}}} + \frac{v_{\text{max}}}{2a} = \frac{402 \text{ m}}{92.4 \text{ m/s}} + \frac{92.4 \text{ m/s}}{2 \left(11.6 \text{ m/s}^2\right)} = 8.33 \text{ s}
\]

**Car B wins the race** by \( 8.55 \text{ s} - 8.33 \text{ s} = 0.22 \text{ s} \).
88. **REASONING AND SOLUTION**  During the first phase of the acceleration,

\[
a_1 = \frac{v}{t_1}
\]

During the second phase of the acceleration,

\[
v = (3.4 \text{ m/s}) - (1.1 \text{ m/s}^2)(1.2 \text{ s}) = 2.1 \text{ m/s}
\]

Then

\[
a_1 = \frac{2.1 \text{ m/s}}{1.5 \text{ s}} = 1.4 \text{ m/s}^2
\]
ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (a) The horizontal component \( v_x \) of the projectile’s velocity remains constant throughout the motion, since the acceleration \( a_x \) in the horizontal direction is zero \( (a_x = 0 \text{ m/s}^2) \). The vertical component \( v_y \), however, changes as the projectile moves. This component is greatest at point 1, decreases to zero at point 2 at the top of the trajectory, and then increases to a magnitude less than that at point 1 as the projectile approaches point 3.

2. (b) The minimum speed of the projectile occurs when it is at the top of its trajectory. At this point the vertical component of its velocity is zero \( (v_y = 0 \text{ m/s}) \). Since there is no acceleration in the \( x \) direction \( (a_x = 0 \text{ m/s}^2) \), the \( x \) component of the projectile’s velocity remains constant at \( v_x = +30 \text{ m/s} \) throughout the motion. Thus, the minimum speed is 30 m/s.

3. (c) The acceleration due to gravity is the same for both balls, despite the fact that they have different velocities.

4. (d) The acceleration of a projectile is the same at all points on the trajectory. It points downward, toward the earth, and has a magnitude of 9.80 m/s\(^2\).

5. (c) Since there is no acceleration in the \( x \) direction \( (a_x = 0 \text{ m/s}^2) \) for projectile motion, the \( x \) component \( v_x \) of the velocity is constant throughout the motion. And, the acceleration due to gravity, \( a_y = -9.80 \text{ m/s}^2 \) (“downward” is the negative direction), also remains constant.

6. (c) The time for a projectile to reach the ground depends only on the \( y \) component (or vertical component) of its variables, i.e., \( y \), \( v_{0y} \), and \( a_y \). These variables are the same for both balls. The fact that Ball 1 is moving horizontally at the top of its trajectory does not play a role in the time it takes for it to reach the ground.

7. (a) Using \( y = -19.6 \text{ m}, a_y = -9.80 \text{ m/s}^2 \), and \( v_{0y} = 0 \text{ m/s} \), Equation 3.5b \( (y = v_{0y}t + \frac{1}{2}a_yt^2) \) can be used to calculate the time \( t \).

8. (b) The time a projectile is in the air is equal to twice the time it takes to fall from its maximum height. Both have the same maximum height, so the time of fall is the same. Therefore, both projectiles are in the air for the same amount of time.
9. (a) The time a projectile is in the air is equal to twice the time it takes to fall from its maximum height. Projectile 1 reaches the greater height, so it spends the greater amount of time in the air.

10. (c) The vertical component (or y component) of the final velocity depends on the y components of the kinematic variables \( y, v_{0y}, \) and \( a_y \) and the time \( t \). These variables are the same for both balls, so they have the same vertical component of the velocity.

11. (a) A person standing on the ground sees a car traveling at +25 m/s. When the driver of the car looks out the window, she sees the person moving in the opposite direction with the same speed, or with a velocity of \(-25 \) m/s.

12. (d) The velocity \( v_{BC} \) of the bus relative to the car is given by the relation

\[
v_{BC} = v_{BG} + v_{GC} = v_{BG} + (-v_{CG}) = +16 \text{ m/s} + (-12 \text{ m/s}) = +4 \text{ m/s}.
\]

13. (d) This answer is arrived at by using the relation \( v_{PG} = v_{PB} + v_{BG} = +2 \text{ m/s} + 16 \text{ m/s} = +18 \text{ m/s} \).

14. (c) The velocity \( v_{PC} \) of the passenger relative to the car is given by \( v_{PC} = v_{PB} + v_{BC} \), according to the subscripting method discussed in Section 3.4. However, the last term on the right of this equation is given by \( v_{BC} = v_{BG} + v_{GC} \). So, \( v_{PC} = v_{PB} + v_{BG} + v_{GC} = +2 \text{ m/s} + 16 \text{ m/s} + (-12 \text{ m/s}) = +6 \text{ m/s} \).

15. (b) The velocity of the jeep relative to you is zero. Thus, the horizontal component of the tire’s velocity relative to you is also zero. Since this component of the velocity never changes as the tire falls, the car cannot hit the tire, regardless of how close the car is to the jeep.

16. The magnitude \( v_{AB} \) of the velocity of car A relative to car B is \( v_{AB} = 34.2 \) m/s. The angle that the velocity \( v_{AB} \) makes with respect to due east is \( \theta = 37.9^\circ \) south of east.
PROBLEMS

1. **SSM REASONING** The displacement is a vector drawn from the initial position to the final position. The magnitude of the displacement is the shortest distance between the positions. Note that it is only the initial and final positions that determine the displacement. The fact that the squirrel jumps to an intermediate position before reaching his final position is not important. The trees are perfectly straight and both growing perpendicular to the flat horizontal ground beneath them. Thus, the distance between the trees and the length of the trunk of the second tree below the squirrel’s final landing spot form the two perpendicular sides of a right triangle, as the drawing shows. To this triangle, we can apply the Pythagorean theorem and determine the magnitude \( A \) of the displacement vector \( \mathbf{A} \).

![Diagram of a right triangle with sides 1.3 m and 2.5 m](image)

**SOLUTION** According to the Pythagorean theorem, we have

\[
A = \sqrt{(1.3 \, \text{m})^2 + (2.5 \, \text{m})^2} = 2.8 \, \text{m}
\]

2. **REASONING** The meteoroid’s speed is the magnitude of its velocity vector, here described in terms of two perpendicular components, one directed toward the east and one directed vertically downward. Let east be the +\( x \) direction, and up be the +\( y \) direction. Then the components of the meteoroid’s velocity are \( v_x = +18.3 \, \text{km/s} \) and \( v_y = -11.5 \, \text{km/s} \). The meteoroid’s speed \( v \) is related to these components by the Pythagorean theorem (Equation 1.7):

\[
v^2 = v_x^2 + v_y^2.
\]

**SOLUTION** From the Pythagorean theorem,

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(+18.3 \, \text{km/s})^2 + (-11.5 \, \text{km/s})^2} = 21.6 \, \text{km/s}
\]

It’s important to note that the negative sign for \( v_y \) becomes a positive sign when this quantity is squared. Forgetting this fact would yield a value for \( v \) that is smaller than \( v_x \), but the magnitude of a vector cannot be smaller than either of its components.
3. **REASONING** To determine the horizontal and vertical components of the launch velocity, we will use trigonometry. To do so, however, we need to know both the launch angle and the magnitude of the launch velocity. The launch angle is given. The magnitude of the launch velocity can be determined from the given acceleration and the definition of acceleration given in Equation 3.2.

**SOLUTION** According to Equation 3.2, we have

\[ a = \frac{v - v_0}{t - t_0} \quad \text{or} \quad 340 \, \text{m/s}^2 = \frac{v - 0 \, \text{m/s}}{0.050 \, \text{s}} \quad \text{or} \quad v = \left(340 \, \text{m/s}^2\right)(0.050 \, \text{s}) \]

Using trigonometry, we find the components to be

\[ v_x = v \cos 51^\circ = \left(340 \, \text{m/s}^2\right)(0.050 \, \text{s}) \cos 51^\circ = 11 \, \text{m/s} \]

\[ v_y = v \sin 51^\circ = \left(340 \, \text{m/s}^2\right)(0.050 \, \text{s}) \sin 51^\circ = 13 \, \text{m/s} \]

4. **REASONING** The displacement is the vector drawn from the initial to the final position. The magnitude of the displacement vector is the shortest distance between the initial and final position.

**SOLUTION** The batter’s initial position is home plate. His final position is at third base. The shortest distance between these two positions is 27.4 m. Therefore, the magnitude of the player’s displacement is 27.4 m.

5. **SSM REASONING** The displacement of the elephant seal has two components; 460 m due east and 750 m downward. These components are mutually perpendicular; hence, the Pythagorean theorem can be used to determine their resultant.

**SOLUTION** From the Pythagorean theorem,

\[ R^2 = (460 \, \text{m})^2 + (750 \, \text{m})^2 \]

Therefore,

\[ R = \sqrt{(460 \, \text{m})^2 + (750 \, \text{m})^2} = 8.8 \times 10^2 \, \text{m} \]

6. **REASONING AND SOLUTION** The horizontal displacement is

\[ x = 19 \, 600 \, \text{m} - 11 \, 200 \, \text{m} = 8400 \, \text{m} \]
The vertical displacement is
\[ y = 4900 \text{ m} - 3200 \text{ m} = 1700 \text{ m} \]
The magnitude of the displacement is therefore,
\[ \Delta r = \sqrt{x^2 + y^2} = \sqrt{(8400 \text{ m})^2 + (1700 \text{ m})^2} = 8600 \text{ m} \]

7. **SSM REASONING AND SOLUTION**
\[ x = r \cos \theta = (162 \text{ km}) \cos 62.3^\circ = 75.3 \text{ km} \]
\[ y = r \sin \theta = (162 \text{ km}) \sin 62.3^\circ = 143 \text{ km} \]

8. **REASONING** Consider first the shopper’s ride up the escalator. Let the diagonal length of the escalator be \( L \), the height of the upper floor be \( H \), and the angle that the escalator makes with respect to the horizontal be \( \theta \) (see the diagram). Because \( L \) is the hypotenuse of the right triangle and \( H \) is opposite the angle \( \theta \), the three quantities are related by the inverse sine function:
\[ \theta = \sin^{-1}\left(\frac{h_o}{h}\right) = \sin^{-1}\left(\frac{H}{L}\right) \quad (1.4) \]

Now consider the entire trip from the bottom to the top of the escalator (a distance \( L \)), and then from the top of the escalator to the store entrance (a distance \( s \)). The right turn between these two parts of the trip means that they are perpendicular (see the diagram). The shopper’s total displacement has a magnitude \( D \), and this serves as the hypotenuse of a right triangle with \( L \) and \( s \). From the Pythagorean theorem, the three sides are related as follows: \( D^2 = L^2 + s^2 \).

**SOLUTION** Solving \( D^2 = L^2 + s^2 \) for the length \( L \) of the escalator gives \( L = \sqrt{D^2 - s^2} \). We now use this result and the relation \( \theta = \sin^{-1}\left(\frac{H}{L}\right) \) to obtain the angle \( \theta \):
\[ \theta = \sin^{-1}\left(\frac{H}{L}\right) = \sin^{-1}\left(\frac{H}{\sqrt{D^2 - s^2}}\right) = \sin^{-1}\left(\frac{6.00 \text{ m}}{\sqrt{(16.0 \text{ m})^2 - (9.00 \text{ m})^2}}\right) = 27.0^\circ \]
9. **SSM REASONING**

a. We designate the direction down and parallel to the ramp as the $+x$ direction, and the table shows the variables that are known. Since three of the five kinematic variables have values, one of the equations of kinematics can be employed to find the acceleration $a_x$.

### $x$-Direction Data

<table>
<thead>
<tr>
<th>$x$</th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$v_{0x}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+12.0 m</td>
<td>?</td>
<td>+7.70 m/s</td>
<td>0 m/s</td>
<td></td>
</tr>
</tbody>
</table>

b. The acceleration vector points down and parallel to the ramp, and the angle of the ramp is $25.0^\circ$ relative to the ground (see the drawing). Therefore, trigonometry can be used to determine the component $a_{\text{parallel}}$ of the acceleration that is parallel to the ground.

**SOLUTION**

a. Equation 3.6a \(v_x^2 = v_{0x}^2 + 2a_x x\) can be used to find the acceleration in terms of the three known variables. Solving this equation for $a_x$ gives

\[
a_x = \frac{v_x^2 - v_{0x}^2}{2x} = \frac{( +7.70 \text{ m/s})^2 - (0 \text{ m/s})^2}{2( +12.0 \text{ m})} = \frac{2.47 \text{ m/s}^2}{2}
\]

b. The drawing shows that the acceleration vector is oriented $25.0^\circ$ relative to the ground. The component $a_{\text{parallel}}$ of the acceleration that is parallel to the ground is

\[
a_{\text{parallel}} = a_x \cos 25.0^\circ = \left( 2.47 \text{ m/s}^2 \right) \cos 25.0^\circ = 2.24 \text{ m/s}^2
\]

10. **REASONING** The component method can be used to determine the magnitude and direction of the bird watcher's displacement. Once the displacement is known, Equation 3.1 can be used to find the average velocity.

**SOLUTION** The following table gives the components of the individual displacements of the bird watcher. The last entry gives the components of the bird watcher's resultant displacement. Due east and due north have been chosen as the positive directions.

<table>
<thead>
<tr>
<th>Displacement</th>
<th>East/West Component</th>
<th>North/South Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.50 km</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-0.75 km</td>
</tr>
<tr>
<td>C</td>
<td>-(2.15 km) \cos 35.0^\circ = -1.76 km</td>
<td>(2.15 km) \sin 35.0^\circ = 1.23 km</td>
</tr>
<tr>
<td>(\mathbf{r}) = A + B + C</td>
<td>-1.26 km</td>
<td>0.48 km</td>
</tr>
</tbody>
</table>
a. From the Pythagorean theorem, we have

\[ r = \sqrt{(-1.26 \text{ km})^2 + (0.48 \text{ km})^2} = 1.35 \text{ km} \]

The angle \( \theta \) is given by

\[ \theta = \tan^{-1}\left( \frac{0.48 \text{ km}}{1.26 \text{ km}} \right) = 21^\circ, \text{ north of west} \]

b. From Equation 3.1, the average velocity is

\[ \bar{v} = \frac{\Delta r}{\Delta t} = \frac{1.35 \text{ km}}{2.50 \text{ h}} = 0.540 \text{ km/h, 21}^\circ \text{ north of west} \]

Note that the direction of the average velocity is, by definition, the same as the direction of the displacement.

11. **REASONING AND SOLUTION**

a. Average speed is defined as the total distance \( d \) covered divided by the time \( \Delta t \) required to cover the distance. The total distance covered by the earth is one-fourth the circumference of its circular orbit around the sun:

\[ d = \frac{1}{4} \times 2\pi (1.50 \times 10^{11} \text{ m}) = 2.36 \times 10^{11} \text{ m} \]

\[ \bar{v} = \frac{d}{\Delta t} = \frac{2.36 \times 10^{11} \text{ m}}{7.89 \times 10^6 \text{ s}} = 2.99 \times 10^4 \text{ m/s} \]

b. The average velocity is defined as the displacement divided by the elapsed time.

In moving one-fourth of the distance around the sun, the earth completes the displacement shown in the figure at the right. From the Pythagorean theorem, the magnitude of this displacement is

\[ \Delta r = \sqrt{r^2 + r^2} = \sqrt{2} r \]

Thus, the magnitude of the average velocity is

\[ \bar{v} = \frac{\Delta r}{\Delta t} = \frac{\sqrt{2} \times 1.50 \times 10^{11} \text{ m}}{7.89 \times 10^6 \text{ s}} = 2.69 \times 10^4 \text{ m/s} \]
12. **REASONING** The motion in the $x$ direction occurs independently of the motion in the $y$ direction. The components of the velocity change from their initial values of $v_{0x}$ and $v_{0y}$ to their final values of $v_x$ and $v_y$. The changes occur, respectively, because of the acceleration components $a_x$ and $a_y$. The final values can be determined with the aid of Equations 3.3a and 3.3b.

**SOLUTION**

a. According to Equation 3.3a, the $x$ component of the velocity is

$$v_x = v_{0x} + a_x t = 5480 \text{ m/s} + (1.20 \text{ m/s}^2)(842 \text{ s}) = 6490 \text{ m/s}$$

b. According to Equation 3.3b, the $y$ component of the velocity is

$$v_y = v_{0y} + a_y t = 0 \text{ m/s} + (8.40 \text{ m/s}^2)(842 \text{ s}) = 7070 \text{ m/s}$$

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13. **SSM REASONING** The vertical component of the ball’s velocity $v_0$ changes as the ball approaches the opposing player. It changes due to the acceleration of gravity. However, the horizontal component does not change, assuming that air resistance can be neglected. Hence, the horizontal component of the ball’s velocity when the opposing player fields the ball is the same as it was initially.

**SOLUTION** Using trigonometry, we find that the horizontal component is

$$v_x = v_0 \cos \theta = (15 \text{ m/s}) \cos 55^\circ = 8.6 \text{ m/s}$$

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14. **REASONING** We will determine the ball’s displacement $y$ in the vertical direction from $y = v_{0y} t + \frac{1}{2} a_y t^2$ (Equation 3.5b). We choose this equation because we know that $v_{0y} = 0.00 \text{ m/s}$ (the ball initially travels horizontally) and that $a_y = -9.80 \text{ m/s}^2$ (assuming that upward is the $+y$ direction). We do not know the value of the ball’s time of flight $t$. However, we can determine it from the motion in the horizontal or $x$ direction.

**SOLUTION** From Equation 3.5b we have that

$$y = v_{0y} t + \frac{1}{2} a_y t^2 = (0.00 \text{ m/s}) t + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2 = \frac{1}{2} a_y t^2$$

The motion in the horizontal direction occurs at a constant velocity of $v_{0x} = +28.0 \text{ m/s}$ and the displacement in the horizontal direction is $x = +19.6 \text{ m}$. Thus, it follows that
\[ x = v_{0x} t \quad \text{or} \quad t = \frac{x}{v_{0x}} = \frac{19.6 \text{ m}}{28.0 \text{ m/s}} = 0.700 \text{ s} \]

Using this value for the time in the expression for \( y \) shows that

\[ y = \frac{1}{2} a_y t^2 = \frac{1}{2} \left(-9.80 \text{ m/s}^2\right) \left(0.700 \text{ s}\right)^2 = -2.40 \text{ m} \]

Since the displacement of the ball is 2.40 m downward, the ball is \[2.40 \text{ m} \] above the court when it leaves the racket.

15. **REASONING**

a. Here is a summary of the data for the skateboarder, using \( v_0 = 6.6 \text{ m/s} \) and \( \theta = 58^\circ \):

<table>
<thead>
<tr>
<th>( y )</th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>-9.80 \text{ m/s}^2</td>
<td>0 \text{ m/s}</td>
<td>+(6.6 \text{ m/s})\sin 58^\circ = +5.6 \text{ m/s}</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( a_x )</th>
<th>( v_x )</th>
<th>( v_{0x} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>0 \text{ m/s}^2</td>
<td>+(6.6 \text{ m/s})\cos 58^\circ = +3.5 \text{ m/s}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We will use the relation \( v_y^2 = v_{0y}^2 + 2a_y y \) (Equation 3.6b) to find the skateboarder’s vertical displacement \( y \) above the end of the ramp. When the skateboarder is at the highest point, his vertical displacement \( y_1 \) above the ground is equal to his initial height \( y_0 \) plus his vertical displacement \( y \) above the end of the ramp: \( y_1 = y_0 + y \). Next we will use \( v_y = v_{0y} + a_y t \) (Equation 3.3b) to calculate the time \( t \) from the launch to the highest point, which, with \( x = v_{0x} t + \frac{1}{2} a_x t^2 \) (Equation 3.5a), will give us his horizontal displacement \( x \).

**SOLUTION**

a. Substituting \( v_y = 0 \text{ m/s} \) into Equation 3.6b and solving for \( y \), we find

\[ (0 \text{ m/s})^2 = v_{0y}^2 + 2a_y y \quad \text{or} \quad y = \frac{-v_{0y}^2}{2a_y} = \frac{-(5.6 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +1.6 \text{ m} \]

Therefore, the highest point \( y_1 \) reached by the skateboarder occurs when \( y_1 = y_0 + y = 1.2 \text{ m} + 1.6 \text{ m} = +2.8 \text{ m} \) above the ground.
b. We now turn to the horizontal displacement, which requires that we first find the elapsed time $t$. Putting $v_y = 0$ m/s into Equation 3.3b and solving for $t$ yields

$$0 \text{ m/s} = v_{0y} + a_y t \quad \text{or} \quad t = \frac{-v_{0y}}{a_y} \quad (1)$$

Given that $a_x = 0$ m/s$^2$, the relation $x = v_{0x} t + \frac{1}{2} a_x t^2$ (Equation 3.5a) reduces to $x = v_{0x} t$. Substituting Equation (1) for $t$ then gives the skateboarder’s horizontal displacement:

$$x = v_{0x} t = v_{0x} \left( -\frac{v_{0y}}{a_y} \right) = -\frac{v_{0x} v_{0y}}{a_y} = -\frac{(+3.5 \text{ m/s})(+5.6 \text{ m/s})}{-9.80 \text{ m/s}} = +2.0 \text{ m}$$

16. **REASONING** The magnitude $v$ of the puck’s velocity is related to its $x$ and $y$ velocity components ($v_x$ and $v_y$) by the Pythagorean theorem (Equation 1.7); $v = \sqrt{v_x^2 + v_y^2}$. The relation $v_x = v_{0x} + a_x t$ (Equation 3.3a) may be used to find $v_x$, since $v_{0x}$, $a_x$, and $t$ are known. Likewise, the relation $v_y = v_{0y} + a_y t$ (Equation 3.3b) may be employed to determine $v_y$, since $v_{0y}$, $a_y$, and $t$ are known. Once $v_x$ and $v_y$ are determined, the angle $\theta$ that the velocity makes with respect to the $+x$ axis can be found by using the inverse tangent function (Equation 1.6); $\theta = \tan^{-1}\left( \frac{v_y}{v_x} \right)$.

**SOLUTION** Using Equations 3.3a and 3.3b, we find that

$$v_x = v_{0x} + a_x t = +1.0 \text{ m/s} + \left( 2.0 \text{ m/s}^2 \right)(0.50 \text{ s}) = +2.0 \text{ m/s}$$

$$v_y = v_{0y} + a_y t = +2.0 \text{ m/s} + \left( -2.0 \text{ m/s}^2 \right)(0.50 \text{ s}) = +1.0 \text{ m/s}$$

The magnitude $v$ of the puck’s velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.0 \text{ m/s})^2 + (1.0 \text{ m/s})^2} = 2.2 \text{ m/s}$$

The angle $\theta$ that the velocity makes with respect to the $+x$ axis is

$$\theta = \tan^{-1}\left( \frac{v_y}{v_x} \right) = \tan^{-1}\left( \frac{1.0 \text{ m/s}}{2.0 \text{ m/s}} \right) = 27^\circ \text{ above the } +x \text{ axis}$$
17. **REASONING** Since the spider encounters no appreciable air resistance during its leap, it can be treated as a projectile. The thickness of the magazine is equal to the spider’s vertical displacement \( y \) during the leap. The relevant data are as follows (assuming upward to be the \(+y\) direction):

\[
\begin{array}{|c|c|c|c|c|}
\hline
y & a_y & v_y & v_{0y} & t \\
\hline
? & -9.80 \text{ m/s}^2 & (0.870 \text{ m/s}) \sin 35.0^\circ = +0.499 \text{ m/s} & 0.0770 \text{ s} \\
\hline
\end{array}
\]

**SOLUTION** We will calculate the spider’s vertical displacement directly from \( y = v_{0y}t + \frac{1}{2}a_yt^2 \) (Equation 3.5b):

\[
y = (0.499 \text{ m/s})(0.0770 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.0770 \text{ s})^2 = 0.0094 \text{ m}
\]

To express this result in millimeters, we use the fact that 1 m equals 1000 mm:

\[
\text{Thickness} = (0.0094 \text{ m})(\frac{1000 \text{ mm}}{1 \text{ m}}) = 9.4 \text{ mm}
\]

18. **REASONING** The motion of the bullet in the horizontal direction occurs at a constant velocity \( v_{0x} = 670 \text{ m/s} \), so that we can use \( x = v_{0x}t \) to determine the displacement \( x \) between the end of the rifle and the bull’s-eye. The magnitude of \( x \) is the distance we seek. However, we also need to know the time of flight \( t \) in order to determine \( x \). We will determine \( t \) from the bullet’s displacement \( y \) in the vertical direction by using \( y = v_{0y}t + \frac{1}{2}a_yt^2 \) (Equation 3.5b). We choose this equation because we know that \( y = -0.025 \text{ m} \) (assuming that upward is the \(+y\) direction), \( v_{0y} = 0.00 \text{ m/s} \) (the bullet initially travels horizontally) and \( a_y = -9.80 \text{ m/s}^2 \). Thus, we know every variable in the equation except \( t \).

**SOLUTION** The displacement \( x \) between the end of the rifle and the bull’s-eye is

\[
x = v_{0x}t
\]

Referring to Equation 3.5b for the vertical motion of the bullet and the fact that \( v_{0y} = 0.00 \text{ m/s} \), we have

\[
y = v_{0y}t + \frac{1}{2}a_yt^2 = (0.00 \text{ m/s})t + \frac{1}{2}a_yt^2 = \frac{1}{2}a_yt^2 \quad \text{or} \quad t = \sqrt{\frac{2y}{a_y}}
\]

Substituting this result into the equation for \( x \) gives
Thus, the distance between the end of the rifle and the bull's-eye is \(48 \text{ m}\).

19. **REASONING**

a. The maximum possible distance that the ball can travel occurs when it is launched at an angle of 45.0°. When the ball lands on the green, it is at the same elevation as the tee, so the vertical component (or \(y\) component) of the ball's displacement is zero. The time of flight is given by the \(y\) variables, which are listed in the table below. We designate "up" as the +\(y\) direction.

<table>
<thead>
<tr>
<th>(y)</th>
<th>(a_y)</th>
<th>(v_y)</th>
<th>(v_{0y})</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>-9.80 m/s²</td>
<td>+30.3 m/s sin 45.0° = +21.4 m/s</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Since three of the five kinematic variables are known, we can employ one of the equations of kinematics to find the time \(t\) that the ball is in the air.

b. The longest hole in one that the golfer can make is equal to the range \(R\) of the ball. This distance is given by the \(x\) variables and the time of flight, as determined in part (a). Once again, three variables are known, so an equation of kinematics can be used to find the range of the ball. The +\(x\) direction is taken to be from the tee to the green.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(a_x)</th>
<th>(v_x)</th>
<th>(v_{0x})</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R = ?)</td>
<td>0 m/s²</td>
<td>+30.3 m/s cos 45.0° = +21.4 m/s</td>
<td>from part a</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION**

a. We will use Equation 3.5b to find the time, since this equation involves the three known variables in the \(y\) direction:

\[
y = v_{0y}t + \frac{1}{2}a_y t^2 = \left(v_{0y} + \frac{1}{2} a_y t\right) t
\]

\[
0 \text{ m} = \left[+21.4 \text{ m/s} + \frac{1}{2}(-9.80 \text{ m/s}^2) t\right] t
\]

Solving this quadratic equation yields two solutions, \(t = 0 \text{ s}\) and \(t = 4.37 \text{ s}\). The first solution represents the situation when the golf ball just begins its flight, so we discard this one. Therefore, \(t = 4.37 \text{ s}\).
Chapter 3  Problems

b. With the knowledge that \( t = 4.37 \) s and the values for \( a_x \) and \( v_{0x} \) (see the \( x \)-direction data table above), we can use Equation 3.5a to obtain the range \( R \) of the golf ball.

\[
x = v_{0x}t + \frac{1}{2}a_xt^2 = (+21.4 \text{ m/s})(4.37 \text{ s}) + \frac{1}{2}(0 \text{ m/s}^2)(4.37 \text{ s})^2 = 93.5 \text{ m}
\]

20. **REASONING** The magnitude (or speed) \( v \) of the ball’s velocity is related to its \( x \) and \( y \) velocity components (\( v_x \) and \( v_y \)) by the Pythagorean theorem: \( v = \sqrt{v_x^2 + v_y^2} \) (Equation 1.7).

The horizontal component \( v_x \) of the ball’s velocity never changes during the flight, since, in the absence of air resistance, there is no acceleration in the \( x \) direction (\( a_x = 0 \text{ m/s}^2 \)). Thus, \( v_x \) is equal to the horizontal component \( v_{0x} \) of the initial velocity, or \( v_x = v_{0x} = v_0 \cos 40.0^\circ \). Since \( v_0 \) is known, \( v_x \) can be determined.

The vertical component \( v_y \) of the ball’s velocity does change during the flight, since the acceleration in the \( y \) direction is that due to gravity (\( a_y = -9.80 \text{ m/s}^2 \)). The relation \( v_y^2 = v_{0y}^2 + 2a_yy \) (Equation 2.6b) may be used to find \( v_y^2 \), since \( a_y \), \( y \), and \( v_{0y} \) are known (\( v_{0y} = v_0 \sin 40.0^\circ \)).

**SOLUTION** The speed \( v \) of the golf ball just before it lands is

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(v_0 \cos 40.0^\circ)^2 + v_{0y}^2 + 2a_yy} = \sqrt{(14.0 \text{ m/s} \cos 40.0^\circ)^2 + (14.0 \text{ m/s} \sin 40.0^\circ)^2 + 2(-9.80 \text{ m/s}^2)(3.00 \text{ m})} = 11.7 \text{ m/s}
\]

21. **SSM REASONING** When the skier leaves the ramp, she exhibits projectile motion. Since we know the maximum height attained by the skier, we can find her launch speed \( v_0 \) using Equation 3.6b, \( v_y^2 = v_{0y}^2 + 2a_yy \), where \( v_{0y} = v_0 \sin 63^\circ \).
**SOLUTION** At the highest point in her trajectory, \( v_y = 0 \). Solving Equation 3.6b for \( v_0_y \) we obtain, taking upward as the positive direction,

\[
v_{0y} = v_y \sin 63^\circ = \sqrt{-2a_y y} \quad \text{or} \quad v_0 = \frac{-2a_y y}{\sin 63^\circ} = \frac{\sqrt{-2(-9.80 \text{ m/s}^2)(13 \text{ m})}}{\sin 63^\circ} = 18 \text{ m/s}
\]

22. **REASONING** The vehicle’s initial velocity is in the +\( y \) direction, and it accelerates only in the +\( x \) direction. Therefore, the \( y \) component of its velocity remains constant at \( v_y = +21.0 \text{ m/s} \). Initially, the \( x \) component of the vehicle’s velocity is zero \( (v_{0x} = 0 \text{ m/s}) \), but it increases at a rate of \( a_x = +0.320 \text{ m/s}^2 \), reaching a final value \( v_x \) given by the relation \( v_x = v_{0x} + a_x t \) (Equation 3.3a) when the pilot shuts off the RCS thruster. Once we have found \( v_x \), we will use the right triangle shown in the drawing to find the magnitude \( v \) and direction \( \theta \) of the vehicle’s final velocity.

**SOLUTION**

a. During the 45.0-second thruster burn, the \( x \) component of the vehicle’s velocity increases from zero to

\[
v_x = v_{0x} + a_x t = (0 \text{ m/s}) + (0.320 \text{ m/s}^2)(45.0 \text{ s}) = 14.4 \text{ m/s}
\]

Applying the Pythagorean theorem to the right triangle in the drawing, we find the magnitude of the vehicle’s velocity to be:

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(14.4 \text{ m/s})^2 + (21.0 \text{ m/s})^2} = 25.5 \text{ m/s}
\]

b. Referring again to the drawing, we see that the \( x \) and \( y \) components of the vehicle’s velocity are related by the tangent of the angle \( \theta \). Therefore, the angle of the vehicle’s final velocity is

\[
\theta = \tan^{-1} \left( \frac{v_x}{v_y} \right) = \tan^{-1} \left( \frac{14.4 \text{ m/s}}{21.0 \text{ m/s}} \right) = 34.4^\circ
\]

23. **SSM REASONING** Since the magnitude of the velocity of the fuel tank is given by

\[
v = \sqrt{v_x^2 + v_y^2}
\]

it is necessary to know the velocity components \( v_x \) and \( v_y \) just before impact. At the instant of release, the empty fuel tank has the same velocity as that of the plane. Therefore, the magnitudes of the initial velocity components of the fuel tank are given
by \( v_{0x} = v_0 \cos \theta \) and \( v_{0y} = v_0 \sin \theta \), where \( v_0 \) is the speed of the plane at the instant of release. Since the \( x \) motion has zero acceleration, the \( x \) component of the velocity of the plane remains equal to \( v_{0x} \) for all later times while the tank is airborne. The \( y \) component of the velocity of the tank after it has undergone a vertical displacement \( y \) is given by Equation 3.6b.

**SOLUTION**

a. Taking up as the positive direction, the velocity components of the fuel tank just before it hits the ground are

\[
v_x = v_{0x} = v \cos \theta = (135 \text{ m/s}) \cos 15^\circ = 1.30 \times 10^2 \text{ m/s}
\]

From Equation 3.6b, we have

\[
v_y = -\sqrt{v_{0y}^2 + 2a_y y} = -\sqrt{(v_0 \sin \theta)^2 + 2a_y y} = -\sqrt{(135 \text{ m/s}) \sin 15.0^\circ \cdot 2(-9.80 \text{ m/s}^2)(-2.00 \times 10^3 \text{ m})} = -201 \text{ m/s}
\]

Therefore, the magnitude of the velocity of the fuel tank just before impact is

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.30 \times 10^2 \text{ m/s})^2 + (201 \text{ m/s})^2} = 239 \text{ m/s}
\]

The velocity vector just before impact is inclined at an angle \( \phi \) with respect to the horizontal. This angle is

\[
\phi = \tan^{-1} \left( \frac{201 \text{ m/s}}{1.30 \times 10^2 \text{ m/s}} \right) = 57.1^\circ
\]

b. As shown in Conceptual Example 10, once the fuel tank in part a rises and falls to the same altitude at which it was released, its motion is identical to the fuel tank in part b. Therefore, the velocity of the fuel tank in part b just before impact is 239 m/s at an angle of 57.1° with respect to the horizontal.

---

24. **REASONING** In the absence of air resistance, the horizontal velocity component never changes from its initial value of \( v_{0x} \). Therefore the horizontal distance \( D \) traveled by the criminal (which must equal or exceed the distance between the two buildings) is the initial velocity times the travel time \( t \), or \( D = v_{0x} t \).

The time \( t \) can be found by noting that the motion of the criminal between the buildings is that of a projectile whose acceleration in the \( y \) direction is that due to gravity \( (a_y = -9.80 \text{ m/s}^2, \text{ assuming downward to be the negative direction}) \). The relation
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\[ y = v_{0y}t + \frac{1}{2}a_y t^2 \] (Equation 3.5b) allows us to determine the time, since \( y \), \( v_{0y} \), and \( a_y \) are known. Since the criminal is initially running in the horizontal direction, \( v_{0y} = 0 \) m/s. Setting \( v_{0y} = 0 \) m/s and solving the equation above for \( t \) yields \( t = \sqrt{\frac{2y}{a_y}} \). In this result, \( y = -2.0 \) m, since downward is the negative direction.

**SOLUTION** The horizontal distance traveled after launch is \( D = v_{0x}t \). Substituting \( t = \sqrt{\frac{2y}{a_y}} \) into this relation gives

\[
D = v_{0x}t = v_{0x} \sqrt{\frac{2y}{a_y}} = (5.3 \text{ m/s}) \sqrt{\frac{2(-2.0 \text{ m})}{-9.8 \text{ m/s}^2}} = 3.4 \text{ m}
\]

25. **REASONING** The data given in the problem are summarized as follows:

<table>
<thead>
<tr>
<th>x-Direction Data</th>
<th>y-Direction Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( a_x )</td>
</tr>
<tr>
<td>4.11 \times 10^6 \text{ m}</td>
<td>?</td>
</tr>
</tbody>
</table>

With these data, we can use Equations 3.5a and 3.5b to determine the acceleration component \( a_x \) and \( a_y \).

**SOLUTION** Using Equations 3.5a and 3.5b, we can solve for the acceleration components and find that

\[
x = v_{0x}t + \frac{1}{2}a_x t^2 \quad \text{or} \quad a_x = \frac{2x - 2v_{0x}t}{t^2} \quad (3.5a)
\]

\[
y = v_{0y}t + \frac{1}{2}a_y t^2 \quad \text{or} \quad a_y = \frac{2y - 2v_{0y}t}{t^2} \quad (3.5b)
\]

\[
a_x = \frac{2(4.11 \times 10^6 \text{ m}) - 2(4370 \text{ m/s})(684 \text{ s})}{(684 \text{ s})^2} = 4.79 \text{ m/s}^2
\]

\[
a_y = \frac{2(6.07 \times 10^6 \text{ m}) - 2(6280 \text{ m/s})(684 \text{ s})}{(684 \text{ s})^2} = 7.59 \text{ m/s}^2
\]

26. **REASONING** In the absence of air resistance, there is no acceleration in the \( x \) direction \( (a_x = 0 \text{ m/s}^2) \), so the range \( R \) of a projectile is given by \( R = v_{0x}t \), where \( v_{0x} \) is the horizontal component of the launch velocity and \( t \) is the time of flight. We will show that \( v_{0x} \) and \( t \) are each proportional to the initial speed \( v_0 \) of the projectile, so the range is proportional to \( v_0^2 \).
Since $v_{0x} = v_0 \cos \theta$, where $\theta$ is the launch angle, we see that $v_{0x}$ is proportional to $v_0$. To show that the time of flight $t$ is also proportional to the launch speed $v_0$, we use the fact that, for a projectile that is launched from and returns to ground level, the vertical displacement is $y = 0$ m. Using the relation $y = v_{0y}t + \frac{1}{2}a_yt^2$ (Equation 3.5b), we have

$$0 = v_{0y}t + \frac{1}{2}a_yt^2 \quad \text{or} \quad 0 = v_{0y}t + \frac{1}{2}a_yt \quad \text{or} \quad t = \frac{-2v_{0y}}{a_y}$$

Thus, the flight time is proportional to the vertical component of the launch velocity $v_{0y}$, which, in turn, is proportional to the launch speed $v_0$ since $v_{0y} = v_0 \sin \theta$.

**SOLUTION** The range of a projectile is given by $R = v_{0x}t$ and, since both $v_{0x}$ and $t$ are proportional to $v_0$, the range is proportional to $v_0^2$. The given range is 23 m. When the launch speed doubles, the range increases by a factor of $2^2 = 4$, since the range is proportional to the square of the speed. Thus, the new range is

$$R = 4(23 \text{ m}) = 92 \text{ m}$$

**SSM REASONING AND SOLUTION** The water exhibits projectile motion. The $x$ component of the motion has zero acceleration while the $y$ component is subject to the acceleration due to gravity. In order to reach the highest possible fire, the displacement of the hose from the building is $x$, where, according to Equation 3.5a (with $a_x = 0$ m/s$^2$),

$$x = v_{0x}t = (v_0 \cos \theta)t$$

with $t$ equal to the time required for the water the reach its maximum vertical displacement. The time $t$ can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_yt$$

When the water has reached its maximum vertical displacement, $v_y = 0$ m/s. Taking up and to the right as the positive directions, we find that

$$t = \frac{-v_{0y}}{a_y} = \frac{-v_0 \sin \theta}{a_y}$$

and

$$x = (v_0 \cos \theta)\left(\frac{-v_0 \sin \theta}{a_y}\right)$$
Therefore, we have
\[ x = \frac{-v_0^2 \cos \theta \sin \theta}{a_y} = \frac{- (25.0 \text{ m/s})^2 \cos 35.0^\circ \sin 35.0^\circ}{-9.80 \text{ m/s}^2} = 30.0 \text{ m} \]

**28. REASONING** We will treat the horizontal and vertical parts of the motion separately. The directions upward and to the right are chosen as the positive directions in the drawing. Ignoring air resistance, we note that there is no acceleration in the horizontal direction. Thus, the horizontal component \(v_{0x}\) of the ball’s initial velocity remains unchanged throughout the motion, and the horizontal component \(x\) of the displacement is simply \(v_{0x}\) times the time \(t\) during which the motion occurs. This is true for both bunted balls. Since the value of \(x\) is the same for both, we have that
\[ x_A = x_B \quad \text{or} \quad v_{0x,A}t_A = v_{0x,B}t_B \quad \text{or} \quad v_{0x,A} = v_{0x,B} \left( \frac{t_A}{t_B} \right) \quad (1) \]

To use this result to calculate \(v_{0x,B}\), it is necessary to determine the times \(t_A\) and \(t_B\). We can accomplish this by applying the equations of kinematics to the vertical part of the motion for each ball. The data for the vertical motion are summarized as follows:

<table>
<thead>
<tr>
<th>(y)-Direction Data, Ball A</th>
<th>(y)-Direction Data, Ball B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_A)</td>
<td>(a_{y,A})</td>
</tr>
<tr>
<td>-1.2 m</td>
<td>-9.80 m/s²</td>
</tr>
</tbody>
</table>

Note that the initial velocity components \(v_{0y}\) are zero, because the balls are bunted horizontally. With these data, Equation 3.5b gives
\[ y = v_{0y}t + \frac{1}{2}a_yt^2 = (0 \text{ m/s})t + \frac{1}{2}a_yt^2 \quad \text{or} \quad t = \frac{\sqrt{2y}}{a_y} \quad (2) \]

**SOLUTION** Using Equation (2) for each ball and substituting the expressions for \(t_A\) and \(t_B\) into Equation (1) gives
\[ v_{0x,B} = v_{0x,A} \left( \frac{t_A}{t_B} \right) = v_{0x,A} \frac{\sqrt{2y_A/a_{y,A}}}{\sqrt{2y_B/a_{y,B}}} \]
\[ = v_{0x,A} \sqrt{\frac{y_A}{y_B}} = (1.9 \text{ m/s}) \sqrt{\frac{1.2 \text{ m}}{1.5 \text{ m}}} = 1.7 \text{ m/s} \]
Note that the accelerations $a_{yA}$ and $a_{yB}$ both equal the acceleration due to gravity and are eliminated algebraically from the calculation.

29. **REASONING** The vertical displacement $y$ of the ball depends on the time that it is in the air before being caught. These variables depend on the $y$-direction data, as indicated in the table, where the $+y$ direction is "up."

<table>
<thead>
<tr>
<th>$y$</th>
<th>$a_y$</th>
<th>$v_y$</th>
<th>$v_{0y}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$-9.80 \text{ m/s}^2$</td>
<td>0 m/s</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Since only two variables in the $y$ direction are known, we cannot determine $y$ at this point. Therefore, we examine the data in the $x$ direction, where $+x$ is taken to be the direction from the pitcher to the catcher.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$v_{0x}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+17.0 \text{ m}$</td>
<td>0 m/s$^2$</td>
<td>+41.0 m/s</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Since this table contains three known variables, the time $t$ can be evaluated by using an equation of kinematics. Once the time is known, it can then be used with the $y$-direction data, along with the appropriate equation of kinematics, to find the vertical displacement $y$.

**SOLUTION** Using the $x$-direction data, Equation 3.5a can be employed to find the time $t$ that the baseball is in the air:

\[ x = v_{0x} t + \frac{1}{2} a_x t^2 = v_{0x} t \quad \text{(since } a_x = 0 \text{ m/s}^2 \text{)} \]

Solving for $t$ gives

\[ t = \frac{x}{v_{0x}} = \frac{+17.0 \text{ m}}{+41.0 \text{ m/s}} = 0.415 \text{ s} \]

The displacement in the $y$ direction can now be evaluated by using the $y$-direction data table and the value of $t = 0.415 \text{ s}$. Using Equation 3.5b, we have

\[ y = v_{0y} t + \frac{1}{2} a_y t^2 = (0 \text{ m/s})(0.415 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.415 \text{ s})^2 = -0.844 \text{ m} \]

The distance that the ball drops is given by the magnitude of this result, so Distance = $0.844 \text{ m}$. 
30. **REASONING** The data for the problem are summarized below. In the tables, we use the symbol $v_0$ to denote the speed with which the ball is thrown and choose upward and to the right as the positive directions.

### $x$-Direction Data

<table>
<thead>
<tr>
<th>$x$</th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$v_{0x}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>183 m</td>
<td>0 m/s$^2$</td>
<td>$v_0 \cos 30.0^\circ$</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Note that $a_x = 0$ m/s$^2$, because air resistance is being ignored. Also note that $y = 0$ m, because the football rises and then returns to the same vertical level from which it was launched. Finally, we have used trigonometry to express the components $v_{0x}$ and $v_{0y}$ of the initial velocity in terms of the speed $v_0$ and the 30.0° launch angle. The key here is to remember that the horizontal and vertical parts of the motion can be treated separately, the time for the motion being the same for each. Since air resistance is being ignored, we can apply the equations of kinematics separately to the motions in the $x$ and $y$ directions.

### $y$-Direction Data

<table>
<thead>
<tr>
<th>$y$</th>
<th>$a_y$</th>
<th>$v_y$</th>
<th>$v_{0y}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>$-9.80$ m/s$^2$</td>
<td>$v_0 \sin 30.0^\circ$</td>
<td>Same as for $x$ direction</td>
<td></td>
</tr>
</tbody>
</table>

SOLUTION Since there is no acceleration in the horizontal direction, motion in that direction is constant-velocity motion, and the horizontal displacement $x$ is simply the initial velocity component $v_{0x}$ times the time $t$:

$$x = v_{0x} t = (v_0 \cos 30.0^\circ) t$$

An expression for $t$ can be obtained by considering the motion in the vertical direction. Thus, we use Equation 3.5b from the equations of kinematics and recognize that the displacement $y$ is zero and $v_{0y} = v_0 \sin 30.0^\circ$:

$$y = v_{0y} t + \frac{1}{2} a_y t^2 \quad \text{or} \quad 0 = (v_0 \sin 30.0^\circ) t + \frac{1}{2} a_y t^2 \quad \text{or} \quad t = \frac{-2v_0 \sin 30.0^\circ}{a_y}$$

Substituting this result for the time into the expression for $x$ gives
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\[ x = (v_0 \cos 30.0^\circ)t = (v_0 \cos 30.0^\circ)\left(-\frac{2v_0 \sin 30.0^\circ}{a_y}\right) = -\frac{2v_0^2 \cos 30.0^\circ \sin 30.0^\circ}{a_y} \]

\[ v_0 = \sqrt{\frac{-x a_y}{2 \cos 30.0^\circ \sin 30.0^\circ}} = \sqrt{\frac{-(183 \text{ m})(-9.80 \text{ m/s}^2)}{2 \cos 30.0^\circ \sin 30.0^\circ}} = 45.5 \text{ m/s} \]

31. **SSM REASONING** The speed of the fish at any time \( t \) is given by \( v = \sqrt{v_x^2 + v_y^2} \), where \( v_x \) and \( v_y \) are the \( x \) and \( y \) components of the velocity at that instant. Since the horizontal motion of the fish has zero acceleration, \( v_x = v_{0x} \) for all times \( t \). Since the fish is dropped by the eagle, \( v_{0x} \) is equal to the horizontal speed of the eagle and \( v_{0y} = 0 \). The \( y \) component of the velocity of the fish for any time \( t \) is given by Equation 3.3b with \( v_{0y} = 0 \). Thus, the speed at any time \( t \) is given by \( v = \sqrt{v_{0x}^2 + (a_y t)^2} \).

**SOLUTION**

a. The initial speed of the fish is \( v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{v_{0x}^2 + 0^2} = v_{0x} \). When the fish's speed doubles, \( v = 2v_{0x} \). Therefore,

\[ 2v_{0x} = \left(v_{0x} + (a_y t)^2\right)^{1/2} \quad \text{or} \quad 4v_{0x}^2 = v_{0x}^2 + (a_y t)^2 \]

Assuming that downward is positive and solving for \( t \), we have

\[ t = \sqrt{\frac{3v_{0x}}{a_y}} = \sqrt{\frac{3 \left(6.0 \text{ m/s}\right)}{9.80 \text{ m/s}^2}} = 1.1 \text{ s} \]

b. When the fish's speed doubles again, \( v = 4v_{0x} \). Therefore,

\[ 4v_{0x} = \left(v_{0x} + (a_y t)^2\right)^{1/2} \quad \text{or} \quad 16v_{0x}^2 = v_{0x}^2 + (a_y t)^2 \]

Solving for \( t \), we have

\[ t = \sqrt{15} \frac{v_{0x}}{a_y} = \sqrt{15} \left(\frac{6.0 \text{ m/s}}{9.80 \text{ m/s}^2}\right) = 2.37 \text{ s} \]

Therefore, the additional time for the speed to double again is \((2.4 \text{ s}) - (1.1 \text{ s}) = 1.3 \text{ s}\).
32. **REASONING** Since we know the launch angle $\theta = 15.0^\circ$, the launch speed $v_0$ can be obtained using trigonometry, which gives the $y$ component of the launch velocity as $v_{0y} = v_0 \sin \theta$. Solving this equation for $v_0$ requires a value for $v_{0y}$, which we can obtain from the vertical height of $y = 13.5$ m by using Equation 3.6b from the equations of kinematics.

**SOLUTION** From Equation 3.6b we have

$$v_y^2 = v_{0y}^2 + 2a_y y$$

$$(0 \text{ m/s})^2 = v_{0y}^2 + 2(-9.80 \text{ m/s}^2)(13.5 \text{ m}) \quad \text{or} \quad v_{0y} = \sqrt{2(9.80 \text{ m/s}^2)(13.5 \text{ m})}$$

Using trigonometry, we find

$$v_0 = \frac{v_{0y}}{\sin 15.0^\circ} = \frac{\sqrt{2(9.80 \text{ m/s}^2)(13.5 \text{ m})}}{\sin 15.0^\circ} = 62.8 \text{ m/s}$$

33. **REASONING** The drawing at the right shows the velocity vector $\mathbf{v}$ of the water at a point below the top of the falls. The components of the velocity are also shown. The angle $\theta$ is given by $\tan \theta = v_y / v_x$, so that

$$v_y = -v_x \tan \theta = -(2.7 \text{ m/s}) \tan 75^\circ$$

Here we have used the fact that the horizontal velocity component $v_x$ remains unchanged at its initial value of 2.7 m/s as the water falls. Knowing the $y$ component of the velocity, we can use Equation 3.6b, ($v_y^2 = v_{0y}^2 + 2a_y y$) to find the vertical distance $y$.

**SOLUTION** Taking $v_{0y} = 0 \text{ m/s}$, and taking upward as the positive direction, we have from Equation 3.5b that

$$y = \frac{v_y^2}{2a_y} = \frac{-(2.7 \text{ m/s}) \tan 75^\circ}{2(-9.80 \text{ m/s}^2)} = -5.2 \text{ m}$$

Therefore, the velocity vector of the water points downward at a 75° angle below the horizontal at a vertical distance of $[5.2 \text{ m}]$ below the edge.

34. **REASONING** We begin by considering the flight time of the ball on the distant planet. Once the flight time is known, we can determine the maximum height and the range of the ball.
The range of a projectile is proportional to the time that the projectile is in the air. Therefore, the flight time on the distant planet 3.5 times larger than on earth. The flight time can be found from Equation 3.3b ($v_y = v_{0y} + a_y t$). When the ball lands, it is at the same level as the tee; therefore, from the symmetry of the motion $v_y = -v_{0y}$. Taking upward and to the right as the positive directions, we find that the flight time on earth would be

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin \theta}{a_y} = \frac{-2(45 \text{ m/s}) \sin 29^\circ}{-9.80 \text{ m/s}^2} = 4.45 \text{ s}$$

Therefore, the flight time on the distant planet is $3.5 \times (4.45 \text{ s}) = 15.6 \text{ s}$. From the symmetry of the problem, we know that this is twice the amount of time required for the ball to reach its maximum height, which, consequently, is $7.80 \text{ s}$.

**SOLUTION**

a. The height $y$ of the ball at any instant is given by Equation 3.4b as the product of the average velocity component in the $y$ direction $\frac{1}{2} (v_{0y} + v_y)$ and the time $t$: $y = \frac{1}{2} (v_{0y} + v_y) t$.

Since the maximum height $H$ is reached when the final velocity component in the $y$ direction is zero ($v_y = 0 \text{ m/s}$), we find that

$$H = \frac{1}{2} v_{0y} t = \frac{1}{2} v_0 \sin 29^\circ t = \frac{1}{2} (45 \text{ m/s}) \sin 29^\circ (7.80 \text{ s}) = 85 \text{ m}$$

b. The range of the ball on the distant planet is

$$x = v_{0x} t = v_0 \cos 29^\circ t = (45 \text{ m/s}) \cos 29^\circ (15.6 \text{ s}) = 610 \text{ m}$$

**35. REASONING** The rocket will clear the top of the wall by an amount that is the height of the rocket as it passes over the wall minus the height of the wall. To find the height of the rocket as it passes over the wall, we separate the rocket’s projectile motion into its horizontal and vertical parts and treat each one separately. From the horizontal part we will obtain the time of flight until the rocket reaches the location of the wall. Then, we will use this time along with the acceleration due to gravity in the equations of kinematics to determine the height of the rocket as it passes over the wall.

**SOLUTION** We begin by finding the horizontal and vertical components of the launch velocity

$$v_{0x} = v_0 \cos 60.0^\circ = (75.0 \text{ m/s}) \cos 60.0^\circ$$

$$v_{0y} = v_0 \sin 60.0^\circ = (75.0 \text{ m/s}) \sin 60.0^\circ$$

Using $v_{0x}$, we can obtain the time of flight, since the distance to the wall is known to be $27.0 \text{ m}$:
The height of the rocket as it clears the wall can be obtained from Equation 3.5b, in which we take upward to be the positive direction. The amount by which the rocket clears the wall can then be obtained:

\[ y = v_{0y} t + \frac{1}{2} a_y t^2 \]

\[ y = (75.0 \text{ m/s})(\sin 60.0^\circ)(0.720 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.720 \text{ s})^2 = 44.2 \text{ m} \]

clearance = 44.2 m – 11.0 m = \boxed{33.2 \text{ m}}

36. **REASONING** We will treat the horizontal and vertical parts of the motion separately. The directions upward and to the right are chosen as the positive directions in the drawing. Ignoring air resistance, we can apply the equations of kinematics to the vertical part of the motion. The data for the vertical motion are summarized in the following tables. Note that the initial velocity component \( v_{0y} \) is zero, because the bullets are fired horizontally. The vertical component \( y \) of the bullets’ displacements are entered in the tables with minus signs, because the bullets move downward in the negative \( y \) direction. The times of flight \( t_A \) and \( t_B \) have been identified in the tables as a matter of convenience.

**y-Direction Data, First Shot**

<table>
<thead>
<tr>
<th>( y )</th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -H_A )</td>
<td>(-9.80 \text{ m/s}^2)</td>
<td>0 m/s</td>
<td>( t_A )</td>
<td></td>
</tr>
</tbody>
</table>

**y-Direction Data, Second Shot**

<table>
<thead>
<tr>
<th>( y )</th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -H_B )</td>
<td>(-9.80 \text{ m/s}^2)</td>
<td>0 m/s</td>
<td>( t_B )</td>
<td></td>
</tr>
</tbody>
</table>

With these data, Equation 3.5b gives

\[ y = v_{0y} t + \frac{1}{2} a_y t^2 = (0 \text{ m/s}) t + \frac{1}{2} a_y t^2 = \frac{1}{2} a_y t^2 \quad (1) \]

**SOLUTION** Applying Equation (1) to both shots, we find that

\[ \frac{y_B}{y_A} = \frac{-H_B}{-H_A} = \frac{1}{2} a_y t_B^2 \quad \text{or} \quad \frac{H_B}{H_A} = \frac{t_B^2}{t_A^2} \quad (2) \]

To use this result to calculate the ratio \( H_B/H_A \), it is necessary to determine the times \( t_A \) and \( t_B \). To do this, we consider the horizontal part of the motion and note that there is no acceleration in the horizontal direction. Therefore, the horizontal component \( v_{0x} \) of the bullet’s initial velocity remains unchanged throughout the motion, and the horizontal
component $x$ of the displacement is simply $v_0x$ times the time $t$ during which the motion occurs. We have that

$$x_A = v_0x_t_A \quad \text{and} \quad x_B = v_0x_t_B \quad \text{or} \quad t_A = \frac{x_A}{v_0x} \quad \text{and} \quad t_B = \frac{x_B}{v_0x}$$

Substituting these results into Equation (2) gives

$$\frac{H_B}{H_A} = \frac{t_B^2}{t_A^2} = \left(\frac{x_B}{v_0x} / \frac{x_A}{v_0x}\right)^2 = \left(\frac{x_B}{x_A}\right)^2$$

It is given that $x_B = 2x_A$, so that we find

$$\frac{H_B}{H_A} = \left(\frac{x_B}{x_A}\right)^2 = \left(\frac{2x_A}{x_A}\right)^2 = 4$$

37. **SSM REASONING**

a. The drawing shows the initial velocity $v_0$ of the package when it is released. The initial speed of the package is 97.5 m/s. The component of its displacement along the ground is labeled as $x$. The data for the $x$ direction are indicated in the data table below.

$$v_0 = 97.5 \text{ m/s} \cos 50.0^\circ = +62.7 \text{ m/s}$$

Since only two variables are known, it is not possible to determine $x$ from the data in this table. A value for a third variable is needed. We know that the time of flight $t$ is the same for both the $x$ and $y$ motions, so let’s now look at the data in the $y$ direction.

**$x$-Direction Data**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$v_{0x}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>0 m/s$^2$</td>
<td>+$(97.5 \text{ m/s}) \cos 50.0^\circ = +62.7 \text{ m/s}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the displacement $y$ of the package points from its initial position toward the ground, so its value is negative, i.e., $y = -732 \text{ m}$. The data in this table, along with the
appropriate equation of kinematics, can be used to find the time of flight \( t \). This value for \( t \) can, in turn, be used in conjunction with the \( x \)-direction data to determine \( x \).

b. The drawing at the right shows the velocity of the package just before impact. The angle that the velocity makes with respect to the ground can be found from the inverse tangent function as \( \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \). Once the time has been found in part (a), the values of \( v_y \) and \( v_x \) can be determined from the data in the tables and the appropriate equations of kinematics.

**SOLUTION**

a. To determine the time that the package is in the air, we will use Equation 3.5b \( \left( y = v_{0y}t + \frac{1}{2}a_y t^2 \right) \) and the data in the \( y \)-direction data table. Solving this quadratic equation for the time yields

\[
t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\frac{1}{2}a_y\right)(-y)}}{2\left(\frac{1}{2}a_y\right)}
\]

\[
t = \frac{-\left(74.7 \text{ m/s}\right) \pm \sqrt{\left(74.7 \text{ m/s}\right)^2 - 4\left(\frac{1}{2}\right)(-9.80 \text{ m/s}^2)(732 \text{ m})}}{2\left(\frac{1}{2}\right)(-9.80 \text{ m/s}^2)} = -6.78 \text{ s} \quad \text{and} \quad 22.0 \text{ s}
\]

We discard the first solution, since it is a negative value and, hence, unrealistic. The displacement \( x \) can be found using \( t = 22.0 \text{ s} \), the data in the \( x \)-direction data table, and Equation 3.5a:

\[
x = v_{0x}t + \frac{1}{2}a_xt^2 = (+62.7 \text{ m/s})(22.0 \text{ s}) + \frac{1}{2}(0 \text{ m/s}^2)(22.0 \text{ s})^2 = 1380 \text{ m}
\]

b. The angle \( \theta \) that the velocity of the package makes with respect to the ground is given by \( \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \). Since there is no acceleration in the \( x \) direction \( (a_x = 0 \text{ m/s}^2) \), \( v_x \) is the same as \( v_{0x} \), so that \( v_x = v_{0x} = +62.7 \text{ m/s} \). Equation 3.3b can be employed with the \( y \)-direction data to find \( v_y \):

\[
v_y = v_{0y} + a_y t = 74.7 \text{ m/s} + \left( -9.80 \text{ m/s}^2 \right)(22.0 \text{ s}) = -141 \text{ m/s}
\]

Therefore,

\[
\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{-141 \text{ m/s}}{+62.7 \text{ m/s}} \right) = -66.0^\circ
\]

where the minus sign indicates that the angle is \(-66.0^\circ\) below the horizontal.
38. **REASONING** Since the vertical height is asked for, we will begin with the vertical part of the motion, treating it separately from the horizontal part. The directions upward and to the right are chosen as the positive directions in the drawing. The data for the vertical motion are summarized in the following table. Note that the initial velocity component $v_{0y}$ is zero, because the marble is thrown horizontally. The vertical component $y$ of the marble’s displacement is entered in the table as $-H$, where $H$ is the height we seek. The minus sign is included, because the marble moves downward in the negative $y$ direction. The vertical component $v_y$ of the final velocity is checked as an important variable in the table, because we are given the angle that the final velocity makes with respect to the horizontal. Ignoring air resistance, we apply the equations of kinematics. With the data indicated in the table, Equation 3.6b becomes

$$v_y^2 = v_{0y}^2 + 2a_yy = (0 m/s)^2 + 2a_y(-H) \quad \text{or} \quad H = \frac{-v_y^2}{2a_y} \quad (1)$$

**SOLUTION** To use Equation (1), we need to determine the vertical component $v_y$ of the final velocity. We are given that the final velocity makes an angle of 65° with respect to the horizontal, as the inset in the drawing shows. Thus, from trigonometry, it follows that

$$\tan 65° = \frac{-v_y}{v_x} \quad \text{or} \quad v_y = -v_x \tan 65° \quad \text{or} \quad v_y = -v_{0x} \tan 65°$$

where the minus sign is included, because $v_y$ points downward in the negative $y$ direction. In the absence of air resistance, there is no acceleration in the $x$ direction, and the horizontal component $v_x$ of the final velocity is equal to the initial value $v_{0x}$. Substituting this result into Equation (1) gives

$$H = \frac{-v_y^2}{2a_y} = \frac{-(v_{0x} \tan 65°)^2}{2a_y} = \frac{-[(15 m/s) \tan 65°]^2}{2 \left(-9.80 m/s^2\right)} = 53 \text{ m}$$
39. **REASONING** As discussed in Conceptual Example 5, the horizontal velocity component of the bullet does not change from its initial value and is equal to the horizontal velocity of the car. The same thing is true here for the tomato. In other words, regardless of its vertical position relative to the ground, the tomato always remains above you as you travel in the convertible. From the symmetry of free fall motion, we know that when you catch the tomato, its velocity will be 11 m/s straight downward. The time $t$ required to catch the tomato can be found by solving Equation 3.3b \( v_y = v_{oy} + a_y t \) \( v_y = -v_{oy} \). Once $t$ is known, the distance that the car moved can be found from $x = v_x t$.

**SOLUTION** Taking upward as the positive direction, we find the flight time of the tomato to be \[ t = \frac{v_y - v_{oy}}{a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2(11 \text{ m/s})}{-9.80 \text{ m/s}^2} = 2.24 \text{ s} \]

Thus, the car moves through a distance of \[ x = v_x t = (25 \text{ m/s })(2.24 \text{ s}) = 56 \text{ m} \]

40. **REASONING** The initial velocity of the diver has a magnitude $v_0$ and direction $\theta_0$ (the angle that the diver makes with respect to the horizontal). These two variables are shown in the drawing.

If we can determine the $x$ and $y$ components ($v_{ox}$ and $v_{oy}$) of her initial velocity, then the magnitude of her initial velocity can be obtained from the Pythagorean theorem \( v_0 = \sqrt{v_{ox}^2 + v_{oy}^2} \), and the angle $\theta_0$ can be found by using the inverse tangent function \[ \theta_0 = \tan^{-1}\left(\frac{v_{oy}}{v_{ox}}\right) \].

**SOLUTION** The drawing at the right also shows the velocity of the diver at the instant she contacts the water. Her speed is given as $v = 8.90 \text{ m/s}$ and her body is extended at an angle of $\theta = 75.0^\circ$ with respect to the horizontal surface of the water.

We'll begin by examining the data for the $x$ direction. Since air resistance is being ignored, there is no acceleration in the $x$ direction, so $a_x = 0 \text{ m/s}^2$. Referring to the drawing, we see that the $x$ component of the final velocity is given by
\[ v_x = +v \cos 75.0^\circ = +(8.90 \text{ m/s}) \cos 75.0^\circ = +2.30 \text{ m/s} \]

**x-Direction Data**

<table>
<thead>
<tr>
<th></th>
<th>( a_x )</th>
<th>( v_x )</th>
<th>( v_{0x} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 m/s(^2)</td>
<td>+2.30 m/s</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Since there is no acceleration in the \( x \) direction, the \( x \) component of the diver's velocity does not change. Thus, \( v_{0x} = v_x = +2.30 \text{ m/s} \).

To find a value for \( v_{0y} \) we look at the data for the \( y \) direction. We are told that the vertical displacement \( y \) of the diver is \( y = -3.00 \text{ m} \), where downward is chosen as the negative direction (see the drawing). The diver's acceleration in the \( y \) direction is that due to gravity, so \( a_y = -9.80 \text{ m/s}^2 \). Finally, the \( y \) component of the diver's final velocity (see the drawing showing \( v_y \)) is \( v_y = -v \sin 75.0^\circ = -(8.90 \text{ m/s}) \sin 75.0^\circ = -8.60 \text{ m/s} \)

**y-Direction Data**

<table>
<thead>
<tr>
<th></th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3.00 \text{ m})</td>
<td>(-9.80 \text{ m/s}^2)</td>
<td>(-8.60 \text{ m/s})</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

Since three variables are known, we can use the relation \( v_{y}^2 = v_{0y}^2 + 2a_y y \) (Equation 3.6b) to find \( v_{0y} \):

\[
v_{0y} = +\sqrt{v_y^2 - 2a_y y} = +\sqrt{(-8.60 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(-3.00 \text{ m})} = +3.89 \text{ m/s}
\]

Since we now know values for \( v_{0x} \) (+2.30 m/s) and \( v_{0y} \) (+3.89 m/s), the Pythagorean theorem can be used to determine the initial speed of the diver:

\[
v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(2.30 \text{ m/s})^2 + (3.89 \text{ m/s})^2} = 4.52 \text{ m/s}
\]

The angle \( \theta_0 \) can be found by using the inverse tangent function:

\[
\theta_0 = \tan^{-1}\left( \frac{v_{0y}}{v_{0x}} \right) = \tan^{-1}\left( \frac{3.89 \text{ m/s}}{2.30 \text{ m/s}} \right) = 59.4^\circ
\]
41. **REASONING** The speed \( v \) of the soccer ball just before the goalie catches it is given by

\[
v = \sqrt{v_x^2 + v_y^2},
\]
where \( v_x \) and \( v_y \) are the \( x \) and \( y \) components of the final velocity of the ball.

The data for this problem are (the \( +x \) direction is from the kicker to the goalie, and the \( +y \) direction is the “up” direction):

### \( x \)-Direction Data

<table>
<thead>
<tr>
<th>( x )</th>
<th>( a_x )</th>
<th>( v_x )</th>
<th>( v_{0x} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+16.8 m</td>
<td>0 m/s(^2)</td>
<td>?</td>
<td>+16.0 m/s ( \cos 28.0^\circ = +14.1 ) m/s</td>
<td></td>
</tr>
</tbody>
</table>

### \( y \)-Direction Data

<table>
<thead>
<tr>
<th>( y )</th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9.80 m/s(^2)</td>
<td>?</td>
<td>+16.0 m/s ( \sin 28.0^\circ = +7.51 ) m/s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since there is no acceleration in the \( x \) direction \( (a_x = 0 \) m/s\(^2\)\), \( v_x \) remains the same as \( v_{0x} \), so \( v_x = v_{0x} = +14.1 \) m/s. The time \( t \) that the soccer ball is in the air can be found from the \( x \)-direction data, since three of the variables are known. With this value for the time and the \( y \)-direction data, the \( y \) component of the final velocity can be determined.

### SOLUTION

Since \( a_x = 0 \) m/s\(^2\), the time can be calculated from Equation 3.5a as

\[
t = \frac{x}{v_{0x}} = \frac{+16.8 \text{ m}}{+14.1 \text{ m/s}} = 1.19 \text{ s}.
\]

The value for \( v_y \) can now be found by using Equation 3.3b with this value of the time and the \( y \)-direction data:

\[
v_y = v_{0y} + a_y t = +7.51 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)(1.19 \text{ s}) = -4.15 \text{ m/s}
\]

The speed of the ball just as it reaches the goalie is

\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(+14.1 \text{ m/s})^2 + (-4.15 \text{ m/s})^2} = 14.7 \text{ m/s}
\]

42. **REASONING** As shown in the drawing, the angle that the velocity vector makes with the horizontal is given by

\[
\tan \theta = \frac{v_y}{v_x}
\]

where, from Equation 3.3b,

\[
v_y = v_{0y} + a_y t = v_0 \sin \theta_0 + a_y t
\]
and, from Equation 3.3a (since $a_x = 0 \text{ m/s}^2$),

$$v_x = v_{0x} = v_0 \cos \theta_0$$

Therefore,

$$\tan \theta = \frac{v_y}{v_x} = \frac{v_0 \sin \theta_0 + a_y t}{v_0 \cos \theta_0}$$

**SOLUTION** Solving for $t$, we find

$$t = \frac{v_0 \left( \cos \theta_0 \tan \theta - \sin \theta_0 \right)}{a_y} = \frac{(29 \text{ m/s})(\cos 36^\circ \tan 18^\circ - \sin 36^\circ)}{-9.80 \text{ m/s}^2} = 0.96 \text{ s}$$

43. **SSM REASONING** The angle $\theta$ can be found from

$$\theta = \tan^{-1} \left( \frac{2400 \text{ m}}{x} \right)$$

where $x$ is the horizontal displacement of the flare. Since $a_x = 0 \text{ m/s}^2$, it follows that $x = (v_0 \cos 30.0^\circ) t$. The flight time $t$ is determined by the vertical motion. In particular, the time $t$ can be found from Equation 3.5b. Once the time is known, $x$ can be calculated.

**SOLUTION** From Equation 3.5b, assuming upward is the positive direction, we have

$$y = -(v_0 \sin 30.0^\circ) t + \frac{1}{2} a_y t^2$$

which can be rearranged to give the following equation that is quadratic in $t$:

$$\frac{1}{2} a_y t^2 - (v_0 \sin 30.0^\circ) t - y = 0$$

Using $y = -2400 \text{ m}$ and $a_y = -9.80 \text{ m/s}^2$ and suppressing the units, we obtain the quadratic equation

$$4.9t^2 + 120t - 2400 = 0$$

Using the quadratic formula, we obtain $t = 13 \text{ s}$. Therefore, we find that

$$x = (v_0 \cos 30.0^\circ) t = (240 \text{ m/s})(\cos 30.0^\circ)(13 \text{ s}) = 2700 \text{ m}$$

Equation (1) then gives

$$\theta = \tan^{-1} \left( \frac{2400 \text{ m}}{2700 \text{ m}} \right) = 42^\circ$$
44. REASONING We can obtain an expression for the car’s initial velocity \( v_0 \) by starting with the relation \( x = v_0t + \frac{1}{2}a_xt^2 \) (Equation 3.5a). Here \( x \) is the horizontal component of the car’s displacement, \( v_{0x} \) is the horizontal component of the car’s initial velocity \( (v_{0x} = v_0 \) for a horizontal launch), and \( a_x \) is its acceleration in the horizontal direction \( (a_x = 0 \text{ m/s}^2 \) for projectile motion). Solving for the initial velocity, we find that

\[
x = v_0t + \frac{1}{2}(0 \text{ m/s}^2)t^2 = v_0t \quad \text{or} \quad v_0 = \frac{x}{t}
\]

The time \( t \) in Equation (1) is known, but \( x \) is not. To find \( x \), we note that at \( t = 1.1 \text{ s} \), the car’s displacement has a magnitude of \( \Delta r = 7.0 \text{ m} \). The displacement \( \Delta r \) of the car has a horizontal component \( x \) and a vertical component \( y \). The magnitude of the displacement is related to \( x \) and \( y \) by the Pythagorean theorem:

\[
(\Delta r)^2 = x^2 + y^2
\]

Solving Equation (2) for \( x \) and substituting the result into Equation (1) gives

\[
v_0 = \frac{x}{t} = \frac{\sqrt{(\Delta r)^2 - y^2}}{t}
\]

To determine \( y \), we turn to the relation \( y = v_{0y}t + \frac{1}{2}a_yt^2 \) (Equation 3.5b). Setting \( v_{0y} = 0 \text{ m/s} \) (again, for a horizontal launch), we find that Equation (3) becomes

\[
y = (0 \text{ m/s})t + \frac{1}{2}a_yt^2 = \frac{1}{2}a_yt^2
\]

SOLUTION Substituting Equation (4) into Equation (3) gives

\[
v_0 = \frac{\sqrt{(\Delta r)^2 - y^2}}{t} = \frac{\sqrt{(\Delta r)^2 - \frac{1}{4}a_y^2t^4}}{t}
\]

Thus, the car’s speed just as it drives off the end of the dock is

\[
v_0 = \frac{\sqrt{(7.0 \text{ m})^2 - \frac{1}{4}(-9.80 \text{ m/s}^2)^2(1.1 \text{ s})^4}}{1.1 \text{ s}} = 3.4 \text{ m/s}
\]

45. REASONING The following drawings show the initial and final velocities of the ski jumper and their scalar components. The initial speed of the ski jumper is given by

\[
v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}
\]

and the angle that the initial velocity makes with the horizontal is

\[
\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)
\]

The scalar components \( v_{0x} \) and \( v_{0y} \) can be determined by using the
equations of kinematics and the data in the following tables. (The +x direction is in the
direction of the horizontal displacement of the skier, and the +y direction is “up.”)

![Initial velocity and Final velocity](image)

### x-Direction Data

<table>
<thead>
<tr>
<th>x</th>
<th>(a_x)</th>
<th>(v_x)</th>
<th>(v_{0x})</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+51.0 m</td>
<td>0 m/s(^2)</td>
<td>+(23.0 m/s) (\cos 43.0^\circ) = +16.8 m/s</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

### y-Direction Data

<table>
<thead>
<tr>
<th>y</th>
<th>(a_y)</th>
<th>(v_y)</th>
<th>(v_{0y})</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9.80 m/s(^2)</td>
<td>-(23.0 m/s) (\sin 43.0^\circ) = -15.7 m/s</td>
<td>?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since there is no acceleration in the \(x\) direction \((a_x = 0 \text{ m/s}^2)\), \(v_{0x}\) is the same as \(v_x\), so we have that \(v_{0x} = v_x = +16.8 \text{ m/s}\). The time that the skier is in the air can be found from the \(x\)-direction data, since three of the variables are known. With the value for the time and the \(y\)-direction data, the \(y\) component of the initial velocity can be determined.

**SOLUTION** Since \(a_x = 0 \text{ m/s}^2\), the time can be determined from Equation 3.5a as

\[
t = \frac{x}{v_{0x}} = \frac{+51.0 \text{ m}}{+16.8 \text{ m/s}} = 3.04 \text{ s}.
\]

The value for \(v_{0y}\) can now be found by using Equation 3.3b with this value of the time and the \(y\)-direction data:

\[
v_{0y} = v_y - a_y t = -15.7 \text{ m/s} - (-9.80 \text{ m/s}^2)(3.04 \text{ s}) = +14.1 \text{ m/s}
\]

The speed of the skier when he leaves the end of the ramp is

\[
v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{( +16.8 \text{ m/s})^2 + (+14.1 \text{ m/s})^2} = 21.9 \text{ m/s}
\]

The angle that the initial velocity makes with respect to the horizontal is

\[
\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{+14.1 \text{ m/s}}{+16.8 \text{ m/s}}\right) = 40.0^\circ
\]
46. **REASONING** The height of each building is the magnitude of the vertical displacement $y$ of the stone that is thrown from it. We will determine $y$ from $y = v_{0y}t + \frac{1}{2}a_yt^2$ (Equation 3.5b). We choose this equation because we know that $v_{0y} = 0.00 \text{ m/s}$ (the stones initially travel horizontally) and that $a_y = -9.80 \text{ m/s}^2$ (assuming that upward is the $+y$ direction). We do not know the value of the fall time $t$ for either stone. However, we can relate the fall time to the horizontal displacement $x$ that each stone has when it lands. This is possible because the motion of the stones in the horizontal direction occurs at a constant velocity $v_{0x}$, so that we can use $x = v_{0x}t$ to determine the displacement $x$.

**SOLUTION** From Equation 3.5b we have that

$$y = v_{0y}t + \frac{1}{2}a_yt^2 = (0.00 \text{ m/s})t + \frac{1}{2}a_yt^2 = \frac{1}{2}a_yt^2$$

The motion in the horizontal direction occurs at a constant velocity of $v_{0x}$, so that the horizontal displacement $x$ and the fall time $t$ can be related as follows:

$$x = v_{0x}t \quad \text{or} \quad t = \frac{x}{v_{0x}}$$

Substituting this result for $t$ into the expression for $y$ gives

$$y = \frac{1}{2}a_yt^2 = \frac{1}{2}a_y \left( \frac{x}{v_{0x}} \right)^2 = \frac{a_yx^2}{2v_{0x}^2}$$

This expression for $y$ applies to each stone with different values for $y$ and $x$ but the same values for $a_y$ and $v_{0x}$. Thus, the ratio $y_{\text{taller}}/y_{\text{shorter}}$ is

$$y_{\text{taller}}/y_{\text{shorter}} = \frac{a_yx_{\text{taller}}^2}{2v_{0x}^2} = \frac{x_{\text{taller}}^2}{2v_{0x}^2}$$

We also know that $x_{\text{taller}} = 2x_{\text{shorter}}$, since the stone thrown from the top of the taller building lands twice as far from the base of the building as does the other stone. Therefore, we find that

$$y_{\text{taller}}/y_{\text{shorter}} = \frac{x_{\text{taller}}^2}{x_{\text{shorter}}^2} = \frac{(2x_{\text{shorter}})^2}{x_{\text{shorter}}^2} = 4$$

We see, then, that the ratio of the height of the taller building to the height of the shorter building is $4$. 

____________________________________________________________________________________________
47. **REASONING AND SOLUTION**  In the absence of air resistance, the bullet exhibits projectile motion. The \( x \) component of the motion has zero acceleration while the \( y \) component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the bullet is given by Equation 3.5a (with \( a_x = 0 \text{ m/s}^2 \)):

\[
x = v_{0x}t = (v_0 \cos \theta)t
\]

with \( t \) equal to the time required for the bullet to reach the target. The time \( t \) can be found by considering the vertical motion. From Equation 3.3b,

\[
y = v_{0y} + a_y t
\]

When the bullet reaches the target, \( v_y = -v_{0y} \). Assuming that up and to the right are the positive directions, we have

\[
t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin \theta}{a_y}
\]

and

\[
x = (v_0 \cos \theta) \left( \frac{-2v_0 \sin \theta}{a_y} \right)
\]

Using the fact that \( 2 \sin \theta \cos \theta = \sin 2\theta \), we have

\[
x = -\frac{2v_0^2 \cos \theta \sin \theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y}
\]

Thus, we find that

\[
\sin 2\theta = -\frac{x a_y}{v_0^2} = -\frac{(91.4 \text{ m})(-9.80 \text{ m/s}^2)}{(427 \text{ m/s})^2} = 4.91 \times 10^{-3}
\]

and

\[
2\theta = 0.281^\circ \text{ or } 2\theta = 180.000^\circ - 0.281^\circ = 179.719^\circ
\]

Therefore,

\[
\theta = 0.141^\circ \text{ and } 89.860^\circ
\]

48. **REASONING** We will treat the horizontal and vertical parts of the motion separately. The range \( R \) is the product of the horizontal component of the initial velocity \( v_{0x} \) and the time of flight \( t \). The time of flight can be obtained from the vertical part of the motion by using Equation 3.5b \( \left( y = v_{0y}t + \frac{1}{2}a_y t^2 \right) \) and the fact that the displacement \( y \) in the vertical direction is zero, since the projectile is launched from and returns to ground level. The expression for the range obtained in this way can then be applied to obtain the desired launch angle for doubling the range.
**SOLUTION**  The range of the projectile is

\[ R = v_0 x t = (v_0 \cos \theta) t \]

Using Equation 3.5b, we obtain the time of flight as

\[ y = 0 = v_0 y t + \frac{1}{2} a_y t^2 \quad \text{or} \quad t = -\frac{2v_0 y}{a_y} = -\frac{2v_0 \sin \theta}{a_y} \]

Substituting this expression for \( t \) into the range expression gives

\[ R = (v_0 \cos \theta) \left( -\frac{2v_0 \sin \theta}{a_y} \right) = -\frac{2v_0^2 \cos \theta \sin \theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y} \]

where we have used the fact that \( 2 \cos \theta \sin \theta = \sin 2\theta \). We can now apply this expression for the range to the initial range \( R_1 \) for \( \theta_1 \) and the range \( R_2 \) for \( \theta_2 \):

\[ R_1 = -\frac{v_0^2 \sin 2\theta_1}{a_y} \quad \text{or} \quad R_2 = -\frac{v_0^2 \sin 2\theta_2}{a_y} \]

Dividing these two expressions gives

\[ \frac{R_2}{R_1} = \frac{-\left(\frac{v_0^2 \sin 2\theta_2}{a_y}\right) / a_y}{-\left(\frac{v_0^2 \sin 2\theta_1}{a_y}\right) / a_y} = \frac{\sin 2\theta_2}{\sin 2\theta_1} = 2 \]

where we have used the fact that \( R_2/R_1 = 2 \). Since \( \theta_1 = 12.0^\circ \), we find that

\[ \sin 2\theta_2 = 2.00 \sin 2\theta_1 = 2.00 \sin 2(12.0^\circ) = 0.813 \]

\[ \theta_2 = \frac{\sin^{-1} 0.813}{2.00} = \boxed{27.2^\circ} \]

**49. SSM REASONING**  Since the horizontal motion is not accelerated, we know that the \( x \) component of the velocity remains constant at 340 m/s. Thus, we can use Equation 3.5a (with \( a_x = 0 \text{m/s}^2 \)) to determine the time that the bullet spends in the building before it is embedded in the wall. Since we know the vertical displacement of the bullet after it enters the building, we can use the flight time in the building and Equation 3.5b to find the \( y \) component of the velocity of the bullet as it enters the window. Then, Equation 3.6b can be used (with \( v_{0,y} = 0 \text{m/s} \)) to determine the vertical displacement \( y \) of the bullet as it passes between the buildings. We can determine the distance \( H \) by adding the magnitude of \( y \) to the vertical distance of 0.50 m within the building.
Once we know the vertical displacement of the bullet as it passes between the buildings, we can determine the time $t_1$ required for the bullet to reach the window using Equation 3.4b. Since the motion in the $x$ direction is not accelerated, the distance $D$ can then be found from $D = v_{0x} t_1$.

**SOLUTION** Assuming that the direction to the right is positive, we find that the time that the bullet spends in the building is (according to Equation 3.5a)

$$ t = \frac{x}{v_{0x}} = \frac{6.9 \text{ m}}{340 \text{ m/s}} = 0.0203 \text{ s} $$

The vertical displacement of the bullet after it enters the building is, taking down as the negative direction, equal to $-0.50 \text{ m}$. Therefore, the vertical component of the velocity of the bullet as it passes through the window is, from Equation 3.5b,

$$ v_{0y} = \frac{y - \frac{1}{2} a_y t^2}{t} = \frac{\frac{1}{2} a_y t}{t} = \frac{\frac{1}{2} (-9.80 \text{ m/s}^2)(0.0203 \text{ s})}{0.0203 \text{ s}} = -24.5 \text{ m/s} $$

The vertical displacement of the bullet as it travels between the buildings is (according to Equation 3.6b with $v_{0y} = 0 \text{ m/s}$)

$$ y = \frac{v_y^2}{2a_y} = \frac{(-24.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = -30.6 \text{ m} $$

Therefore, the distance $H$ is

$$ H = 30.6 \text{ m} + 0.50 \text{ m} = 31 \text{ m} $$

The time for the bullet to reach the window, according to Equation 3.4b, is

$$ t_1 = \frac{2y}{v_{0y} + v_y} = \frac{2y}{v_y} = \frac{2(-30.6 \text{ m})}{(-24.5 \text{ m/s})} = 2.50 \text{ s} $$

Hence, the distance $D$ is given by

$$ D = v_{0x} t_1 = (340 \text{ m/s})(2.50 \text{ s}) = 850 \text{ m} $$

50. **REASONING** The angle $\theta$ is the angle that the balloon’s initial velocity $v_0$ makes with the horizontal, and can be found from the horizontal and vertical components of the initial velocity:
\[
\tan \theta = \frac{v_{0y}}{v_{0x}} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) \quad (1)
\]

(Note: because the balloon follows a curved trajectory, \(\theta\) is not related in this fashion to the horizontal and vertical components \(x\) and \(y\) of the balloon’s displacement.) We will use \(x = v_{0x}t + \frac{1}{2}a_xt^2\) (Equation 3.5a) and \(y = v_{0y}t + \frac{1}{2}a_yt^2\) (Equation 3.5b) to find expressions for the horizontal and vertical components \((v_{0x}, v_{0y})\) of the balloon’s initial velocity, and then combine those results with Equation (1) to find \(\theta\). For the initial horizontal velocity component, with \(a_x = 0 \text{ m/s}^2\) since air resistance is being ignored, Equation 3.5a gives

\[
x = v_{0x}t + \frac{1}{2}(0 \text{ m/s}^2)t^2 = v_{0x}t \quad \text{or} \quad v_{0x} = \frac{x}{t} \quad (2)
\]

For the initial vertical velocity component, we obtain from Equation 3.5b that

\[
y = v_{0y}t + \frac{1}{2}a_yt^2 \quad \text{or} \quad v_{0y} = \frac{y - \frac{1}{2}a_yt^2}{t} \quad (3)
\]

In part \(b\), the initial speed \(v_0\) of the second balloon can be found by first noting that it is related to the \(x\) component \(v_{0x}\) of the balloon’s initial velocity by \(v_{0x} = v_0 \cos \theta\). Since there is no acceleration in the \(x\) direction \((a_x = 0 \text{ m/s}^2)\), \(v_{0x}\) is equal to the \(x\) component of the balloon’s displacement \((x = +35.0 \text{ m})\) divided by the time \(t\) that the second balloon is in the air, or \(v_{0x} = x/t\). Thus, the relation \(v_{0x} = v_0 \cos \theta\) can be written as

\[
\frac{x}{t} = v_0 \cos \theta \quad (4)
\]

In Equation (4), \(x = +35.0 \text{ m}\) and \(\theta\) is known from the result of part (a). The time of flight \(t\) is related to the \(y\) component of the balloon’s displacement by \(y = v_{0y}t + \frac{1}{2}a_yt^2\) (Equation 3.5b). Since \(v_{0y} = v_0 \sin \theta\), Equation 3.5b can be expressed as

\[
y = v_{0y}t + \frac{1}{2}a_yt^2 = (v_0 \sin \theta)t + \frac{1}{2}a_yt^2 \quad (5)
\]

Equations (4) and (5) provide everything necessary to find the initial speed \(v_0\) of the water balloon for the second launch.

**SOLUTION**

a. Substituting Equations (2) for \(v_{0x}\) and (3) for \(v_{0y}\) into Equation (1) gives an expression for the balloon’s initial direction \(\theta\):
The balloon leaves the roof of Jackson, 15.0 m above the ground, and hits halfway up Walton, a point \( \frac{1}{2} \) (22.0 m) = 11.0 m above the ground. The balloon’s height at impact is therefore 15.0 m − 11.0 m = 4.0 m below its launch height. Taking up as the positive direction, the vertical displacement of the balloon is therefore \( y = -4.0 \) m, and its horizontal displacement is \( x = +35.0 \) m, the distance between the buildings. Therefore, the angle at which the first balloon is launched is

\[
\theta = \tan^{-1}\left( \frac{\frac{1}{2}a_y t^2}{\frac{1}{2}a_y t^2} \right) = \tan^{-1}\left( \frac{y}{\frac{1}{2}a_y t^2} \right) = \tan^{-1}\left( \frac{y}{x} \right)
\]

The balloon leaves the roof of Jackson, 15.0 m above the ground, and hits halfway up Walton, a point \( \frac{1}{2} \) (22.0 m) = 11.0 m above the ground. The balloon’s height at impact is therefore 15.0 m − 11.0 m = 4.0 m below its launch height. Taking up as the positive direction, the vertical displacement of the balloon is therefore \( y = -4.0 \) m, and its horizontal displacement is \( x = +35.0 \) m, the distance between the buildings. Therefore, the angle at which the first balloon is launched is

\[
\theta = \tan^{-1}\left[ \frac{-4.0 \text{ m} - \frac{1}{2}(9.80 \text{ m/s}^2)(2.0 \text{ s})^2}{35.0 \text{ m}} \right] = 24^\circ
\]

b. The angle \( \theta \) is that found in part (a), and the vertical displacement is the difference in heights between the Walton & Jackson dorms: \( y = 22.0 \text{ m} - 15.0 \text{ m} = +7.0 \text{ m} \). Solving Equation (4) for the elapsed time \( t \) and substituting the result into Equation (5) gives

\[
y = (v_0 \sin \theta) t + \frac{1}{2} a_y t^2 = (v_0 \sin \theta) \left( \frac{x}{v_0 \cos \theta} \right) + \frac{1}{2} a_y \left( \frac{x}{v_0 \cos \theta} \right)^2
\]

Solving this equation for the initial speed \( v_0 \) of the second balloon gives

\[
v_0^2 = \frac{x}{\cos \theta} \sqrt{\frac{1}{2}a_y \left( y - x \sin \theta \right)} = 35.0 \text{ m} \cos 24^\circ \sqrt{\frac{1}{2}(9.80 \text{ m/s}^2) \left( \frac{7.0 \text{ m} - (35.0 \text{ m}) \sin 24^\circ}{\cos 24^\circ} \right)} = 29 \text{ m/s}
\]

51. **REASONING** We can use information about the motion of clown \( A \) and the collision to determine the initial velocity components for clown \( B \). Once the initial velocity components are known, the launch speed \( v_{0B} \) and the launch angle \( \theta_B \) for clown \( B \) can be determined.

**SOLUTION** From Equation 3.5b \( \left( y = v_{0y} + \frac{1}{2}a_y t^2 \right) \) we can find the time of flight until the collision. Taking upward as positive and noting for clown \( A \) that \( v_{0y} = (9.00 \text{ m/s}) \sin 75.0^\circ = 8.693 \text{ m/s} \), we have

\[
1.00 \text{ m} = (8.693 \text{ m/s})t + \frac{1}{2}(9.80 \text{ m/s}^2) t^2
\]

Rearranging this result and suppressing the units gives
The quadratic equation reveals that
\[
t = \frac{8.693 \pm \sqrt{(-8.693)^2 - 4(4.90)(1.00)}}{2(4.90)} = 1.650 \text{ s or} \ 0.1237 \text{ s}
\]

Using these values for \( t \) with the magnitudes \( v_{0xA} \) and \( v_{0xB} \) of the horizontal velocity components for each clown, we note that the horizontal distances traveled by each clown must add up to 6.00 m. Thus,
\[
v_{0xA}t + v_{0xB}t = 6.00 \text{ m} \quad \text{or} \quad v_{0xB} = \frac{6.00 \text{ m}}{t} - v_{0xA}
\]

Using \( v_{0xA} = (9.00 \text{ m/s}) \cos 75.0^\circ = 2.329 \text{ m/s} \), we find
\[
v_{0xB} = \frac{6.00 \text{ m}}{1.650 \text{ s}} - 2.329 \text{ m/s} = 1.307 \text{ m/s} \quad \text{or} \quad v_{0xB} = \frac{6.00 \text{ m}}{0.1237 \text{ s}} - 2.329 \text{ m/s} = 46.175 \text{ m/s}
\]

The vertical component of clown B’s velocity is \( v_{0yB} \) and must be the same as that for clown A, since each clown has the same vertical displacement of 1.00 m at the same time. Hence, \( v_{0yB} = 8.693 \text{ m/s} \) (see above). The launch speed of clown B, finally, is
\[
v_{0B} = \sqrt{v_{0xB}^2 + v_{0yB}^2}
\]

Thus, we find
\[
v_{0B} = \sqrt{(1.307 \text{ m/s})^2 + (8.693 \text{ m/s})^2} = 8.79 \text{ m/s}
\]

or
\[
v_{0B} = \sqrt{(46.175 \text{ m/s})^2 + (8.693 \text{ m/s})^2} = 47.0 \text{ m/s}
\]

For these two possible launch speeds, we find the corresponding launch angles using the following drawings, neither of which is to scale:
\[
\theta = \tan^{-1} \left( \frac{8.693 \, \text{m/s}}{1.307 \, \text{m/s}} \right) = 81.5^\circ \\
\theta = \tan^{-1} \left( \frac{8.693 \, \text{m/s}}{46.175 \, \text{m/s}} \right) = 10.7^\circ 
\]

Since the problem states that \( \theta_B > 45^\circ \), the solution is \( v_{0b} = 8.79 \, \text{m/s} \) and \( \theta_B = 81.5^\circ \).

52. **REASONING** The time it takes for John to pass Chad is the distance between them divided by John’s speed \( v_{JC} \) relative to Chad. We need, then to find John’s velocity relative to Chad.

**SOLUTION** In computing the necessary relative velocity, we define the following symbols:

- \( v_{JC} \) = John’s velocity relative to Chad
- \( v_{JG} \) = John’s velocity relative to the ground
- \( v_{GC} \) = The ground’s velocity relative to Chad
- \( v_{CG} \) = Chad’s velocity relative to the ground

John’s velocity relative to Chad is

\[
v_{JC} = v_{JG} + v_{GC} = v_{JG} - v_{CG} = (4.50 \, \text{m/s, north}) - (4.00 \, \text{m/s, north}) = 0.50 \, \text{m/s, north}
\]

Here we have made use of the fact that \( v_{GC} = -v_{CG} \). The time it takes for John to pass Chad is

\[
t = \frac{95 \, \text{m}}{v_{JC}} = \frac{95 \, \text{m}}{0.50 \, \text{m/s}} = 190 \, \text{s}
\]

53. **SSM REASONING** The velocity \( v_{SG} \) of the swimmer relative to the ground is the vector sum of the velocity \( v_{SW} \) of the swimmer relative to the water and the velocity \( v_{WG} \) of the water relative to the ground as shown at the right:

\[
v_{SG} = v_{SW} + v_{WG}.
\]

The component of \( v_{SG} \) that is parallel to the width of the river determines how fast the swimmer is moving across the river; this parallel component is \( v_{SW} \). The time for the swimmer to cross the river is equal to the width of the river divided by the magnitude of this velocity component.
The component of \( \mathbf{v}_{SG} \) that is parallel to the direction of the current determines how far the swimmer is carried downstream; this component is \( \mathbf{v}_{WG} \). Since the motion occurs with constant velocity, the distance that the swimmer is carried downstream while crossing the river is equal to the magnitude of \( \mathbf{v}_{WG} \) multiplied by the time it takes for the swimmer to cross the river.

**SOLUTION**

a. The time \( t \) for the swimmer to cross the river is

\[
t = \frac{\text{width}}{v_{SW}} = \frac{2.8 \times 10^3 \text{ m}}{1.4 \text{ m/s}} = 2.0 \times 10^3 \text{ s}
\]

b. The distance \( x \) that the swimmer is carried downstream while crossing the river is

\[
x = v_{WG} t = (0.91 \text{ m/s})(2.0 \times 10^3 \text{ s}) = 1.8 \times 10^3 \text{ m}
\]

54. **REASONING** There are three velocities involved:

- \( \mathbf{v}_{NB} \) = velocity of Neil relative to Barbara
- \( \mathbf{v}_{NG} \) = velocity of Neil relative to the Ground (3.2 m/s, due west)
- \( \mathbf{v}_{BG} \) = velocity of Barbara relative to the Ground (4.0 m/s, due south)

Ordering the vectors \( \mathbf{v}_{NB}, \mathbf{v}_{NG} \) and \( \mathbf{v}_{GB} \) by their subscripts in the manner discussed in Section 3.4 of the text, we see that \( \mathbf{v}_{NB} = \mathbf{v}_{NG} + \mathbf{v}_{GB} \). Note that this ordering involves \( \mathbf{v}_{GB} \), the velocity of the ground relative to Barbara. According to the discussion in Section 3.4, \( \mathbf{v}_{GB} \) is related to \( \mathbf{v}_{BG} \) by \( \mathbf{v}_{BG} = -\mathbf{v}_{GB} \). Thus, the three vectors listed above are related by \( \mathbf{v}_{NB} = \mathbf{v}_{NG} + (-\mathbf{v}_{BG}) \). The drawing shows this relationship.

Since the three vectors form a right triangle, the magnitudes of the vectors are related by the Pythagorean theorem, which we will use to obtain Neil’s speed relative to Barbara. Trigonometry can be used to determine the angle \( \theta \).

**SOLUTION** Using the Pythagorean theorem, we find that the speed \( \mathbf{v}_{NB} \) of Neil relative to Barbara is

\[
\mathbf{v}_{NB} = \sqrt{v_{NG}^2 + (-v_{BG})^2} = \sqrt{(3.2 \text{ m/s})^2 + (-4.0 \text{ m/s})^2} = 5.1 \text{ m/s}
\]
The angle $\theta$ can be found from trigonometry:

$$\theta = \cos^{-1} \left( \frac{v_{NG}}{v_{NB}} \right) = \cos^{-1} \left( \frac{3.2 \text{ m/s}}{5.1 \text{ m/s}} \right) = 51^\circ \text{ (north of west)}$$

55. **REASONING** There are three velocities involved:

- $v_{TG}$ = the velocity of the Truck relative to the Ground
- $v_{TP}$ = the velocity of the Truck relative to the Police car
- $v_{PG}$ = the velocity of the Police car relative to the Ground

Of these three vectors, we know the directions of $v_{PG}$ (north) and $v_{TG}$ (west), and the magnitudes of $v_{PG}$ (29 m/s) and $v_{TP}$ (48 m/s). Ordering these vectors by their subscripts in the manner discussed in Section 3.4 of the text, we have $v_{TG} = v_{TP} + v_{PG}$. The triangle formed by this vector sum is a right triangle, because two of the vectors ($v_{TG}$ and $v_{PG}$) are perpendicular to one another. The third vector, $v_{TP}$, must be the triangle’s hypotenuse. The magnitudes of the three vectors are thus related to one another by the Pythagorean theorem, which we will use to obtain the truck’s speed relative to the ground.

**SOLUTION**

a. To represent the vector sum $v_{TG} = v_{TP} + v_{PG}$ graphically, the vectors being added together, $v_{TP}$ and $v_{PG}$, must be drawn tail-to-head (see the drawing). The resultant vector $v_{TG}$ runs from the tail of $v_{TP}$ to the head of $v_{PG}$. The direction of $v_{TP}$ shows that, relative to the police car, the truck is moving both west and south.

b. As the drawing shows, $v_{TP}$ is the hypotenuse of the vector right triangle. Therefore, the Pythagorean theorem gives

$$v_{TP}^2 = v_{TG}^2 + v_{PG}^2 \quad \text{or} \quad v_{TG} = \sqrt{v_{TP}^2 - v_{PG}^2}$$

The speed of the truck relative to the ground is

$$v_{TG} = \sqrt{(48 \text{ m/s})^2 - (29 \text{ m/s})^2} = 38 \text{ m/s}$$

This is equivalent to 85 mph.
56. **REASONING** When you use the speed ramp, the time it takes you to cover the ground distance \( d = 105 \text{ m} \) is the distance divided by your speed relative to the ground. To determine your speed relative to the ground we will refer to the following relative velocities:

- \( v_{YG} \) = Your velocity relative to the Ground
- \( v_{YR} \) = Your velocity relative to the speed Ramp
- \( v_{RG} \) = The speed Ramp’s velocity relative to the Ground

**SOLUTION** While you are on the ramp, your speed \( v_{YG} \) relative to the ground is the magnitude of the velocity \( v_{YG} \). The desired time \( t \) is

\[
 t = \frac{d}{v_{YG}}
\]

Using the relative velocities defined in the **REASONING**, your velocity relative to the ground is given by

\[
 v_{YG} = v_{YR} + v_{RG}
\]

Since you walk with respect to the ramp at the same rate that you walk on the ground, your velocity \( v_{YR} \) with respect to the ramp has a magnitude of \( v_{YR} = \frac{105 \text{ m}}{75 \text{ s}} = 1.4 \text{ m/s} \).

Moreover, we know that the magnitude of the ramp’s velocity relative to the ground is \( v_{RG} = 2.0 \text{ m/s} \). Since you and the ramp are moving in the same direction, the velocities \( v_{YR} \) and \( v_{RG} \) have the same direction (assumed to be the positive direction), and we can write that

\[
 v_{YG} = v_{YR} + v_{RG} = (+1.4 \text{ m/s}) + (+2.0 \text{ m/s}) = +3.4 \text{ m/s}
\]

Thus, using our expression for the time, we obtain

\[
 t = \frac{d}{v_{YG}} = \frac{105 \text{ m}}{3.4 \text{ m/s}} = 31 \text{ s}
\]

57. **REASONING** Let \( v_{HB} \) represent the velocity of the hawk relative to the balloon and \( v_{BG} \) represent the velocity of the balloon relative to the ground. Then, as indicated by Equation 3.7, the velocity of the hawk relative to the ground is \( v_{HG} = v_{HB} + v_{BG} \). Since the vectors \( v_{HB} \) and \( v_{BG} \) are at right angles to each other, the vector addition can be carried out using the Pythagorean theorem.
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**SOLUTION** Using the drawing at the right, we have from the Pythagorean theorem,

\[
v_{HG} = \sqrt{v_{HB}^2 + v_{BG}^2} = \sqrt{(2.0 \text{ m/s})^2 + (6.0 \text{ m/s})^2} = 6.3 \text{ m/s}
\]

The angle \( \theta \) is

\[
\theta = \tan^{-1} \left( \frac{v_{HB}}{v_{BG}} \right) = \tan^{-1} \left( \frac{2.0 \text{ m/s}}{6.0 \text{ m/s}} \right) = 18^\circ, \text{ north of east}
\]

58. **REASONING** The time it takes for the passenger to walk the distance on the boat is the distance divided by the passenger’s speed \( v_{PB} \) relative to the boat. The time it takes for the passenger to cover the distance on the water is the distance divided by the passenger’s speed \( v_{PW} \) relative to the water. The passenger’s velocity relative to the boat is given. However, we need to determine the passenger’s velocity relative to the water.

**SOLUTION**

a. In determining the velocity of the passenger relative to the water, we define the following symbols:

\[
\begin{align*}
v_{PW} &= \text{Passenger’s velocity relative to the water} \\
v_{PB} &= \text{Passenger’s velocity relative to the boat} \\
v_{BW} &= \text{Boat’s velocity relative to the water}
\end{align*}
\]

The passenger’s velocity relative to the water is

\[
v_{PW} = v_{PB} + v_{BW} = (1.5 \text{ m/s, north}) + (5.0 \text{ m/s, south}) = 3.5 \text{ m/s, south}
\]

b. The time it takes for the passenger to walk a distance of 27 m on the boat is

\[
t = \frac{27 \text{ m}}{v_{PB}} = \frac{27 \text{ m}}{1.5 \text{ m/s}} = 18 \text{ s}
\]

c. The time it takes for the passenger to cover a distance of 27 m on the water is

\[
t = \frac{27 \text{ m}}{v_{PW}} = \frac{27 \text{ m}}{3.5 \text{ m/s}} = 7.7 \text{ s}
\]

59. **SSM REASONING** The velocity \( v_{AB} \) of train A relative to train B is the vector sum of the velocity \( v_{AG} \) of train A relative to the ground and the velocity \( v_{GB} \) of the ground relative to train B, as indicated by Equation 3.7: \( v_{AB} = v_{AG} + v_{GB} \). The values of \( v_{AG} \) and
\( \mathbf{v}_{BG} \) are given in the statement of the problem. We must also make use of the fact that \( \mathbf{v}_{GB} = -\mathbf{v}_{BG} \).

**SOLUTION**

a. Taking east as the positive direction, the velocity of \( A \) relative to \( B \) is, according to Equation 3.7,

\[
\mathbf{v}_{AB} = \mathbf{v}_{AG} + \mathbf{v}_{GB} = \mathbf{v}_{AG} - \mathbf{v}_{BG} = (+13 \text{ m/s}) - (-28 \text{ m/s}) = +41 \text{ m/s}
\]

The positive sign indicates that the direction of \( \mathbf{v}_{AB} \) is due east.

b. Similarly, the velocity of \( B \) relative to \( A \) is

\[
\mathbf{v}_{BA} = \mathbf{v}_{BG} + \mathbf{v}_{GA} = \mathbf{v}_{BG} - \mathbf{v}_{AG} = (-28 \text{ m/s}) - (+13 \text{ m/s}) = -41 \text{ m/s}
\]

The negative sign indicates that the direction of \( \mathbf{v}_{BA} \) is due west.

---

60. **REASONING** Using subscripts to make the relationships among the three relative velocities clear, we have the following:

- \( \mathbf{v}_{PG} \), the velocity of the Plane relative to the Ground (unknown speed, due west)
- \( \mathbf{v}_{PA} \), the velocity of the Plane relative to the Air (245 m/s, unknown direction)
- \( \mathbf{v}_{AG} \), the velocity of the Air relative to the Ground (38.0 m/s, due north)

Arranging the subscripts as shown in Section 3.4 of the text, we find that the velocity of the plane relative to the ground is the vector sum of the other two velocities: \( \mathbf{v}_{PG} = \mathbf{v}_{PA} + \mathbf{v}_{AG} \). This vector sum may be illustrated as follows:

As the three relative velocity vectors form a right triangle, we will apply trigonometry to find the direction \( \theta \) that the pilot should aim the plane relative to due west.

**SOLUTION** The magnitudes of the two known sides of the vector right triangle are \( v_{PA} \) (245 m/s) and \( v_{AG} \) (38.0 m/s) which are, respectively, the hypotenuse and the side opposite the angle \( \theta \) (see the drawing). The sine of the angle \( \theta \) is therefore the ratio of \( v_{AG} \) to \( v_{PA} \), so that the direction the pilot should head the plane is

\[
\theta = \sin^{-1} \left( \frac{v_{AG}}{v_{PA}} \right) = \sin^{-1} \left( \frac{38.0 \text{ m/s}}{245 \text{ m/s}} \right) = 8.92^\circ, \text{ south of west}
\]
61. **REASONING AND SOLUTION** The velocity of the raindrops relative to the train is given by

\[ v_{RT} = v_{RG} + v_{GT} \]

where \( v_{RG} \) is the velocity of the raindrops relative to the ground and \( v_{GT} \) is the velocity of the ground relative to the train.

Since the train moves horizontally, and the rain falls vertically, the velocity vectors are related as shown in the figure at the right. Then

\[ v_{GT} = v_{RG} \tan \theta = (5.0 \text{ m/s}) (\tan 25^\circ) = 2.3 \text{ m/s} \]

The train is moving at a speed of \( 2.3 \text{ m/s} \)

62. **REASONING** The relative velocities in this problem are:

\[ v_{PW} = \text{velocity of the Passenger relative to the Water} \]

\[ v_{PB} = \text{velocity of the Passenger relative to the Boat (2.50 m/s, due east)} \]

\[ v_{BW} = \text{velocity of the Boat relative to the Water (5.50 m/s, at 38.0^\circ north of east)} \]

The velocities are shown in the drawing are related by the subscripting method discussed in Section 3.4:

\[ v_{PW} = v_{PB} + v_{BW} \]

We will determine the magnitude and direction of \( v_{PW} \) from the equation above by using the method of scalar components.

**SOLUTION** The table below lists the scalar components of the three vectors.

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x ) Component</th>
<th>( y ) Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{PB} )</td>
<td>+2.50 m/s</td>
<td>0 m/s</td>
</tr>
<tr>
<td>( v_{BW} )</td>
<td>+((5.50 \text{ m/s}) \cos 38.0^\circ = +4.33 \text{ m/s})</td>
<td>+((5.50 \text{ m/s}) \sin 38.0^\circ = +3.39 \text{ m/s})</td>
</tr>
<tr>
<td>( v_{PW} = v_{PB} + v_{BW} )</td>
<td>+2.50 m/s + 4.33 m/s = +6.83 m/s</td>
<td>0 m/s + 3.39 m/s = +3.39 m/s</td>
</tr>
</tbody>
</table>
The magnitude of \( v_{PW} \) can be found by applying the Pythagorean theorem to the \( x \) and \( y \) components:

\[
v_{PW} = \sqrt{(6.83 \text{ m/s})^2 + (3.39 \text{ m/s})^2} = 7.63 \text{ m/s}
\]

The angle \( \theta \) (see the drawings) that \( v_{PW} \) makes with due east is

\[
\theta = \tan^{-1}\left(\frac{3.39 \text{ m/s}}{6.83 \text{ m/s}}\right) = 26.4^\circ \text{ north of east}
\]

63. **SSM REASONING** The velocity \( v_{PM} \) of the puck relative to Mario is the vector sum of the velocity \( v_{PI} \) of the puck relative to the ice and the velocity \( v_{IM} \) of the ice relative to Mario as indicated by Equation 3.7: \( v_{PM} = v_{PI} + v_{IM} \). The values of \( v_{MI} \) and \( v_{PI} \) are given in the statement of the problem. In order to use the data, we must make use of the fact that \( v_{IM} = -v_{MI} \), with the result that \( v_{PM} = v_{PI} - v_{MI} \).

**SOLUTION** The first two rows of the following table give the east/west and north/south components of the vectors \( v_{PI} \) and \( -v_{MI} \). The third row gives the components of their resultant \( v_{PM} = v_{PI} - v_{MI} \). Due east and due north have been taken as positive.

<table>
<thead>
<tr>
<th>Vector</th>
<th>East/West Component</th>
<th>North/South Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{PI} )</td>
<td>(-(11.0 \text{ m/s}) \sin 22^\circ = -4.1 \text{ m/s})</td>
<td>(-(11.0 \text{ m/s}) \cos 22^\circ = -10.2 \text{ m/s})</td>
</tr>
<tr>
<td>( -v_{MI} )</td>
<td>0</td>
<td>+7.0 \text{ m/s}</td>
</tr>
</tbody>
</table>

\[
v_{PM} = v_{PI} - v_{MI} \quad -4.1 \text{ m/s} \quad -3.2 \text{ m/s}
\]

Now that the components of \( v_{PM} \) are known, the Pythagorean theorem can be used to find the magnitude.

\[
v_{PM} = \sqrt{(-4.1 \text{ m/s})^2 + (-3.2 \text{ m/s})^2} = 5.2 \text{ m/s}
\]

The direction of \( v_{PM} \) is found from

\[
\phi = \tan^{-1}\left(\frac{4.1 \text{ m/s}}{3.2 \text{ m/s}}\right) = 52^\circ \text{ west of south}
\]
64. **REASONING AND SOLUTION** While flying west, the airplane has a ground speed of

\[ v_{PG} = 2.40 \times 10^2 \text{ m/s} - 57.8 \text{ m/s} = 182 \text{ m/s} \]

and requires time \( t_W = x/(182 \text{ m/s}) \) to reach the turn-around point. While flying east the airplane has a ground speed of

\[ v_{PG} = 2.40 \times 10^2 \text{ m/s} + 57.8 \text{ m/s} = 298 \text{ m/s} \]

and requires \( t_E = x/(298 \text{ m/s}) \) to return home. Now the total time for the trip is

\[ t = t_W + t_E = 6.00 \text{ h} = 2.16 \times 10^4 \text{ s}, \]

so

\[ x/182 + x/298 = 2.16 \times 10^4 \text{ m} \]

or \( x = \frac{2.44 \times 10^6 \text{ m}}{= 2440 \text{ km}} \)

65. **SSM REASONING** The relative velocities in this problem are:

\[ v_{PS} = \text{velocity of the Passenger relative to the Shore} \]
\[ v_{P2} = \text{velocity of the Passenger relative to Boat 2 (1.20 m/s, due east)} \]
\[ v_{2S} = \text{velocity of Boat 2 relative to the Shore} \]
\[ v_{21} = \text{velocity of Boat 2 relative to Boat 1 (1.60 m/s, at 30.0° north of east)} \]
\[ v_{1S} = \text{velocity of Boat 1 relative to the Shore (3.00 m/s, due north)} \]

The velocity \( v_{PS} \) of the passenger relative to the shore is related to \( v_{P2} \) and \( v_{2S} \) by (see the method of subscripting discussed in Section 3.4):

\[ v_{PS} = v_{P2} + v_{2S} \]

But \( v_{2S} \), the velocity of Boat 2 relative to the shore, is related to \( v_{21} \) and \( v_{1S} \) by

\[ v_{2S} = v_{21} + v_{1S} \]

Substituting this expression for \( v_{2S} \) into the first equation yields

\[ v_{PS} = v_{P2} + v_{21} + v_{1S} \]

This vector sum is shown in the diagram. We will determine the magnitude of \( v_{PS} \) from the equation above by using the method of scalar components.

**SOLUTION** The table below lists the scalar components of the four vectors in the drawing.
### Table 1

<table>
<thead>
<tr>
<th>Vector</th>
<th>(x) Component</th>
<th>(y) Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{P2})</td>
<td>+1.20 m/s</td>
<td>0 m/s</td>
</tr>
<tr>
<td>(v_{21})</td>
<td>((1.60 \text{ m/s}) \cos 30.0^\circ = +1.39 \text{ m/s})</td>
<td>((1.60 \text{ m/s}) \sin 30.0^\circ = +0.80 \text{ m/s})</td>
</tr>
<tr>
<td>(v_{1S})</td>
<td>0 m/s</td>
<td>+3.00 m/s</td>
</tr>
</tbody>
</table>

\[v_{PS} = v_{P2} + v_{21} + v_{1S}\] 
\[= +1.20 \text{ m/s} + 1.39 \text{ m/s} + 0.80 \text{ m/s} + 3.00 \text{ m/s} = +3.80 \text{ m/s}\]

The magnitude of \(v_{PS}\) can be found by applying the Pythagorean theorem to its \(x\) and \(y\) components:

\[v_{PS} = \sqrt{(2.59 \text{ m/s})^2 + (3.80 \text{ m/s})^2} = 4.60 \text{ m/s}\]

66. **REASONING**  The magnitude and direction of the initial velocity \(v_0\) can be obtained using the Pythagorean theorem and trigonometry, once the \(x\) and \(y\) components of the initial velocity \(v_{0x}\) and \(v_{0y}\) are known. These components can be calculated using Equations 3.3a and 3.3b.

**SOLUTION**  Using Equations 3.3a and 3.3b, we obtain the following results for the velocity components:

\[v_{0x} = v_x - a_x t = 3775 \text{ m/s} - (5.10 \text{ m/s}^2)(565 \text{ s}) = 893.5 \text{ m/s}\]

\[v_{0y} = v_y - a_y t = 4816 \text{ m/s} - (7.30 \text{ m/s}^2)(565 \text{ s}) = 691.5 \text{ m/s}\]

Using the Pythagorean theorem and trigonometry, we find

\[v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(893.5 \text{ m/s})^2 + (691.5 \text{ m/s})^2} = 1130 \text{ m/s}\]

\[\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{691.5 \text{ m/s}}{893.5 \text{ m/s}}\right) = 37.7^\circ\]

67. **REASONING**  Trigonometry indicates that the \(x\) and \(y\) components of the dolphin’s velocity are related to the launch angle \(\theta\) according to \(\tan \theta = v_y / v_x\).

**SOLUTION**  Using trigonometry, we find that the \(y\) component of the dolphin’s velocity is

\[v_y = v_x \tan \theta = v_x \tan 35^\circ = (7.7 \text{ m/s}) \tan 35^\circ = 5.4 \text{ m/s}\]
68. **REASONING** The upward direction is chosen as positive. Since the ballast bag is released from rest relative to the balloon, its initial velocity relative to the ground is equal to the velocity of the balloon relative to the ground, so that \( v_{0y} = 3.0 \text{ m/s} \). Time required for the ballast to reach the ground can be found by solving Equation 3.5b for \( t \).

**SOLUTION** Using Equation 3.5b, we have
\[
\frac{1}{2}a_y t^2 + v_{0y} t - y = 0 \quad \text{or} \quad \frac{1}{2}(-9.80 \text{ m/s}^2) t^2 + (3.0 \text{ m/s}) t - (-9.5 \text{ m}) = 0
\]
This equation is quadratic in \( t \), and \( t \) may be found from the quadratic formula. Using the quadratic formula, suppressing the units, and discarding the negative root, we find
\[
t = \frac{-3.0 \pm \sqrt{(3.0)^2 - 4(-9.90)(9.5)}}{2(-9.90)} = 1.7 \text{ s}
\]

69. **REASONING** The time that the ball spends in the air is determined by its vertical motion. The time required for the ball to reach the lake can be found by solving Equation 3.5b for \( t \). The motion of the golf ball is characterized by constant velocity in the \( x \) direction and accelerated motion (due to gravity) in the \( y \) direction. Thus, the \( x \) component of the velocity of the golf ball is constant, and the \( y \) component of the velocity at any time \( t \) can be found from Equation 3.3b. Once the \( x \) and \( y \) components of the velocity are known for a particular time \( t \), the speed can be obtained from \( v = \sqrt{v_x^2 + v_y^2} \).

**SOLUTION**

a. Since the ball rolls off the cliff horizontally, \( v_{0y} = 0 \). If the origin is chosen at top of the cliff and upward is assumed to be the positive direction, then the vertical component of the ball's displacement is \( y = -15.5 \text{ m} \). Thus, Equation 3.5b gives
\[
t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-15.5 \text{ m})}{(-9.80 \text{ m/s}^2)}} = 1.78 \text{ s}
\]

b. Since there is no acceleration in the \( x \) direction, \( v_x = v_{0x} = 11.4 \text{ m/s} \). The \( y \) component of the velocity of the ball just before it strikes the water is, according to Equation 3.3b,
\[
v_y = v_{0y} + a_y t = [0 + (-9.80 \text{ m/s}^2)(1.78 \text{ s})] = -17.4 \text{ m/s}
\]
The speed of the ball just before it strikes the water is, therefore,
\[
v = \sqrt{v_x^2 + v_y^2} = \sqrt{(11.4 \text{ m/s})^2 + (-17.4 \text{ m/s})^2} = 20.8 \text{ m/s}
\]
70. **REASONING** As shown in Example 8 in the text, the range \( R \) of a projectile is given by 
\[ R = v_{0x}t, \]
where \( v_{0x} = v_0 \cos \theta \) is the horizontal component of the launch velocity of magnitude \( v_0 \) and launch angle \( \theta \), and \( t \) is the total flight time of the projectile. In the absence of air resistance, the time required for the projectile to rise is equal to the time required for the projectile to fall; therefore, the total flight time \( t \) is equal to twice the time \( t_H \) required for a projectile to reach its maximum vertical displacement \( H \). The time \( t_H \) can be found from \( v_y = v_{0y} + a_y t \) (Equation 3.3b) by setting \( v_y = 0 \) and solving for \( t = t_H \).

**SOLUTION**

a. With upward taken as the positive direction, the time \( t_H \) for the greyhound to reach its maximum vertical displacement \( H \) is given by Equation 3.3b as
\[ t_H = -\frac{v_{0y}}{a_y} = -\frac{v_0 \sin 31.0^\circ}{a_y} = -\frac{(10.0 \text{ m/s}) \sin 31.0^\circ}{-9.80 \text{ m/s}^2} = 0.526 \text{ s} \]

The range of the leap is, therefore,
\[ R = v_{0x}t = (v_0 \cos 31.0^\circ)t = [(10.0 \text{ m/s}) \cos 31.0^\circ][2(0.526 \text{ s})] = 9.02 \text{ m} \]

b. Ignoring air resistance, we know that the time for the projectile to rise from ground level to its maximum height is equal to the time for the projectile to fall back to ground level. Therefore, the greyhound is in the air for \( 2(0.526 \text{ s}) = 1.05 \text{ s} \).

71. **SSM REASONING** Once the diver is airborne, he moves in the \( x \) direction with constant velocity while his motion in the \( y \) direction is accelerated (at the acceleration due to gravity). Therefore, the magnitude of the \( x \) component of his velocity remains constant at 1.20 m/s for all times \( t \). The magnitude of the \( y \) component of the diver's velocity after he has fallen through a vertical displacement \( y \) can be determined from Equation 3.6b:
\[ v_y^2 = v_{0y}^2 + 2a_y y \]
Since the diver runs off the platform horizontally, \( v_{0y} = 0 \text{ m/s} \). Once the \( x \) and \( y \) components of the velocity are known for a particular vertical displacement \( y \), the speed of the diver can be obtained from \( v = \sqrt{v_x^2 + v_y^2} \).

**SOLUTION** For convenience, we will take downward as the positive \( y \) direction. After the diver has fallen 10.0 m, the \( y \) component of his velocity is, from Equation 3.6b,
\[ v_y = \sqrt{v_{0y}^2 + 2a_y y} = \sqrt{0^2 + 2(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 14.0 \text{ m/s} \]
Therefore,
\[ v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.20 \text{ m/s})^2 + (14.0 \text{ m/s})^2} = 14.1 \text{ m/s} \]
72. **REASONING** When the ball is thrown straight up with an initial speed \(v_0\), the maximum height \(y\) that it reaches can be found by using with the relation \(v_y^2 = v_{0y}^2 + 2a_y y\) (Equation 3.6b). Since the ball is thrown straight up, \(v_{0y} = v_0\), where \(v_0\) is the initial speed of the ball. Also, the speed of the ball is momentarily zero at its maximum height, so \(v_y = 0\) m/s at that point. The acceleration \(a_y\) is that due to gravity, so the only unknown besides \(y\) is the initial speed \(v_0\) of the ball. To determine \(v_0\) we will employ Equation 3.6b a second time, but now it will be applied to the case where the ball is thrown upward at an angle of 52º above the horizontal. In this case the maximum height reached by the ball is \(y_1 = 7.5\) m, the initial speed in the \(y\) direction is \(v_{0y} = v_0 \sin 52^\circ\), and the \(y\)-component of the speed at the maximum height is \(v_y = 0\) m/s.

**SOLUTION** We will start with the relation \(v_y^2 = v_{0y}^2 + 2a_y y\) (Equation 3.6b) to find the maximum height \(y\) that the ball attains when it is thrown straight up. Solving this equation for \(y\), and substituting in \(v_{0y} = v_0\) and \(v_y = 0\) m/s gives

\[
y = -\frac{v_0^2}{2a_y}
\]  

(1)

To determine \(v_0\), we now apply the equation \(v_y^2 = v_{0y}^2 + 2a_y y\) to the situation where the ball is thrown upward at an angle of 52º relative to the horizontal. In this case we note that \(v_y = v_0 \sin 52^\circ\), \(v_y = 0\) m/s, and \(y = y_1\) (the maximum height of 7.5 m reached by the ball). Solving for \(v_0^2\), we find

\[
v_y^2 = (v_0 \sin 52^\circ)^2 + 2a_y y_1
\]

or

\[
v_0^2 = \frac{-2a_y y_1}{\sin^2 52^\circ}
\]

Substituting this expression for \(v_0^2\) into Equation (1) gives

\[
y = -\frac{v_0^2}{2a_y} = -\frac{-2a_y y_1}{2a_y \sin^2 52^\circ} = \frac{y_1}{\sin^2 52^\circ} = \frac{7.5 \text{ m}}{\sin^2 52^\circ} = [12 \text{ m}]
\]
73. **REASONING** The drawing shows the trajectory of the ball, along with its initial speed $v_0$ and vertical displacement $y$. The angle that the initial velocity of the ball makes with the ground is 35.0°. The known data are shown in the following table:

<table>
<thead>
<tr>
<th>$y$</th>
<th>$a_y$</th>
<th>$v_y$</th>
<th>$v_{0y}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5.50 m</td>
<td>−9.80 m/s²</td>
<td></td>
<td>$(46.0 \text{ m/s}) \sin 35.0° = +26.4 \text{ m/s}$</td>
<td>?</td>
</tr>
</tbody>
</table>

Since three of the kinematic variables are known, we will employ the appropriate equation of kinematics to determine the time of flight.

**SOLUTION** Equation 3.5b \( y = v_{0y} t + \frac{1}{2} a_y t^2 \) relates the time \( t \) to the three known variables. The terms in this equation can be rearranged to as to place it in a standard quadratic form: \( \frac{1}{2} a_y t^2 + v_{0y} t - y = 0 \). The solution of this quadratic equation is

\[
 t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\frac{1}{2} a_y\right)(-y)}}{2\left(\frac{1}{2} a_y\right)}
\]

\[
 t = -\frac{(-26.4 \text{ m/s}) \pm \sqrt{(-26.4 \text{ m/s})^2 - 4\left(\frac{1}{2}\right)(-9.80 \text{ m/s}^2)(-5.50 \text{ m})}}{-9.80 \text{ m/s}^2} = 0.217 \text{ s or 5.17 s}
\]

The first solution \( t = 0.217 \text{ s} \) corresponds to the situation where the ball is moving upward and has a displacement of \( y = +5.50 \text{ m} \). The second solution represents the later time when the ball is moving downward and its displacement is also \( y = +5.50 \text{ m} \) (see the drawing). This is the solution we seek, so \( t = \boxed{5.17 \text{ s}} \).

74. **REASONING** There are three velocities involved:

\[ v_{CR} = \text{the initial velocity of the Car relative to the Road} \]
\[ v_{CT} = \text{the velocity of the Car relative to the first Truck} \]
\[ v_{TR} = \text{the velocity of the first Truck relative to the Road (speed = 11 m/s)} \]
Ordering these vectors by their subscripts in the manner discussed in Section 3.4 of the text, we see that \( \mathbf{v}_{\text{CR}} = \mathbf{v}_{\text{CT}} + \mathbf{v}_{\text{TR}} \). The velocity \( \mathbf{v}_{\text{TR}} \) is known, and we will use the equations of kinematics to find \( \mathbf{v}_{\text{CT}} \).

The two trucks have the same velocity relative to the road, and thus have zero velocity relative to one another. Therefore, we will analyze the sports car’s jump across the 15-m gap as if both trucks were stationary. Neglecting air resistance, we treat the car as a projectile and find the initial speed \( v_0 \) it must have, relative to the trucks, in order to reach the flat trailer. (This speed is, in fact, the speed \( v_{\text{CT}} \) that we need to find.) We will use the relation \( x = v_{0x}t + \frac{1}{2}a_xt^2 \) (Equation 3.5a) to find the elapsed time. With \( a_x = 0 \text{ m/s}^2 \), Equation 3.5a becomes \( x = v_{0x}t + \frac{1}{2}(0 \text{ m/s}^2)t^2 = v_{0x}t \). Thus the car is in the air for \( t \) seconds, where

\[
t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos \theta} \tag{1}
\]

In Equation (1), \( x = 15 \text{ m} \) and \( \theta = 16^\circ \), but \( v_0 \) and \( t \) are not known. The heights of the trailer and the ramp are the same, so the car’s vertical displacement is zero. To use this fact, we turn to \( y = v_{0y}t + \frac{1}{2}a_yt^2 \) (Equation 3.5b) and substitute both \( y = 0 \text{ m} \) and \( v_{0y} = v_0 \sin \theta \):

\[
0 = v_{0y}t + \frac{1}{2}a_yt^2 \quad \text{or} \quad v_{0y} = -\frac{1}{2}a_yt \quad \text{or} \quad v_0 \sin \theta = -\frac{1}{2}a_yt \tag{2}
\]

To eliminate the time, we substitute \( t \) from Equation (1) into Equation (2) and find that

\[
v_0 \sin \theta = -\frac{1}{2}a_y \left( \frac{x}{v_0 \cos \theta} \right) \quad \text{or} \quad v_0^2 = \frac{-a_yx}{2 \cos \theta \sin \theta} \quad \text{or} \quad v_0 = \sqrt{\frac{-a_yx}{2 \cos \theta \sin \theta}} \tag{3}
\]

**SOLUTION**  Taking up as the positive direction, we use Equation (3) to calculate the speed \( v_{\text{CT}} = v_0 \) of the car relative to the trucks:

\[
v_{\text{CT}} = v_0 = \sqrt{\frac{-a_yx}{2 \cos \theta \sin \theta}} = \sqrt{\frac{-(-9.80 \text{ m/s}^2)(15 \text{ m})}{2 \cos 16^\circ \sin 16^\circ}} = 17 \text{ m/s}
\]

Since the ramp alters the direction of the car’s velocity but not its magnitude, the initial jump speed \( v_0 \) is also the magnitude of the car’s velocity \( v_{\text{CT}} \) relative to the truck before the car reaches the ramp: \( v_0 = v_{\text{CT}} \). The sports car must overtake the truck at a speed of at least 17 m/s relative to the truck, so that the car’s minimum required speed \( v_{\text{CR}} \) relative to the road is

\[
v_{\text{CR}} = v_{\text{CT}} + v_{\text{TR}} = 17 \text{ m/s} + 11 \text{ m/s} = 28 \text{ m/s}
\]
75. **SSM REASONING**  The horizontal distance covered by stone 1 is equal to the distance covered by stone 2 after it passes point \( P \) in the following diagram. Thus, the distance \( \Delta x \) between the points where the stones strike the ground is equal to \( x_2 \), the horizontal distance covered by stone 2 when it reaches \( P \). In the diagram, we assume up and to the right are positive.

\[ x_2 = v_{0x}t_p = (v_0 \cos \theta)t_p \]

For the vertical motion of stone 2, \( v_y = v_0 \sin \theta + a_y t \). Solving for \( t \) gives

\[ t = \frac{v_y - v_0 \sin \theta}{a_y} \]

When stone 2 reaches \( P \), \( v_y = -v_0 \sin \theta \), so the time required to reach \( P \) is

\[ t_p = \frac{-2v_0 \sin \theta}{a_y} \]

Then,

\[ x_2 = v_{0x}t_p = (v_0 \cos \theta) \left( \frac{-2v_0 \sin \theta}{a_y} \right) \]
76. **REASONING** The three relative velocities in this situation are

\[ \mathbf{v}_{TG} = \text{the velocity of the Truck relative to the Ground (unknown)} \]
\[ \mathbf{v}_{TC} = \text{the velocity of the Truck relative to the Car (24.0 m/s, 52.0° north of east)} \]
\[ \mathbf{v}_{CG} = \text{the velocity of the Car relative to the Ground (16.0 m/s, due north)} \]

The velocity of the truck relative to the ground may be expressed as the vector sum of the other two velocities: \( \mathbf{v}_{TG} = \mathbf{v}_{TC} + \mathbf{v}_{CG} \).

Note the fashion in which the “middle” subscripts on the right side of the equals sign are matched. See the diagram for an illustration of this vector sum.

Because the vectors do not form a right triangle, we will utilize the component method of vector addition to determine the eastward and northward components of the resultant vector \( \mathbf{v}_{TG} \).

For convenience, we will take east as the +x direction and north as the +y direction. Once we know \( \mathbf{v}_{TG,x} \) and \( \mathbf{v}_{TG,y} \), we will use the Pythagorean theorem to calculate the magnitude of the truck’s velocity relative to the ground, which, from the diagram, we expect to be larger than \( \mathbf{v}_{TC} = 24.0 \text{ m/s} \).

**SOLUTION** The vector \( \mathbf{v}_{CG} \) points due north, and thus has no x component. Therefore the x component of \( \mathbf{v}_{TG} \) is equal to the x component of \( \mathbf{v}_{TC} \):

\[ \mathbf{v}_{TG,x} = \mathbf{v}_{TC,x} + \mathbf{v}_{CG,x} = \mathbf{v}_{TC} \cos 52.0^\circ + 0 \text{ m/s} \]
\[ = (24.0 \text{ m/s}) \cos 52.0^\circ = 14.8 \text{ m/s} \]

Next, we find the y component of the truck’s velocity relative to the ground. Noting that the vector \( \mathbf{v}_{CG} \) points due north, so that \( \mathbf{v}_{CG,y} = \mathbf{v}_{CG} \), we have

\[ \mathbf{v}_{TG,y} = \mathbf{v}_{TC,y} + \mathbf{v}_{CG,y} = \mathbf{v}_{TC} \sin 52.0^\circ + \mathbf{v}_{CG} \]
\[ = (24.0 \text{ m/s}) \sin 52.0^\circ + 16.0 \text{ m/s} = 34.9 \text{ m/s} \]

The magnitude of \( \mathbf{v}_{TG} \) is found from the Pythagorean theorem:
77. **Reasoning** Using the data given in the problem, we can find the maximum flight time \( t \) of the ball using Equation 3.5b \( (y = v_{0y}t + \frac{1}{2}a_y t^2) \). Once the flight time is known, we can use the definition of average velocity to find the minimum speed required to cover the distance \( x \) in that time.

**Solution** Equation 3.5b is quadratic in \( t \) and can be solved for \( t \) using the quadratic formula. According to Equation 3.5b, the maximum flight time is (with upward taken as the positive direction)

\[
t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\frac{1}{2}\right)a_y(-y)}}{2\left(\frac{1}{2}\right)a_y} = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2a_yy}}{a_y}
\]

\[
= \frac{-15.0 \text{ m/s} \sin 50.0^\circ \pm \sqrt{\left[(15.0 \text{ m/s} \sin 50.0^\circ)^2 + 2(-9.80 \text{ m/s}^2)(2.10 \text{ m})\right]}}{-9.80 \text{ m/s}^2}
\]

\[
= 0.200 \text{ s and 2.145 s}
\]

where the first root corresponds to the time required for the ball to reach a vertical displacement of \( y = +2.10 \text{ m} \) as it travels upward, and the second root corresponds to the time required for the ball to have a vertical displacement of \( y = +2.10 \text{ m} \) as the ball travels upward and then downward. The desired flight time \( t \) is 2.145 s.

During the 2.145 s, the horizontal distance traveled by the ball is

\[
x = v_x t = (v_0 \cos \theta)t = [(15.0 \text{ m/s} \cos 50.0^\circ)(2.145 \text{ s})] = 20.68 \text{ m}
\]

Thus, the opponent must move \( 20.68 \text{ m} - 10.0 \text{ m} = 10.68 \text{ m} \) in \( 2.145 \text{ s} - 0.30 \text{ s} = 1.845 \text{ s} \). The opponent must, therefore, move with a minimum average speed of

\[
\bar{v}_\text{min} = \frac{10.68 \text{ m}}{1.845 \text{ s}} = 5.79 \text{ m/s}
\]

78. **Reasoning** The velocity \( \mathbf{v}_{\text{OW}} \) of the object relative to the water is the vector sum of the velocity \( \mathbf{v}_{\text{OS}} \) of the object relative to the ship and the velocity \( \mathbf{v}_{\text{SW}} \) of the ship relative to the water, as indicated by Equation 3.7: \( \mathbf{v}_{\text{OW}} = \mathbf{v}_{\text{OS}} + \mathbf{v}_{\text{SW}} \). The value of \( \mathbf{v}_{\text{SW}} \) is given in
the statement of the problem. We can find the value of $v_{OS}$ from the fact that we know the position of the object relative to the ship at two different times. The initial position is $r_{OS1}$, and the final position is $r_{OS2}$. Since the object moves with constant velocity,

$$v_{OS} = \frac{\Delta r_{OS}}{\Delta t} = \frac{r_{OS2} - r_{OS1}}{\Delta t} \tag{1}$$

**SOLUTION** The first two rows of the following table give the east/west and north/south components of the vectors $r_{OS2}$ and $-r_{OS1}$. The third row of the table gives the components of $\Delta r_{OS} = r_{OS2} - r_{OS1}$. Due east and due north have been taken as positive.

<table>
<thead>
<tr>
<th>Vector</th>
<th>East/West Component</th>
<th>North/South Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{OS2}$</td>
<td>$-(1120 \text{ m}) \cos 57.0^\circ$</td>
<td>$-(1120 \text{ m}) \sin 57.0^\circ$</td>
</tr>
<tr>
<td></td>
<td>$= -6.10 \times 10^2 \text{ m}$</td>
<td>$= -9.39 \times 10^2 \text{ m}$</td>
</tr>
<tr>
<td>$-r_{OS1}$</td>
<td>$-(2310 \text{ m}) \cos 32.0^\circ$</td>
<td>$(2310 \text{ m}) \sin 32.0^\circ$</td>
</tr>
<tr>
<td></td>
<td>$= -1.96 \times 10^3 \text{ m}$</td>
<td>$= 1.22 \times 10^3 \text{ m}$</td>
</tr>
<tr>
<td>$\Delta r_{OS}$</td>
<td>$r_{OS2} - r_{OS1}$</td>
<td>$-2.57 \times 10^3 \text{ m}$</td>
</tr>
</tbody>
</table>

Now that the components of $\Delta r_{OS}$ are known, the Pythagorean theorem can be used to find the magnitude.

$$\Delta r_{OS} = \sqrt{(-2.57 \times 10^3 \text{ m})^2 + (2.81 \times 10^2 \text{ m})^2} = 2.59 \times 10^3 \text{ m}$$

The direction of $\Delta r_{OS}$ is found from

$$\phi = \tan^{-1}\left(\frac{2.81 \times 10^2 \text{ m}}{2.57 \times 10^3 \text{ m}}\right) = 6.24^\circ$$

Therefore, from Equation (1),

$$v_{OS} = \frac{\Delta r_{OS}}{\Delta t} = \frac{r_{OS2} - r_{OS1}}{\Delta t} = \frac{2.59 \times 10^3 \text{ m}}{360 \text{ s}} = 7.19 \text{ m/s, } 6.24^\circ \text{ north of west}$$

Now that $v_{OS}$ is known, we can find $v_{OW}$, as indicated by Equation 3.7: $v_{OW} = v_{OS} + v_{SW}$. The following table summarizes the vector addition:
### Vector Components

<table>
<thead>
<tr>
<th>Vector</th>
<th>East/West Component</th>
<th>North/South Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{v}_{OS} )</td>
<td>(-(7.19 \text{ m/s}) \cos 6.24^\circ = -7.15 \text{ m/s})</td>
<td>((7.19 \text{ m/s}) \sin 6.24^\circ = 0.782 \text{ m/s})</td>
</tr>
<tr>
<td>( \mathbf{v}_{SW} )</td>
<td>+4.20 \text{ m/s}</td>
<td>0 \text{ m/s}</td>
</tr>
</tbody>
</table>

\[
\mathbf{v}_{OW} = \mathbf{v}_{OS} + \mathbf{v}_{SW} = -2.95 \text{ m/s} + 0.782 \text{ m/s} = -2.17 \text{ m/s}
\]

Now that the components of \( \mathbf{v}_{OW} \) are known, the Pythagorean theorem can be used to find the magnitude.

\[
\mathbf{v}_{OW} = \sqrt{(-2.95 \text{ m/s})^2 + (0.782 \text{ m/s})^2} = 3.05 \text{ m/s}
\]

The direction of \( \mathbf{v}_{OW} \) is found from

\[
\tan \phi = \frac{0.782 \text{ m/s}}{2.95 \text{ m/s}} \Rightarrow \phi = 14.8^\circ \text{ north of west}
\]

### Reasoning

The drawing shows the trajectory of the ball, along with its initial speed \( v_0 \), horizontal displacement \( x \), and vertical displacement \( y \). The angle that the initial velocity of the ball makes with the horizontal is \( \theta \). The known data are shown in the tables below:

#### x-Direction Data

<table>
<thead>
<tr>
<th>( x )</th>
<th>( a_x )</th>
<th>( v_x )</th>
<th>( v_{0x} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+26.9 m</td>
<td>0 \text{ m/s}^2</td>
<td>( + (19.8 \text{ m/s}) \cos \theta )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### y-Direction Data

<table>
<thead>
<tr>
<th>( y )</th>
<th>( a_y )</th>
<th>( v_y )</th>
<th>( v_{0y} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2.74 m</td>
<td>-9.80 \text{ m/s}^2</td>
<td>( + (19.8 \text{ m/s}) \sin \theta )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are only two known variables in each table, so we cannot directly use the equations of kinematics to find the angle \( \theta \). Our approach will be to first use the \( x \) direction data and obtain an expression for the time of flight \( t \) in terms of \( x \) and \( v_{0x} \). We will then enter this
expression for $t$ into the $y$-direction data table. The four variables in this table, $y$, $a_y$, $v_{0y}$, and $t$, can be related by using the appropriate equation of kinematics. This equation can then be solved for the angle $\theta$.

SOLUTION Using the $x$-direction data, Equation 3.5a can be employed to find the time $t$ that the ball is in the air:

$$x = v_{0x}t + \frac{1}{2}a_xt^2 = v_{0x}t \quad \text{(since } a_x = 0 \text{ m/s}^2)$$

Solving for $t$ gives

$$t = \frac{x}{v_{0x}} = \frac{+26.9 \text{ m}}{(19.8 \text{ m/s})\cos \theta}$$

Using the expression above for the time $t$ and the data in the $y$-direction data table, the displacement in the $y$ direction can be written with the aid of Equation 3.5b:

$$y = v_{0y}t + \frac{1}{2}a_yt^2$$

$$+2.74 \text{ m} = \left[\left(\frac{19.8 \text{ m/s}}{}\sin \theta\right)\left(\frac{+26.9 \text{ m}}{(19.8 \text{ m/s})\cos \theta}\right) + \frac{1}{2}(19.8 \text{ m/s})\cos \theta\right]^2$$

Evaluating the numerical factors and using the fact that $\sin \theta/\cos \theta = \tan \theta$, the equation above becomes

$$+2.74 \text{ m} = (+26.9 \text{ m})\tan \theta + \frac{(-9.04 \text{ m})}{\cos^2 \theta}$$

Using $\frac{1}{\cos^2 \theta} = 1 + \tan^2 \theta$, this equation can be rearranged and placed into a quadratic form:

$$(-9.04 \text{ m})\tan^2 \theta + (26.9 \text{ m})\tan \theta - 11.8 \text{ m} = 0$$

The solutions to this quadratic equation are

$$\tan \theta = \frac{-26.9 \text{ m} \pm \sqrt{(26.9 \text{ m})^2 - 4(-9.04 \text{ m})(-11.8 \text{ m})}}{2(-9.04 \text{ m})} = 0.535 \quad \text{and} \quad 2.44$$

The two angles are $\theta_1 = \tan^{-1} 0.535 = 28.1^\circ$ and $\theta_2 = \tan^{-1} 2.44 = 67.7^\circ$
1. (b) If only one force acts on the object, it is the net force; thus, the net force must be nonzero. Consequently, the velocity would change, according to Newton’s first law, and could not be constant.

2. (d) This situation violates the first law, which predicts that the rabbit’s foot tends to remain in place where it was when the car begins accelerating. The car would leave the rabbit’s foot behind. That is, the rabbit’s foot would swing away from, not toward, the windshield.

3. (e) Newton’s first law states that an object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force. All three statements are consistent with the first law.

4. (a) Newton’s second law with a net force of $7560 \text{ N} - 7340 \text{ N} = 220 \text{ N}$ due north gives the answer directly.

5. (c) Newton’s second law gives the answer directly, provided the net force is calculated by vector addition of the two given forces. The direction of the net force gives the direction of the acceleration.

6. (e) Newton’s second law gives the answer directly. One method is to determine the total acceleration by vector addition of the two given components. The net force has the same direction as the acceleration.

7. (e) Answers a and b are false, according to the third law, which states that whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body. It does not matter whether one of the bodies is stationary or whether it collapses. Answer c is false, because according to the third law, Sam and his sister experience forces of equal magnitudes during the push-off. Since Sam has the greater mass, he flies off with the smaller acceleration, according to the second law. Answer d is false, because in catching and throwing the ball each astronaut applies a force to it, and, according to the third law, the ball applies an oppositely directed force of equal magnitude to each astronaut. These reaction forces accelerate the astronauts away from each other, so that the distance between them increases.

8. (b) Newton’s third law indicates that Paul and Tom apply forces of equal magnitude to each other. According to Newton’s second law, the magnitude of each of these forces is the mass times the magnitude of the acceleration. Thus, we have $m_{\text{Paul}}a_{\text{Paul}} = m_{\text{Tom}}a_{\text{Tom}}$, or $m_{\text{Paul}}/m_{\text{Tom}} = a_{\text{Tom}}/a_{\text{Paul}}$. 
9. (e) Newton’s law of gravitation gives the answer directly. According to this law the weight is directly proportional to the mass of the planet, so twice the mass means twice the weight. However, this law also indicates that the weight is inversely proportional to the square of the planet’s radius, so three times the radius means one ninth the weight. Together, these two factors mean that the weight on the planet is $2/9$ or 0.222 times your earth-weight.

10. (c) Newton’s law of gravitation gives the answer, provided that the distance between the centers of the spheres is used for $r$ ($r = 0.50 \text{ m} + 1.20 \text{ m} + 0.80 \text{ m}$), rather than the distance between the surfaces of the spheres.

11. (a) The answer follows directly from the fact that weight $W$ is given by $W = mg$, where $m$ is the mass and $g$ is the acceleration due to the earth’s gravity. Thus, $m = (784 \text{ N})/(9.80 \text{ m/s}^2) = 80.0 \text{ kg}$. The mass is the same on Mercury as on Earth, because mass is an intrinsic property of matter.

12. (d) What matters is the direction of the elevator’s acceleration. When the acceleration is upward, the apparent weight is greater than the true weight. When the acceleration is downward, the apparent weight is less than the true weight. In both possibilities the acceleration points upward.

13. (b) According to Newton’s third law, the pusher and the wall exert forces of equal magnitude but opposite directions on each other. The normal force is the component of the wall’s force that is perpendicular to the wall. Thus, it has the same magnitude as the component of the pusher’s force that is perpendicular to the wall. As a result, the normal forces are ranked in the same order as the perpendicular components of the pusher’s forces. The smallest perpendicular component is in B, and the largest is in C.

14. (a) The static frictional force is balancing the component of the block’s weight that points down the slope of the incline. This component is smallest in B and greatest in A.

15. (b) The static frictional force that blocks A and B exert on each other has a magnitude $f$. The force that B exerts on A is directed to the right (the positive direction), while the force that A exerts on B is directed to the left. Blocks B and C also exert static frictional forces on each other, but these forces have a magnitude $2f$, because the normal force pressing B and C together is twice the normal force pressing A and B together. The force that C exerts on B is directed to the right, while the force that B exerts on C is directed to the left. In summary, then, block A experiences a single frictional force $+f$, which is the net frictional force; block B experiences two frictional forces, $-f$ and $+2f$, the net frictional force being $-f + 2f = +f$; block C experiences a single frictional force $+2f$, which is the net frictional force. It follows that $f_{s,A} = f_{s,B} = f_{s,C}/2$.

16. (c) The magnitude of the kinetic frictional force is proportional to the magnitude of the normal force. The normal force is smallest in B, because the vertical component of $F$ compensates for part of the block’s weight. In contrast, the normal force is greatest in C, because the vertical component of $F$ adds to the weight of the block.
17. (d) Acceleration is inversely proportional to mass, according to Newton’s second law. This law also indicates that acceleration is directly proportional to the net force. The frictional force is the net force acting on a block, and its magnitude is directly proportional to the magnitude of the normal force. However, in each of the pictures the normal force is directly proportional to the weight and, thus, the mass of a block. The inverse proportionality of the acceleration to mass and the direct proportionality of the net force to mass offset each other. The result is that the deceleration is the same in each case.

18. (e) In B the tension \( T \) is the smallest, because three rope segments support the weight \( W \) of the block, with the result that \( 3T = W \), or \( T = W/3 \). In A the tension is the greatest, because only one rope segment supports the weight of the block, with the result that \( T = W \).

19. (c) Since the engines are shut down and since nothing is nearby to exert a force, the net force acting on the probe is zero, and its acceleration must be zero, according to Newton’s second law. With zero acceleration the probe is in equilibrium.

20. (a) The hallmark of an object in equilibrium is that it has no acceleration. Therefore, an object in equilibrium need not be at rest. It can be moving with a constant velocity.

21. (b) Since the object is not in equilibrium, it must be accelerating. Newton’s second law, in turn, implies that a net force must be present to cause the acceleration. Whether the net forces arises from a single force, two perpendicular forces, or three forces is not important, because only the net force appears in the second law.

22. (d) The block is at rest and, therefore, in equilibrium. According to Newton’s second law, then, the net force acting on the block in a direction parallel to the inclined surface of the incline must be zero. This means that the force of static friction directed up the incline must balance the component of the block’s weight directed down the incline \( [(8.0 \text{ kg})(9.8 \text{ m/s}^2) \sin 22^\circ = 29 \text{ N}] \).

23. (b) Since the boxes move at a constant velocity, they have no acceleration and are, therefore, in equilibrium. According to Newton’s second law, the net force acting on each box must be zero. Thus, Newton’s second law applied to each box gives two equations in two unknowns, the magnitude of the tension in the rope between the boxes and the kinetic frictional force that acts on each box. Note that the frictional forces acting on the boxes are identical, because the boxes are identical. Solving these two equations shows that the tension is one-half of the applied force.

24. 31 kg·m/s^2

25. 517 N
CHAPTER 4 \(\textit{FORCES AND NEWTON'S LAWS OF MOTION}\)

PROBLEMS

1. **REASONING AND SOLUTION** According to Newton's second law, the acceleration is \(a = \Sigma F/m\). Since the pilot and the plane have the same acceleration, we can write

\[
\left( \frac{\Sigma F}{m} \right)_{\text{Pilot}} = \left( \frac{\Sigma F}{m} \right)_{\text{Plane}} \quad \text{or} \quad (\Sigma F)_{\text{Pilot}} = m_{\text{Pilot}} \left( \frac{\Sigma F}{m} \right)_{\text{Plane}}
\]

Therefore, we find

\[
(\Sigma F)_{\text{Pilot}} = (78 \text{ kg}) \left( \frac{3.7 \times 10^4 \text{ N}}{3.1 \times 10^4 \text{ kg}} \right) = 93 \text{ N}
\]

2. **REASONING** Newton's second law of motion gives the relationship between the net force \(\Sigma F\) and the acceleration \(a\) that it causes for an object of mass \(m\). The net force is the vector sum of all the external forces that act on the object. Here the external forces are the drive force, the force due to the wind, and the resistive force of the water.

**SOLUTION** We choose the direction of the drive force (due west) as the positive direction. Solving Newton's second law \((\Sigma F = ma)\) for the acceleration gives

\[
a = \frac{\Sigma F}{m} = \frac{+4100 \text{ N} - 800 \text{ N} - 1200 \text{ N}}{6800 \text{ kg}} = +0.31 \text{ m/s}^2
\]

The positive sign for the acceleration indicates that its direction is **due west**.

3. **REASONING** In each case, we will apply Newton's second law. Remember that it is the net force that appears in the second law. The net force is the vector sum of both forces.

**SOLUTION**

a. We will use Newton's second law, \(\Sigma F_x = ma_x\), to find the force \(F_2\). Taking the positive \(x\) direction to be to the right, we have

\[
F_1 + F_2 = ma_x \quad \text{so} \quad F_2 = ma_x - F_1
\]
\[ F_2 = (3.0 \text{ kg})(+5.0 \text{ m/s}^2) - (9.0 \text{ N}) = +6 \text{ N} \]

b. Applying Newton’s second law again gives
\[ F_2 = ma - F_1 = (3.0 \text{ kg})(-5.0 \text{ m/s}^2) - (9.0 \text{ N}) = -24 \text{ N} \]

c. An application of Newton’s second law gives
\[ F_2 = ma - F_1 = (3.0 \text{ kg})(0 \text{ m/s}^2) - (9.0 \text{ N}) = -9.0 \text{ N} \]

4. **REASONING** According to Newton’s second law, Equation 4.1, the average net force \( \Sigma F \) is equal to the product of the object’s mass \( m \) and the average acceleration \( \bar{a} \). The average acceleration is equal to the change in velocity divided by the elapsed time (Equation 2.4), where the change in velocity is the final velocity \( v \) minus the initial velocity \( v_0 \).

**SOLUTION** The average net force exerted on the car and riders is
\[
\Sigma F = m\bar{a} = m\frac{v-v_0}{t-t_0} = \left(5.5 \times 10^3 \text{ kg}\right)\frac{45 \text{ m/s} - 0 \text{ m/s}}{7.0 \text{ s}} = 3.5 \times 10^4 \text{ N}
\]

5. **SSM REASONING** The magnitude \( \Sigma F \) of the net force acting on the kayak is given by Newton’s second law as \( \Sigma F = ma \) (Equation 4.1), where \( m \) is the combined mass of the person and kayak, and \( a \) is their acceleration. Since the initial and final velocities, \( v_0 \) and \( v \), and the displacement \( x \) are known, we can employ one of the equations of kinematics from Chapter 2 to find the acceleration.

**SOLUTION** Solving Equation 2.9 \( v^2 = v_0^2 + 2ax \) from the equations of kinematics for the acceleration, we have
\[
a = \frac{v^2-v_0^2}{2x}
\]

Substituting this result into Newton’s second law gives
\[
\Sigma F = ma = m\left(\frac{v^2-v_0^2}{2x}\right) = (73 \text{ kg})\left[\frac{(0.60 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(0.41 \text{ m})}\right] = 32 \text{ N}
\]

6. **REASONING** The time \( t \) required for the projectile to come up to a final speed \( v \), starting from an initial speed \( v_0 \), can be obtained with the aid of Equation 2.4 of the equations of kinematics, which is \( v = v_0 + at \). To use this equation, however, we need a value for the acceleration \( a \). We can obtain this value with the aid of Newton’s second law.
SOLUTION Solving Equation 2.4 for the time \( t \) gives

\[
t = \frac{v - v_0}{a}
\]

According to Newton’s second law, the net force is \( \Sigma F = ma \), which can be solved for the acceleration to show that

\[
a = \frac{\Sigma F}{m}
\]

Substituting the expression for \( a \) into the expression for \( t \) gives

\[
t = \frac{v - v_0}{(\Sigma F)/m} = \frac{m(v - v_0)}{\Sigma F} = \frac{(5.0 \text{ kg})[(4.0 \times 10^3 \text{ m/s}) - (0 \text{ m/s})]}{4.9 \times 10^5 \text{ N}} = 0.041 \text{ s}
\]

where we have used the fact that the projection starts from rest so that \( t_0 = 0 \text{ s} \).

7. **SSM REASONING AND SOLUTION** The acceleration required is

\[
a = \frac{v^2 - v_0^2}{2x} = \frac{- (15.0 \text{ m/s})^2}{2(50.0 \text{ m})} = -2.25 \text{ m/s}^2
\]

Newton's second law then gives the magnitude of the net force as

\[
F = ma = (1580 \text{ kg})(2.25 \text{ m/s}^2) = 3560 \text{ N}
\]

8. **REASONING** The magnitudes of the initial \( (v_0 = 0 \text{ m/s}) \) and final \( (v = 805 \text{ m/s}) \) velocities are known. In addition, data is given for the mass and the thrust, so that Newton’s second law can be used to determine the acceleration of the probe. Therefore, kinematics Equation 2.4 \( (v = v_0 + at) \) can be used to determine the time \( t \).

**SOLUTION** Solving Equation 2.4 for the time gives

\[
t = \frac{v - v_0}{a}
\]

Newton’s second law gives the acceleration as \( a = (\Sigma F)/m \). Using this expression in Equation 2.4 gives

\[
t = \frac{(v - v_0)}{(\Sigma F)/m} = \frac{m(v - v_0)}{\Sigma F} = \frac{(474 \text{ kg})(805 \text{ m/s} - 0 \text{ m/s})}{56 \times 10^3 \text{ N}} = 6.8 \times 10^6 \text{ s}
\]

Since one day contains \( 8.64 \times 10^4 \text{ s} \), the time is

\[
t = \left(6.8 \times 10^6 \text{ s}\right) \frac{1 \text{ day}}{8.64 \times 10^4 \text{ s}} = 79 \text{ days}
\]
9. **REASONING** Let due east be chosen as the positive direction. Then, when both forces point due east, Newton's second law gives

$$\sum F = m a_1$$

where $a_1 = 0.50 \text{ m/s}^2$. When $F_A$ points due east and $F_B$ points due west, Newton's second law gives

$$\sum F = m a_2$$

where $a_2 = 0.40 \text{ m/s}^2$. These two equations can be used to find the magnitude of each force.

**SOLUTION**

a. Adding Equations 1 and 2 gives

$$F_A = \frac{m(a_1 + a_2)}{2} = \frac{(8.0 \text{ kg})(0.50 \text{ m/s}^2 + 0.40 \text{ m/s}^2)}{2} = 3.6 \text{ N}$$

b. Subtracting Equation 2 from Equation 1 gives

$$F_B = \frac{m(a_1 - a_2)}{2} = \frac{(8.0 \text{ kg})(0.50 \text{ m/s}^2 - 0.40 \text{ m/s}^2)}{2} = 0.40 \text{ N}$$

10. **REASONING** From Newton's second law, we know that the net force $\sum F$ acting on the electron is directly proportional to its acceleration, so in part a we will first find the electron's acceleration. The problem text gives the electron's initial velocity ($v_0 = +5.40 \times 10^5 \text{ m/s}$) and final velocity ($v = +2.10 \times 10^6 \text{ m/s}$), as well as its displacement ($x = +0.038 \text{ m}$) during the interval of acceleration. The elapsed time is not known, so we will use Equation 2.9 ($v^2 = v_0^2 + 2ax$) to calculate the electron's acceleration. Then we will find the net force acting on the electron from Equation 4.1 ($\sum F = ma$) and the electron's mass. Because $F_1$ points in the $+x$ direction and $F_2$ points in the $-x$ direction, the net force acting on the electron is $\sum F = F_1 - F_2$. In part b of the problem, we will rearrange this expression to obtain the magnitude of the second electric force.

**SOLUTION**

a. Solving Equation 2.9 for the electron's acceleration, we find that
Chapter 4 Problems

11. **REASONING** According to Newton's second law ($\sum F = ma$), the acceleration of the object is given by $a = \frac{\sum F}{m}$, where $\sum F$ is the net force that acts on the object. We must first find the net force that acts on the object, and then determine the acceleration using Newton's second law.

**SOLUTION** The following table gives the $x$ and $y$ components of the two forces that act on the object. The third row of that table gives the components of the net force.

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$-Component</th>
<th>$y$-Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>40.0 N</td>
<td>0 N</td>
</tr>
<tr>
<td>$F_2$</td>
<td>(60.0 N) cos 45.0° = 42.4 N</td>
<td>(60.0 N) sin 45.0° = 42.4 N</td>
</tr>
</tbody>
</table>

$\sum F = F_1 + F_2$

82.4 N 42.4 N

The magnitude of $\sum F$ is given by the Pythagorean theorem as

$$\sum F = \sqrt{(82.4 \text{ N})^2 + (42.4 \text{ N})^2} = 92.7 \text{ N}$$

The angle $\theta$ that $\sum F$ makes with the $+x$ axis is

$$\theta = \tan^{-1} \left( \frac{42.4 \text{ N}}{82.4 \text{ N}} \right) = 27.2^\circ$$

According to Newton's second law, the magnitude of the acceleration of the object is

$$a = \frac{\sum F}{m} = \frac{92.7 \text{ N}}{3.00 \text{ kg}} = 30.9 \text{ m/s}^2$$
Since Newton's second law is a vector equation, we know that the direction of the right hand side must be equal to the direction of the left hand side. In other words, the direction of the acceleration \(a\) is the same as the direction of the net force \(\Sigma F\). Therefore, the direction of the acceleration of the object is \(27.2^\circ\) above the +x axis.

12. **REASONING** The net force \(\Sigma F\) has a horizontal component \(\Sigma F_x\) and a vertical component \(\Sigma F_y\). Since these components are perpendicular, the Pythagorean theorem applies (Equation 1.7), and the magnitude of the net force is \(\Sigma F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}\). Newton’s second law allows us to express the components of the net force acting on the ball in terms of its mass and the horizontal and vertical components of its acceleration: \(\Sigma F_x = ma_x\), \(\Sigma F_y = ma_y\) (Equations 4.2a and 4.2b).

**SOLUTION** Combining the Pythagorean theorem with Newton’s second law, we obtain the magnitude of the net force acting on the ball:

\[
\Sigma F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(ma_x)^2 + (ma_y)^2} = m\sqrt{a_x^2 + a_y^2}
\]

\[
= (0.430 \text{ kg})\sqrt{(810 \text{ m/s}^2)^2 + (1100 \text{ m/s}^2)^2} = 590 \text{ N}
\]

13. **SSM REASONING** To determine the acceleration we will use Newton’s second law \(\Sigma F = ma\). Two forces act on the rocket, the thrust \(T\) and the rocket’s weight \(W\), which is \(mg = (4.50 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2) = 4.41 \times 10^6 \text{ N}\). Both of these forces must be considered when determining the net force \(\Sigma F\). The direction of the acceleration is the same as the direction of the net force.

**SOLUTION** In constructing the free-body diagram for the rocket we choose upward and to the right as the positive directions. The free-body diagram is as follows:

The \(x\) component of the net force is

\[
\Sigma F_x = T \cos 55.0^\circ
\]

\[
= (7.50 \times 10^6 \text{ N}) \cos 55.0^\circ = 4.30 \times 10^6 \text{ N}
\]

The \(y\) component of the net force is

\[
\Sigma F_y = T \sin 55.0^\circ - W = (7.50 \times 10^6 \text{ N}) \sin 55.0^\circ - 4.41 \times 10^6 \text{ N} = 1.73 \times 10^6 \text{ N}
\]
The magnitudes of the net force and of the acceleration are

\[ \Sigma F = \sqrt{\left( \Sigma F_x \right)^2 + \left( \Sigma F_y \right)^2} \]

\[ a = \frac{\sqrt{\left( \Sigma F_x \right)^2 + \left( \Sigma F_y \right)^2}}{m} = \frac{\sqrt{(4.30 \times 10^6 \text{ N})^2 + (1.73 \times 10^6 \text{ N})^2}}{4.50 \times 10^5 \text{ kg}} = 10.3 \text{ m/s}^2 \]

The direction of the acceleration is the same as the direction of the net force. Thus, it is directed above the horizontal at an angle of

\[ \theta = \tan^{-1}\left( \frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1}\left( \frac{1.73 \times 10^6 \text{ N}}{4.30 \times 10^6 \text{ N}} \right) = 21.9^\circ \]

14. **REASONING** The net force \( \Sigma F \) is specified by Newton’s second law as \( \Sigma F = ma \), where \( m \) is the mass of the ball, which is given, and \( a \) is the ball’s acceleration. The acceleration is not given. However, it can be obtained with the aid of \( v = v_0 + at \) (Equation 2.4), because the final velocity \( v \) and the initial velocity \( v_0 \) are known, as is the time \( t \).

**SOLUTION** Solving Equation 2.4 for the acceleration, we obtain

\[ a = \frac{v - v_0}{t} \]

Substituting this result into Newton’s second law we obtain

\[ \Sigma F = ma = m \left( \frac{v - v_0}{t} \right) = (0.38 \text{ kg}) \left[ \frac{(-2.0 \text{ m/s}) - (+2.1 \text{ m/s})}{3.3 \times 10^{-3} \text{ s}} \right] = -470 \text{ N} \]

The answer is negative, indicating that the force is directed away from the cushion.

15. **REASONING** The acceleration of the sky diver can be obtained directly from Newton’s second law as the net force divided by the sky diver’s mass. The net force is the vector sum of the sky diver’s weight and the drag force.

**SOLUTION** From Newton’s second law, \( \Sigma F = ma \) (Equation 4.1), the sky diver’s acceleration is

\[ a = \frac{\Sigma F}{m} \]
The free-body diagram shows the two forces acting on the sky diver, his weight $W$ and the drag force $f$. The net force is $\Sigma F = f - W$. Thus, the acceleration can be written as

$$a = \frac{f - W}{m}$$

The acceleration of the sky diver is

$$a = \frac{f - W}{m} = \frac{1027 \text{ N} - 915 \text{ N}}{93.4 \text{ kg}} = \frac{+1.20 \text{ m/s}^2}{93.4 \text{ kg}}$$

Note that the acceleration is positive, indicating that it points upward.

16. **REASONING** Since there is only one force acting on the man in the horizontal direction, it is the net force. According to Newton’s second law, Equation 4.1, the man must accelerate under the action of this force. The factors that determine this acceleration are (1) the magnitude and (2) the direction of the force exerted on the man, and (3) the mass of the man.

When the woman exerts a force on the man, the man exerts a force of equal magnitude, but opposite direction, on the woman (Newton’s third law). It is the only force acting on the woman in the horizontal direction, so, as is the case with the man, she must accelerate. The factors that determine her acceleration are (1) the magnitude and (2) the direction of the force exerted on her, and (3) the her mass.

**SOLUTION**

a. The acceleration of the man is, according to Equation 4.1, equal to the net force acting on him divided by his mass.

$$a_{\text{man}} = \frac{\Sigma F}{m} = \frac{45 \text{ N}}{82 \text{ kg}} = \frac{0.55 \text{ m/s}^2}{(\text{due east})}$$

b. The acceleration of the woman is equal to the net force acting on her divided by her mass.

$$a_{\text{woman}} = \frac{\Sigma F}{m} = \frac{45 \text{ N}}{48 \text{ kg}} = \frac{0.94 \text{ m/s}^2}{(\text{due west})}$$

17. **REASONING** According to Newton’s second law, the acceleration of the probe is $a = \Sigma F/m$. Using this value for the acceleration in Equation 2.8 and noting that the probe starts from rest ($v_0 = 0 \text{ m/s}$), we can write the distance traveled by the probe as

$$x = v_0 t + \frac{1}{2} at^2 = \frac{1}{2} \left( \frac{\Sigma F}{m} \right) t^2$$

This equation is the basis for our solution.
**SOLUTION** Since each engine produces the same amount of force or thrust \( T \), the net force is \( \Sigma F = 2T \) when the engines apply their forces in the same direction and \( \Sigma F = \sqrt{T^2 + T^2} = \sqrt{2}T \) when they apply their forces perpendicularly. Thus, we write the distances traveled in the two situations as follows:

\[
x = \frac{1}{2} \left( \frac{2T}{m} \right) t^2 \quad \text{and} \quad x = \frac{1}{2} \left( \frac{\sqrt{2}T}{m} \right) t^2_{\perp}
\]

Engines fired in the same direction

Engines fired perpendicularly

Since the distances are the same, we have

\[
\frac{1}{2} \left( \frac{2T}{m} \right) t^2 = \frac{1}{2} \left( \frac{\sqrt{2}T}{m} \right) t^2_{\perp}
\]

or \( \sqrt{2} t^2 = t^2_{\perp} \)

The firing time when the engines apply their forces perpendicularly is, then,

\[
t_{\perp} = \left( \frac{\sqrt{2}}{2} \right) t = \left( \frac{\sqrt{2}}{2} \right)(28 \text{ s}) = \frac{33}{s}
\]

18. **REASONING** For both the tug and the asteroid, Equation 2.8 \( (x = v_0 t + \frac{1}{2} at^2) \) applies with \( v_0 = 0 \) m/s, since both are initially at rest. In applying this equation, we must be careful and use the proper acceleration for each object. Newton’s second law indicates that the acceleration is given by \( a = \Sigma F/m \). In this expression, we note that the magnitudes of the net forces acting on the tug and the asteroid are the same, according to Newton’s action-reaction law. The masses of the tug and the asteroid are different, however. Thus, the distance traveled for either object is given by, where we use for \( \Sigma F \) only the magnitude of the pulling force

\[
x = v_0 t + \frac{1}{2} at^2 = \frac{1}{2} \left( \frac{\Sigma F}{m} \right) t^2
\]

**SOLUTION** Let \( L \) be the initial distance between the tug and the asteroid. When the two objects meet, the distances that each has traveled must add up to equal \( L \). Therefore,

\[
L = x_T + x_A = \frac{1}{2} a_T t^2 + \frac{1}{2} a_A t^2
\]

\[
L = \frac{1}{2} \left( \frac{\Sigma F}{m_T} \right) t^2 + \frac{1}{2} \left( \frac{\Sigma F}{m_A} \right) t^2 = \frac{1}{2} \Sigma F \left( \frac{1}{m_T} + \frac{1}{m_A} \right) t^2
\]

Solving for the time \( t \) gives

\[
t = \sqrt{\frac{2L}{\Sigma F \left( \frac{1}{m_T} + \frac{1}{m_A} \right)}} = \sqrt{\frac{2(450 \text{ m})}{\left( 490 \text{ N} \right) \left( \frac{1}{3500 \text{ kg}} + \frac{1}{6200 \text{ kg}} \right)}} = 64 \text{ s}
\]
We first determine the acceleration of the boat. Then, using Newton’s second law, we can find the net force $\sum \mathbf{F}$ that acts on the boat. Since two of the three forces are known, we can solve for the unknown force $\mathbf{F}_w$ once the net force $\sum \mathbf{F}$ is known.

**SOLUTION**

Let the direction due east be the positive $x$ direction and the direction due north be the positive $y$ direction. The $x$ and $y$ components of the initial velocity of the boat are then

$$v_{0x} = (2.00 \text{ m/s}) \cos 15.0^\circ = 1.93 \text{ m/s}$$

$$v_{0y} = (2.00 \text{ m/s}) \sin 15.0^\circ = 0.518 \text{ m/s}$$

Thirty seconds later, the $x$ and $y$ velocity components of the boat are

$$v_x = (4.00 \text{ m/s}) \cos 35.0^\circ = 3.28 \text{ m/s}$$

$$v_y = (4.00 \text{ m/s}) \sin 35.0^\circ = 2.29 \text{ m/s}$$

Therefore, according to Equations 3.3a and 3.3b, the $x$ and $y$ components of the acceleration of the boat are

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{3.28 \text{ m/s} - 1.93 \text{ m/s}}{30.0 \text{ s}} = 4.50 \times 10^{-2} \text{ m/s}^2$$

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{2.29 \text{ m/s} - 0.518 \text{ m/s}}{30.0 \text{ s}} = 5.91 \times 10^{-2} \text{ m/s}^2$$

Thus, the $x$ and $y$ components of the net force that act on the boat are

$$\sum F_x = ma_x = (325 \text{ kg}) (4.50 \times 10^{-2} \text{ m/s}^2) = 14.6 \text{ N}$$

$$\sum F_y = ma_y = (325 \text{ kg}) (5.91 \times 10^{-2} \text{ m/s}^2) = 19.2 \text{ N}$$

The following table gives the $x$ and $y$ components of the net force $\sum \mathbf{F}$ and the two known forces that act on the boat. The fourth row of that table gives the components of the unknown force $\mathbf{F}_w$.

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$-Component</th>
<th>$y$-Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \mathbf{F}$</td>
<td>14.6 N</td>
<td>19.2 N</td>
</tr>
<tr>
<td>$\mathbf{F}_1$</td>
<td>$(31.0 \text{ N}) \cos 15.0^\circ = 29.9 \text{ N}$</td>
<td>$(31.0 \text{ N}) \sin 15.0^\circ = 8.02 \text{ N}$</td>
</tr>
<tr>
<td>$\mathbf{F}_2$</td>
<td>$-(23.0 \text{ N}) \cos 15.0^\circ = -22.2 \text{ N}$</td>
<td>$-(23.0 \text{ N}) \sin 15.0^\circ = -5.95 \text{ N}$</td>
</tr>
</tbody>
</table>

$$\mathbf{F}_w = \sum \mathbf{F} - \mathbf{F}_1 - \mathbf{F}_2 = 14.6 \text{ N} - 29.9 \text{ N} + 22.2 \text{ N} = 6.9 \text{ N}$$

The magnitude of $\mathbf{F}_w$ is given by the Pythagorean theorem as
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The angle \( \theta \) that \( \mathbf{F}_W \) makes with the \( x \) axis is

\[
\theta = \tan^{-1} \left( \frac{17.1 \, \text{N}}{6.9 \, \text{N}} \right) = 68^\circ
\]

Therefore, the direction of \( \mathbf{F}_W \) is \( 68^\circ \), north of east.

20. \textit{REASONING} The gravitational force acting on each object is specified by Newton’s law of universal gravitation. The acceleration of each object when released can be determined with the aid of Newton’s second law. We recognize that the gravitational force is the only force acting on either object, so that it is the net force to use when applying the second law.

\textit{SOLUTION}

a. The magnitude of the gravitational force exerted on the rock by the earth is given by Equation 4.3 as

\[
F_{\text{rock}} = \frac{G m_{\text{earth}} m_{\text{rock}}}{r_{\text{earth}}^2} = \frac{(6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \, \text{kg})(5.0 \, \text{kg})}{(6.38 \times 10^6 \, \text{m})^2} = 49 \, \text{N}
\]

The magnitude of the gravitational force exerted on the pebble by the earth is

\[
F_{\text{pebble}} = \frac{G m_{\text{earth}} m_{\text{pebble}}}{r_{\text{earth}}^2} = \frac{(6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \, \text{kg})(3.0 \times 10^{-4} \, \text{kg})}{(6.38 \times 10^6 \, \text{m})^2} = 2.9 \times 10^{-3} \, \text{N}
\]

b. According to the second law, the magnitude of the acceleration of the rock is equal to the gravitational force exerted on the rock divided by its mass.

\[
a_{\text{rock}} = \frac{F_{\text{rock}}}{m_{\text{rock}}} = \frac{G m_{\text{earth}}}{r_{\text{earth}}^2} = \frac{(6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \, \text{kg})}{(6.38 \times 10^6 \, \text{m})^2} = 9.80 \, \text{m/s}^2
\]

According to the second law, the magnitude of the acceleration of the pebble is equal to the gravitational force exerted on the pebble divided by its mass.
21. REASONING AND SOLUTION
a. According to Equation 4.4, the weight of an object of mass \( m \) on the surface of Mars would be given by

\[
W = \frac{GM_M m}{R_M^2}
\]

where \( M_M \) is the mass of Mars and \( R_M \) is the radius of Mars. On the surface of Mars, the weight of the object can be given as \( W = mg \) (see Equation 4.5), so

\[
mg = \frac{GM_M m}{R_M^2}
\]

or

\[
g = \frac{GM_M}{R_M^2}
\]

Substituting values, we have

\[
g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(6.46 \times 10^{23} \text{ kg})}{(3.39 \times 10^6 \text{ m})^2} = 3.75 \text{ m/s}^2
\]

b. According to Equation 4.5,

\[
W = mg = (65 \text{ kg})(3.75 \text{ m/s}^2) = 2.4 \times 10^2 \text{ N}
\]

22. REASONING The magnitude of the gravitational force that each part exerts on the other is given by Newton’s law of gravitation as \( F = \frac{G m_1 m_2}{r^2} \). To use this expression, we need the masses \( m_1 \) and \( m_2 \) of the parts, whereas the problem statement gives the weights \( W_1 \) and \( W_2 \). However, the weight is related to the mass by \( W = mg \), so that for each part we know that \( m = W/g \).

SOLUTION The gravitational force that each part exerts on the other is

\[
F = \frac{G m_1 m_2}{r^2} = \frac{G (W_1/g)(W_2/g)}{r^2}
\]

\[
= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(11000 \text{ N})(3400 \text{ N})}{(9.80 \text{ m/s}^2)^2 (12 \text{ m})^2} = 1.8 \times 10^{-7} \text{ N}
\]
23. **REASONING** The earth exerts a gravitational force on the raindrop, and simultaneously the raindrop exerts a gravitational force on the earth. This gravitational force is equal in magnitude to the gravitational force that the earth exerts on the raindrop. The forces that the raindrop and the earth exert on each other are Newton’s third law (action–reaction) forces. Newton’s law of universal gravitation specifies the magnitude of both forces.

**SOLUTION**

a. The magnitude of the gravitational force exerted on the raindrop by the earth is given by Equation 4.3:

\[
F_{\text{raindrop}} = \frac{G m_{\text{earth}} m_{\text{raindrop}}}{r_{\text{earth}}^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(5.2 \times 10^{-7} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2} = 5.1 \times 10^{-6} \text{ N}
\]

b. The magnitude of the gravitational force exerted on the earth by the raindrop is

\[
F_{\text{earth}} = \frac{G m_{\text{earth}} m_{\text{raindrop}}}{r_{\text{earth}}^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(5.2 \times 10^{-7} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2} = 5.1 \times 10^{-6} \text{ N}
\]

24. **REASONING** Newton’s law of gravitation shows how the weight \(W\) of an object of mass \(m\) is related to the mass \(M\) and radius \(r\) of the planet on which the object is located: \(W = GMm/r^2\). In this expression \(G\) is the universal gravitational constant. Using the law of gravitation, we can express the weight of the object on each planet, set the two weights equal, and obtain the desired ratio.

**SOLUTION** According to Newton’s law of gravitation, we have

\[
\frac{GM_A m}{r_A^2} = \frac{GM_B m}{r_B^2}
\]

The mass \(m\) of the object, being an intrinsic property, is the same on both planets and can be eliminated algebraically from this equation. The universal gravitational constant can likewise be eliminated algebraically. As a result, we find that

\[
\frac{M_A}{r_A^2} = \frac{M_B}{r_B^2} \quad \text{or} \quad \frac{M_A}{M_B} = \frac{r_B^2}{r_A^2}
\]

\[
\frac{r_A}{r_B} = \sqrt{\frac{M_A}{M_B}} = \sqrt{0.60} = 0.77
\]
25. **SSM REASONING** Newton’s law of universal gravitation indicates that the gravitational force that each uniform sphere exerts on the other has a magnitude that is inversely proportional to the square of the distance between the centers of the spheres. Therefore, the maximum gravitational force between two uniform spheres occurs when the centers of the spheres are as close together as possible, that is, when the spherical surfaces touch. Then the distance between the centers of the spheres is the sum of the two radii.

**SOLUTION** When the bowling ball and the billiard ball are touching, the distance between their centers is \( r = r_{\text{Bowling}} + r_{\text{Billiard}} \). Using this expression in Newton’s law of universal gravitation gives

\[
F = \frac{G m_{\text{Bowling}} m_{\text{Billiard}}}{r^2} = \frac{G m_{\text{Bowling}} m_{\text{Billiard}}}{(r_{\text{Bowling}} + r_{\text{Billiard}})^2}
\]

\[
= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.2 \text{ kg})(0.38 \text{ kg})}{(0.11 \text{ m} + 0.028 \text{ m})^2} = 9.6 \times 10^{-9} \text{ N}
\]

26. **REASONING** As discussed in Conceptual Example 7, the same net force is required on the moon as on the earth. This net force is given by Newton’s second law as \( \Sigma F = ma \), where the mass \( m \) is the same in both places. Thus, from the given mass and acceleration, we can calculate the net force. On the moon, the net force comes about due to the drive force and the opposing frictional force. Since the drive force is given, we can find the frictional force.

**SOLUTION** Newton’s second law, with the direction of motion taken as positive, gives

\[
\Sigma F = ma \quad \text{or} \quad (1430 \text{ N}) - f = (5.90 \times 10^3 \text{ kg})(0.220 \text{ m/s}^2)
\]

Solving for the frictional force \( f \), we find

\[
f = (1430 \text{ N}) - (5.90 \times 10^3 \text{ kg})(0.220 \text{ m/s}^2) = 130 \text{ N}
\]

27. **SSM REASONING AND SOLUTION** According to Equations 4.4 and 4.5, the weight of an object of mass \( m \) at a distance \( r \) from the center of the earth is

\[
mg = \frac{G M_E m}{r^2}
\]

In a circular orbit that is \( 3.59 \times 10^7 \text{ m} \) above the surface of the earth (radius = \( 6.38 \times 10^6 \text{ m} \), mass = \( 5.98 \times 10^{24} \text{ kg} \)), the total distance from the center of the earth is \( r = 3.59 \times 10^7 \text{ m} + 6.38 \times 10^6 \text{ m} \). Thus the acceleration \( g \) due to gravity is
\[ g = \frac{GM_E}{r^2} = \frac{(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^2 \text{kg})}{(3.59 \times 10^7 \text{m} + 6.38 \times 10^6 \text{m})^2} = 0.223 \text{ m/s}^2 \]

28. **REASONING AND SOLUTION**

The magnitude of the net force acting on the moon is found by the Pythagorean theorem to be

\[ F = \sqrt{F_{SM}^2 + F_{EM}^2} \]

Newton's law of gravitation applied to the sun-moon (the units have been suppressed)

\[ F_{SM} = \frac{Gm_sm_M}{r_{SM}^2} = \frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})(7.35 \times 10^{22})}{(1.50 \times 10^{11})^2} = 4.34 \times 10^{20} \text{ N} \]

A similar application to the earth-moon gives

\[ F_{EM} = \frac{Gm_em_M}{r_{EM}^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(7.35 \times 10^{22})}{(3.85 \times 10^{8})^2} = 1.98 \times 10^{20} \text{ N} \]

The net force on the moon is then

\[ F = \sqrt{(4.34 \times 10^{20} \text{ N})^2 + (1.98 \times 10^{20} \text{ N})^2} = 4.77 \times 10^{20} \text{ N} \]

29. **REASONING** Each particle experiences two gravitational forces, one due to each of the remaining particles. To get the net gravitational force, we must add the two contributions, taking into account the directions. The magnitude of the gravitational force that any one particle exerts on another is given by Newton’s law of gravitation as \( F = Gm_1m_2 / r^2 \). Thus, for particle A, we need to apply this law to its interaction with particle B and with particle C. For particle B, we need to apply the law to its interaction with particle A and with particle C. Lastly, for particle C, we must apply the law to its interaction with particle A and with particle B. In considering the directions, we remember that the gravitational force between two particles is always a force of attraction.

**SOLUTION** We begin by calculating the magnitude of the gravitational force for each pair of particles:
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\[
F_{AB} = \frac{Gm_A m_B}{r^2} = \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \cdot (363 \text{ kg})(517 \text{ kg})}{(0.500 \text{ m})^2} = 5.007 \times 10^{-5} \text{ N}
\]

\[
F_{BC} = \frac{Gm_B m_C}{r^2} = \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \cdot (517 \text{ kg})(154 \text{ kg})}{(0.500 \text{ m})^2} = 8.497 \times 10^{-5} \text{ N}
\]

\[
F_{AC} = \frac{Gm_A m_C}{r^2} = \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \cdot (363 \text{ kg})(154 \text{ kg})}{(0.500 \text{ m})^2} = 6.629 \times 10^{-6} \text{ N}
\]

In using these magnitudes we take the direction to the right as positive.

a. Both particles B and C attract particle A to the right, the net force being

\[
F_A = F_{AB} + F_{AC} = 5.007 \times 10^{-5} \text{ N} + 6.629 \times 10^{-6} \text{ N} = 5.67 \times 10^{-5} \text{ N, right}
\]

b. Particle C attracts particle B to the right, while particle A attracts particle B to the left, the net force being

\[
F_B = F_{BC} - F_{AB} = 8.497 \times 10^{-5} \text{ N} - 5.007 \times 10^{-5} \text{ N} = 3.49 \times 10^{-5} \text{ N, right}
\]

c. Both particles A and B attract particle C to the left, the net force being

\[
F_C = F_{AC} + F_{BC} = 6.629 \times 10^{-6} \text{ N} + 8.497 \times 10^{-5} \text{ N} = 9.16 \times 10^{-5} \text{ N, left}
\]

30. REASONING The weight of a person on the earth is the gravitational force \( F_{\text{earth}} \) that it exerts on the person. The magnitude of this force is given by Equation 4.3 as

\[
F_{\text{earth}} = G \frac{m_{\text{earth}} m_{\text{person}}}{r_{\text{earth}}^2}
\]

where \( r_{\text{earth}} \) is the distance from the center of the earth to the person. In a similar fashion, the weight of the person on another planet is

\[
F_{\text{planet}} = G \frac{m_{\text{planet}} m_{\text{person}}}{r_{\text{planet}}^2}
\]

We will use these two expressions to obtain the weight of the traveler on the planet.

SOLUTION Dividing \( F_{\text{planet}} \) by \( F_{\text{earth}} \) we have
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\[ \frac{F_{\text{planet}}}{F_{\text{earth}}} = \frac{G \frac{m_{\text{planet}} m_{\text{person}}}{r_{\text{planet}}^2}}{G \frac{m_{\text{earth}} m_{\text{person}}}{r_{\text{earth}}^2}} = \left( \frac{m_{\text{planet}}}{m_{\text{earth}}} \right) \left( \frac{r_{\text{earth}}}{r_{\text{planet}}} \right)^2 \]

or

\[ F_{\text{planet}} = F_{\text{earth}} \left( \frac{m_{\text{planet}}}{m_{\text{earth}}} \right) \left( \frac{r_{\text{earth}}}{r_{\text{planet}}} \right)^2 \]

Since we are given that \( \frac{m_{\text{planet}}}{m_{\text{earth}}} = 3 \) and \( \frac{r_{\text{earth}}}{r_{\text{planet}}} = \frac{1}{2} \), the weight of the space traveler on the planet is

\[ F_{\text{planet}} = (540.0 \text{ N})(3) \left( \frac{1}{2} \right)^2 = 405.0 \text{ N} \]

31. **REASONING** According to Equation 4.4, the weights of an object of mass \( m \) on the surfaces of planet A (mass = \( M_A \), radius = \( R \)) and planet B (mass = \( M_B \), radius = \( R \)) are

\[ W_A = \frac{G M_A m}{R^2} \quad \text{and} \quad W_B = \frac{G M_B m}{R^2} \]

The difference between these weights is given in the problem.

**SOLUTION** The difference in weights is

\[ W_A - W_B = \frac{G M_A m}{R^2} - \frac{G M_B m}{R^2} = \frac{G m}{R^2} (M_A - M_B) \]

Rearranging this result, we find

\[ M_A - M_B = \frac{(W_A - W_B) R^2}{G m} = \frac{(3620 \text{ N})(1.33 \times 10^7 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5450 \text{ kg})} = 1.76 \times 10^{24} \text{ kg} \]

32. **REASONING** The following drawing shows the point between the earth and the moon where the gravitational force exerted on the spacecraft by the earth balances that exerted by the moon.
The magnitude of the gravitational force exerted on the spacecraft by the earth is

$$F_{\text{earth}} = G \frac{m_{\text{earth}} m_{\text{spacecraft}}}{r^2}$$

whereas that exerted on the spacecraft by the moon is

$$F_{\text{moon}} = G \frac{m_{\text{moon}} m_{\text{spacecraft}}}{(r_{\text{earth-moon}} - r)^2}$$

By setting these two expressions equal to each other (since the gravitational forces balance), we will be able to find the distance $r$.

**SOLUTION** Setting $F_{\text{earth}}$ equal to $F_{\text{moon}}$, we have

$$G \frac{m_{\text{earth}} m_{\text{spacecraft}}}{r^2} = G \frac{m_{\text{moon}} m_{\text{spacecraft}}}{(r_{\text{earth-moon}} - r)^2}$$

Solving this expression for $r$ gives

$$r = \frac{(r_{\text{earth-moon}}) \sqrt{m_{\text{earth}}}}{1 + \sqrt{m_{\text{earth}}/m_{\text{moon}}}} = \frac{(3.85 \times 10^8 \text{ m}) \sqrt{81.4}}{1 + \sqrt{81.4}} = 3.47 \times 10^8 \text{ m}$$

33. **REASONING** The gravitational attraction between the planet and the moon is governed by Newton’s law of gravitation $F = GMm/r^2$ (Equation 4.3), where $M$ is the planet’s mass and $m$ is the moon’s mass. Because the magnitude of this attractive force varies inversely with the square of the distance $r$ between the center of the moon and the center of the planet, the maximum force $F_{\text{max}}$ occurs at the minimum distance $r_{\text{min}}$, and the minimum force $F_{\text{min}}$ at
the maximum distance \( r_{\text{max}} \). The problem text states that \( F_{\text{max}} \) exceeds \( F_{\text{min}} \) by 11\%, or \( F_{\text{max}} = 1.11 F_{\text{min}} \). This expression can be rearranged to give the ratio of the forces: \( F_{\text{max}} / F_{\text{min}} = 1.11 \). We will use Equation 4.3 to compute the desired distance ratio in terms of this force ratio.

**SOLUTION** From Equation 4.3, the ratio of the maximum gravitational force to the minimum gravitational force is

\[
\frac{F_{\text{max}}}{F_{\text{min}}} = \frac{GMm}{r_{\text{max}}^2} = \frac{1}{r_{\text{min}}^2} = \frac{1}{r_{\text{max}}^2}
\]

Taking the square root of both sides of this expression and substituting the ratio of forces \( F_{\text{max}} / F_{\text{min}} = 1.11 \) yields the ratio of distances:

\[
\frac{r_{\text{max}}}{r_{\text{min}}} = \sqrt{\frac{F_{\text{max}}}{F_{\text{min}}} = \sqrt{1.11} = 1.05}
\]

The moon’s maximum distance from the center of the planet is therefore about 5\% larger than its minimum distance.

---

**34. REASONING** This situation is exactly like holding a ball in your hand and releasing it. The ball falls downward from rest and has the acceleration due to gravity. Because of this acceleration, the ball picks up speed as it falls. The only difference on the neutron star is that the acceleration due to gravity is not 9.8 m/s\(^2\) as it is on earth. Assuming the acceleration remains constant during the fall, the equations of kinematics can be used.

**SOLUTION** Since the gravitational force is assumed to be constant, the acceleration will be constant and the final velocity \( v \) of the object can be calculated from \( v^2 = v_0^2 + 2ay \) (Equation 2.9). To obtain the acceleration \( a \), we turn to Equations 4.4 and 4.5 and find that the acceleration due to gravity at the surface of the neutron star is

\[
a = \frac{Gm}{r^2} = \frac{(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2)(2.0 \times 10^{30} \text{kg})}{(5.0 \times 10^3 \text{m})^2} = 5.3 \times 10^{12} \text{ m/s}^2
\]

Solving Equation 2.9 for \( v \) (with \( v_0 = 0 \text{ m/s} \) since the object falls from rest) and using \( a = 5.3 \times 10^{12} \text{ m/s}^2 \) yield

\[
v = \sqrt{2ay} = \sqrt{2(5.3 \times 10^{12} \text{ m/s}^2)(0.010 \text{ m})} = 3.3 \times 10^5 \text{ m/s}
\]
35. **Reasoning** The gravitational force that the sun exerts on a person standing on the earth is given by Equation 4.3 as \( F_{\text{sun}} = \frac{G M_{\text{sun}} m}{r_{\text{sun-earth}}^2} \), where \( M_{\text{sun}} \) is the mass of the sun, \( m \) is the mass of the person, and \( r_{\text{sun-earth}} \) is the distance from the sun to the earth. Likewise, the gravitational force that the moon exerts on a person standing on the earth is given by \( F_{\text{moon}} = \frac{G M_{\text{moon}} m}{r_{\text{moon-earth}}^2} \), where \( M_{\text{moon}} \) is the mass of the moon and \( r_{\text{moon-earth}} \) is the distance from the moon to the earth. These relations will allow us to determine whether the sun or the moon exerts the greater gravitational force on the person.

**Solution** Taking the ratio of \( F_{\text{sun}} \) to \( F_{\text{moon}} \), and using the mass and distance data from the inside of the text’s front cover, we find

\[
\frac{F_{\text{sun}}}{F_{\text{moon}}} = \frac{\frac{G M_{\text{sun}} m}{r_{\text{sun-earth}}^2}}{\frac{G M_{\text{moon}} m}{r_{\text{moon-earth}}^2}} = \left( \frac{M_{\text{sun}}}{M_{\text{moon}}} \right) \left( \frac{r_{\text{moon-earth}}}{r_{\text{sun-earth}}} \right)^2
\]

\[
= \left( \frac{1.99 \times 10^{30} \text{ kg}}{7.35 \times 10^{22} \text{ kg}} \right) \left( \frac{3.85 \times 10^8 \text{ m}}{1.50 \times 10^{11} \text{ m}} \right)^2 = 178
\]

Therefore, the sun exerts the greater gravitational force.

36. **Reasoning** According to Equation 4.4, the weights of an object of mass \( m \) on the surface of a planet (mass = \( M \), radius = \( R \)) and at a height \( H \) above the surface are

\[
W = \frac{G M m}{R^2} \quad \text{and} \quad W_H = \frac{G M m}{(R + H)^2}
\]

The fact that \( W \) is one percent less than \( W_H \) tells us that the \( W_H/W = 0.9900 \), which is the starting point for our solution.

**Solution** The ratio \( W_H/W \) is

\[
\frac{W_H}{W} = \left( \frac{R + H}{R} \right)^2 = \frac{R^2}{(R + H)^2} = \frac{1}{(1 + H/R)^2} = 0.9900
\]

Solving for \( H/R \) gives

\[
1 + \frac{H}{R} = \sqrt{0.9900} \quad \text{or} \quad \frac{H}{R} = \frac{1}{\sqrt{0.9900}} = 0.0050
\]
37. **REASONING** We place the third particle (mass \( m_3 \)) as shown in the following drawing:

![Diagram of three particles](image)

The magnitude of the gravitational force that one particle exerts on another is given by Newton’s law of gravitation as \( F = Gm_1m_2/r^2 \). Before the third particle is in place, this law indicates that the force on each particle has a magnitude \( F_{\text{before}} = Gm_2m/L^2 \). After the third particle is in place, each of the first two particles experiences a greater net force, because the third particle also exerts a gravitational force on them.

**SOLUTION** For the particle of mass \( m \), we have

\[
\frac{F_{\text{after}}}{F_{\text{before}}} = \frac{Gm_3 + Gm_2}{D^2 + L^2} = \frac{L^2m_3}{2mD^2} + 1
\]

For the particle of mass \( 2m \), we have

\[
\frac{F_{\text{after}}}{F_{\text{before}}} = \frac{G2mm_3 + Gm_2}{(L-D)^2 + L^2} = \frac{L^2m_3}{m(L-D)^2} + 1
\]

Since \( F_{\text{after}}/F_{\text{before}} = 2 \) for both particles, we have

\[
\frac{L^2m_3}{2mD^2} + 1 = \frac{L^2m_3}{m(L-D)^2} + 1 \quad \text{or} \quad 2D^2 = (L-D)^2
\]

Expanding and rearranging this result gives \( D^2 + 2LD - L^2 = 0 \), which can be solved for \( D \) using the quadratic formula:

\[
D = \frac{-2L \pm \sqrt{(2L)^2 - 4(1)(-L^2)}}{2(1)} = 0.414L \quad \text{or} \quad -2.414L
\]

The negative solution is discarded because the third particle lies on the +x axis between \( m \) and \( 2m \). Thus, \( D = 0.414L \).

38. **REASONING** In each case the object is in equilibrium. According to Equation 4.9b, \( \Sigma F_y = 0 \), the net force acting in the \( y \) (vertical) direction must be zero. The net force is composed of the weight of the object(s) and the normal force exerted on them.
SOLUTION

a. There are three vertical forces acting on the crate: an upward normal force \( F_N \) that the floor exerts, the weight \(-m_1g\) of the crate, and the weight \(-m_2g\) of the person standing on the crate. Since the weights act downward, they are assigned negative numbers. Setting the sum of these forces equal to zero gives

\[
\sum F_y = F_N + (-m_1g) + (-m_2g) = 0
\]

The magnitude of the normal force is

\[
F_N = m_1g + m_2g = (35 \text{ kg} + 65 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N}
\]

b. There are only two vertical forces acting on the person: an upward normal force \( F_N \) that the crate exerts and the weight \(-m_2g\) of the person. Setting the sum of these forces equal to zero gives

\[
\sum F_y = F_N + (-m_2g) = 0
\]

The magnitude of the normal force is

\[
F_N = m_2g = (65 \text{ kg})(9.80 \text{ m/s}^2) = 640 \text{ N}
\]

39. SSM REASONING  In order to start the crate moving, an external agent must supply a force that is at least as large as the maximum value \( f_s^{\text{MAX}} = \mu_s F_N \), where \( \mu_s \) is the coefficient of static friction (see Equation 4.7). Once the crate is moving, the magnitude of the frictional force is very nearly constant at the value \( f_k = \mu_k F_N \), where \( \mu_k \) is the coefficient of kinetic friction (see Equation 4.8). In both cases described in the problem statement, there are only two vertical forces that act on the crate; they are the upward normal force \( F_N \), and the downward pull of gravity (the weight) \( mg \). Furthermore, the crate has no vertical acceleration in either case. Therefore, if we take upward as the positive direction, Newton's second law in the vertical direction gives \( F_N - mg = 0 \), and we see that, in both cases, the magnitude of the normal force is \( F_N = mg \).

SOLUTION

a. Therefore, the applied force needed to start the crate moving is

\[
f_s^{\text{MAX}} = \mu_s mg = (0.760)(60.0 \text{ kg})(9.80 \text{ m/s}^2) = 447 \text{ N}
\]

b. When the crate moves in a straight line at constant speed, its velocity does not change, and it has zero acceleration. Thus, Newton's second law in the horizontal direction becomes
\( P - f_k = 0 \), where \( P \) is the required pushing force. Thus, the applied force required to keep the crate sliding across the dock at a constant speed is

\[
P = f_k = \mu_k m g = (0.410)(60.0 \text{ kg})(9.80 \text{ m/s}^2) = 241 \text{ N}
\]

40. **REASONING AND SOLUTION** According to Equation 3.3b, the acceleration of the astronaut is \( a_y = (v_y - v_{0y})/t = v_y / t \), where we have used the fact that \( v_{0y} = 0 \text{ m/s} \) since the rocket blasts off from rest. The apparent weight and the true weight of the astronaut are related according to Equation 4.6. Direct substitution gives

\[
F_N = mg + ma_y = m \left( g + a_y \right) = m \left( g + \frac{v_y}{t} \right)
\]

\[
= (57 \text{ kg}) \left( 9.80 \text{ m/s}^2 + \frac{45 \text{ m/s}}{15 \text{ s}} \right) = 7.3 \times 10^2 \text{ N}
\]

41. **REASONING** As the drawing shows, the normal force \( F_N \) points perpendicular to the hill, while the weight \( W \) points vertically down. Since the car does not leave the surface of the hill, there is no acceleration in this perpendicular direction. Therefore, the magnitude of the perpendicular component of the weight \( W \cos \theta \) must equal the magnitude of the normal force, \( F_N = W \cos \theta \). Thus, the magnitude of the normal force is less than the magnitude of the weight. As the hill becomes steeper, \( \theta \) increases, and \( \cos \theta \) decreases. Consequently, the normal force decreases as the hill becomes steeper. The magnitude of the normal force does not depend on whether the car is traveling up or down the hill.

**SOLUTION**

a. From the **REASONING**, we have that \( F_N = W \cos \theta \). The ratio of the magnitude of the normal force to the magnitude \( W \) of the weight is

\[
\frac{F_N}{W} = \frac{W \cos \theta}{W} = \cos 15^\circ = 0.97
\]

b. When the angle is \( 35^\circ \), the ratio is

\[
\frac{F_N}{W} = \frac{W \cos \theta}{W} = \cos 35^\circ = 0.82
\]
42. **REASONING AND SOLUTION**  The apparent weight is
\[ F_N = m_w(g + a) \]

We need to find the acceleration \( a \). Let \( T \) represent the force applied by the hoisting cable. Newton's second law applied to the elevator gives
\[ T - (m_w + m_e)g = (m_w + m_e)a \]

Solving for \( a \) gives
\[ a = \frac{T - (m_w + m_e)g}{m_w + m_e} = \frac{9410 \text{ N}}{60.0 \text{ kg} + 815 \text{ kg}} = 9.80 \text{ m/s}^2 = 0.954 \text{ m/s}^2 \]

Now the apparent weight is
\[ F_N = (60.0 \text{ kg})(9.80 \text{ m/s}^2 + 0.954 \text{ m/s}^2) = 645 \text{ N} \]

43. **REASONING**  As shown in the free-body diagram below, three forces act on the car: the static frictional force \( f_s \) (directed up the hill), the normal force \( F_N \) (directed perpendicular to the road), and its weight \( mg \). As it sits on the hill, the car has an acceleration of zero \( (a_x = a_y = 0 \text{ m/s}^2) \). Therefore, the net force acting on the car in the \( x \) direction must be zero \( (\Sigma F_x = 0) \) and the net force in the \( y \) direction must be zero \( (\Sigma F_y = 0) \). These two relations will allow us to find the normal force and the static frictional force.

**SOLUTION**

a. Applying Newton’s second law to the \( y \) direction \( (\Sigma F_y = 0) \) yields
\[ \Sigma F_y = +F_N - mg \cos 15^\circ = 0 \]  \hspace{1cm} (4.2b)

where the term \( -mg \cos 15^\circ \) is the \( y \) component of the car’s weight (negative, because this component points along the negative \( y \) axis). Solving for the magnitude \( F_N \) of the normal force, we obtain
\[ F_N = mg \cos 15^\circ = (1700 \text{ kg})(9.80 \text{ m/s}^2) \cos 15^\circ = 1.6 \times 10^4 \text{ N} \]

b. Applying Newton’s second law to the \( x \) direction \( (\Sigma F_x = 0) \) gives
\[ \Sigma F_x = +mg \sin 15^\circ - f_s = 0 \]  \hspace{1cm} (4.2a)

where the term \( mg \sin 15^\circ \) is the \( x \) component of the car’s weight. Solving this expression for the static frictional force gives
\[ f_s = mg \sin 15^\circ = (1700 \text{ kg})(9.80 \text{ m/s}^2) \sin 15^\circ = 4.3 \times 10^3 \text{ N} \]
44. **REASONING**
   a. Since the refrigerator does not move, the static frictional force must be equal in magnitude, but opposite in direction, to the horizontal pushing force that the person exerts on the refrigerator.

   b. The magnitude of the maximum static frictional force is given by Equation 4.7 as 
   \[ f_s^{\text{MAX}} = \mu_s F_N \]. This is also the largest possible force that the person can exert on the refrigerator before it begins to move. Thus, the factors that determine this force magnitude are the coefficient of static friction \( \mu_s \) and the magnitude \( F_N \) of the normal force (which is equal to the weight of the refrigerator in this case).

**SOLUTION**
   a. Since the refrigerator does not move, it is in equilibrium, and the magnitude of the static frictional force must be equal to the magnitude of the horizontal pushing force. Thus, the magnitude of the static frictional force is \( 267 \text{ N} \). The direction of this force must be opposite to that of the pushing force, so the static frictional force is in the \( +x \) direction.

   b. The magnitude of the largest pushing force is given by Equation 4.7 as
   \[ f_s^{\text{MAX}} = \mu_s F_N = \mu_s mg = (0.65)(57 \text{ kg})(9.80 \text{ m/s}^2) = 360 \text{ N} \]

45. **SSM REASONING** In each of the three cases under consideration the kinetic frictional force is given by \( f_k = \mu_k F_N \). However, the normal force \( F_N \) varies from case to case. To determine the normal force, we use Equation 4.6 \( F_N = mg + ma \) and thereby take into account the acceleration of the elevator. The normal force is greatest when the elevator accelerates upward (\( a \) positive) and smallest when the elevator accelerates downward (\( a \) negative).

**SOLUTION**
   a. When the elevator is stationary, its acceleration is \( a = 0 \text{ m/s}^2 \). Using Equation 4.6, we can express the kinetic frictional force as
   \[ f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a) \]
   \[ = (0.360)(6.00 \text{ kg})\left[(9.80 \text{ m/s}^2) + (0 \text{ m/s}^2)\right] = 21.2 \text{ N} \]

   b. When the elevator accelerates upward, \( a = +1.20 \text{ m/s}^2 \). Then,
   \[ f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a) \]
   \[ = (0.360)(6.00 \text{ kg})\left[(9.80 \text{ m/s}^2) + (1.20 \text{ m/s}^2)\right] = 23.8 \text{ N} \]
c. When the elevator accelerates downward, \( a = -1.20 \text{ m/s}^2 \). Then,

\[
f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a)
\]

\[
= (0.360)(6.00 \text{ kg}) \left[ (9.80 \text{ m/s}^2) + (-1.20 \text{ m/s}^2) \right] = 18.6 \text{ N}
\]

46. **REASONING** It is the static friction force that accelerates the cup when the plane accelerates. The maximum possible magnitude of this force will determine the maximum acceleration, according to Newton’s second law.

**SOLUTION** According to Newton’s second law and Equation 4.7 for the maximum static frictional force, we have

\[
\Sigma F = f_{\text{max}}^s = \mu_s F_N = \mu_s mg = ma
\]

In this result, we have used the fact that the magnitude of the normal force is \( F_N = mg \), since the plane is flying horizontally and the normal force acting on the cup balances the cup’s weight. Solving for the acceleration \( a \) gives

\[
a = \mu_s g = (0.30)(9.80 \text{ m/s}^2) = 2.9 \text{ m/s}^2
\]

47. **REASONING** The magnitude of the kinetic frictional force is given by Equation 4.8 as the coefficient of kinetic friction times the magnitude of the normal force. Since the slide into second base is horizontal, the normal force is vertical. It can be evaluated by noting that there is no acceleration in the vertical direction and, therefore, the normal force must balance the weight.

To find the player’s initial velocity \( v_0 \), we will use kinematics. The time interval for the slide into second base is given as \( t = 1.6 \text{ s} \). Since the player comes to rest at the end of the slide, his final velocity is \( v = 0 \text{ m/s} \). The player’s acceleration \( a \) can be obtained from Newton’s second law, since the net force is the kinetic frictional force, which is known from part (a), and the mass is given. Since \( t, v, \) and \( a \) are known and we seek \( v_0 \), the appropriate kinematics equation is Equation 2.4 \((v = v_0 + at)\).

**SOLUTION**

a. Since the normal force \( F_N \) balances the weight \( mg \), we know that \( F_N = mg \). Using this fact and Equation 4.8, we find that the magnitude of the kinetic frictional force is

\[
f_k = \mu_k F_N = \mu_k mg = (0.49)(81 \text{ kg})(9.8 \text{ m/s}^2) = 390 \text{ N}
\]
b. Solving Equation 2.4 \((v = v_0 + at)\) for \(v_0\) gives \(v_0 = v - at\). Taking the direction of the player’s slide to be the positive direction, we use Newton’s second law and Equation 4.8 for the kinetic frictional force to write the acceleration \(a\) as follows:

\[
 a = \frac{\Sigma F}{m} = -\mu_k mg = -\mu_k g
\]

The acceleration is negative, because it points opposite to the player’s velocity, since the player slows down during the slide. Thus, we find for the initial velocity that

\[
v_0 = v - (-\mu_k g)t = 0 \text{ m/s} \left[ -\left( 0.49 \left( 9.8 \text{ m/s}^2 \right) \right) (1.6 \text{ s}) \right] = +7.7 \text{ m/s}
\]

48. **REASONING AND SOLUTION** The deceleration produced by the frictional force is

\[
 a = -\frac{\vec{f}_k}{m} = -\mu_k mg = -\mu_k g
\]

The speed of the automobile after 1.30 s have elapsed is given by Equation 2.4 as

\[
v = v_0 + at = v_0 + (-\mu_k g)t = 16.1 \text{ m/s} - (0.720)(9.80 \text{ m/s}^2)(1.30 \text{ s}) = 6.9 \text{ m/s}
\]

49. **SSM REASONING AND SOLUTION** The free-body diagram is shown at the right. The forces that act on the picture are the pressing force \(P\), the normal force \(F_N\) exerted on the picture by the wall, the weight \(mg\) of the picture, and the force of static friction \(f_{s\text{MAX}}\). The maximum magnitude for the frictional force is given by Equation 4.7: \(f_{s\text{MAX}} = \mu_s F_N\). The picture is in equilibrium, and, if we take the directions to the right and up as positive, we have in the \(x\) direction

\[
 \sum F_x = P - F_N = 0 \quad \text{or} \quad P = F_N
\]

and in the \(y\) direction

\[
 \sum F_y = f_{s\text{MAX}} - mg = 0 \quad \text{or} \quad f_{s\text{MAX}} = mg
\]

Therefore,

\[
 f_{s\text{MAX}} = \mu_s F_N = mg
\]

But since \(F_N = P\), we have

\[
 \mu_s P = mg
\]

Solving for \(P\), we have

\[
 P = \frac{mg}{\mu_s} = \frac{(1.10 \text{ kg})(9.80 \text{ m/s}^2)}{0.660} = 16.3 \text{ N}
\]
50. **REASONING** We assume the car accelerates in the +x direction. The air resistance force $f_A$ opposes the car’s motion (see the free-body diagram). The frictional force is static, because the tires do not slip, and points in the direction of the car’s acceleration. The reason for this is that without friction the car’s wheels would simply spin in place, and the car’s acceleration would be severely limited. The frictional force has its maximum value $f_s^{\text{MAX}}$ because we seek the maximum acceleration before slipping occurs. Applying Newton’s second law (\(\Sigma F_x = ma_x\), Equation 4.2a) to the horizontal motion gives

\[
f_s^{\text{MAX}} - f_A = ma_x
\]

where $a_x$ is the maximum acceleration we seek. The air resistance force $f_A$ is given, and we will find the maximum static frictional force from Equation 4.7 \(f_s^{\text{MAX}} = \mu F_N\). Because the car’s acceleration has no vertical component, the net vertical force acting on the car must be zero, so the upward normal force $F_N$ must balance the two downward forces, the car’s weight $W$ and the downforce $D$:

\[
F_N = W + D = mg + D
\]

**SOLUTION** According to Equation 4.7 \(f_s^{\text{MAX}} = \mu F_N\) and Equation (2), the maximum static frictional force the track can exert on the car is

\[
f_s^{\text{MAX}} = \mu F_N = \mu (mg + D)
\]

Now solving Equation (1) for the car’s acceleration $a_x$ and then substituting Equation (3) for the static frictional force $f_s^{\text{MAX}}$, we obtain

\[
a_x = \frac{f_s^{\text{MAX}} - f_A}{m} = \frac{\mu_s (mg + D) - f_A}{m}
\]

\[
= \frac{(0.87)\left[(690 \text{ kg})(9.80 \text{ m/s}^2) + 4060 \text{ N}\right] - 1190 \text{ N}}{690 \text{ kg}} = 12 \text{ m/s}^2
\]
51. **REASONING** The free-body diagram for the box is shown on the left in the following drawing. On the right the same drawing is repeated, except that the pushing force $P$ is resolved into its horizontal and vertical components.

Since the block is moving at a constant velocity, it has no acceleration, and Newton’s second law indicates that the net vertical and net horizontal forces must separately be zero.

**SOLUTION** Taking upward and to the right as the positive directions, we write the zero net vertical and horizontal forces as follows:

\[
\begin{align*}
F_N - mg - P \sin \theta &= 0 \\
P \cos \theta - f_k &= 0
\end{align*}
\]

From the equation for the horizontal forces, we have $P \cos \theta = f_k$. But the kinetic frictional force is $f_k = \mu_k F_N$. Furthermore, from the equation for the vertical forces, we have $F_N = mg + P \sin \theta$. With these substitutions, we obtain

\[
P \cos \theta = f_k = \mu_k F_N = \mu_k (mg + P \sin \theta)
\]

Solving for $P$ gives

\[
P = \frac{\mu_k mg}{\cos \theta - \mu_k \sin \theta}
\]

The necessary pushing force becomes infinitely large when the denominator in this expression is zero. Hence, we find that $\cos \theta - \mu_k \sin \theta = 0$, which can be rearranged to show that

\[
\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{1}{\mu_k} \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{1}{0.41} \right) = 68^\circ
\]
52. **REASONING** The free-body diagram for the helicopter is shown in the drawing. Since the velocity is constant, the acceleration is zero and the helicopter is at equilibrium. Therefore, according to Newton’s second law, the net force acting on the helicopter is zero.

**SOLUTION** Since the net force is zero, the components of the net force in the vertical and horizontal directions are separately zero. Referring to the free-body diagram, we can see, then, that

\[
\begin{align*}
L \cos 21.0^\circ - W &= 0 \\
L \sin 21.0^\circ - R &= 0
\end{align*}
\]

a. Equation (1) gives

\[
L = \frac{W}{\cos 21.0^\circ} = \frac{53800 \text{ N}}{\cos 21.0^\circ} = 57600 \text{ N}
\]

b. Equation (2) gives

\[
R = L \sin 21.0^\circ = (57600 \text{ N}) \sin 21.0^\circ = 20600 \text{ N}
\]

53. **SSM REASONING** In order for the object to move with constant velocity, the net force on the object must be zero. Therefore, the north/south component of the third force must be equal in magnitude and opposite in direction to the 80.0 N force, while the east/west component of the third force must be equal in magnitude and opposite in direction to the 60.0 N force. Therefore, the third force has components: 80.0 N due south and 60.0 N due east. We can use the Pythagorean theorem and trigonometry to find the magnitude and direction of this third force.

**SOLUTION** The magnitude of the third force is

\[
F_3 = \sqrt{(80.0 \text{ N})^2 + (60.0 \text{ N})^2} = 1.00 \times 10^2 \text{ N}
\]

The direction of \(F_3\) is specified by the angle \(\theta\) where

\[
\theta = \tan^{-1} \left( \frac{80.0 \text{ N}}{60.0 \text{ N}} \right) = 53.1^\circ, \text{ south of east}
\]
54. **REASONING** The drawing assumes that upward is the +y direction and shows the I-beam together with the three forces that act on it: its weight $W$ and the tension $T$ in each of the cables. Since the I-beam is moving upward at a constant velocity, its acceleration is zero and it is in equilibrium. Therefore, according to Equation 4.9b, the net force in the vertical (or y) direction must be zero, so that $\sum F_y = 0$. This relation will allow us to find the magnitude of the tension.

**SOLUTION** With up as the +y direction, the vertical component of the tension in each cable is $T \sin 70.0^\circ$, and Equation 4.9b becomes

$$\sum F_y = T \sin 70.0^\circ + T \sin 70.0^\circ - 8.00 \times 10^3 \text{ N} = 0$$

Solving this equation for the tension gives $T = 4260 \text{ N}$. $W = -8.00 \times 10^3 \text{ N}$

55. **REASONING** The drawing shows the two forces, $T$ and $T'$, that act on the tooth. To obtain the net force, we will add the two forces using the method of components (see Section 1.8).

**SOLUTION** The table lists the two vectors and their $x$ and $y$ components:

<table>
<thead>
<tr>
<th>Vector</th>
<th>$x$ component</th>
<th>$y$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$+T \cos 16.0^\circ$</td>
<td>$-T \sin 16.0^\circ$</td>
</tr>
<tr>
<td>$T'$</td>
<td>$-T' \cos 16.0^\circ$</td>
<td>$-T' \sin 16.0^\circ$</td>
</tr>
<tr>
<td>$T + T'$</td>
<td>$+T \cos 16.0^\circ - T' \cos 16.0^\circ$</td>
<td>$-T \sin 16.0^\circ - T' \sin 16.0^\circ$</td>
</tr>
</tbody>
</table>

Since we are given that $T = T' = 21.0 \text{ N}$, the sum of the $x$ components of the forces is

$$\sum F_x = +T \cos 16.0^\circ - T' \cos 16.0^\circ = +(21 \text{ N}) \cos 16.0^\circ - (21 \text{ N}) \cos 16.0^\circ = 0 \text{ N}$$

The sum of the $y$ components is

$$\sum F_y = -T \sin 16.0^\circ - T' \sin 16.0^\circ = -(21 \text{ N}) \sin 16.0^\circ - (21 \text{ N}) \sin 16.0^\circ = -11.6 \text{ N}$$

The magnitude $F$ of the net force exerted on the tooth is

$$F = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(0 \text{ N})^2 + (-11.6 \text{ N})^2} = 11.6 \text{ N}$$
56. **REASONING** At first glance there seems to be very little information given. However, it is enough. In part a of the drawing the bucket is hanging stationary and, therefore, is in equilibrium. The forces acting on it are its weight and the two tension forces from the rope. There are two tension forces from the rope, because the rope is attached to the bucket handle at two places. These three forces must balance, which will allow us to determine the weight of the bucket. In part b of the drawing, the bucket is again in equilibrium, since it is traveling at a constant velocity and, therefore, has no acceleration. The forces acting on the bucket now are its weight and a single tension force from the rope, and they again must balance. In part b, there is only a single tension force, because the rope is attached to the bucket handle only at one place. This will allow us to determine the tension in part b, since the weight is known.

**SOLUTION** Let \( W \) be the weight of the bucket, and let \( T \) be the tension in the rope as the bucket is being pulled up at a constant velocity. The free-body diagrams for the bucket in parts a and b of the drawing are as follows:

Since the bucket in part a is in equilibrium, the net force acting on it is zero. Taking upward to be the positive direction, we have

\[
\sum F = 92.0 \text{ N} + 92.0 \text{ N} - W = 0 \quad \text{or} \quad W = 184 \text{ N}
\]

Similarly, in part b we have

\[
\sum F = T - W = 0 \quad \text{or} \quad T = W = 184 \text{ N}
\]

57. **SSM REASONING** The worker is standing still. Therefore, he is in equilibrium, and the net force acting on him is zero. The static frictional force \( f_s \) that prevents him from slipping points upward along the roof, an angle of \( \theta \) degrees above the horizontal; we choose this as the \(-x\) direction (see the free-body diagram). The normal force \( F_N \) is perpendicular to the roof and thus has no \( x \) component. But the gravitational force \( mg \) of
the earth on the worker points straight down, and thus has a component parallel to the roof. We will use this free-body diagram and find the worker’s mass \( m \) by applying Newton’s second law with the acceleration equal to zero.

**SOLUTION** The static frictional force \( f_s \) points in the \(-x\) direction, and the \( x\) component of the worker’s weight \( mg \) points in the \(+x\) direction. Because there are no other forces with \( x\) components, and the worker’s acceleration is zero, these two forces must balance each other. The \( x\) component of the worker’s weight is \( mg \sin \theta \), therefore \( f_s = mg \sin \theta \). Solving this relation for the worker’s mass, we obtain

\[
m = \frac{f_s}{g \sin \theta} = \frac{390 \text{ N}}{(9.80 \text{ m/s}^2)(\sin 36^\circ)} = 68 \text{ kg}
\]

58. **REASONING** Since the motion in the horizontal direction occurs at a constant velocity, there is no horizontal acceleration. Since there is no motion in the vertical direction, there is also no vertical acceleration. Therefore, the stuntman is in equilibrium, and Newton’s second law indicates that the net force in the horizontal direction is zero and that the net force in the vertical direction is also zero.

**SOLUTION** Consider a free body diagram for the stuntman with the \( x\) axis parallel to the ground and the \(+y\) axis vertically upward. We assume that the stuntman is moving in the \(+x\) direction. Newton’s second law written for no motion along the \( y\)-axis is \( \Sigma F_y = 0 \) or

\[
F_N - mg = 0
\]

This gives the normal force to be

\[
F_N = mg = (109 \text{ kg})(9.80 \text{ m/s}^2)
\]

Newton's second law for constant-velocity motion in the \( x\) direction is \( \Sigma F_x = 0 \) or

\[
T - f_k = 0
\]

where \( T \) and \( f_k \) denote the cable tension and the kinetic frictional force, respectively. Then

\[
T = f_k = \mu_k F_N = (0.870)(109 \text{ kg})(9.80 \text{ m/s}^2) = 929 \text{ N}
\]

59. **REASONING** The sum of the angles the right and left surfaces make with the horizontal and the angle between the two surface must be 180.0°. Therefore, the angle that the left surface makes with respect to the horizontal is 180.0° – 90.0° – 45.0° = 45.0°. In the free-body diagram of the wine bottle, the \( x\) axis is the horizontal. The force each surface exerts on the bottle is perpendicular to the surfaces, so both forces are directed 45.0° above the horizontal. Letting the surface on the right be surface 1, and the surface on the left be surface 2, the forces \( F_1 \) and \( F_2 \) are as shown in the free-body diagram. The third force acting on the
bottle is $W$, its weight or the gravitational force exerted on it by the earth. We will apply Newton’s second law and analyze the vertical forces in this free-body diagram to determine the magnitude $F$ of the forces $F_1$ and $F_2$, using the fact that the bottle is in equilibrium.

**SOLUTION** The vertical components of the forces exerted by the surfaces are $F_{1y} = F_1 \sin 45.0\degree$ and $F_{2y} = F_2 \sin 45.0\degree$. But the forces $F_1$ and $F_2$ have the same magnitude $F$, so the two vertical components become $F_{1y} = F_{2y} = F \sin 45.0\degree$. Because the bottle is in equilibrium, the upward forces must balance the downward force:

$$F_{1y} + F_{2y} = W \quad \text{or} \quad F \sin 45.0\degree + F \sin 45.0\degree = W \quad \text{or} \quad 2F \sin 45.0\degree = W$$

Therefore,

$$F = \frac{W}{2 \sin 45.0\degree} = \frac{mg}{2 \sin 45.0\degree} = \frac{(1.40 \text{ kg})(9.80 \text{ m/s}^2)}{2 \sin 45.0\degree} = 9.70 \text{ N}$$

60. **REASONING** The free-body diagram in the drawing at the right shows the forces that act on the clown (weight = $W$). In this drawing, note that $P$ denotes the pulling force. Since the rope passes around three pulleys, forces of magnitude $P$ are applied both to the clown’s hands and his feet. The normal force due to the floor is $F_N$, and the maximum static frictional force is $f_s^{\text{MAX}}$. At the instant just before the clown’s feet move, the net vertical and net horizontal forces are zero, according to Newton’s second law, since there is no acceleration at this instant.

**SOLUTION** According to Newton’s second law, with upward and to the right chosen as the positive directions, we have

$$F_N + P - W = 0 \quad \text{and} \quad f_s^{\text{MAX}} - P = 0$$

From the horizontal-force equation we find $P = f_s^{\text{MAX}}$. But $f_s^{\text{MAX}} = \mu_s F_N$. From the vertical-force equation, the normal force is $F_N = W - P$. With these substitutions, it follows that

$$P = f_s^{\text{MAX}} = \mu_s F_N = \mu_s (W - P)$$

Solving for $P$ gives

$$P = \frac{\mu_s W}{1 + \mu_s} = \frac{(0.53)(890 \text{ N})}{1 + 0.53} = 310 \text{ N}$$
61. **REASONING** The drawing at the right shows the forces on boxes 1, 2, and 3. Since the boxes are at rest, they are in equilibrium. According to Equation 4.9b, the net force in the vertical, or \( y \), direction is zero, \( \Sigma F_y = 0 \). There are two unknowns in this problem, the normal force that the table exerts on box 1 and the tension in the rope that connects boxes 2 and 3. To determine these unknowns we will apply the relation \( \Sigma F_y = 0 \) twice, once to the boxes on the left of the pulley and once to the box on the right.

**SOLUTION** There are four forces acting on the two boxes on the left. The boxes are in equilibrium, so that the net force must be zero. Choosing the \(+y\) direction as being the upward direction, we have that

\[
\Sigma F_y = -W_1 - W_2 + F_N + T = 0
\]

(1)

where \( W_1 \) and \( W_2 \) are the magnitudes of the weights of the boxes, \( F_N \) is the magnitude of the normal force that the table exerts on box 1, and \( T \) is the magnitude of the tension in the rope. We know the weights. To find the unknown tension, note that box 3 is also in equilibrium, so that the net force acting on it must be zero.

\[
\Sigma F_y = -W_3 + T = 0 \quad \text{so that} \quad T = W_3
\]

Substituting this expression for \( T \) into Equation (1) and solving for the normal force gives

\[
F_N = W_1 + W_2 - W_3 = 55 \text{ N} + 35 \text{ N} - 28 \text{ N} = 62 \text{ N}
\]

62. **REASONING** Since the wire beneath the limb is at rest, it is in equilibrium and the net force acting on it must be zero. Three forces comprise the net force, the 151-N force from the limb, the 447-N tension force from the left section of the wire, and the tension force \( T \) from the right section of the wire. We will resolve the forces into components and set the sum of the \( x \) components and the sum of the \( y \) components separately equal to zero. In so doing we will obtain two equations containing the unknown quantities, which are the horizontal and vertical components of the tension force \( T \). These two equations will be solved simultaneously to give values for the two unknowns. Knowing the components of the tension force, we can determine its magnitude and direction.

**SOLUTION** Let \( T_x \) and \( T_y \) be the horizontal and vertical components of the tension force. The free-body diagram for the wire beneath the limb is shown below. Taking upward and
to the right as the positive directions, we find for the $x$ components of the forces that

$$\Sigma F_x = T_x - (447 \text{ N}) \cos 14.0^\circ = 0$$

$$T_x = (447 \text{ N}) \cos 14.0^\circ = 434 \text{ N}$$

For the $y$ components of the forces we have

$$\Sigma F_y = T_y + (447 \text{ N}) \sin 14.0^\circ - 151 \text{ N} = 0$$

$$T_y = -(447 \text{ N}) \sin 14.0^\circ + 151 \text{ N} = 43 \text{ N}$$

The magnitude of the tension force is

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{(434 \text{ N})^2 + (43 \text{ N})^2} = 436 \text{ N}$$

Since the components of the tension force and the angle $\theta$ are related by $\tan \theta = T_y / T_x$, we find that

$$\theta = \tan^{-1} \left( \frac{T_y}{T_x} \right) = \tan^{-1} \left( \frac{43 \text{ N}}{434 \text{ N}} \right) = 5.7^\circ$$

**63. SSM REASONING** There are four forces that act on the chandelier; they are the forces of tension $T$ in each of the three wires, and the downward force of gravity $mg$. Under the influence of these forces, the chandelier is at rest and, therefore, in equilibrium. Consequently, the sum of the $x$ components as well as the sum of the $y$ components of the forces must each be zero. The figure below shows a quasi-free-body diagram for the chandelier and the force components for a suitable system of $x$, $y$ axes. Note that the diagram only shows one of the forces of tension; the second and third tension forces are not shown in the interest of clarity. The triangle at the right shows the geometry of one of the cords, where $\ell$ is the length of the cord, and $d$ is the distance from the ceiling.

We can use the forces in the $y$ direction to find the magnitude $T$ of the tension in any one wire.

**SOLUTION** Remembering that there are three tension forces, we see from the diagram that
3T \sin \theta = mg \quad \text{or} \quad T = \frac{mg}{3 \sin \theta} = \frac{mg}{3(d / \ell)} = \frac{mg \ell}{3d}

Therefore, the magnitude of the tension in any one of the cords is

\[ T = \frac{(44 \text{ kg})(9.80 \text{ m/s}^2)(2.0 \text{ m})}{3(1.5 \text{ m})} = 1.9 \times 10^2 \text{ N} \]

---

64. **REASONING** The block is in equilibrium in each case, because the block has a constant velocity and is not accelerating. A zero acceleration is the hallmark of equilibrium. At equilibrium, the net force is zero (i.e., the forces balance to zero), and we will obtain the magnitude of the pushing force by utilizing this fact as it pertains to the vertical or \( y \) direction. We will use Equation 4.9b \( (\Sigma F_y = 0) \) for this purpose.

Note that the direction of the kinetic frictional force is not the same in each case. The frictional force always opposes the relative motion between the surface of the block and the wall. Therefore, when the block slides upward, the frictional force points downward. When the block slides downward, the frictional force points upward. These directions are shown in the free-body diagrams (not to scale) for the two cases. In these drawings \( W \) is the weight of the block and \( f_k \) is the kinetic frictional force.

In each case the magnitude of the frictional force is the same. It is given by Equation 4.8 as \( f_k = \mu_k F_N \), where \( \mu_k \) is the coefficient of kinetic friction and \( F_N \) is the magnitude of the normal force. The coefficient of kinetic friction does not depend on the direction of the motion. Furthermore, the magnitude of the normal force in each case is the component of the pushing force that is perpendicular to the wall, or \( F_N = P \sin \theta \).

**SOLUTION** Using Equations 4.9b to describe the balance of forces that act on the block in the \( y \) direction and referring to the free-body diagrams, we have

**Upward motion** \( \Sigma F_y = P \cos \theta - W - f_k = 0 \)

**Downward motion** \( \Sigma F_y = P \cos \theta - W + f_k = 0 \)

According to Equation 4.8, the magnitude of the kinetic frictional force is \( f_k = \mu_k F_N \), where we have pointed out in the **REASONING** that the magnitude of the normal force is \( F_N = P \sin \theta \). Substituting into the equations for \( \Sigma F_y \) in the two cases, we obtain
\[ \Sigma F_y = P \cos \theta - W - \mu_k P \sin \theta = 0 \]

**Upward motion**

\[ P = \frac{W}{\cos \theta - \mu_k \sin \theta} = \frac{39.0 \, \text{N}}{\cos 30.0^\circ - (0.250) \sin 30.0^\circ} = 52.6 \, \text{N} \]

\[ \Sigma F_y = P \cos \theta - W + \mu_k P \sin \theta = 0 \]

**Downward motion**

\[ P = \frac{W}{\cos \theta + \mu_k \sin \theta} = \frac{39.0 \, \text{N}}{\cos 30.0^\circ + (0.250) \sin 30.0^\circ} = 39.4 \, \text{N} \]

65. **REASONING** The toboggan has a constant velocity, so it has no acceleration and is in equilibrium. Therefore, the forces acting on the toboggan must balance, that is, the net force acting on the toboggan must be zero. There are three forces present, the kinetic frictional force, the normal force from the inclined surface, and the weight \( mg \) of the toboggan. Using Newton’s second law with the acceleration equal to zero, we will obtain the kinetic friction coefficient.

**SOLUTION** In drawing the free-body diagram for the toboggan we choose the +\( x \) axis to be parallel to the hill surface and downward, the +\( y \) direction being perpendicular to the hill surface. We also use \( f_k \) to symbolize the frictional force. Since the toboggan is in equilibrium, the zero net force components in the \( x \) and \( y \) directions are

\[ \Sigma F_x = mg \sin 8.00^\circ - \mu_k F_N = 0 \]

\[ \Sigma F_y = F_N - mg \cos 8.00^\circ = 0 \]

In the first of these expressions we have used Equation 4.8 for \( f_k \) to express the kinetic frictional force. Solving the second equation for the normal force \( F_N \) and substituting into the first equation gives

\[ mg \sin 8.00^\circ - \mu_k mg \cos 8.00^\circ = 0 \quad \text{or} \quad \mu_k = \frac{\sin 8.00^\circ}{\cos 8.00^\circ} = \tan 8.00^\circ = 0.141 \]
66. **REASONING** The diagram at the right shows the force $F$ that the ground exerts on the end of a crutch. This force, as mentioned in the statement of the problem, acts along the crutch and, therefore, makes an angle $\theta$ with respect to the vertical. The horizontal and vertical components of this force are also shown. The horizontal component, $F \sin \theta$, is the static frictional force that prevents the crutch from slipping on the floor, so $f_s = F \sin \theta$. The largest value that the static frictional force can have before the crutch begins to slip is then given by $f_s^{\text{MAX}} = F \sin \theta^{\text{MAX}}$. We also know from Section 4.9 (see Equation 4.7) that the maximum static frictional force is related to the magnitude $F_N$ of the normal force by $f_s^{\text{MAX}} = \mu_s F_N$, where $\mu_s$ is the coefficient of static friction. These two relations will allow us to find $\theta^{\text{MAX}}$.

**SOLUTION** The magnitude of the maximum static frictional force is given by $f_s^{\text{MAX}} = \mu_s F_N$. But, as mentioned in the **REASONING** section, $f_s^{\text{MAX}}$ is also the horizontal component of the force $F$, so $f_s^{\text{MAX}} = F \sin \theta^{\text{MAX}}$. The vertical component of $F$ is $F \cos \theta^{\text{MAX}}$ and is the magnitude $F_N$ of the normal force that the ground exerts on the crutch. Thus, we have

$$\frac{f_s^{\text{MAX}}}{F \sin \theta^{\text{MAX}}} = \frac{\mu_s F_N}{F \cos \theta^{\text{MAX}}}$$

The force $F$ can be algebraically eliminated from this equation, leaving

$$\frac{\sin \theta^{\text{MAX}}}{\cos \theta^{\text{MAX}}} = \mu_s \quad \text{or} \quad \tan \theta^{\text{MAX}} = \mu_s$$

The maximum angle that a crutch can have is

$$\theta^{\text{MAX}} = \tan^{-1}(\mu_s) = \tan^{-1}(0.90) = 42^\circ$$

67. **SSM REASONING** When the bicycle is coasting straight down the hill, the forces that act on it are the normal force $F_N$ exerted by the surface of the hill, the force of gravity $mg$, and the force of air resistance $R$. When the bicycle climbs the hill, there is one additional force; it is the applied force that is required for the bicyclist to climb the hill at constant speed. We can use our knowledge of the motion of the bicycle down the hill to find $R$. Once $R$ is known, we can analyze the motion of the bicycle as it climbs the hill.

**SOLUTION** The following figure (on the left) shows the free-body diagram for the forces during the downhill motion. The hill is inclined at an angle $\theta$ above the horizontal. The figure (on the right) also shows these forces resolved into components parallel to and perpendicular to the line of motion.
Since the bicyclist is traveling at a constant velocity, his acceleration is zero. Therefore, according to Newton’s second law, we have \( \sum F_x = 0 \) and \( \sum F_y = 0 \). Taking the direction up the hill as positive, we have \( \sum F_x = R - mg \sin \theta = 0 \), or
\[
R = mg \sin \theta = (80.0 \text{ kg})(9.80 \text{ m/s}^2) \sin 15.0^\circ = 203 \text{ N}
\]
When the bicyclist climbs the same hill at constant speed, an applied force \( P \) must push the system up the hill. Since the speed is the same, the magnitude of the force of air resistance will remain 203 N. However, the air resistance will oppose the motion by pointing down the hill. The figure at the right shows the resolved forces that act on the system during the uphill motion.
Using the same sign convention as above, we have \( \sum F_x = P - mg \sin \theta - R = 0 \), or
\[
P = R + mg \sin \theta = 203 \text{ N} + 203 \text{ N} = 406 \text{ N}
\]

68. **REASONING** Because the kite line is straight, the distance between the kite and the person holding the line is \( L = 43 \text{ m} \), as shown in the right part of the drawing. In order to find the height \( h \) of the kite, we need to know the angle \( \theta \) that the kite line makes with the horizontal (see the following drawing). Once that is known, \( h = L \sin \theta \) will give the kite’s height relative to the person. The key to finding the angle \( \theta \) is the realization that the tension force \( T \) exerted on the kite by the line is parallel to the line itself. Therefore, the tension \( T \) is directed at an angle \( \theta \) below the horizontal (see the free-body diagram below). To find the angle \( \theta \), it is sufficient, then, to find one of the components of the tension force, either \( T_x \) or \( T_y \), because the magnitude \( T \) of the tension is known. With values for \( T_x \) or \( T_y \) and \( T \), we can use either the sine or cosine function to determine \( \theta \).

Both the wind’s force \( f \) and the tension \( T \) have horizontal and vertical components, while the weight force \( W \) is purely vertical. Therefore, there are only two horizontal forces acting on the kite (\( T_x, f_x \)), but three vertical forces (\( T_y, f_y, W \)), so it will be easier to calculate \( T_x \), the horizontal component of the tension. Because the kite is stationary, the horizontal component of the tension must balance the horizontal component of the force exerted on the kite by the wind:
\[
T_x = f_x
\]
SOLUTION From the free-body diagram of the kite, we can see that the $x$ components of the tension and air resistance forces are $T_x = T \cos \theta$ and $f_x = f \cos 56^\circ$. Substituting these expressions into Equation (1), we find

$$T_x = f_x \quad \text{or} \quad T \cos \theta = f \cos 56^\circ \quad \text{or} \quad \cos \theta = \frac{f \cos 56^\circ}{T}$$

$$\theta = \cos^{-1} \left( \frac{f \cos 56^\circ}{T} \right) = \cos^{-1} \left[ \frac{(19 \text{ N}) \cos 56^\circ}{16 \text{ N}} \right] = 48^\circ$$

Therefore, the kite’s height relative to the person is

$$h = L \sin \theta = (43 \text{ m}) \sin 48^\circ = 32 \text{ m}$$

69. REASONING The weight of the part of the washcloth off the table is $m_{\text{off}} g$. At the instant just before the washcloth begins to slide, this weight is supported by a force that has magnitude equal to $f_s^{\text{MAX}}$, which is the static frictional force that the table surface applies to the part of the washcloth on the table. This force is transmitted “around the bend” in the washcloth hanging over the edge by the tension forces between the molecules of the washcloth, in much the same way that a force applied to one end of a rope is transmitted along the rope as it passes around a pulley.

SOLUTION Since the static frictional supports the weight of the washcloth off the table, we have $f_s^{\text{MAX}} = m_{\text{off}} g$. The static frictional force is $f_s^{\text{MAX}} = \mu_s F_N$. The normal force $F_N$ is applied by the table to the part of the washcloth on the table and has a magnitude equal to the weight of that part of the washcloth. This is so, because the table is assumed to be horizontal and the part of the washcloth on it does not accelerate in the vertical direction. Thus, we have

$$f_s^{\text{MAX}} = \mu_s F_N = \mu_s m_{\text{on}} g = m_{\text{off}} g$$
The magnitude $g$ of the acceleration due to gravity can be eliminated algebraically from this result, giving $\mu_s m_{on} = m_{off}$. Dividing both sides by $m_{on} + m_{off}$ gives

$$\mu_s \left( \frac{m_{on}}{m_{on} + m_{off}} \right) = \frac{m_{off}}{m_{on} + m_{off}} \quad \text{or} \quad \mu_s f_{on} = f_{off}$$

where we have used $f_{on}$ and $f_{off}$ to denote the fractions of the washcloth on and off the table, respectively. Since $f_{on} + f_{off} = 1$, we can write the above equation on the right as

$$\mu_s \left( 1 - f_{off} \right) = f_{off} \quad \text{or} \quad f_{off} = \frac{\mu_s}{1 + \mu_s} = \frac{0.40}{1 + 0.40} = 0.29$$

70. **REASONING** In addition to the upward buoyant force $B$ and the downward resistive force $R$, a downward gravitational force $mg$ acts on the submarine (see the free-body diagram), where $m$ denotes the mass of the submarine. Because the submarine is not in contact with any rigid surface, no normal force is exerted on it. We will calculate the acceleration of the submarine from Newton’s second law, using the free-body diagram as a guide.

**SOLUTION** Choosing up as the positive direction, we sum the forces acting on the submarine to find the net force $\Sigma F$. According to Newton’s second law, the acceleration $a$ is

$$a = \frac{\Sigma F}{m} = \frac{B - R - mg}{m} = \frac{16140 \text{ N} - 1030 \text{ N} - (1450 \text{ kg})(9.80 \text{ m/s}^2)}{1450 \text{ kg}} = \frac{0.62 \text{ m/s}^2}{1}$$

where the positive value indicates that the direction is upward.

71. **SSM REASONING** We can use the appropriate equation of kinematics to find the acceleration of the bullet. Then Newton's second law can be used to find the average net force on the bullet.

**SOLUTION** According to Equation 2.4, the acceleration of the bullet is

$$a = \frac{v - v_0}{t} = \frac{715 \text{ m/s} - 0 \text{ m/s}}{2.50 \times 10^{-3} \text{ s}} = 2.86 \times 10^5 \text{ m/s}^2$$

Therefore, the net average force on the bullet is

$$\sum F = ma = (15 \times 10^{-3} \text{ kg})(2.86 \times 10^5 \text{ m/s}^2) = 4290 \text{ N}$$
72. **REASONING**
   a. Since the fish is being pulled up at a constant speed, it has no acceleration. According to Newton’s second law, the net force acting on the fish must be zero. We will use this fact to determine the weight of the heaviest fish that can be pulled up.

   b. When the fish has an upward acceleration, we can still use Newton’s second law to find the weight of the heaviest fish. However, because the fish has an acceleration, we will see that the maximum weight is less than that in part (a).

**SOLUTION**
   a. There are two forces acting on the fish (taking the upward vertical direction to be the $+y$ direction): the maximum force of $+45$ N due to the line, and the weight $-W$ of the fish (negative, because the weight points down). Newton’s second law ($\Sigma F_y = 0$, Equation 4.9b) gives

   \[ \Sigma F_y = +45 \text{ N} - W = 0 \quad \text{or} \quad W = 45 \text{ N} \]

   b. Since the fish has an upward acceleration $a_y$, Newton’s second law ($\Sigma F_y = ma_y$, Equation 4.2b) becomes

   \[ \Sigma F_y = +45 \text{ N} - W' = ma_y \]

   Where $W'$ is the weight of the heaviest fish that can be pulled up with an acceleration. Solving this equation for $W'$ gives

   \[ W' = +45 \text{ N} - ma_y \]

   The mass $m$ of the fish is the magnitude $W'$ of its weight divided by the magnitude $g$ of the acceleration due to gravity (see Equation 4.5), or $m = W'/g$. Substituting this relation for $m$ into the previous equation gives

   \[ W' = +45 \text{ N} - \left( \frac{W'}{g} \right) a_y \]

   Solving this equation for $W'$ yields

   \[ W' = \frac{45 \text{ N}}{1 + \frac{a_y}{g}} = \frac{45 \text{ N}}{1 + \frac{2.0 \text{ m/s}^2}{9.80 \text{ m/s}^2}} = 37 \text{ N} \]

73. **REASONING** According to Newton’s second law, the acceleration has the same direction as the net force and a magnitude given by $a = \Sigma F/m$. 
SOLUTION  Since the two forces are perpendicular, the magnitude of the net force is given by the Pythagorean theorem as \( \Sigma F = \sqrt{(40.0 \text{ N})^2 + (60.0 \text{ N})^2} \). Thus, according to Newton’s second law, the magnitude of the acceleration is

\[
a = \frac{\Sigma F}{m} = \frac{\sqrt{(40.0 \text{ N})^2 + (60.0 \text{ N})^2}}{4.00 \text{ kg}} = 18.0 \text{ m/s}^2
\]

The direction of the acceleration vector is given by

\[
\theta = \tan^{-1}\left(\frac{60.0 \text{ N}}{40.0 \text{ N}}\right) = 56.3^\circ \text{ above the } +x \text{ axis}
\]

74. REASONING  In the absence of air resistance, the two forces acting on the sensor are its weight \( W \) and the tension \( T \) in the towing cable (see the free-body diagram). We see that \( T_x \) is the only horizontal force acting on the sensor, and therefore Newton’s second law \( \Sigma F_x = ma_x \) (Equation 4.2a) gives \( T_x = ma_x \). Because the vertical component of the sensor’s acceleration is zero, the vertical component of the cable’s tension \( T \) must balance the sensor’s weight: \( T_y = W = mg \). We thus have sufficient information to calculate the horizontal and vertical components of the tension force \( T \), and therefore to calculate its magnitude \( T \) from the Pythagorean theorem: \( T^2 = T_x^2 + T_y^2 \).

SOLUTION  Given that \( T_x = ma_x \) and that \( T_y = mg \), the Pythagorean theorem yields the magnitude \( T \) of the tension in the cable:

\[
T = \sqrt{T_x^2 + T_y^2} = \sqrt{(ma)^2 + (mg)^2} = \sqrt{m^2a^2 + m^2g^2} = m\sqrt{a^2 + g^2}
\]

\[
= (129 \text{ kg})\sqrt{(2.84 \text{ m/s}^2)^2 + (9.80 \text{ m/s}^2)^2} = 1320 \text{ N}
\]

75. REASONING AND SOLUTION  From Newton's second law and the equation: \( v = v_0 + at \), we have

\[
F = ma = m\frac{v - v_0}{t}
\]

a. When the skier accelerates from rest \( (v_0 = 0 \text{ m/s}) \) to a speed of 11 m/s in 8.0 s, the required net force is

\[
F = m\frac{v - v_0}{t} = (73 \text{ kg})\frac{(11 \text{ m/s}) - 0 \text{ m/s}}{8.0 \text{ s}} = 1.0 \times 10^2 \text{ N}
\]
b. When the skier lets go of the tow rope and glides to a halt \((v = 0 \text{ m/s})\) in 21 s, the net force acting on the skier is

\[
F = m \frac{v - v_0}{t} = (73 \text{ kg}) \frac{0 \text{ m/s} - (11 \text{ m/s})}{21 \text{ s}} = -38 \text{ N}
\]

The magnitude of the net force is \(38 \text{ N}\).

76. **REASONING** Newton’s second law can be used in both parts of this problem, because it applies no matter what value the acceleration. In part (a) the acceleration is non-zero. In part (b) the acceleration is zero, since the motion occurs at a constant acceleration.

**SOLUTION**

a. In the vertical direction, Newton’s second law is written as \(\Sigma F_y = ma_y\). Thus, assuming upward is the positive direction, denoting the cable tension by \(T\), and the man’s weight by \(W = mg\), we have

\[
\frac{T - mg}{\Sigma F_y} = ma_y
\]

Solving this expression for \(T\) reveals that

\[
T = ma_y + mg = mg \left(\frac{a_y}{g} + 1\right) = (822 \text{ N}) \left(\frac{1.10 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right) = 914 \text{ N}
\]

b. The acceleration of the man is zero when his velocity is constant. Using the expression obtained for the tension in part a, we obtain

\[
T = ma_y + mg = m \left(0 \text{ m/s}^2\right) + mg = 822 \text{ N}
\]

77. **REASONING** The only horizontal force acting on the boat and trailer is the tension in the hitch; therefore, it is the net force. According to Newton’s second law, the tension (or the net force) equals the mass times the acceleration. The mass is known, and the acceleration can be found by applying an appropriate equation of kinematics from Chapter 3.

**SOLUTION** Assume that the boat and trailer are moving in the +\(x\) direction. Newton’s second law is \(\Sigma F_x = ma_x\) (see Equation 4.2a), where the net force is just the tension +\(T\) in the hitch, so \(\Sigma F_x = T\). Thus,

\[
T = ma_x
\]  

(1)

Since the initial and final velocities, \(v_{0x}\) and \(v_{x}\), and the time \(t\) are known, we may use Equation 3.3a from the equations of kinematics to relate these variables to the acceleration:
\[ v_x = v_{0x} + a_x t \]  \hspace{1cm} (3.3a)

Solving Equation (3.3a) for \( a_x \) and substituting the result into Equation (1), we find that

\[ T = ma_x = m \left( \frac{v_x - v_{0x}}{t} \right) = (410 \text{ kg}) \left( \frac{11 \text{ m/s} - 0 \text{ m/s}}{28 \text{ s}} \right) = 160 \text{ N} \]

78. **REASONING** The forces acting on the motorcycle are the normal force \( F_N \), the 3150-N propulsion force, the 250-N force of air resistance, and the weight, which is \( mg = (292 \text{ kg})(9.80 \text{ m/s}^2) = 2860 \text{ N} \). All of these forces must be considered when determining the net force for use with Newton’s second law to determine the acceleration. In particular, we note that the motion occurs along the ramp and that both the propulsion force and air resistance are directed parallel to the ramp surface. In contrast, the normal force and the weight do not act parallel to the ramp. The normal force is perpendicular to the ramp surface, while the weight acts vertically downward. However, the weight does have a component along the ramp.

**SOLUTION** In drawing the free-body diagram for the motorcycle we choose the +x axis to be parallel to the ramp surface and upward, the +y direction being perpendicular to the ramp surface. The diagram is shown at the right. Since the motorcycle accelerates along the ramp and we seek only that acceleration, we can ignore the forces that point along the y axis (the normal force \( F_N \) and the \( y \) component of the weight). The \( x \) component of Newton’s second law is

\[ \Sigma F_x = 3150 \text{ N} - (2860 \text{ N})\sin 30.0^\circ - 250 \text{ N} = ma_x \]

Solving for the acceleration \( a_x \) gives

\[ a_x = \frac{3150 \text{ N} - (2860 \text{ N})\sin 30.0^\circ - 250 \text{ N}}{292 \text{ kg}} = 5.03 \text{ m/s}^2 \]

79. **SSM REASONING** The speed of the skateboarder at the bottom of the ramp can be found by solving Equation 2.9 \((v^2 = v_0^2 + 2ax\), where \( x \) is the distance that the skater moves down the ramp) for \( v \). The figure at the right shows the free-body diagram for the skateboarder. The net force \( \Sigma F \), which accelerates the skateboarder down the ramp, is the component of
the weight that is parallel to the incline: \( \Sigma F = mg \sin \theta \). Therefore, we know from Newton's second law that the acceleration of the skateboarder down the ramp is

\[
a = \frac{\Sigma F}{m} = \frac{mg \sin \theta}{m} = g \sin \theta
\]

**SOLUTION**  Thus, the speed of the skateboarder at the bottom of the ramp is

\[
v = \sqrt{v_0^2 + 2ax} = \sqrt{v_0^2 + 2g \sin \theta x} = \sqrt{(2.6 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(6.0 \text{ m}) \sin 18^\circ} = 6.6 \text{ m/s}
\]

80. **REASONING**  Since we assume that there is no frictional force resisting the airplane’s motion, the only horizontal force acting on the airplane arises because of the tension (magnitude = \( T \)) in the cable. From Newton’s second law \( \Sigma F = Ma \) (Equation 4.1), we conclude that the airplane’s acceleration is given by \( a = \Sigma F/M = T/M \), where \( M \) is the mass of the airplane. The harder the man pulls on the cable, the greater the tension \( T \), and the greater the airplane’s acceleration. According to Newton’s third law, however, the cable also exerts an opposing horizontal force of magnitude \( T \) on the man. Thus, if he is to keep his footing, \( T \) cannot exceed the maximum force of static friction \( f_s^{\text{MAX}} \) the runway exerts on him. Therefore, the airplane’s acceleration is greatest when these two forces have equal magnitudes: \( T = f_s^{\text{MAX}} \) (see the free-body diagram of the man). The maximum static frictional force the runway can exert is determined by the relation \( f_s^{\text{MAX}} = \mu_s F_N \) (Equation 4.7). Because the man has no acceleration in the vertical direction, the normal force must balance the downward pull of gravity: \( F_N = mg \).

**SOLUTION**  Combining \( F_N = mg \) and \( f_s^{\text{MAX}} = \mu_s F_N \) (Equation 4.7), we obtain the maximum tension in the cable:

\[
T = f_s^{\text{MAX}} = \mu_s F_N = \mu_s mg
\]

(1)

We can now substitute Equation (1) for the tension into Newton’s second law \( (a = T/M) \), and calculate the maximum possible acceleration of the airplane:

\[
a = \frac{T}{M} = \frac{\mu_s mg}{M} = \frac{(0.77)(85 \text{ kg})(9.80 \text{ m/s}^2)}{109 000 \text{ kg}} = 5.9 \times 10^{-3} \text{ m/s}^2
\]
81. **REASONING AND SOLUTION** If the +x axis is taken to be parallel to and up the ramp, then \( \Sigma F_x = m a_x \) gives

\[
T - f_k - mg \sin 30.0^\circ = ma_x
\]

where \( f_k = \mu_k F_N \). Hence,

\[
T = ma_x + \mu_k F_N + mg \sin 30.0^\circ \quad (1)
\]

Also, \( \Sigma F_y = ma_y \) gives

\[
F_N - mg \cos 30.0^\circ = 0
\]

since no acceleration occurs in this direction. Then

\[
F_N = mg \cos 30.0^\circ \quad (2)
\]

Substitution of Equation (2) into Equation (1) yields

\[
T = ma_x + \mu_k mg \cos 30.0^\circ + mg \sin 30.0^\circ
\]

\[
T = (205 \text{ kg})(0.800 \text{ m/s}^2) + (0.900)(205 \text{ kg})(9.80 \text{ m/s}^2)\cos 30.0^\circ
\]

\[
+ (205 \text{ kg})(9.80 \text{ m/s}^2)\sin 30.0^\circ = 2730 \text{ N}
\]

82. **REASONING** To determine the man’s upward acceleration by means of Newton’s second law, we first need to identify all of the forces exerted on him and then construct a free-body diagram. The earth pulls down on the man with a gravitational force \( W = mg \). Once he begins accelerating upward, he is no longer in contact with the ground, so there is no normal force acting on him. The pulling force \( P \) that he exerts on the rope does not appear in his free-body diagram because it is not a force exerted on him. Each end of the rope exerts a tension force on him. If we assume that the rope is massless, and ignore friction between the rope and the branch, then the magnitude of the tension \( T \) is the same everywhere in the rope. Because the man pulls down on the free end intentionally and on the other end inadvertently (because it is tied around his waist), Newton’s third law predicts that both ends of the rope pull upward on him. The third law predicts that the free end of the rope pulls up on the man with a force exactly equal in magnitude to that of the 358-N pulling force. Thus, in addition to the downward gravitational force, there are two upward tension forces with magnitudes \( T = 358 \text{ N} \) acting on the man, as illustrated in the free-body diagram.

**SOLUTION** Taking up as the positive direction and applying Newton’s second law to the man’s free-body diagram yields

\[
\Sigma F = 2T - W = ma
\]

Solving for the acceleration \( a \), we find

\[
a = \frac{2T - W}{m} = \frac{2T - mg}{m} = \frac{2T}{m} - g = \frac{2(358 \text{ N})}{72.0 \text{ kg}} - 9.80 \text{ m/s}^2 = 0.14 \text{ m/s}^2
\]
83. **REASONING AND SOLUTION**

The system is shown in the drawing. We will let $m_1 = 21.0 \text{ kg}$, and $m_2 = 45.0 \text{ kg}$. Then, $m_1$ will move upward, and $m_2$ will move downward. There are two forces that act on each object; they are the tension $T$ in the cord and the weight $mg$ of the object. The forces are shown in the free-body diagrams at the far right.

We will take upward as the positive direction. If the acceleration of $m_1$ is $a$, then the acceleration of $m_2$ must be $-a$. From Newton's second law, we have for $m_1$ that

$$\sum F_y = T - m_1 g = m_1 a \quad (1)$$

and for $m_2$

$$\sum F_y = T - m_2 g = -m_2 a \quad (2)$$

a. Eliminating $T$ between these two equations, we obtain

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \left( \frac{45.0 \text{ kg} - 21.0 \text{ kg}}{45.0 \text{ kg} + 21.0 \text{ kg}} \right) (9.80 \text{ m/s}^2) = 3.56 \text{ m/s}^2$$

b. Eliminating $a$ between Equations (1) and (2), we find

$$T = \frac{2m_1 m_2}{m_1 + m_2} g = \left[ \frac{2(21.0 \text{ kg})(45.0 \text{ kg})}{21.0 \text{ kg} + 45.0 \text{ kg}} \right] (9.80 \text{ m/s}^2) = 281 \text{ N}$$

84. **REASONING** Newton's second law, Equation 4.2a, can be used to find the tension in the coupling between the cars, since the mass and acceleration are known. The tension in the coupling between the 30th and 31st cars is responsible for providing the acceleration for the 20 cars from the 31st to the 50th car. The tension in the coupling between the 49th and 50th cars is responsible only for pulling one car, the 50th.

**SOLUTION**

a. The tension $T$ between the 30th and 31st cars is

$$T_x = (\text{Mass of 20 cars}) a_x \quad (4.2a)$$

$$= (20 \text{ cars}) \left(6.8 \times 10^3 \text{ kg/car}\right) \left(8.0 \times 10^{-2} \text{ m/s}^2\right) = 1.1 \times 10^4 \text{ N}$$
b. The tension \( T \) between the 49\(^{th} \) and 50\(^{th} \) cars is

\[
T_x = (\text{Mass of 1 car})a_x
\]

\[
= (1 \text{ car})(6.8 \times 10^3 \text{ kg/car})(8.0 \times 10^{-2} \text{ m/s}^2) = 5.4 \times 10^2 \text{ N}
\]

85. **REASONING** Since we assume that there is no frictional force resisting the airplane’s motion, the only horizontal force acting on the airplane arises because of the tension (magnitude = \( T \)) in the cable. The airplane (mass = \( M \)) undergoes a horizontal acceleration caused by the horizontal component \( T_x = T \cos \theta \) of the tension force, where \( \theta \) is the angle that the cable makes with the horizontal. From Newton’s second law, the acceleration of the airplane is

\[
a_x = \frac{\Sigma F_x}{M} = \frac{T \cos \theta}{M}
\]

(1)

The maximum tension in the cable is limited by the condition that the man’s feet must not slip. When the man pulls as hard as possible without slipping, the horizontal component of the tension acting on him matches the maximum static frictional force: \( T_x = T \cos \theta = f_s^{\text{MAX}} \) (see the free-body diagram of the man). The maximum static frictional force itself is given by \( f_s^{\text{MAX}} = \mu_s F_N \) (Equation 4.7). Together, these two relations yield

\[
T \cos \theta = \mu_s F_N
\]

(2)

To evaluate the magnitude \( F_N \) of the normal force that acts on the man, we must consider Newton’s third law. This law indicates that when the man (mass = \( m \)) pulls up on the cable, the cable pulls down on him (see the free-body diagram of the man). This additional downward force increases the upward normal force \( F_N \) the runway exerts on him. Applying Newton’s second law to the vertical direction in this diagram, with zero acceleration, we see that \( \Sigma F_y = F_N - T_y - mg = F_N - T \sin \theta - mg = 0 \). Solving for \( F_N \) yields

\[
F_N = T \sin \theta + mg
\]

(3)

Substituting Equation (3) into Equation (2) yields an expression in which the tension \( T \) is the only unknown quantity:

\[
T \cos \theta = \mu_s (T \sin \theta + mg)
\]

(4)

Rearranging Equation (4) and solving for the tension in the cable, we find:
\[ T \cos \theta = \mu_s T \sin \theta + \mu_s mg \quad \text{or} \quad T \left( \cos \theta - \mu_s \sin \theta \right) = \mu_s mg \]

\[ T = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta} \quad (5) \]

Equation (5) may be substituted into Equation (1) for the airplane’s acceleration:

\[ a_x = \frac{T \cos \theta}{M} = \frac{\mu_s mg \cos \theta}{M \left( \cos \theta - \mu_s \sin \theta \right)} \quad (6) \]

**SOLUTION**  We apply Equation (6) to calculate the acceleration of the airplane:

\[
a_x = \frac{\mu_s mg \cos \theta}{M \left( \cos \theta - \mu_s \sin \theta \right)} = \frac{(0.77)(85 \text{ kg})(9.80 \text{ m/s}^2) \cos 9.0^\circ}{(109,000 \text{ kg})(\cos 9.0^\circ - 0.77 \sin 9.0^\circ)} = 6.7 \times 10^{-3} \text{ m/s}^2
\]

86. **REASONING**  The free-body diagrams for the large cube (mass = \(M\)) and the small cube (mass = \(m\)) are shown in the following drawings. In the case of the large cube, we have omitted the weight and the normal force from the surface, since the play no role in the solution (although they do balance).

In these diagrams, note that the two blocks exert a normal force on each other; the large block exerts the force \(F_N\) on the smaller block, while the smaller block exerts the force \(-F_N\) on the larger block. In accord with Newton’s third law these forces have opposite directions and equal magnitudes \(F_N\). Under the influence of the forces shown, the two blocks have the same acceleration \(a\). We begin our solution by applying Newton’s second law to each one.

**SOLUTION**  According to Newton’s second law, we have

\[
\Sigma F = P - F_N = Ma \quad \text{Large block} \quad F_N = ma \quad \text{Small block}
\]

Substituting \(F_N = ma\) into the large-block expression and solving for \(P\) gives

\[
P = (M + m) a
\]

For the smaller block to remain in place against the larger block, the static frictional force must balance the weight of the smaller block, so that \(f_{s_{\text{MAX}}} = mg\). But \(f_{s_{\text{MAX}}}\) is given by
\[ f_s^{\text{MAX}} = \mu_s F_N, \] where, from the Newton’s second law, we know that \( F_N = ma \). Thus, we have \( \mu_s ma = mg \) or \( a = g/\mu_s \). Using this result in the expression for \( P \) gives

\[
P = (M + m)a = \frac{(M + m)g}{\mu_s} = \frac{(25 \text{ kg} + 4.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.71} = 4.0 \times 10^2 \text{ N}
\]

87. **REASONING** As the free-body diagram shows, there are two forces acting on the fireman as he slides down the pole: his weight \( W \) and the kinetic frictional force \( f_k \). The kinetic frictional force opposes the motion of the fireman, so it points upward in the \( +y \) direction. In accord with Newton’s second law, the net force, which is the sum of these two forces, is equal to the fireman’s mass times his acceleration. His mass, and therefore his weight, is known, but his acceleration is not. We will turn to one of the equations of kinematics from Chapter 3 to determine the acceleration.

**SOLUTION** Newton’s second law \( (\Sigma F_y = ma_y, \text{Equation 4.2b}) \) can be applied to this situation:

\[
\Sigma F_y = +f_k - W = ma_y
\]

The magnitude \( W \) of the fireman’s weight can be expressed in terms of his mass as \( W = mg \) (Equation 4.5), where \( g \) is magnitude of the acceleration due to gravity. Solving the equation above for the magnitude of the kinetic frictional force, and using \( W = mg \), gives

\[
f_k = ma_y + W = ma_y + mg
\]

(1)

Since the initial and final velocities, \( v_{0y} \) and \( v_y \), and the displacement \( y \) are known, we will use Equation 3.6b from the equations of kinematics to relate these variables to the acceleration: \( v_y^2 = v_{0y}^2 + 2a_y y \). Solving this equation for \( a_y \) and substituting the result into Equation 1 gives

\[
f_k = m \left( \frac{v_y^2 - v_{0y}^2}{2y} \right) + mg
\]

We note that the fireman slides down the pole, so his displacement is negative, or \( y = -4.0 \text{ m} \). The magnitude of the kinetic frictional force is, then,

\[
f_k = m \left( \frac{v_y^2 - v_{0y}^2}{2y} \right) + mg
\]

\[
= (86 \text{ kg}) \left[ \frac{(1.4 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(-4.0 \text{ m})} \right] + (86 \text{ kg})(9.80 \text{ m/s}^2) = 820 \text{ N}
\]
88. **REASONING**

Consider the forces that act on each block. Only one force contributes to the horizontal net force acting on block 1, as shown in the free-body diagram. This is the force \(-P\) with which block 2 pushes on block 1. The minus sign in the free-body diagram indicates the direction of the force is to the left. This force is part of the action-reaction pair of forces that is consistent with Newton’s third law. Block 1 pushes forward and to the right against block 2, and block 2 pushes backward and to the left against block 1 with an oppositely directed force of equal magnitude.

Two forces contribute to the horizontal net force acting on block 2, as shown in the free-body diagram. One is the force \(P\) with which block 1 pushes on block 2. According to Newton’s third law, this force has the same magnitude but the opposite direction as the force with which block 2 pushes on block 1. The other force is the kinetic frictional force \(f_k\), which points to the left, in opposition to the relative motion between the block and the surface on which it slides.

Both blocks decelerate, the magnitude of the deceleration being the same for each block. They have the same deceleration, because they are pressed together. Since the blocks are moving to the right in the drawing, the acceleration vector points to the left, for it reflects the slowing down of the motion. In Case A and in Case B we will apply Newton’s second law separately to each block in order to relate the net force to the acceleration.

**SOLUTION** Referring to the free-body diagram for block 1, we write Newton’s second law as follows:

$$-P = m_1 (-a)$$

Where \(a\) is the magnitude of the acceleration. The minus sign appears on the right side of this equation because the acceleration, being a deceleration, points to the left, in the negative direction. Referring to the free-body diagram for block 2, we write Newton’s second law as follows:

$$P - f_k = m_2 (-a)$$

Solving Equation (1) for \(a\) gives \(a = P / m_1\). Substituting this result into Equation (2) gives

$$P - f_k = m_2 \left( \frac{-P}{m_1} \right) \quad \text{or} \quad P = \frac{m_1 f_k}{m_1 + m_2}$$
Substituting this result for \( P \) into \( \frac{a}{m} = \frac{f_k}{m_1(m_1 + m_2)} \) gives

\[
a = \frac{P}{m_1} = \frac{m_1 f_k}{m_1(m_1 + m_2)} = \frac{f_k}{m_1 + m_2}
\]

We can now use these results to calculate \( P \) and \( a \) in both cases.

a. Case A

\[
P = \frac{m_1 f_k}{m_1 + m_2} = \frac{(3.0 \text{ kg})(5.8 \text{ N})}{3.0 \text{ kg} + 3.0 \text{ kg}} = 2.9 \text{ N}
\]

Case B

\[
P = \frac{m_1 f_k}{m_1 + m_2} = \frac{(6.0 \text{ kg})(5.8 \text{ N})}{6.0 \text{ kg} + 3.0 \text{ kg}} = 3.9 \text{ N}
\]

b. Case A

\[
a = \frac{-f_k}{m_1 + m_2} = \frac{-5.8 \text{ N}}{3.0 \text{ kg} + 3.0 \text{ kg}} = -0.97 \text{ m/s}^2
\]

The magnitude of the acceleration is \( 0.97 \text{ m/s}^2 \).

Case B

\[
a = \frac{-f_k}{m_1 + m_2} = \frac{-5.8 \text{ N}}{6.0 \text{ kg} + 3.0 \text{ kg}} = -0.64 \text{ m/s}^2
\]

The magnitude of the acceleration is \( 0.64 \text{ m/s}^2 \).

89. **SSM REASONING** The shortest time to pull the person from the cave corresponds to the maximum acceleration, \( a_y \), that the rope can withstand. We first determine this acceleration and then use kinematic Equation 3.5b \( (y = v_0t + \frac{1}{2}a_y t^2) \) to find the time \( t \).

**SOLUTION** As the person is being pulled from the cave, there are two forces that act on him; they are the tension \( T \) in the rope that points vertically upward, and the weight of the person \( mg \) that points vertically downward. Thus, if we take upward as the positive direction, Newton's second law gives \( \sum F_y = T - mg = ma_y \). Solving for \( a_y \), we have

\[
a_y = \frac{T}{m} - g = \frac{T}{W/g} - g = \frac{569 \text{ N}}{(5.20 \times 10^2 \text{ N})/(9.80 \text{ m/s}^2)} - 9.80 \text{ m/s}^2 = 0.92 \text{ m/s}^2
\]

Therefore, from Equation 3.5b with \( v_{0y} = 0 \text{ m/s} \), we have \( y = \frac{1}{2}a_y t^2 \). Solving for \( t \), we find

\[
t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(35.1 \text{ m})}{0.92 \text{ m/s}^2}} = 8.7 \text{ s}
\]
90. **REASONING** The time \( t \) required for the sled to undergo a displacement \( x \) down the slope, starting with an initial velocity \( v_{0x} \), can be obtained with the aid of Equation 3.5a of the equations of kinematics, which is \( x = v_{0x}t + \frac{1}{2}a_xt^2 \). To use this equation, however, we need a value for the acceleration \( a_x \). We can obtain this value by using Newton’s second law.

**SOLUTION** Solving Equation 2.8 for the time \( t \) and using the fact that \( v_{0x} = 0 \text{ m/s} \) (the sled starts from rest), we find that

\[
x = \frac{1}{2}a_xt^2 \quad \text{or} \quad t = \frac{2x}{a_x} \tag{1}
\]

In using Newton’s second law to find the acceleration \( a_x \), we take the \( x \) axis to be parallel to the slope, with +\( x \) direction pointing down the slope. Then, \( \Sigma F_x = ma_x \) gives

\[
F_w + mg \sin \theta - \mu_k F_N = ma_x \tag{2}
\]

where the force from the wind is \( F_w \), the component of the weight of the person and sled that points down the slope \((\theta = 30.0^\circ)\) is \( mg \sin \theta \), and the kinetic frictional force is \( f_k = \mu_k F_N \). For the \( y \) direction (perpendicular to the slope, with +\( y \) directed away from the slope), \( \Sigma F_y = ma_y \) gives

\[
F_N - mg \cos \theta = 0 \tag{3}
\]

where we have used the fact that there is zero acceleration of the sled in the \( y \) direction. Solving for the normal force \( F_N \) gives

\[
F_N = mg \cos \theta \tag{4}
\]

Substituting this result into Equation (2), we find that

\[
F_w + mg \sin \theta - \mu_k \frac{mg \cos \theta}{F_N} = ma_x
\]

Thus, the value of the acceleration that we need is

\[
a_x = \frac{F_w}{m} + g \sin \theta - \mu_k g \cos \theta
\]

\[
= \frac{105 \text{ N}}{65.0 \text{ kg}} + (9.80 \text{ m/s}^2 \sin 30.0^\circ - 0.150(9.80 \text{ m/s}^2 \cos 30.0^\circ) = 5.24 \text{ m/s}^2
\]

Returning to Equation (1), we find that the time required for the sled to travel down the slope is

\[
t = \frac{2x}{a_x} = \frac{2(175 \text{ m})}{5.24 \text{ m/s}^2} = 8.17 \text{ s}
\]
91. **REASONING AND SOLUTION**
   a. Newton's second law for block 1 (10.0 kg) is
   
   \[ T = m_1 a \]  
   
   \(^{(1)}\)
   
   Block 2 (3.00 kg) has two ropes attached each carrying a tension \( T \). Also, block 2 only travels half the distance that block 1 travels in the same amount of time so its acceleration is only half of block 1's acceleration. Newton's second law for block 2 is then
   
   \[ 2T - m_2 g = -\frac{1}{2} m_2 a \]  
   
   \(^{(2)}\)
   
   Solving Equation (1) for \( a \), substituting into Equation (2), and rearranging gives
   
   \[ T = \frac{1}{2} m_2 g \frac{1}{1 + \frac{1}{4} \left( m_2 / m_1 \right)} = 13.7 \text{ N} \]
   
   b. Using this result in Equation (1) yields
   
   \[ a = \frac{T}{m_1} = \frac{13.7 \text{ N}}{10.0 \text{ kg}} = 1.37 \text{ m/s}^2 \]

92. **REASONING AND SOLUTION**
   a. The force acting on the sphere which accelerates it is the horizontal component of the tension in the string. Newton's second law for the horizontal motion of the sphere gives
   
   \[ T \sin \theta = ma \]
   
   The vertical component of the tension in the string supports the weight of the sphere so
   
   \[ T \cos \theta = mg \]
   
   Eliminating \( T \) from the above equations results in \( a = g \tan \theta \).
   
   b. \( a = g \tan \theta = (9.80 \text{ m/s}^2) \tan 10.0^\circ = 1.73 \text{ m/s}^2 \)
   
   c. Rearranging the result of part a and setting \( a = 0 \text{ m/s}^2 \) gives
   
   \[ \theta = \tan^{-1} \left( \frac{a}{g} \right) = 0^\circ \]

93. **REASONING AND SOLUTION** The penguin comes to a halt on the horizontal surface because the kinetic frictional force opposes the motion and causes it to slow down. The time required for the penguin to slide to a halt \( (v = 0 \text{ m/s}) \) after entering the horizontal patch of ice is, according to Equation 2.4,
We must, therefore, determine the acceleration of the penguin as it slides along the horizontal patch (see the following drawing).

For the penguin sliding on the horizontal patch of ice, we find from free-body diagram B and Newton's second law in the $x$ direction (motion to the right is taken as positive) that

$$
\sum F_x = -f_{k2} = ma_x \quad \text{or} \quad a_x = \frac{-f_{k2}}{m} = -\mu_k \frac{F_{N2}}{m}
$$

In the $y$ direction in free-body diagram B, we have $\sum F_y = F_{N2} - mg = 0$, or $F_{N2} = mg$. Therefore, the acceleration of the penguin is

$$a_x = \frac{-\mu_k mg}{m} = -\mu_k g \quad \text{(1)}$$

Equation (1) indicates that, in order to find the acceleration $a_x$, we must find the coefficient of kinetic friction.

We are told in the problem statement that the coefficient of kinetic friction between the penguin and the ice is the same for the incline as for the horizontal patch. Therefore, we can use the motion of the penguin on the incline to determine the coefficient of friction and use it in Equation (1).

For the penguin sliding down the incline, we find from free-body diagram A (see the previous drawing) and Newton's second law (taking the direction of motion as positive) that

$$\sum F_x = mg \sin \theta - f_{k1} = ma_x = 0 \quad \text{or} \quad f_{k1} = mg \sin \theta \quad \text{(2)}$$

Here, we have used the fact that the penguin slides down the incline with a constant velocity, so that it has zero acceleration. From Equation 4.8, we know that $f_{k1} = \mu_k F_{N1}$. Applying Newton's second law in the direction perpendicular to the incline, we have
\[ \Sigma F_y = F_{N1} - mg \cos \theta = 0 \quad \text{or} \quad F_{N1} = mg \cos \theta \]

Therefore, \( f_{k1} = \mu_k mg \cos \theta \), so that according to Equation (2), we find

\[ f_{k1} = \mu_k mg \cos \theta = mg \sin \theta \]

Solving for the coefficient of kinetic friction, we have

\[ \mu_k = \frac{\sin \theta}{\cos \theta} = \tan \theta \]

Finally, the time required for the penguin to slide to a halt after entering the horizontal patch of ice is

\[ t = \frac{-v_0}{a_x} = \frac{-v_0}{-\mu_k g} = \frac{v_0}{g \tan \theta} = \frac{1.4 \text{ m/s}}{(9.80 \text{ m/s}^2) \tan 6.9^\circ} = 1.2 \text{ s} \]

94. \textbf{REASONING AND SOLUTION}

a. The static frictional force is responsible for accelerating the top block so that it does not slip against the bottom one. The maximum force that can be supplied by friction is

\[ f_s^{\text{MAX}} = \mu_s F_N = \mu_s m_1 g \]

Newton's second law requires that \( f_s^{\text{MAX}} = m_1 a \), so

\[ a = \mu_s g \]

The force necessary to cause BOTH blocks to have this acceleration is

\[ F = (m_1 + m_2) a = (m_1 + m_2) \mu_s g \]

\[ F = (5.00 \text{ kg} + 12.0 \text{ kg})(0.600)(9.80 \text{ m/s}^2) = 1.00 \times 10^2 \text{ N} \]

b. The maximum acceleration that the two block combination can have before slipping occurs is

\[ a = \frac{F}{17.0 \text{ kg}} \]

Newton's second law applied to the 5.00 kg block is

\[ F - \mu_s m_1 g = m_1 a = (5.00 \text{ kg}) \frac{F}{17.0 \text{ kg}} \]

Hence

\[ F = 41.6 \text{ N} \]
95. **REASONING** With air resistance neglected, only two forces act on the bungee jumper at this instant (see the free-body diagram): the bungee cord pulls up on her with a force \( B \), and the earth pulls down on her with a gravitational force \( mg \). Because we know the jumper’s mass and acceleration, we can apply Newton’s second law to this free-body diagram and solve for \( B \).

**SOLUTION** We will take the direction of the jumper’s acceleration (downward) as negative. Then, the net force acting on the bungee jumper is \( \Sigma F = B - mg \). With Newton’s second law \( (\Sigma F = ma) \), this becomes \( ma = B - mg \). We now solve for \( B \):

\[
B = ma + mg = m(a + g) = (55 \text{ kg})(-7.6 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 120 \text{ N}
\]

As indicated in the free-body diagram, the direction of the force applied by the bungee cord is upward.

96. **REASONING** We do not have sufficient information to calculate the average net force applied to the fist by taking the vector sum of the individually applied forces. However, we have the mass \( m \) of the fist, as well as its initial velocity \( (v_0 = 0 \text{ m/s, since the fist starts from rest}) \), final velocity \( (v = 8.0 \text{ m/s}) \), and the elapsed time \( (\Delta t = 0.15 \text{ s}) \). Therefore we can use Equation 2.4 \( \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} \) to determine the average acceleration \( \bar{a} \) of the fist and then use Equation 4.1 \( (\Sigma \vec{F} = m\bar{a}) \) to find the average net force \( \Sigma \vec{F} \) applied to the fist.

**SOLUTION** Inserting the relation \( \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} \) into Newton’s second law yields the average net force applied to the fist:

\[
\Sigma \vec{F} = m\bar{a} = m\left(\frac{v - v_0}{\Delta t}\right) = (0.70 \text{ kg})\left( \frac{8.0 \text{ m/s} - 0 \text{ m/s}}{0.15 \text{ s}} \right) = 37 \text{ N}
\]

97. **REASONING AND SOLUTION**

a. The apparent weight of the person is given by Equation 4.6 as

\[
F_N = mg + ma = (95.0 \text{ kg})(9.80 \text{ m/s}^2 + 1.80 \text{ m/s}^2) = 1.10 \times 10^3 \text{ N}
\]

b. \( F_N = (95.0 \text{ kg})(9.80 \text{ m/s}^2) = 931 \text{ N} \)

c. \( F_N = (95.0 \text{ kg})(9.80 \text{ m/s}^2 - 1.30 \text{ m/s}^2) = 808 \text{ N} \)
98. **REASONING** Newton’s second law gives the acceleration as $a = (\Sigma F)/m$. Since we seek only the horizontal acceleration, it is the $x$ component of this equation that we will use; $a_x = (\Sigma F_x)/m$. For completeness, however, the free-body diagram will include the vertical forces also (the normal force $F_N$ and the weight $W$).

**SOLUTION** The free-body diagram is shown at the right, where

\[
F_1 = 59.0 \, \text{N}
\]
\[
F_2 = 33.0 \, \text{N}
\]
\[
\theta = 70.0^\circ
\]

When $F_1$ is replaced by its $x$ and $y$ components, we obtain the free body diagram in the following drawing.

Choosing right to be the positive direction, we have

\[
a_x = \frac{\Sigma F_x}{m} = \frac{F_1 \cos \theta - F_2}{m}
\]

\[
a_x = \frac{(59.0 \, \text{N}) \cos 70.0^\circ - (33.0 \, \text{N})}{7.00 \, \text{kg}} = -1.83 \, \text{m/s}^2
\]

Thus, the horizontal acceleration has a magnitude of $1.83 \, \text{m/s}^2$, and the minus sign indicates that it points to the left.

99. **SSM REASONING** The book is kept from falling as long as the total static frictional force balances the weight of the book. The forces that act on the book are shown in the following free-body diagram, where $P$ is the magnitude of the pressing force applied by each hand.
In this diagram, note that there are two pressing forces, one from each hand. Each hand also applies a static frictional force, and, therefore, two static frictional forces are shown. The maximum static frictional force is related in the usual way to a normal force $F_N$, but in this problem the normal force is provided by the pressing force, so that $F_N = P$.

**SOLUTION** Since the frictional forces balance the weight, we have

$$2f_s^{\text{MAX}} = 2(\mu_s F_N) = 2(\mu_s P) = W$$

Solving for $P$, we find that

$$P = \frac{W}{2\mu_s} = \frac{31 \text{ N}}{2(0.40)} = 39 \text{ N}$$

100. **REASONING** Suppose the bobsled is moving along the $+x$ direction. There are two forces acting on it that are parallel to its motion; a force $F_x$ propelling it forward and a force of $-450 \text{ N}$ that is resisting its motion. The net force is the sum of these two forces. According to Newton’s second law, Equation (4.2a), the net force is equal to the mass of the bobsled times its acceleration. Since the mass and acceleration are known, we can use the second law to determine the magnitude of the propelling force.

**SOLUTION**

a. Newton’s second law states that

$$\sum F_x = ma_x$$

Solving this equation for $F_x$ gives

$$F_x = ma_x + 450 \text{ N} = (270 \text{ kg})(2.4 \text{ m/s}^2) + 450 \text{ N} = 1100 \text{ N}$$

b. The magnitude of the net force that acts on the bobsled is

$$\Sigma F_x = ma_x = (270 \text{ kg})(2.4 \text{ m/s}^2) = 650 \text{ N}$$

101. **REASONING AND SOLUTION** The acceleration needed so that the craft touches down with zero velocity is

$$a = \frac{v^2 - v_0^2}{2s} = \frac{-(18.0 \text{ m/s})^2}{2(-165 \text{ m})} = 0.982 \text{ m/s}^2$$

Newton's second law applied in the vertical direction gives

$$F - mg = ma$$

Then

$$F = m(a + g) = (1.14 \times 10^4 \text{ kg})(0.982 \text{ m/s}^2 + 1.60 \text{ m/s}^2) = 29,400 \text{ N}$$
102. **REASONING** The reading on the bathroom scale is proportional to the normal force it exerts on the man. When he simply stands on the scale, his acceleration is zero, so the normal force pushing up on him balances the downward pull of gravity: $F_{N1} = mg$ (see the free-body diagram). Thus, the first reading on the scale is his actual mass $m$, the ratio of the normal force the scale exerts on him to the acceleration due to gravity: First reading $= m = F_{N1}/g = 92.6$ kg. With the chin-up bar helping to support him, the normal force exerted on him by the scale decreases, and the second reading is the ratio of the reduced normal force $F_{N2}$ to the acceleration due to gravity: Second reading $= F_{N2}/g = 75.1$ kg. Lastly, we note that the magnitude $P$ of the force the chin-up bar exerts on the man is exactly equal to the magnitude $P$ of the force that the man exerts on the chin-up bar. This prediction is due to Newton’s third law. Therefore, it is a value for $P$ that we seek.

**SOLUTION** When the man is pulling down on the chin-up bar, there are two upward forces acting on him (see the second part of the drawing), and he is still at rest, so the sum of these two forces balances the downward pull of gravity: $F_{N2} + P = mg$, or $P = mg - F_{N2}$. Since the second reading on the scale is equal to $F_{N2}/g$, the normal force the scale exerts on him is $F_{N2} = $ (Second reading)$g$. Thus we obtain the magnitude $P$ of the force the man exerts on the chin-up bar:

$$P = mg - F_{N2} = mg - (\text{Second reading})g = (m - \text{Second reading})g$$

$$= (92.6 \text{ kg} - 75.1 \text{ kg})(9.80 \text{ m/s}^2) = 172 \text{ N}$$

103. **SSM REASONING** If we assume that the acceleration is constant, we can use Equation 2.4 ($v = v_0 + at$) to find the acceleration of the car. Once the acceleration is known, Newton's second law ($\sum F = ma$) can be used to find the magnitude and direction of the net force that produces the deceleration of the car.

**SOLUTION** We choose due east to be the positive direction. Given this choice, the average acceleration of the car is, according to Equation 2.4,

$$a = \frac{v - v_0}{t} = \frac{17.0 \text{ m/s} - 27.0 \text{ m/s}}{8.00 \text{ s}} = -1.25 \text{ m/s}^2$$

where the minus sign indicates that the acceleration points due west. According to Newton's Second law, the net force on the car is

$$\sum F = ma = (1380 \text{ kg})(-1.25 \text{ m/s}^2) = -1730 \text{ N}$$
104. **REASONING** The net force acting on the ball can be calculated using Newton's second law. Before we can use Newton's second law, however, we must use Equation 2.9 from the equations of kinematics to determine the acceleration of the ball.

**SOLUTION** According to Equation 2.9, the acceleration of the ball is given by

$$a = \frac{v^2 - v_0^2}{2x}$$

Thus, the magnitude of the net force on the ball is given by

$$\sum F = ma = m \left(\frac{v^2 - v_0^2}{2x}\right) = (0.058 \text{ kg}) \left[\frac{(45 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(0.44 \text{ m})}\right] = 130 \text{ N}$$

105. **REASONING AND SOLUTION**

a. The mass of an object is an intrinsic property of the object and is independent of its location in the universe. Thus, on earth, the traveler’s mass is \(m = 115 \text{ kg}\). According to Equation 4.5, the weight of the space traveler on earth is

$$W = mg = (115 \text{ kg})(9.80 \text{ m/s}^2) = 1.13 \times 10^3 \text{ N}$$

b. In interplanetary space where there are no nearby planetary objects, the gravitational force exerted on the space traveler is zero and \(g = 0 \text{ m/s}^2\). Therefore, the weight is \(W = 0 \text{ N}\). Since the mass of an object is an intrinsic property of the object and is independent of its location in the universe, the mass of the space traveler is still \(m = 115 \text{ kg}\).

106. **REASONING** The magnitude of the gravitational force exerted on the satellite by the earth is given by Equation 4.3 as \(F = Gm_{\text{satellite}}m_{\text{earth}}/r^2\), where \(r\) is the distance between the satellite and the center of the earth. This expression also gives the magnitude of the gravitational force exerted on the earth by the satellite. According to Newton’s second law, the magnitude of the earth’s acceleration is equal to the magnitude of the gravitational force exerted on it divided by its mass. Similarly, the magnitude of the satellite’s acceleration is equal to the magnitude of the gravitational force exerted on it divided by its mass.
**FORCES AND NEWTON'S LAWS OF MOTION**

**SOLUTION**

a. The magnitude of the gravitational force exerted on the satellite when it is a distance of two earth radii from the center of the earth is

\[
F = \frac{G m_{\text{satellite}} m_{\text{earth}}}{r^2} = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)(425 \text{ kg})(5.98 \times 10^{24} \text{ kg})}{\left(2 \left(6.38 \times 10^6 \text{ m}\right)\right)^2} = 1.04 \times 10^3 \text{ N}
\]

b. The magnitude of the gravitational force exerted on the earth when it is a distance of two earth radii from the center of the satellite is

\[
F = \frac{G m_{\text{satellite}} m_{\text{earth}}}{r^2} = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)(425 \text{ kg})(5.98 \times 10^{24} \text{ kg})}{\left(2 \left(6.38 \times 10^6 \text{ m}\right)\right)^2} = 1.04 \times 10^3 \text{ N}
\]

c. The acceleration of the satellite can be obtained from Newton’s second law.

\[
a_{\text{satellite}} = \frac{F}{m_{\text{satellite}}} = \frac{1.04 \times 10^3 \text{ N}}{425 \text{ kg}} = 2.45 \text{ m/s}^2
\]

d. The acceleration of the earth can also be obtained from Newton’s second law.

\[
a_{\text{earth}} = \frac{F}{m_{\text{earth}}} = \frac{1.04 \times 10^3 \text{ N}}{5.98 \times 10^{24} \text{ kg}} = 1.74 \times 10^{-22} \text{ m/s}^2
\]

**SSM REASONING** The free-body diagrams for Robin (mass = \(m\)) and for the chandelier (mass = \(M\)) are given at the right. The tension \(T\) in the rope applies an upward force to both. Robin accelerates upward, while the chandelier accelerates downward, each acceleration having the same magnitude. Our solution is based on separate applications of Newton’s second law to Robin and the chandelier.

**SOLUTION** Applying Newton’s second law, we find

\[
T - mg = ma \quad \text{and} \quad T - Mg = -Ma
\]

In these applications we have taken upward as the positive direction, so that Robin’s acceleration is \(a\), while the chandelier’s acceleration is \(-a\). Solving the Robin-Hood equation for \(T\) gives

\[
T = mg + ma
\]

Substituting this expression for \(T\) into the Chandelier equation gives
Chapter 4 Problems

\[ mg + ma - Mg = -Ma \quad \text{or} \quad a = \left( \frac{M - m}{M + m} \right) g \]

a. Robin’s acceleration is

\[ a = \left( \frac{M - m}{M + m} \right) g = \left[ \frac{(195 \text{ kg}) - (77.0 \text{ kg})}{(195 \text{ kg}) + (77.0 \text{ kg})} \right] (9.80 \text{ m/s}^2) = 4.25 \text{ m/s}^2 \]

b. Substituting the value of \( a \) into the expression for \( T \) gives

\[ T = mg + ma = (77.0 \text{ kg})(9.80 \text{ m/s}^2 + 4.25 \text{ m/s}^2) = 1080 \text{ N} \]

108. **REASONING**  Kinetic friction is the only horizontal force that acts on the skater. It acts opposite to the direction of motion, slowing him down. We take the direction of the motion of the skater as the positive direction. Therefore, the kinetic frictional force points in the negative direction. According to Newton's second law \((\Sigma F = ma)\) with \(\Sigma F = -f_k\), the deceleration is \(a = -f_k / m\).

The magnitude of the frictional force that acts on the skater is, according to Equation 4.8, \(f_k = \mu_k F_N\) where \(\mu_k\) is the coefficient of kinetic friction between the ice and the skate blades. There are only two vertical forces that act on the skater; they are the upward normal force \(F_N\) and the downward pull of gravity (the weight) \(mg\). Since the skater has no vertical acceleration, Newton's second law in the vertical direction (with upward as the positive direction) gives \(F_N - mg = 0\). Therefore, the magnitude of the normal force is \(F_N = mg\), and the deceleration is given by

\[ a = \frac{-f_k}{m} = \frac{-\mu_k F_N}{m} = \frac{-\mu_k mg}{m} = -\mu_k g \]

**SOLUTION**

a. Direct substitution into the previous expression gives

\[ a = -\mu_k g = -0.100(9.80 \text{ m/s}^2) = -0.980 \text{ m/s}^2 \]

The magnitude of this deceleration is \(0.980 \text{ m/s}^2\). Since the skater is moving in the positive direction and is slowing down, the direction of the acceleration is in the negative direction and is **opposite to the direction of the motion**.

b. The displacement \(x\) through which the skater will slide before he comes to rest can be obtained from \(v^2 = v_0^2 + 2ax\) (Equation 2.9). Since the skater comes to rest, we know that \(v = 0 \text{ m/s}\). Solving for \(x\), we obtain

\[ (0 \text{ m/s})^2 = v_0^2 + 2ax \quad \text{or} \quad x = \frac{-v_0^2}{2a} = \frac{-(7.60 \text{ m/s})^2}{2(-0.980 \text{ m/s}^2)} = 29.5 \text{ m} \]
109. **REASONING** Static friction determines the magnitude of the applied force at which either the upper or lower block begins to slide. For the upper block the static frictional force is applied only by the lower block. For the lower block, however, separate static frictional forces are applied by the upper block and by the horizontal surface. The maximum magnitude of any of the individual frictional forces is given by Equation 4.7 as the coefficient of static friction times the magnitude of the normal force.

**SOLUTION** We begin by drawing the free-body diagram for the lower block.

This diagram shows that three horizontal forces act on the lower block, the applied force, and the two maximum static frictional forces, one from the upper block and one from the horizontal surface. At the instant that the lower block just begins to slide, the blocks are in equilibrium and the applied force is balanced by the two frictional forces, with the result that

\[ F_{\text{Applied}} = f_{s, \text{from } A}^{\text{MAX}} + f_{s, \text{from surface}}^{\text{MAX}} \]  

(1)

According to Equation 4.7, the magnitude of the maximum frictional force from the surface is

\[ f_{s, \text{from surface}}^{\text{MAX}} = \mu_s F_N = \mu_s 2mg \]  

(2)

Here, we have recognized that the normal force \( F_N \) from the horizontal surface must balance the weight \( 2mg \) of both blocks.

It remains now to determine the magnitude of the maximum frictional force \( f_{s, \text{from } A}^{\text{MAX}} \) from the upper block. To this end, we draw the free-body diagram for the upper block at the instant that it just begins to slip due to the 47.0-N applied force. At this instant the block is in equilibrium, so that the frictional force from the lower block \( B \) balances the 47.0-N force. Thus, \( f_{s, \text{from } B}^{\text{MAX}} = 47.0 \text{ N} \), and according to Equation 4.7, we have

\[ f_{s, \text{from } B}^{\text{MAX}} = \mu_s F_N = \mu_s mg = 47.0 \text{ N} \]

Here, we have recognized that the normal force \( F_N \) from the lower block must balance the weight \( mg \) of only the upper block. This result tells us that \( \mu_s mg = 47.0 \text{ N} \). To determine
we invoke Newton’s third law to conclude that the magnitudes of the frictional forces at the A-B interface are equal, since they are action-reaction forces. Thus, 

\[ f_{s, \text{from } A}^{\text{MAX}} = \mu_s mg \]. Substituting this result and Equation (2) into Equation (1) gives

\[ F_{\text{Applied}} = f_{s, \text{from } A}^{\text{MAX}} + f_{s, \text{from surface}}^{\text{MAX}} = \mu_s mg + \mu_s 2mg = 3(47.0 \text{ N}) = 141 \text{ N} \]

110. **REASONING** Since the mountain climber is at rest, she is in equilibrium and the net force acting on her must be zero. Three forces comprise the net force, her weight, and the tension forces from the left and right sides of the rope. We will resolve the forces into components and set the sum of the \( x \) components and the sum of the \( y \) components separately equal to zero. In so doing we will obtain two equations containing the unknown quantities, the tension \( T_L \) in the left side of the rope and the tension \( T_R \) in the right side. These two equations will be solved simultaneously to give values for the two unknowns.

**SOLUTION** Using \( W \) to denote the weight of the mountain climber and choosing right and upward to be the positive directions, we have the following free-body diagram for the climber:

For the \( x \) components of the forces we have

\[ \Sigma F_x = T_R \sin 80.0^\circ - T_L \sin 65.0^\circ = 0 \]

For the \( y \) components of the forces we have

\[ \Sigma F_y = T_R \cos 80.0^\circ + T_L \cos 65.0^\circ - W = 0 \]

Solving the first of these equations for \( T_R \), we find that

\[ T_R = T_L \frac{\sin 65.0^\circ}{\sin 80.0^\circ} \]

Substituting this result into the second equation gives

\[ T_L \frac{\sin 65.0^\circ}{\sin 80.0^\circ} \cos 80.0^\circ + T_L \cos 65.0^\circ - W = 0 \quad \text{or} \quad T_L = 1.717 W \]

Using this result in the expression for \( T_R \) reveals that

\[ T_R = T_L \frac{\sin 65.0^\circ}{\sin 80.0^\circ} = (1.717W) \frac{\sin 65.0^\circ}{\sin 80.0^\circ} = 1.580 W \]

Since the weight of the climber is \( W = 535 \text{ N} \), we find that

\[ T_L = 1.717 W = 1.717(535 \text{ N}) = 919 \text{ N} \]

\[ T_R = 1.580 W = 1.580(535 \text{ N}) = 845 \text{ N} \]
111. **REASONING** The tension in each coupling bar is responsible for accelerating the objects behind it. The masses of the cars are \( m_1 \), \( m_2 \), and \( m_3 \). We can use Newton’s second law to express the tension in each coupling bar, since friction is negligible:

\[
T_A = (m_1 + m_2 + m_3) a \\
T_B = (m_2 + m_3) a \\
T_C = m_3 a
\]

In these expressions \( a = 0.12 \text{ m/s}^2 \) remains constant. Consequently, the tension in a given bar will change only if the total mass of the objects accelerated by that bar changes as a result of the luggage transfer. Using \( \Delta \) (Greek capital delta) to denote a change in the usual fashion, we can express the changes in the above tensions as follows:

\[
\Delta T_A = \left[ \Delta (m_1 + m_2 + m_3) \right] a \\
\Delta T_B = \left[ \Delta (m_2 + m_3) \right] a \\
\Delta T_C = (\Delta m_3) a
\]

**SOLUTION**

a. Moving luggage from car 2 to car 1 does not change the total mass \( m_1 + m_2 + m_3 \), so \( \Delta(m_1 + m_2 + m_3) = 0 \text{ kg} \) and \( \Delta T_A = 0 \text{ N} \).

The transfer from car 2 to car 1 causes the total mass \( m_2 + m_3 \) to decrease by 39 kg, so \( \Delta(m_2 + m_3) = -39 \text{ kg} \) and

\[
\Delta T_B = \left[ \Delta (m_2 + m_3) \right] a = (-39 \text{ kg})(0.12 \text{ m/s}^2) = -4.7 \text{ N}
\]

The transfer from car 2 to car 1 does not change the mass \( m_3 \), so \( \Delta m_3 = 0 \text{ kg} \) and \( \Delta T_C = 0 \text{ N} \).

b. Moving luggage from car 2 to car 3 does not change the total mass \( m_1 + m_2 + m_3 \), so \( \Delta(m_1 + m_2 + m_3) = 0 \text{ kg} \) and \( \Delta T_A = 0 \text{ N} \).

The transfer from car 2 to car 3 does not change the total mass \( m_2 + m_3 \), so \( \Delta(m_2 + m_3) = 0 \text{ kg} \) and \( \Delta T_B = 0 \text{ N} \).

The transfer from car 2 to car 3 causes the mass \( m_3 \) to increase by 39 kg, so \( \Delta m_3 = +39 \text{ kg} \) and

\[
\Delta T_C = (\Delta m_3) a = (+39 \text{ kg})(0.12 \text{ m/s}^2) = +4.7 \text{ N}
\]
112. **REASONING** The box comes to a halt because the kinetic frictional force and the component of its weight parallel to the incline oppose the motion and cause the box to slow down. The displacement of the box as it travels up the incline can be found by solving the relation \( v_x^2 = v_{0x}^2 + 2a_x x \) (Equation 3.6a) for \( x \). The initial and final velocities are known, but the acceleration is not. We will use Newton’s second law to find the acceleration.

**SOLUTION** Solving Equation 3.6a for the displacement of the box, and noting that the box’s final velocity is \( v_x = 0 \text{ m/s} \), we have that

\[
x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{-v_{0x}^2}{2a_x}
\]

(1)

To find the acceleration \( a_x \), we start with the above drawing, which represents the free-body diagram for the box. It shows the resolved components of the forces that act on the box. If we take the direction up the incline as the positive \( x \) direction, then Newton's second law (Equation 4.2a) gives

\[
\sum F_x = -mg \sin \theta - f_k = ma_x
\]

(2)

where \( m \) is the mass of the box, \( g \) is the magnitude of the acceleration due to gravity, \( \theta \) is the angle of the incline, and \( f_k \) is the magnitude of the kinetic frictional force. Solving Equation (2) for the acceleration, we have that

\[
a_x = \frac{-mg \sin \theta - f_k}{m}
\]

(3)

The magnitude \( f_k \) of the kinetic frictional force is related to the coefficient of kinetic friction \( \mu_k \) and the magnitude \( F_N \) of the normal force by Equation 4.8:

\[
f_k = \mu_k F_N
\]

Substituting this expression for \( f_k \) into Equation (3) yields

\[
a_x = \frac{-mg \sin \theta - \mu_k F_N}{m}
\]

(4)

To find \( F_N \), we note that there is no acceleration of the box perpendicular to the plane (in the \( y \) direction), so that \( a_y = 0 \text{ m/s}^2 \). Thus, Newton’s second law, applied to the \( y \)-direction, can be written as \( \sum F_y = ma_y = 0 \) (see Equation 4.9b). Noting that there are two forces that act on the box in the \( y \) direction, \( -mg \cos \theta \) and \( +F_N \) (see the free-body diagram above), Equation 4.9b becomes

\[
\sum F_y = -mg \cos \theta + F_N = 0
\]

(5)
Solving this equation for $F_N$ and substituting the result into Equation (4) yields

$$a_x = \frac{-mg \sin \theta - \mu_k F_N}{m} = -\mu_k g \sin \theta - \mu_k \left( \mu_k g \cos \theta \right) = -g \sin \theta - \mu_k g \cos \theta$$

We can now substitute this expression for $a_x$ into Equation (1), with the result that

$$x = \frac{-v_{0x}^2}{2a_x} = \frac{-v_{0x}^2}{2 \left( -g \sin \theta - \mu_k g \cos \theta \right)} = \frac{v_{0x}^2}{2g \left( \sin \theta + \mu_k \cos \theta \right)}$$

Therefore, the displacement of the box is

$$x = \frac{v_{0x}^2}{2g \left( \sin \theta + \mu_k \cos \theta \right)} = \frac{(1.50 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)[\sin 15.0^\circ + (0.180) \cos 15.0^\circ]} = 0.265 \text{ m}$$

113. **REASONING** Equations 3.5a \((x = v_{0x}t + \frac{1}{2} a_x t^2)\) and 3.5b \((y = v_{0y}t + \frac{1}{2} a_y t^2)\) give the displacements of an object under the influence of constant accelerations $a_x$ and $a_y$. We can add these displacements as vectors to find the magnitude and direction of the resultant displacement. To use Equations 3.5a and 3.5b, however, we must have values for $a_x$ and $a_y$. We can obtain these values from Newton’s second law, provided that we combine the given forces to calculate the $x$ and $y$ components of the net force acting on the duck, and it is here that our solution begins.

**SOLUTION** Let the directions due east and due north, respectively, be the $+x$ and $+y$ directions. Then, the components of the net force are

$$\Sigma F_x = 0.10 \text{ N} + (0.20 \text{ N}) \cos 52^\circ = 0.2231 \text{ N}$$

$$\Sigma F_y = -(0.20 \text{ N}) \sin 52^\circ = -0.1576 \text{ N}$$

According to Newton’s second law, the components of the acceleration are

$$a_x = \frac{\Sigma F_x}{m} = \frac{0.2231 \text{ N}}{2.5 \text{ kg}} = 0.08924 \text{ m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{-0.1576 \text{ N}}{2.5 \text{ kg}} = -0.06304 \text{ m/s}^2$$

From Equations 3.5a and 3.5b, we now obtain the displacements in the $x$ and $y$ directions:

$$x = v_{0x}t + \frac{1}{2} a_x t^2 = (0.11 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(0.08924 \text{ m/s}^2)(3.0 \text{ s})^2 = 0.7316 \text{ m}$$

$$y = v_{0y}t + \frac{1}{2} a_y t^2 = (0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-0.06304 \text{ m/s}^2)(3.0 \text{ s})^2 = -0.2837 \text{ m}$$

The magnitude of the resultant displacement is
The direction of the resultant displacement is
\[ \theta = \tan^{-1}\left(\frac{0.2837 \text{ m}}{0.7316 \text{ m}}\right) = 21^\circ \text{ south of east} \]

114. **REASONING AND SOLUTION** The figure at the right shows the three spheres with sphere 3 being the sphere of unknown mass. Sphere 3 feels a force \( \mathbf{F}_{31} \) due to the presence of sphere 1, and a force \( \mathbf{F}_{32} \) due to the presence of sphere 2. The net force on sphere 3 is the resultant of \( \mathbf{F}_{31} \) and \( \mathbf{F}_{32} \).

Note that since the spheres form an equilateral triangle, each interior angle is 60°. Therefore, both \( \mathbf{F}_{31} \) and \( \mathbf{F}_{32} \) make a 30° angle with the vertical line as shown. Furthermore, \( \mathbf{F}_{31} \) and \( \mathbf{F}_{32} \) have the same magnitude given by

\[ F = \frac{GMm_3}{r^2} \]

where \( M \) is the mass of either sphere 1 or 2 and \( m_3 \) is the mass of sphere 3. The components of the two forces are shown in the following drawings:

Clearly, the horizontal components of the two forces add to zero. Thus, the net force on sphere 3 is the resultant of the vertical components of \( \mathbf{F}_{31} \) and \( \mathbf{F}_{32} \):

\[ F_3 = 2F \cos \theta = 2 \frac{GMm_3}{r^2} \cos \theta \]

The acceleration of sphere 3 is given by Newton's second law:

\[ a_3 = \frac{F_3}{m_3} = 2 \frac{GM}{r^2} \cos \theta = 2 \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) (2.80 \text{ kg})}{(1.20 \text{ m})^2} \cos 30.0^\circ = 2.25 \times 10^{-10} \text{ m/s}^2 \]
115. [SSM] **REASONING** Let us assume that the skater is moving horizontally along the +x axis. The time \( t \) it takes for the skater to reduce her velocity to \( v_x = +2.8 \text{ m/s} \) from \( v_{0x} = +6.3 \text{ m/s} \) can be obtained from one of the equations of kinematics:

\[
v_x = v_{0x} + a_x t \tag{3.3a}
\]

The initial and final velocities are known, but the acceleration is not. We can obtain the acceleration from Newton’s second law \( (\Sigma F_x = ma_x, \text{ Equation 4.2a}) \) in the following manner. The kinetic frictional force is the only horizontal force that acts on the skater, and, since it is a resistive force, it acts opposite to the direction of the motion. Thus, the net force in the \( x \) direction is \( \Sigma F_x = -f_k \), where \( f_k \) is the magnitude of the kinetic frictional force. Therefore, the acceleration of the skater is \( a_x = \Sigma F_x / m = -f_k / m \).

The magnitude of the frictional force is \( f_k = \mu_k F_N \) (Equation 4.8), where \( \mu_k \) is the coefficient of kinetic friction between the ice and the skate blades and \( F_N \) is the magnitude of the normal force. There are two vertical forces acting on the skater: the upward-acting normal force \( F_N \) and the downward pull of gravity (her weight) \( mg \). Since the skater has no vertical acceleration, Newton's second law in the vertical direction gives (taking upward as the positive direction) \( \Sigma F_y = F_N - mg = 0 \). Therefore, the magnitude of the normal force is \( F_N = mg \) and the magnitude of the acceleration is

\[
a_x = -\frac{f_k}{m} = -\frac{\mu_k F_N}{m} = -\frac{\mu_k mg}{g} = -\mu_k g
\]

**SOLUTION**

Solving the equation \( v_x = v_{0x} + a_x t \) for the time and substituting the expression above for the acceleration yields

\[
t = \frac{v_x - v_{0x}}{a_x} = \frac{v_x - v_{0x}}{-\mu_k g} = \frac{2.8 \text{ m/s} - 6.3 \text{ m/s}}{-0.081 \left(9.80 \text{ m/s}^2\right)} = 4.4 \text{ s}
\]

116. **REASONING AND SOLUTION**

a. According to Equation 4.5, the mass of each block is as follows:

- **Lighter block**
  \[
m_1 = \frac{W_1}{g} = \frac{412 \text{ N}}{9.80 \text{ m/s}^2} = 42.0 \text{ kg}
\]

- **Heavier block**
  \[
m_2 = \frac{W_2}{g} = \frac{908 \text{ N}}{9.80 \text{ m/s}^2} = 92.7 \text{ kg}
\]

The rope exerts a tension, \( T \), acting upward on each block. Applying Newton's second law to the lighter block (block 1) gives
\[ T - m_1 g = m_1 a \]

Similarly, for the heavier block (block 2)

\[ T - m_2 g = -m_2 a \]

Subtracting the second equation from the first and rearranging yields

\[ a = \left( \frac{m_2 - m_1}{m_2 + m_1} \right) g = \left( \frac{92.7 \text{ kg} - 42.0 \text{ kg}}{92.7 \text{ kg} + 42.0 \text{ kg}} \right) (9.80 \text{ m/s}^2) = 3.69 \text{ m/s}^2 \]

b. The tension in the rope is now 908 N since the tension is the reaction to the applied force exerted by the hand. Newton's second law applied to the lighter block is

\[ T - m_1 g = m_1 a \]

Solving for \( a \) gives

\[ a = \frac{T}{m_1} - g = \frac{(908 \text{ N})}{42.0 \text{ kg}} - 9.80 \text{ m/s}^2 = 11.8 \text{ m/s}^2 \]

c. In the first case, the inertia of BOTH blocks affects the acceleration whereas, in the second case, only the lighter block's inertia remains.

117. [SSM] REASONING AND SOLUTION

a. The left mass (mass 1) has a tension \( T_1 \) pulling it up. Newton's second law gives

\[ T_1 - m_1 g = m_1 a \]  \hspace{1cm} (1)

The right mass (mass 3) has a different tension, \( T_3 \), trying to pull it up. Newton's second for it is

\[ T_3 - m_3 g = -m_3 a \]  \hspace{1cm} (2)

The middle mass (mass 2) has both tensions acting on it along with friction. Newton's second law for its horizontal motion is

\[ T_3 - T_1 - \mu_k m_2 g = m_2 a \]  \hspace{1cm} (3)

Solving Equation (1) and Equation (2) for \( T_1 \) and \( T_3 \), respectively, and substituting into Equation (3) gives

\[ a = \frac{\left( m_3 - m_1 - \mu_k m_2 \right) g}{m_1 + m_2 + m_3} \]

Hence,

\[ a = \frac{\left[ 25.0 \text{ kg} - 10.0 \text{ kg} - (0.100)(80.0 \text{ kg}) \right] (9.80 \text{ m/s}^2)}{10.0 \text{ kg} + 80.0 \text{ kg} + 25.0 \text{ kg}} = 0.60 \text{ m/s}^2 \]
b. From part a:

\[
T_1 = m_1(g + a) = (10.0 \text{ kg})(9.80 \text{ m/s}^2 + 0.60 \text{ m/s}^2) = 104 \text{ N}
\]

\[
T_3 = m_3(g - a) = (25.0 \text{ kg})(9.80 \text{ m/s}^2 - 0.60 \text{ m/s}^2) = 230 \text{ N}
\]

118. **REASONING** The following figure shows the crate on the incline and the free body diagram for the crate. The diagram at the far right shows all the forces resolved into components that are parallel and perpendicular to the surface of the incline. We can analyze the motion of the crate using Newton's second law. The coefficient of friction can be determined from the resulting equations.

**SOLUTION** Since the crate is at rest, it is in equilibrium and its acceleration is zero in all directions. If we take the direction down the incline as positive, Newton's second law indicates that

\[
\sum F_x = P \cos \theta + mg \sin \theta - f_s^{\text{MAX}} = 0
\]

According to Equation 4.7, \( f_s^{\text{MAX}} = \mu_s F_N \). Therefore, we have

\[
P \cos \theta + mg \sin \theta - \mu_s F_N = 0
\]

The expression for the normal force can be found from analyzing the forces that are perpendicular to the incline. Taking up to be positive, we have

\[
\sum F_y = P \sin \theta + F_N - mg \cos \theta = 0 \quad \text{or} \quad F_N = mg \cos \theta - P \sin \theta
\]

Equation (1) then becomes

\[
P \cos \theta + mg \sin \theta - \mu_s (mg \cos \theta - P \sin \theta) = 0
\]

Solving for the coefficient of static friction, we find that

\[
\mu_s = \frac{P \cos \theta + mg \sin \theta}{mg \cos \theta - P \sin \theta} = \frac{(535 \text{ N}) \cos 20.0^\circ + (225 \text{ kg})(9.80 \text{ m/s}^2) \sin 20.0^\circ}{(225 \text{ kg})(9.80 \text{ m/s}^2) \cos 20.0^\circ - (535 \text{ N}) \sin 20.0^\circ} = 0.665
\]
119. **REASONING** The diagram shows the two applied forces that act on the crate. These two forces, plus the kinetic frictional force $f_k$ constitute the net force that acts on the crate. Once the net force has been determined, Newtons’ second law, $\Sigma F = ma$ (Equation 4.1) can be used to find the acceleration of the crate.

**SOLUTION** The sum of the applied forces is $F = F_1 + F_2$. The $x$-component of this sum is $F_x = F_1 \cos 55.0^\circ + F_2 = (88.0 \text{ N}) \cos 55.0^\circ + 54.0 \text{ N} = 104 \text{ N}$. The $y$-component of $F$ is $F_y = F_1 \sin 55.0^\circ = (88.0 \text{ N}) \sin 55.0^\circ = 72.1 \text{ N}$. The magnitude of $F$ is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(104 \text{ N})^2 + (72.1 \text{ N})^2} = 127 \text{ N}$$

Since the crate starts from rest, it moves along the direction of $F$. The kinetic frictional force $f_k$ opposes the motion, so it points opposite to $F$. The net force acting on the crate is the sum of $F$ and $f_k$. The magnitude $a$ of the crate’s acceleration is equal to the magnitude $\Sigma F$ of the net force divided by the mass $m$ of the crate

$$a = \frac{\Sigma F}{m} = \frac{-f_k + F}{m} \quad (4.1)$$

According to Equation 4.8, the magnitude $f_k$ of the kinetic frictional force is given by $f_k = \mu_k F_N$, where $F_N$ is the magnitude of the normal force. In this situation, $F_N$ is equal to the magnitude of the crate’s weight, so $F_N = mg$. Thus, the $x$-component of the acceleration is

$$a = \frac{-\mu_k mg + F}{m} = \frac{-0.350 \times (25.0 \text{ kg}) \times (9.80 \text{ m/s}^2) + 127 \text{ N}}{25.0 \text{ kg}} = 1.65 \text{ m/s}^2$$

The crate moves along the direction of $F$, whose $x$ and $y$ components have been determined previously. Therefore, the acceleration is also along $F$. The angle $\phi$ that $F$ makes with the $x$-axis can be found using the inverse tangent function:

$$\phi = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{F_1 \sin 55.0^\circ}{F_1 \cos 55.0^\circ + F_2} \right)$$

$$= \tan^{-1} \left[ \frac{(88.0 \text{ N}) \sin 55.0^\circ}{(88.0 \text{ N}) \cos 55.0^\circ + 54.0 \text{ N}} \right] = 34.6^\circ \text{ above the } x \text{ axis}$$
CHAPTER 5  
DYNAMICS OF UNIFORM CIRCULAR MOTION

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (c) The velocity of car A has a constant magnitude (speed) and direction. Since its velocity is constant, car A does not have an acceleration. The velocity of car B is continually changing direction during the turn. Therefore, even though car B has a constant speed, it has an acceleration (known as a centripetal acceleration).

2. (d) The centripetal (or “center-seeking”) acceleration of the car is perpendicular to its velocity and points toward the center of the circle that the road follows.

3. (b) The magnitude of the centripetal acceleration is equal to \( \frac{v^2}{r} \), where \( v \) is the speed of the object and \( r \) is the radius of the circular path. Since the radius of the track is smaller at A compared to B, the centripetal acceleration of the car at A has a greater magnitude.

4. (a) The magnitude \( a_c \) of the centripetal acceleration is given by \( a_c = \frac{v^2}{r} \).

5. (d) The acceleration (known as the centripetal acceleration) and the net force (known as the centripetal force) have the same direction and point toward the center of the circular path.

6. (a) According to the discussion in Example 7 in Section 5.3, the maximum speed that the cylinder can have is given by \( v_{\text{max}} = \sqrt{\mu_s gr} \), where \( \mu_s \) is the coefficient of static friction, \( g \) is the acceleration due to gravity, and \( r \) is the radius of the path.

7. (d) The radius of path 1 is twice that of path 2. The tension in the cord is the centripetal force. Since the centripetal force is inversely proportional to the radius \( r \) of the path, \( T_1 \) must be one-half of \( T_2 \).

8. (a) The centripetal force is given by \( F_c = \frac{mv^2}{r} \). The centripetal forces for particles 1, 2 and 3 are, respectively, \( 4m_0v^2_0/r_0 \), \( 3m_0v^2_0/r_0 \), and \( 2m_0v^2_0/r_0 \).

9. (d) The centripetal force is directed along the radius and toward the center of the circular path. The component \( F_N \sin \theta \) of the normal force is directed along the radius and points toward the center of the path.

10. (a) The magnitude of the centripetal force is given by \( F_c = \frac{mv^2}{r} \). The two cars have the same speed \( v \) and the radius \( r \) of the turn is the same. The cars also have the same mass \( m \), even though they have different weights due to the different accelerations due to gravity. Therefore, the centripetal accelerations are the same.
11. (e) The centripetal force acting on a satellite is provided by the gravitational force. The magnitude of the gravitational force is inversely proportional to the radius squared \(1/r^2\), so if the radius is doubled, the gravitational force is one fourth as great; \(1/2^2 = 1/4\).

12. The orbital speed is \(v = 1.02 \times 10^3\) m/s.

13. (b) The magnitude of the centripetal force acting on the astronaut is equal to her apparent weight. The centripetal force is given by Equation 5.3 as \(F_c = mv^2/r\), which depends on the square \((v^2)\) of the astronaut’s speed and inversely \((1/r)\) on the radius of the ring. According to Equation 5.1, \(r = vT/(2\pi)\), the radius is directly proportional to the speed. Thus, the centripetal force is directly proportional to the speed \(v\) of the astronaut. As the astronaut walks from the inner ring to the outer ring, her speed doubles and so does her apparent weight.

14. (d) The skier at A is speeding up, so the direction of the acceleration, and hence the net force, must be parallel to the skier’s velocity. At B the skier is momentarily traveling at a constant speed on a circular path of radius \(r\). The direction of the net force, called the centripetal force, must be toward the center of the path. At C the skier is in free-fall, so the net force, which is the gravitational force, is straight downward.

15. (b) According to Newton’s second law, the net force, \(F_N - mg\), must equal the mass \(m\) times the centripetal acceleration \(v^2/r\).
CHAPTER 5 | DYNAMICS OF UNIFORM CIRCULAR MOTION

PROBLEMS

1. **REASONING** The magnitude $a_c$ of the car’s centripetal acceleration is given by Equation 5.2 as $a_c = \frac{v^2}{r}$, where $v$ is the speed of the car and $r$ is the radius of the track.

The radius is $r = 2.6 \times 10^3$ m. The speed can be obtained from Equation 5.1 as the circumference $(2\pi r)$ of the track divided by the period $T$ of the motion. The period is the time for the car to go once around the track ($T = 360$ s).

**SOLUTION** Since $a_c = \frac{v^2}{r}$ and $v = \left(\frac{2\pi r}{T}\right)$, the magnitude of the car’s centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{2\pi r}{r} = 4\pi^2 r \left(\frac{2.6 \times 10^3 \text{ m}}{(360 \text{ s})^3}\right) = 0.79 \text{ m/s}^2$$

2. **REASONING** According to $a_c = \frac{v^2}{r}$ (Equation 5.2), the magnitude $a_c$ of the centripetal acceleration depends on the speed $v$ of the object and the radius $r$ of its circular path. In Example 2 the object is moving on a path whose radius is infinitely large; in other words, the object is moving along a straight line.

**SOLUTION** Using Equation 5.2, we find the following values for the magnitude of the centripetal acceleration:

- **Example 1**
  $$a_c = \frac{v^2}{r} = \frac{(12 \text{ m/s})^2}{0.50 \text{ m}} = 290 \text{ m/s}^2$$

- **Example 2**
  $$a_c = \frac{v^2}{r} = \frac{(35 \text{ m/s})^2}{\infty} = 0 \text{ m/s}^2$$

- **Example 3**
  $$a_c = \frac{v^2}{r} = \frac{(2.3 \text{ m/s})^2}{1.8 \text{ m}} = 2.9 \text{ m/s}^2$$
3. **REASONING AND SOLUTION** Let \( s \) represent the length of the path of the pebble after it is released. From Conceptual Example 2, we know that the pebble will fly off tangentially. Therefore, the path \( s \) is perpendicular to the radius \( r \) of the circle. Thus, the distances \( r, s, \) and \( d \) form a right triangle with hypotenuse \( d \) as shown in the figure at the right. From the figure we see that

\[
\cos \alpha = \frac{r}{d} = \frac{r}{10r} = \frac{1}{10} \quad \text{or} \quad \alpha = \cos^{-1}\left(\frac{1}{10}\right) = 84^\circ
\]

Furthermore, from the figure, we see that \( \alpha + \theta + 35^\circ = 180^\circ \). Therefore,

\[
\theta = 145^\circ - \alpha = 145^\circ - 84^\circ = 61^\circ
\]

4. **REASONING** In each case, the magnitude of the centripetal acceleration is given by \( a_c = \frac{v^2}{r} \) (Equation 5.2). Therefore, we will apply this expression to each boat and set the centripetal accelerations equal. The resulting equation can be solved for the desired ratio.

**SOLUTION** Using Equation 5.2 for the centripetal acceleration of each boat, we have

\[
a_{cA} = \frac{v_A^2}{r_A} \quad \text{and} \quad a_{cB} = \frac{v_B^2}{r_B}
\]

Setting the two centripetal accelerations equal gives

\[
\frac{v_A^2}{r_A} = \frac{v_B^2}{r_B}
\]

Solving for the ratio of the speeds gives

\[
\frac{v_A}{v_B} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{120 \text{ m}}{240 \text{ m}}} = 0.71
\]

5. **SSM REASONING** The speed of the plane is given by Equation 5.1: \( v = \frac{2\pi r}{T} \), where \( T \) is the period or the time required for the plane to complete one revolution.

**SOLUTION** Solving Equation 5.1 for \( T \) we have

\[
T = \frac{2\pi r}{v} = \frac{2\pi (2850 \text{ m})}{110 \text{ m/s}} = 160 \text{ s}
\]
6. **REASONING** Blood traveling through the aortic arch follows a circular path with a diameter of 5.0 cm and, therefore, a radius of \( r = 2.5 \text{ cm} = 0.025 \text{ m} \). We know the speed \( v \) of the blood flow, so the relation \( a_c = \frac{v^2}{r} \) (Equation 5.2) will give the magnitude of the blood’s centripetal acceleration.

**SOLUTION** With a blood flow speed of \( v = 0.32 \text{ m/s} \), the magnitude of the centripetal acceleration in the aortic arch is

\[
a_c = \frac{v^2}{r} = \frac{(0.32 \text{ m/s})^2}{0.025 \text{ m}} = 4.1 \text{ m/s}^2
\]

7. **REASONING** Since the tip of the blade moves on a circular path, it experiences a centripetal acceleration whose magnitude \( a_c \) is given by Equation 5.2 as, \( a_c = \frac{v^2}{r} \), where \( v \) is the speed of blade tip and \( r \) is the radius of the circular path. The radius is known, and the speed can be obtained by dividing the distance that the tip travels by the time \( t \) of travel. Since an angle of 90\(^\circ\) corresponds to one fourth of the circumference of a circle, the distance is \( \frac{1}{4}(2\pi r) \).

**SOLUTION** Since \( a_c = \frac{v^2}{r} \) and \( v = \frac{1}{4}(2\pi r)/t = \pi r/(2t) \), the magnitude of the centripetal acceleration of the blade tip is

\[
a_c = \frac{v^2}{r} = \frac{(\pi r/2t)^2}{r} = \frac{\pi^2 r^2}{4t^2} = \frac{\pi^2 (0.45 \text{ m})}{4(0.40 \text{ s})^2} = 6.9 \text{ m/s}^2
\]

8. **REASONING** The centripetal acceleration is given by Equation 5.2 as \( a_c = \frac{v^2}{r} \). The value of the radius \( r \) is given, so to determine \( a_c \) we need information about the speed \( v \). But the speed is related to the period \( T \) by \( v = \frac{(2\pi r)}{T} \), according to Equation 5.1. We can substitute this expression for the speed into Equation 5.2 and see that

\[
a_c = \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}
\]

**SOLUTION** To use the expression obtained in the reasoning, we need a value for the period \( T \). The period is the time for one revolution. Since the container is turning at 2.0 revolutions per second, the period is \( T = \frac{1 \text{ s}}{2.0 \text{ revolutions}} = 0.50 \text{ s} \). Thus, we find that the centripetal acceleration is

\[
a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (0.12 \text{ m})}{(0.50 \text{ s})^2} = 19 \text{ m/s}^2
\]
9. **REASONING AND SOLUTION** Since the magnitude of the centripetal acceleration is given by Equation 5.2, \( a_c = \frac{v^2}{r} \), we can solve for \( r \) and find that
\[
r = \frac{v^2}{a_c} = \frac{(98.8 \text{ m/s})^2}{3.00(9.80 \text{ m/s}^2)} = 332 \text{ m}
\]

10. **REASONING** The centripetal acceleration for any point that is a distance \( r \) from the center of the disc is, according to Equation 5.2, \( a_c = \frac{v^2}{r} \). From Equation 5.1, we know that \( v = \frac{2\pi r}{T} \) where \( T \) is the period of the motion. Combining these two equations, we obtain
\[
a_c = \frac{(2\pi r / T)^2}{r} = 4\pi^2 \frac{r}{T^2}
\]

**SOLUTION** Using the above expression for \( a_c \), the ratio of the centripetal accelerations of the two points in question is
\[
\frac{a_2}{a_1} = \frac{4\pi^2 r_2 / T_2^2}{4\pi^2 r_1 / T_1^2} = \frac{r_2 / T_2^2}{r_1 / T_1^2}
\]
Since the disc is rigid, all points on the disc must move with the same period, so \( T_1 = T_2 \). Making this cancellation and solving for \( a_2 \), we obtain
\[
a_2 = a_1 \frac{r_2}{r_1} = \left(120 \text{ m/s}^2\right) \left(\frac{0.050 \text{ m}}{0.030 \text{ m}}\right) = 2.0 \times 10^2 \text{ m/s}^2
\]
Note that even though \( T_1 = T_2 \), it is not true that \( v_1 = v_2 \). Thus, the simplest way to approach this problem is to express the centripetal acceleration in terms of the period \( T \) which cancels in the final step.

11. **REASONING AND SOLUTION** The sample makes one revolution in time \( T \) as given by
\[
T = 2\pi v / r
\]
The speed is
\[
v^2 = ra_c = (5.00 \times 10^{-2} \text{ m})(6.25 \times 10^3)(9.80 \text{ m/s}^2) \quad \text{so that} \quad v = 55.3 \text{ m/s}
\]
The period is
\[
T = 2\pi (5.00 \times 10^{-2} \text{ m})/(55.3 \text{ m/s}) = 5.68 \times 10^{-3} \text{ s} = 9.47 \times 10^{-5} \text{ min}
\]
The number of revolutions per minute = \( 1/T = 10,600 \text{ rev/min} \).
12. **Reasoning and Solution**

a. At the equator a person travels in a circle whose radius equals the radius of the earth, \( r = R_e = 6.38 \times 10^6 \) m, and whose period of rotation is \( T = 1 \) day = 86 400 s. We have

\[
v = 2\pi R_e/T = 464 \text{ m/s}
\]

The centripetal acceleration is

\[
a_c = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{6.38 \times 10^6 \text{ m}} = 3.37 \times 10^{-2} \text{ m/s}^2
\]

b. At 30.0° latitude a person travels in a circle of radius,

\[
r = R_e \cos 30.0° = 5.53 \times 10^6 \text{ m}
\]

Thus,

\[
v = 2\pi r/T = 402 \text{ m/s}
\]

and

\[
a_c = \frac{v^2}{r} = 2.92 \times 10^{-2} \text{ m/s}^2
\]

13. **SSM Reasoning** In Example 3, it was shown that the magnitudes of the centripetal acceleration for the two cases are

<table>
<thead>
<tr>
<th>Radius</th>
<th>( a_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>33 m</td>
<td>35 m/s²</td>
</tr>
<tr>
<td>24 m</td>
<td>48 m/s²</td>
</tr>
</tbody>
</table>

According to Newton's second law, the centripetal force is \( F_c = ma_c \) (see Equation 5.3).

**Solution**

a. Therefore, when the sled undergoes the turn of radius 33 m,

\[
F_c = ma_c = (350 \text{ kg})(35 \text{ m/s}^2) = 1.2 \times 10^4 \text{ N}
\]

b. Similarly, when the radius of the turn is 24 m,

\[
F_c = ma_c = (350 \text{ kg})(48 \text{ m/s}^2) = 1.7 \times 10^4 \text{ N}
\]

14. **Reasoning** The person feels the centripetal force acting on his back. This force is \( F_c = mv^2/r \), according to Equation 5.3. This expression can be solved directly to determine the radius \( r \) of the chamber.

**Solution** Solving Equation 5.3 for the radius \( r \) gives

\[
r = \frac{mv^2}{F_c} = \frac{(83 \text{ kg})(3.2 \text{ m/s})^2}{560 \text{ N}} = 1.5 \text{ m}
\]
15. **REASONING** At the maximum speed, the maximum centripetal force acts on the tires, and static friction supplies it. The magnitude of the maximum force of static friction is specified by Equation 4.7 as \( f_s^{\text{MAX}} = \mu_s F_N \), where \( \mu_s \) is the coefficient of static friction and \( F_N \) is the magnitude of the normal force. Our strategy, then, is to find the normal force, substitute it into the expression for the maximum frictional force, and then equate the result to the centripetal force, which is \( F_c = mv^2/r \), according to Equation 5.3. This will lead us to an expression for the maximum speed that we can apply to each car.

**SOLUTION** Since neither car accelerates in the vertical direction, we can conclude that the car’s weight \( mg \) is balanced by the normal force, so \( F_N = mg \). From Equations 4.7 and 5.3 it follows that

\[
f_s^{\text{MAX}} = \mu_s F_N = \mu_s mg = F_c = \frac{mv^2}{r}
\]

Thus, we find that

\[
\mu_s mg = \frac{mv^2}{r} \quad \text{or} \quad v = \sqrt{\mu_s gr}
\]

Applying this result to car A and car B gives

\[
v_A = \sqrt{\mu_{s,A} gr} \quad \text{and} \quad v_B = \sqrt{\mu_{s,B} gr}
\]

In these two equations, the radius \( r \) does not have a subscript, since the radius is the same for either car. Dividing the two equations and noting that the terms \( g \) and \( r \) are eliminated algebraically, we see that

\[
\frac{v_B}{v_A} = \frac{\sqrt{\mu_{s,B} gr}}{\sqrt{\mu_{s,A} gr}} = \frac{\sqrt{\mu_{s,B}}}{\sqrt{\mu_{s,A}}} \quad \text{or} \quad v_B = v_A \sqrt{\frac{\mu_{s,B}}{\mu_{s,A}}} = (25 \text{ m/s}) \sqrt{\frac{0.85}{1.1}} = 22 \text{ m/s}
\]

16. **REASONING** The centripetal force that acts on the skater is \( F_c = mv^2/r \) (Equation 5.3). This expression can be solved directly to determine the mass \( m \).

**SOLUTION** Solving Equation 5.3 for the mass \( m \) gives

\[
m = \frac{F_c r}{v^2} = \frac{(460 \text{ N})(31 \text{ m})}{(14 \text{ m/s})^2} = 73 \text{ kg}
\]
17. **REASONING AND SOLUTION** The force \( P \) supplied by the man will be largest when the partner is at the lowest point in the swing. The diagram at the right shows the forces acting on the partner in this situation. The centripetal force necessary to keep the partner swinging along the arc of a circle is provided by the resultant of the force supplied by the man and the weight of the partner. From the diagram, we see that

\[
P - mg = \frac{mv^2}{r}
\]

Therefore,

\[
P = \frac{mv^2}{r} + mg
\]

Since the weight of the partner, \( W \), is equal to \( mg \), it follows that \( m = (W/g) \) and

\[
P = \left(\frac{(W/g)v^2}{r}\right) + W = \left[\frac{(475 \text{ N})}{(9.80 \text{ m/s}^2)}\right] (4.00 \text{ m/s})^2 + (475 \text{ N}) = 594 \text{ N}
\]

18. **REASONING** The centripetal force \( F_c \) that keeps the car (mass = \( m \), speed = \( v \)) on the curve (radius = \( r \)) is \( F_c = \frac{mv^2}{r} \) (Equation 5.3). The maximum force of static friction \( F_{s, \text{MAX}} \) provides this centripetal force. Thus, we know that \( \frac{F_{s, \text{MAX}}}{m} = \frac{mv^2}{r} \), which can be solved for the speed to show that \( v = \sqrt{r \frac{F_{s, \text{MAX}}}{m}} \). We can apply this result to both the dry road and the wet road and, in so doing, obtain the desired wet-road speed.

**SOLUTION** Applying the expression \( v = \sqrt{\frac{r F_{s, \text{MAX}}}{m}} \) to both road conditions gives

\[
V_{\text{dry}} = \sqrt{r \frac{F_{s, \text{MAX}}}{m}} \quad \text{and} \quad V_{\text{wet}} = \sqrt{r \frac{F_{s, \text{wet}}}{m}}
\]

We divide the two equations in order to eliminate the unknown mass \( m \) and unknown radius \( r \) algebraically, and we remember that \( \frac{F_{s, \text{wet}}}{F_{s, \text{dry}}} = \frac{1}{3} \). Therefore,

\[
\frac{V_{\text{wet}}}{V_{\text{dry}}} = \sqrt{\frac{r \frac{F_{s, \text{wet}}}{m}}{r \frac{F_{s, \text{dry}}}{m}}} = \frac{\frac{F_{s, \text{wet}}}{F_{s, \text{dry}}}}{r} = \frac{1}{3}
\]

Solving for \( V_{\text{wet}} \), we obtain

\[
V_{\text{wet}} = \frac{V_{\text{dry}}}{\sqrt{3}} = \frac{21 \text{ m/s}}{\sqrt{3}} = 12 \text{ m/s}
\]
19. **REASONING** The centripetal force is the name given to the net force pointing toward the center of the circular path. At the lowest point the net force consists of the tension in the arm pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as \( F_c = \frac{mv^2}{r} \).

**SOLUTION**

(a) The centripetal force is

\[
F_c = \frac{mv^2}{r} = \frac{(9.5 \text{ kg})(2.8 \text{ m/s})^2}{0.85 \text{ m}} = 88 \text{ N}
\]

(b) Using \( T \) to denote the tension in the arm, at the bottom of the circle we have

\[
F_c = T - mg = \frac{mv^2}{r}
\]

\[
T = mg + \frac{mv^2}{r} = (9.5 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(9.5 \text{ kg})(2.8 \text{ m/s})^2}{0.85 \text{ m}} = 181 \text{ N}
\]

20. **REASONING** When the penny is rotating with the disk (and not sliding relative to it), it is the static frictional force that provides the centripetal force required to keep the penny moving on a circular path. The magnitude \( f_s^{\text{MAX}} \) of the maximum static frictional force is given by \( f_s^{\text{MAX}} = \mu_s F_N \) (Equation 4.7), where \( F_N \) is the magnitude of the normal force and \( \mu_s \) is the coefficient of static friction. Solving this relation for \( \mu_s \) gives

\[
\mu_s = \frac{f_s^{\text{MAX}}}{F_N}
\]  

(1)

Since the maximum centripetal force that can act on the penny is the maximum static frictional force, we have \( F_c = f_s^{\text{MAX}} \). Since \( F_c = \frac{mv^2}{r} \) (Equation 5.3), it follows that \( f_s^{\text{MAX}} = \frac{mv^2}{r} \). Substituting this expression into Equation (1) yields

\[
\mu_s = \frac{f_s^{\text{MAX}}}{F_N} = \frac{\frac{mv^2}{r}}{F_N}
\]  

(2)

The speed of the penny can be determined from the period \( T \) of the motion and the radius \( r \) according to \( v = \frac{2\pi r}{T} \) (Equation 5.1). Furthermore, since the penny does not accelerate in the vertical direction, the upward normal force must be balanced by the downward-pointing weight, so that \( F_N = mg \), where \( g \) is the acceleration due to gravity. Substituting these two expressions for \( v \) and \( F_N \) into Equation (2) gives
\[ \mu_s = \frac{mv^2}{rF_N} = \frac{\mu T^2}{r \left( \rho mg \right)} = \frac{4\pi^2 r}{g T^2} \]  

**SOLUTION**  Using Equation (3), we find that the coefficient of static friction required to keep the penny rotating on the disk is

\[ \mu_s = \frac{4\pi^2 r}{g T^2} = \frac{4\pi^2 (0.150 \text{ m})}{(9.80 \text{ m/s}^2)(1.80 \text{ s})^2} = 0.187 \]

21. **SSM REASONING**  Let \( v_0 \) be the initial speed of the ball as it begins its projectile motion. Then, the centripetal force is given by Equation 5.3: \( F_c = \frac{mv_0^2}{r} \). We are given the values for \( m \) and \( r \); however, we must determine the value of \( v_0 \) from the details of the projectile motion after the ball is released.

In the absence of air resistance, the \( x \) component of the projectile motion has zero acceleration, while the \( y \) component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the ball is given by Equation 3.5a (with \( a_x = 0 \text{ m/s}^2 \)):

\[ x = v_{0x} t = (v_0 \cos \theta) t \]

with \( t \) equal to the flight time of the ball while it exhibits projectile motion. The time \( t \) can be found by considering the vertical motion. From Equation 3.3b,

\[ v_y = v_{0y} + a_y t \]

After a time \( t \), \( v_y = -v_{0y} \). Assuming that up and to the right are the positive directions, we have

\[ t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin \theta}{a_y} \]

and

\[ x = (v_0 \cos \theta) \left( \frac{-2v_0 \sin \theta}{a_y} \right) \]

Using the fact that \( 2\sin \theta \cos \theta = \sin 2\theta \), we have

\[ x = -\frac{2v_0^2 \cos \theta \sin \theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y} \]  

Equation (1) (with upward and to the right chosen as the positive directions) can be used to determine the speed \( v_0 \) with which the ball begins its projectile motion. Then Equation 5.3 can be used to find the centripetal force.
**SOLUTION** Solving equation (1) for \(v_0\), we have
\[
v_0 = \sqrt{-\frac{x a_y}{\sin 2\theta}} = \sqrt{\frac{-(86.75 \text{ m})(-9.80 \text{ m/s}^2)}{\sin 2(41^\circ)}} = 29.3 \text{ m/s}
\]

Then, from Equation 5.3,
\[
F_C = \frac{mv^2}{r} = \frac{(7.3 \text{ kg})(29.3 \text{ m/s})^2}{1.8 \text{ m}} = 3500 \text{ N}
\]

22. **REASONING** The coefficient \(\mu_s\) of static friction is related to the magnitude \(f_{s,\text{MAX}}\) of the maximum static frictional force and the magnitude \(F_N\) of the normal force acting on the car by \(f_{s,\text{MAX}} = \mu_s F_N\) (Equation 4.7), so that:
\[
\mu_s = \frac{f_{s,\text{MAX}}}{F_N}
\]  
Equation (1)

The car is going around an unbanked curve, so the centripetal force \(F_c = \frac{mv^2}{r}\) (Equation 5.3) must be horizontal. The static frictional force is the only horizontal force, so it serves as the centripetal force. The maximum centripetal force occurs when \(F_c = f_{s,\text{MAX}}\). Therefore, the maximum speed \(v\) the car can have without slipping is related to \(f_{s,\text{MAX}}\) by
\[
F_c = f_{s,\text{MAX}} = \frac{mv^2}{r}
\]  
Equation (2)

Substituting Equation (2) into Equation (1) yields
\[
\mu_s = \frac{mv^2}{r} \frac{r}{F_N} = \frac{mv^2}{F_N}
\]  
Equation (3)

In part a the car is subject to two downward-pointing forces, its weight \(W\) and the downforce \(D\). The vertical acceleration of the car is zero, so the upward normal force must balance the two downward forces: \(F_N = W + D\). Combining this relation with Equation (3), we obtain an expression for the coefficient of static friction:
\[
\mu_s = \frac{r}{F_N} = \frac{mv^2}{W + D} = \frac{mv^2}{r(mg + D)}
\]  
Equation (4)

**SOLUTION**
a. Since the downforce is \(D = 11 \text{ 000 N}\), Equation (4) gives the coefficient of static friction as
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\[
\mu_s = \frac{mv^2}{r(mg + D)} = \frac{(830 \text{ kg})(58 \text{ m/s})^2}{(160 \text{ m})[(830 \text{ kg})(9.80 \text{ m/s}^2) + 11000 \text{ N}]} = 0.91
\]

b. The downforce is now absent \((D = 0 \text{ N})\). Solving Equation (4) for the speed of the car, we find that

\[
v = \sqrt{\frac{\mu_s r (mg + D)}{m}} = \sqrt{\frac{\mu_s r (mg + 0 \text{ N})}{m}} = \sqrt{\frac{\mu_s r (mg)}{m}} = \sqrt{\mu_s rg} = \sqrt{(0.91)(160 \text{ m})(9.80 \text{ m/s}^2)} = 38 \text{ m/s}
\]

23. **REASONING**

a. The free body diagram shows the swing ride and the two forces that act on a chair: the tension \(T\) in the cable, and the weight \(mg\) of the chair and its occupant. We note that the chair does not accelerate vertically, so the net force \(\sum F_y\) in the vertical direction must be zero, \(\sum F_y = 0\). The net force consists of the upward vertical component of the tension and the downward weight of the chair. The fact that the net force is zero will allow us to determine the magnitude of the tension.

\[
\begin{align*}
&
\text{mg} \\
&60.0^\circ \text{T} \\
&15.0 \text{ m} \\
& \sum F_y = 0
\end{align*}
\]

b. According to Newton’s second law, the net force \(\sum F_x\) in the horizontal direction is equal to the mass \(m\) of the chair and its occupant times the centripetal acceleration \(a_c = v^2/r\), so that \(\sum F_x = ma_c = mv^2/r\). There is only one force in the horizontal direction, the horizontal component of the tension, so it is the net force. We will use Newton’s second law to find the speed \(v\) of the chair.

**SOLUTION**

a. The vertical component of the tension is \(+T \cos 60.0^\circ\), and the weight is \(-mg\), where we have chosen “up” as the + direction. Since the chair and its occupant have no vertical acceleration, we have that \(\sum F_y = 0\), so
+T \cos 60.0^\circ - mg = 0 \quad \sum F_y \quad (1)

Solving for the magnitude $T$ of the tension gives

$$
T = \frac{mg}{\cos 60.0^\circ} = \frac{(179 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 60.0^\circ} = 3510 \text{ N}
$$

b. The horizontal component of the tension is $+T \sin 60.0^\circ$, where we have chosen the direction to the left in the diagram as the + direction. Since the chair and its occupant have a centripetal acceleration in this direction, we have

$$
\frac{T \sin 60.0^\circ}{\sum F_x} = ma_x = m \left( \frac{v^2}{r} \right) \quad (2)
$$

From the drawing we see that the radius $r$ of the circular path is $r = (15.0 \text{ m}) \sin 60.0^\circ = 13.0 \text{ m}$. Solving Equation (2) for the speed $v$ gives

$$
v = \sqrt{\frac{rT \sin 60.0^\circ}{m}} = \sqrt{\frac{(13.0 \text{ m})(3510 \text{ N}) \sin 60.0^\circ}{179 \text{ kg}}} = 14.9 \text{ m/s}
$$

24. **REASONING** The angle $\theta$ at which a friction-free curve is banked depends on the radius $r$ of the curve and the speed $v$ with which the curve is to be negotiated, according to Equation 5.4: $\tan \theta = v^2/(rg)$. For known values of $\theta$ and $r$, the safe speed is

$$
v = \sqrt{rg \tan \theta}
$$

Before we can use this result, we must determine $\tan \theta$ for the banking of the track.

**SOLUTION** The drawing at the right shows a cross-section of the track. From the drawing we have

$$
\tan \theta = \frac{18 \text{ m}}{53 \text{ m}} = 0.34
$$

a. Therefore, the smallest speed at which cars can move on this track without relying on friction is

$$
v_{\min} = \sqrt{(112 \text{ m})(9.80 \text{ m/s}^2)(0.34)} = 19 \text{ m/s}
$$

b. Similarly, the largest speed is

$$
v_{\max} = \sqrt{(165 \text{ m})(9.80 \text{ m/s}^2)(0.34)} = 23 \text{ m/s}
$$
25. **REASONING**  From the discussion on banked curves in Section 5.4, we know that a car can safely round a banked curve without the aid of static friction if the angle \( \theta \) of the banked curve is given by \( \tan \theta = \frac{v_0^2}{rg} \), where \( v_0 \) is the speed of the car and \( r \) is the radius of the curve (see Equation 5.4). The maximum speed that a car can have when rounding an unbanked curve is \( v_0 = \sqrt{\mu_gr} \) (see Example 7). By combining these two relations, we can find the angle \( \theta \).

**SOLUTION**  The angle of the banked curve is \( \theta = \tan^{-1} \left( \frac{v_0^2}{rg} \right) \). Substituting the expression \( v_0 = \sqrt{\mu_gr} \) into this equation gives

\[
\theta = \tan^{-1} \left( \frac{\sqrt{\mu_gr}}{r} \right) = \tan^{-1} \left( \frac{\mu_g}{r} \right) = \tan^{-1} (0.81) = 39^\circ
\]

26. **REASONING**  We will treat this situation as a circular turn on a banked surface, with the angle \( \theta \) that the rider leans serving as the banking angle. The banking angle \( \theta \) is related to the speed \( \nu \) of the watercraft, the radius \( r \) of the curve and the magnitude \( g \) of the acceleration due to gravity by \( \tan \theta = \frac{\nu^2}{rg} \) (Equation 5.4). If the rider is closer to the seawall than \( r \), she will hit the wall while making the turn. Therefore, the minimum distance at which she must begin the turn is \( r \), the minimum turn radius (see the drawing).

**SOLUTION**  Solving Equation 5.4 for \( r \), we obtain the minimum distance:

\[
r = \frac{\nu^2}{g \tan \theta} = \frac{(26 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \tan 22^\circ} = 170 \text{ m}
\]

27. **REASONING**  The relation \( \tan \theta = \frac{\nu^2}{rg} \) (Equation 5.4) determines the banking angle \( \theta \) that a banked curve of radius \( r \) must have if a car is to travel around it at a speed \( \nu \) without relying on friction. In this expression \( g \) is the magnitude of the acceleration due to gravity. We will solve for \( \nu \) and apply the result to each curve. The fact that the radius of each curve is the same will allow us to determine the unknown speed.
**SOLUTION** According to Equation 5.4, we have

\[
\tan \theta = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{rg \tan \theta}
\]

Applying this result for the speed to each curve gives

\[
v_A = \sqrt{rg \tan \theta_A} \quad \text{and} \quad v_B = \sqrt{rg \tan \theta_B}
\]

Note that the terms \( r \) and \( g \) are the same for each curve. Therefore, these terms are eliminated algebraically when we divide the two equations. We find, then, that

\[
\frac{v_B}{v_A} = \frac{\tan \theta_B}{\tan \theta_A} \quad \text{or} \quad v_B = v_A \sqrt{\frac{\tan \theta_B}{\tan \theta_A}} = (18 \text{ m/s}) \sqrt{\frac{\tan 19^\circ}{\tan 13^\circ}} = 22 \text{ m/s}
\]

28. **REASONING** The distance \( d \) is related to the radius \( r \) of the circle on which the car travels by \( d = r \sin 50.0^\circ \) (see the drawing).

We can obtain the radius by noting that the car experiences a centripetal force that is directed toward the center of the circular path. This force is provided by the component, \( F_N \cos 50.0^\circ \), of the normal force that is parallel to the radius. Setting this force equal to the mass \( m \) of the car times the centripetal acceleration \( a_c = \frac{v^2}{r} \) gives \( F_N \cos 50.0^\circ = ma_c = \frac{mv^2}{r} \). Solving for the radius \( r \) and substituting it into the relation \( d = r \sin 50.0^\circ \) gives

\[
d = \frac{r}{\sin 50.0^\circ} = \frac{F_N \cos 50.0^\circ}{\sin 50.0^\circ} = \frac{mv^2}{(F_N \cos 50.0^\circ)(\sin 50.0^\circ)} \quad (1)
\]

The magnitude \( F_N \) of the normal force can be obtained by observing that the car has no vertical acceleration, so the net force in the vertical direction must be zero, \( \sum F_y = 0 \). The net force consists of the upward vertical component of the normal force and the downward weight of the car. The vertical component of the normal force is \( +F_N \sin 50.0^\circ \), and the weight is \(-mg\), where we have chosen the “up” direction as the + direction. Thus, we have that

\[
+ F_N \sin 50.0^\circ - mg = 0
\]

From (2)
Solving this equation for $F_N$ and substituting it into the equation above will yield the distance $d$.

**SOLUTION** Solving Equation (2) for $F_N$ and substituting the result into Equation (1) gives

\[
d = \frac{mv^2}{\left(F_N \cos 50.0^\circ\right)(\sin 50.0^\circ)} = \frac{mv^2}{mg \left(\sin 50.0^\circ\right)(\cos 50.0^\circ)}\]

\[
= \frac{v^2}{g \cos 50.0^\circ} = \frac{(34.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \cos 50.0^\circ} = 184 \text{ m}
\]

29. **REASONING** The lifting force $L$ is perpendicular to the jet’s wings. When the jet banks at an angle $\theta$ above the horizontal, therefore, the lifting force tilts an angle $\theta$ from the vertical (see the free-body diagram). Because the jet has no vertical acceleration during the horizontal turn, the upward vertical component $L \cos \theta$ of the lifting force balances the jet’s weight: $L \cos \theta = mg$, where $m$ is the jet’s mass and $g$ is the acceleration due to gravity. Therefore, the magnitude of the lifting force is $L = mg / \cos \theta$.

At this point we know $m$ and $g$, but not the banking angle $\theta$. Since the jet follows a horizontal circle, the centripetal force must be horizontal. The only horizontal force acting on the jet is the horizontal component $L \sin \theta$ of the lifting force, so this must be the centripetal force. The situation is completely analogous to that of a car driving around a banked curve without the assistance of friction. The relation $\tan \theta = v^2/(rg)$ (Equation 5.4), therefore, expresses the relationship between the jet’s unknown banking angle $\theta$, its speed $v$, the radius $r$ of the turn, and $g$, all of which are known.

**SOLUTION** The magnitude of the lifting force is

\[
L = \frac{mg}{\cos \theta} = \frac{(2.00 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2)}{\cos \theta}
\]

Solving the relation $\tan \theta = v^2/(rg)$ (Equation 5.4) for the angle $\theta$, we obtain

\[
\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{(123 \text{ m/s})^2}{(3810 \text{ m})(9.80 \text{ m/s}^2)}\right] = 22.1^\circ
\]
Substituting this value for $\theta$ into Equation (1) for the lifting force gives

$$L = \frac{mg}{\cos \theta} = \frac{(2.00 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 22.1^\circ} = 2.12 \times 10^6 \text{ N}$$

30. **REASONING** The centripetal force $F_c$ required to keep an object of mass $m$ that moves with speed $v$ on a circle of radius $r$ is $F_c = mv^2 / r$ (Equation 5.3). From Equation 5.1, we know that $v = 2\pi r / T$, where $T$ is the period or the time for the suitcase to go around once. Therefore, the centripetal force can be written as

$$F_c = \frac{m(2\pi r / T)^2}{r} = \frac{4m\pi^2 r}{T^2} \quad (1)$$

This expression can be solved for $T$. However, we must first find the centripetal force that acts on the suitcase.

**SOLUTION** Three forces act on the suitcase. They are the weight $mg$ of the suitcase, the force of static friction $f_s^{\text{MAX}}$, and the normal force $F_N$ exerted on the suitcase by the surface of the carousel. The following drawing shows the free body diagram for the suitcase.

In this diagram, the $y$ axis is along the vertical direction. The force of gravity acts, then, in the $-y$ direction. The centripetal force that causes the suitcase to move on its circular path is provided by the net force in the $+x$ direction in the diagram. From the diagram, we can see that only the forces $F_N$ and $f_s^{\text{MAX}}$ have horizontal components. Thus, we have $F_c = f_s^{\text{MAX}} \cos \theta - F_N \sin \theta$, where the minus sign indicates that the $x$ component of $F_N$ points to the left in the diagram. Using Equation 4.7 for the maximum static frictional force, we can write this result as in Equation (2).

$$F_c = \mu_s F_N \cos \theta - F_N \sin \theta = F_N (\mu_s \cos \theta - \sin \theta) \quad (2)$$

If we apply Newton's second law in the $y$ direction, we see from the diagram that

$$F_N \cos \theta + f_s^{\text{MAX}} \sin \theta - mg = ma_y = 0 \quad \text{or} \quad F_N \cos \theta + \mu_s F_N \sin \theta - mg = 0$$

where we again have used Equation 4.7 for the maximum static frictional force. Solving for the normal force, we find

$$F_N = \frac{mg}{\cos \theta + \mu_s \sin \theta}$$
Using this result in Equation (2), we obtain the magnitude of the centripetal force that acts on the suitcase:

\[ F_c = F_N \left( \mu_s \cos \theta - \sin \theta \right) = \frac{mg \left( \mu_s \cos \theta - \sin \theta \right)}{\cos \theta + \mu_s \sin \theta} \]

With this expression for the centripetal force, Equation (1) becomes

\[ \frac{mg \left( \mu_s \cos \theta - \sin \theta \right)}{\cos \theta + \mu_s \sin \theta} = \frac{4m \pi^2 r}{T^2} \]

Solving for the period \( T \), we find

\[ T = \sqrt{\frac{4\pi^2 r \left( \cos \theta + \mu_s \sin \theta \right)}{g \left( \mu_s \cos \theta - \sin \theta \right)}} = \sqrt{\frac{4\pi^2 (11.0 \text{ m}) \left( \cos 36.0^\circ + 0.760 \sin 36.0^\circ \right)}{(9.80 \text{ m/s}^2) \left( 0.760 \cos 36.0^\circ - \sin 36.0^\circ \right)}} = 45 \text{ s} \]

31. **Reasoning** The speed \( v \) of a satellite in circular orbit about the earth is given by \( v = \sqrt{\frac{GM_E}{r}} \) (Equation 5.5), where \( G \) is the universal gravitational constant, \( M_E \) is the mass of the earth, and \( r \) is the radius of the orbit. The radius is measured from the center of the earth, not the surface of the earth, to the satellite. Therefore, the radius is found by adding the height of the satellite above the surface of the earth to the radius of the earth \((6.38 \times 10^6 \text{ m})\).

**Solution** First we add the orbital heights to the radius of the earth to obtain the orbital radii. Then we use Equation 5.5 to calculate the speeds.

**Satellite A** \( r_A = 6.38 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m} = 6.74 \times 10^6 \text{ m} \)

\[ v = \sqrt{\frac{GM_E}{r_A}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.74 \times 10^6 \text{ m}}} = 7690 \text{ m/s} \]

**Satellite B** \( r_A = 6.38 \times 10^6 \text{ m} + 720 \times 10^3 \text{ m} = 7.10 \times 10^6 \text{ m} \)

\[ v = \sqrt{\frac{GM_E}{r_A}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{7.10 \times 10^6 \text{ m}}} = 7500 \text{ m/s} \]

32. **Reasoning** The speed \( v \) of a satellite in a circular orbit of radius \( r \) about the earth is given by \( v = \sqrt{\frac{GM_E}{r}} \) (Equation 5.5). In this expression \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) is the universal gravitational constant and \( M_E = 5.98 \times 10^{24} \text{ kg} \) is the mass of the earth. The
orbital radius of a synchronous satellite in earth orbit is calculated in Example 11 to be $r = 4.23 \times 10^7$ m.

**SOLUTION** Using Equation 5.5, we find that the speed of a synchronous satellite in earth orbit is

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\left(\frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2}{4.23 \times 10^7 \text{ m}}\right)(5.98 \times 10^{24} \text{ kg})} = 3070 \text{ m/s}$$

33. **SSM REASONING** Equation 5.5 gives the orbital speed for a satellite in a circular orbit around the earth. It can be modified to determine the orbital speed around any planet $P$ by replacing the mass of the earth $M_E$ by the mass of the planet $M_P$:

$$v = \sqrt{\frac{GM_P}{r}}$$

**SOLUTION** The ratio of the orbital speeds is, therefore,

$$\frac{v_2}{v_1} = \sqrt{\frac{GM_P}{r_2}} \frac{r_1}{GM_P} \frac{r_1}{r_2} \frac{1}{r_1}$$

Solving for $v_2$ gives

$$v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = (1.70 \times 10^4 \text{ m/s}) \sqrt{\frac{5.25 \times 10^6 \text{ m}}{8.60 \times 10^6 \text{ m}}} = 1.33 \times 10^4 \text{ m/s}$$

34. **REASONING AND SOLUTION** The normal force exerted by the wall on each astronaut is the centripetal force needed to keep him in the circular path, i.e., $F_c = \frac{mv^2}{r}$. Rearranging and letting $F_c = (1/2)mg$ yields

$$r = \frac{2v^2}{g} = 2(35.8 \text{ m/s})^2/(9.80 \text{ m/s}^2) = 262 \text{ m}$$

35. **REASONING** The speed of the satellite is given by Equation 5.1 as $v = \frac{2\pi r}{T}$. Since we are given that the period is $T = 1.20 \times 10^4$ s, it will be possible to determine the speed from Equation 5.1 if we can determine the radius $r$ of the orbit. To find the radius, we will use Equation 5.6, which relates the period to the radius according to $T = 2\pi r^{3/2} / \sqrt{GM_E}$, where $G$ is the universal gravitational constant and $M_E$ is the mass of the earth.

**SOLUTION** According to Equation 5.1, the orbital speed is

$$v = \frac{2\pi r}{T}$$
To find a value for the radius, we begin with Equation 5.6:

\[ T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \quad \text{or} \quad r^{3/2} = \frac{T\sqrt{GM_E}}{2\pi} \]

Next, we square both sides of the result for \( r^{3/2} \):

\[ (r^{3/2})^2 = \left( \frac{T\sqrt{GM_E}}{2\pi} \right)^2 \quad \text{or} \quad r^3 = \frac{T^2GM_E}{4\pi^2} \]

We can now take the cube root of both sides of the expression for \( r^3 \) in order to determine \( r \):

\[ r = \sqrt[3]{\frac{T^2GM_E}{4\pi^2}} = \sqrt[3]{\frac{(1.20 \times 10^4 \text{ s})^2 (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{4\pi^2}} = 1.13 \times 10^7 \text{ m} \]

With this value for the radius, we can use Equation 5.1 to obtain the speed:

\[ v = \frac{2\pi r}{T} = \frac{2\pi(1.13 \times 10^7 \text{ m})}{1.20 \times 10^4 \text{ s}} = 5.92 \times 10^3 \text{ m/s} \]

36. **REASONING** The period of a satellite is given by \( T = 2\pi r^{3/2} / \sqrt{GM_E} \) (Equation 5.6), where \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) is the universal gravitational constant.

**SOLUTION** Using Equation 5.6, we find that the period is

\[ T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} = \frac{2\pi \left[ 2(6.38 \times 10^6 \text{ m}) \right]^{3/2}}{\sqrt{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} = 1.43 \times 10^4 \text{ s} \]

37. **SSM REASONING** Equation 5.2 for the centripetal acceleration applies to both the plane and the satellite, and the centripetal acceleration is the same for each. Thus, we have

\[ a_c = \frac{v_{\text{plane}}^2}{r_{\text{plane}}} = \frac{v_{\text{satellite}}^2}{r_{\text{satellite}}} \quad \text{or} \quad v_{\text{plane}} = \left( \frac{r_{\text{plane}}}{r_{\text{satellite}}} \right)^{1/2} v_{\text{satellite}} \]

The speed of the satellite can be obtained directly from Equation 5.5.

**SOLUTION** Using Equation 5.5, we can express the speed of the satellite as

\[ v_{\text{satellite}} = \sqrt{\frac{GM_E}{r_{\text{satellite}}}} \]
Substituting this expression into the expression obtained in the reasoning for the speed of the plane gives

$$v_{\text{plane}} = \left( \frac{r_{\text{plane}}}{r_{\text{satellite}}} \right) v_{\text{satellite}} = \left( \frac{r_{\text{plane}}}{r_{\text{satellite}}} \right) \sqrt{\frac{Gm}{r_{\text{satellite}}}} = \sqrt{\frac{r_{\text{plane}} Gm}{r_{\text{satellite}}}}$$

$$v_{\text{plane}} = \sqrt{\frac{(15 \text{ m})(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.7 \times 10^6 \text{ m}}} = 12 \text{ m/s}$$

38. **REASONING** The satellite’s true weight $W$ when at rest on the surface of the planet is the gravitational force the planet exerts on it. This force is given by $W = GM_p m / r^2$ (Equation 4.4), where $G$ is the universal gravitational constant, $M_p$ is the mass of the planet, $m$ is the mass of the satellite, and $r$ is the distance between the satellite and the center of the planet. When the satellite is at rest on the planet’s surface, its distance from the planet’s center is $R_p$, the radius of the planet, so we have $W = GM_p m / R_p^2$. The satellite’s mass $m$ is given, as is the planet’s radius $R_p$. But we must use the relation $T = \frac{2\pi r^{3/2}}{\sqrt{GM_p}}$ (Equation 5.6) to determine the planet’s mass $M_p$ in terms of the satellite’s orbital period $T$ and orbital radius $r$. Squaring both sides of Equation 5.6 and solving for $M_p$, we obtain

$$T^2 = \frac{2^2 \pi^2 (r^{3/2})^2}{(\sqrt{GM_p})^2} = \frac{4\pi^2 r^3}{GM_p} \quad \text{or} \quad M_p = \frac{4\pi^2 r^3}{GT^2} \quad (1)$$

Substituting Equation (1) into $W = \frac{GM_p m}{R_p^2}$ (Equation 4.4), we find that

$$W = \frac{Gm}{R_p^2} (M_p) = \frac{Gm}{R_p^2} \left( \frac{4\pi^2 r^3}{GT^2} \right) = \frac{4\pi^2 r^3 m}{R_p^2 T^2} \quad (2)$$

**SOLUTION** All of the quantities in Equation (2), except for the period $T$, are given in SI base units, so we must convert the period from hours to seconds, the SI base unit for time: $T = (2.00 \text{ h})(3600 \text{ s}/(1 \text{ h})) = 7.20 \times 10^3 \text{ s}$. The satellite’s orbital radius $r$ in Equation (2) is the distance between the satellite and the center of the orbit, which is the planet’s center. Therefore, the orbital radius is the sum of the planet’s radius $R_p$ and the satellite’s height $h$ above the planet’s surface: $r = R_p + h = 4.15 \times 10^6 \text{ m} + 4.1 \times 10^5 \text{ m} = 4.56 \times 10^6 \text{ m}$. We now use Equation (2) to calculate the satellite’s true weight at the planet’s surface:

$$W = \frac{4\pi^2 r^3 m}{R_p^2 T^2} = \frac{4\pi^2 (4.56 \times 10^6 \text{ m})^3 (5850 \text{ kg})}{(4.15 \times 10^6 \text{ m})^2 (7.20 \times 10^3 \text{ s})^2} = 2.45 \times 10^4 \text{ N}$$
39. **REASONING** The speed $v$ of a planet orbiting a star is given by $v = \sqrt{\frac{GM_S}{r}}$ (Equation 5.5), where $M_S$ is the mass of the star, $G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ is the universal gravitational constant, and $r$ is the orbital radius. This expression can be solved for $M_S$. However, the orbital radius $r$ is not known, so we will use the relation $v = \frac{2\pi r}{T}$ (Equation 5.1) to eliminate $r$ in favor of the known quantities $v$ and $T$ (the period). Returning to Equation 5.5, we square both sides and solve for the mass of the star:

$$\frac{GM_S}{r} = v^2 \quad \text{or} \quad M_S = \frac{rv^2}{G} \quad (1)$$

Then, solving $v = \frac{2\pi r}{T}$ for $r$ yields $r = \frac{vT}{2\pi}$, which we substitute into Equation (1):

$$M_S = \frac{rv^2}{G} = \left(\frac{vT}{2\pi}\right) \frac{v^2}{G} = \frac{v^3T}{2\pi G} \quad (2)$$

We will use Equation (2) to calculate the mass of the star in part $a$. In part $b$, we will solve Equation (2) for the orbital period $T$ of the faster planet, which should be shorter than that of the slower planet.

**SOLUTION**

a. The speed of the slower planet is $v = 43.3 \text{ km/s} = 43.3 \times 10^3 \text{ m/s}$. Its orbital period in seconds is $T = (7.60 \text{ yr})[(3.156 \times 10^7 \text{ s})/(1 \text{ yr})] = 2.40 \times 10^8 \text{ s}$. Substituting these values into Equation (2) yields the mass of the star:

$$M_S = \frac{v^3T}{2\pi G} = \frac{(43.3 \times 10^3 \text{ m/s})^3 (2.40 \times 10^8 \text{ s})}{2\pi (6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = 4.65 \times 10^{31} \text{ kg}$$

This is roughly 23 times the mass of the sun.

b. Solving Equation (2) for the orbital period $T$, we obtain

$$\frac{v^3T}{2\pi G} = M_S \quad \text{or} \quad T = \frac{2\pi GM_S}{v^3} \quad (3)$$

The speed of the faster planet is $v = 58.6 \text{ km/s} = 58.6 \times 10^3 \text{ m/s}$. Equation (3) now gives the orbital period of the faster planet in seconds:

$$T = \frac{2\pi (6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.65 \times 10^{31} \text{ kg})}{(58.6 \times 10^3 \text{ m/s})^3} = 9.69 \times 10^7 \text{ s}$$
Lastly, we convert the period from seconds to years:

\[ T = \left(9.69 \times 10^7 \text{ s}\right) \frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} = 3.07 \text{ yr} \]

40. **REASONING AND SOLUTION**

a. The centripetal acceleration of a point on the rim of chamber A is the artificial acceleration due to gravity,

\[ a_A = v_A^2/r_A = 10.0 \text{ m/s}^2 \]

A point on the rim of chamber A moves with a speed \( v_A = 2\pi r_A/T \) where \( T \) is the period of revolution, 60.0 s. Substituting the second equation into the first and rearranging yields

\[ r_A = a_A T^2/(4\pi^2) = 912 \text{ m} \]

b. Now

\[ r_B = r_A/4.00 = 228 \text{ m} \]

c. A point on the rim of chamber B has a centripetal acceleration \( a_B = v_B^2/r_B \). The point moves with a speed \( v_B = 2\pi r_B/T \). Substituting the second equation into the first yields

\[ a_B = \frac{4\pi^2 r_B}{T^2} = \frac{4\pi^2 (228 \text{ m})}{(60.0 \text{ s})^2} = 2.50 \text{ m/s}^2 \]

41. **SSM REASONING** As the motorcycle passes over the top of the hill, it will experience a centripetal force, the magnitude of which is given by Equation 5.3: \( F_c = mv^2/r \). The centripetal force is provided by the net force on the cycle + driver system. At that instant, the net force on the system is composed of the normal force, which points upward, and the weight, which points downward. Taking the direction toward the center of the circle (downward) as the positive direction, we have \( F_c = mg - F_N \). This expression can be solved for \( F_N \), the normal force.

**SOLUTION**

a. The magnitude of the centripetal force is

\[ F_c = \frac{mv^2}{r} = \frac{(342 \text{ kg})(25.0 \text{ m/s})^2}{126 \text{ m}} = 1.70 \times 10^3 \text{ N} \]

b. The magnitude of the normal force is

\[ F_N = mg - F_c = (342 \text{ kg})(9.80 \text{ m/s}^2) - 1.70 \times 10^3 \text{ N} = 1.66 \times 10^3 \text{ N} \]
42. **REASONING** The normal force (magnitude $F_N$) that the pilot’s seat exerts on him is part of the centripetal force that keeps him on the vertical circular path. However, there is another contribution to the centripetal force, as the drawing at the right shows. This additional contribution is the pilot’s weight (magnitude $W$). To obtain the ratio $F_N/W$, we will apply Equation 5.3, which specifies the centripetal force as $F_c = mv^2/r$.

**SOLUTION** Noting that the direction upward (toward the center of the circular path) is positive in the drawing, we see that the centripetal force is $F_c = F_N - W$. Thus, from Equation 5.3 we have

$$F_c = F_N - W = \frac{mv^2}{r}$$

The weight is given by $W = mg$ (Equation 4.5), so we can divide the expression for the centripetal force by the expression for the weight and obtain that

$$F_c = \frac{F_N - W}{W} = \frac{mv^2}{mgr} \quad \text{or} \quad \frac{F_N}{W} - 1 = \frac{v^2}{gr}$$

Solving for the ratio $F_N/W$, we find that

$$\frac{F_N}{W} = 1 + \frac{v^2}{gr} = 1 + \frac{(230 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(690 \text{ m})} = 8.8$$

43. **REASONING** The centripetal force is the name given to the net force pointing toward the center of the circular path. At point 3 at the top the net force pointing toward the center of the circle consists of the normal force and the weight, both pointing toward the center. At point 1 at the bottom the net force consists of the normal force pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as $F_c = mv^2/r$.

**SOLUTION** At point 3 we have

$$F_c = F_N + mg = \frac{mv^3}{r}$$

At point 1 we have

$$F_c = F_N - mg = \frac{mv^1}{r}$$
Subtracting the second equation from the first gives

\[ 2mg = \frac{mv_3^2}{r} - \frac{mv_1^2}{r} \]

Rearranging gives

\[ v_3^2 = 2gr + v_1^2 \]

Thus, we find that

\[ v_3 = \sqrt{2(9.80 \text{ m/s}^2)(3.0 \text{ m}) + (15 \text{ m/s})^2} = 17 \text{ m/s} \]

---

44. **REASONING** The rider’s speed \( v \) at the top of the loop is related to the centripetal force acting on her by \( F_c = \frac{mv^2}{r} \) (Equation 5.3). The centripetal force \( F_c \) is the net force, which is the sum of the two vertical forces: \( W \) (her weight) and \( F_N \) (the magnitude of the normal force exerted on her by the electronic sensor). Both forces are illustrated in the “Top of loop” free-body diagram. Because both forces point in the same direction, the magnitude of the centripetal force is \( F_c = mg + F_N \). Thus, we have that \( mg + F_N = \frac{mv^2}{r} \). We will solve this relation to find the speed \( v \) of the rider. The reading on the sensor at the top of the loop gives the magnitude \( F_N = 350 \text{ N} \) of the downward normal force. Her weight \( mg \) is equal to the reading on the sensor when level and stationary (see the “Stationary” free-body diagram).

**SOLUTION** Solving \( mg + F_N = \frac{mv^2}{r} \) for the speed \( v \), we obtain

\[ v^2 = \frac{r(mg + F_N)}{m} \quad \text{or} \quad v = \sqrt{\frac{r(mg + F_N)}{m}} \]

The only quantity not yet known is the rider’s mass \( m \), so we will calculate it from her weight \( W \) by using the relation \( W = mg \) (Equation 4.5). Thus, we find that \( m = \frac{W}{g} = \frac{770 \text{ N}}{(9.80 \text{ m/s}^2)} = 79 \text{ kg} \). The speed of the rider at the top of the loop is

\[ v = \sqrt{\frac{r(mg + F_N)}{m}} = \sqrt{\frac{(21 \text{ m})(770 \text{ N} + 350 \text{ N})}{79 \text{ kg}}} = 17 \text{ m/s} \]
45. **REASONING** The magnitude $F_c$ of the centripetal force is $F_c = \frac{mv^2}{r}$ (Equation 5.3). Since the speed $v$, the mass $m$, and the radius $r$ are fixed, the magnitude of the centripetal force is the same at each point on the circle. When the ball is at the three o’clock position, the force of gravity, acting downward, is perpendicular to the stick and cannot contribute to the centripetal force. (See Figure 5.20 in the text, point 2, for a similar situation.) At this point, only the tension of $T = 16$ N contributes to the centripetal force. Considering that the centripetal force is the same everywhere, we can conclude that it has a magnitude of 16 N everywhere.

At the twelve o’clock position the tension $T$ and the force of gravity $mg$ both act downward (the negative direction) toward the center of the circle, with the result that the centripetal force at this point is $-T - mg$. (See Figure 5.20, point 3.) At the six o’clock position the tension points upward toward the center of the circle, while the force of gravity points downward, with the result that the centripetal force at this point is $T - mg$. (See Figure 5.20, point 1.)

**SOLUTION** Assuming that upward is the positive direction, we find at the twelve and six o’clock positions that

**Twelve o’clock**

$-T - mg = -16$ N

Centripetal force

$T = 16$ N $- (0.20$ kg $)(9.80$ m/s$^2$ ) $= 14$ N

**Six o’clock**

$T - mg = 16$ N

Centripetal force

$T = 16$ N $+ (0.20$ kg $)(9.80$ m/s$^2$ ) $= 18$ N

---

46. **REASONING** When the stone is whirled in a horizontal circle, the centripetal force is provided by the tension $T_h$ in the string and is given by Equation 5.3 as

$$\frac{m v^2}{r} = T_h$$

(1)

where $m$ and $v$ are the mass and speed of the stone, and $r$ is the radius of the circle. When the stone is whirled in a vertical circle, the maximum tension occurs when the stone is at the lowest point in its path. The free-body diagram shows the forces that act on the stone in this situation: the tension $T_v$ in the string and the weight $mg$ of the stone. The centripetal force is the net force that points toward the center of the circle. Setting the centripetal force equal to $\frac{mv^2}{r}$, as per Equation 5.3, we have
Here, we have assumed upward to be the positive direction. We are given that the maximum tension in the string in the case of vertical motion is 15.0% larger than that in the case of horizontal motion. We can use this fact, along with Equations 1 and 2, to find the speed of the stone.

**SOLUTION** Since the maximum tension in the string in the case of vertical motion is 15.0% larger than that in the horizontal motion, \( T_v = (1.000 + 0.150) T_h \). Substituting the values of \( T_h \) and \( T_v \) from Equations (1) and (2) into this relation gives

\[
T_v = (1.000 + 0.150) T_h
\]

\[
\frac{mv^2}{r} + mg = (1.000 + 0.150) \left( \frac{mv^2}{r} \right)
\]

Solving this equation for the speed \( v \) of the stone yields

\[
v = \sqrt{\frac{gr}{0.150}} = \sqrt{\frac{(9.80 \text{ m/s}^2) \times (1.10 \text{ m})}{0.150}} = 8.48 \text{ m/s}
\]

---

47. **REASONING** Because the crest of the hill is a circular arc, the motorcycle’s speed \( v \) is related to the centripetal force \( F_c \) acting on the motorcycle: \( F_c = \frac{mv^2}{r} \) (Equation 5.3), where \( m \) is the mass of the motorcycle and \( r \) is the radius of the circular crest. Solving Equation 5.3 for the speed, we obtain \( v^2 = F_c r / m \) or \( v = \sqrt{F_c r / m} \). The free-body diagram shows that two vertical forces act on the motorcycle. One is the weight \( mg \) of the motorcycle, which points downward. The other is the normal force \( F_N \) exerted by the road. The normal force points directly opposite the motorcycle’s weight. Note that the motorcycle’s weight must be greater than the normal force. The reason for this is that the centripetal force is the net force produced by \( mg \) and \( F_N \) and must point toward the center of the circle, which lies below the motorcycle. Only if the magnitude \( mg \) of the weight exceeds the magnitude \( F_N \) of the normal force will the centripetal force point downward. Therefore, we can express the magnitude of the centripetal force as \( F_c = mg - F_N \). With this identity, the relation

\[
v = \sqrt{\frac{(mg - F_N)r}{m}}
\]

becomes

\[
v = \sqrt{\frac{(mg - F_N)r}{m}}
\]
SOLUTION When the motorcycle rides over the crest sufficiently fast, it loses contact with the road. At that point, the normal force $F_N$ is zero. In that case, Equation (1) yields the motorcycle’s maximum speed:

$$v = \sqrt{\frac{(mg - 0)r}{m}} = \sqrt{\frac{mg \cdot gr}{m}} = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(45.0 \text{ m})} = 21.0 \text{ m/s}$$

48. REASONING The drawing at the right shows the two forces that act on a piece of clothing just before it loses contact with the wall of the cylinder. At that instant the centripetal force is provided by the normal force $F_N$ and the radial component of the weight. From the drawing, the radial component of the weight is given by

$$mg \cos \phi = mg \cos (90^\circ - \theta) = mg \sin \theta$$

Therefore, with inward taken as the positive direction, Equation 5.3 ($F_c = mv^2 / r$) gives

$$F_N + mg \sin \theta = \frac{mv^2}{r}$$

At the instant that a piece of clothing loses contact with the surface of the drum, $F_N = 0 \text{ N}$, and the above expression becomes

$$mg \sin \theta = \frac{mv^2}{r}$$

According to Equation 5.1, $v = \frac{2\pi}{T}$, and with this substitution we obtain

$$g \sin \theta = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

This expression can be solved for the period $T$. Since the period is the required time for one revolution, the number of revolutions per second can be found by calculating $1 / T$.

SOLUTION Solving for the period, we obtain

$$T = \sqrt{\frac{4\pi^2 r}{g \sin \theta}} = 2\pi \sqrt{\frac{r}{g \sin \theta}} = 2\pi \sqrt{\frac{0.32 \text{ m}}{(9.80 \text{ m/s}^2) \sin 70.0^\circ}} = 1.17 \text{ s}$$

Therefore, the number of revolutions per second that the cylinder should make is

$$\frac{1}{T} = \frac{1}{1.17 \text{ s}} = 0.85 \text{ rev/s}$$
49. **REASONING** The magnitude $F_c$ of the centripetal force that acts on the skater is given by Equation 5.3 as $F_c = \frac{mv^2}{r}$, where $m$ and $v$ are the mass and speed of the skater, and $r$ is the distance of the skater from the pivot. Since all of these variables are known, we can find the magnitude of the centripetal force.

**SOLUTION** The magnitude of the centripetal force is

$$F_c = \frac{mv^2}{r} = \frac{(80.0 \text{ kg})(6.80 \text{ m/s})^2}{6.10 \text{ m}} = 606 \text{ N}$$

50. **REASONING** Two pieces of information are provided. One is the fact that the magnitude of the centripetal acceleration $a_c$ is 9.80 m/s$^2$. The other is that the space station should not rotate faster than two revolutions per minute. This rate of twice per minute corresponds to thirty seconds per revolution, which is the minimum value for the period $T$ of the motion. With these data in mind, we will base our solution on Equation 5.2, which gives the centripetal acceleration as $a_c = \frac{v^2}{r}$, and on Equation 5.1, which specifies that the speed $v$ on a circular path of radius $r$ is $v = \frac{2\pi r}{T}$.

**SOLUTION** From Equation 5.2, we have

$$a_c = \frac{v^2}{r} \quad \text{or} \quad r = \frac{v^2}{a_c}$$

Substituting $v = \frac{2\pi r}{T}$ into this result and solving for the radius gives

$$r = \frac{v^2}{a_c} = \left(\frac{2\pi r}{T}\right)^2 \quad \text{or} \quad r = \frac{a_c T^2}{4\pi^2} = \frac{\left(9.80 \text{ m/s}^2\right)(30.0 \text{ s})^2}{4\pi^2} = 223 \text{ m}$$

51. **REASONING** The relationship between the magnitude $a_c$ of the centripetal acceleration and the period $T$ of the tip of a moving clock hand can be obtained by using Equations 5.2 and 5.1:

$$a_c = \frac{v^2}{r} \quad (5.2) \quad \quad v = \frac{2\pi r}{T} \quad (5.1)$$

The period is the time it takes a clock hand to go once around the circle. In these expressions, $v$ is the speed of the tip of the hand and $r$ is the length of the hand. Substituting Equation 5.1 into Equation 5.2 yields
\[ a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \quad (1) \]

**SOLUTION** The period of the second hand is \( T_{\text{second}} = 60 \text{ s} \). The period of the minute hand is \( T_{\text{minute}} = 1 \text{ h} = 3600 \text{ s} \). Using Equation (1), we find that the ratio of the centripetal acceleration of the tip of the second hand to that of the minute hand is

\[
\frac{a_{c, \text{second}}}{a_{c, \text{minute}}} = \frac{\frac{4\pi^2 r}{T_{\text{second}}^2}}{\frac{4\pi^2 r}{T_{\text{minute}}^2}} = \frac{T_{\text{minute}}^2}{T_{\text{second}}^2} = \frac{(3600 \text{ s})^2}{(60 \text{ s})^2} = 3600
\]

52. **REASONING AND SOLUTION**

a. In terms of the period of the motion, the centripetal force is written as

\[ F_c = 4\pi^2 mr/T^2 = 4\pi^2 \left(0.0120 \text{ kg}\right)\left(0.100 \text{ m}\right)/\left(0.500 \text{ s}\right)^2 = 0.189 \text{ N} \]

b. The centripetal force varies as the square of the speed. Thus, doubling the speed would increase the centripetal force by a factor of \( 2^2 = 4 \).

53. **REASONING** In Section 5.5 it is shown that the period \( T \) of a satellite in a circular orbit about the earth is given by (see Equation 5.6)

\[ T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \]

where \( r \) is the radius of the orbit, \( G \) is the universal gravitational constant, and \( M_E \) is the mass of the earth. The ratio of the periods of satellites A and B is, then,

\[ \frac{T_A}{T_B} = \frac{\frac{2\pi r_A^{3/2}}{\sqrt{GM_E}}}{\frac{2\pi r_B^{3/2}}{\sqrt{GM_E}}} = \frac{r_A^{3/2}}{r_B^{3/2}} \]

We do not know the radii \( r_A \) and \( r_B \). However we do know that the speed \( v \) of a satellite is equal to the circumference \( (2\pi r) \) of its orbit divided by the period \( T \), so \( v = 2\pi r/T \).

**SOLUTION** Solving the relation \( v = 2\pi r/T \) for \( r \) gives \( r = vT/2\pi \). Substituting this value for \( r \) into Equation (1) yields
\[ \frac{T_A}{T_B} = \frac{v_A^{3/2}}{v_B^{3/2}} = \left( \frac{v_A}{v_B} \right)^{3/2} = \left( \frac{v_A}{v_B} \right)^{3/2} \]

Squaring both sides of this equation, algebraically solving for the ratio \( T_A/T_B \), and using the fact that \( v_A = 3v_B \) gives

\[ \frac{T_A}{T_B} = \frac{v_B^3}{v_A^3} = \frac{v_B^3}{(3v_B)^3} = \frac{1}{27} \]

54. **REASONING** The centripetal acceleration depends on the speed \( v \) and the radius \( r \) of the curve, according to \( a_c = v^2/r \) (Equation 5.2). The speeds of the cars are the same, and since they are negotiating the same curve, the radius is the same. Therefore, the cars have the same centripetal acceleration. However, the magnitude \( F_c \) of the centripetal force depends on the mass \( m \) of the car, as well as the speed and the radius of the curve, according to \( F_c = mv^2/r \) (Equation 5.3). Since the speed and the radius are the same for each car, the car with the greater mass, which is car B, experiences the greater centripetal acceleration.

**SOLUTION** We find the following values for the magnitudes of the centripetal accelerations and forces:

**Car A**

\[ a_c = \frac{v^2}{r} = \frac{(27 \text{ m/s})^2}{120 \text{ m}} = 6.1 \text{ m/s}^2 \]

\[ F_c = \frac{m_A v^2}{r} = \frac{(1100 \text{ kg})(27 \text{ m/s})^2}{120 \text{ m}} = 6700 \text{ N} \]

**Car B**

\[ a_c = \frac{v^2}{r} = \frac{(27 \text{ m/s})^2}{120 \text{ m}} = 6.1 \text{ m/s}^2 \]

\[ F_c = \frac{m_B v^2}{r} = \frac{(1600 \text{ kg})(27 \text{ m/s})^2}{120 \text{ m}} = 9700 \text{ N} \]

55. **SSM REASONING** According to Equation 5.3, the magnitude \( F_c \) of the centripetal force that acts on each passenger is \( F_c = mv^2/r \), where \( m \) and \( v \) are the mass and speed of a passenger and \( r \) is the radius of the turn. From this relation we see that the speed is given by \( v = \sqrt{F_c r/m} \). The centripetal force is the net force required to keep each passenger moving on the circular path and points toward the center of the circle. With the aid of a free-body diagram, we will evaluate the net force and, hence, determine the speed.
**SOLUTION** The free-body diagram shows a passenger at the bottom of the circular dip. There are two forces acting: her downward-acting weight $mg$ and the upward-acting force $2mg$ that the seat exerts on her. The net force is $+2mg - mg = +mg$, where we have taken “up” as the positive direction. Thus, $F_c = mg$. The speed of the passenger can be found by using this result in the equation above.

Substituting $F_c = mg$ into the relation $v = \sqrt{\frac{F_c r}{m}}$ yields

$$v = \sqrt{\frac{F_c r}{m}} = \sqrt{\left(\frac{mg}{m}\right)r} = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 14.0 \text{ m/s}$$

56. **REASONING** The astronaut in the chamber is subjected to a centripetal acceleration $a_c$ that is given by $a_c = \frac{v^2}{r}$ (Equation 5.2). In this expression $v$ is the speed at which the astronaut in the chamber moves on the circular path of radius $r$. We can solve this relation for the speed.

**SOLUTION** Using Equation 5.2, we have

$$a_c = \frac{v^2}{r} \quad \text{or} \quad v = \sqrt{a_c r} = \sqrt{7.5(9.80 \text{ m/s}^2)}(15 \text{ m}) = 33 \text{ m/s}$$

57. **SSM** **REASONING AND SOLUTION** The centripetal acceleration for any point on the blade a distance $r$ from center of the circle, according to Equation 5.2, is $a_c = \frac{v^2}{r}$. From Equation 5.1, we know that $v = 2\pi r / T$ where $T$ is the period of the motion. Combining these two equations, we obtain

$$a_c = \frac{(2\pi r / T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

a. Since the turbine blades rotate at 617 rev/s, all points on the blades rotate with a period of $T = (1/617) \text{ s} = 1.62 \times 10^{-3} \text{ s}$. Therefore, for a point with $r = 0.020 \text{ m}$, the magnitude of the centripetal acceleration is

$$a_c = \frac{4\pi^2 (0.020 \text{ m})}{(1.62 \times 10^{-3} \text{ s})^2} = 3.0 \times 10^5 \text{ m/s}^2$$

b. Expressed as a multiple of $g$, this centripetal acceleration is

$$a_c = \left(3.0 \times 10^5 \text{ m/s}^2\right) \left(\frac{1.00 \text{ g}}{9.80 \text{ m/s}^2}\right) = 3.1 \times 10^4 \text{ g}$$
58. **REASONING** The magnitude of the centripetal acceleration of any point on the helicopter blade is given by Equation 5.2, \( a_C = \frac{v^2}{r} \), where \( r \) is the radius of the circle on which that point moves. From Equation 5.1: \( v = \frac{2\pi r}{T} \). Combining these two expressions, we obtain

\[
\frac{4\pi^2}{T^2} \]

All points on the blade move with the same period \( T \).

**SOLUTION** The ratio of the centripetal acceleration at the end of the blade (point 1) to that which exists at a point located 3.0 m from the center of the circle (point 2) is

\[
\frac{a_{c1}}{a_{c2}} = \frac{4\pi^2 r_1 / T^2}{4\pi^2 r_2 / T^2} = \frac{r_1}{r_2} = \frac{6.7 \text{ m}}{3.0 \text{ m}} = 2.2
\]

59. **REASONING** The drawing shows the block (mass \( m \)) hanging as it does when the van goes around the curve. Two forces act on the block, the tension \( T \) in the string and its weight \( mg \). The upward vertical component of the tension is \( T \cos \theta \). The horizontal component of the tension is \( T \sin \theta \) and points to the left, toward the center of the circular curve. Since the block does not accelerate in the vertical direction, the upward vertical component of the tension balances the downward-directed weight. The horizontal component of the tension provides the centripetal force that keeps the block moving on its circular path in the horizontal plane.

**SOLUTION** Since the horizontal component of the tension provides the centripetal force, Equation 5.3 can be written as follows:

\[
\frac{T \sin \theta}{\text{Centripetal force}} = \frac{mv^2}{r}
\]

where \( v \) is the speed of the van and \( r \) is the radius of the curve. Since the vertical component of the tension balances the weight, we have

\[
T \cos \theta = mg
\]

Dividing Equation (1) by Equation (2), we obtain

\[
\frac{T \sin \theta}{T \cos \theta} = \frac{mv^2 / r}{mg} \quad \text{or} \quad \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{v^2}{rg}
\]

This result indicates that

\[
\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{(28 \text{ m/s})^2}{(150 \text{ m})(9.80 \text{ m/s}^2)}\right] = 28^\circ
\]
60. **REASONING** We seek the ratio \( m_1/m_2 \) of the masses. The mass appears in the centripetal force expression \( F_c = m v^2 / r \) (Equation 5.3), so we begin by applying it to each of the particles. The centripetal force \( F_c \) arises because of the tension in each section of the rod. The tension in the inner section is greater than the tension in the outer section because the inner section not only keeps particle 1 on its circular path, but also helps to keep particle 2 on its circular path. We note the following three items for use in our solution:

- The drawing shows the rod and the particles. The tension in the inner section has a magnitude of \( T_I \), whereas the tension in the outer section has a magnitude of \( T_O \). The tension in the inner section causes a leftward-pointing force of magnitude \( T_I \) to be exerted on particle 1. Similarly, the tension in the outer section causes a leftward-pointing force of magnitude \( T_O \) to be exerted on particle 2. However, the tension in the outer section also causes a rightward-pointing force of magnitude \( T_O \) to be exerted on particle 1.

- Since particle 1 is located at the rod’s center and particle 2 is located at the rod’s end, the distances \( r_1 \) and \( r_2 \) of the particles from the end (axis) about which the rod is rotating are related according to \( r_1/r_2 = 1/2 \).

- The speeds \( v_1 \) and \( v_2 \) of the particles are also related, since the rod and the particles rotate as a rigid unit. Consider that each particle travels the circumference of its circle in the same time. Since the circumference is \( 2\pi r \) and \( r_1/r_2 = 1/2 \), it follows that \( v_1/v_2 = 1/2 \).

**SOLUTION** Applying Equation 5.3 to each particle, we obtain

\[
\frac{T_I - T_O}{\text{Centripetal force}} = \frac{m_1 v_1^2}{r_1} \quad \text{and} \quad \frac{T_O}{\text{Centripetal force}} = \frac{m_2 v_2^2}{r_2}
\]

Dividing the left equation by the right equation gives

\[
\frac{T_I - T_O}{T_O} = \frac{m_1 v_1^2 / r_1}{m_2 v_2^2 / r_2} = \frac{m_1 v_1^2 r_2}{m_2 v_2^2 r_1} \quad \text{or} \quad \frac{T_I - T_O}{T_O} = \frac{m_1 v_1^2 r_2}{m_2 v_2^2 r_1}
\]

Into this result we can now substitute \( T_I/T_O = 3 \) and \( v_1/v_2 = 1/2 \) and \( r_1/r_2 = 1/2 \):

\[
3 - 1 = \frac{m_1}{m_2} \left( \frac{1}{2} \right)^2 \left( \frac{2}{1} \right) \quad \text{or} \quad 2 = \frac{m_1}{m_2} \left( \frac{1}{2} \right) \quad \text{or} \quad \frac{m_1}{m_2} = 4
\]
61. **REASONING** If the effects of gravity are not ignored in Example 5, the plane will make an angle $\theta$ with the vertical as shown in figure A below. The figure B shows the forces that act on the plane, and figure C shows the horizontal and vertical components of these forces.

![Diagram of forces](image)

From figure C we see that the resultant force in the horizontal direction is the horizontal component of the tension in the guideline and provides the centripetal force. Therefore,

$$T \sin \theta = \frac{mv^2}{r}$$

From figure A, the radius $r$ is related to the length $L$ of the guideline by $r = L \sin \theta$; therefore,

$$T \sin \theta = \frac{mv^2}{L \sin \theta} \tag{1}$$

The resultant force in the vertical direction is zero: $T \cos \theta - mg = 0$, so that

$$T \cos \theta = mg \tag{2}$$

From equation (2) we have

$$T = \frac{mg}{\cos \theta} \tag{3}$$

Equation (3) contains two unknown, $T$ and $\theta$. First we will solve equations (1) and (3) simultaneously to determine the value(s) of the angle $\theta$. Once $\theta$ is known, we can calculate the tension using equation (3).

**SOLUTION** Substituting equation (3) into equation (1):

$$\left( \frac{mg}{\cos \theta} \right) \sin \theta = \frac{mv^2}{L \sin \theta}$$

Thus,
Using the fact that \( \cos^2 \theta + \sin^2 \theta = 1 \), equation (4) can be written

\[
\frac{1 - \cos^2 \theta}{\cos \theta} = \frac{v^2}{gL}
\]

or

\[
\frac{1}{\cos \theta} - \cos \theta = \frac{v^2}{gL}
\]

This can be put in the form of an equation that is quadratic in \( \cos \theta \). Multiplying both sides by \( \cos \theta \) and rearranging yields:

\[
\cos^2 \theta + \frac{v^2}{gL} \cos \theta - 1 = 0
\]

Equation (5) is of the form

\[
x^2 + bx + c = 0
\]

with \( x = \cos \theta \), \( a = 1 \), \( b = \frac{v^2}{gL} \), and \( c = -1 \). The solution to equation (6) is found from the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

When \( v = 19.0 \) m/s, \( b = 2.17 \). The positive root from the quadratic formula gives \( x = \cos \theta = 0.391 \). Substitution into equation (3) yields

\[
T = \frac{mg}{\cos \theta} = \frac{(0.900 \text{ kg})(9.80 \text{ m/s}^2)}{0.391} = 23 \text{ N}
\]

When \( v = 38.0 \) m/s, \( b = 8.67 \). The positive root from the quadratic formula gives \( x = \cos \theta = 0.114 \). Substitution into equation (3) yields

\[
T = \frac{mg}{\cos \theta} = \frac{(0.900 \text{ kg})(9.80 \text{ m/s}^2)}{0.114} = 77 \text{ N}
\]
1. (e) When the force is perpendicular to the displacement, as in C, there is no work. When the force points in the same direction as the displacement, as in B, the maximum work is done. When the force points at an angle with respect to the displacement but has a component in the direction of the displacement, as in A, the work has a value between zero and the maximum work.

2. (b) Work is positive when the force has a component in the direction of the displacement. The force shown has a component along the \(-x\) and along the \(+y\) axis. Therefore, displacements in these two directions involve positive work.

3. (c) The work is given by \(W = (F \cos \theta)s\), which is zero when \(F = 0\, \text{N}\), \(s = 0\, \text{m}\), or \(\theta = 90^\circ\).

4. 78 kg·m²/s²

5. (a) The kinetic energy is \(KE = \frac{1}{2}mv^2\). Since the velocity components are perpendicular, the Pythagorean theorem indicates that \(v^2 = v_{\text{East}}^2 + v_{\text{North}}^2\). Therefore,

\[
KE = \frac{1}{2} m \left( v_{\text{East}}^2 + v_{\text{North}}^2 \right) = \frac{1}{2} \left( 3.00 \, \text{kg} \right) \left[ (5.00 \, \text{m/s})^2 + (8.00 \, \text{m/s})^2 \right]
\]

6. 115 m/s

7. (e) The work-energy theorem states that \(W = KE_f - KE_0\). Since work is done, the kinetic energy changes. Since kinetic energy is \(KE = \frac{1}{2} mv^2\), the speed \(v\) must also change. Since the instantaneous speed is the magnitude of the instantaneous velocity, the velocity must also change.

8. (d) The work-energy theorem indicates that when the net force acting on the particle does negative work, the kinetic energy decreases. Since each force does negative work, the work done by the net force must be negative, and the kinetic energy must decrease. But when the kinetic energy decreases, the speed must also decrease, since the kinetic energy is proportional to the square of the speed. Since it is stated that the speed increases, this answer is not possible.

9. (b) The work-energy theorem states that the net work done on the particle is equal to the change in the particle’s kinetic energy. However, the speed does not change. Therefore, the kinetic energy does not change, because kinetic energy is proportional to the square of the speed. According to the work-energy theorem, the net work is zero, which will be the case if \(W_1 = -W_2\).
10. (d) Since the block starts from rest, the work energy theorem is

\[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} (2.70 \text{ kg}) v_t^2 \]

The final speed can be obtained from this expression, since the work done by the net force is

\[ W = (75.0 \text{ N}) (\cos 38.0^\circ) (6.50 \text{ m}) + (54.0 \text{ N}) (\cos 0.0^\circ) (6.50 \text{ m}) + (93.0 \text{ N}) (\cos 65.0^\circ) (6.50 \text{ m}) \]

11. (c) The gravitational force is a conservative force, and a conservative force does no net work on an object moving around a closed path.

12. (e) Since air resistance always opposes the motion, it is a force that is always directed opposite to the displacement. Therefore, negative work is done.

13. (d) The velocity is constant. Therefore, the speed is also constant, and so is the kinetic energy. However, the total mechanical energy is the kinetic plus the gravitational potential energy, and the car moves up the hill, gaining potential energy as it goes. Thus, in the circumstance described mechanical energy cannot be conserved.

14. (c) The ball comes to a halt at B, when all of its initial kinetic energy is converted into potential energy.

15. (a) The stone’s motion is an example of projectile motion, in which the final vertical height is the same as the initial vertical height. Because friction and air resistance are being ignored, mechanical energy is conserved. Since the initial and final vertical heights are the same, \( \Delta \text{PE} = 0 \text{ J} \). Since energy is conserved, it follows that \( \Delta \text{KE} = 0 \text{ J} \) also.

16. (b) The total mechanical energy is the kinetic energy plus the gravitational potential energy:

\[ E = \frac{1}{2} m v^2 + mgh = \frac{1}{2} (88.0 \text{ kg})(19.0 \text{ m/s})^2 + (88.0 \text{ kg})(9.80 \text{ m/s}^2)(55.0 \text{ m}) \]

Since friction and air resistance are being ignored, the total mechanical energy is conserved, which means that it has the same value, no matter what the height above sea level is.

17. (d) Since the track is frictionless, the conservation of mechanical energy applies. We take the initial vertical level of the car as the zero level for gravitational potential energy. The highest vertical level that the car can attain occurs when the final speed of the car is zero and all of the initial kinetic energy is converted into gravitational potential energy. According to the conservation principle, \( \text{KE}_0 = mgh \), or \( h = \frac{\text{KE}_0}{mg} = \frac{2.2 \text{ J}}{(0.50 \text{ kg})(9.80 \text{ m/s}^2)} = 0.45 \text{ m} \). Thus, the car can pass over hills A, B, C, and D but not hill E.

18. (a) Using the ground as the zero level for measuring height and applying the energy conservation principle gives the following result:

\[ \frac{1}{2} m (7.00 \text{ m/s})^2 + \frac{1}{2} m (9.00 \text{ m/s})^2 + mg (11.0 \text{ m}) = \frac{1}{2} m v_f^2 \]

The mass \( m \) of the ball can be eliminated algebraically.
19. 7.07 m/s

20. (d) The work done by the net nonconservative force is 
\[ W_{nc} = KE_f + PE_f - KE_0 - PE_0. \]

21. (b) The principle of conservation of mechanical energy applies only if the work done by the net nonconservative force is zero, 
\[ W_{nc} = 0 \text{ J}. \] When only a single nonconservative force is present, it is the net nonconservative force, and a force that is perpendicular to the displacement does no work.

22. (a) A single nonconservative force is the net nonconservative force, and the work it does is given by 
\[ W_{nc} = \Delta KE + \Delta PE. \] Since the velocity is constant, \( \Delta KE = 0 \text{ J.} \) Therefore, 
\[ W_{nc} = \Delta PE = mg(h_f - h_0). \] But \( h_f - h_0 = -325 \text{ m}, \) since \( h_f \) is smaller than \( h_0. \) Thus, 
\[ W_{nc} = (92.0 \text{ kg})(9.80 \text{ m/s}^2)(-325 \text{ m}). \]

23. 71.5 J

24. 12 370 J

25. 344 W

26. 130 N

27. 376 J
CHAPTER 6  WORK AND ENERGY

PROBLEMS

1. **SSM REASONING AND SOLUTION** We will assume that the tug-of-war rope remains parallel to the ground, so that the force that moves team B is in the same direction as the displacement. According to Equation 6.1, the work done by team A is

\[
W = (F \cos \theta)s = (1100 \text{ N})(\cos 0^\circ)(2.0 \text{ m}) = 2.2 \times 10^3 \text{ J}
\]

2. **REASONING**
   a. We will use \( W = (F \cos \theta)s \) (Equation 6.1) to determine the work \( W \). The force (magnitude = \( F \)) doing the work is the force exerted on you by the elevator floor. This force is the normal force and has a magnitude \( F_N \), so \( F = F_N \) in Equation 6.1. To determine the normal force, we will use the fact that the elevator is moving at a constant velocity and apply Newton’s second law with the acceleration set to zero. Since the force exerted by the elevator and the displacement (magnitude = \( s \)) are in the same direction on the upward part of the trip, the angle between them is \( \theta = 0^\circ \), with the result that the work done by the force is positive.

   b. To determine the normal force, we will again use the fact that the elevator is moving at a constant velocity and apply Newton’s second law with the acceleration set to zero. Since the force exerted by the elevator and the displacement are in opposite directions on the downward part of the trip, the angle between them is \( \theta = 180^\circ \), and so the work done by the force is negative.

**SOLUTION**
   a. The free-body diagram at the right shows the three forces that act on you: \( W \) is your weight, \( W_b \) is the weight of your belongings, and \( F_N \) is the normal force exerted on you by the floor of the elevator. Since you are moving upward at a constant velocity, your acceleration is zero, you are in equilibrium, and the net force in the \( y \) direction must be zero:

\[
\frac{F_N - W - W_b}{\sum F_y} = 0
\]

Therefore, the magnitude of the normal force is \( F_N = W + W_b \). The work done by the normal force is
\[ W = (F_N \cos \theta) s = (W + W_g)(\cos \theta) s \]
\[ = (685 \text{ N} + 915 \text{ N})(\cos 0^\circ)(15.2 \text{ m}) = 24300 \text{ J} \]

b. During the downward trip, you are still in equilibrium since the elevator is moving with a constant velocity. The magnitude of the normal force is now \( F_N = W \). The work done by the normal force is

\[ W = (F_N \cos \theta) s = (W \cos \theta) s = (685 \text{ N})(\cos 180^\circ)(15.2 \text{ m}) = -10400 \text{ J} \]

3. **REASONING AND SOLUTION** The work done by the retarding force is given by Equation 6.1: \( W = (F \cos \theta) s \). Since the force is a retarding force, it must point opposite to the direction of the displacement, so that \( \theta = 180^\circ \). Thus, we have

\[ W = (F \cos \theta) s = (3.0 \times 10^3 \text{ N})(\cos 180^\circ)(850 \text{ m}) = -2.6 \times 10^6 \text{ J} \]

The work done by this force is **negative**, because the retarding force is directed opposite to the direction of the displacement of the truck.

4. **REASONING**

a. The work done by the gravitational force is given by Equation 6.1 as \( W = (F \cos \theta) s \). The gravitational force points downward, opposite to the upward vertical displacement of 4.60 m. Therefore, the angle \( \theta \) is 180º.

b. The work done by the escalator is done by the upward normal force that the escalator exerts on the man. Since the man is moving at a constant velocity, he is in equilibrium, and the net force acting on him must be zero. This means that the normal force must balance the man’s weight. Thus, the magnitude of the normal force is \( F_N = mg \), and the work that the escalator does is also given by Equation 6.1. However, since the normal force and the upward vertical displacement point in the same direction, the angle \( \theta \) is 0º.

**SOLUTION**

a. According to Equation 6.1, the work done by the gravitational force is

\[ W = (F \cos \theta) s = (mg \cos \theta) s \]
\[ = (75.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 180^\circ)(4.60 \text{ m}) = -3.38 \times 10^3 \text{ J} \]

b. The work done by the escalator is
\[ W = (F \cos \theta) s = (F_N \cos \theta) s \]
\[ = (75.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 0^\circ)(4.60 \text{ m}) = 3.38 \times 10^3 \text{ J} \]

5. **REASONING AND SOLUTION** Solving Equation 6.1 for the angle \( \theta \), we obtain
\[ \theta = \cos^{-1}\left( \frac{W}{Fs} \right) = \cos^{-1}\left[ \frac{1.10 \times 10^3 \text{ J}}{(30.0 \text{ N})(50.0 \text{ m})} \right] = 42.8^\circ \]

6. **REASONING** The drawing shows three of the forces that act on the cart: \( F \) is the pushing force that the shopper exerts, \( f \) is the frictional force that opposes the motion of the cart, and \( mg \) is its weight. The displacement \( s \) of the cart is also shown. Since the cart moves at a constant velocity along the +\( x \) direction, it is in equilibrium. The net force acting on it in this direction is zero, \( \Sigma F_x = 0 \).

This relation can be used to find the magnitude of the pushing force. The work done by a constant force is given by Equation 6.1 as \( W = (F \cos \theta)s \), where \( F \) is the magnitude of the force, \( s \) is the magnitude of the displacement, and \( \theta \) is the angle between the force and the displacement.

**SOLUTION**

a. The \( x \)-component of the net force is zero, \( \Sigma F_x = 0 \), so that

\[ \frac{F \cos 29.0^\circ - f}{\Sigma F_x} = 0 \]  
(4.9a)

The magnitude of the force that the shopper exerts is \( F = \frac{f}{\cos 29.0^\circ} = \frac{48.0 \text{ N}}{\cos 29.0^\circ} = 54.9 \text{ N} \).

b. The work done by the pushing force \( F \) is

\[ W = (F \cos \theta)s = (54.9 \text{ N})(\cos 29.0^\circ)(22.0 \text{ m}) = 1060 \text{ J} \]  
(6.1)

c. The angle between the frictional force and the displacement is \( 180^\circ \), so the work done by the frictional force \( f \) is

\[ W = (f \cos \theta)s = (48.0 \text{ N})(\cos 180^\circ)(22.0 \text{ m}) = -1060 \text{ J} \]
d. The angle between the weight of the cart and the displacement is 90°, so the work done by the weight \( mg \) is
\[
W = (mg \cos \theta)s = (16.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 90°)(22.0 \text{ m}) = 0 \text{ J}
\]

7. **SSM REASONING AND SOLUTION**
   
a. In both cases, the lift force \( L \) is perpendicular to the displacement of the plane, and, therefore, does no work. As shown in the drawing in the text, when the plane is in the dive, there is a component of the weight \( W \) that points in the direction of the displacement of the plane. When the plane is climbing, there is a component of the weight that points opposite to the displacement of the plane. Thus, since the thrust \( T \) is the same for both cases, the net force in the direction of the displacement is greater for the case where the plane is diving. Since the displacement \( s \) is the same in both cases, more net work is done during the dive.
   
b. The work done during the dive is \( W_{\text{dive}} = (T + W \cos 75°)s \), while the work done during the climb is \( W_{\text{climb}} = (T + W \cos 115°)s \). Therefore, the difference between the net work done during the dive and the climb is
\[
W_{\text{dive}} - W_{\text{climb}} = (T + W \cos 75°)s - (T + W \cos 115°)s = Ws(\cos 75° - \cos 115°)
\]
\[
= (5.9 \times 10^4 \text{ N})(1.7 \times 10^3 \text{ m})(\cos 75° - \cos 115°) = 6.8 \times 10^7 \text{ J}
\]

8. **REASONING AND SOLUTION** According to Equation 6.1, \( W = Fs \cos \theta \), the work is
   
a. \( W = (94.0 \text{ N})(35.0 \text{ m}) \cos 25.0° = 2980 \text{ J} \)
   
b. \( W = (94.0 \text{ N})(35.0 \text{ m}) \cos 0° = 3290 \text{ J} \)

9. **REASONING** The amount of work \( W \) done by a wind force \( F \) is given by \( W = (F \cos \theta)s \) (Equation 6.1), where \( F \) is the magnitude of the force, \( \theta \) is the angle between the direction of the force and the direction of the boat’s motion (which is due north), and \( s \) is the magnitude of the boat’s displacement. In the two scenarios described, the magnitudes of both wind forces \( F_1 \) and \( F_2 \) are the same and equal \( F \), but the displacements and angles are different. Therefore, we can express the work done in the first case as \( W_1 = (F \cos \theta_1)s_1 \) and that done in the second case as \( W_2 = (F \cos \theta_2)s_2 \). We will use these two expressions, along with the fact that the wind does the same amount of work in both cases \( (W_1 = W_2) \) to find the angle \( \theta_1 \) between due north and the direction of the wind’s force.
SOLUTION Setting the two expressions for work done by the wind equal, and solving for the angle $\theta_1$, we find

$$\left( F \cos \theta_1 \right) s_1 = \left( F \cos \theta_2 \right) s_2 \quad \text{or} \quad \cos \theta_1 = \frac{s_2 \cos \theta_2}{s_1} \quad \text{or} \quad \theta_1 = \cos^{-1} \left( \frac{s_2 \cos \theta_2}{s_1} \right)$$

In the second case, the force of the wind points due north, the same direction that the boat sails. Therefore, the second angle is zero ($\theta_2 = 0^\circ$), and angle that the force of the wind makes with due north is

$$\theta_1 = \cos^{-1} \left[ \frac{(47 \text{ m})(\cos 0^\circ)}{52 \text{ m}} \right] = 25^\circ$$

10. REASONING The four forces that act on the box are the pushing force (magnitude $= P$), the frictional force (magnitude $= f_k$), the normal force exerted by the floor (magnitude $= F_N$), and the weight $mg$ of the box. The kinetic frictional force is related to the normal force by $f_k = \mu_k F_N$ (Equation 4.8), where $\mu_k = 0.25$ is the coefficient of kinetic friction. In this particular case, the box does not accelerate in the vertical direction, so that the vertically-directed forces must balance to zero. Since the normal force points upward and the weight points downward, it follows that $F_N = mg$. For each of the four forces we will determine the work $W$ as $W = (F \cos \theta)s$ (Equation 6.1), where $F$ is the magnitude of the force, $\theta$ is the angle between the direction of the force and the displacement, and $s = 7.0 \text{ m}$ is the magnitude of the displacement.

SOLUTION The pushing force and the displacement point in the same direction, so that $\theta = 0.0^\circ$. Equation 6.1 reveals that the work done by the pushing force is

$$W_P = (P \cos \theta)s = (160 \text{ N})(\cos 0.0^\circ)(7.0 \text{ m}) = 1.1 \times 10^3 \text{ J}$$

The magnitude of the kinetic frictional force is $f_k = \mu_k F_N = \mu_k mg$ according to Equation 4.8 and the fact that $F_N = mg$. The frictional force and the displacement point in the opposite directions, so that $\theta = 180^\circ$. Thus, according to Equation 6.1, the work done by the frictional force is

$$W_f = (\mu_k mg \cos \theta)s = 0.25(55 \text{ kg})(9.80 \text{ m/s}^2)(\cos 180.0^\circ)(7.0 \text{ m}) = -940 \text{ J}$$

Both the normal force and the weight of the box are perpendicular to the displacement, so for these two forces $\theta = 90.0^\circ$. According to Equation 6.1, the work for both forces is

$$W = (F \cos \theta)s = (F \cos 90.0^\circ)(7.0 \text{ m}) = 0.0 \text{ J}$$
11. **REASONING** The net work done by the pushing force and the frictional force is zero, and our solution is focused on this fact. Thus, we express this net work as $W_p + W_f = 0$, where $W_p$ is the work done by the pushing force and $W_f$ is the work done by the frictional force. We will substitute for each individual work using Equation 6.1 \[ W = (F \cos \theta) s \] and solve the resulting equation for the magnitude $P$ of the pushing force.

**SOLUTION** According to Equation 6.1, the work done by the pushing force is

$$W_p = (P \cos 30.0^\circ) s = 0.866 \ P s$$

The frictional force opposes the motion, so the angle between the force and the displacement is $180^\circ$. Thus, the work done by the frictional force is

$$W_f = (f_k \cos 180^\circ) s = -f_k s$$

Equation 4.8 indicates that the magnitude of the kinetic frictional force is $f_k = \mu_k F_N$, where $F_N$ is the magnitude of the normal force acting on the crate. The free-body diagram shows the forces acting on the crate. Since there is no acceleration in the vertical direction, the $y$ component of the net force must be zero:

$$F_N \sin 30.0^\circ - mg - P \sin 30.0^\circ = 0$$

Therefore,

$$F_N = mg + P \sin 30.0^\circ$$

It follows, then, that the magnitude of the frictional force is

$$f_k = \mu_k F_N = \mu_k (mg + P \sin 30.0^\circ)$$

Suppressing the units, we find that the work done by the frictional force is

$$W_f = -f_k s = - (0.200)[(1.00 \times 10^2 \text{ kg})(9.80 \text{ m/s}^2) + 0.500 P]s = -(0.100P + 196)s$$

Since the net work is zero, we have

$$W_p + W_f = 0.866 \ P s - (0.100P + 196)s = 0$$

Eliminating $s$ algebraically and solving for $P$ gives $P = 256 \text{ N}$.
12. **REASONING AND SOLUTION** The net work done on the car is

\[
W_T = W_F + W_f + W_g + W_N
\]

\[
W_T = F_s \cos 0.0^\circ + f_s \cos 180^\circ - mg_s \sin 5.0^\circ + F_Ns \cos 90^\circ
\]

Rearranging this result gives

\[
F = \frac{W_T}{s} + f + mg \sin 5.0^\circ
\]

\[
= \frac{150 \times 10^3 \text{ J}}{290 \text{ m}} + 524 \text{ N} + (1200 \text{ kg}) \left(9.80 \text{ m/s}^2\right) \sin 5.0^\circ = 2.07 \times 10^3 \text{ N}
\]

13. **REASONING** The work done by the catapult \(W_{\text{catapult}}\) is one contribution to the work done by the net external force that changes the kinetic energy of the plane. The other contribution is the work done by the thrust force of the plane’s engines \(W_{\text{thrust}}\). According to the work-energy theorem (Equation 6.3), the work done by the net external force \(W_{\text{catapult}} + W_{\text{thrust}}\) is equal to the change in the kinetic energy. The change in the kinetic energy is the given kinetic energy of \(4.5 \times 10^7\) J at lift-off minus the initial kinetic energy, which is zero since the plane starts at rest. The work done by the thrust force can be determined from Equation 6.1 \([W = (F \cos \theta)s]\), since the magnitude \(F\) of the thrust is \(2.3 \times 10^5\) N and the magnitude \(s\) of the displacement is 87 m. We note that the angle \(\theta\) between the thrust and the displacement is 0º, because they have the same direction. In summary, we will calculate \(W_{\text{catapult}}\) from \(W_{\text{catapult}} + W_{\text{thrust}} = \text{KE}_f - \text{KE}_0\).

**SOLUTION** According to the work-energy theorem, we have

\[
W_{\text{catapult}} + W_{\text{thrust}} = \text{KE}_f - \text{KE}_0
\]

Using Equation 6.1 and noting that \(\text{KE}_0 = 0\) J, we can write the work energy theorem as follows:

\[
W_{\text{catapult}} + (F \cos \theta)s = \text{KE}_f
\]

Solving for \(W_{\text{catapult}}\) gives

\[
W_{\text{catapult}} = \text{KE}_f - (F \cos \theta)s
\]

\[
= 4.5 \times 10^7 \text{ J} - (2.3 \times 10^5 \text{ N}) \cos 0^\circ (87 \text{ m}) = 2.5 \times 10^7 \text{ J}
\]
14. **REASONING** The work-energy theorem is \( W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \) (Equation 6.3), where \( W \) is the work done by a net force acting on an object of mass \( m \) and \( v_0 \) and \( v_f \) are the object’s initial and final speeds, respectively. The work done by a force is \( W = (F \cos \theta) s \) (Equation 6.1), where \( F \) is the magnitude of the force, \( \theta \) is the angle between the direction of the force and the displacement and \( s \) is the magnitude of the displacement. Thus, we can also write the work-energy theorem as

\[
(F \cos \theta) s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \tag{1}
\]

In this expression, we know that \( F = 6800 \text{ N}, \theta = 0.0^\circ \) (since the net force acts parallel to the ball’s motion), \( s = 0.010 \text{ m}, m = 0.045 \text{ kg}, \) and \( v_0 = 0.0 \text{ m/s} \) (since the ball is at rest initially). Thus, we can solve Equation (1) for \( v_f \), which is the speed with which the ball leaves the club.

**SOLUTION** Using the fact that \( v_0 = 0.0 \text{ m/s} \) in Equation (1) gives

\[
(F \cos \theta) s = \frac{1}{2} m v_f^2 \quad \text{or} \quad v_f = \sqrt{\frac{2(F \cos \theta) s}{m}}
\]

The ball leaves the club with a speed of

\[
v_f = \sqrt{\frac{2(6800 \text{ N})(\cos 0.0^\circ)(0.010 \text{ m})}{0.045 \text{ kg}}} = 55 \text{ m/s}
\]

15. **SSM REASONING** The car’s kinetic energy depends upon its mass and speed via \( KE = \frac{1}{2} m v^2 \) (Equation 6.2). The total amount of work done on the car is equal to the difference between its final and initial kinetic energies: \( W = KE_f - KE_0 \) (Equation 6.3). We will use these two relationships to determine the car’s mass.

**SOLUTION** Combining \( KE = \frac{1}{2} m v^2 \) (Equation 6.2) and \( W = KE_f - KE_0 \) (Equation 6.3), we obtain

\[
W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \quad \text{or} \quad \frac{1}{2} m (v_f^2 - v_0^2) = W
\]

Solving this expression for the car’s mass \( m \), and noting that 185 kJ = 185 000 J, we find that

\[
m = \frac{2W}{v_f^2 - v_0^2} = \frac{2(185 \text{ 000 J})}{(28.0 \text{ m/s})^2 - (23.0 \text{ m/s})^2} = 1450 \text{ kg}
\]
16. **REASONING**

a. Because we know the flea’s mass \( m \), we can find its speed \( v_f \) once we know its kinetic energy \( KE_f = \frac{1}{2}mv_f^2 \) (Equation 6.2) at the instant it leaves the ground. The work-energy theorem \( W = KE_f - KE_0 \) (Equation 6.3) relates \( KE_f \) to the total work \( W \) done on the flea during push-off. Because it starts from rest, the flea has no initial kinetic energy, and we see that the total work done on the flea is equal to its final kinetic energy: \( W = KE_f - (0 \, \text{J}) = KE_f = \frac{1}{2}mv_f^2 \). Since air resistance and the flea’s weight are being ignored, the only force doing work on the flea is the upward force \( F_{\text{ground}} \) exerted by the ground, so \( W_{\text{ground}} = W = \frac{1}{2}mv_f^2 \).

b. We will make use of the definition of work \( W_{\text{ground}} = (F_{\text{ground}} \cos \theta)s \) (Equation 6.1) to calculate the flea’s upward displacement \( s \) while the force \( F_{\text{ground}} \) is doing work on the flea.

**SOLUTION**

a. Solving \( W_{\text{ground}} = \frac{1}{2}mv_f^2 \) for the final speed \( v_f \) of the flea, we find

\[
\frac{1}{2}mv_f^2 = W_{\text{ground}} \quad \text{or} \quad v_f^2 = \frac{2W_{\text{ground}}}{m} \quad \text{or} \quad v_f = \sqrt{\frac{2W_{\text{ground}}}{m}}
\]

Therefore,

\[
v_f = \sqrt{\frac{2\left(+2.4 \times 10^{-4} \, \text{J}\right)}{1.9 \times 10^{-4} \, \text{kg}}} = 1.6 \, \text{m/s}
\]

b. The force \( F_{\text{ground}} \) of the ground on the flea is upward, the same as the direction of the flea’s displacement \( s \) while it is pushing off. Therefore, the angle \( \theta \) between \( F_{\text{ground}} \) and \( s \) is zero. Solving \( W_{\text{ground}} = (F_{\text{ground}} \cos \theta)s \) for the displacement magnitude \( s \), we obtain

\[
s = \frac{W_{\text{ground}}}{F_{\text{ground}} \cos \theta} = \frac{+2.4 \times 10^{-4} \, \text{J}}{(0.38 \, \text{N})(\cos 0^\circ)} = 6.3 \times 10^{-4} \, \text{m}
\]

17. **REASONING** The work \( W \) done by the net external force acting on the skier can be found from the work-energy theorem in the form of \( W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \) (Equation 6.3) because the final and initial kinetic energies of the skier can be determined. The kinetic energy \( \left(\frac{1}{2}mv^2\right) \) is increasing, because the skier’s speed is increasing. Thus, the work will be
positive, reflecting the fact that the net external force must be in the same direction as the displacement of the skier to make the skier pick up speed.

**SOLUTION** The work done by the net external force acting on the skier is

\[ W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

\[ = \frac{1}{2}(70.3 \text{ kg})(11.3 \text{ m/s})^2 - \frac{1}{2}(70.3 \text{ kg})(6.10 \text{ m/s})^2 = 3.2 \times 10^3 \text{ J} \]

18. **REASONING AND SOLUTION** From the work-energy theorem, Equation 6.3,

\[ W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2} m \left( v_f^2 - v_0^2 \right) \]

a. \[ W = \frac{1}{2} (7420 \text{ kg}) \left[ (8450 \text{ m/s})^2 - (2820 \text{ m/s})^2 \right] = 2.35 \times 10^{11} \text{ J} \]

b. \[ W = \frac{1}{2} (7420 \text{ kg}) \left[ (2820 \text{ m/s})^2 - (8450 \text{ m/s})^2 \right] = -2.35 \times 10^{11} \text{ J} \]

19. **SSM REASONING** The work done to launch either object can be found from Equation 6.3, the work-energy theorem, \( W = \text{KE}_f - \text{KE}_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \).

**SOLUTION**

a. The work required to launch the hammer is

\[ W = \frac{1}{2} \frac{1}{2} m \left( v_f^2 - v_0^2 \right) = \frac{1}{2} (7.3 \text{ kg}) \left[ (29 \text{ m/s})^2 - (0 \text{ m/s})^2 \right] = 3.1 \times 10^3 \text{ J} \]

b. Similarly, the work required to launch the bullet is

\[ W = \frac{1}{2} m \left( v_f^2 - v_0^2 \right) = \frac{1}{2} (0.0026 \text{ kg}) \left[ (410 \text{ m/s})^2 - (0 \text{ m/s})^2 \right] = 2.2 \times 10^2 \text{ J} \]

20. **REASONING** To find the coefficient of kinetic friction \( \mu_k \), we need to find the force of kinetic friction \( f_k \) and the normal force \( F_N \) (see Equation 4.8, \( f_k = \mu_k F_N \)). The normal force and the weight \( mg \) of the sled balance, since they are the only two forces acting vertically and the sled does not accelerate in the vertical direction. The force of kinetic friction can be obtained from the work \( W_f \) done by the frictional force, according to Equation 6.1 \( [W_f = (f_k \cos \theta) s] \), where \( s \) is the magnitude of the displacement. To find the work, we will employ the work-energy theorem, as given in Equation 6.3 \( (W = \text{KE}_f - \text{KE}_0) \). In this equation \( W \) is the work done by the net force, but the normal force and the weight balance,
so the net force is that due to the pulling force $P$ and the frictional force. As a result $W = W_{\text{pull}} + W_f$.

**SOLUTION** According to the work-energy theorem, we have

$$W = W_{\text{pull}} + W_f = KE_f - KE_0$$

Using Equation 6.1\[W = (F \cos \theta) s\] to express each work contribution, writing the kinetic energy as $\frac{1}{2} mv^2$, and noting that the initial kinetic energy is zero (the sled starts from rest), we obtain

$$\frac{(P \cos 0^\circ) s}{W_{\text{pull}}} + \frac{(f_k \cos 180^\circ) s}{W_f} = \frac{1}{2} mv^2$$

The angle $\theta$ between the force and the displacement is $0^\circ$ for the pulling force (it points in the same direction as the displacement) and $180^\circ$ for the frictional force (it points opposite to the displacement). Equation 4.8 indicates that the magnitude of the frictional force is $f_k = \mu_k F_N$, and we know that the magnitude of the normal force is $F_N = mg$. With these substitutions the work-energy theorem becomes

$$\frac{(P \cos 0^\circ) s}{W_{\text{pull}}} + \frac{(\mu_k mg \cos 180^\circ) s}{W_f} = \frac{1}{2} mv^2$$

Solving for the coefficient of kinetic friction gives

$$\mu_k = \frac{\frac{1}{2} mv^2 - (P \cos 0^\circ) s}{(mg \cos 180^\circ) s} = \frac{\frac{1}{2} (16 \text{ kg})(2.0 \text{ m/s})^2 - (24 \text{ N})(8.0 \text{ m})}{-(16 \text{ kg})(9.80 \text{ m/s}^2)(8.0 \text{ m})} = 0.13$$

21. **REASONING**

a. The work $W$ done by the force is related to the change in the asteroid’s kinetic energy by the work-energy theorem $W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2$ (Equation 6.3). Since the final speed of the asteroid is $v_f = 5500$ m/s and is less than the initial speed of $v_0 = 7100$ m/s, the work obtained from Equation 6.3 will be negative. This is consistent with the fact that, since the asteroid is slowing down, it is decelerating, so that the force must point in a direction that is opposite to the direction of the displacement.

b. Since the work $W$ is known from part (a), we can use $W = (F \cos \theta) s$ (Equation 6.1) to determine the magnitude $F$ of the force. The direction of the force is opposite to the displacement of the asteroid, so the angle between them is $\theta = 180^\circ$. 


**SOLUTION**

a. The work-energy theorem states that the work done by the force acting on the asteroid is equal to its final kinetic energy minus its initial kinetic energy:

\[
W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

\[
= \frac{1}{2}(4.5 \times 10^4 \text{ kg})(5500 \text{ m/s})^2 - \frac{1}{2}(4.5 \times 10^4 \text{ kg})(7100 \text{ m/s})^2 = -4.5 \times 10^{11} \text{ J}
\]

b. The work done by the force acting on the asteroid is given by \( W = (F \cos \theta) s \) (Equation 6.1). Since the force acts to slow down the asteroid, the force and the displacement vectors point in opposite directions, so the angle between them is \( \theta = 180^\circ \). The magnitude of the force is

\[
F = \frac{W}{s \cos \theta} = \frac{-4.5 \times 10^{11} \text{ J}}{(1.8 \times 10^6 \text{ m}) \cos 180^\circ} = 2.5 \times 10^5 \text{ N}
\]

---

22. **REASONING** Since the person has an upward acceleration, there must be a net force acting in the upward direction. The net force \( \Sigma F_y \) is related to the acceleration \( a_y \) by Newton’s second law, \( \Sigma F_y = ma_y \), where \( m \) is the mass of the person. This relation will allow us to determine the tension in the cable. The work done by the tension and the person’s weight can be found directly from the definition of work, Equation 6.1.

**SOLUTION**

a. The free-body diagram at the right shows the two forces that act on the person. Applying Newton’s second law, we have

\[
\frac{T - mg}{\Sigma F_y} = ma_y
\]

Solving for the magnitude of the tension in the cable yields

\[
T = m(a_y + g) = (79 \text{ kg})(0.70 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 8.3 \times 10^2 \text{ N}
\]

b. The work done by the tension in the cable is

\[
W_T = (T \cos \theta) s = (8.3 \times 10^2 \text{ N}) \cos 0^\circ (11 \text{ m}) = 9.1 \times 10^3 \text{ J}
\]

c. The work done by the person’s weight is

\[
W_w = (mg \cos \theta) s = (79 \text{ kg})(9.8 \text{ m/s}^2) \cos 180^\circ (11 \text{ m}) = -8.5 \times 10^3 \text{ J}
\]
d. The work-energy theorem relates the work done by the two forces to the change in the kinetic energy of the person. The work done by the two forces is \( W = W_T + W_W \):

\[
\frac{W_T + W_W}{W} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2
\]  

(6.3)

Solving this equation for the final speed of the person gives

\[
v_f = \sqrt{v_0^2 + \frac{2}{m} (W_T + W_W)}
\]

\[
= \sqrt{(0 \text{ m/s})^2 + \frac{2}{79 \text{ kg}} (9.1 \times 10^3 \text{ J} - 8.5 \times 10^3 \text{ J})} = 4 \text{ m/s}
\]

23. **SSM REASONING** When the satellite goes from the first to the second orbit, its kinetic energy changes. The net work that the external force must do to change the orbit can be found from the work-energy theorem: \( W = KE_f - KE_0 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \). The speeds \( v_f \) and \( v_0 \) can be obtained from Equation 5.5 for the speed of a satellite in a circular orbit of radius \( r \). Given the speeds, the work energy theorem can be used to obtain the work.

**SOLUTION** According to Equation 5.5, \( v = \sqrt{\frac{GM_E}{r}} \). Substituting into the work-energy theorem, we have

\[
W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} m \left( v_f^2 - v_0^2 \right) = \frac{1}{2} m \left[ \left( \frac{GM_E}{r_f} \right)^2 - \left( \frac{GM_E}{r_0} \right)^2 \right] = \frac{GM_E m}{2} \left( \frac{1}{r_f} - \frac{1}{r_0} \right)
\]

Therefore,

\[
W = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(6200 \text{ kg})}{2}
\]

\[
\times \left( \frac{1}{7.0 \times 10^6 \text{ m}} - \frac{1}{3.3 \times 10^7 \text{ m}} \right) = 1.4 \times 10^{11} \text{ J}
\]

24. **REASONING** The initial speed \( v_0 \) of the skier can be obtained by applying the work-energy theorem: \( W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \) (Equation 6.3). This theorem indicates that the initial
kinetic energy \( \frac{1}{2} m v_0^2 \) of the skier is related to the skier’s final kinetic energy \( \frac{1}{2} m v_f^2 \) and the work \( W \) done on the skier by the kinetic frictional force according to

\[
\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 - W
\]

Solving for the skier’s initial speed gives

\[
v_0 = \sqrt{v_f^2 - \frac{2W}{m}} \tag{1}
\]

The work done by the kinetic frictional force is given by \( W = (f_k \cos \theta)s \) (Equation 6.1), where \( f_k \) is the magnitude of the kinetic frictional force and \( s \) is the magnitude of the skier’s displacement. Because the kinetic frictional force points opposite to the displacement of the skier, \( \theta = 180^\circ \). According to Equation 4.8, the kinetic frictional force has a magnitude of \( f_k = \mu_k F_N \), where \( \mu_k \) is the coefficient of kinetic friction and \( F_N \) is the magnitude of the normal force. Thus, the work can be expressed as

\[
W = (f_k \cos \theta)s = (\mu_k F_N \cos 180^\circ)s
\]

Substituting this expression for \( W \) into Equation (1), we have that

\[
v_0 = \sqrt{v_f^2 - \frac{2\mu_k F_N (\cos 180^\circ)s}{m}} \tag{2}
\]

Since the skier is sliding on level ground, the magnitude of the normal force is equal to the weight \( mg \) of the skier (see Example 10 in Chapter 4), so \( F_N = mg \). Substituting this relation into Equation (2) gives

\[
v_0 = \sqrt{v_f^2 - \frac{2\mu_k mg (\cos 180^\circ)s}{m}} = \sqrt{v_f^2 - 2\mu_k g (\cos 180^\circ)s}
\]

**SOLUTION** Since the skier comes to a halt, \( v_f = 0 \text{ m/s} \). Therefore, the initial speed is

\[
v_0 = \sqrt{v_f^2 - 2\mu_k g (\cos 180^\circ)s} = \sqrt{(0 \text{ m/s})^2 - 2(0.050)(9.80 \text{ m/s}^2)(\cos 180^\circ)(21 \text{ m})} = 4.5 \text{ m/s}
\]

**25. SSM REASONING** According to the work-energy theorem, the kinetic energy of the sled increases in each case because work is done on the sled. The work-energy theorem is given by Equation 6.3: \( W = KE_f - KE_0 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \). The work done on the sled is
given by Equation 6.1: \( W = (F \cos \theta)s \). The work done in each case can, therefore, be expressed as

\[
W_1 = (F \cos 0^\circ)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \Delta KE_1
\]

and

\[
W_2 = (F \cos 62^\circ)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \Delta KE_2
\]

The fractional increase in the kinetic energy of the sled when \( \theta = 0^\circ \) is

\[
\frac{\Delta KE_1}{KE_0} = \frac{(F \cos 0^\circ)s}{KE_0} = 0.38
\]

Therefore,

\[
Fs = (0.38) KE_0
\]

The fractional increase in the kinetic energy of the sled when \( \theta = 62^\circ \) is

\[
\frac{\Delta KE_2}{KE_0} = \frac{(F \cos 62^\circ)s}{KE_0} = \frac{Fs}{KE_0} (cos 62^\circ)
\]

Equation (1) can be used to substitute for \( Fs \) in Equation (2).

**SOLUTION** Combining Equations (1) and (2), we have

\[
\frac{\Delta KE_2}{KE_0} = \frac{Fs}{KE_0} (cos 62^\circ) = \frac{(0.38) KE_0}{KE_0} (cos 62^\circ) = (0.38)(cos 62^\circ) = 0.18
\]

Thus, the sled's kinetic energy would increase by \( 18\% \).

26. **REASONING**

a. The magnitude \( s \) of the displacement that occurs while the snowmobile coasts to a halt is the distance that we seek. The work \( W \) done by the net external force \( F \) acting on the snowmobile during the coasting phase is given by \( W = (F \cos \theta)s \) (Equation 6.1), and we can use this expression to obtain \( s \). To do so, we will need to evaluate \( W \). This we will do with the aid of the work-energy theorem as given by \( W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \) (Equation 6.3), since we know the mass \( m \) and the final and initial velocities \( v_f \) and \( v_0 \), respectively. The net external force \( F \) during the coasting phase is just the horizontal force of kinetic friction (magnitude = \( f_k \)), since the drive force is shut off and the vertical forces balance (there is no vertical acceleration). This frictional force has a magnitude of 205 N. This follows because, while the snowmobile is moving at a constant 5.50 m/s, it is not accelerating, and the drive force must be balancing the frictional force, which exists both before and after the drive force is shut off. Finally, to use \( W = (F \cos \theta)s \) (Equation 6.1) to determine \( s \), we will need a value for the angle \( \theta \) between the frictional force and the displacement. The frictional force points opposite to the displacement, so \( \theta = 180^\circ \).
b. By using only the work-energy theorem it is not possible to determine the time \( t \) during which the snowmobile coasts to a halt. To determine \( t \) we need to use the equations of kinematics. We can use these equations, if we assume that the acceleration during the coasting phase is constant. The acceleration is determined by the force of friction (assumed constant) and the mass of the snowmobile, according to Newton’s second law. For example, we can use \( s = \frac{1}{2} (v_0 + v_f) t \) (Equation 2.7) to obtain \( t \) once we know the magnitude of the displacement \( s \), which is the distance in which the snowmobile coasts to a halt.

**SOLUTION**

a. According to Equation 6.3, the work energy theorem, is

\[
W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2
\]

As discussed in the **REASONING**, we know that \( W = (f_k \cos \theta) s \). Substituting this result into the work-energy theorem gives

\[
(f_k \cos \theta) s = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \quad (1)
\]

In Equation (1), \( f_k = 205 \text{ N} \) (see the **REASONING**), the angle \( \theta \) between the frictional force and the displacement is \( \theta = 180^\circ \) (see the **REASONING**), the final speed is \( v_f = 0 \text{ m/s} \) (the snowmobile coasts to a halt), and the initial speed is given as \( v_0 = 5.50 \text{ m/s} \). Solving Equation (1) for \( s \) gives

\[
s = \frac{m(v_f^2 - v_0^2)}{2f_k \cos \theta} = \frac{(136 \text{ kg})[(0 \text{ m/s})^2 - (5.50 \text{ m/s})^2]}{2(205 \text{ N}) \cos 180^\circ} = 10.0 \text{ m}
\]

b. As explained in the **REASONING**, we can use \( s = \frac{1}{2} (v_0 + v_f) t \) (Equation 2.7) to determine the time \( t \) during which the snowmobile coasts to a halt. Solving this equation for \( t \) gives

\[
t = \frac{2s}{v_0 + v_f} = \frac{2(10.0 \text{ m})}{5.50 \text{ m/s} + 0 \text{ m/s}} = 3.64 \text{ s}
\]

27. **REASONING AND SOLUTION** The net work done on the plane can be found from the work-energy theorem:

\[
W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 = \frac{1}{2} m(v_f^2 - v_0^2)
\]

The tension in the guideline provides the centripetal force \( (mv^2/r) \) required to keep the plane moving in a circular path. Since the tension in the guideline becomes four times greater,
\[ T_f = 4 \ T_0 \quad \text{or} \quad \frac{mv_f^2}{r_f} = 4 \ \frac{mv_0^2}{r_0} \]

Solving for \( v_f^2 \) gives

\[ v_f^2 = \frac{4v_0^2 r_f}{r_0} \]

Substitution of this expression into the work-energy theorem gives

\[ W = \frac{1}{2} m \left[ \frac{4v_0^2 r_f}{r_0} - v_0^2 \right] = \frac{1}{2} mv_0^2 \left[ 4 \left( \frac{r_f}{r_0} \right) - 1 \right] \]

Therefore,

\[ W = \frac{1}{2} \ (0.90 \text{ kg})(22 \text{ m/s})^2 \left[ 4 \left( \frac{14 \text{ m}}{16 \text{ m}} \right) - 1 \right] = 5.4 \times 10^2 \text{ J} \]

28. **REASONING** It is useful to divide this problem into two parts. The first part involves the skier moving on the snow. We can use the work-energy theorem to find her speed when she comes to the edge of the cliff. In the second part she leaves the snow and falls freely toward the ground. We can again employ the work-energy theorem to find her speed just before she lands.

**SOLUTION** The drawing at the right shows the three forces that act on the skier as she glides on the snow. The forces are: her weight \( mg \), the normal force \( F_N \), and the kinetic frictional force \( f_k \). Her displacement is labeled as \( s \). The work-energy theorem, Equation 6.3, is

\[ W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \]

where \( W \) is the work done by the net external force that acts on the skier. The work done by each force is given by Equation 6.1, \( W = (F \cos \theta)s \), so the work-energy theorem becomes

\[ \frac{(mg \cos 65.0^\circ)s + (f_k \cos 180^\circ)s + (F_N \cos 90^\circ)s}{W} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \]

Since \( \cos 90^\circ = 0 \), the third term on the left side can be eliminated. The magnitude \( f_k \) of the kinetic frictional force is given by Equation 4.8 as \( f_k = \mu_k F_N \). The magnitude \( F_N \) of the normal force can be determined by noting that the skier does not leave the surface of the slope, so \( a_y = 0 \text{ m/s}^2 \). Thus, we have that \( \Sigma F_y = 0 \), so
\[
\frac{F_N - mg \cos 25.0^\circ}{\Sigma F_y} = 0 \quad \text{or} \quad F_N = mg \cos 25.0^\circ
\]

The magnitude of the kinetic frictional force becomes \( f_k = \mu_k F_N = \mu_k mg \cos 25.0^\circ \). Substituting this result into the work-energy theorem, we find that

\[
\left( mg \cos 65.0^\circ \right)s + \left( \mu_k mg \cos 25.0^\circ \right)\left( \cos 180^\circ \right)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

Algebraically eliminating the mass \( m \) of the skier from every term, setting \( \cos 180^\circ = -1 \) and \( v_0 = 0 \) m/s, and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{2gs \left( \cos 65.0^\circ - \mu_k \cos 25.0^\circ \right)}
\]

\[= \sqrt{2 \left( 9.80 \text{ m/s}^2 \right) \left( 10.4 \text{ m} \right) \left[ \cos 65.0^\circ - (0.200)\cos 25.0^\circ \right]} = 7.01 \text{ m/s}
\]

The drawing at the right shows her displacement \( s \) during free fall. Note that the displacement is a vector that starts where she leaves the slope and ends where she touches the ground. The only force acting on her during the free fall is her weight \( mg \). The work-energy theorem, Equation 6.3, is

\[
W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

The work \( W \) is that done by her weight, so the work-energy theorem becomes

\[
\frac{(mg \cos \theta)s}{W} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

In this expression \( \theta \) is the angle between her weight (which points vertically downward) and her displacement. Note from the drawing that \( s \cos \theta = 3.50 \) m. Algebraically eliminating the mass \( m \) of the skier from every term in the equation above and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{v_0^2 + 2g(s \cos \theta)}
\]

\[= \sqrt{\left( 7.01 \text{ m/s} \right)^2 + 2 \left( 9.80 \text{ m/s}^2 \right) \left( 3.50 \text{ m} \right)} = 10.9 \text{ m/s}
\]
29. **REASONING AND SOLUTION**  
The vertical height of the skier is \( h = s \sin 14.6^\circ \) so

\[
\Delta PE = mgs \sin 14.6^\circ
\]

\[
= (75.0 \text{ kg})(9.80 \text{ m/s}^2)(2830 \text{ m}) \sin 14.6^\circ
\]

\[
= 5.24 \times 10^5 \text{ J}
\]

30. **REASONING** At a height \( h \) above the ground, the gravitational potential energy of each clown is given by \( \text{PE} = mgh \) (Equation 6.5). At a height \( h_j \), Juggle’s potential energy is, therefore, \( \text{PE}_j = m_jgh_j \), where \( m_j \) is Juggle’s mass. Similarly, Bangles’ potential energy can be expressed in terms of his mass \( m_B \) and height \( h_B \): \( \text{PE}_B = m_Bgh_B \). We will use the fact that these two amounts of potential energy are equal (\( \text{PE}_j = \text{PE}_B \)) to find Juggle’s mass \( m_j \).

**SOLUTION** Setting the gravitational potential energies of both clowns equal, we solve for Juggle’s mass and obtain

\[
m_j = \frac{m_B h_B}{h_j} \quad \text{or} \quad m_j = \frac{m_B h_B}{h_j} = \frac{(86 \text{ kg})(2.5 \text{ m})}{(3.3 \text{ m})} = 65 \text{ kg}
\]

31. **REASONING** The work done by the weight of the basketball is given by Equation 6.1 as \( W = (F \cos \theta)s \), where \( F = mg \) is the magnitude of the weight, \( \theta \) is the angle between the weight and the displacement, and \( s \) is the magnitude of the displacement. The drawing shows that the weight and displacement are parallel, so that \( \theta = 0^\circ \). The potential energy of the ball is given by Equation 6.5 as \( \text{PE} = mgh \), where \( h \) is the height of the ball above the ground.

**SOLUTION**

a. The work done by the weight of the basketball is

\[
W = (F \cos \theta)s = mg (\cos 0^\circ)(h_0 - h_f) = (0.60 \text{ kg})(9.80 \text{ m/s}^2)(6.1 \text{ m} - 1.5 \text{ m}) = 27 \text{ J}
\]

b. The potential energy of the ball, relative to the ground, when it is released is

\[
\text{PE}_0 = mgh_0 = (0.60 \text{ kg})(9.80 \text{ m/s}^2)(6.1 \text{ m}) = 36 \text{ J} \quad (6.5)
\]

c. The potential energy of the ball, relative to the ground, when it is caught is

\[
\text{PE}_f = mgh_f = (0.60 \text{ kg})(9.80 \text{ m/s}^2)(1.5 \text{ m}) = 8.8 \text{ J} \quad (6.5)
\]
d. The change in the ball’s gravitational potential energy is

\[ \Delta PE = PE_f - PE_0 = 8.8 \text{ J} - 36 \text{ J} = -27 \text{ J} \]

We see that the change in the gravitational potential energy is equal to \(-27 \text{ J} = -W\), where \(W\) is the work done by the weight of the ball (see part a).

32. **REASONING** Gravitational potential energy is \(PE = mgh\) (Equation 6.5), where \(m\) is the mass, \(g\) is the magnitude of the acceleration due to gravity, and \(h\) is the height. The change in the potential energy is the final minus the initial potential energy. The weight is \(mg\), so it can be obtained from the change in the potential energy.

**SOLUTION** The change in the pole-vaulter's potential energy is

\[ PE_f - PE_0 = mgh_f - mgh_0 \]

With \(h_f = 0 \text{ m}\) and \(h_0 = 5.80 \text{ m}\) we find that

\[ mg = \frac{PE_f - PE_0}{h_f - h_0} = \frac{0 \text{ J} - 3.70 \times 10^3 \text{ J}}{0 \text{ m} - 5.80 \text{ m}} = 6.38 \times 10^2 \text{ N} \]

33. **SSM REASONING** During each portion of the trip, the work done by the resistive force is given by Equation 6.1, \(W = (F \cos \theta)s\). Since the resistive force always points opposite to the displacement of the bicyclist, \(\theta = 180^\circ\); hence, on each part of the trip, \(W = (F \cos 180^\circ)s = -Fs\). The work done by the resistive force during the round trip is the algebraic sum of the work done during each portion of the trip.

**SOLUTION**

a. The work done during the round trip is, therefore,

\[ W_{\text{total}} = W_1 + W_2 = -F_1s_1 - F_2s_2 \]

\[ = -(3.0 \text{ N})(5.0 \times 10^3 \text{ m}) - (3.0 \text{ N})(5.0 \times 10^3 \text{ m}) = -3.0 \times 10^4 \text{ J} \]

b. Since the work done by the resistive force over the closed path is not zero, we can conclude that the resistive force is not a conservative force.

34. **REASONING** The gravitational force on Rocket Man is conservative, but the force generated by the propulsion unit is nonconservative. Therefore, the work \(W_p\) done by the propulsive force is equal to the net work done by all external nonconservative forces: \(W_p = W_{nc}\). We will use \(W_{nc} = E_f - E_0\) (Equation 6.8) to calculate \(W_{nc}\) in terms of Rocket Man’s initial mechanical energy and final mechanical energy. Because Rocket Man’s speed
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and height increase after he leaves the ground, both his kinetic energy $KE = \frac{1}{2}mv^2$ (Equation 6.2) and potential energy $PE = mgh$ (Equation 6.5) increase. Therefore, his final mechanical energy $E_f = KE_f + PE_f$ is greater than his initial mechanical energy $E_0 = KE_0 + PE_0$, and the work $W_p = W_{nc} = E_f - E_0$ done by the propulsive force is positive.

**SOLUTION** We make the simplifying assumption that Rocket Man’s initial height is $h_0 = 0$ m. Thus, his initial potential energy $PE_0 = mgh_0$ is zero, and his initial kinetic energy $KE_0 = \frac{1}{2}mv_0^2$ is also zero (because he starts from rest). This means he has no initial mechanical energy, and we see that the work done by the propulsive force is equal to his final mechanical energy: $W_p = E_f - 0 = E_f$. Expressing Rocket Man’s final mechanical energy $E_f$ in terms of his final kinetic energy and his final potential energy, we obtain the work done by the propulsive force:

$$W_p = \frac{1}{2}mv_f^2 + mgh_f$$

$$= \frac{1}{2}(136 \text{ kg})(5.0 \text{ m/s})^2 + (136 \text{ kg})(9.80 \text{ m/s}^2)(16 \text{ m}) = +2.3 \times 10^4 \text{ J}$$

35. **SSM REASONING AND SOLUTION**

a. The work done by non-conservative forces is given by Equation 6.7b as

$$W_{nc} = \Delta KE + \Delta PE \quad \text{so} \quad \Delta PE = W_{nc} - \Delta KE$$

Now

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(55.0 \text{ kg})[(6.00 \text{ m/s})^2 - (1.80 \text{ m/s})^2] = 901 \text{ J}$$

and

$$\Delta PE = 80.0 \text{ J} - 265 \text{ J} - 901 \text{ J} = -1086 \text{ J}$$

b. $\Delta PE = mg(h - h_0)$ so

$$h - h_0 = \frac{\Delta PE}{mg} = \frac{-1086 \text{ J}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = -2.01 \text{ m}$$

This answer is negative, so that the skater’s vertical position has changed by $2.01 \text{ m}$ and the skater is below the starting point.

36. **REASONING** The girl’s gravitational potential energy $PE = mgh$ (Equation 6.5) increases as she rises. This increase comes at the expense of her kinetic energy $KE = \frac{1}{2}mv^2$ (Equation 6.2), which decreases as she rises. Because air resistance is negligible, all of the
kinetic energy she loses is transformed into potential energy, so that her total mechanical
energy \( E = KE + PE \) remains constant. We will use this principle to determine the distance
she rises during this interval.

**SOLUTION** The conservation principle gives

\[
\frac{KE_f + PE_f}{E_f} = \frac{KE_0 + PE_0}{E_0}
\]

or

\[
PE_f - PE_0 = KE_0 - KE_f
\]

or

\[
mg(h_f - h_0) = KE_0 - KE_f
\]

Solving this expression for \( h_f - h_0 \), which is the distance she rises, we obtain

\[
h_f - h_0 = \frac{KE_0 - KE_f}{mg} = \frac{440 \text{ J} - 210 \text{ J}}{(35 \text{ kg})(9.80 \text{ m/s}^2)} = 0.67 \text{ m}
\]

37. **SSM REASONING** The only two forces that act on the gymnast are his weight and the
force exerted on his hands by the high bar. The latter is the (non-conservative) reaction
force to the force exerted on the bar by the gymnast, as predicted by Newton's third law.
This force, however, does no work because it points perpendicular to the circular path of
motion. Thus, \( W_{nc} = 0 \) J, and we can apply the principle of conservation of mechanical
energy.

**SOLUTION** The conservation principle gives

\[
\frac{\frac{1}{2}mv_f^2 + mgh_f}{E_f} = \frac{\frac{1}{2}mv_0^2 + mgh_0}{E_0}
\]

Since the gymnast's speed is momentarily zero at the top of the swing, \( v_0 = 0 \) m/s. If we
take \( h_f = 0 \) m at the bottom of the swing, then \( h_0 = 2r \), where \( r \) is the radius of the circular
path followed by the gymnast's waist. Making these substitutions in the above expression
and solving for \( v_f \), we obtain

\[
v_f = \sqrt{2gh_0} = \sqrt{2g(2r)} = \sqrt{2(9.80 \text{ m/s}^2)(2 \times 1.1 \text{ m})} = 6.6 \text{ m/s}
\]

38. **REASONING** The distance \( h \) in the drawing in the text is the difference between the
skateboarder’s final and initial heights (measured, for example, with respect to the ground),
or \( h = h_f - h_0 \). The difference in the heights can be determined by using the conservation of
mechanical energy. This conservation law is applicable because nonconservative forces are
negligible, so the work done by them is zero (\( W_{nc} = 0 \) J). Thus, the skateboarder’s final total
mechanical energy \( E_f \) is equal to his initial total mechanical energy \( E_0 \):

\[
\frac{\frac{1}{2}mv_f^2 + mgh_f}{E_f} = \frac{\frac{1}{2}mv_0^2 + mgh_0}{E_0}
\]

(6.9b)
Solving Equation 6.9b for \( h_f - h_0 \), we find that
\[
\frac{h_f - h_0}{h} = \frac{\frac{1}{2} v_0^2 - \frac{1}{2} v_f^2}{g}
\]

**SOLUTION** Using the fact that \( v_0 = 5.4 \text{ m/s} \) and \( v_f = 0 \text{ m/s} \) (since the skateboarder comes to a momentary rest), the distance \( h \) is
\[
h = \frac{\frac{1}{2} v_0^2 - \frac{1}{2} v_f^2}{g} = \frac{\frac{1}{2} (5.4 \text{ m/s})^2 - \frac{1}{2} (0 \text{ m/s})^2}{9.80 \text{ m/s}^2} = 1.5 \text{ m}
\]

39. **REASONING** We can find the landing speed \( v_f \) from the final kinetic energy \( KE_f \), since \( KE_f = \frac{1}{2} m v_f^2 \). Furthermore, we are ignoring air resistance, so the conservation of mechanical energy applies. This principle states that the mechanical energy with which the pebble is initially launched is also the mechanical energy with which it finally strikes the ground. Mechanical energy is kinetic energy plus gravitational potential energy. With respect to potential energy we will use ground level as the zero level for measuring heights. The pebble initially has kinetic and potential energies. When it strikes the ground, however, the pebble has only kinetic energy, its potential energy having been converted into kinetic energy. It does not matter how the pebble is launched, because only the vertical height determines the gravitational potential energy. Thus, in each of the three parts of the problem the same amount of potential energy is converted into kinetic energy. As a result, the speed that we will calculate from the final kinetic energy will be the same in parts (a), (b), and (c).

**SOLUTION**

a. Applying the conservation of mechanical energy in the form of Equation 6.9b, we have
\[
\frac{\frac{1}{2} m v_f^2 + mgh_f}{\text{Final mechanical energy at ground level}} = \frac{\frac{1}{2} m v_0^2 + mgh_0}{\text{Initial mechanical energy at top of building}}
\]
Solving for the final speed gives
\[
v_f = \sqrt{v_0^2 + 2g (h_0 - h_f)} = \sqrt{(14.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)[(31.0 \text{ m}) - (0 \text{ m})]} = 28.3 \text{ m/s}
\]

b. The calculation and answer are the same as in part (a).

c. The calculation and answer are the same as in part (a).

40. **REASONING**
a. and b. Since there is no friction, there are only two forces acting on each box, the gravitational force (its weight) and the normal force. The normal force, being perpendicular to the displacement of a box, does no work. The gravitational force does work. However, it
is a conservative force, so the principle of conservation of mechanical energy applies to the motion of each box, and we can use it to determine the speed in each case. This principle reveals that the final speed depends only on the initial speed, and the initial and final heights. The final speed does not depend on the mass of the box or on the steepness of the slope. Thus, both boxes reach B with the same speed.

c. The kinetic energy is given by the expression $KE = \frac{1}{2}mv^2$ (Equation 6.2). Both boxes have the same speed $v$ when they reach B. However, the heavier box has the greater mass. Therefore, the heavier box has the greater kinetic energy.

**SOLUTION**

a. The conservation of mechanical energy states that

$$\frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_A^2 + mgh_A$$  \hspace{1cm} (6.9b)

Solving for the speed $v_B$ at B gives

$$v_B = \sqrt{v_A^2 + 2g(h_A - h_B)} = \sqrt{(0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(4.5 \text{ m} - 1.5 \text{ m})} = 7.7 \text{ m/s}$$

b. The speed of the heavier box is the same as that of the lighter box: $7.7 \text{ m/s}$.

c. The ratio of the kinetic energies is

$$\frac{(KE_B)_{\text{heavier box}}}{(KE_B)_{\text{lighter box}}} = \frac{\left(\frac{1}{2}mv_B^2\right)_{\text{heavier box}}}{\left(\frac{1}{2}mv_B^2\right)_{\text{lighter box}}} = \frac{m_{\text{heavier box}}}{m_{\text{lighter box}}} = \frac{44 \text{ kg}}{11 \text{ kg}} = 4.0$$

41. **REASONING** Since air resistance is being neglected, the only force that acts on the golf ball is the conservative gravitational force (its weight). Since the maximum height of the trajectory and the initial speed of the ball are known, the conservation of mechanical energy can be used to find the kinetic energy of the ball at the top of the highest point. The conservation of mechanical energy can also be used to find the speed of the ball when it is 8.0 m below its highest point.

**SOLUTION**

a. The conservation of mechanical energy, Equation 6.9b, states that

$$\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0$$

Solving this equation for the final kinetic energy, $KE_f$, yields
\[ KE_f = \frac{1}{2}mv_f^2 + mg(h_0 - h_f) \]
\[ = \frac{1}{2} (0.0470 \text{ kg})(52.0 \text{ m/s})^2 + (0.0470 \text{ kg})(9.80 \text{ m/s}^2)(0 \text{ m} - 24.6 \text{ m}) = 52.2 \text{ J} \]

b. The conservation of mechanical energy, Equation 6.9b, states that
\[ \frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0 \]

The mass \( m \) can be eliminated algebraically from this equation, since it appears as a factor in every term. Solving for \( v_f \) and noting that the final height is \( h_f = 24.6 \text{ m} - 8.0 \text{ m} = 16.6 \text{ m} \), we have that
\[ v_f = \sqrt{v_0^2 + 2g(h_0 - h_f)} = \sqrt{(52.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0 \text{ m} - 16.6 \text{ m})} = 48.8 \text{ m/s} \]

42. **REASONING** If air resistance is ignored, the only nonconservative force that acts on the person is the tension in the rope. However, since the tension always points perpendicular to the circular path of the motion, it does no work, and the principle of conservation of mechanical energy applies.

**SOLUTION** Let the initial position of the person be at the top of the cliff, a distance \( h_0 \) above the water. Then, applying the conservation principle to path 2, we have
\[ \frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0 \]

Solving this relation for the speed at the release point, we have
\[ v_0 = \sqrt{v_f^2 + 2g(h_f - h_0)} \]

But \( v_f = 13.0 \text{ m/s} \), since the speed of entry is the same for either path. Therefore,
\[ v_0 = \sqrt{v_f^2 + 2g(h_f - h_0)} = \sqrt{(13.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(-5.20 \text{ m})} = 8.2 \text{ m/s} \]

43. **SSM REASONING** To find the maximum height \( H \) above the end of the track we will analyze the projectile motion of the skateboarder after she leaves the track. For this analysis we will use the principle of conservation of mechanical energy, which applies because friction and air resistance are being ignored. In applying this principle to the projectile motion, however, we will need to know the speed of the skateboarder when she leaves the
track. Therefore, we will begin by determining this speed, also using the conservation principle in the process. Our approach, then, uses the conservation principle twice.

**SOLUTION** Applying the conservation of mechanical energy in the form of Equation 6.9b, we have

\[
\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0
\]

We designate the flat portion of the track as having a height \( h_0 = 0 \) m and note from the drawing that its end is at a height of \( h_f = 0.40 \) m above the ground. Solving for the final speed at the end of the track gives

\[
v_f = \sqrt{v_0^2 + 2g(h_0 - h_f)} = \sqrt{(5.4 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0 \text{ m} - (0.40 \text{ m})} = 4.6 \text{ m/s}
\]

This speed now becomes the initial speed \( v_0 = 4.6 \text{ m/s} \) for the next application of the conservation principle. At the maximum height of her trajectory she is traveling horizontally with a speed \( v_i \) that equals the horizontal component of her launch velocity. Thus, for the next application of the conservation principle \( v_i = (4.6 \text{ m/s}) \cos 48^\circ \). Applying the conservation of mechanical energy again, we have

\[
\frac{1}{2}mv_i^2 + mg[H - (0.40 \text{ m})] = \frac{1}{2}mv_0^2 + mg(0.40 \text{ m})
\]

Recognizing that \( h_0 = 0.40 \) m and \( h_f = 0.40 \) m + \( H \) and solving for \( H \) give

\[
H = \frac{v_0^2 - v_i^2}{2g} = \frac{(4.6 \text{ m/s})^2 - [(4.6 \text{ m/s}) \cos 48^\circ]^2}{2(9.80 \text{ m/s}^2)} = 0.60 \text{ m}
\]

**REASONING** We can apply the conservation of mechanical energy to this problem. To do so, we will need the initial speed \( v_0 \) of the ball at the instant it is released and the final speed \( v_f \) at the highest point on its upward path. The speed \( v_0 \) is the speed that the ball has on its circular path. Since the ball moves three times around the circle’s circumference (which is \( 2\pi \) times the radius of the circle) in one second, its speed on the circular path is

\[
v_0 = \frac{3[2\pi(1.5 \text{ m})]}{1.0 \text{ s}} = 28.3 \text{ m/s}
\]

The speed \( v_f \) is \( v_f = 0.0 \) m/s, since the ball comes momentarily to a halt at the highest point of its upward path.
**SOLUTION** The ball is released when it is \( h_0 = 0.75 \text{m} \) above the ground and traveling vertically upward. Its initial speed is

The conservation of energy requires that

\[
\frac{1}{2} mv_f^2 + mgh_f = \frac{1}{2} mv_0^2 + mgh_0
\]

or

\[
h_f = \frac{v_0^2}{2g} + h_0
\]

where we have used the fact that \( v_f = 0.0 \text{ m/s} \). Thus, we find that the maximum height is

\[
h_f = \frac{v_0^2}{2g} + h_0 = \frac{(28.3 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} + 0.75 \text{ m} = 42 \text{ m}
\]

---

45. **SSM REASONING** Gravity is the only force acting on the vaulters, since friction and air resistance are being ignored. Therefore, the net work done by the nonconservative forces is zero, and the principle of conservation of mechanical energy holds.

**SOLUTION** Let \( E_{2f} \) represent the total mechanical energy of the second vaulter at the ground, and \( E_{20} \) represent the total mechanical energy of the second vaulter at the bar. Then, the principle of mechanical energy is written for the second vaulter as

\[
\frac{1}{2} mv_{2f}^2 + mgh_{2f} = \frac{1}{2} mv_{20}^2 + mgh_{20}
\]

Since the mass \( m \) of the vaulter appears in every term of the equation, \( m \) can be eliminated algebraically. The quantity \( h_{20} \) is \( h_{20} = h \), where \( h \) is the height of the bar. Furthermore, when the vaulter is at ground level, \( h_{2f} = 0 \text{ m} \). Solving for \( v_{20} \) we have

\[
v_{20} = \sqrt{v_{2f}^2 - 2gh} \tag{1}
\]

In order to use Equation (1), we must first determine the height \( h \) of the bar. The height \( h \) can be determined by applying the principle of conservation of mechanical energy to the first vaulter on the ground and at the bar. Using notation similar to that above, we have

\[
\frac{1}{2} mv_{1f}^2 + mgh_{1f} = \frac{1}{2} mv_{10}^2 + mgh_{10}
\]

Where \( E_{10} \) corresponds to the total mechanical energy of the first vaulter at the bar. The height of the bar is, therefore,

\[
h = h_{10} = \frac{v_{1f}^2 - v_{10}^2}{2g} = \frac{(8.90 \text{ m/s})^2 - (1.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 3.99 \text{ m}
\]
The speed at which the second vaulter clears the bar is, from Equation (1),
\[ v_{20} = \sqrt{\frac{v_{2f}^2}{2g} - 2gh} = \sqrt{(9.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(3.99 \text{ m})} = 1.7 \text{ m/s} \]

46. **REASONING**  The drawing at the right shows the object (mass \( m \)) in its initial position on the vertical string of length \( L \). The object’s initial speed is \( v_0 \). The drawing also shows the object in its final position, where the string makes an angle \( \theta \) with the vertical and the object is a vertical distance \( H \) above its starting point. In this position the object’s final speed is \( v_f \). To determine \( \theta \) we will use the cosine function \( \cos \theta = \frac{L-H}{L} \). To do so, however, we will need a value for the distance \( H \). To obtain this value we will use the conservation of mechanical energy.

**SOLUTION** Using the cosine function, we have
\[
\cos \theta = \frac{L-H}{L} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{L-H}{L}\right) \quad (1)
\]

The conservation of mechanical energy (Equation 6.9b) specifies that
\[
\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0 \quad \text{or} \quad \frac{1}{2}mv_f^2 + mg(h_f - h_0) = \frac{1}{2}mv_0^2 \quad (2)
\]

In this expression we know that \( v_f = 0.0 \text{ m/s} \) since the object comes to a momentary halt in its final position. We also know (see the drawing) that \( h_f - h_0 = H \). Thus, Equation (2) becomes
\[
mgH = \frac{1}{2}mv_0^2 \quad \text{or} \quad H = \frac{v_0^2}{2g} = \frac{(2.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.20 \text{ m}
\]

With this value for \( H \) we can use Equation (1) to determine the angle \( \theta \):
\[
\theta = \cos^{-1}\left(\frac{L-H}{L}\right) = \cos^{-1}\left[\frac{(0.75 \text{ m})-(0.20 \text{ m})}{0.75 \text{ m}}\right] = 43^\circ
\]

47. **REASONING**  We are neglecting air resistance and friction, which eliminates two nonconservative forces from consideration. Another nonconservative force acting on the semitrailer is the normal force exerted by the ramp. But this force does no work because it is perpendicular to the ramp and the semitrailer’s velocity. Hence, the semi-trailer’s total
mechanical energy is conserved, and we will apply the energy conservation principle to
determine the initial velocity \( v_0 \) of the truck.

**SOLUTION** The truck coasts to a stop, so we know that \( v_f = 0 \) m/s. Therefore, the energy conservation principle gives

\[
\frac{1}{2} m v_f^2 + mgh_f = \frac{1}{2} m v_0^2 + mgh_0
\]

or \( mgh_f = \frac{1}{2} m v_0^2 \) or \( g(h_f - h_0) = \frac{1}{2} v_0^2 \) (1)

Note that \( h_f - h_0 \) is the vertical height up which the truck climbs on the ramp, as the drawing indicates. The drawing also indicates that \( h_f - h_0 \) is related to the distance \( d \) the truck coasts along the ramp and the ramp’s angle \( \theta \) of inclination. The side \( h_f - h_0 \) is opposite the angle \( \theta \), and \( d \) is the hypotenuse, so we solve \( \sin \theta = \frac{h_f - h_0}{d} \) (Equation 1.1) to find that \( h_f - h_0 = d \sin \theta \).

Substituting \( d \sin \theta \) for \( h_f - h_0 \) in Equation (1), we obtain

\[ gd \sin \theta = \frac{1}{2} v_0^2 \quad \text{or} \quad v_0^2 = 2gd \sin \theta \quad \text{or} \quad v_0 = \sqrt{2gd \sin \theta} \]

When the semitrailer enters the ramp, therefore, its speed is

\[ v_0 = \sqrt{2\left(9.80 \text{ m/s}^2\right)(154 \text{ m}) \sin 14.0^\circ} = 27.0 \text{ m/s} \]

48. **REASONING** The principle of conservation of mechanical energy provides the solution to both parts of this problem. It applies in both cases because the tracks are frictionless (and we assume that air resistance is negligible).

When the block on the longer track reaches its maximum height, its final total mechanical energy consists only of gravitational potential energy, because it comes to a momentary halt at this point. Thus, its final speed is zero, and, as a result, its kinetic energy is also zero. In contrast, when the block on the shorter track reaches the top of its trajectory after leaving the track, its final total mechanical energy consists of part kinetic and part potential energy. At the top of the trajectory the block is still moving horizontally, with a velocity that equals the horizontal velocity component that it had when it left the track (assuming that air resistance is negligible). Thus, at the top of the trajectory the block has some kinetic energy.

We can expect that the height \( H \) is greater than the height \( H_1 + H_2 \). To see why, note that both blocks have the same initial kinetic energy, since each has the same initial speed. Moreover, the total mechanical energy is conserved. Therefore, the initial kinetic energy is converted to potential energy as each block moves upward. On the longer track, the final total mechanical energy is all potential energy, whereas on the shorter track it is only part potential energy, as discussed previously. Since gravitational potential energy is
proportional to height, the final height is greater for the block with the greater potential energy, or the one on the longer track.

**SOLUTION**  The two inclines in the problem are as follows:

![Diagram of blocks on inclines](image)

a. **Longer track:** Applying the principle of conservation of mechanical energy for the longer track gives

\[
\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0
\]

Recognizing that \(v_f = 0\) m/s, \(h_f = H\), and \(h_0 = 0\) m, we find that

\[
\frac{1}{2}m(0 \text{ m/s})^2 + mgH = \frac{1}{2}mv_0^2 + mg(0 \text{ m})
\]

\[
H = \frac{v_0^2}{2g} = \frac{(7.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 2.50 \text{ m}
\]

b. **Shorter track:** We apply the conservation principle to the motion of the block after it leaves the track, in order to calculate the final height at the top of the trajectory, where \(h_f = H_1 + H_2\) and the final speed is \(v_f\). For the initial position at the end of the track we have \(h_0 = H_1\) and an initial speed of \(v_{0T}\). Thus, it follows that

\[
\frac{1}{2}mv_f^2 + mg(H_1 + H_2) = \frac{1}{2}mv_{0T}^2 + mgH_1
\]

Solving for \(H_1 + H_2\) gives

\[
H_1 + H_2 = \frac{v_{0T}^2 + 2gH_1 - v_f^2}{2g}
\]

At the top of its trajectory the block is moving horizontally with a velocity that equals the horizontal component of the velocity with which it left the track, that is, \(v_f = v_{0T} \cos \theta\). Thus, a value for \(v_{0T}\) is needed and can be obtained by applying the conservation principle to the motion of the block that takes place on the track.
For the motion on the track the final speed is the speed \( v_{0T} \) at which the block leaves the track, and the final height is \( h_f = H_1 = 1.25 \text{ m} \). The initial speed is the given value of \( v_0 = 7.00 \text{ m/s} \), and the initial height is \( h_0 = 0 \text{ m} \). The conservation principle reveals that

\[
\frac{1}{2}mv^2_{0T} + mgH_1 = \frac{1}{2}mv^2_0 + mg(0 \text{ m})
\]

Final total mechanical energy  

\[
v_{0T} = \sqrt{v^2_0 - 2gH_1} = \sqrt{(7.00 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(1.25 \text{ m})} = 4.95 \text{ m/s}
\]

For use in Equation (1) we find, then, that the speed with which the block leaves the track is \( v_{0T} = 4.95 \text{ m/s} \) and the final speed at the top of the trajectory is \( v_f = v_{0T} \cos \theta = (4.95 \text{ m/s}) \cos \theta \). In addition, \( h_f = H_1 + H_2 \) at the top of the trajectory, and \( h_0 = H_1 = 1.25 \text{ m} \) for the initial position at the end of the track. Thus, Equation (1) reveals that

\[
H_1 + H_2 = \frac{v^2_{0T} + 2gH_1 - v^2_f}{2g} = \frac{(4.95 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(1.25 \text{ m}) - [(4.95 \text{ m/s}) \cos 50.0^\circ]^2}{2(9.80 \text{ m/s}^2)} = 1.98 \text{ m}
\]

As expected, \( H_1 + H_2 \) is less than \( H \), and the block rises to a greater height on the longer track.

49. **REASONING** If air resistance is ignored, the only nonconservative force that acts on the skier is the normal force exerted on the skier by the snow. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved. Our solution will be based on this fact.

**SOLUTION** The conservation of mechanical energy (Equation 6.9b) specifies that

\[
\frac{1}{2}mv^2_0 + mgh_0 = \frac{1}{2}mv^2_f + mgh_f
\]

Since the skier starts from rest \( v_0 = 0 \text{ m/s} \). Let \( h_f \) define the zero level for heights, then the final gravitational potential energy is zero. This gives

\[
mgh_0 = \frac{1}{2}mv^2_f
\]

(1)
At the crest of the second hill, the two forces that act on the skier are the normal force and the weight of the skier. The resultant of these two forces provides the necessary centripetal force to keep the skier moving along the circular arc of the hill. When the skier just loses contact with the snow, the normal force is zero and the weight of the skier must provide the necessary centripetal force.

\[ mg = \frac{mv_f^2}{r} \]  

so that \( v_f^2 = gr \)  

(2)

Substituting this expression for \( v_f^2 \) into Equation (1) gives

\[ mgh_0 = \frac{1}{2}mgr \]

or

\[ h_0 = \frac{r}{2} = \frac{36 \text{ m}}{2} = 18 \text{ m} \]

50. **REASONING AND SOLUTION** If air resistance is ignored, the only nonconservative force that acts on the person is the normal force exerted on the person by the surface. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved.

\[ \frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f \]

(1)

where the final state pertains to the position where the person leaves the surface. Since the person starts from rest \( v_0 = 0 \text{ m/s} \). Since the radius of the surface is \( r \), \( h_0 = r \), and \( h_f = r \cos \theta_f \) where \( \theta_f \) is the angle at which the person leaves the surface. Equation (1) becomes

\[ mgr = \frac{1}{2}mv_f^2 + mg \left( r \cos \theta_f \right) \]

(2)

In general, as the person slides down the surface, the two forces that act on him are the normal force \( F_N \) and the weight \( mg \). The centripetal force required to keep the person moving in the circular path is the resultant of \( F_N \) and the radial component of the weight, \( mg \cos \theta \).

When the person leaves the surface, the normal force is zero, and the radial component of the weight provides the centripetal force.

\[ mg \cos \theta_f = \frac{mv_f^2}{r} \]

\[ v_f^2 = gr \cos \theta_f \]

(3)
Substituting this expression for $v_f^2$ into Equation (2) gives

$$mgr = \frac{1}{2} mg (r \cos \theta_f) + mg (r \cos \theta_f)$$

Solving for $\theta_f$ gives

$$\theta_f = \cos^{-1}\left(\frac{2}{3}\right) = 48^\circ$$

51. **SSM REASONING**

a. Since there is no air friction, the only force that acts on the projectile is the conservative gravitational force (its weight). The initial and final speeds of the ball are known, so the conservation of mechanical energy can be used to find the maximum height that the projectile attains.

b. When air resistance, a nonconservative force, is present, it does negative work on the projectile and slows it down. Consequently, the projectile does not rise as high as when there is no air resistance. The work-energy theorem, in the form of Equation 6.6, may be used to find the work done by air friction. Then, using the definition of work, Equation 6.1, the average force due to air resistance can be found.

**SOLUTION**

a. The conservation of mechanical energy, as expressed by Equation 6.9b, states that

$$E_f = E_0$$

$$\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0$$

The mass $m$ can be eliminated algebraically from this equation since it appears as a factor in every term. Solving for the final height $h_f$ gives

$$h_f = \frac{1}{2}\left(v_0^2 - v_f^2\right) + h_0$$

Setting $h_0 = 0$ m and $v_f = 0$ m/s, the final height, in the absence of air resistance, is

$$h_f = \frac{v_0^2}{2g} = \frac{(18.0 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 16.5 \text{ m}$$

b. The work-energy theorem is

$$W_{nc} = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)$$

(6.6)
where $W_{nc}$ is the nonconservative work done by air resistance. According to Equation 6.1, the work can be written as $W_{nc} = (\overline{F_R} \cos 180^\circ) s$, where $\overline{F_R}$ is the average force of air resistance. As the projectile moves upward, the force of air resistance is directed downward, so the angle between the two vectors is $\theta = 180^\circ$ and $\cos \theta = -1$. The magnitude $s$ of the displacement is the difference between the final and initial heights, $s = h_f - h_0 = 11.8$ m. With these substitutions, the work-energy theorem becomes

$$-\overline{F_R} s = \frac{1}{2} m (v_f^2 - v_o^2) + mg (h_f - h_0)$$

Solving for $\overline{F_R}$ gives

$$\overline{F_R} = \frac{\frac{1}{2} m (v_f^2 - v_o^2) + mg (h_f - h_0)}{-s}$$

$$= \frac{\frac{1}{2} (0.750 \text{ kg})(0 \text{ m/s})^2 - (18.0 \text{ m/s})^2 + (0.750 \text{ kg})(9.80 \text{ m/s}^2)(11.8 \text{ m})}{-(11.8 \text{ m})} = 2.9 \text{ N}$$

52. **REASONING** The ball’s initial speed and vertical position are given, as are the ball’s final speed and vertical position. Therefore, we can use the work-energy theorem as given in Equation 6.8 to determine the work done by the nonconservative force of air resistance.

**SOLUTION** According to Equation 6.8, the work $W_{nc}$ done by the nonconservative force of air resistance is

$$W_{nc} = \left(\frac{1}{2} m v_f^2 + mgh_f\right) - \left(\frac{1}{2} m v_o^2 + mgh_0\right)$$

Thus, we find that

$$W_{nc} = \left[\frac{1}{2} (0.600 \text{ kg})(4.20 \text{ m/s})^2 + (0.600 \text{ kg})(9.80 \text{ m/s}^2)(3.10 \text{ m})\right] - \left[\frac{1}{2} (0.600 \text{ kg})(7.20 \text{ m/s})^2 + (0.600 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})\right] = -3.8 \text{ J}$$

53. **REASONING** As the firefighter slides down the pole from a height $h_0$ to the ground ($h_f = 0$ m), his potential energy decreases to zero. At the same time, his kinetic energy increases as he speeds up from rest ($v_0 = 0$ m/s) to a final speed $v_f$. However, the upward nonconservative force of kinetic friction $f_k$, acting over a downward displacement $h_0$, does a negative amount of work on him: $W_{nc} = \left(\overline{f_k} \cos 180^\circ\right) h_0 = -f_k h_0$ (Equation 6.1). This work
decreases his total mechanical energy $E$. Applying the work-energy theorem (Equation 6.8), with $h_f = 0$ m and $v_0 = 0$ m/s, we obtain

$$W_{nc} = \left(\frac{1}{2}mv_f^2 + mgh_f\right) - \left(\frac{1}{2}mv_0^2 + mgh_0\right)$$  \hspace{1cm} (6.8)

$$-f_k h_0 = \left(\frac{1}{2}mv_f^2 + 0\right) - \left(0 + mgh_0\right) = \frac{1}{2}mv_f^2 - mgh_0$$  \hspace{1cm} (1)

**SOLUTION** Solving Equation (1) for the height $h_0$ gives

$$mgh_0 - f_k h_0 = \frac{1}{2}mv_f^2 \quad \text{or} \quad h_0 (mg - f_k) = \frac{1}{2}mv_f^2 \quad \text{or} \quad h_0 = \frac{mv_f^2}{2(mg - f_k)}$$

The distance $h_0$ that the firefighter slides down the pole is, therefore,

$$h_0 = \frac{(93 \text{ kg})(3.4 \text{ m/s})^2}{2\left[(93 \text{ kg})(9.80 \text{ m/s}^2) - 810 \text{ N}\right]} = 5.3 \text{ m}$$

54. **REASONING** We will use the work-energy theorem $W_{nc} = E_f - E_0$ (Equation 6.8) to find the speed of the student. $W_{nc}$ is the work done by the kinetic frictional force and is negative because the force is directed opposite to the displacement of the student.

**SOLUTION** The work-energy theorem states that

$$W_{nc} = \left(\frac{1}{2}mv_f^2 + mgh_f\right) - \left(\frac{1}{2}mv_0^2 + mgh_0\right)$$  \hspace{1cm} (6.8)

Solving for the final speed gives

$$v_f = \sqrt{\frac{2W_{nc}}{m} + v_0^2 - 2g(h_f - h_0)}$$

$$= \sqrt{\frac{2(-6.50 \times 10^3 \text{ J})}{83.0 \text{ kg}} + (0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(-11.8 \text{ m})} = 8.6 \text{ m/s}$$

55. **SSM REASONING** The work-energy theorem can be used to determine the net work done on the car by the nonconservative forces of friction and air resistance. The work-energy theorem is, according to Equation 6.8,
\[ W_{nc} = \left( \frac{1}{2} mv_f^2 + mgh_f \right) - \left( \frac{1}{2} mv_0^2 + mgh_0 \right) \]

The nonconservative forces are the force of friction, the force of air resistance, and the force provided by the engine. Thus, we can rewrite Equation 6.8 as

\[ W_{\text{friction}} + W_{\text{air}} + W_{\text{engine}} = \left( \frac{1}{2} mv_f^2 + mgh_f \right) - \left( \frac{1}{2} mv_0^2 + mgh_0 \right) \]

This expression can be solved for \( W_{\text{friction}} + W_{\text{air}} \).

**SOLUTION** We will measure the heights from sea level, where \( h_0 = 0 \) m. Since the car starts from rest, \( v_0 = 0 \) m/s. Thus, we have

\[
W_{\text{friction}} + W_{\text{air}} = m \left( \frac{1}{2} v_f^2 + gh_f \right) - W_{\text{engine}} \\
= (1.50 \times 10^3 \text{ kg}) \left[ \frac{1}{2} (27.0 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(2.00 \times 10^2 \text{ m}) \right] - (4.70 \times 10^6 \text{ J}) \\
= -1.21 \times 10^6 \text{ J}
\]

56. **REASONING** According to Equation 6.6, the work \( W_{nc} \) done by nonconservative forces, such as kinetic friction and air resistance, is equal to the object’s change in kinetic energy, \( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \), plus the change in its potential energy, \( mgh_f - mgh_0 \):

\[
W_{nc} = \left( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \right) + (mgh_f - mgh_0) \tag{6.6}
\]

This relation will be used in both parts (a) and (b) of the problem.

**SOLUTION**

a. Since nonconservative forces are absent, the work done by them is zero, so \( W_{nc} = 0 \) J. In this case, we can algebraically rearrange Equation 6.6 to show that the initial total mechanical energy (kinetic energy plus potential energy) \( E_0 \) at the top is equal to the final total mechanical energy \( E_f \) at the bottom:

\[
\frac{1}{2} mv_f^2 + mgh_f = \frac{1}{2} mv_0^2 + mgh_0
\]

Solving for the final speed \( v_f \) of the skeleton and rider gives

\[
v_f = \sqrt{v_0^2 + 2g(h_0 - h_f)}
\]

The initial speed of the rider is \( v_0 = 0 \) m/s, and the drop in height is \( h_0 - h_f = 126 \text{ m} \), so the final speed at the bottom of the run is

\[
v_f = \sqrt{0^2 + 2(9.80 \text{ m/s}^2)(126 \text{ m})} = 49.7 \text{ m/s}
\]
b. The work $W_{nc}$ done by the nonconservative forces follows directly from Equation 6.6:

$$W_{nc} = \frac{1}{2} m (v_f^2 - v_0^2) + mg (h_f - h_0)$$

$$= \frac{1}{2} (118 \text{ kg}) [(40.5 \text{ m/s})^2 - (0 \text{ m/s})^2] + (118 \text{ kg})(9.80 \text{ m/s}^2)(-126 \text{ m}) = -4.89 \times 10^4 \text{ J}$$

Note that the difference in heights, $h_f - h_0 = -126 \text{ m}$, is a negative number because the final height $h_f$ is less than the initial height $h_0$.

57. **REASONING** When launched with the minimum initial speed $v_0$, the puck has just enough initial kinetic energy to reach the teammate with a final speed $v_f$ of zero. All of the initial kinetic energy has served the purpose of compensating for the work done by friction in opposing the motion. Friction is a nonconservative force, so to determine $v_0$ we will use the work-energy theorem in the form of Equation 6.8: $W_{nc} = \left( \frac{1}{2} m v_f^2 + mgh_f \right) - \left( \frac{1}{2} m v_0^2 + mgh_0 \right)$. $W_{nc}$ is the work done by the net nonconservative force, which, in this case, is just the kinetic frictional force. This work can be expressed using Equation 6.1 as $W_{nc} = (f_k \cos \theta) s$, where $f_k$ is the magnitude of the kinetic frictional force and $s$ is the magnitude of the displacement, or the distance between the players. We are given neither the frictional force nor the distance between the players, but we are given the initial speed that enables the puck to travel halfway. This information will be used to evaluate $W_{nc}$.

**SOLUTION** Noting that a hockey rink is flat ($h_f = h_0$), we write Equation 6.8 as follows

$$W_{nc} = \left( \frac{1}{2} m v_f^2 + mgh_f \right) - \left( \frac{1}{2} m v_0^2 + mgh_0 \right) = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 = -\frac{1}{2} mv_0^2$$

where we have used the fact that the final speed is $v_f = 0 \text{ m/s}$ when the initial speed $v_0$ has its minimum value. According to Equation 6.1 the work done on the puck by the kinetic frictional force is

$$W_{nc} = (f_k \cos \theta) s = (f_k \cos 180^\circ) s = -f_k s$$

In this expression the angle $\theta$ is $180^\circ$, because the frictional force points opposite to the displacement. With this substitution Equation (1) becomes

$$-f_k s = -\frac{1}{2} mv_0^2 \quad \text{or} \quad f_k s = \frac{1}{2} mv_0^2$$

To evaluate the term $f_k s$ we note that Equation (2) also applies to the failed attempt at the pass, so that

$$f_k \left( \frac{1}{2} s \right) = \frac{1}{2} m (1.7 \text{ m/s})^2 \quad \text{or} \quad f_k s = m (1.7 \text{ m/s})^2$$

Substituting this result into Equation (2) gives
58. **REASONING AND SOLUTION** The force exerted by the bat on the ball is the only nonconservative force acting. The work due to this force is

\[ W_{nc} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 + mg (h_f - h_0) \]

Taking \( h_0 = 0 \) m at the level of the bat, \( v_0 = 40.0 \) m/s just before the bat strikes the ball and \( v_f \) to be the speed of the ball at \( h_f = 25.0 \) m, we have

\[
v_f = \sqrt{\frac{2W_{nc}}{m} + v_0^2 - 2gh_f} = \sqrt{\frac{2(70.0 \text{ J})}{0.140 \text{ kg}} + (40.0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(25.0 \text{ m})} = 45.9 \text{ m/s}
\]

59. **REASONING** We can determine the force exerted on the diver by the water if we first find the work done by the force. This is because the work can be expressed using Equation 6.1 as \( W_{nc} = (F \cos \theta)s \), where \( F \) is the magnitude of the force from the water and \( s \) is the magnitude of the displacement in the water. Since the force that the water exerts is nonconservative, we have included the subscript “nc” in labeling the work. To calculate the work we will use the work-energy theorem in the form of Equation 6.8:

\[
W_{nc} = (\frac{1}{2}mv_f^2 + mgh_f) - (\frac{1}{2}mv_0^2 + mgh_0)
\]

In this theorem \( W_{nc} \) is the work done by the net nonconservative force, which, in this case, is just the force exerted by the water.

**SOLUTION** We write Equation 6.8 as follows:

\[
W_{nc} = \underbrace{\left( \frac{1}{2}mv_f^2 + mgh_f \right)}_{\text{Final mechanical energy}} - \underbrace{\left( \frac{1}{2}mv_0^2 + mgh_0 \right)}_{\text{Initial mechanical energy}} = mgh_f - mgh_0
\]

where we have used the fact that the diver is at rest initially and finally, so \( v_0 = v_f = 0 \) m/s. According to Equation 6.1 the work done on the diver by the force of the water is

\[
W_{nc} = (F \cos \theta)s = (F \cos 180^\circ)s = -Fs
\]

The angle \( \theta \) between the force of the water and the displacement is 180°, because the force opposes the motion. Substituting this result into Equation (1) gives

\[
-Fs = mgh_f - mgh_0
\]
Identifying the final position under the water as \( h_f = 0 \) m and using upward as the positive direction, we know that the initial position on the tower must be \( h_0 = 3.00 \) m + \( 1.10 \) m = \( 4.10 \) m. Thus, solving Equation (2) for \( F \), we find that

\[
F = \frac{mg(h_0 - h_f)}{s} = \frac{(67.0 \text{ kg})(9.80 \text{ m/s}^2)[(4.10 \text{ m}) - (0 \text{ m})]}{1.10 \text{ m}} = 2450 \text{ N}
\]

60. **REASONING AND SOLUTION** According to the work-energy theorem as given in Equation 6.8, we have

\[
W_{nc} = \left(\frac{1}{2}mv_f^2 + mgh_f\right) - \left(\frac{1}{2}mv_0^2 + mgh_0\right)
\]

The metal piece starts at rest and is at rest just as it barely strikes the bell, so that \( v_f = v_0 = 0 \) m/s. In addition, \( h_f = h \) and \( h_0 = 0 \) m, while \( W_{nc} = 0.25\left(\frac{1}{2}Mv^2\right) \), where \( M \) and \( v \) are the mass and speed of the hammer. Thus, the work-energy theorem becomes

\[
0.25\left(\frac{1}{2}Mv^2\right) = mgh
\]

Solving for the speed of the hammer, we find

\[
v = \sqrt{\frac{2mgh}{0.25M}} = \sqrt{\frac{2(0.400 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m})}{0.25(9.00 \text{ kg})}} = 4.17 \text{ m/s}
\]

61. **SSM REASONING** After the wheels lock, the only nonconservative force acting on the truck is friction. The work done by this conservative force can be determined from the work-energy theorem. According to Equation 6.8,

\[
W_{nc} = E_f - E_0 = \left(\frac{1}{2}mv_f^2 + mgh_f\right) - \left(\frac{1}{2}mv_0^2 + mgh_0\right)
\]

where \( W_{nc} \) is the work done by friction. According to Equation 6.1, \( W = (F \cos \theta)s \); since the force of kinetic friction points opposite to the displacement, \( \theta = 180^\circ \). According to Equation 4.8, the kinetic frictional force has a magnitude of \( f_k = \mu_k F_N \), where \( \mu_k \) is the coefficient of kinetic friction and \( F_N \) is the magnitude of the normal force. Thus,

\[
W_{nc} = (f_k \cos \theta)s = \mu_k F_N (\cos 180^\circ)s = -\mu_k F_N s
\]
Since the truck is sliding down an incline, we refer to the free-body diagram in order to determine the magnitude of the normal force $F_N$. The free-body diagram at the right shows the three forces that act on the truck. They are the normal force $F_N$, the force of kinetic friction $f_k$, and the weight $mg$. The weight has been resolved into its components, and these vectors are shown as dashed arrows. From the free body diagram and the fact that the truck does not accelerate perpendicular to the incline, we can see that the magnitude of the normal force is given by

$$F_N = mg \cos 15.0^\circ$$

Therefore, Equation (2) becomes

$$W_{nc} = -\mu_k (mg \cos 15.0^\circ) s$$

Combining Equations (1), (2), and (3), we have

$$-\mu_k (mg \cos 15.0^\circ) s = \left( \frac{1}{2} m v_f^2 + mgh_f \right) - \left( \frac{1}{2} m v_0^2 + mgh_0 \right)$$

This expression may be solved for $s$.

**SOLUTION** When the car comes to a stop, $v_f = 0$ m/s. If we take $h_f = 0$ m at ground level, then, from the drawing at the right, we see that $h_0 = s \sin 15.0^\circ$. Equation (4) then gives

$$s = \frac{v_0^2}{2g \left( \mu_k \cos 15.0^\circ - \sin 15.0^\circ \right)} = \frac{(11.1 \text{ m/s})^2}{2 \left( 9.80 \text{ m/s}^2 \right) \left[ (0.750) \cos 15.0^\circ - \sin 15.0^\circ \right]} = 13.5 \text{ m}$$

62. **REASONING** The average power $\overline{P}$ is given directly as the work $W$ done divided by the time $t$ that it takes to do the work: $\overline{P} = \frac{W}{t}$ (Equation 10a). The work can be obtained by realizing that this situation is just like that of a constant force pushing an object in a straight line, the force pointing in the same direction as the displacement of the object. This is because the force is always applied parallel to the motion of the crank handle. Thus, the work can be calculated from $W = (F \cos \theta) s$ (Equation 6.1), where $s$ is the magnitude of the displacement and $\theta$ is the angle between the direction of the force and the displacement.

**SOLUTION** When the person turns the crank through one revolution, the handle moves a distance equal to the circumference of a circle of radius $r$, so that the magnitude of the
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displacement in Equation 6.1 is \( s = 2\pi r \). Thus, the work done for one revolution of the handle is

\[
W = (F \cos \theta) s = (F \cos \theta)(2\pi r)
\]

The average power, then, is

\[
\bar{P} = \frac{W}{t} = \frac{(F \cos \theta)(2\pi r)}{t}
\]

Remembering that the force is always applied parallel to the motion of the handle so that \( \theta = 0.0^\circ \), we find that the average power expended is

\[
\bar{P} = \frac{(F \cos \theta)(2\pi r)}{t} = \frac{(22 \text{ N})(\cos 0.0^\circ)(2\pi)(0.28 \text{ m})}{1.3 \text{ s}} = \frac{3.0 \times 10^4 \text{ W}}{}
\]

63. SSM REASONING The work \( W \) done is equal to the average power \( \bar{P} \) multiplied by the time \( t \), or

\[
W = \bar{P}t
\]

The average power is the average power generated per kilogram of body mass multiplied by Armstrong’s mass. The time of the race is the distance \( s \) traveled divided by the average speed \( \bar{v} \), or \( t = s / \bar{v} \) (see Equation 2.1).

SOLUTION

a. Substituting \( t = s / \bar{v} \) into Equation 6.10a gives

\[
W = \bar{P}t = \bar{P} \left( \frac{s}{\bar{v}} \right) = \left( \frac{6.50 \text{ W}}{\text{kg}} \right) \left( \frac{75.0 \text{ kg}}{\text{g}} \right) \left( \frac{135 \times 10^3 \text{ m}}{12.0 \text{ m/s}} \right) = \frac{5.48 \times 10^6 \text{ J}}{}
\]

b. Since 1 joule = 2.389 \times 10^{-4} nutritional calories, the work done is

\[
W = 5.48 \times 10^6 \text{ joules} = \left( \frac{5.48 \times 10^6 \text{ joules}}{1 \text{ joule}} \right) \left( \frac{2.389 \times 10^{-4} \text{ nutritional calories}}{1 \text{ joule}} \right) = \frac{1.31 \times 10^3 \text{ nutritional calories}}{}
\]

64. REASONING The work \( W \) done is given by Equation 6.1 as \( W = (F \cos \theta) s \), where \( F \) is the magnitude of the force that you exert on the rowing bar, \( s \) is the magnitude of the bar’s displacement, and \( \theta \) is the angle between the force and the displacement. Solving this equation for \( F \) gives
\[ F = \frac{W}{(\cos \theta)s} \]  \hspace{1cm} (6.1)

The work is also equal to the average power \( \bar{P} \) multiplied by the time \( t \), or

\[ W = \bar{P}t \]  \hspace{1cm} (6.10a)

Substituting Equation (6.10a) into (6.1) gives

\[ F = \frac{W}{(\cos \theta)s} = \frac{\bar{P}t}{(\cos \theta)s} \]

**SOLUTION** The angle \( \theta \) is 0°, since the direction of the pulling force is the same as the displacement of the rowing bar. Thus, the magnitude of the force exerted on the handle is

\[ F = \frac{\bar{P}t}{(\cos \theta)s} = \frac{(82 \text{ W})(1.5 \text{ s})}{(\cos 0°)(1.2 \text{ m})} = 1.0 \times 10^2 \text{ N} \]

---

65. **REASONING** The average power is given by Equation 6.10b (\( \bar{P} = \frac{\text{Change in energy}}{\text{Time}} \)). The time is given. Since the road is level, there is no change in the gravitational potential energy, and the change in energy refers only to the kinetic energy. According to Equation 6.2, the kinetic energy is \( \frac{1}{2}mv^2 \). The speed \( v \) is given, but the mass \( m \) is not. However, we can obtain the mass from the given weights, since the weight is \( mg \).

**SOLUTION**

a. Using Equations 6.10b and 6.2, we find that the average power is

\[ \bar{P} = \frac{\text{Change in energy}}{\text{Time}} = \frac{\text{KE}_f - \text{KE}_0}{\text{Time}} = \frac{\frac{1}{2}mv^2_f - \frac{1}{2}mv^2_0}{\text{Time}} \]

Since the car starts from rest, \( v_0 = 0 \text{ m/s} \), and since the weight is \( W = mg \), the mass is \( m = W/g \). Therefore, the average power is

\[ \bar{P} = \frac{\frac{1}{2}mv^2_f - \frac{1}{2}mv^2_0}{\text{Time}} = \frac{Wv^2_f}{2g(\text{Time})} \]  \hspace{1cm} (1)

Using the given values for the weight, final speed, and the time, we find that

\[ \bar{P} = \frac{Wv^2_f}{2g(\text{Time})} = \frac{(9.0 \times 10^3 \text{ N})(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(5.6 \text{ s})} = 3.3 \times 10^4 \text{ W (44 hp)} \]

b. From Equation (1) it follows that

\[ \bar{P} = \frac{Wv^2_f}{2g(\text{Time})} = \frac{(1.4 \times 10^4 \text{ N})(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(5.6 \text{ s})} = 5.1 \times 10^4 \text{ W (68 hp)} \]
66. **REASONING** The average power is defined as the work divided by the time, Equation 6.10a, so both the work and time must be known. The time is given. The work can be obtained with the aid of the work-energy theorem as formulated in $W_{nc} = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)$ (Equation 6.6). $W_{nc}$ is the work done by the lifting force acting on the helicopter. In using this equation, we note that two types of energy are changing: the kinetic energy $\left(\frac{1}{2}mv^2\right)$ and the gravitational potential energy $(mgh)$. The kinetic energy is increasing, because the speed of the helicopter is increasing. The gravitational potential energy is increasing, because the height of the helicopter is increasing.

**SOLUTION** The average power is

$$\bar{P} = \frac{W_{nc}}{t} \quad (6.10a)$$

where $W_{nc}$ is the work done by the nonconservative lifting force and $t$ is the time. The work is related to the helicopter’s kinetic and potential energies by Equation 6.6:

$$W_{nc} = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)$$

Thus, the average power is

$$\bar{P} = \frac{W_{nc}}{t} = \frac{\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)}{t} = \frac{\frac{1}{2}m(v_f^2 - v_0^2) + mg(h_f - h_0)}{t}$$

$$\bar{P} = \frac{\frac{1}{2}(810 \text{ kg})\left[(7.0 \text{ m/s})^2 - (0 \text{ m/s})^2\right] + (810 \text{ kg})(9.80 \text{ m/s}^2)(8.2 \text{ m} - 0 \text{ m})}{3.5 \text{ s}} = 2.4 \times 10^4 \text{ W}$$

67. **REASONING** The average power developed by the cheetah is equal to the work done by the cheetah divided by the elapsed time (Equation 6.10a). The work, on the other hand, can be related to the change in the cheetah’s kinetic energy by the work-energy theorem, Equation 6.3.

**SOLUTION**

a. The average power is

$$\bar{P} = \frac{W}{t} \quad (6.10a)$$

where $W$ is the work done by the cheetah. This work is related to the change in the cheetah’s kinetic energy by Equation 6.3, $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$, so the average power is
\[ \bar{P} = \frac{W}{t} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \]

\[ = \frac{1}{2}(110 \text{ kg})(27 \text{ m/s})^2 - \frac{1}{2}(110 \text{ kg})(0 \text{ m/s})^2 \]

\[ = \frac{1.0 \times 10^4 \text{ W}}{4.0 \text{ s}} = 2500 \text{ W} \]

b. The power, in units of horsepower (hp), is

\[ \bar{P} = (1.0 \times 10^4 \text{ W}) \left( \frac{1 \text{ hp}}{745.7 \text{ W}} \right) = 13 \text{ hp} \]

68. **REASONING** The average power generated by the tension in the cable is equal to the work done by the tension divided by the elapsed time (Equation 6.10a). The work can be related to the change in the lift’s kinetic and potential energies via the work-energy theorem.

**SOLUTION**

The average power is

\[ \bar{P} = \frac{W_{nc}}{t} \quad \text{(6.10a)} \]

where \( W_{nc} \) is the work done by the nonconservative tension force. This work is related to the lift’s kinetic and potential energies by Equation 6.6, \( W_{nc} = \left( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \right) + (mgh_f - mgh_0) \), so the average power is

\[ \bar{P} = \frac{W_{nc}}{t} = \frac{\left( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \right) + (mgh_f - mgh_0)}{t} \]

Since the lift moves at a constant speed, \( v_0 = v_f \), the average power becomes

\[ \bar{P} = \frac{mg(h_f - h_0)}{t} = \frac{(4 \text{ skiers}) \left( 65 \text{ kg/skier} \right) \left( 9.80 \text{ m/s}^2 \right) (140 \text{ m})}{(2 \text{ min}) \left( 60 \text{ s} \right) \left( 1 \text{ min} \right)} = 3.0 \times 10^3 \text{ W} \]

69. **SSM REASONING AND SOLUTION** In the following drawings, the positive direction is to the right. When the boat is not pulling a skier, the engine must provide a force \( F_1 \) to overcome the resistive force of the water \( F_R \). Since the boat travels with constant speed, these forces must be equal in magnitude and opposite in direction.

\[ -F_R + F_1 = ma = 0 \]

or

\[ F_R = F_1 \quad \text{(1)} \]
When the boat is pulling a skier, the engine must provide a force $F_2$ to balance the resistive force of the water $F_R$ and the tension in the tow rope $T$.

$$-F_R + F_2 - T = ma = 0$$

or

$$F_2 = F_R + T \quad (2)$$

The average power is given by $P = Fv$, according to Equation 6.11 in the text. Since the boat moves with the same speed in both cases, $v_1 = v_2$, and we have

$$\frac{P_1}{F_1} = \frac{P_2}{F_2} \quad \text{or} \quad F_2 = F_1 \frac{P_2}{P_1}$$

Using Equations (1) and (2), this becomes

$$F_1 + T = F_1 \frac{P_2}{P_1}$$

Solving for $T$ gives

$$T = F_1 \left( \frac{P_2}{P_1} - 1 \right)$$

The force $F_1$ can be determined from $P_1 = F_1 v_1$, thereby giving

$$T = \frac{P_1}{v_1} \left( \frac{P_2}{P_1} - 1 \right) = \frac{7.50 \times 10^4 \text{ W}}{12 \text{ m/s}} \left( \frac{8.30 \times 10^4 \text{ W}}{7.50 \times 10^4 \text{ W}} - 1 \right) = 6.7 \times 10^2 \text{ N}$$

70. **REASONING AND SOLUTION** The following drawings show the free-body diagrams for the car in going both up and down the hill. The force $F_R$ is the combined force of air resistance and friction, and the forces $F_U$ and $F_D$ are the forces supplied by the engine in going uphill and downhill respectively.

*Going up the hill*

*Going down the hill*

Writing Newton's second law in the direction of motion for the car as it goes uphill, taking uphill as the positive direction, we have
\[ F_U - F_R - mg \sin \theta = ma = 0 \]

Solving for \( F_U \), we have
\[ F_U = F_R + mg \sin \theta \]

Similarly, when the car is going downhill, Newton's second law in the direction of motion gives
\[ F_R - F_D - mg \sin \theta = ma = 0 \]

so that
\[ F_D = F_R - mg \sin \theta \]

Since the car needs 47 hp more to sustain the constant uphill velocity than the constant downhill velocity, we can write
\[ P_U = P_D + \Delta P \]

where \( P_U \) is the power needed to sustain the constant uphill velocity, \( P_D \) is the power needed to sustain the constant downhill velocity, and \( \Delta P = 47 \text{ hp} \). In terms of SI units,
\[ \Delta P = (47 \text{ hp}) \left( \frac{746 \text{ W}}{1 \text{ hp}} \right) = 3.51 \times 10^4 \text{ W} \]

Using Equation 6.11 \( (P = F \nu) \), the equation \( P_U = P_D + \Delta P \) can be written as
\[ R_U \nu = R_D \nu + \Delta P \]

Using the expressions for \( F_U \) and \( F_D \), we have
\[ (F_R + mg \sin \theta) \nu = (F_R - mg \sin \theta) \nu + \Delta P \]

Solving for \( \theta \), we find
\[ \theta = \sin^{-1} \left( \frac{\Delta P}{2mg \nu} \right) = \sin^{-1} \left( \frac{3.51 \times 10^4 \text{ W}}{2(1900 \text{ kg})(9.80 \text{ m/s}^2)(27 \text{ m/s})} \right) = 2.0^\circ \]

Therefore, the angle \( \theta \) is approximately 2.0 degrees.

---

71. **REASONING** The area under the force-versus-displacement graph over any displacement interval is equal to the work done over that displacement interval.

**SOLUTION**

a. Since the area under the force-versus-displacement graph from \( s = 0 \) to 0.50 m is greater for bow 1, it follows that \( \text{bow 1 requires more work to draw the bow full} \).
b. The work required to draw bow 1 is equal to the area of the triangular region under the force-versus-displacement curve for bow 1. Since the area of a triangle is equal one-half times the base of the triangle times the height of the triangle, we have

\[
W_1 = \frac{1}{2}(0.50\text{ m})(350\text{ N}) = 88\text{ J}
\]

For bow 2, we note that each small square under the force-versus-displacement graph has an area of

\[
(0.050\text{ m})(40.0\text{ N}) = 2.0\text{ J}
\]

We estimate that there are approximately 31.3 squares under the force-versus-displacement graph for bow 2; therefore, the total work done is

\[
(31.3\text{ squares})(\frac{2.0\text{ J}}{\text{square}}) = 63\text{ J}
\]

Therefore, the additional work required to stretch bow 1 as compared to bow 2 is

\[
88\text{ J} - 63\text{ J} = 25\text{ J}
\]

**Reasoning** When the force varies with the displacement, the work is the area beneath the graph of the force component \(F \cos \theta\) along the displacement as a function of the magnitude \(s\) of the displacement. Here, the shape of this area is a triangle. The area of a triangle is one-half times the base times the height of the triangle.

**Solution** The base of the triangle is 1.60 m, and the “height” is 62.0 N. Therefore, the area, which is the work, is

\[
W = \frac{1}{2}\text{ Base} \times \text{Height} = \frac{1}{2}(1.60\text{ m})(62.0\text{ N}) = 49.6\text{ J}
\]

**Reasoning** The area under the force-versus-displacement graph over any displacement interval is equal to the work done over that interval. From Example 14, we know that the total work done in drawing back the string of the bow is 60.5 J, corresponding to a total area under the curve of 242 squares (each square represents 0.250 J of work). The percentage of the total work done over any displacement interval is, therefore,

\[
\text{Percentage} = \frac{\text{Work done during interval}}{\text{Total work done during all intervals}} \times 100 = \frac{\text{Number of squares under curve during interval}}{\text{Total number of squares under entire curve}} \times 100
\]

**Solution**

a. We estimate that there are 130 squares under the curve from \(s = 0\) to 0.306 m; therefore, the percentage of work done during the interval in question is
b. Similarly, we estimate that there are 112 squares under the curve in the interval from $s = 0.306 \text{ m}$ to 0.500 m; therefore, the percentage of work done during the interval in question is

$$\frac{112 \text{ squares}}{242 \text{ squares}} \times 100 = 46\%$$

74. **REASONING** The work done during each interval is equal to the area under the force vs. displacement curve over that interval. In the interval from 2.0 to 4.0 m, the area under the curve has two distinct regions, one triangular and the other rectangular. The total work done from 2.0 to 4.0 m is the sum of the areas of these two regions.

**SOLUTION**

a. Since the area under the curve from 0 to 1.0 m is triangular, the area under the curve over this interval is $(1/2) \times \text{height} \times \text{base}:

$$W_{01} = \frac{1}{2} (+6.0 \text{ N})(1.0 \text{ m} - 0 \text{ m}) = 3.0 \text{ J}$$

b. Since there is no area under the curve from 1.0 to 2.0 m, the work done is zero. This is reasonable since there is no net force acting on the object during this interval.

c. In the interval from 2.0 to 4.0 m, the area under the curve has two distinct regions. From 2.0 to 3.0 m, the area is triangular in shape, while from 3.0 to 4.0 m the region is rectangular. The total work done from 2.0 to 4.0 m is the sum of the areas of these two regions.

$$W_{23} = \frac{1}{2} (-6.0 \text{ N})(3.0 \text{ m} - 2.0 \text{ m}) = -3.0 \text{ J}$$

and

$$W_{34} = (-6.0 \text{ N})(4.0 \text{ m} - 3.0 \text{ m}) = -6.0 \text{ J}$$

Thus, the total work done in the interval from 2.0 to 4.0 m is

$$W_{24} = W_{23} + W_{34} = (-3.0 \text{ J}) + (-6.0 \text{ J}) = -9.0 \text{ J}$$

Notice in the interval from 2.0 to 4.0 m, the area under the curve (and hence the work done) is negative, because the force is negative in this interval.

75. **REASONING** The final speed $v_f$ of the object is related to its final kinetic energy by $\text{KE}_f = \frac{1}{2} m v_f^2$ (Equation 6.2). The object’s mass $m$ is given, and we will use the work-energy theorem $W = \text{KE}_f - \text{KE}_0$ (Equation 6.3) to determine its final kinetic energy. We are helped by the fact that the object starts from rest, and therefore has no initial kinetic energy $\text{KE}_0$. 
Thus, the net work done on the object equals its final kinetic energy: 

$$W = KE_f = \frac{1}{2} mv_f^2.$$ 

In a graph of $F \cos \theta$ versus $s$, such as that given, the net work $W$ is the total area under the graph. For the purpose of calculation, it is convenient to divide this area into two pieces, a triangle (from $s = 0$ m to $s = 10.0$ m) and a rectangle (from $s = 10.0$ m to $s = 20.0$ m). Both pieces have the same width (10.0 m) and height (10.0 N), so the triangle has half the area of the rectangle.

**SOLUTION**

Solving $W = \frac{1}{2} mv_f^2$ for the final speed of the object, we obtain $v_f = \sqrt{\frac{2W}{m}}$.

The net work $W$ is the sum of the work $W_{0,10}$ done on the object from $s = 0$ m to $s = 10.0$ m (the triangular area) and the work $W_{10,20}$ done on the object from $s = 10.0$ m to $s = 20.0$ m (the rectangular area). We calculate the work $W_{10,20}$ by multiplying the width (10.0 m) and height (10.0 N) of the rectangle: $W_{10,20} = (\text{Width})(\text{Height})$. The triangle’s area, which is the work $W_{0,10}$, is half this amount: $W_{0,10} = \frac{1}{2} W_{10,20}$. Therefore, the net work $W = W_{0,10} + W_{10,20}$ done during the entire interval from $s = 0$ m to $s = 20.0$ m is

$$W = \frac{1}{2} W_{10,20} + W_{10,20} = \frac{3}{2} W_{10,20} = \frac{3}{2} (\text{Width})(\text{Height})$$

We can now calculate the final speed of the object from $v_f = \sqrt{\frac{2W}{m}}$:

$$v_f = \sqrt{\frac{2 \left( \frac{3}{2} \right)(\text{Width})(\text{Height})}{m}} = \sqrt{\frac{3(10.0 \text{ m})(10.0 \text{ N})}{6.00 \text{ kg}}} = 7.07 \text{ m/s}$$

76. **REASONING**

The work done by the tension in the cable is given by Equation 6.1 as $W = (T \cos \theta)s$. Since the elevator is moving upward at a constant velocity, it is in equilibrium, and the magnitude $T$ of the tension must be equal to the magnitude $mg$ of the elevator’s weight; $T = mg$.

**SOLUTION**

a. The tension and the displacement vectors point in the same direction (upward), so the angle between them is $\theta = 0^\circ$. The work done by the tension is

$$W = (T \cos \theta)s = (mg \cos \theta)s$$

$$= (1200 \text{ kg})(9.80 \text{ m/s}^2) \cos 0^\circ (35 \text{ m}) = 4.1 \times 10^5 \text{ J}$$

b. The weight and the displacement vectors point in opposite directions, so the angle between them is $\theta = 180^\circ$. The work done by the weight is

$$W = (mg \cos \theta)s = (1200 \text{ kg})(9.80 \text{ m/s}^2) \cos 180^\circ (35 \text{ m}) = -4.1 \times 10^5 \text{ J}$$
77. **SSM REASONING** No forces other than gravity act on the rock since air resistance is being ignored. Thus, the net work done by nonconservative forces is zero, \( W_{nc} = 0 \) J. Consequently, the principle of conservation of mechanical energy holds, so the total mechanical energy remains constant as the rock falls.

If we take \( h = 0 \) m at ground level, the gravitational potential energy at any height \( h \) is, according to Equation 6.5, \( PE = mgh \). The kinetic energy of the rock is given by Equation 6.2: \( KE = \frac{1}{2}mv^2 \). In order to use Equation 6.2, we must have a value for \( v^2 \) at each desired height \( h \). The quantity \( v^2 \) can be found from Equation 2.9 with \( v_0 = 0 \) m/s, since the rock is released from rest. At a height \( h \), the rock has fallen through a distance \((20.0 \, \text{m}) - h\), and according to Equation 2.9, \( v^2 = 2ay = 2a[(20.0 \, \text{m}) - h] \). Therefore, the kinetic energy at any height \( h \) is given by \( KE = ma[(20.0 \, \text{m}) - h] \). The total energy at any height \( h \) is the sum of the kinetic energy and potential energy at the particular height.

**SOLUTION** The calculations are performed below for \( h = 10.0 \) m. The table that follows also shows the results for \( h = 20.0 \) m and \( h = 0 \) m.

\[
PE = mgh = (2.00 \, \text{kg})(9.80 \, \text{m/s}^2)(10.0 \, \text{m}) = 196 \, \text{J}
\]

\[
KE = ma[(20.0 \, \text{m}) - h] = (2.00 \, \text{kg})(9.80 \, \text{m/s}^2)(20.0 \, \text{m} - 10.0 \, \text{m}) = 196 \, \text{J}
\]

\[
E = KE + PE = 196 \, \text{J} + 196 \, \text{J} = 392 \, \text{J}
\]

<table>
<thead>
<tr>
<th>( h ) (m)</th>
<th>KE (J)</th>
<th>PE (J)</th>
<th>E (J)</th>
</tr>
</thead>
<tbody>
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<td>392</td>
<td>392</td>
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<tr>
<td>10.0</td>
<td>196</td>
<td>196</td>
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</tr>
<tr>
<td>0</td>
<td>392</td>
<td>0</td>
<td>392</td>
</tr>
</tbody>
</table>

Note that in each case, the value of \( E \) is the same, because mechanical energy is conserved.

78. **REASONING** The work \( W_{nc} \) done by the nonconservative force exerted on the surfer by the wave is given by Equation 6.6, which states that this work is equal to the surfer’s change in kinetic energy, \( \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \), plus the change in her potential energy, \( mgh_f - mgh_0 \):

\[
W_{nc} = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + (mgh_f - mgh_0) \tag{6.6}
\]

All the terms in this equation are known, so we can evaluate the work.
**SOLUTION** We note that the difference between her final and initial heights is 
\[ h_f - h_0 = -2.7 \text{ m}, \] negative because her initial height \( h_0 \) is greater than her final height \( h_f \). Thus, the work done by the nonconservative force is:

\[
W_{nc} = \frac{1}{2}m(v_f^2 - v_0^2) + mg(h_f - h_0)
\]

\[
= \frac{1}{2}(59 \text{ kg})[(9.5 \text{ m/s})^2 - (1.4 \text{ m/s})^2] + (59 \text{ kg})(9.80 \text{ m/s}^2)(-2.7 \text{ m}) = 1.0 \times 10^3 \text{ J}
\]

---

**79. SSM REASONING** The work \( W \) done by the tension in the tow rope is given by Equation 6.1 as \( W = (F \cos \theta)s \), where \( F \) is the magnitude of the tension, \( s \) is the magnitude of the skier’s displacement, and \( \theta \) is the angle between the tension and the displacement vectors. The magnitude of the displacement (or the distance) is the speed \( v \) of the skier multiplied by the time \( t \) (see Equation 2.1), or \( s = vt \).

**SOLUTION** Substituting \( s = vt \) into the expression for the work, \( W = (F \cos \theta)s \), we have \( W = (F \cos \theta)vt \). Since the skier moves parallel to the boat and since the tow rope is parallel to the water, the angle between the tension and the skier’s displacement is \( \theta = 37.0^\circ \). Thus, the work done by the tension is

\[
W = (F \cos \theta)vt = [(135 \text{ N}) \cos 37.0^\circ](9.30 \text{ m/s})(12.0 \text{ s}) = 1.20 \times 10^4 \text{ J}
\]

---

**80. REASONING** The change in gravitational potential energy for both the adult and the child is \( \Delta PE = mgh_f - mgh_0 \), where we have used Equation 6.5. Therefore, \( \Delta PE = mg(h_f - h_0) \). In this expression \( h_f - h_0 \) is the vertical height of the second floor above the first floor, and its value is not given. However, we know that it is the same for both staircases, a fact that will play the central role in our solution.

**SOLUTION** Solving \( \Delta PE = mg(h_f - h_0) \) for \( h_f - h_0 \), we obtain

\[
h_f - h_0 = \frac{(\Delta PE)_{Adult}}{m_{Adult}g} \quad \text{and} \quad h_f - h_0 = \frac{(\Delta PE)_{Child}}{m_{Child}g}
\]

Since \( h_f - h_0 \) is the same for the adult and the child, we have

\[
\frac{(\Delta PE)_{Adult}}{m_{Adult}g} = \frac{(\Delta PE)_{Child}}{m_{Child}g}
\]

Solving this result for \( (\Delta PE)_{Child} \) gives

\[
(\Delta PE)_{Child} = \frac{(\Delta PE)_{Adult}}{m_{Child}} m_{Child} = \frac{(2.00 \times 10^3 \text{ J})(18.0 \text{ kg})}{81.0 \text{ kg}} = 444 \text{ J}
\]
81. **REASONING** The work done by a constant force of magnitude $F$ that makes an angle $\theta$ with a displacement of magnitude $s$ is given by $W = (F \cos \theta) s$ (Equation 6.1). The magnitudes and directions of the pulling forces exerted by the husband and wife differ, but the displacement of the wagon is the same in both cases. Therefore, we can express the work $W_H$ done by the husband’s pulling force as $W_H = (F_H \cos \theta_H) s$. Similarly, the work done by the wife can be written as $W_W = (F_W \cos \theta_W) s$. We know the directional angles of both forces, the magnitude $F_H$ of the husband’s pulling force, and that both forces do the same amount of work ($W_W = W_H$).

**SOLUTION** Setting the two expressions for work equal to one another, and solving for the magnitude $F_W$ of the wife’s pulling force, we find

$$
\frac{(F_W \cos \theta_W) s}{W_W} = \frac{(F_H \cos \theta_H) s}{W_H}
$$

or

$$
F_W = \frac{F_H \cos \theta_H}{\cos \theta_W} = \frac{(67 \text{ N}) (\cos 58^\circ)}{\cos 38^\circ} = 45 \text{ N}
$$

82. **REASONING** The power required to accelerate the glider is found from $P = \frac{\text{Work}}{\text{Time}}$ (Equation 6.10a). The net work done on the glider is equal to the change in its kinetic energy, according to the work-energy theorem $W = KE_f - KE_0$ (Equation 6.3). The glider accelerates from rest, so its initial kinetic energy is zero. Therefore, the average power required is $P = \frac{KE_f}{\text{Time}} = \frac{\frac{1}{2}mv_f^2}{t} = \frac{mv_f^2}{2t}$. The constant tension in the cable produces a constant acceleration in the glider, permitting us to determine the time from the expression $x = \frac{1}{2}(v_0 + v_f)t$ (Equation 2.7), where $x$ is the glider’s displacement.

**SOLUTION** Since the glider starts from rest, we have $v_0 = 0 \text{ m/s}$. Therefore, solving $x = \frac{1}{2}(v_0 + v_f)t$ for the time $t$, we obtain $t = \frac{2x}{v_0 + v_f} = \frac{2x}{0 \text{ m/s} + v_f} = \frac{2x}{v_f}$.

Substituting $\frac{2x}{v_f}$ for $t$ in $P = \frac{mv_f^2}{2t}$, we find that the average power required of the winch is

$$
P = \frac{mv_f^2}{2 \left( \frac{2x}{v_f} \right)} = \frac{mv_f^3}{4x} = \frac{(184 \text{ kg})(26.0 \text{ m/s})^3}{4(48.0 \text{ m})} = 1.68 \times 10^4 \text{ W}
$$

83. **SSM** **REASONING AND SOLUTION**

a. The lost mechanical energy is

$$
E_{\text{lost}} = E_0 - E_f
$$
The ball is dropped from rest, so its initial energy is purely potential. The ball is momentarily at rest at the highest point in its rebound, so its final energy is also purely potential. Then

\[ E_{\text{lost}} = mgh_0 - mgh_f = (0.60 \text{ kg})(9.80 \text{ m/s}^2)[(1.05 \text{ m}) - (0.57 \text{ m})] = 2.8 \text{ J} \]

b. The work done by the player must compensate for this loss of energy.

\[ E_{\text{lost}} = \left( F \cos \theta \right) s \quad \Rightarrow \quad F = \frac{E_{\text{lost}}}{(\cos \theta) s} = \frac{2.8 \text{ J}}{(\cos 0^\circ)(0.080 \text{ m})} = 35 \text{ N} \]

84. **REASONING**

a. Because the kinetic frictional force, a nonconservative force, is present, it does negative work on the skier. The work-energy theorem, in the form of Equation 6.6, may be used to find the work done by this force.

b. Once the work done by the kinetic frictional force is known, the magnitude of the kinetic frictional force can be determined by using the definition of work, Equation 6.1, since the magnitude of the skier’s displacement is known.

**SOLUTION**

a. The work \( W_{\text{nc}} \) done by the kinetic frictional force is related to the object’s kinetic and potential energies by Equation 6.6:

\[ W_{\text{nc}} = \left( \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \right) + (mgh_f - mgh_0) \]

The initial height of the skier at the bottom of the hill is \( h_0 = 0 \text{ m} \), and the final height is \( h_f = s \sin 25^\circ \) (see the drawing). Thus, the work is

\[ W_{\text{nc}} = \left( \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \right) + mg\left( s \sin 25^\circ - h_0 \right) \]

\[ = \frac{1}{2}(63 \text{ kg})(4.4 \text{ m/s})^2 - \frac{1}{2}(63 \text{ kg})(6.6 \text{ m/s})^2 \]

\[ + (63 \text{ kg})(9.80 \text{ m/s}^2)[(1.9 \text{ m}) \sin 25^\circ - 0 \text{ m}] = -270 \text{ J} \]

b. The work done by the kinetic frictional force is, according to Equation 6.1, \( W_{\text{nc}} = (f_k \cos 180^\circ)s \), where \( f_k \) is the magnitude of the kinetic frictional force, and \( s \) is the magnitude of the skier’s displacement. The displacement of the skier is up the hill and the kinetic frictional force is directed down the hill, so the angle between the two vectors is \( \theta = 180^\circ \) and \( \cos \theta = -1 \). Solving the equation above for \( f_k \), we have

\[ f_k = \frac{W_{\text{nc}}}{-s} = \frac{-270 \text{ J}}{-1.9 \text{ m}} = 140 \text{ N} \]
85. **REASONING** Friction and air resistance are being ignored. The normal force from the slide is perpendicular to the motion, so it does no work. Thus, no net work is done by nonconservative forces, and the principle of conservation of mechanical energy applies.

**SOLUTION** Applying the principle of conservation of mechanical energy to the swimmer at the top and the bottom of the slide, we have

\[
\frac{1}{2} mv_f^2 + mgh_f = \frac{1}{2} mv_0^2 + mgh_0
\]

If we let \( h \) be the height of the bottom of the slide above the water, \( h_f = h \), and \( h_0 = H \). Since the swimmer starts from rest, \( v_0 = 0 \) m/s, and the above expression becomes

\[
\frac{1}{2} v_f^2 + gh = gH
\]

Solving for \( H \), we obtain

\[
H = h + \frac{v_f^2}{2g}
\]

Before we can calculate \( H \), we must find \( v_f \) and \( h \). Since the velocity in the horizontal direction is constant,

\[
v_f = \frac{\Delta x}{\Delta t} = \frac{5.00 \text{ m}}{0.500 \text{ s}} = 10.0 \text{ m/s}
\]

The vertical displacement of the swimmer after leaving the slide is, from Equation 3.5b (with down being negative),

\[
y = \frac{1}{2} a_y t^2 = \frac{1}{2} (-9.80 \text{ m/s}^2)(0.500 \text{ s})^2 = -1.23 \text{ m}
\]

Therefore, \( h = 1.23 \text{ m} \). Using these values of \( v_f \) and \( h \) in the above expression for \( H \), we find

\[
H = h + \frac{v_f^2}{2g} = 1.23 \text{ m} + \frac{(10.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 6.33 \text{ m}
\]

86. **REASONING AND SOLUTION** At the bottom of the circular path of the swing, the centripetal force is provided by the tension in the rope less the weight of the swing and rider. That is,

\[
\frac{mv^2}{r} = T - mg
\]

\[
F_c
\]
Solving for the mass yields 

\[ m = \frac{T}{\frac{v^2}{r} + g} \]

The energy of the swing is conserved if friction is ignored. The initial energy, \( E_0 \), when the swing is released is completely potential energy and is \( E_0 = mgh_0 \), where

\[ h_0 = r(1 - \cos 60.0^\circ) = \frac{1}{2}r \]

is the vertical height of the swing. At the bottom of the path the swing's energy is entirely kinetic and is

\[ E_f = \frac{1}{2}mv^2 \]

Setting \( E_0 = E_f \) and solving for \( v \) gives

\[ v = \sqrt{2gh_0} = \sqrt{gr} \]

The expression for the mass now becomes

\[ m = \frac{T}{2g} = \frac{8.00 \times 10^2 \text{ N}}{2(9.80 \text{ m/s}^2)} = 40.8 \text{ kg} \]

---

87. **REASONING** The magnitude \( f_k \) of the average kinetic friction force exerted on the skateboarder is related to the amount of work \( W_k \) the done by that force and the distance \( s \) he slides along the ramp according to \( W_k = (f_k \cos \theta)s \) (Equation 6.1). Solving this equation for the kinetic friction force, we find

\[ f_k = \frac{W_k}{s \cos \theta} \]  

\[ \text{(1)} \]

The ramp is semicircular, so sliding from the top to the bottom means sliding one-fourth of the circumference of a circle with a radius of \( r = 2.70 \text{ m} \). Therefore, \( s = \frac{1}{4}(2\pi r) = \frac{1}{2}\pi r \). Because the kinetic friction force is always directed opposite the skateboarder’s velocity, the angle \( \theta \) in Equation (1) is 180°. Thus, Equation (1) becomes

\[ f_k = \frac{2W_k}{\pi r} \]  

\[ \text{(2)} \]
To complete the solution, we will use the work-energy theorem

\[ W_{nc} = \left( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \right) + (mgh_f - mgh_0) \]  

(Equation 6.6) to find the work \( W_k \) done by the nonconservative kinetic friction force. We can do this because the normal force exerted by the ramp, which is also a nonconservative force, is always perpendicular to the skateboarder’s velocity. Consequently, the normal force does no work, so the net work \( W_{nc} \) done by all nonconservative forces is just the work \( W_k \) done by the kinetic friction force:

\[ W_{nc} = W_k = \left( \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \right) + (mgh_f - mgh_0) = \frac{1}{2} m(v_f^2 - v_0^2) + mg(h_f - h_0) \]  

(3)

**SOLUTION** Substituting Equation (3) into Equation (2), we obtain

\[ f_k = \frac{-2W_k}{\pi r} = -\frac{2}{\pi} \left[ \frac{1}{2} m(v_f^2 - v_0^2) + mg(h_f - h_0) \right] \]  

(4)

The skateboarder falls from rest, so we have \( v_0 = 0 \) m/s. He starts a distance \( h \) above the top of the ramp, which itself is a distance \( r \) above the bottom of the ramp. His vertical displacement, therefore, is \( h_f - h_0 = -(h + r) \). The algebraic sign of this displacement is negative because he moves downward. Suppressing units, Equation (4) then yields

\[ f_k = \frac{-2}{\pi} \left[ \frac{1}{2} m(v_f^2 - 0) - mg(h + r) \right] = -\frac{mv_f^2 - 2mg(h + r)}{\pi r} = \frac{2mg(h + r) - mv_f^2}{\pi r} \]  

(5)

Thus, the average force of kinetic friction acting on the skateboarder as he slides down the ramp is

\[ f_k = \frac{2(61.0 \text{ kg})(9.80 \text{ m/s}^2)(1.80 \text{ m} + 2.70 \text{ m}) - (61.0 \text{ kg})(6.40 \text{ m/s})^2}{\pi(2.70 \text{ m})} = 3.40 \times 10^2 \text{ N} \]
1. (b) Kinetic energy, $\frac{1}{2}mv^2$, is a scalar quantity and is the same for both cars. Momentum, $mv$, is a vector quantity that has a magnitude and a direction. The two cars have different directions, so they have different momenta.

2. (d) Momentum is a vector quantity that has a magnitude and a direction. The magnitudes ($m_0v_0$) and directions (due north) are the same for both runners.

3. The magnitude of the total momentum is $47,700 \text{ kg} \cdot \text{m/s}$.

4. (d) According to the impulse-momentum theorem, the impulse is equal to the final momentum minus the initial momentum. Therefore, the impulses are the same in both cases, since the final momenta are the same and the initial momenta are the same. The impulse is also the product of the net average force and the time of impact. Since the impulses are equal and the time of impact is larger for the air mattress than for the ground, the air mattress exerts a smaller net average force on the high jumper.

5. The impulse is $+324 \text{ kg} \cdot \text{m/s}$.

6. (c) According to the impulse-momentum theorem, the impulse is equal to the change in the particle’s momentum. The magnitude of the momentum change is the same in regions A and C; therefore, in these regions the particle experiences impulses of the same magnitude. The momentum in region B is constant. Therefore, the change in momentum is zero, and so is the impulse.

7. (b) According to the impulse-momentum theorem, the net average force is equal to the change in the particle’s momentum divided by the time interval. This ratio is greatest in region C. The ratio is equal to zero in region B, since the change in the particle’s momentum is zero there.

8. Magnitude of net average force in Region A = $1.0 \text{ N}$, Magnitude of net average force in Region C = $4.0 \text{ N}$

9. (d) Since there are no external forces acting on the rocket, momentum is conserved before, during, and after the separation. This means that the momentum of the rocket before the separation ($+150,000 \text{ kg} \cdot \text{m/s}$) is equal to the sum of the momenta of the two stages after the separation [$+250,000 \text{ kg} \cdot \text{m/s} + (-100,000 \text{ kg} \cdot \text{m/s}) = +150,000 \text{ kg} \cdot \text{m/s}$].
10. (a) The net external force acting on the ball/earth system is zero. The gravitational forces that the ball and earth exert on each other are internal forces, or forces that the objects within the system exert on each other. The space probe is also an isolated system, since there are no external forces acting on it.

11. (c) The net external force that acts on the objects is zero, so the total linear momentum is conserved. Therefore, the total momentum before the collision (+12 kg·m/s − 2 kg·m/s = +10 kg·m/s) equals the total momentum after the collision (−4 kg·m/s + 14 kg·m/s = +10 kg·m/s).

12. (b) The collision between the objects is an elastic collision, so the total kinetic energy is conserved. Therefore, the total kinetic energy before the collision (8 J + 6 J = 14 J) is equal to the total kinetic energy after the collision (−4 J + 14 J = 14 J).

13. (c) Since the net external force acting on the two objects during the collision is zero, the total linear momentum of the system is conserved. In other words, the total linear momentum before the collision (−3 kg·m/s + 4 kg·m/s = +1 kg·m/s) equals the total linear momentum after the collision (1 kg·m/s). Furthermore, some kinetic energy is lost during a completely inelastic collision, which is the case here. The final kinetic energy (4 J) is less than the total initial kinetic energy (1 J + 6 J = 7 J).

14. (d) The net external force acting on the two objects during the collision is zero, so the total momentum of the system is conserved. In two dimensions this means that the $x$-component of the initial total momentum (+16 kg·m/s) is equal to the $x$-component of the final total momentum. Since the $x$-component of the final momentum of object 1 is +6 kg·m/s, the $x$-component of the final momentum of object 2 must be +10 kg·m/s (+16 kg·m/s − 6 kg·m/s = +10 kg·m/s).

15. (c) The net external force acting on the two objects during the collision is zero, so the total momentum of the system is conserved. In two dimensions this means that the $y$-component of the initial total momentum (0 kg·m/s) is equal to the $y$-component of the final total momentum. Since the $y$-component of the final momentum of object 1 is −5 kg·m/s, then the $y$-component of the final momentum of object 2 must be +5 kg·m/s [0 kg·m/s − (−5 kg·m/s) = +5 kg·m/s].

16. $x$ coordinate of the center of mass = −1.5 m,
$y$ coordinate of the center of mass = 0 m

17. Velocity of center of mass = 2.0 m/s, along the $+x$ axis
CHAPTER 7 | IMPULSE AND MOMENTUM

PROBLEMS

1. **REASONING** The impulse that the wall exerts on the skater can be found from the impulse-momentum theorem, Equation 7.4. The average force $\bar{F}$ exerted on the skater by the wall is the only force exerted on her in the horizontal direction, so it is the net force; $\Sigma F = \bar{F}$.

   **SOLUTION** From Equation 7.4, the average force exerted on the skater by the wall is
   \[
   \bar{F} = \frac{mv_f - mv_0}{\Delta t} = \frac{(46 \text{ kg})(-1.2 \text{ m/s}) - (46 \text{ kg})(0 \text{ m/s})}{0.80 \text{ s}} = -69 \text{ N}
   \]

   From Newton's third law, the average force exerted on the wall by the skater is equal in magnitude and opposite in direction to this force. Therefore,
   \[
   \text{Force exerted on wall} = +69 \text{ N}
   \]

   The plus sign indicates that this force points opposite to the velocity of the skater.

2. **REASONING** According to the impulse-momentum theorem, the rocket’s final momentum $mv_f$ differs from its initial momentum by an amount equal to the impulse $(\Sigma \bar{F})\Delta t$ of the net force exerted on it: $(\Sigma \bar{F})\Delta t = mv_f - mv_0$ (Equation 7.4). We are ignoring gravitational and frictional forces, so this impulse is due entirely to the force generated by the motor. The magnitude of the motor’s impulse is given as $(\Sigma \bar{F})\Delta t = 29.0 \text{ N} \cdot \text{s}$, so we will obtain the rocket’s final speed by using Equation 7.4.

   **SOLUTION** The rocket starts from rest, so $v_0 = 0 \text{ m/s}$, and the impulse-momentum theorem becomes $(\Sigma \bar{F})\Delta t = mv_f$. Therefore, the rocket’s final speed $v_f$ is the magnitude of the motor impulse divided by the rocket’s mass:
   \[
   v_f = \frac{(\Sigma \bar{F})\Delta t}{m} = \frac{29.0 \text{ N} \cdot \text{s}}{0.175 \text{ kg}} = 166 \text{ m/s}
   \]

3. **REASONING** The impulse that the roof of the car applies to the hailstones can be found from the impulse-momentum theorem, Equation 7.4. Two forces act on the hailstones, the average force $\bar{F}$ exerted by the roof, and the weight of the hailstones. Since it is assumed that $\bar{F}$ is much greater than the weight of the hailstones, the net average force $(\Sigma \bar{F})$ is equal to $\bar{F}$.
**SOLUTION** From Equation 7.4, the impulse that the roof applies to the hailstones is:

\[
\vec{F} \Delta t = m(v_f - v_0) = m(v_f - v_0)
\]

Solving for \( \vec{F} \) (with \( u \) taken to be the positive direction) gives

\[
\vec{F} = \left( \frac{m}{\Delta t} \right) (v_f - v_0) = (0.060 \text{ kg/s}) \left[ (+15 \text{ m/s}) - (-15 \text{ m/s}) \right] = +1.8 \text{ N}
\]

This is the average force exerted on the hailstones by the roof of the car. The positive sign indicates that this force points upward. From Newton's third law, the average force exerted by the hailstones on the roof is equal in magnitude and opposite in direction to this force. Therefore,

\[
\text{Force on roof} = -1.8 \text{ N}
\]

The negative sign indicates that this force points downward.

---

**4. REASONING** The impulse-momentum theorem, as expressed in Equation 7.4, states that the impulse acting on each car is equal to the final momentum of the car minus its initial momentum:

\[
\left( \Sigma \vec{F} \right) \Delta t = m(v_f - v_0) \quad \text{or} \quad \Sigma \vec{F} = \frac{m(v_f - v_0)}{\Delta t}
\]

where \( \Sigma \vec{F} \) is the net average force that acts on the car, and \( \Delta t \) is the time interval during which the force acts.

**SOLUTION** We assume that the velocity of each car points in the +x direction. The net average force acting on each car is:

- **Car A**
  \[
  \Sigma \vec{F} = \frac{m(v_f - v_0)}{\Delta t} = \frac{(1400 \text{ kg})(+27 \text{ m/s}) - (1400 \text{ kg})(0 \text{ m/s})}{9.0 \text{ s}} = +4200 \text{ N}
  \]

- **Car B**
  \[
  \Sigma \vec{F} = \frac{m(v_f - v_0)}{\Delta t} = \frac{(1900 \text{ kg})(+27 \text{ m/s}) - (1900 \text{ kg})(0 \text{ m/s})}{9.0 \text{ s}} = +5700 \text{ N}
  \]

---

**SOLUTION** The impulse that the volleyball player applies to the ball can be found from the impulse-momentum theorem, Equation 7.4. Two forces act on the volleyball while it’s being spiked: an average force \( \vec{F} \) exerted by the player, and the weight of the ball. As in Example 1, we will assume that \( \vec{F} \) is much greater than the weight of the ball, so the weight can be neglected. Thus, the net average force \( \Sigma \vec{F} \) is equal to \( \vec{F} \).
**SOLUTION**  From Equation 7.4, the impulse that the player applies to the volleyball is

\[
\overline{F} \Delta t = m(v_f - v_0) \\
\text{Impulse} \quad \text{Final} \quad \text{Initial}
\]

\[
= m(v_f - v_0) = (0.35 \text{ kg}) \left[ (-21 \text{ m/s}) - (+4.0 \text{ m/s}) \right] = -8.7 \text{ kg} \cdot \text{m/s}
\]

The minus sign indicates that the direction of the impulse is the same as that of the final velocity of the ball.

6. **REASONING** To determine the total momentum of the two-arrow system, we will find the momentum of each arrow and add the two values. However, we need to remember that momentum is a vector quantity. Each momentum vector has a direction, and these directions must be taken into account when the two vectors are added.

**SOLUTION** According to Equation 7.2, the magnitude of each arrow’s momentum is the mass \(m\) times the magnitude \(v\) of the velocity:

\[
p = mv = (0.100 \text{ kg})(30.0 \text{ m/s}) = 3.00 \text{ kg} \cdot \text{m/s}
\]

The momentum vectors of the arrows are perpendicular, so the magnitude of the total momentum is given by the Pythagorean theorem

\[
P_T = \sqrt{p_1^2 + p_2^2} = \sqrt{\left(3.00 \text{ kg} \cdot \text{m/s}\right)^2 + \left(3.00 \text{ kg} \cdot \text{m/s}\right)^2} = 4.24 \text{ kg} \cdot \text{m/s}
\]

Momentum has the same direction as the velocity. Thus, one momentum vector points due east and the other due south. As a result, the direction of the total momentum is south of east. The angle of the total momentum vector as measured from due east is

\[
\theta = \tan^{-1} \left( \frac{3.00 \text{ kg} \cdot \text{m/s}}{3.00 \text{ kg} \cdot \text{m/s}} \right) = 45.0^\circ \text{ south of east}
\]

7. **REASONING AND SOLUTION**

a. According to Equation 7.4, the impulse-momentum theorem, \((\Sigma F) \Delta t = m(v_f - v_0)\). Since the only horizontal force exerted on the puck is the force \(\overline{F}\) exerted by the goalie, \(\Sigma \overline{F} = \overline{F}\). Since the goalie catches the puck, \(v_f = 0 \text{ m/s}\). Solving for the average force exerted on the puck, we have

\[
\overline{F} = \frac{m(v_f - v_0)}{\Delta t} = \frac{(0.17 \text{ kg})[(0 \text{ m/s}) - (+65 \text{ m/s})]}{5.0 \times 10^{-3} \text{ s}} = -2.2 \times 10^3 \text{ N}
\]

By Newton’s third law, the force exerted on the goalie by the puck is equal in magnitude and opposite in direction to the force exerted on the puck by the goalie. Thus, the average force exerted on the goalie is \(+2.2 \times 10^3 \text{ N}\).
b. If, instead of catching the puck, the goalie slaps it with his stick and returns the puck straight back to the player with a velocity of \(-65 \text{ m/s}\), then the average force exerted on the puck by the goalie is

\[
\bar{F} = \frac{m(v_f - v_0)}{\Delta t} = \frac{(0.17 \text{ kg})(-65 \text{ m/s}) - (+65 \text{ m/s})}{5.0 \times 10^{-3} \text{ s}} = -4.4 \times 10^3 \text{ N}
\]

The average force exerted on the goalie by the puck is thus \(+4.4 \times 10^3 \text{ N}\).

The answer in part (b) is twice that in part (a). This is consistent with the conclusion of Conceptual Example 3. The change in the momentum of the puck is greater when the puck rebounds from the stick. Thus, the puck exerts a greater impulse, and hence a greater force, on the goalie.

8. **Reasoning** We will apply the impulse momentum theorem as given in Equation 7.4 to solve this problem. From this theorem we know that, for a given change in momentum, greater forces are associated with shorter time intervals. Therefore, we expect that the force in the stiff-legged case will be greater than in the knees-bent case.

**Solution**

a. Assuming that upward is the positive direction, we find from the impulse-momentum theorem that

\[
\Sigma \bar{F} = \frac{mv_f - mv_0}{\Delta t} = \frac{(75 \text{ kg})(0 \text{ m/s}) - (75 \text{ kg})(-6.4 \text{ m/s})}{2.0 \times 10^{-3} \text{ s}} = +2.4 \times 10^5 \text{ N}
\]

b. Again using the impulse-momentum theorem, we find that

\[
\Sigma \bar{F} = \frac{mv_f - mv_0}{\Delta t} = \frac{(75 \text{ kg})(0 \text{ m/s}) - (75 \text{ kg})(-6.4 \text{ m/s})}{0.10 \text{ s}} = +4.8 \times 10^3 \text{ N}
\]

c. The net average force acting on the man is \(\Sigma \bar{F} = \bar{F}_{\text{ground}} + W\), where \(\bar{F}_{\text{ground}}\) is the average upward force exerted on the man by the ground and \(W\) is the downward-acting weight of the man. It follows, then, that \(\bar{F}_{\text{ground}} = \Sigma \bar{F} - W\). Since the weight is \(W = -mg\), we have

**Stiff - legged** \(\bar{F}_{\text{ground}} = \Sigma \bar{F} - W\)

\[= +2.4 \times 10^5 \text{ N} - \left[-(75 \text{ kg})(9.80 \text{ m/s}}^2\right] = +2.4 \times 10^5 \text{ N}\]

**Knees - bent** \(\bar{F}_{\text{ground}} = \Sigma \bar{F} - W\)

\[= +4.8 \times 10^3 \text{ N} - \left[-(75 \text{ kg})(9.80 \text{ m/s}}^2\right] = +5.5 \times 10^3 \text{ N}\]
9. **REASONING** The impulse-momentum theorem (Equation 7.4) states that the impulse of an applied force is equal to the change in the momentum of the object to which the force is applied. We will use this theorem to determine the final momentum from the given value of the initial momentum. The impulse is the average force times the time interval during which the force acts, according to Equation 7.1. The force and the time interval during which it acts are given, so we can calculate the impulse.

**SOLUTION** According to the impulse-momentum theorem, the impulse applied by the retrorocket is

\[ J = mv_f - mv_0 \]  

(7.4)

The impulse is \( J = \overline{F} \Delta t \) (Equation 7.1), which can be substituted into Equation 7.4 to give

\[ \overline{F} \Delta t = mv_f - mv_0 \quad \text{or} \quad mv_f = \overline{F} \Delta t + mv_0 \]

where \( mv_f \) is the final momentum. Taking the direction in which the probe is traveling as the positive direction, we have that the initial momentum is \( mv_0 = +7.5 \times 10^7 \text{ kg} \cdot \text{m/s} \) and the force is \( \overline{F} = -2.0 \times 10^6 \text{ N} \). The force is negative, because it points opposite to the direction of the motion. With these data, we find that the final momentum after the retrorocket ceases to fire is

\[ mv_f = \overline{F} \Delta t + mv_0 = (-2.0 \times 10^6 \text{ N})(12 \text{ s}) + 7.5 \times 10^7 \text{ kg} \cdot \text{m/s} = 5.1 \times 10^7 \text{ kg} \cdot \text{m/s} \]

10. **REASONING** During the time interval \( \Delta t \), a mass \( m \) of water strikes the turbine blade. The incoming water has a momentum \( mv_0 \) and that of the outgoing water is \( mv_f \). In order to change the momentum of the water, an impulse \( (\Sigma \overline{F}) \Delta t \) is applied to it by the stationary turbine blade. Now \( (\Sigma \overline{F}) \Delta t = \overline{F} \Delta t \), since only the force of the blade is assumed to act on the water in the horizontal direction. These variables are related by the impulse-momentum theorem, \( \overline{F} \Delta t = mv_f - mv_0 \), which can be solved to find the average force \( \overline{F} \) exerted on the water by the blade.

**SOLUTION** Solving the impulse-momentum theorem for the average force gives

\[ \overline{F} = \frac{mv_f - mv_0}{\Delta t} = \frac{m}{\Delta t} (v_f - v_0) \]

The ratio \( m/(\Delta t) \) is the mass of water per second that strikes the blade, or 30.0 kg/s, so the average force is

\[ \overline{F} = \frac{m}{\Delta t} (v_f - v_0) = (30.0 \text{ kg/s})[(16.0 \text{ m/s}) - (+16.0 \text{ m/s})] = -960 \text{ N} \]

The magnitude of the average force is \( 960 \text{ N} \).
11. **REASONING** We will divide this problem into two parts, because the forces acting on the student change abruptly at the instant of impact. In the first part, the student falls freely from rest, under the sole influence of the conservative gravitational force. Thus, the student’s total mechanical energy $E$ is conserved up to the instant of impact. We will use the energy conservation principle to determine the student’s initial height $H$ in terms of the student’s velocity $v_{\text{impact}}$ at that instant. The second part of the student’s motion begins at impact, when the force $F_{\text{ground}}$ due to the ground overwhelms the gravitational force and brings the student to rest. The force of the ground is nonconservative, so instead of the energy conservation principle, we will apply the impulse-momentum theorem $F_{\text{ground}} \Delta t = m v_f - m v_0$ (Equation 7.4) to analyze the collision. Because this time interval begins at impact, $v_0$ is the student’s impact velocity: $v_0 = v_{\text{impact}}$.

**SOLUTION** We begin with the energy conservation principle $\frac{1}{2} m v_0^2 + mgh_0 = \frac{1}{2} m v_f^2 + mgh_f$ (Equation 6.9b) applied to the student’s fall to the ground. Falling from rest implies $v_0 = 0$ m/s, and the student’s final velocity is the impact velocity: $v_f = v_{\text{impact}}$. Thus, we have

$$0 + \frac{1}{2} m v_{\text{impact}}^2 + mgh_f \quad \text{or} \quad g(h_0 - h_f) = \frac{1}{2} v_{\text{impact}}^2 \quad \text{or} \quad H = \frac{v_{\text{impact}}^2}{2g} \quad (1)$$

For the student’s collision with the ground, the impulse-momentum theorem gives $F_{\text{ground}} \Delta t = m v_f - m v_0$ (Equation 7.4). The collision brings the student to rest, so we know that $v_f = 0$ m/s, and Equation 7.4 becomes $F_{\text{ground}} \Delta t = -m v_0$. Solving for the impact speed $v_0$, we obtain

$$v_0 = v_{\text{impact}} = -\frac{F_{\text{ground}} \Delta t}{m} \quad (2)$$

Substituting Equation (2) into Equation (1) yields

$$H = \frac{v_{\text{impact}}^2}{2g} = \left( -\frac{F_{\text{ground}} \Delta t}{m} \right)^2 \left( \frac{F_{\text{ground}} \Delta t}{m} \right)^2 = \left( \frac{(+18 \ 000 \text{ N})(0.040 \text{ s})}{2 \cdot (9.80 \text{ m/s}^2)(63 \text{ kg})} \right)^2 = 6.7 \text{ m}$$

12. **REASONING** The impulse applied to the golf ball by the floor can be found from Equation 7.4, the impulse-momentum theorem: $(\Sigma \mathbf{F}) \Delta t = m v_f - m v_0$. Two forces act on the golf ball, the average force $\mathbf{F}$ exerted by the floor, and the weight of the golf ball. Since $\mathbf{F}$ is much greater than the weight of the golf ball, the net average force $(\Sigma \mathbf{F})$ is equal to $\mathbf{F}$. Only the vertical component of the ball's momentum changes during impact with the floor. In order to use Equation 7.4 directly, we must first find the vertical components of the initial and final velocities. We begin, then, by finding these velocity components.
**SOLUTION**  The following figures show the initial and final velocities of the golf ball.

If we take up as the positive direction, then the vertical components of the initial and final velocities are, respectively, $v_{0y} = -v_0 \cos 30.0^\circ$ and $v_{fy} = +v_f \cos 30.0^\circ$. Then, from Equation 7.4 the impulse is

\[
\vec{F} \Delta t = m(v_{fy} - v_{0y}) = m[(+v_f \cos 30.0^\circ) - (-v_0 \cos 30.0^\circ)]
\]

Since $v_0 = v_f = 45 \text{ m/s}$, the impulse applied to the golf ball by the floor is

\[
\vec{F} \Delta t = 2mv_0 \cos 30.0^\circ = 2(0.047 \text{ kg})(45 \text{ m/s})(\cos 30.0^\circ) = 3.7 \text{ N} \cdot \text{s}
\]

13. **REASONING**  This is a problem in vector addition, and we will use the component method for vector addition. Using this method, we will add the components of the individual momenta in the direction due north to obtain the component of the vector sum in the direction due north. We will obtain the component of the vector sum in the direction due east in a similar fashion from the individual components in that direction. For each jogger the momentum is the mass times the velocity.

**SOLUTION**  Assuming that the directions north and east are positive, the components of the joggers’ momenta are as shown in the following table:

<table>
<thead>
<tr>
<th>85 kg jogger</th>
<th>55 kg jogger</th>
</tr>
</thead>
<tbody>
<tr>
<td>(85 kg)(2.0 m/s) = 170 kg·m/s</td>
<td>(55 kg)(3.0 m/s)\cos 32^\circ = 140 kg·m/s</td>
</tr>
<tr>
<td>0 kg·m/s</td>
<td>(55 kg)(3.0 m/s)\sin 32^\circ = 87 kg·m/s</td>
</tr>
</tbody>
</table>

Using the Pythagorean theorem, we find that the magnitude of the total momentum is

\[
\sqrt{(310 \text{ kg} \cdot \text{m} / \text{s})^2 + (87 \text{ kg} \cdot \text{m} / \text{s})^2} = 322 \text{ kg} \cdot \text{m} / \text{s}
\]
Chapter 7   Problems

The total momentum vector points north of east by an angle \( \theta \), which is given by

\[
\theta = \tan^{-1} \left( \frac{87 \text{ kg} \cdot \text{m/s}}{310 \text{ kg} \cdot \text{m/s}} \right) = 16^\circ
\]

14. **REASONING** The height from which the basketball was dropped can be found by solving \( v_y^2 = v_{0y}^2 + 2a_y y \) (Equation 3.6b) for the displacement \( y \). Since the ball was dropped from rest, we know that \( v_{0y} = 0 \). From the definition of momentum (Equation 7.2), it follows that the speed \( v_y \) of the ball just before striking the floor is given by \( v_y = p/m \).

**SOLUTION** Therefore, taking downward as the positive direction, we have

\[
y = \frac{v_y^2}{2a_y} = \frac{(p/m)^2}{2(9.80 \text{ m/s}^2)} = \frac{[(3.1 \text{ kg} \cdot \text{m/s}) / (0.60 \text{ kg})]^2}{2(9.80 \text{ m/s}^2)} = 1.4 \text{ m}
\]

15. **REASONING AND SOLUTION** The excess weight of the truck is due to the force exerted on the truck by the sand. Newton's third law requires that this force be equal in magnitude to the force exerted on the sand by the truck. In time \( t \), a mass \( m \) of sand falls into the truck bed and comes to rest. The impulse is

\[
\vec{F} \Delta t = m(v_f - v_0) \quad \text{so} \quad \vec{F} = \frac{m(v_f - v_0)}{\Delta t}
\]

The sand gains a speed \( v_0 \) in falling a height \( h \) so

\[
v_0 = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(2.00 \text{ m})} = 6.26 \text{ m/s}
\]

The velocity of the sand just before it hits the truck is \( v_0 = -6.26 \text{ m/s} \), where the downward direction is taken to be the negative direction. The final velocity of the sand is \( v_f = 0 \text{ m/s} \). Thus, the average force exerted on the sand is

\[
\vec{F} = \left( \frac{m}{\Delta t} \right)(v_f - v_0) = (55.0 \text{ kg/s})[(0 \text{ m/s}) - (-6.26 \text{ m/s})] = +344 \text{ N}
\]

16. **REASONING** Bonzo and Ender constitute an isolated system. Therefore, the principle of conservation of linear momentum holds. Since both members of the system are initially stationary, the system's initial linear momentum is zero and must remain zero during and after any interaction between Bonzo and Ender, since they interact via internal (not external) forces.
**SOLUTION**

a. Since the total momentum of the system is conserved, and the total momentum of the system is zero, Bonzo and Ender must have equal but opposite linear momenta. Bonzo has the larger mass, since he has the smaller recoil velocity.

b. Conservation of linear momentum gives $0 = m_{\text{Bonzo}}v_{\text{Bonzo}} + m_{\text{Ender}}v_{\text{Ender}}$. Solving for the ratio of the masses, we have

$$\frac{m_{\text{Bonzo}}}{m_{\text{Ender}}} = -\frac{v_{\text{Ender}}}{v_{\text{Bonzo}}} = -\frac{-2.5 \text{ m/s}}{1.5 \text{ m/s}} = 1.7$$

**17. REASONING** The total momentum of the two-cart system is the sum of their individual momenta. In part a, therefore, we will use $P_f = m_1v_{f1} + m_2v_{f2}$ to calculate the system’s total momentum $P_f$ when both carts are rolling. In order to find the initial velocity $v_{01}$ of the first cart in part b, we need to know the system’s initial momentum $P_0 = m_1v_{01} + m_2v_{02}$. According to the principle of conservation of linear momentum, this is equal to the system’s final momentum $P_f$ if the system is isolated. Let’s see if it is. The attractive magnetic forces felt by the carts are internal forces and do not, therefore, affect the application of the conservation principle. Friction is negligible, and the external gravitational force and normal forces balance out on the level track. Therefore, the net external force on the system is zero, and the system is indeed isolated. We conclude that the system’s initial momentum $P_0$ is identical to the final momentum $P_f$ found in part a.

**SOLUTION**

a. We calculate the system’s final momentum directly from the relation $P_f = m_1v_{f1} + m_2v_{f2}$, taking care to use the algebraic signs indicating the directions of the velocity and momentum vectors:

$$P_f = (2.3 \text{ kg})(+4.5 \text{ m/s}) + (1.5 \text{ kg})(-1.9 \text{ m/s}) = +7.5 \text{ kg} \cdot \text{m/s}$$

b. The system’s initial momentum is the sum of the carts’ initial momenta: $P_0 = m_1v_{01} + m_2v_{02}$. But the second cart is held initially at rest, so $v_{02} = 0 \text{ m/s}$. Setting the initial momentum of the system equal to its final momentum, we have $P_0 = P_f = m_1v_{01}$. Solving for the first cart’s initial velocity, we obtain

$$v_{01} = \frac{P_f}{m_1} = \frac{+7.5 \text{ kg} \cdot \text{m/s}}{2.3 \text{ kg}} = +3.3 \text{ m/s}$$

**18. REASONING** Since friction between the disks and the air-hockey table is negligible, and the weight of each disk is balanced by an upward-acting normal force, the net external force acting on the disks and spring is zero. Therefore, the two-disk system (including the spring) is an isolated system, and the total linear momentum of the system remains constant.
With $v_{01}$ and $v_{f1}$ denoting the initial and final velocities of disk 1 and $v_{02}$ and $v_{f2}$ denoting the initial and final velocities of disk 2, the conservation of linear momentum states that

$$\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum after}} = \frac{m_1 v_{01} + m_2 v_{02}}{\text{Total momentum before}}$$

spring is released

spring is released

Solving this equation for $v_{f2}$ yields the final velocity of disk 2.

**SOLUTION** Using the fact that $v_{01} = v_{02} = v_0 = (+5.0 \text{ m/s})$, and remembering that disk 1 comes to a halt after the spring is released ($v_{f1} = 0 \text{ m/s}$), we have

$$v_{f2} = \frac{m_1 v_{01} + m_2 v_{02} - m_1 v_{f1}}{m_2} = \frac{m_1 v_0 + m_2 v_0 - m_1 v_{f1}}{m_2}$$

$$= \frac{(1.2 \text{ kg})(+5.0 \text{ m/s}) + (2.4 \text{ kg})(+5.0 \text{ m/s}) - (1.2 \text{ kg})(0 \text{ m/s})}{2.4 \text{ kg}} = 7.5 \text{ m/s}$$

19. **SSM REASONING** The system consists of the lumberjack and the log. For this system, the sum of the external forces is zero. This is because the weight of the system is balanced by the corresponding normal force (provided by the buoyant force of the water) and the water is assumed to be frictionless. The lumberjack and the log, then, constitute an isolated system, and the principle of conservation of linear momentum holds.

**SOLUTION**

a. The total linear momentum of the system before the lumberjack begins to move is zero, since all parts of the system are at rest. Momentum conservation requires that the total momentum remains zero during the motion of the lumberjack.

$$\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum}} = \frac{0}{\text{Initial momentum}}$$

just before the jump

Here the subscripts "1" and "2" refer to the first log and lumberjack, respectively. Let the direction of motion of the lumberjack be the positive direction. Then, solving for $v_{1f}$ gives

$$v_{f1} = -\frac{m_2 v_{f2}}{m_1} = -\frac{(98 \text{ kg})(+3.6 \text{ m/s})}{230 \text{ kg}} = -1.5 \text{ m/s}$$

The minus sign indicates that the first log recoils as the lumberjack jumps off.

b. Now the system is composed of the lumberjack, just before he lands on the second log, and the second log. Gravity acts on the system, but for the short time under consideration while the lumberjack lands, the effects of gravity in changing the linear momentum of the system are negligible. Therefore, to a very good approximation, we can say that the linear momentum of the system is very nearly conserved. In this case, the initial momentum is not
zero as it was in part (a); rather the initial momentum of the system is the momentum of the lumberjack just before he lands on the second log. Therefore,

\[ m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02} \]

In this expression, the subscripts "1" and "2" now represent the second log and lumberjack, respectively. Since the second log is initially at rest, \( v_{01} = 0 \). Furthermore, since the lumberjack and the second log move with a common velocity, \( v_{f1} = v_{f2} = v_f \). The statement of momentum conservation then becomes

\[ m_1 v_f + m_2 v_f = m_2 v_{02} \]

Solving for \( v_f \), we have

\[ v_f = \frac{m_2 v_{02}}{m_1 + m_2} = \frac{(98 \text{ kg})(+3.6 \text{ m/s})}{230 \text{ kg} + 98 \text{ kg}} = +1.1 \text{ m/s} \]

The positive sign indicates that the system moves in the same direction as the original direction of the lumberjack's motion.

20. **REASONING** Let the total amount of gas in the completely filled propulsion unit be \( m_{\text{total}} \), and the amount of gas ejected during the space-walk be \( m_{\text{ejected}} \). The total mass of the gas is the sum of the mass \( m_{\text{ejected}} \) of the ejected gas and the mass \( m_{\text{left}} \) of the gas left over at the end of the space-walk: \( m_{\text{total}} = m_{\text{ejected}} + m_{\text{left}} \). The mass of the leftover gas, in turn, is the difference between the mass \( m_1 = 165 \text{ kg} \) of the astronaut with the partially full propulsion unit and the mass \( m_{\text{empty}} = 146 \text{ kg} \) of the astronaut with the completely empty propulsion unit: \( m_{\text{left}} = m_1 - m_{\text{empty}} = 165 \text{ kg} - 146 \text{ kg} = 19 \text{ kg} \). Thus, the percentage of gas propellant in the completely filled propulsion unit that is depleted during the space walk is

\[ \text{Percent depleted} = \frac{m_{\text{ejected}}}{m_{\text{total}}} \times 100\% = \frac{m_{\text{ejected}}}{m_{\text{ejected}} + m_{\text{left}}} \times 100\% \] (1)

We will find \( m_{\text{ejected}} \) by considering the space-walk. During the space-walk, the astronaut, the propulsion unit, and the gas form an isolated system. Although the propulsion unit and the ejected gas exert forces on one another, as do the astronaut and the propulsion unit, these are internal forces. The parts of the system interact only with each other, so the net external force on the system is zero. Hence, the total momentum of the system is conserved. We will apply the momentum conservation principle \( m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02} \) (Equation 7.7b), with the following identifications:

\[ m_1 = 165 \text{ kg} \text{ (mass of astronaut and partially empty propulsion unit)} \]
\[ m_2 = m_{\text{ejected}} \text{ ? (mass of ejected gas)} \]
\[ v_{f1} = -0.39 \text{ m/s} \text{ (final velocity of astronaut and partially empty propulsion unit)} \]
\[ v_{f2} = +32 \text{ m/s} \text{ (final velocity of ejected gas)} \]
No part of the system is moving before the gas is ejected, so we also have $v_{01} = v_{02} = 0 \text{ m/s}$. We will determine the unknown mass $m_2 = m_{\text{ejected}}$ of the ejected gas from Equation 7.7b.

**SOLUTION** The momentum conservation principle yields the mass $m_2 = m_{\text{ejected}}$ of the ejected gas:

$$\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum after gas is ejected}} = \frac{0}{\text{Total momentum before gas is ejected}} \quad \text{or} \quad m_2 = \frac{-m_1 v_{f1}}{v_{f2}} = \frac{-(165 \text{ kg})(-0.39 \text{ m/s})}{(+32 \text{ m/s})} = 2.0 \text{ kg}$$

Equation (1) then yields the percent of propellant gas depleted during the space-walk:

$$\text{Percent depleted} = \frac{m_{\text{ejected}}}{m_{\text{ejected}} + m_{\text{left}}} \times 100\% = \frac{2.0 \text{ kg}}{2.0 \text{ kg} + 19 \text{ kg}} \times 100\% = 9.5\%$$

21. **REASONING** The two-stage rocket constitutes the system. The forces that act to cause the separation during the explosion are, therefore, forces that are internal to the system. Since no external forces act on this system, it is isolated and the principle of conservation of linear momentum applies:

$$\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum after separation}} = \frac{(m_1 + m_2)v_0}{\text{Total momentum before separation}}$$

where the subscripts "1" and "2" refer to the lower and upper stages, respectively. This expression can be solved for $v_{f1}$.

**SOLUTION** Solving for $v_{f1}$ gives

$$v_{f1} = \frac{(m_1 + m_2)v_0 - m_2 v_{f2}}{m_1}$$

$$= \frac{2400 \text{ kg} + 1200 \text{ kg} \ (4900 \text{ m/s}) - (1200 \text{ kg})(5700 \text{ m/s})}{2400 \text{ kg}} = +4500 \text{ m/s}$$

Since $v_{f1}$ is positive, its direction is the same as the rocket before the explosion.

22. **REASONING** We consider the boy and the skateboard as a single system. Friction between the skateboard and the sidewalk is minimal, so there is no net horizontal force acting on the system. Therefore, the horizontal component of the system’s linear momentum remains
constant while the boy pushes off from the skateboard: \(m_1v_{f1x} + m_2v_{f2x} = m_1v_{01x} + m_2v_{02x}\) (Equation 7.9a). We will solve this equation to find the final velocity of the skateboard.

**SOLUTION** The data given in the problem may be summarized as follows:

<table>
<thead>
<tr>
<th></th>
<th>Mass</th>
<th>Horizontal velocity component</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
<td>Final</td>
<td></td>
</tr>
<tr>
<td>Boy</td>
<td>(m_1 = 40.0) kg</td>
<td>(v_{01x} = v_0 = +5.30) m/s</td>
<td>(v_{f1x} = +(6.00) m/s)(cos 9.50°) = +5.92 m/s</td>
</tr>
<tr>
<td>Skateboard</td>
<td>(m_2 = 2.50) kg</td>
<td>(v_{02x} = v_0 = +5.30) m/s</td>
<td>(v_{f2x} = ?)</td>
</tr>
</tbody>
</table>

Solving equation 7.9a for the final horizontal component of the skateboard’s velocity, we obtain

\[
m_2v_{f2x} = m_1v_0 + m_2v_0 - m_1v_{f1x} \quad \text{or} \quad v_{f2x} = \frac{m_1(v_0 - v_{f1x}) + m_2v_0}{m_2}
\]

Thus,

\[
v_{f2x} = \frac{m_1(v_0 - v_{f1x})}{m_2} + v_0 = \frac{(40.0\) kg}(+5.30\) m/s - 5.92 m/s\) + 5.30 m/s = -4.6 m/s
\]

23. **REASONING** For the system consisting of the female character, the gun and the bullet, the sum of the external forces is zero, because the weight of each object is balanced by a corresponding upward (normal) force, and we are ignoring friction. The female character, the gun and the bullet, then, constitute an isolated system, and the principle of conservation of linear momentum applies.

**SOLUTION** a. The total momentum of the system before the gun is fired is zero, since all parts of the system are at rest. Momentum conservation requires that the total momentum remains zero after the gun has been fired.

\[
\frac{m_1v_{f1} + m_2v_{f2}}{m_1v_{f1}} + m_2v_{f2} = \frac{0}{m_2v_{f2}}
\]

where the subscripts 1 and 2 refer to the woman (plus gun) and the bullet, respectively. Solving for \(v_{f1}\), the recoil velocity of the woman (plus gun), gives

\[
v_{f1} = -\frac{m_2v_{f2}}{m_1} = \frac{-(0.010\) kg}(720 m/s) = -0.14 m/s
\]

51 kg
b. Repeating the calculation for the situation in which the woman shoots a blank cartridge, we have

\[ v_{f1} = \frac{-m_2 v_{f2}}{m_1} = \frac{-(5.0 \times 10^{-4} \text{ kg})(720 \text{ m/s})}{51 \text{ kg}} = -7.1 \times 10^{-3} \text{ m/s} \]

In both cases, the minus sign means that the bullet and the woman move in opposite directions when the gun is fired. The total momentum of the system remains zero, because momentum is a vector quantity, and the momenta of the bullet and the woman have equal magnitudes, but opposite directions.

24. **REASONING** We will divide the problem into two parts: (a) the motion of the freely falling block after it is dropped from the building and before it collides with the bullet, and (b) the collision of the block with the bullet.

During the falling phase we will use an equation of kinematics that describes the velocity of the block as a function of time (which is unknown). During the collision with the bullet, the external force of gravity acts on the system. This force changes the momentum of the system by a negligibly small amount since the collision occurs over an extremely short time interval. Thus, to a good approximation, the sum of the external forces acting on the system during the collision is negligible, so the linear momentum of the system is conserved. The principle of conservation of linear momentum can be used to provide a relation between the momenta of the system before and after the collision. This relation will enable us to find a value for the time it takes for the bullet/block to reach the top of the building.

**SOLUTION** Falling from rest \((v_{0,\text{block}} = 0 \text{ m/s})\), the block attains a final velocity \(v_{\text{block}}\) just before colliding with the bullet. This velocity is given by Equation 2.4 as

\[ v_{\text{block}} = v_{0,\text{block}} + at \]

where \(a\) is the acceleration due to gravity \((a = -9.8 \text{ m/s}^2)\) and \(t\) is the time of fall. The upward direction is assumed to be positive. Therefore, the final velocity of the falling block is

\[ v_{\text{block}} = at \]

During the collision with the bullet, the total linear momentum of the bullet/block system is conserved, so we have that

\[ \left( m_{\text{bullet}} + m_{\text{block}} \right) v_f = m_{\text{bullet}} v_{\text{bullet}} + m_{\text{block}} v_{\text{block}} \]

Here \(v_f\) is the final velocity of the bullet/block system after the collision, and \(v_{\text{bullet}}\) and \(v_{\text{block}}\) are the initial velocities of the bullet and block just before the collision. We note that the bullet/block system reverses direction, rises, and comes to a momentary halt at the top of
the building. This means that \( v_f \), the final velocity of the bullet/block system after the collision must have the same magnitude as \( v_{\text{block}} \), the velocity of the falling block just before the bullet hits it. Since the two velocities have opposite directions, it follows that \( v_f = -v_{\text{block}} \). Substituting this relation and Equation (1) into Equation (2) gives

\[
(m_{\text{bullet}} + m_{\text{block}})(-at) = m_{\text{bullet}} v_{\text{bullet}} + m_{\text{block}} (at)
\]

Solving for the time, we find that

\[
t = \frac{-m_{\text{bullet}} v_{\text{bullet}}}{a(m_{\text{bullet}} + 2m_{\text{block}})} = \frac{-0.015 \text{ kg}(810 \text{ m/s})}{(-9.80 \text{ m/s}^2)[(0.015 \text{ kg}) + 2(1.8 \text{ kg})]} = 0.34 \text{ s}
\]

25. **REASONING** No net external force acts on the plate parallel to the floor; therefore, the component of the momentum of the plate that is parallel to the floor is conserved as the plate breaks and flies apart. Initially, the total momentum parallel to the floor is zero. After the collision with the floor, the component of the total momentum parallel to the floor must remain zero. The drawing in the text shows the pieces in the plane parallel to the floor just after the collision. Clearly, the linear momentum in the plane parallel to the floor has two components; therefore the linear momentum of the plate must be conserved in each of these two mutually perpendicular directions. Using the drawing in the text, with the positive directions taken to be up and to the right, we have

\[
x \text{ direction} \quad -m_1 v_1 \sin 25.0^\circ + m_2 v_2 \cos 45.0^\circ = 0 \quad (1)
\]

\[
y \text{ direction} \quad m_1 v_1 \cos 25.0^\circ + m_2 v_2 \sin 45.0^\circ - m_3 v_3 = 0 \quad (2)
\]

These equations can be solved simultaneously for the masses \( m_1 \) and \( m_2 \).

**SOLUTION** Using the values given in the drawing for the velocities after the plate breaks, we have,

\[
-m_1 (3.00 \text{ m/s}) \sin 25.0^\circ + m_2 (1.79 \text{ m/s}) \cos 45.0^\circ = 0 \quad (1)
\]

\[
m_1 (3.00 \text{ m/s}) \cos 25.0^\circ + m_2 (1.79 \text{ m/s}) \sin 45.0^\circ - (1.30 \text{ kg})(3.07 \text{ m/s}) = 0 \quad (2)
\]

Subtracting (2) from (1), and noting that \( \cos 45.0^\circ = \sin 45.0^\circ \), gives \( m_1 = 1.00 \text{ kg} \).

Substituting this value into either (1) or (2) then yields \( m_2 = 1.00 \text{ kg} \).

26. **REASONING** The only forces acting on the two men following push-off are the force due to the tension in the ropes and the gravitational force. The tension force acts perpendicular to the swinging motion and, thus, does no work. The gravitational force is conservative.
Therefore, the principle of conservation of mechanical energy applies to the swinging motion of both men. Moreover, Adolf and Ed constitute an isolated system during push-off, since the gravitational force is balanced by the tension force from the rope for each man. Therefore, no net external force acts on the system, and the principle of conservation of linear momentum holds during push-off. We will use both conservation principles.

**SOLUTION** To find the height \( h_E \) to which Ed rises after push-off, we use conservation of mechanical energy. We measure \( h_E \) relative to Ed’s initial position, where the height is zero. Note that at the top of his swing, Ed’s speed is momentarily zero and he has no kinetic energy:

\[
\frac{m_E g h_E}{2} = \frac{1}{2} m_E v_E^2 \quad \text{or} \quad h_E = \frac{v_E^2}{2g} \quad (1)
\]

Next, we use conservation of momentum to find Ed’s speed \( v_E \) just after push-off. In doing so, we note that before push-off both men are at rest and, therefore, have zero momentum:

\[
\frac{m_E v_E + m_A v_A}{2} = 0 \quad \text{or} \quad v_E = -\frac{m_A v_A}{m_E} \quad (2)
\]

Substituting Equation (2) into Equation (1) gives

\[
h_E = \frac{v_E^2}{2g} = \frac{(-m_A v_A / m_E)^2}{2g} = \frac{m_A^2 v_A^2}{m_E^2 2g} \quad (3)
\]

To use Equation (3), we need a value for Adolf’s speed \( v_A \) just after push-off. We can obtain it by using conservation of mechanical energy and the height \( h_A \) to which Adolf swings above his initial or zero height. Note that at the top of his swing, Adolf’s speed is momentarily zero and he has no kinetic energy:

\[
\frac{m_A g h_A}{2} = \frac{1}{2} m_A v_A^2 \quad \text{or} \quad v_A = \sqrt{2 g h_A} \quad (4)
\]

Substituting Equation (4) into Equation (3) gives

\[
h_E = \frac{m_A^2 v_A^2}{m_E^2 2g} \quad \frac{m_A^2 \left(\sqrt{2 g h_A}\right)^2}{m_E^2 2g} \quad = \frac{m_A^2 h_A}{m_E} = \frac{(120 \text{ kg})^2 \cdot (0.65 \text{ m})}{(78 \text{ kg})^2} = 1.5 \text{ m}
\]
27. **REASONING**  The cannon and the shell constitute the system. Since no external force hinders the motion of the system after the cannon is unbolted, conservation of linear momentum applies in that case. If we assume that the burning gun powder imparts the same kinetic energy to the system in each case, we have sufficient information to develop a mathematical description of this situation, and solve it for the velocity of the shell fired by the loose cannon.

**SOLUTION**  For the case where the cannon is unbolted, momentum conservation gives

\[
\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum after shell is fired}} = \frac{0}{\text{Initial momentum of system}}
\]

where the subscripts "1" and "2" refer to the cannon and shell, respectively. In both cases, the burning gun powder imparts the same kinetic energy to the system. When the cannon is bolted to the ground, only the shell moves and the kinetic energy imparted to the system is

\[
\text{KE} = \frac{1}{2} m_{\text{shell}} v_{\text{shell}}^2 = \frac{1}{2} (85.0 \text{ kg})(551 \text{ m/s})^2 = 1.29 \times 10^7 \text{ J}
\]

The kinetic energy imparted to the system when the cannon is unbolted has the same value and can be written using the same notation as in equation (1):

\[
\text{KE} = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2
\]

Solving equation (1) for \(v_{f1}\), the velocity of the cannon after the shell is fired, and substituting the resulting expression into Equation (2) gives

\[
\text{KE} = \frac{m_2^2 v_{f2}^2}{2m_1} + \frac{1}{2} m_2 v_{f2}^2
\]

Solving equation (3) for \(v_{f2}\) gives

\[
v_{f2} = \sqrt{\frac{2 \text{KE}}{m_2 \left( \frac{m_2}{m_1} + 1 \right)}} = \sqrt{\frac{2(1.29 \times 10^7 \text{ J})}{(85.0 \text{ kg}) \left( \frac{85.0 \text{ kg}}{5.80 \times 10^3 \text{ kg}} + 1 \right)}} = +547 \text{ m/s}
\]

28. **REASONING**  Together, Ashley and Miranda constitute an isolated system, since their combined weight is balanced by an upward normal force, and friction is negligible. The total momentum of the system is, therefore, conserved when Miranda hops onto the tube. We will use the momentum conservation principle \(m_1 v_{f1} + m_2 v_{f2} = m_1 v_{01} + m_2 v_{02}\) (Equation 7.7b) to analyze this one-dimensional collision. We are ignoring the mass and momentum of the inner tube.
**SOLUTION** After Miranda \((m_2 = 58 \text{ kg})\) jumps onto the inner tube, she and Ashley \((m_1 = 71 \text{ kg})\) both have the same final velocity: \(v_f = v_{f1} = v_{f2}\). Making this substitution in Equation 7.7b, and solving for their common final velocity, we obtain

\[
\frac{m_1 v_f + m_2 v_f}{\text{Total momentum after Miranda hops on}} = \frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum before Miranda hops on}}
\]

Their common velocity after Miranda hops on is, therefore,

\[
v_f = \frac{m_1 v_{f1} + m_2 v_{f2}}{m_1 + m_2} = \frac{(71 \text{ kg})(+2.7 \text{ m/s}) + (58 \text{ kg})(+4.5 \text{ m/s})}{71 \text{ kg} + 58 \text{ kg}} = +3.5 \text{ m/s}
\]

The common speed is the magnitude of this value or \(3.5 \text{ m/s}\).

29. **REASONING** a. During the collision between the bullet and the wooden block, linear momentum is conserved, since no net external force acts on the bullet and the block. The weight of each is balanced by the tension in the suspension wire, and the forces that the bullet and block exert on each other are internal forces. This conservation law will allow us to find the speed of the bullet/block system immediately after the collision.

b. Just after the collision, the bullet/block rise up, ultimately reaching a final height \(h_f\) before coming to a momentary rest. During this phase, the tension in the wire (a nonconservative force) does no work, since it acts perpendicular to the motion. Thus, the work done by nonconservative forces is zero, and the total mechanical energy of the system is conserved. An application of this conservation law will enable us to determine the height \(h_f\).

**SOLUTION** a. The principle of conservation of linear momentum states that the total momentum after the collision is equal to that before the collision.

\[
(\text{momentum after collision}) = (\text{momentum before collision})
\]

Solving this equation for the speed \(v_f\) of the bullet/block system just after the collision gives

\[
v_f = \frac{m_{\text{bullet}} v_{0,\text{bullet}} + m_{\text{block}} v_{0,\text{block}}}{m_{\text{bullet}} + m_{\text{block}}}
\]

\[
= \frac{(0.00250 \text{ kg})(425 \text{ m/s}) + (0.215 \text{ kg})(0 \text{ m/s})}{0.00250 \text{ kg} + 0.215 \text{ kg}} = 4.89 \text{ m/s}
\]

b. Just after the collision, the total mechanical energy of the system is all kinetic energy, since we take the zero-level for the gravitational potential energy to be at the initial height of the block. As the bullet/block system rises, kinetic energy is converted into potential energy.
At the highest point, the total mechanical energy is all gravitational potential energy. Since the total mechanical energy is conserved, we have

\[
\left( m_{\text{bullet}} + m_{\text{block}} \right) g h_f = \frac{1}{2} \left( m_{\text{bullet}} + m_{\text{block}} \right) v_f^2
\]

Solving this expression for the height \( h_f \) gives

\[
h_f = \frac{\frac{1}{2} v_f^2}{g} = \frac{1}{2} \left( \frac{4.89 \text{ m/s}}{9.80 \text{ m/s}^2} \right)^2 = 1.22 \text{ m}
\]

30. **REASONING** Since momentum is conserved, the total momentum of the two-object system after the collision must be the same as it was before the collision. Momentum is mass times velocity. Since one of the objects is at rest initially, the total initial momentum comes only from the moving object.

Let \( m_1 \) and \( v_{01} \) be, respectively, the mass and initial velocity of the moving object before the collision. In addition, \( m \) and \( v_f \) are the total mass and final velocity of the two objects (which stick together) after the collision. The conservation of linear momentum can be written as

\[
m v_f = m_1 v_{01}
\]

Solving this equation for \( v_f \), the final velocity of the two-object system gives

\[
v_f = \frac{m_1 v_{01}}{m}
\]

**SOLUTION**

**Large-mass object (8.0 kg) moving initially:** Assume that, before the collision, the object is moving in the + direction so that \( v_{01} = +25 \text{ m/s} \). Then,

\[
v_f = \frac{m_1 v_{01}}{m} = \frac{(8.0 \text{ kg}) (25 \text{ m/s})}{3.0 \text{ kg} + 8.0 \text{ kg}} = +18 \text{ m/s}
\]

The final speed is \[18 \text{ m/s}].

**Small-mass object (3.0 kg) moving initially:**

\[
v_f = \frac{m_1 v_{01}}{m} = \frac{(3.0 \text{ kg}) (25 \text{ m/s})}{3.0 \text{ kg} + 8.0 \text{ kg}} = +6.8 \text{ m/s}
\]

The final speed is \[6.8 \text{ m/s}].
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31. **REASONING** Batman and the boat with the criminal constitute the system. Gravity acts on this system as an external force; however, gravity acts vertically, and we are concerned only with the horizontal motion of the system. If we neglect air resistance and friction, there are no external forces that act horizontally; therefore, the total linear momentum in the horizontal direction is conserved. When Batman collides with the boat, the horizontal component of his velocity is zero, so the statement of conservation of linear momentum in the horizontal direction can be written as

$$\frac{(m_1 + m_2)v_f}{m_1 + m_2} = \frac{m_1v_{01} + 0}{m_1}$$

Here, $m_1$ is the mass of the boat, and $m_2$ is the mass of Batman. This expression can be solved for $v_f$, the velocity of the boat after Batman lands in it.

**SOLUTION** Solving for $v_f$ gives

$$v_f = \frac{m_1v_{01}}{m_1 + m_2} = \frac{(510 \text{ kg})(+11 \text{ m/s})}{510 \text{ kg} + 91 \text{ kg}} = +9.3 \text{ m/s}$$

The plus sign indicates that the boat continues to move in its initial direction of motion.

32. **REASONING** The weight of each vehicle is balanced by the normal force exerted by the road. Assuming that friction and other resistive forces can be ignored, we will treat the two-vehicle system as an isolated system and apply the principle of conservation of linear momentum.

**SOLUTION** Using $v_{0, \text{car}}$ and $v_{0, \text{SUV}}$ to denote the velocities of the vehicles before the collision and applying the principle of conservation of linear momentum, we have

$$0 = \frac{m_{\text{car}}v_{0, \text{car}} + m_{\text{SUV}}v_{0, \text{SUV}}}{m_{\text{car}} + m_{\text{SUV}}}$$

Note that the total momentum of both vehicles after the collision is zero, because the collision brings each vehicle to a halt. Solving this result for $v_{0, \text{SUV}}$ and taking the direction in which the car moves as the positive direction gives

$$v_{0, \text{SUV}} = \frac{-m_{\text{car}}v_{0, \text{car}}}{m_{\text{SUV}}} = \frac{-(1100 \text{ kg})(32 \text{ m/s})}{2500 \text{ kg}} = -14 \text{ m/s}$$

This result is negative, since the velocity of the sport utility vehicle is opposite to that of the car, which has been chosen to be positive. The speed of the sport utility vehicle is the magnitude of $v_{0, \text{SUV}}$ or $14 \text{ m/s}$.
33. **REASONING** The system consists of the two balls. The total linear momentum of the two-ball system is conserved because the net external force acting on it is zero. The principle of conservation of linear momentum applies whether or not the collision is elastic.

\[
\frac{m_1 v_{f1} + m_2 v_{f2}}{\text{Total momentum after collision}} = \frac{m_1 v_{o1} + 0}{\text{Total momentum before collision}}
\]

When the collision is elastic, the kinetic energy is also conserved during the collision

\[
\frac{\frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2}{\text{Total kinetic energy after collision}} = \frac{\frac{1}{2}m_1 v_{o1}^2 + 0}{\text{Total kinetic energy before collision}}
\]

**SOLUTION**

a. The final velocities for an elastic collision are determined by simultaneously solving the above equations for the final velocities. The procedure is discussed in Example 7 in the text, and leads to Equations 7.8a and 7.8b. According to Equation 7.8:

\[
v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{o1} \quad \text{and} \quad v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{o1}
\]

Let the initial direction of motion of the 5.00-kg ball define the positive direction. Substituting the values given in the text, these equations give

- **5.00-kg ball**
  \[
v_{f1} = \left(\frac{5.00 \text{ kg} - 7.50 \text{ kg}}{5.00 \text{ kg} + 7.50 \text{ kg}}\right)(2.00 \text{ m/s}) = \frac{-0.400 \text{ m/s}}{}
\]

- **7.50-kg ball**
  \[
v_{f2} = \left(\frac{2(5.00 \text{ kg})}{5.00 \text{ kg} + 7.50 \text{ kg}}\right)(2.00 \text{ m/s}) = \frac{+1.60 \text{ m/s}}{}
\]

The signs indicate that, after the collision, the 5.00-kg ball reverses its direction of motion, while the 7.50-kg ball moves in the direction in which the 5.00-kg ball was initially moving.

b. When the collision is completely inelastic, the balls stick together, giving a composite body of mass \(m_1 + m_2\) that moves with a velocity \(v_f\). The statement of conservation of linear momentum then becomes

\[
\frac{(m_1 + m_2)v_f}{\text{Total momentum after collision}} = \frac{m_1 v_{o1} + 0}{\text{Total momentum before collision}}
\]

The final velocity of the two balls after the collision is, therefore,

\[
v_f = \frac{m_1 v_{o1}}{m_1 + m_2} = \frac{(5.00 \text{ kg})(2.00 \text{ m/s})}{5.00 \text{ kg} + 7.50 \text{ kg}} = \frac{+0.800 \text{ m/s}}{}
\]
34. **REASONING** The net external force acting on the two-puck system is zero (the weight of each ball is balanced by an upward normal force, and we are ignoring friction due to the layer of air on the hockey table). Therefore, the two pucks constitute an isolated system, and the principle of conservation of linear momentum applies.

**SOLUTION** Conservation of linear momentum requires that the total momentum is the same before and after the collision. Since linear momentum is a vector, the $x$ and $y$ components must be conserved separately. Using the drawing in the text, momentum conservation in the $x$ direction yields

$$m_A v_{0A} = m_A v_{fA} (\cos 65^\circ) + m_B v_{fB} (\cos 37^\circ) \tag{1}$$

while momentum conservation in the $y$ direction yields

$$0 = m_A v_{fA} (\sin 65^\circ) - m_B v_{fB} (\sin 37^\circ) \tag{2}$$

Solving equation (2) for $v_{fB}$, we find that

$$v_{fB} = \frac{m_A v_{fA} (\sin 65^\circ)}{m_B (\sin 37^\circ)} \tag{3}$$

Substituting equation (3) into Equation (1) leads to

$$m_A v_{0A} = m_A v_{fA} (\cos 65^\circ) + \left[ \frac{m_A v_{fA} (\sin 65^\circ)}{\sin 37^\circ} \right] (\cos 37^\circ)$$

a. Solving for $v_{fA}$ gives

$$v_{fA} = \frac{v_{0A}}{\cos 65^\circ + \left( \frac{\sin 65^\circ}{\tan 37^\circ} \right)} = \frac{+5.5 \text{ m/s}}{\cos 65^\circ + \left( \frac{\sin 65^\circ}{\tan 37^\circ} \right)} = 3.4 \text{ m/s}$$

b. From equation (3), we find that

$$v_{fB} = \frac{(0.025 \text{ kg})(3.4 \text{ m/s})(\sin 65^\circ)}{(0.050 \text{ kg})(\sin 37^\circ)} = 2.6 \text{ m/s}$$

35. **SSM REASONING** We obtain the desired percentage in the usual way, as the kinetic energy of the target (with the projectile in it) divided by the projectile’s incident kinetic energy, multiplied by a factor of 100. Each kinetic energy is given by Equation 6.2 $\frac{1}{2} mv^2$, where $m$ and $v$ are mass and speed, respectively. Data for the masses are given, but the speeds are not provided. However, information about the speeds can be obtained by using the principle of conservation of linear momentum.
**SOLUTION** We define the following quantities:

- \( KE_{TP} \) = kinetic energy of the target with the projectile in it
- \( KE_{0P} \) = kinetic energy of the incident projectile
- \( m_p \) = mass of incident projectile = 0.20 kg
- \( m_T \) = mass of target = 2.50 kg
- \( v_f \) = speed at which the target with the projectile in it flies off after being struck
- \( v_{0P} \) = speed of incident projectile

The desired percentage is

\[
\text{Percentage} = \frac{KE_{TP}}{KE_{0P}} \times 100 = \frac{1}{2} \left( \frac{m_T + m_p}{m_p} \right) v_f^2 \times 100\%
\]  

(1)

According to the momentum-conservation principle, we have

\[
\frac{(m_T + m_p) \cdot v_f}{\text{Total momentum of target and projectile after target is struck}} = \frac{0 + m_p \cdot v_{0P}}{\text{Total momentum of target and projectile before target is struck}}
\]

Note that the target is stationary before being struck and, hence, has zero initial momentum. Solving for the ratio \( v_f / v_{0P} \), we find that

\[
\frac{v_f}{v_{0P}} = \frac{m_p}{m_T + m_p}
\]

Substituting this result into Equation (1) gives

\[
\text{Percentage} = \frac{1}{2} \left( \frac{m_T + m_p}{m_p} \right) \left( \frac{m_p}{m_T + m_p} \right)^2 \times 100\%
\]

\[
= \frac{m_p}{m_T + m_p} \times 100\% = \frac{0.20 \text{ kg}}{2.50 \text{ kg} + 0.20 \text{ kg}} \times 100\% = 7.4\%
\]

36. **REASONING** According to the momentum-conservation principle, the final total momentum is the same as the initial total momentum. The initial total momentum is the vector sum of the two initial momentum vectors of the objects. One of the vectors (\( \mathbf{p}_{0A} \)) points due east and one (\( \mathbf{p}_{0B} \)) due north, so that they are perpendicular (see the following drawing on the left).
Based on the conservation principle, the direction of the final total momentum $P_f$ must be the same as the direction of the initial total momentum. From the drawing on the right, we can see that the initial total momentum has a component $p_{0A}$ pointing due east and a component $p_{0B}$ pointing due north. The final total momentum $P_f$ has these same components and, therefore, must point north of east at an angle $\theta$.

**SOLUTION** Based on the conservation of linear momentum, we know that the magnitude of the final total momentum is the same as the magnitude of the initial total momentum. Using the Pythagorean theorem with the initial momentum vectors of the two objects, we find that the magnitude of the final total momentum is

$$P_f = \sqrt{P_{0A}^2 + P_{0B}^2} = \sqrt{(m_A v_{0A})^2 + (m_B v_{0B})^2}$$

$$P_f = \sqrt{(17.0 \text{ kg})^2 (8.00 \text{ m/s})^2 + (29.0 \text{ kg})^2 (5.00 \text{ m/s})^2} = 199 \text{ kg} \cdot \text{m/s}$$

The direction is given by

$$\theta = \tan^{-1}\left(\frac{p_{0B}}{p_{0A}}\right) = \tan^{-1}\left(\frac{m_B v_{0B}}{m_A v_{0A}}\right) = \tan^{-1}\left[\frac{(29.0 \text{ kg})(5.00 \text{ m/s})}{(17.0 \text{ kg})(8.00 \text{ m/s})}\right] = 46.8^\circ \text{ north of east}$$

**37. REASONING** The velocity of the second ball just after the collision can be found from Equation 7.8b (see Example 7). In order to use Equation 7.8b, however, we must know the velocity of the first ball just before it strikes the second ball. Since we know the impulse delivered to the first ball by the pool stick, we can use the impulse-momentum theorem (Equation 7.4) to find the velocity of the first ball just before the collision.

**SOLUTION** According to the impulse-momentum theorem, $F \Delta t = m v_f - m v_0$, and setting $v_0 = 0 \text{ m/s}$ and solving for $v_f$, we find that the velocity of the first ball after it is struck by the pool stick and just before it hits the second ball is
\[ v_f = \frac{\bar{F} \Delta t}{m} = \frac{+1.50 \text{ N} \cdot \text{s}}{0.165 \text{ kg}} = 9.09 \text{ m/s} \]

Substituting values into Equation 7.8b (with \( v_{01} = 9.09 \text{ m/s} \)), we have

\[ v_{f2} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{01} = \left( \frac{2m}{m + m} \right) v_{01} = v_{01} = +9.09 \text{ m/s} \]

38. **REASONING**

a. Since air resistance is negligible as the ball swings downward, the work done by nonconservative forces is zero, \( W_{nc} = 0 \text{ J} \). The force due to the tension in the wire is perpendicular to the motion and, therefore, does no work. Thus, the total mechanical energy, which is the sum of the kinetic and potential energies, is conserved (see Section 6.5). The conservation of mechanical energy will be used to find the speed of the ball just before it collides with the block.

b. When the ball collides with the stationary block, the collision is elastic. This means that, during the collision, the total kinetic energy of the system is conserved. The horizontal or \( x \)-component of the total momentum is conserved, because the horizontal surface is frictionless, and so the net average external force acting on the ball-block system in the horizontal direction is zero. The total kinetic energy is conserved, since the collision is known to be elastic.

**SOLUTION**

a. As the ball falls, the total mechanical energy is conserved. Thus, the total mechanical energy at the top of the swing is equal to that at the bottom:

\[
\frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 v^2 \]

In this expression \( L \) is the length of the wire, \( m_1 \) is the mass of the ball, \( g \) is the magnitude of the acceleration due to gravity, and \( v \) is the speed of the ball just before the collision. We have chosen the zero-level for the potential energy to be at ground level, so the initial potential energy of the ball is \( m_1 g L \). Solving for the speed \( v \) of the ball gives

\[ v = \sqrt{2gL} = \sqrt{2(9.80 \text{ m/s}^2)(1.20 \text{ m})} = 4.85 \text{ m/s} \]

At the bottom of the swing the ball is moving horizontally and to the right, which we take to be the +\( x \) direction (see the drawing in the text). Thus the velocity of the ball just before impact is \( v_x = +4.85 \text{ m/s} \).
b. The conservation of linear momentum and the conservation of the total kinetic energy can be used to describe the behavior of the system during the elastic collision. This situation is identical to that in Example 7 in Section 7.3, so Equation 7.8a applies:

\[ v_f = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_x \]

where \( v_f \) is the final velocity of the ball after the collision, \( m_1 \) and \( m_2 \) are, respectively, the masses of the ball and block, and \( v_x \) is the velocity of the ball just before the collision. Since \( v_x = +4.85 \text{ m/s} \), we find that

\[ v_f = \left( \frac{1.60 \text{ kg} - 2.40 \text{ kg}}{1.60 \text{ kg} + 2.40 \text{ kg}} \right) (+4.85 \text{ m/s}) = -0.97 \text{ m/s} \]

The minus sign indicates that the ball rebounds to the left after the collision.

39. **REASONING** Considering the boat and the stone as a single system, there is no net external horizontal force acting on the stone-boat system, and thus the horizontal component of the system’s linear momentum is conserved: \( m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x} + m_2 v_{02x} \) (Equation 7.9a). We have taken east as the positive direction.

**SOLUTION** We will use the following symbols in solving the problem:

- \( m_1 \) = mass of the stone = 0.072 kg
- \( v_{f1} \) = final speed of the stone = 11 m/s
- \( v_{01} \) = initial speed of the stone = 13 m/s
- \( m_2 \) = mass of the boat
- \( v_{f2} \) = final speed of the boat = 2.1 m/s
- \( v_{02} \) = initial speed of the boat = 0 m/s

Because the boat is initially at rest, \( v_{02x} = 0 \text{ m/s} \), and Equation 7.9a reduces to \( m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{01x} \). Solving for the mass \( m_2 \) of the boat, we obtain

\[ m_2 v_{f2x} = m_1 v_{01x} - m_1 v_{f1x} \quad \text{or} \quad m_2 = \frac{m_1 (v_{01x} - v_{f1x})}{v_{f2x}} \]

As noted above, the boat’s final velocity is horizontal, so \( v_{f2x} = v_{f2} \). The horizontal components of the stone’s initial and final velocities are, respectively, \( v_{01x} = v_{01} \cos 15^\circ \) and \( v_{f1x} = v_{f1} \cos 12^\circ \) (see the drawing).

Thus, the mass of the boat must be
40. **REASONING** The system in this problem consists of the coal and the car. There is no net external force acting on this system as far as the horizontal direction is concerned; the gravitational force acts in the vertical (not the horizontal) direction and the weight of the car (and its contents) is balanced by the normal force supplied by the track. Thus, linear momentum in the horizontal direction is conserved.

**SOLUTION** We seek the final speed \( v_f \) of the coal/car system after the coal has come to rest in the car. Since linear momentum is conserved in the horizontal direction during the "collision," we have

\[
\frac{(m_{\text{coal}} + m_{\text{car}})v_f}{m_{\text{coal}} + m_{\text{car}}} = \frac{m_{\text{coal}}v_{\text{coal}} \cos 25.0^\circ + m_{\text{car}}v_{\text{car}}}{m_{\text{coal}} + m_{\text{car}}}
\]

Solving this equation for \( v_f \), we obtain

\[
v_f = \frac{m_{\text{coal}}v_{\text{coal}} \cos 25.0^\circ + m_{\text{car}}v_{\text{car}}}{m_{\text{coal}} + m_{\text{car}}}
\]

\[
= \frac{(150 \text{ kg})(0.80 \text{ m/s}) \cos 25.0^\circ + (440 \text{ kg})(0.50 \text{ m/s})}{150 \text{ kg} + 440 \text{ kg}} = 0.56 \text{ m/s}
\]

41. **SSM REASONING** The two skaters constitute the system. Since the net external force acting on the system is zero, the total linear momentum of the system is conserved. In the \( x \) direction (the east/west direction), conservation of linear momentum gives \( P_{fx} = P_{0x} \), or

\[
(m_1 + m_2)v_f \cos \theta = m_1v_{01}
\]

Note that since the skaters hold onto each other, they move away with a common velocity \( v_f \). In the \( y \) direction, \( P_{fy} = P_{0y} \), or

\[
(m_1 + m_2)v_f \sin \theta = m_2v_{02}
\]

These equations can be solved simultaneously to obtain both the angle \( \theta \) and the velocity \( v_f \).

**SOLUTION**
a. Division of the equations above gives

\[
\theta = \tan^{-1}\left( \frac{m_2v_{02}}{m_1v_{01}} \right) = \tan^{-1}\left[ \frac{(70.0 \text{ kg})(7.00 \text{ m/s})}{(50.0 \text{ kg})(3.00 \text{ m/s})} \right] = 73.0^\circ
\]
b. Solution of the first of the momentum equations gives

\[ v_f = \frac{m_1 v_{01}}{(m_1 + m_2) \cos \theta} = \frac{(50.0 \text{ kg})(3.00 \text{ m/s})}{(50.0 \text{ kg} + 70.0 \text{ kg})(\cos 73.0^\circ)} = 4.28 \text{ m/s} \]

42. **REASONING**

a. The conservation of linear momentum can be applied to this three-object system (the bullet and the two blocks), even though the second collision occurs later than the first one. Since there is no friction between the blocks and the horizontal surface, air resistance is negligible, and the weight of each block is balanced by a normal force, the net external force acting on this system is zero, and the conservation of linear momentum applies. This principle will allow us to determine the velocity of the second block after the bullet imbeds itself.

b. The total kinetic energy of the three-body system is not conserved. Both collisions are inelastic, and the collision with block 2 is completely inelastic since the bullet comes to rest within the block. As with any inelastic collision, the total kinetic energy after the collisions is less than that before the collisions.

**SOLUTION**

a. The conservation of linear momentum states that the total momentum of the system after the collisions [see part (b) of the drawing] is equal to that before the collisions [part (a) of the drawing]:

\[ \frac{m_{\text{block 1}} v_{\text{block 1}} + (m_{\text{block 2}} + m_{\text{bullet}}) v_{\text{block 2}}}{\text{Total momentum after collisions}} = \frac{m_{\text{bullet}} v_{\text{bullet}}}{\text{Total momentum before collisions}} \]

Solving for the velocity \( v_{\text{block 2}} \) of block 2 after the collisions gives

\[ v_{\text{block 2}} = \frac{m_{\text{bullet}} v_{\text{bullet}} - m_{\text{block 1}} v_{\text{block 1}}}{m_{\text{block 2}} + m_{\text{bullet}}} \]

\[ = \frac{(4.00 \times 10^{-3} \text{ kg})(+355 \text{ m/s}) - (1.150 \text{ kg})(+0.550 \text{ m/s})}{1.530 \text{ kg} + 4.00 \times 10^{-3} \text{ kg}} = +0.513 \text{ m/s} \]
b. The ratio of the total kinetic energy (KE) after the collisions to that before the collisions is

\[
\frac{KE_{\text{after}}}{KE_{\text{before}}} = \frac{\frac{1}{2}m_{\text{block 1}}v_{\text{block 1}}^2 + \frac{1}{2}(m_{\text{block 2}} + m_{\text{bullet}})v_{\text{block 2}}^2}{\frac{1}{2}m_{\text{bullet}}v_{\text{bullet}}^2} = \frac{\frac{1}{2}(1.150 \text{ kg})(0.550 \text{ m/s})^2 + \frac{1}{2}(1.530 \text{ kg} + 4.00 \times 10^{-3} \text{ kg})(0.513 \text{ m/s})^2}{\frac{1}{2}(4.00 \times 10^{-3} \text{ kg})(355 \text{ m/s})^2} = 1.49 \times 10^{-3}
\]

43. **REASONING** The ratio of the kinetic energy of the hydrogen atom after the collision to that of the electron before the collision is

\[
\frac{KE_{\text{hydrogen, after collision}}}{KE_{\text{electron, before collision}}} = \frac{\frac{1}{2}m_{\text{H}}v_{f,H}^2}{\frac{1}{2}m_{\text{e}}v_{0,e}^2}
\]

where \(v_{f,H}\) is the final speed of the hydrogen atom, and \(v_{0,e}\) is the initial speed of the electron. The ratio \(m_{\text{H}}/m_{\text{e}}\) of the masses is known. Since the electron and the stationary hydrogen atom experience an elastic head-on collision, we can employ Equation 7.8b to determine how \(v_{f,H}\) is related to \(v_{0,e}\).

**SOLUTION** According to Equation 7.8b, the final speed \(v_{f,H}\) of the hydrogen atom after the collision is related to the initial speed \(v_{0,e}\) of the electron by

\[
v_{f,H} = \left(\frac{2m_{\text{e}}}{m_{\text{e}} + m_{\text{H}}}\right)v_{0,e}
\]

Substituting this expression into the ratio of the kinetic energies gives

\[
\frac{KE_{\text{hydrogen, after collision}}}{KE_{\text{electron, before collision}}} = \frac{\frac{1}{2}m_{\text{H}}v_{f,H}^2}{\frac{1}{2}m_{\text{e}}v_{0,e}^2} = \frac{m_{\text{H}} \left(\frac{2m_{\text{e}}}{m_{\text{e}} + m_{\text{H}}}\right)^2}{m_{\text{e}}}
\]

The right hand side of this equation can be algebraically rearranged to give

\[
\frac{KE_{\text{hydrogen, after collision}}}{KE_{\text{electron, before collision}}} = \left(\frac{m_{\text{H}}}{m_{\text{e}}}\right) \left(\frac{2}{1 + \frac{m_{\text{H}}}{m_{\text{e}}}\right)^2} = (1837) \left(\frac{2}{1 + 1837}\right)^2 = 2.175 \times 10^{-3}
\]
44. **REASONING** The total linear momentum of the sled and person is conserved, since no net external force acts in the horizontal direction during the collision. We will use the conservation principle to determine the velocity of the person and the sled as they move away. Then, knowing this velocity, we can use the equations of kinematics and Newton’s second law to find the coefficient of kinetic friction.

**SOLUTION**

a. According to momentum conservation, we have

\[
\frac{(m_{\text{person}} + m_{\text{sled}}) v_f}{\text{Total momentum in horizontal direction after collision}} = \frac{m_{\text{person}} v_{\text{person}}}{\text{Total momentum in horizontal direction before collision}}
\]

where \( v_f \) denotes the velocity of the sled and the person as they move away. Solving this equation for \( v_f \) gives

\[
v_f = \frac{m_{\text{person}} v_{\text{person}}}{m_{\text{person}} + m_{\text{sled}}} = \frac{(60.0 \text{ kg})(+3.80 \text{ m/s})}{60.0 \text{ kg} + 12.0 \text{ kg}} = +3.17 \text{ m/s}
\]

b. It is the kinetic frictional force that causes the sled and the rider to come to a halt after coasting 30.0 m. According to Equation 4.8, the magnitude of the frictional force is \( f_k = \mu_k F_N \), where \( F_N \) is the magnitude of the normal force. Since there is no acceleration in the vertical direction, we know that the normal force and the weight are balanced, so that \( F_N = (m_{\text{person}} + m_{\text{sled}}) g \) and \( f_k = \mu_k (m_{\text{person}} + m_{\text{sled}}) g \). Newton’s second law then gives the acceleration as

\[
a = -\frac{\mu_k (m_{\text{person}} + m_{\text{sled}}) g}{m_{\text{person}} + m_{\text{sled}}} = -\mu_k g
\]

In Equation (1) the minus sign is present because the sled is slowing down, so the acceleration points opposite to the velocity. According to Equation 2.9 of the equations of kinematics, we have

\[
v^2 = v_0^2 + 2ax \quad \text{or} \quad (0.00 \text{ m/s})^2 = (+3.17 \text{ m/s})^2 + 2\left[-\mu_k (9.80 \text{ m/s}^2)\right](30.0 \text{ m})
\]

Solving for \( \mu_k \) gives

\[
\mu_k = \frac{(3.17 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(30.0 \text{ m})} = 0.0171
\]

45. **SSM REASONING** The two balls constitute the system. The tension in the wire is the only nonconservative force that acts on the ball. The tension does no work since it is perpendicular to the displacement of the ball. Since \( W_{\text{nc}} = 0 \text{ J} \), the principle of conservation
of mechanical energy holds and can be used to find the speed of the 1.50-kg ball just before
the collision. Momentum is conserved during the collision, so the principle of conservation
of momentum can be used to find the velocities of both balls just after the collision. Once
the collision has occurred, energy conservation can be used to determine how high each ball
rises.

**SOLUTION**

a. Applying the principle of energy conservation to the 1.50-kg ball, we have

\[
\frac{1}{2} m v_1^2 + m g h_1 = \frac{1}{2} m v_0^2 + m g h_0
\]

If we measure the heights from the lowest point in the swing, \( h_f = 0 \) m, and the expression
above simplifies to

\[
\frac{1}{2} m v_f^2 = \frac{1}{2} m v_0^2 + m g h_0
\]

Solving for \( v_f \), we have

\[
v_f = \sqrt{v_0^2 + 2 g h_0} = \sqrt{(5.00 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.300 \text{ m})} = 5.56 \text{ m/s}
\]

b. If we assume that the collision is elastic, then the velocities of both balls just after the
collision can be obtained from Equations 7.8a and 7.8b:

\[
v_{f1} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{01} \quad \text{and} \quad v_{f2} = \left( \frac{2 m_1}{m_1 + m_2} \right) v_{01}
\]

Since \( v_{01} \) corresponds to the speed of the 1.50-kg ball just before the collision, it is equal to
the quantity \( v_f \) calculated in part (a). With the given values of \( m_1 = 1.50 \) kg and
\( m_2 = 4.60 \) kg, and the value of \( v_{01} = 5.56 \) m/s obtained in part (a), Equations 7.8a and
7.8b yield the following values:

\[
v_{f1} = -2.83 \text{ m/s} \quad \text{and} \quad v_{f2} = +2.73 \text{ m/s}
\]

The minus sign in \( v_{f1} \) indicates that the first ball reverses its direction as a result of the
collision.

c. If we apply the conservation of mechanical energy to either ball after the collision we have

\[
\frac{1}{2} m v_f^2 + m g h_f = \frac{1}{2} m v_0^2 + m g h_0
\]

where \( v_0 \) is the speed of the ball just after the collision, and \( h_f \) is the final height to which the
ball rises. For either ball, \( h_0 = 0 \) m, and when either ball has reached its maximum height,
\( v_f = 0 \) m/s. Therefore, the expression of energy conservation reduces to
Thus, the heights to which each ball rises after the collision are

\[ h_f = \frac{v_{01}^2}{2g} \quad \text{or} \quad h_f = \frac{v_{02}^2}{2g} \]

Thus, the heights to which each ball rises after the collision are

- **1.50-kg ball**
  \[ h_f = \frac{v_{01}^2}{2g} = \frac{(2.83 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.409 \text{ m} \]

- **4.60-kg ball**
  \[ h_f = \frac{v_{02}^2}{2g} = \frac{(2.73 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.380 \text{ m} \]

46. **REASONING AND SOLUTION** During the elastic collision both the total kinetic energy and the total momentum of the balls are conserved. The conservation of momentum gives

\[ mv_{f1} + mv_{f2} = mv_{01} + mv_{02} \]

where ball 1 has \( v_{01} = +7.0 \text{ m/s} \) and ball 2 has \( v_{02} = -4.0 \text{ m/s} \). Hence,

\[ v_{f1} + v_{f2} = v_{01} + v_{02} = 3.0 \text{ m/s} \]

The conservation of kinetic energy gives

\[ \frac{1}{2}mv_{f1}^2 + \frac{1}{2}mv_{f2}^2 = \frac{1}{2}mv_{01}^2 + \frac{1}{2}mv_{02}^2 \]

or

\[ v_{f1}^2 + v_{f2}^2 = v_{01}^2 + v_{02}^2 = 65 \text{ m}^2/\text{s}^2 \]

Solving the first equation for \( v_{f1} \) and substituting into the second gives

\[ 2v_{f2}^2 - (6.0 \text{ m/s})v_{f2} - 56 \text{ m}^2/\text{s}^2 = 0 \]

The quadratic formula yields two solutions, \( v_{f2} = 7.0 \text{ m/s} \) and \( v_{f2} = -4.0 \text{ m/s} \).

The first equation now gives \( v_{f1} = -4.0 \text{ m/s} \) and \( v_{f1} = +7.0 \text{ m/s} \). Hence, the 7.0-m/s-ball has a final velocity of \([-4.0 \text{ m/s}]\), opposite its original direction.

The 4.0-m/s-ball has a final velocity of \([+7.0 \text{ m/s}]\), opposite its original direction.
47. **Reasoning and Solution** Initially, the ball has a total mechanical energy given by $E_0 = mgh_0$. After one bounce it reaches the top of its trajectory with an energy of

$$E_1 = 0.900 \, E_0 = 0.900 \, mgh_0$$

After two bounces it has energy

$$E_2 = (0.900)^2 mgh_0$$

After $N$ bounces it has a remaining energy of

$$E_N = (0.900)^N mgh_0$$

In order to just reach the sill the ball must have $E_N = mgh$ where $h = 2.44$ m. Hence,

$$(0.900)^N mgh_0 = mgh \quad \text{or} \quad (0.900)^N = h/h_0$$

Taking the log of both sides gives

$$N \log(0.900) = \log(h/h_0)$$

Then

$$N = \frac{\log \left( \frac{2.44 \text{ m}}{6.10 \text{ m}} \right)}{\log (0.900)} = 8.7$$

The ball can make 8 bounces and still reach the sill.

48. **Reasoning** The velocity of the center of mass of this two-particle system is

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \quad \text{(Equation 7.11)}.$$  

Since we know that $v_1 = +4.6 \text{ m/s}$, $v_2 = -6.1 \text{ m/s}$, and $v_{cm} = 0.0 \text{ m/s}$, Equation 7.11 can be solved directly for the ratio $m_1/m_2$.

**Solution** With $v_{cm} = 0.0 \text{ m/s}$, Equation 7.11 becomes

$$v_{cm} = 0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \quad \text{or} \quad 0 = m_1 v_1 + m_2 v_2 \quad \text{or} \quad \frac{m_1}{m_2} = -\frac{v_2}{v_1}$$

Therefore, it follows that

$$\frac{m_1}{m_2} = -\frac{v_2}{v_1} = -\frac{-6.1 \text{ m/s}}{+4.6 \text{ m/s}} = 1.3$$
49. **REASONING AND SOLUTION**  The velocity of the center of mass of a system is given by Equation 7.11. Using the data and the results obtained in Example 5, we obtain the following:

a. The velocity of the center of mass of the two-car system before the collision is

\[
(v_{cm})_{\text{before}} = \frac{m_1v_{01} + m_2v_{02}}{m_1 + m_2}
\]

\[
= \frac{(65 \times 10^3 \text{ kg})(+0.80 \text{ m/s}) + (92 \times 10^3 \text{ kg})(+1.2 \text{ m/s})}{65 \times 10^3 \text{ kg} + 92 \times 10^3 \text{ kg}} = +1.0 \text{ m/s}
\]

b. The velocity of the center of mass of the two-car system after the collision is

\[
(v_{cm})_{\text{after}} = \frac{m_1v_{f1} + m_2v_{f2}}{m_1 + m_2} = v_f = +1.0 \text{ m/s}
\]

c. The answer in part (b) should be the same as the common velocity \(v_f\). Since the cars are coupled together, every point of the two-car system, including the center of mass, must move with the same velocity.

50. **REASONING**  Using Equation 7.10, we can calculate the location of the center of mass of John and Barbara:

\[
x_{cm} = \frac{m_Jx_J + m_Bx_B}{m_J + m_B}
\]

By calculating John and Barbara’s center of mass before and after they change positions, we can determine how far and in what direction their center of mass move as a result of the switch.

**SOLUTION**

**Before**

\[
x_{cm} = \frac{m_Jx_J + m_Bx_B}{m_J + m_B} = \frac{(86 \text{ kg})(9.0 \text{ m}) + (55 \text{ kg})(2.0 \text{ m})}{86 \text{ kg} + 55 \text{ kg}} = 6.3 \text{ m}
\]

**After**

\[
x_{cm} = \frac{m_Jx_J + m_Bx_B}{m_J + m_B} = \frac{(86 \text{ kg})(2.0 \text{ m}) + (55 \text{ kg})(9.0 \text{ m})}{86 \text{ kg} + 55 \text{ kg}} = 4.7 \text{ m}
\]

The center of mass moves by an amount 6.3 m – 4.7 m = 1.6 m. Since it moves from the 6.3-m point to the 4.7-m point, the center of mass moves toward the origin.
51. **REASONING** We will use the relation 
\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \] (Equation 7.10) to determine the mass of the smaller star. Let the line joining the centers of the two stars be the \(x\) axis, and let the position of the larger star’s center be \(x_0 = 0 \text{ m}\) (see the drawing). Then the position of the smaller star’s center is \(x_2 = x_1 + d = d\), where \(d\) is the distance separating the centers of the stars.

**SOLUTION** With the substitutions \(x_1 = 0 \text{ m}\) and \(x_2 = d\), Equation 7.10 simplifies to 
\[ x_{cm} = \frac{0 + m_2 d}{m_1 + m_2} = \frac{m_2 d}{m_1 + m_2} \]

Solving this relation for the mass \(m_2\) of the smaller star, we obtain 
\[ m_1 + m_2 = \frac{m_2 d}{x_{cm}} \text{ or } m_2 = \frac{d}{x_{cm}} - \frac{m_1}{x_{cm}} \]

Thus, the mass of the smaller star is 
\[ m_2 = \frac{m_1}{d} = \frac{3.70 \times 10^{30} \text{ kg}}{7.17 \times 10^{11} \text{ m}} = 1.51 \times 10^{30} \text{ kg} \]

52. **REASONING** The coordinates \(x_{cm}\) and \(y_{cm}\) of the molecule’s center of mass are given by 
\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \text{ and } y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \] (Equation 7.10), where we make the following identifications:

<table>
<thead>
<tr>
<th>Atom</th>
<th>Mass</th>
<th>(x)-coordinate</th>
<th>(y)-coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen (left side)</td>
<td>(m_1 = m_O)</td>
<td>(x_1 = -d \sin 60.0^\circ)</td>
<td>(y_1 = +d \cos 60.0^\circ)</td>
</tr>
<tr>
<td>Oxygen (right side)</td>
<td>(m_2 = m_O)</td>
<td>(x_2 = +d \sin 60.0^\circ)</td>
<td>(y_2 = +d \cos 60.0^\circ)</td>
</tr>
<tr>
<td>Sulfur</td>
<td>(m_3 = m_S = 2m_O)</td>
<td>(x_3 = 0 \text{ nm})</td>
<td>(y_3 = 0 \text{ nm})</td>
</tr>
</tbody>
</table>

Here, we have used \(d = 0.143 \text{ nm}\) to represent the distance between the center of the sulfur atom and the center of either oxygen atom.
**SOLUTION**

a. Making substitutions into \( x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \) (Equation 7.10) from the data table above, we obtain the \( x \) coordinate of the molecule’s center of mass:

\[
x_{cm} = \frac{-m_O d \sin 60.0^\circ + m_O d \sin 60.0^\circ + m_S (0)}{m_O + m_O + m_S} = 0 \text{ nm}
\]

This result makes sense, because the sulfur atom’s \( x \) coordinate is \( x_3 = 0 \text{ nm} \), and the oxygen atoms, which have equal masses, have \( x \) coordinates of equal magnitude but opposite signs: \( x_1 = -x_2 \).

b. The \( y \) coordinate of the sulfur atom is zero (\( y_3 = 0 \text{ nm} \)), but the \( y \) coordinates of the two oxygen atoms are both positive, so the center of mass of the sulfur dioxide molecule has a positive \( y \) coordinate:

\[
y_{cm} = \frac{+m_O d \cos 60.0^\circ + m_O d \cos 60.0^\circ + m_S (0 \text{ nm}) - 2m_O d \cos 60.0^\circ}{2m_O + 2m_O} = \frac{+2 m_O d \cos 60.0^\circ}{4 m_O} = \frac{+d \cos 60.0^\circ}{2} = \frac{(0.143 \text{ nm}) \cos 60.0^\circ}{2} = +0.0358 \text{ nm}
\]

53. **SSM REASONING**  Let \( m \) be Al’s mass, which means that Jo’s mass is 168 kg – \( m \). Since friction is negligible and since the downward-acting weight of each person is balanced by the upward-acting normal force from the ice, the net external force acting on the two-person system is zero. Therefore, the system is isolated, and the conservation of linear momentum applies. The initial total momentum must be equal to the final total momentum.

**SOLUTION**  Applying the principle of conservation of linear momentum and assuming that the direction in which Al moves is the positive direction, we find

\[
\begin{align*}
\text{Initial total momentum} & \quad m(0 \text{ m/s}) + (168 \text{ kg} - m)(0 \text{ m/s}) \\
\text{Final total momentum} & \quad m(0.90 \text{ m/s}) + (168 \text{ kg} - m)(-1.2 \text{ m/s})
\end{align*}
\]

Solving this equation for \( m \), we find that

\[
0 = m(0.90 \text{ m/s}) - (168 \text{ kg})(1.2 \text{ m/s}) + m(1.2 \text{ m/s})
\]

\[
m = \frac{(168 \text{ kg})(1.2 \text{ m/s})}{0.90 \text{ m/s} + 1.2 \text{ m/s}} = 96 \text{ kg}
\]
54. **REASONING** Since all of the collisions are elastic, the total mechanical energy of the ball is conserved. However, since gravity affects its vertical motion, its linear momentum is not conserved. If $h_f$ is the maximum height of the ball on its final bounce, conservation of energy gives

$$\frac{1}{2}m v_f^2 + mgh_f = \frac{1}{2}m v_0^2 + mgh_0$$

Solving this equation for $h_f$ gives

$$h_f = \frac{v_0^2 - v_f^2}{2g} + h_0$$

**SOLUTION** In order to use this expression, we must obtain the values for the velocities $v_0$ and $v_f$. The initial velocity has only a horizontal component, $v_0 = v_{0x}$. The final velocity also has only a horizontal component since the ball is at the top of its trajectory, $v_f = v_{fx}$. No forces act in the horizontal direction so the momentum of the ball in this direction is conserved, hence $v_0 = v_f$. Therefore,

$$h_f = h_0 = [3.00 \text{ m}]$$

55. **REASONING** Since the collision is an elastic collision, both the linear momentum and kinetic energy of the two-vehicle system are conserved. The final velocities of the car and van are given in terms of the initial velocity of the car by Equations 7.8a and 7.8b.

**SOLUTION**

a. The final velocity $v_{f1}$ of the car is given by Equation 7.8a as

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{01}$$

where $m_1$ and $m_2$ are, respectively, the masses of the car and van, and $v_{01}$ is the initial velocity of the car. Thus,

$$v_{f1} = \left(\frac{715 \text{ kg} - 1055 \text{ kg}}{715 \text{ kg} + 1055 \text{ kg}}\right) (+2.25 \text{ m/s}) = -0.432 \text{ m/s}$$

b. The final velocity of the van is given by Equation 7.8b:

$$v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{0i} = \left[\frac{2 (715 \text{ kg})}{715 \text{ kg} + 1055 \text{ kg}}\right] (+2.25 \text{ m/s}) = +1.82 \text{ m/s}$$
56. **REASONING** During the collision, the bat exerts an impulse on the ball. The impulse is the product of the average force that the bat exerts and the time of contact. According to the impulse-momentum theorem, the impulse is also equal to the change in the momentum of the ball. We will use these two relations to determine the average force exerted by the bat on the ball.

**SOLUTION** The impulse \( J \) is given by Equation 7.1 as \( J = \vec{F} \Delta t \), where \( \vec{F} \) is the average force that the bat exerts on the ball and \( \Delta t \) is the time of contact. According to the impulse-momentum theorem, Equation 7.4, the net average impulse \( (\sum \vec{F}) \Delta t \) is equal to the change in the ball’s momentum; \( (\sum \vec{F}) \Delta t = m v_f - m v_0 \). Since we are ignoring the weight of the ball, the bat’s force is the net force, so \( \sum \vec{F} = \vec{F} \). Substituting this value for the net average force into the impulse-momentum equation and solving for the average force gives

\[
\vec{F} = \frac{m v_f - m v_0}{\Delta t} = \frac{(0.149 \text{ kg})(-45.6 \text{ m/s}) - (0.149 \text{ kg})(+40.2 \text{ m/s})}{1.10 \times 10^{-3} \text{ s}} = -11600 \text{ N}
\]

where the positive direction for the velocity has been chosen as the direction of the incoming ball.

57. **SSM** **REASONING** According to Equation 7.1, the impulse \( J \) produced by an average force \( \vec{F} \) is \( J = \vec{F} \Delta t \), where \( \Delta t \) is the time interval during which the force acts. We will apply this definition for each of the forces and then set the two impulses equal to one another. The fact that one average force has a magnitude that is three times as large as that of the other average force will then be used to obtain the desired time interval.

**SOLUTION** Applying Equation 7.1, we write the impulse of each average force as follows:

\[
J_1 = \vec{F}_1 \Delta t_1 \quad \text{and} \quad J_2 = \vec{F}_2 \Delta t_2
\]

But the impulses \( J_1 \) and \( J_2 \) are the same, so we have that \( \vec{F}_1 \Delta t_1 = \vec{F}_2 \Delta t_2 \). Writing this result in terms of the magnitudes of the forces gives

\[
\vec{F}_1 \Delta t_1 = \vec{F}_2 \Delta t_2 \quad \text{or} \quad \Delta t_2 = \left( \frac{\vec{F}_1}{\vec{F}_2} \right) \Delta t_1
\]

The ratio of the force magnitudes is given as \( \vec{F}_1 / \vec{F}_2 = 3 \), so we find that

\[
\Delta t_2 = \left( \frac{\vec{F}_1}{\vec{F}_2} \right) \Delta t_1 = 3 \times 3.2 \text{ ms} = 9.6 \text{ ms}
\]
58. REASONING AND SOLUTION  The momentum is zero before the beat. Conservation of momentum requires that it is also zero after the beat; thus

\[ 0 = m_p v_p + m_b v_b \]

so that

\[ v_p = -(m_b/m_p)v_b = -(0.050 \text{ kg}/85 \text{ kg})(0.25 \text{ m/s}) = -1.5 \times 10^{-4} \text{ m/s} \]

59. REASONING AND SOLUTION  Equation 7.10 gives the center of mass of this two-atom system as

\[ x_{cm} = \frac{m_c x_c + m_o x_o}{m_c + m_o} \]

If we take the origin at the center of the carbon atom, then \( x_c = 0 \) m, and we have

\[ x_{cm} = \frac{m_c x_c + m_o x_o}{m_c + m_o} = \frac{x_o}{(m_c / m_o) + 1} = 1.13 \times 10^{-10} \text{ m} \]

60. REASONING  Friction is negligible and the downward-pointing weight of the wagon and its contents is balanced by the upward-pointing normal force from the ground. Therefore, the net external force acting on the wagon and its contents is zero, and the principle of conservation of linear momentum applies. The total linear momentum of the system, then, must remain the same before and after the rock is thrown.

Let’s assume that \( m \) and \( v_{01} \) are, respectively the mass and initial velocity of the wagon, rider, and rock before the rock is thrown. In addition, let \( m_{\text{rock}} \) be the mass of the rock, \( v_{\text{f, rock}} \) be the final velocity of the rock after it is thrown, and \( v_f \) be the final velocity of the wagon and rider (without the rock) after the rock is thrown. The conservation of linear momentum states that

\[ \left( m - m_{\text{rock}} \right) v_f + m_{\text{rock}} v_{\text{f, rock}} = m v_{01} \]

Solving this equation for \( v_f \) gives

\[ v_f = \frac{m v_{01} - m_{\text{rock}} v_{\text{f, rock}}}{m - m_{\text{rock}}} \]

SOLUTION  In applying the momentum conservation principle, we assume that the forward direction is positive. Therefore the velocity of the wagon, rider, and rock before the rock is thrown is \( v_{01} = +0.500 \text{ m/s} \).
Rock thrown forward: When the rock is thrown forward, its velocity is +16 m/s. The final velocity of the wagon is

\[
v_f = \frac{m v_{01} - m_{\text{rock}} v_{\text{rock}}}{m - m_{\text{rock}}} = \frac{(95.0 \text{ kg})(+0.500 \text{ m/s}) - (0.300 \text{ kg})(+16.0 \text{ m/s})}{95.0 \text{ kg} - 0.300 \text{ kg}} = +0.451 \text{ m/s}
\]

The final speed of the wagon (which is the magnitude of its velocity) is \(0.451 \text{ m/s}\).

Rock thrown backward: When the rock is thrown backward, its velocity is \(-16.0 \text{ m/s}\). In this case the final velocity of the wagon is

\[
v_f = \frac{m v_{01} - m_{\text{rock}} v_{\text{rock}}}{m - m_{\text{rock}}} = \frac{(95.0 \text{ kg})(+0.500 \text{ m/s}) - (0.300 \text{ kg})(-16.0 \text{ m/s})}{95.0 \text{ kg} - 0.300 \text{ kg}} = +0.552 \text{ m/s}
\]

The final speed of the wagon is \(0.552 \text{ m/s}\).

61. **REASONING** For use in our solution we define the following masses and initial speeds of the bullets:

<table>
<thead>
<tr>
<th></th>
<th>Mass</th>
<th>Initial Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullet 1</td>
<td>(m_1 = 4.50 \times 10^{-3} \text{ kg})</td>
<td>(v_{01} = 324 \text{ m/s})</td>
</tr>
<tr>
<td>Bullet 2</td>
<td>(m_2 = 4.50 \times 10^{-3} \text{ kg})</td>
<td>(v_{02} = 324 \text{ m/s})</td>
</tr>
<tr>
<td>Bullet 3</td>
<td>(m_3 = ?)</td>
<td>(v_{03} = 575 \text{ m/s})</td>
</tr>
</tbody>
</table>

The drawing shows the bullets just after they are fired from the guns. They collide at the origin of the \(x, y\) axes and form a stationary lump. Assuming that the momentum-conservation principle applies, we can conclude that the total momentum of the three-bullet system is the same after the bullets collide as before they collide. Applying this principle will allow us to determine the unknown mass.

**SOLUTION** Applying the momentum-conservation principle separately in terms of the \(x\) and \(y\) components of the total momentum, we have

\[
x \text{ Component} \quad 0 = m_1 v_{01} \cos 30.0^\circ - m_3 v_{03} \cos 30.0^\circ
\]

\[
y \text{ Component} \quad 0 = -m_1 v_{01} \sin 30.0^\circ - m_3 v_{03} \sin 30.0^\circ + m_2 v_2
\]
Both the \(x\) and \(y\) components of the total momentum of the three-bullet system are zero after the collision, since the bullets form a stationary lump. The \(x\) component of the initial momentum of bullet 1 is positive and the \(y\) component is negative, because this bullet is fired to the right and downward in the drawing. The \(x\) and \(y\) components of the initial momentum of bullet 3 are negative, because this bullet is fired to the left and downward in the drawing. Either of the two equations presented above can be solved for the unknown mass \(m_3\). From the equation for the \(x\) component of the total momentum, we find that

\[
m_3 = \frac{m_1 v_{01}}{v_{03}} = \frac{(4.50 \times 10^{-3} \text{ kg})(324 \text{ m/s})}{575 \text{ m/s}} = 2.54 \times 10^{-3} \text{ kg}
\]

62. **REASONING** During the breakup, the linear momentum of the system is conserved, since the force causing the breakup is an internal force. We will assume that the \(+x\) axis is along the original line of motion (before the breakup), and the \(+y\) axis is perpendicular to this line and points upward. We will apply the conservation of linear momentum twice, once for the momentum components along the \(x\) axis and again for the momentum components along the \(y\) axis.

**SOLUTION** The mass of each piece of the rocket after breakup is \(m\), and so the mass of the rocket before breakup is \(2m\). Applying the conservation of momentum theorem along the original line of motion (the \(x\) axis) gives

\[
m v_1 \cos 30.0^\circ + m v_2 \cos 60.0^\circ = 2m v_0 \quad \text{or} \quad v_1 \cos 30.0^\circ + v_2 \cos 60.0^\circ = 2v_0 \quad (1)
\]

Applying the conservation of momentum along the \(y\) axis gives

\[
m v_1 \sin 30.0^\circ - m v_2 \sin 60.0^\circ = 0 \quad \text{or} \quad v_2 = \frac{v_1 \sin 30.0^\circ}{\sin 60.0^\circ} \quad (2)
\]

a. To find the speed \(v_1\) of the first piece, we substitute the value for \(v_2\) from Equation (2) into Equation (1). The result is

\[
v_1 \cos 30.0^\circ + \left(\frac{v_1 \sin 30.0^\circ}{\sin 60.0^\circ}\right) \cos 60.0^\circ = 2v_0
\]

Solving for \(v_1\) and setting \(v_0 = 45.0 \text{ m/s}\) yields

\[
v_1 = \frac{2v_0}{\cos 30.0^\circ + \left(\frac{\sin 30.0^\circ}{\sin 60.0^\circ}\right) \cos 60.0^\circ} = \frac{2(45.0 \text{ m/s})}{\cos 30.0^\circ + \left(\frac{\sin 30.0^\circ}{\sin 60.0^\circ}\right) \cos 60.0^\circ} = 77.9 \text{ m/s}
\]
b. The speed $v_2$ of the second piece can be found by substituting $v_1 = 77.9 \text{ m/s}$ into Equation (2):

$$v_2 = \frac{v_1 \sin 30.0^\circ}{\sin 60.0^\circ} = \frac{(77.9 \text{ m/s}) \sin 30.0^\circ}{\sin 60.0^\circ} = 45.0 \text{ m/s}$$

63. **REASONING** During the time that the skaters are pushing against each other, the sum of the external forces acting on the two-skater system is zero, because the weight of each skater is balanced by a corresponding normal force and friction is negligible. The skaters constitute an isolated system, so the principle of conservation of linear momentum applies. We will use this principle to find an expression for the ratio of the skater’s masses in terms of their recoil velocities. We will then obtain expressions for the recoil velocities by noting that each skater, after pushing off, comes to rest in a certain distance. The recoil velocity, acceleration, and distance are related by Equation 2.9 of the equations of kinematics.

**SOLUTION** While the skaters are pushing against each other, the total linear momentum of the two-skater system is conserved:

$$m_1 v_{f1} + m_2 v_{f2} = 0$$

Solving this expression for the ratio of the masses gives

$$\frac{m_1}{m_2} = -\frac{v_{f2}}{v_{f1}} \quad \text{(1)}$$

For each skater the (initial) recoil velocity $v_{f1}$, final velocity $v$, acceleration $a$, and displacement $x$ are related by Equation 2.9 of the equations of kinematics: $v^2 = v_{i}^2 + 2ax$. Solving for the recoil velocity gives $v_{f1} = \pm \sqrt{v_{i}^2 - 2ax}$. If we assume that skater 1 recoils in the positive direction and skater 2 recoils in the negative direction, the recoil velocities are

**Skater 1**

$$v_{f1} = +\sqrt{v_{i}^2 - 2ax_i}$$

**Skater 2**

$$v_{f2} = -\sqrt{v_{i}^2 - 2ax_2}$$

Substituting these expressions into Equation (1) gives

$$\frac{m_1}{m_2} = -\frac{\pm \sqrt{v_{i}^2 - 2ax_2}}{\sqrt{v_{i}^2 - 2ax_i}} = \frac{\sqrt{v_{i}^2 - 2ax_2}}{\sqrt{v_{i}^2 - 2ax_i}} \quad \text{(2)}$$
Since the skaters come to rest, their final velocities are zero, so \( v_1 = v_2 = 0 \) m/s. We also know that their accelerations have the same magnitudes. This means that \( a_2 = -a_1 \), where the minus sign denotes that the acceleration of skater 2 is opposite that of skater 1, since they are moving in opposite directions and are both slowing down. Finally, we are given that skater 1 glides twice as far as skater 2. Thus, the displacement of skater 1 is related to that of skater 2 by \( x_1 = -2x_2 \), where, the minus sign denotes that the skaters move in opposite directions. Substituting these values into Equation (2) yields

\[
\frac{m_1}{m_2} = \sqrt{\frac{(0 \text{ m/s})^2 - 2(-a_1)(x_2)}{(0 \text{ m/s})^2 - 2a_1(-2x_2)}} = \frac{1}{\sqrt{2}} = 0.707
\]

64. **REASONING** The mass of each part of the seated human figure will be treated as if it were all located at the corresponding center of mass point. See the drawing given in the text. In effect, then, the problem deals with the three-particle system shown in the drawing at the right. To determine the \( x \) and \( y \) coordinates of the center of mass of this system, we will employ equations analogous to Equation 7.10. The values for the masses are \( m_1 = 41 \text{ kg}, m_2 = 17 \text{ kg}, \text{ and } m_3 = 9.9 \text{ kg} \).

**SOLUTION** The \( x \) and \( y \) coordinates of the center of mass for the three-particle system in the drawing are

\[
x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(41 \text{ kg})(0 \text{ m}) + (17 \text{ kg})(0.17 \text{ m}) + (9.9 \text{ kg})(0.43 \text{ m})}{41 \text{ kg} + 17 \text{ kg} + 9.9 \text{ kg}} = 0.11 \text{ m}
\]

\[
y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(41 \text{ kg})(0.39 \text{ m}) + (17 \text{ kg})(0 \text{ m}) + (9.9 \text{ kg})(-0.26 \text{ m})}{41 \text{ kg} + 17 \text{ kg} + 9.9 \text{ kg}} = 0.20 \text{ m}
\]

65. **SSM REASONING** We will define the system to be the platform, the two people and the ball. Since the ball travels nearly horizontally, the effects of gravity are negligible. Momentum is conserved. Since the initial momentum of the system is zero, it must remain zero as the ball is thrown and caught. While the ball is in motion, the platform will recoil in such a way that the total momentum of the system remains zero. As the ball is caught, the system must come to rest so that the total momentum remains zero. The distance that the
platform moves before coming to rest again can be determined by using the expressions for momentum conservation and the kinematic description for this situation.

**SOLUTION** While the ball is in motion, we have

\[ Mv + mv = 0 \]  

where \( M \) is the combined mass of the platform and the two people, \( V \) is the recoil velocity of the platform, \( m \) is the mass of the ball, and \( v \) is the velocity of the ball.

The distance that the platform moves is given by

\[ x = Vt \]  

where \( t \) is the time that the ball is in the air. The time that the ball is in the air is given by

\[ t = \frac{L}{v-V} \]  

where \( L \) is the length of the platform, and the quantity \((v - V)\) is the velocity of the ball relative to the platform. Remember, both the ball and the platform are moving while the ball is in the air. Combining equations (2) and (3) gives

\[ x = \left( \frac{V}{v-V} \right) L \]  

From equation (1) the ratio of the velocities is \( \frac{V}{v} = -\frac{m}{M} \). Equation (4) then gives

\[ x = \frac{(V/v)L}{1-(V/v)} = \frac{(-m/M)L}{1+(m/M)} = -\frac{mL}{M+m} = \frac{(6.0 \text{ kg})(2.0 \text{ m})}{118 \text{ kg} + 6.0 \text{ kg}} = -0.097 \text{ m} \]

The minus sign indicates that displacement of the platform is in the opposite direction to the displacement of the ball. The distance moved by the platform is the magnitude of this displacement, or \( 0.097 \text{ m} \).
1. (d) Using Equation 8.1 \((\theta = \text{Arc length} / \text{Radius})\) to calculate the angle (in radians) that each object subtends at your eye shows that \(\theta_{\text{Moon}} = 9.0 \times 10^{-3} \text{ rad}\), \(\theta_{\text{Pea}} = 7.0 \times 10^{-3} \text{ rad}\), and \(\theta_{\text{Dime}} = 25 \times 10^{-3} \text{ rad}\). Since \(\theta_{\text{Pea}}\) is less than \(\theta_{\text{Moon}}\), the pea does not completely cover your view of the moon. However, since \(\theta_{\text{Dime}}\) is greater than \(\theta_{\text{Moon}}\), the dime does completely cover your view of the moon.

2. 2.20 cm

3. 38.2 s

4. (a) An angular acceleration of zero means that the angular velocity has the same value at all times, as in statements A or B. However, statement C is also consistent with a zero angular acceleration, because if the angular displacement does not change as time passes, then the angular velocity remains constant at a value of 0 rad/s.

5. (c) A non-zero angular acceleration means that the angular velocity is either increasing or decreasing. The angular velocity is not constant.

6. (b) Since values are given for the initial angular velocity \(\omega_0\), the final angular velocity \(\omega\), and the time \(t\), Equation 8.6 \[ \theta = \frac{1}{2} (\omega_0 + \omega) t \] can be used to calculate the angular displacement \(\theta\).

7. 32 rad/s

8. 88 rad

9. (c) According to Equation 8.9 \((v_T = r\omega)\), the tangential speed is proportional to the radius \(r\) when the angular speed \(\omega\) is constant, as it is for the earth. As the elevator rises, the radius, which is your distance from the center of the earth, increases, and so does your tangential speed.

10. (b) According to Equation 8.9 \((v_T = r\omega)\), the tangential speed is proportional to the radius \(r\) when the angular speed \(\omega\) is constant, as it is for the merry-go-round. Thus, the angular speed of the second child is \(v_T = (2.2 \text{ m/s}) \left(\frac{2.1 \text{ m}}{1.4 \text{ m}}\right)\).
11. 367 rad/s²

12. (e) According to Newton’s second law, the centripetal force is given by \( F_c = ma_c \), where \( m \) is the mass of the ball and \( a_c \) is the centripetal acceleration. The centripetal acceleration is given by Equation 8.11 as \( a_c = r\omega^2 \), where \( r \) is the radius and \( \omega \) is the angular speed. Therefore, \( F_c = m r \omega^2 \), and the centripetal force is proportional to the radius when the mass and the angular speed are fixed, as they are in this problem. As a result,
\[
F_c = (1.7 \text{ N}) \left( \frac{33 \text{ cm}}{12 \text{ cm}} \right).
\]

13. (d) Since the angular speed \( \omega \) is constant, the angular acceleration \( \alpha \) is zero, according to Equation 8.4. Since \( \alpha = 0 \text{ rad/s}^2 \), the tangential acceleration \( a_T \) is zero, according to Equation 8.10. The centripetal acceleration \( a_c \), however, is not zero, since it is proportional to the square of the angular speed, according to Equation 8.11, and the angular speed is not zero.

14. 17.8 m/s²

15. (a) The number \( N \) of revolutions is the distance \( s \) traveled divided by the circumference \( 2\pi r \) of a wheel: \( N = s/(2\pi r) \).

16. 27.0 m/s
1. **REASONING** The average angular velocity is equal to the angular displacement divided by the elapsed time (Equation 8.2). Thus, the angular displacement of the baseball is equal to the product of the average angular velocity and the elapsed time. However, the problem gives the travel time in seconds and asks for the displacement in radians, while the angular velocity is given in revolutions per minute. Thus, we will begin by converting the angular velocity into radians per second.

**SOLUTION** Since $2\pi$ rad = 1 rev and 1 min = 60 s, the average angular velocity $\bar{\omega}$ (in rad/s) of the baseball is

$$\bar{\omega} = \left( \frac{330 \text{ rev}}{\text{min}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 35 \text{ rad/s}$$

Since the average angular velocity of the baseball is equal to the angular displacement $\Delta \theta$ divided by the elapsed time $\Delta t$, the angular displacement is

$$\Delta \theta = \bar{\omega} \Delta t = (35 \text{ rad/s})(0.60 \text{ s}) = 21 \text{ rad}$$

(8.2)

2. **REASONING** The average angular velocity $\bar{\omega}$ has the same direction as $\theta - \theta_0$, because $\bar{\omega} = \frac{\theta - \theta_0}{t - t_0}$ according to Equation 8.2. If $\theta$ is greater than $\theta_0$, then $\bar{\omega}$ is positive. If $\theta$ is less than $\theta_0$, then $\bar{\omega}$ is negative.

**SOLUTION** The average angular velocity is given by Equation 8.2 as $\bar{\omega} = \frac{\theta - \theta_0}{t - t_0}$, where $t - t_0 = 2.0 \text{ s}$ is the elapsed time:

(a) $$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{0.75 \text{ rad} - 0.45 \text{ rad}}{2.0 \text{ s}} = +0.15 \text{ rad/s}$$

(b) $$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{0.54 \text{ rad} - 0.94 \text{ rad}}{2.0 \text{ s}} = -0.20 \text{ rad/s}$$

(c) $$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{4.2 \text{ rad} - 5.4 \text{ rad}}{2.0 \text{ s}} = -0.60 \text{ rad/s}$$

(d) $$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{3.8 \text{ rad} - 3.0 \text{ rad}}{2.0 \text{ s}} = +0.4 \text{ rad/s}$$
3. **REASONING** The average angular velocity \( \bar{\omega} \) is defined as the angular displacement \( \Delta \theta \) divided by the elapsed time \( \Delta t \) during which the displacement occurs: \( \bar{\omega} = \Delta \theta / \Delta t \) (Equation 8.2). This relation can be used to find the average angular velocity of the earth as it spins on its axis and as it orbits the sun.

**SOLUTION**

a. As the earth spins on its axis, it makes 1 revolution \((2\pi \text{ rad})\) in a day. Assuming that the positive direction for the angular displacement is the same as the direction of the earth’s rotation, the angular displacement of the earth in one day is \((\Delta \theta)_{\text{spin}} = +2\pi \text{ rad}\). The average angular velocity is (converting 1 day to seconds):

\[
\bar{\omega} = \frac{(\Delta \theta)_{\text{spin}}}{(\Delta t)_{\text{spin}}} = \frac{+2\pi \text{ rad}}{(1 \text{ day}) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)} = 7.3 \times 10^{-5} \text{ rad/s}
\]

b. As the earth orbits the sun, the earth makes 1 revolution \((2\pi \text{ rad})\) in one year. Taking the positive direction for the angular displacement to be the direction of the earth’s orbital motion, the angular displacement in one year is \((\Delta \theta)_{\text{orbit}} = +2\pi \text{ rad}\). The average angular velocity is (converting 365.25 days to seconds):

\[
\bar{\omega} = \frac{(\Delta \theta)_{\text{orbit}}}{(\Delta t)_{\text{orbit}}} = \frac{+2\pi \text{ rad}}{(365 \frac{1}{4} \text{ days}) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)} = 2.0 \times 10^{-7} \text{ rad/s}
\]

4. **REASONING AND SOLUTION**

According to Equation 8.2, \( \bar{\omega} = \Delta \theta / \Delta t \). Since the angular speed of the sun is constant, \( \bar{\omega} = \omega \). Solving for \( \Delta t \), we have

\[
\Delta t = \frac{\Delta \theta}{\omega} = \left( \frac{2\pi \text{ rad}}{1.1 \times 10^{-15} \text{ rad/s}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right) \left( \frac{1 \text{ y}}{365.25 \text{ day}} \right) = 1.8 \times 10^8 \text{ y}
\]

5. **SSM REASONING AND SOLUTION** Since there are \(2\pi \) radians per revolution and it is stated in the problem that there are 100 grads in one-quarter of a circle, we find that the number of grads in one radian is

\[
(1.00 \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{100 \text{ grad}}{0.250 \text{ rev}} \right) = 63.7 \text{ grad}
\]
6. **REASONING** The relation between the final angular velocity \( \omega \), the initial angular velocity \( \omega_0 \), and the angular acceleration \( \alpha \) is given by Equation 8.4 (with \( t_0 = 0 \) s) as

\[
\omega = \omega_0 + \alpha t
\]

If \( \alpha \) has the same sign as \( \omega_0 \), then the angular speed, which is the magnitude of the angular velocity \( \omega \), is increasing. On the other hand, If \( \alpha \) and \( \omega_0 \) have opposite signs, then the angular speed is decreasing.

**SOLUTION** According to Equation 8.4, we know that \( \omega = \omega_0 + \alpha t \). Therefore, we find:

(a) \( \omega = +12 \text{ rad/s} + (+3.0 \text{ rad/s}^2)(2.0 \text{ s}) = +18 \text{ rad/s} \). The angular speed is 18 rad/s.

(b) \( \omega = +12 \text{ rad/s} + (-3.0 \text{ rad/s}^2)(2.0 \text{ s}) = +6.0 \text{ rad/s} \). The angular speed is 6.0 rad/s.

(c) \( \omega = -12 \text{ rad/s} + (+3.0 \text{ rad/s}^2)(2.0 \text{ s}) = -6.0 \text{ rad/s} \). The angular speed is 6.0 rad/s.

(d) \( \omega = -12 \text{ rad/s} + (-3.0 \text{ rad/s}^2)(2.0 \text{ s}) = -18 \text{ rad/s} \). The angular speed is 18 rad/s.

7. **REASONING** The average angular acceleration has the same direction as \( \omega - \omega_0 \), because

\[
\bar{\alpha} = \frac{\omega - \omega_0}{t - t_0},
\]

according to Equation 8.4. If \( \omega \) is greater than \( \omega_0 \), \( \bar{\alpha} \) is positive. If \( \omega \) is less than \( \omega_0 \), \( \bar{\alpha} \) is negative.

**SOLUTION** The average angular acceleration is given by Equation 8.4 as \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} \), where \( t - t_0 = 4.0 \text{ s} \) is the elapsed time.

(a) \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{+5.0 \text{ rad/s} - 2.0 \text{ rad/s}}{4.0 \text{ s}} = +0.75 \text{ rad/s}^2 \)

(b) \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{+2.0 \text{ rad/s} - 5.0 \text{ rad/s}}{4.0 \text{ s}} = -0.75 \text{ rad/s}^2 \)

(c) \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{-3.0 \text{ rad/s} - (-7.0 \text{ rad/s})}{4.0 \text{ s}} = +1.0 \text{ rad/s}^2 \)

(d) \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} = \frac{-4.0 \text{ rad/s} - (+4.0 \text{ rad/s})}{4.0 \text{ s}} = -2.0 \text{ rad/s}^2 \)
8. **REASONING** The jet is maintaining a distance of \( r = 18.0 \text{ km} \) from the air traffic control tower by flying in a circle. The angle that the jet’s path subtends while its nose crosses over the moon is the same as the angular width \( \theta \) of the moon. The corresponding distance the jet travels is the length of arc \( s \) subtended by the moon’s diameter. We will use the relation \( s = r\theta \) (Equation 8.1) to determine the distance \( s \).

**SOLUTION** In order to use the relation \( s = r\theta \) (Equation 8.1), the angle \( \theta \) must be expressed in radians, as it is. The result will have the same units as \( r \). Because \( s \) is required in meters, we first convert \( r \) to meters:

\[
r = (18.0 \text{ km}) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 1.8 \times 10^4 \text{ m}
\]

Therefore, the distance that the jet travels while crossing in front of the moon is

\[
s = r\theta = (1.80 \times 10^4 \text{ m}) \left(9.04 \times 10^{-3} \text{ rad}\right) = 163 \text{ m}
\]

9. **SSM REASONING** Equation 8.4 \( \bar{\alpha} = (\omega - \omega_0) / t \) indicates that the average angular acceleration is equal to the change in the angular velocity divided by the elapsed time. Since the wheel starts from rest, its initial angular velocity is \( \omega_0 = 0 \text{ rad/s} \). Its final angular velocity is given as \( \omega = 0.24 \text{ rad/s} \). Since the average angular acceleration is given as \( \bar{\alpha} = 0.030 \text{ rad/s}^2 \), Equation 8.4 can be solved to determine the elapsed time \( t \).

**SOLUTION** Solving Equation 8.4 for the elapsed time gives

\[
t = \frac{\omega - \omega_0}{\bar{\alpha}} = \frac{0.24 \text{ rad/s} - 0 \text{ rad/s}}{0.030 \text{ rad/s}^2} = 8.0 \text{ s}
\]

10. **REASONING** The distance \( s \) traveled by a spot on the outer edge of a disk of radius \( r \) can be determined from the angular displacement \( \Delta \theta \) (in radians) of the disk by using \( \Delta \theta = s/r \) (Equation 8.1). The radius is given as \( r = 0.15 \text{ m} \). The angular displacement is not given. However, the angular velocity is given as \( \omega = 1.4 \text{ rev/s} \) and the elapsed time as \( \Delta t = 45 \text{ s} \), so the angular displacement can be obtained from the definition of angular velocity as \( \omega = \Delta \theta / \Delta t \) (Equation 8.2). We must remember that Equation 8.1 is only valid when \( \Delta \theta \) is expressed in radians. It will, therefore, be necessary to convert the given angular velocity from rev/s into rad/s.

**SOLUTION** From Equation 8.1 we have

\[
s = r\Delta \theta
\]

Using Equation 8.2, we can write the displacement as \( \Delta \theta = \omega \Delta t \). With this substitution Equation 8.1 becomes
11. **REASONING** The average angular velocity \( \bar{\omega} \) is defined as the angular displacement \( \Delta \theta \) divided by the elapsed time \( \Delta t \) during which the displacement occurs: \( \bar{\omega} = \Delta \theta / \Delta t \) (Equation 8.2). Solving for the elapsed time gives \( \Delta t = \Delta \theta / \bar{\omega} \). We are given \( \Delta \theta \) and can calculate \( \bar{\omega} \) from the fact that the earth rotates on its axis once every 24.0 hours.

**SOLUTION** The sun itself subtends an angle of \( 9.28 \times 10^{-3} \) rad. When the sun moves a distance equal to its diameter, it moves through an angle that is also \( 9.28 \times 10^{-3} \) rad; thus, \( \Delta \theta = 9.28 \times 10^{-3} \) rad. The average angular velocity \( \bar{\omega} \) at which the sun appears to move across the sky is the same as that of the earth rotating on its axis, \( \bar{\omega}_{\text{earth}} \), so \( \bar{\omega} = \bar{\omega}_{\text{earth}} \).

Since the earth makes one revolution \( (2\pi \text{ rad}) \) every 24.0 h, its average angular velocity is

\[
\bar{\omega}_{\text{earth}} = \frac{\Delta \theta_{\text{earth}}}{\Delta t_{\text{earth}}} = \frac{2\pi \text{ rad}}{24.0 \text{ h}} = \frac{2\pi \text{ rad}}{(24.0 \text{ h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)} = 7.27 \times 10^{-5} \text{ rad/s}
\]

The time it takes for the sun to move a distance equal to its diameter is

\[
\Delta t = \frac{\Delta \theta}{\bar{\omega}_{\text{earth}}} = \frac{9.28 \times 10^{-3} \text{ rad}}{7.27 \times 10^{-5} \text{ rad/s}} = 128 \text{ s (a little over 2 minutes)}
\]

12. **REASONING** It does not matter whether the arrow is aimed closer to or farther away from the axis. The blade edge sweeps through the open angular space as a rigid unit. This means that a point closer to the axis has a smaller distance to travel along the circular arc in order to bridge the angular opening and correspondingly has a smaller tangential speed. A point farther from the axis has a greater distance to travel along the circular arc but correspondingly has a greater tangential speed. These speeds have just the right values so that all points on the blade edge bridge the angular opening in the same time interval.

The rotational speed of the blades must not be so fast that one blade rotates into the open angular space while part of the arrow is still there. A faster arrow speed means that the arrow spends less time in the open space. Thus, the blades can rotate more quickly into the open space without hitting the arrow, so the maximum value of the angular speed \( \omega \) increases with increasing arrow speed \( v \).

A longer arrow traveling at a given speed means that some part of the arrow is in the open space for a longer time. To avoid hitting the arrow, then, the blades must rotate more
slowly. Thus, the maximum value of the angular speed $\omega$ decreases with increasing arrow length $L$.

The time during which some part of the arrow remains in the open angular space is $t_{\text{Arrow}}$. The time it takes for the edge of a propeller blade to rotate through the open angular space between the blades is $t_{\text{Blade}}$. The maximum angular speed is the angular speed such that these two times are equal.

**SOLUTION** The time during which some part of the arrow remains in the open angular space is the time it takes the arrow to travel at a speed $v$ through a distance equal to its own length $L$. This time is $t_{\text{Arrow}} = L/v$. The time it takes for the edge to rotate at an angular speed $\omega$ through the angle $\theta$ between the blades is $t_{\text{Blade}} = \theta/\omega$. The maximum angular speed is the angular speed such that these two times are equal. Therefore, we have

$$\frac{L}{v} = \frac{\theta}{\omega}$$

In this expression we note that the value of the angular opening is $\theta = 60.0^\circ$, which is $\theta = \frac{1}{6}(2\pi)$ rad $= \frac{1}{3} \pi$ rad. Solving the expression for $\omega$ gives

$$\omega = \frac{\theta v}{L} = \frac{\pi v}{3L}$$

Substituting the given values for $v$ and $L$ into this result, we find that

a. $\omega = \frac{\pi v}{3L} = \frac{\pi (75.0 \text{ m/s})}{3(0.71 \text{ m})} = 111 \text{ rad/s}$

b. $\omega = \frac{\pi v}{3L} = \frac{\pi (91.0 \text{ m/s})}{3(0.71 \text{ m})} = 134 \text{ rad/s}$

c. $\omega = \frac{\pi v}{3L} = \frac{\pi (91.0 \text{ m/s})}{3(0.81 \text{ m})} = 118 \text{ rad/s}$

13. **REASONING AND SOLUTION** The people meet at time $t$. At this time the magnitudes of their angular displacements must total $2\pi$ rad.

$$\theta_1 + \theta_2 = 2\pi \text{ rad}$$

Then

$$\omega_1 t + \omega_2 t = 2\pi \text{ rad}$$

$$t = \frac{2\pi \text{ rad}}{\omega_1 + \omega_2} = \frac{2\pi \text{ rad}}{1.7 \times 10^{-3} \text{ rad/s} + 3.4 \times 10^{-3} \text{ rad/s}} = 1200 \text{ s}$$
14. **REASONING AND SOLUTION**  The angular displacements of the astronauts are equal.

For A  \[ \theta = \frac{s_A}{r_A} \]  \hspace{1cm} (8.1)

For B  \[ \theta = \frac{s_B}{r_B} \]

Equating these two equations for \( \theta \) and solving for \( s_B \) gives

\[ s_B = (\frac{r_B}{r_A})s_A = \frac{(1.10 \times 10^3 \text{ m})}{(3.20 \times 10^2 \text{ m})}(2.40 \times 10^2 \text{ m}) = 825 \text{ m} \]

15. **REASONING**  The time required for the bullet to travel the distance \( d \) is equal to the time required for the discs to undergo an angular displacement of 0.240 rad. The time can be found from Equation 8.2; once the time is known, the speed of the bullet can be found using Equation 2.2.

**SOLUTION**  From the definition of average angular velocity:

\[ \bar{\omega} = \frac{\Delta \theta}{\Delta t} \]

the required time is

\[ \Delta t = \frac{\Delta \theta}{\bar{\omega}} = \frac{0.240 \text{ rad}}{95.0 \text{ rad/s}} = 2.53 \times 10^{-3} \text{ s} \]

Note that \( \bar{\omega} = \omega \) because the angular speed is constant. The (constant) speed of the bullet can then be determined from the definition of average speed:

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{d}{\Delta t} = \frac{0.850 \text{ m}}{2.53 \times 10^{-3} \text{ s}} = 336 \text{ m/s} \]

16. **REASONING**  We can apply the definition of the average angular acceleration directly to this problem. As defined, the average angular acceleration \( \bar{\alpha} \) is \( \bar{\alpha} = \frac{\omega - \omega_0}{t - t_0} \) (Equation 8.4).

In this definition, the symbols \( \omega \) and \( \omega_0 \) are the final and initial angular velocities, respectively, and the symbols \( t \) and \( t_0 \) are the final and initial times, respectively.

**SOLUTION**  Using Equation 8.4 and assuming that the initial time is \( t_0 = 0 \text{ s} \), we have

\[ \bar{\alpha} = \frac{\omega - \omega_0}{t} \]

Solving for the time \( t \) gives

\[ t = \frac{\omega - \omega_0}{\bar{\alpha}} = \frac{5.2 \text{ rad/s} - 0 \text{ rad/s}}{4.0 \text{ rad/s}^2} = 1.3 \text{ s} \]
17. **SSM REASONING AND SOLUTION**
   a. If the propeller is to appear stationary, each blade must move through an angle of 120° or $2\pi/3$ rad between flashes. The time required is
   
   \[ t = \frac{\theta}{\omega} = \frac{(2\pi/3) \text{ rad}}{(16.7 \text{ rev/s})(2\pi \text{ rad/1 rev})} = 2.00 \times 10^{-2} \text{s} \]
   
   b. The next shortest time occurs when each blade moves through an angle of 240°, or $4\pi/3$ rad, between successive flashes. This time is twice the value that we found in part a, or $4.00 \times 10^{-2}$ s.

18. **REASONING AND SOLUTION**
   The figure at the right shows the relevant angles and dimensions for either one of the celestial bodies under consideration.
   
   a. Using the figure above
   
   \[ \theta_{\text{moon}} = \frac{s_{\text{moon}}}{r_{\text{moon}}} = \frac{3.48 \times 10^6 \text{ m}}{3.85 \times 10^8 \text{ m}} = 9.04 \times 10^{-3} \text{ rad} \]
   
   \[ \theta_{\text{sun}} = \frac{s_{\text{sun}}}{r_{\text{sun}}} = \frac{1.39 \times 10^9 \text{ m}}{1.50 \times 10^{11} \text{ m}} = 9.27 \times 10^{-3} \text{ rad} \]
   
   b. Since the sun subtends a slightly larger angle than the moon, as measured by a person standing on the earth, the sun cannot be completely blocked by the moon. Therefore, a "total" eclipse of the sun is not really total.
   
   c. The following drawing shows the relevant geometry:
The apparent circular area of the sun as measured by a person standing on the earth is given by: 

\[ A_{\text{sun}} = \pi R_{\text{sun}}^2 \]

where \( R_{\text{sun}} \) is the radius of the sun. The apparent circular area of the sun that is blocked by the moon is 

\[ A_{\text{blocked}} = \pi R_b^2 \]

where \( R_b \) is shown in the figure above. Also from the figure above, it follows that

\[ R_{\text{sun}} = \frac{1}{2} s_{\text{sun}} \quad \text{and} \quad R_b = \frac{1}{2} s_b \]

Therefore, the fraction of the apparent circular area of the sun that is blocked by the moon is

\[
\frac{A_{\text{blocked}}}{A_{\text{sun}}} = \frac{\frac{\pi R_b^2}{\pi R_{\text{sun}}^2}}{\left(\frac{s_b}{s_{\text{sun}}}\right)^2} = \left(\frac{\theta_{\text{moon}}}{\theta_{\text{sun}}}\right)^2
\]

\[
= \left(\frac{9.04 \times 10^{-3} \text{ rad}}{9.27 \times 10^{-3} \text{ rad}}\right)^2 = 0.951
\]

The moon blocks out 95.1 percent of the apparent circular area of the sun.

19. **REASONING** The golf ball must travel a distance equal to its diameter in a maximum time equal to the time required for one blade to move into the position of the previous blade.

**SOLUTION** The time required for the golf ball to pass through the opening between two blades is given by 

\[ \Delta t = \Delta \theta / \omega \]

with \( \omega = 1.25 \text{ rad/s} \) and \( \Delta \theta = (2\pi \text{ rad})/16 = 0.393 \text{ rad} \). Therefore, the ball must pass between two blades in a maximum time of

\[ \Delta t = \frac{0.393 \text{ rad}}{1.25 \text{ rad/s}} = 0.314 \text{ s} \]

The minimum linear speed of the ball is

\[ v = \frac{\Delta x}{\Delta t} = \frac{4.50 \times 10^{-2} \text{ m}}{0.314 \text{ s}} = 1.43 \times 10^{-1} \text{ m/s} \]

20. **REASONING** We know that the skater’s final angular velocity is \( \omega = 0.0 \text{ rad/s} \), since she comes to a stop. We also that her initial angular velocity is \( \omega_0 = +15 \text{ rad/s} \) and that her angular displacement while coming to a stop is \( \theta = +5.1 \text{ rad} \). With these three values, we can use \( \omega^2 = \omega_0^2 + 2\alpha \theta \) (Equation 8.8) to calculate her angular acceleration \( \alpha \). Then, knowing \( \alpha \), we can use \( \omega = \omega_0 + \alpha t \) (Equation 8.4) to determine the time \( t \) during which she comes to a halt.

**SOLUTION**

a. With \( \omega = 0.0 \text{ rad/s} \) for the skater’s final angular velocity, Equation 8.8 becomes

\[ 0 = \omega_0^2 + 2\alpha \theta \]

Solving for \( \alpha \), we obtain
b. With \( \omega = 0.0 \text{ rad/s} \) for the skater’s final angular velocity, Equation 8.4 becomes \( 0 = \omega_0 + \alpha t \). Solving for \( t \), we obtain

\[
t = -\frac{\omega_0}{\alpha} = -\frac{-15 \text{ rad/s}}{-22 \text{ rad/s}^2} = 0.68 \text{ s}
\]

21. **REASONING** The angular displacement is given as \( \theta = 0.500 \text{ rev} \), while the initial angular velocity is given as \( \omega_0 = 3.00 \text{ rev/s} \) and the final angular velocity as \( \omega = 5.00 \text{ rev/s} \). Since we seek the time \( t \), we can use Equation 8.6 \( \theta = \frac{1}{2} (\omega_0 + \omega) t \) from the equations of rotational kinematics to obtain it.

**SOLUTION** Solving Equation 8.6 for the time \( t \), we find that

\[
t = \frac{2\theta}{\omega_0 + \omega} = \frac{2(0.500 \text{ rev})}{3.00 \text{ rev/s} + 5.00 \text{ rev/s}} = 0.125 \text{ s}
\]

22. **REASONING** We know that the final angular speed is \( \omega = 1420 \text{ rad/s} \). We also that the initial angular speed is \( \omega_0 = 420 \text{ rad/s} \) and that the time during which the change in angular speed occurs is \( t = 5.00 \text{ s} \). With these three values, we can use \( \theta = \frac{1}{2} (\omega + \omega_0) t \) (Equation 8.6) to calculate the angular displacement \( \theta \). Then, we can use \( \omega = \omega_0 + \alpha t \) (Equation 8.4) to determine the angular acceleration \( \alpha \).

**SOLUTION**

a. Using Equation 8.6, we find that

\[
\theta = \frac{1}{2} (\omega + \omega_0) t = \frac{1}{2} (1420 \text{ rad/s} + 420 \text{ rad/s})(5.00 \text{ s}) = 4.60 \times 10^3 \text{ rad}
\]

b. Solving Equation 8.4 for \( \alpha \), we find that

\[
\alpha = \frac{\omega - \omega_0}{t} = \frac{1420 \text{ rad/s} - 420 \text{ rad/s}}{5.00 \text{ s}} = 2.00 \times 10^2 \text{ rad/s}^2
\]

23. **REASONING** We are given the turbine’s angular acceleration \( \alpha \), final angular velocity \( \omega \), and angular displacement \( \theta \). Therefore, we will employ \( \omega^2 = \omega_0^2 + 2\alpha\theta \) (Equation 8.8) in order to determine the turbine’s initial angular velocity \( \omega_0 \) for part a. However, in order to
make the units consistent, we will convert the angular displacement $\theta$ from revolutions to radians before substituting its value into Equation 8.8. In part $b$, the elapsed time $t$ is the only unknown quantity. We can, therefore, choose from among $\omega = \omega_0 + \alpha t$ (Equation 8.4), $\theta = \frac{1}{2}(\omega_0 + \omega)t$ (Equation 8.6), or $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ (Equation 8.7) to find the elapsed time. Of the three, Equation 8.4 offers the least algebraic complication, so we will solve it for the elapsed time $t$.

**SOLUTION**

a. One revolution is equivalent to $2\pi$ radians, so the angular displacement of the turbine is

$$\theta = \left(2870 \text{ rev}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1.80 \times 10^4 \text{ rad}$$

Solving $\omega^2 = \omega_0^2 + 2\alpha \theta$ (Equation 8.8) for the square of the initial angular velocity, we obtain $\omega_0^2 = \omega^2 - 2\alpha \theta$, or

$$\omega_0 = \sqrt{\omega^2 - 2\alpha \theta} = \sqrt{(137 \text{ rad/s})^2 - 2 \left(0.140 \text{ rad/s}^2\right) \left(1.80 \times 10^4 \text{ rad}\right)} = 117 \text{ rad/s}$$

b. Solving $\omega = \omega_0 + \alpha t$ (Equation 8.4) for the elapsed time, we find that

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{137 \text{ rad/s} - 117 \text{ rad/s}}{0.140 \text{ rad/s}^2} = 140 \text{ s}$$

24. **REASONING** The angular displacement is given by Equation 8.6 as the product of the average angular velocity and the time

$$\theta = \bar{\omega} t = \frac{1}{2}(\omega_0 + \omega) t$$

This value for the angular displacement is greater than $\omega_0 t$. When the angular displacement $\theta$ is given by the expression $\theta = \omega_0 t$, it is assumed that the angular velocity remains constant at its initial (and smallest) value of $\omega_0$ for the entire time, which does not, however, account for the additional angular displacement that occurs because the angular velocity is increasing.

The angular displacement is also less than $\omega t$. When the angular displacement is given by the expression $\theta = \omega t$, it is assumed that the angular velocity remains constant at its final (and largest) value of $\omega$ for the entire time, which does not account for the fact that the wheel was rotating at a smaller angular velocity during the time interval.
**SOLUTION**

a. If the angular velocity is constant and equals the initial angular velocity \( \omega_0 \), then \( \omega = \omega_0 \) and the angular displacement is

\[
\theta = \omega_0 t = (+220 \text{ rad/s})(10.0 \text{ s}) = +2200 \text{ rad}
\]

b. If the angular velocity is constant and equals the final angular velocity \( \omega \), then \( \omega = \omega \) and the angular displacement is

\[
\theta = \omega t = (+280 \text{ rad/s})(10.0 \text{ s}) = +2800 \text{ rad}
\]

c. Using the definition of average angular velocity, we have

\[
\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(+220 \text{ rad/s} + 280 \text{ rad/s})(10.0 \text{ s}) = +2500 \text{ rad}
\] (8.6)

25. **SSM REASONING**

a. The time \( t \) for the wheels to come to a halt depends on the initial and final velocities, \( \omega_\text{f} \) and \( \omega \), and the angular displacement \( \theta \): \( \theta = \frac{1}{2}(\omega_\text{f} + \omega)t \) (see Equation 8.6). Solving for the time yields

\[
t = \frac{2\theta}{\omega_\text{f} + \omega}
\]

b. The angular acceleration \( \alpha \) is defined as the change in the angular velocity, \( \omega - \omega_\text{f} \), divided by the time \( t \):

\[
\alpha = \frac{\omega - \omega_\text{f}}{t}
\] (8.4)

**SOLUTION**

a. Since the wheel comes to a rest, \( \omega = 0 \text{ rad/s} \). Converting 15.92 revolutions to radians (1 rev = 2\(\pi\) rad), the time for the wheel to come to rest is

\[
t = \frac{2\theta}{\omega_\text{f} + \omega} = \frac{2(15.92 \text{ rev})(\frac{2\pi \text{ rad}}{1 \text{ rev}})}{+20.0 \text{ rad/s} + 0 \text{ rad/s}} = 10.0 \text{ s}
\]

b. The angular acceleration is

\[
\alpha = \frac{\omega - \omega_\text{f}}{t} = \frac{0 \text{ rad/s} - 20.0 \text{ rad/s}}{10.0 \text{ s}} = -2.00 \text{ rad/s}^2
\]
26. **REASONING AND SOLUTION** The angular acceleration is found for the first circumstance.

\[
\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \left(\frac{3.14 \times 10^4 \text{ rad/s}^2}{2(1.88 \times 10^4 \text{ rad})}\right) = 2.33 \times 10^4 \text{ rad/s}^2
\]

For the second circumstance

\[
t = \frac{\omega - \omega_0}{\alpha} = \frac{7.85 \times 10^4 \text{ rad/s}}{2.33 \times 10^4 \text{ rad/s}^2} = 3.37 \text{ s}
\]

27. **SSM REASONING** The equations of kinematics for rotational motion cannot be used directly to find the angular displacement, because the final angular velocity (not the initial angular velocity), the acceleration, and the time are known. We will combine two of the equations, Equations 8.4 and 8.6 to obtain an expression for the angular displacement that contains the three known variables.

**SOLUTION** The angular displacement of each wheel is equal to the average angular velocity multiplied by the time

\[
\theta = \frac{1}{2} \left( \omega_0 + \omega \right) t
\]

(8.6)

The initial angular velocity \( \omega_0 \) is not known, but it can be found in terms of the angular acceleration and time, which are known. The angular acceleration is defined as (with \( t_0 = 0 \) s)

\[
\alpha = \frac{\omega - \omega_0}{t} \quad \text{or} \quad \omega_0 = \omega - \alpha t
\]

(8.4)

Substituting this expression for \( \omega_0 \) into Equation 8.6 gives

\[
\theta = \frac{1}{2} \left( \left( \omega - \alpha t \right) + \omega \right) t = \omega t - \frac{1}{2} \alpha t^2
\]

\[
= (+74.5 \text{ rad/s})(4.50 \text{ s}) - \frac{1}{2}(+6.70 \text{ rad/s}^2)(4.50 \text{ s})^2 = +267 \text{ rad}
\]

28. **REASONING** Equation 8.8 \( \omega^2 = \omega_0^2 + 2\alpha t \) from the equations of rotational kinematics can be employed to find the final angular velocity \( \omega \). The initial angular velocity is \( \omega_0 = 0 \text{ rad/s} \) since the top is initially at rest, and the angular acceleration is given as \( \alpha = 12 \text{ rad/s}^2 \). The angle \( \theta \) (in radians) through which the pulley rotates is not given, but it can be obtained from Equation 8.1 \( \theta = s/r \), where the arc length \( s \) is the 64-cm length of the string and \( r \) is the 2.0-cm radius of the top.
SOLUTION Solving Equation 8.8 for the final angular velocity gives

\[ \omega = \pm \sqrt{\omega_0^2 + 2\alpha \theta} \]

We choose the positive root, because the angular acceleration is given as positive and the top is at rest initially. Substituting \( \theta = s/r \) from Equation 8.1 gives

\[ \omega = + \sqrt{\left(0 \text{ rad/s}\right)^2 + 2 \left(12 \text{ rad/s}^2\right) \left(\frac{64 \text{ cm}}{2.0 \text{ cm}}\right)} = 28 \text{ rad/s} \]

29. REASONING There are three segments to the propeller’s angular motion, and we will calculate the angular displacement for each separately. In these calculations we will remember that the final angular velocity for one segment is the initial velocity for the next segment. Then, we will add the separate displacements to obtain the total.

SOLUTION For the first segment the initial angular velocity is \( \omega_0 = 0 \text{ rad/s} \), since the propeller starts from rest. Its acceleration is \( \alpha = 2.90 \times 10^{-3} \text{ rad/s}^2 \) for a time \( t = 2.10 \times 10^{-3} \text{ s} \). Therefore, we can obtain the angular displacement \( \theta_1 \) from Equation 8.7 of the equations of rotational kinematics as follows:

[First segment]

\[ \theta_1 = \omega_0 t + \frac{1}{2} \alpha t^2 = \left(0 \text{ rad/s}\right) \left(2.10 \times 10^{-3} \text{ s}\right) + \frac{1}{2} \left(2.90 \times 10^{-3} \text{ rad/s}^2\right) \left(2.10 \times 10^{-3} \text{ s}\right)^2 \]

\[ = 6.39 \times 10^{-3} \text{ rad} \]

The initial angular velocity for the second segment is the final velocity for the first segment, and according to Equation 8.4, we have

\[ \omega = \omega_0 + \alpha t = 0 \text{ rad/s} + \left(2.90 \times 10^{-3} \text{ rad/s}^2\right) \left(2.10 \times 10^{-3} \text{ s}\right) = 6.09 \text{ rad/s} \]

Thus, during the second segment, the initial angular velocity is \( \omega_0 = 6.09 \text{ rad/s} \) and remains constant at this value for a time of \( t = 1.40 \times 10^{-3} \text{ s} \). Since the velocity is constant, the angular acceleration is zero, and Equation 8.7 gives the angular displacement \( \theta_2 \) as

[Second segment]

\[ \theta_2 = \omega_0 t + \frac{1}{2} \alpha t^2 = \left(6.09 \text{ rad/s}\right) \left(1.40 \times 10^{-3} \text{ s}\right) + \frac{1}{2} \left(0 \text{ rad/s}^2\right) \left(1.40 \times 10^{-3} \text{ s}\right)^2 = 8.53 \times 10^{-3} \text{ rad} \]

During the third segment, the initial angular velocity is \( \omega_0 = 6.09 \text{ rad/s} \), the final velocity is \( \omega = 4.00 \text{ rad/s} \), and the angular acceleration is \( \alpha = -2.30 \times 10^{-3} \text{ rad/s}^2 \). When the propeller picked up speed in segment one, we assigned positive values to the acceleration and subsequent velocity. Therefore, the deceleration or loss in speed here in segment three means that the acceleration has a negative value. Equation 8.8 \( \left(\omega^2 = \omega_0^2 + 2\alpha \theta_3\right) \) can be used to find the angular displacement \( \theta_3 \). Solving this equation for \( \theta_3 \) gives
30. **REASONING** The average angular velocity is defined as the angular displacement divided by the elapsed time (Equation 8.2). Therefore, the angular displacement is equal to the product of the average angular velocity and the elapsed time. The elapsed time is given, so we need to determine the average angular velocity. We can do this by using the graph of angular velocity versus time that accompanies the problem.

**SOLUTION** The angular displacement $\Delta \theta$ is related to the average angular velocity $\bar{\omega}$ and the elapsed time $\Delta t$ by Equation 8.2, $\Delta \theta = \bar{\omega} \Delta t$. The elapsed time is given as 8.0 s.

To obtain the average angular velocity, we need to extend the graph that accompanies this problem from a time of 5.0 s to 8.0 s. It can be seen from the graph that the angular velocity increases by $+3.0 \text{ rad/s}$ during each second. Therefore, when the time increases from 5.0 to 8.0 s, the angular velocity increases from $+6.0 \text{ rad/s}$ to $6 \text{ rad/s} + 3 \times (3.0 \text{ rad/s}) = +15 \text{ rad/s}$. A graph of the angular velocity from 0 to 8.0 s is shown at the right. The average angular velocity during this time is equal to one half the sum of the initial and final angular velocities:

$$\bar{\omega} = \frac{1}{2} (\omega_i + \omega) = \frac{1}{2} (-9.0 \text{ rad/s} + 15 \text{ rad/s}) = +3.0 \text{ rad/s}$$

The angular displacement of the wheel from 0 to 8.0 s is

$$\Delta \theta = \bar{\omega} \Delta t = (+3.0 \text{ rad/s})(8.0 \text{ s}) = +24 \text{ rad}$$

31. **REASONING** According to Equation 3.5b, the time required for the diver to reach the water, assuming free-fall conditions, is $t = \sqrt{\frac{2y}{a_y}}$. If we assume that the "ball" formed by the diver is rotating at the instant that she begins falling vertically, we can use Equation 8.2 to calculate the number of revolutions made on the way down.
**Chapter 8  Problems**  

**SOLUTION** Taking upward as the positive direction, the time required for the diver to reach the water is

\[ t = \sqrt{\frac{2(-8.3 \text{ m})}{-9.80 \text{ m/s}^2}} = 1.3 \text{ s} \]

Solving Equation 8.2 for \( \Delta \theta \), we find

\[ \Delta \theta = \bar{\omega} \Delta t = (1.6 \text{ rev/s})(1.3 \text{ s}) = 2.1 \text{ rev} \]

32. **REASONING** The time required for the change in the angular velocity to occur can be found by solving Equation 8.4 for \( t \). In order to use Equation 8.4, however, we must know the initial angular velocity \( \omega_0 \). Equation 8.6 can be used to find the initial angular velocity.

**SOLUTION** From Equation 8.6 we have

\[ \theta = \frac{1}{2}(\omega_0 + \omega)t \]

Solving for \( \omega_0 \) gives

\[ \omega_0 = \frac{2\theta}{t} - \omega \]

Since the angular displacement \( \theta \) is zero, \( \omega_0 = -\omega \). Solving \( \omega = \omega_0 + \alpha t \) (Equation 8.4) for \( t \) and using the fact that \( \omega_0 = -\omega \) give

\[ t = \frac{2\omega}{\alpha} = \frac{2(-25.0 \text{ rad/s})}{-4.00 \text{ rad/s}^2} = 12.5 \text{ s} \]

33. **SSM REASONING** The angular displacement of the child when he catches the horse is, from Equation 8.2, \( \theta_c = \omega_c t \). In the same time, the angular displacement of the horse is, from Equation 8.7 with \( \omega_0 = 0 \text{ rad/s} \), \( \theta_h = \frac{1}{2} \alpha t^2 \). If the child is to catch the horse \( \theta_c = \theta_h + (\pi/2) \).

**SOLUTION** Using the above conditions yields

\[ \frac{1}{2} \alpha t^2 - \omega_c t + \frac{1}{2} \pi = 0 \]

or

\[ \frac{1}{2} (0.0100 \text{ rad/s}^2) t^2 - (0.250 \text{ rad/s}) t + \frac{1}{2} (\pi \text{ rad}) = 0 \]

The quadratic formula yields \( t = 7.37 \text{ s} \) and \( 42.6 \text{ s} \); therefore, the shortest time needed to catch the horse is \( t = 7.37 \text{ s} \).
34. **REASONING** The angular acceleration $\alpha$ gives rise to a tangential acceleration $a_T$ according to $a_T = r\alpha$ (Equation 8.10). Moreover, it is given that $a_T = g$, where $g$ is the magnitude of the acceleration due to gravity.

**SOLUTION** Let $r$ be the radial distance of the point from the axis of rotation. Then, according to Equation 8.10, we have

$$g = r\alpha$$

Thus,

$$r = \frac{g}{\alpha} = \frac{9.80 \text{ m/s}^2}{12.0 \text{ rad/s}^2} = 0.817 \text{ m}$$

35. **REASONING** The tangential speed $v_T$ of a point on a rigid body rotating at an angular speed $\omega$ is given by $v_T = r\omega$ (Equation 8.9), where $r$ is the radius of the circle described by the moving point. (In this equation $\omega$ must be expressed in rad/s.) Therefore, the angular speed of the bacterial motor sought in part $a$ is $\omega = v_T / r$. Since we are considering a point on the rim, $r$ is the radius of the motor itself. In part $b$, we seek the elapsed time $t$ for an angular displacement of one revolution at the constant angular velocity $\omega$ found in part $a$. We will use $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ (Equation 8.7) to calculate the elapsed time.

**SOLUTION**

a. The angular speed of the bacterial motor is, from $\omega = v_T / r$ (Equation 8.9),

$$\omega = \frac{v_T}{r} = \frac{2.3 \times 10^{-5} \text{ m/s}}{1.5 \times 10^{-8} \text{ m}} = 1500 \text{ rad/s}$$

b. The bacterial motor is spinning at a constant angular velocity, so it has no angular acceleration. Substituting $\alpha = 0 \text{ rad/s}^2$ into $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ (Equation 8.7), and solving for the elapsed time yields

$$\theta = \omega_0 t + \frac{1}{2} \left(0 \text{ rad/s}^2\right) t^2 = \omega_0 t \quad \text{or} \quad t = \frac{\theta}{\omega_0}$$

The fact that the motor has a constant angular velocity means that its initial and final angular velocities are equal: $\omega_0 = \omega = 1500 \text{ rad/s}$, the value calculated in part $a$, assuming a counterclockwise or positive rotation. The angular displacement $\theta$ is one revolution, or $2\pi$ radians, so the elapsed time is

$$t = \frac{\theta}{\omega_0} = \frac{2\pi \text{ rad}}{1500 \text{ rad/s}} = 4.2 \times 10^{-3} \text{ s}$$
36. **REASONING** In one lap, the car undergoes an angular displacement of $2\pi$ radians. The average angular speed is the magnitude of the average angular velocity $\bar{\omega}$, which is the angular displacement $\Delta \theta$ divided by the elapsed time $\Delta t$. Therefore, since the time for one lap is given, we can apply the definition of average angular velocity $\bar{\omega} = \frac{\Delta \theta}{\Delta t}$ (Equation 8.2). In addition, the given average tangential speed $\bar{v}_T$ and the average angular speed are related by $\bar{v}_T = r \bar{\omega}$ (Equation 8.9), which we can use to obtain the radius $r$ of the track.

**SOLUTION**

a. Using the facts that $\Delta \theta = 2\pi$ rad and $\Delta t = 18.9 \text{ s}$ in Equation 8.2, we find that the average angular speed is

$$\bar{\omega} = \frac{\Delta \theta}{\Delta t} = \frac{2\pi \text{ rad}}{18.9 \text{ s}} = 0.332 \text{ rad/s}$$

b. Solving Equation 8.9 for the radius $r$ gives

$$r = \frac{\bar{v}_T}{\bar{\omega}} = \frac{42.6 \text{ m/s}}{0.332 \text{ rad/s}} = 128 \text{ m}$$

Notice that the unit "rad," being dimensionless, does not appear in the final answer.

37. **SSM REASONING** The angular speed $\omega$ and tangential speed $v_T$ are related by Equation 8.9 ($v_T = r \omega$), and this equation can be used to determine the radius $r$. However, we must remember that this relationship is only valid if we use radian measure. Therefore, it will be necessary to convert the given angular speed in rev/s into rad/s.

**SOLUTION** Solving Equation 8.9 for the radius gives

$$r = \frac{v_T}{\omega} = \frac{54 \text{ m/s}}{(47 \text{ rev/s}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right)} = 0.18 \text{ m}$$

where we have used the fact that 1 rev corresponds to $2\pi$ rad to convert the given angular speed from rev/s into rad/s.

38. **REASONING** The angular speed $\omega$ of the reel is related to the tangential speed $v_T$ of the fishing line by $v_T = r \omega$ (Equation 8.9), where $r$ is the radius of the reel. Solving this equation for $\omega$ gives $\omega = v_T / r$. The tangential speed of the fishing line is just the distance $x$ it travels divided by the time $t$ it takes to travel that distance, or $v_T = x/t$. 
**SOLUTION** Substituting \(v_T = x/t\) into \(\omega = v_T / r\) and noting that 3.0 cm = 3.0 \(\times\) 10\(^{-2}\) m, we find that

\[
\omega = \frac{v_T}{r} = \frac{x}{t} = \frac{2.6\text{ m}}{9.5\text{ s}} = \frac{9.1\text{ rad/s}}{3.0 \times 10^{-2}\text{ m}}
\]

39. **REASONING** The length of tape that passes around the reel is just the average tangential speed of the tape times the time \(t\). The average tangential speed \(\bar{v}_t\) is given by Equation 8.9 \((\bar{v}_t = r\bar{\omega})\) as the radius \(r\) times the average angular speed \(\bar{\omega}\) in rad/s.

**SOLUTION** The length \(L\) of tape that passes around the reel in \(t = 13\) s is \(L = \bar{v}_t t\). Using Equation 8.9 to express the tangential speed, we find

\[
L = \bar{v}_t t = r\bar{\omega}t = (0.014\text{ m})(3.4\text{ rad/s})(13\text{ s}) = 0.62\text{ m}
\]

40. **REASONING AND SOLUTION**

a. A person living in Ecuador makes one revolution (2\(\pi\) rad) every 23.9 hr (8.60 \(\times\) 10\(^4\) s). The angular speed of this person is \(\omega = (2\pi\text{ rad})/(8.60 \times 10^4\text{ s}) = 7.31 \times 10^{-5}\) rad/s. According to Equation 8.9, the tangential speed of the person is, therefore,

\[
v_T = r\omega = \left(6.38 \times 10^6\text{ m}\right)(7.31 \times 10^{-5}\text{ rad/s}) = 4.66 \times 10^2\text{ m/s}
\]

b. The relevant geometry is shown in the drawing at the right. Since the tangential speed is one-third of that of a person living in Ecuador, we have,

\[
\frac{v_T}{3} = r\theta\omega
\]

or

\[
r\theta = \frac{v_T}{3\omega} = \frac{4.66 \times 10^2\text{ m/s}}{3\left(7.31 \times 10^{-5}\text{ rad/s}\right)} = 2.12 \times 10^6\text{ m}
\]

The angle \(\theta\) is, therefore,

\[
\theta = \cos^{-1}\left(\frac{r\theta}{r}\right) = \cos^{-1}\left(\frac{2.12 \times 10^6\text{ m}}{6.38 \times 10^6\text{ m}}\right) = 70.6^\circ
\]
41. **REASONING** The tangential speed $v_T$ of a point on the “equator” of the baseball is given by Equation 8.9 as $v_T = r\omega$, where $r$ is the radius of the baseball and $\omega$ is its angular speed. The radius is given in the statement of the problem. The (constant) angular speed is related to that angle $\theta$ through which the ball rotates by Equation 8.2 as $\omega = \theta/t$, where we have assumed for convenience that $\theta_0 = 0$ rad when $t_0 = 0$ s. Thus, the tangential speed of the ball is

$$v_T = r\omega = r\left(\frac{\theta}{t}\right)$$

The time $t$ that the ball is in the air is equal to the distance $x$ it travels divided by its linear speed $v$, $t = x/v$, so the tangential speed can be written as

$$v_T = r\left(\frac{\theta}{x/v}\right) = \frac{r\theta v}{x}$$

**SOLUTION** The tangential speed of a point on the equator of the baseball is

$$v_T = \frac{r\theta v}{x} = \frac{(3.67 \times 10^{-2} \text{ m})(49.0 \text{ rad})(42.5 \text{ m/s})}{16.5 \text{ m}} = [4.63 \text{ m/s}]$$

42. **REASONING** The linear speed $v_1$ with which the bucket moves down the well is the same as the linear speed of the rope holding the bucket. The rope, in turn, is wrapped around the barrel of the hand crank and unwinds without slipping. This ensures that the rope’s linear speed is the same as the tangential speed $v_T = r_1\omega$ (Equation 8.9) of a point on the surface of the barrel, where $\omega$ and $r_1$ are the barrel’s angular speed and radius, respectively. Therefore, we have $v_1 = r_1\omega$. When applied to the linear speed $v_2$ of the crank handle and the radius $r_2$ of the circle the handle traverses, Equation 8.9 yields $v_2 = r_2\omega$. We can use the same symbol $\omega$ to represent the angular speed of the barrel and the angular speed of the hand crank, since both make the same number of revolutions in any given amount of time. Lastly, we note that the radii $r_1$ of the crank barrel and $r_2$ of the hand crank’s circular motion are half of the respective diameters $d_1 = 0.100 \text{ m}$ and $d_2 = 0.400 \text{ m}$ shown in the text drawing.

**SOLUTION** Solving the relations $v_1 = r_1\omega$ and $v_2 = r_2\omega$ for the angular speed $\omega$ and the linear speed $v_1$ of the bucket, we obtain

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} \quad \text{or} \quad v_1 = \frac{v_2 r_1}{r_2} = \frac{v_2}{\frac{1}{2}d_1} = \frac{v_2 d_1}{d_2}$$

The linear speed of the bucket, therefore, is

$$v_1 = \frac{(1.20 \text{ m/s})(0.100 \text{ m})}{(0.400 \text{ m})} = 0.300 \text{ m/s}$$
43. **REASONING AND SOLUTION** The following figure shows the initial and final states of the system.

![Initial and final configurations](image)

a. From the principle of conservation of mechanical energy:

\[ E_0 = E_f \]

Initially the system has only gravitational potential energy. If the level of the hinge is chosen as the zero level for measuring heights, then: \( E_0 = mgh_0 = mgL \). Just before the object hits the floor, the system has only kinetic energy.

Therefore

\[ mgL = \frac{1}{2}mv^2 \]

Solving for \( v \) gives

\[ v = \sqrt{2gL} \]

From Equation 8.9, \( v_T = r\omega \). Solving for \( \omega \) gives \( \omega = v_T/r \). As the object rotates downward, it travels in a circle of radius \( L \). Its speed just before it strikes the floor is its tangential speed. Therefore,

\[ \omega = \frac{v_T}{r} = \frac{v}{L} = \frac{\sqrt{2gL}}{L} = \sqrt{\frac{2g}{L}} = \sqrt{\frac{2(9.80 \text{ m/s}^2)}{1.50 \text{ m}}} = \boxed{3.61 \text{ rad/s}} \]

b. From Equation 8.10:

\[ a_T = r\alpha \]

Solving for \( \alpha \) gives \( \alpha = a_T/r \). Just before the object hits the floor, its tangential acceleration is the acceleration due to gravity. Thus,

\[ \alpha = \frac{a_T}{r} = \frac{g}{L} = \frac{9.80 \text{ m/s}^2}{1.50 \text{ m}} = \boxed{6.53 \text{ rad/s}^2} \]
44. **REASONING AND SOLUTION** The stone leaves the circular path with a horizontal speed

\[ v_0 = v_T = r \omega \]

so \( \omega = v_0 / r \). We are given that \( r = x / 30 \) so \( \omega = 30 v_0 / x \). Kinematics gives \( x = v_0 t \). With this substitution for \( x \) the expression for \( \omega \) becomes \( \omega = 30 / t \). Kinematics also gives for the vertical displacement \( y \) that

\[ y = v_{0y} t + \frac{1}{2} a_y t^2 \] (Equation 3.5b). In Equation 3.5b we know that \( v_{0y} = 0 \) m/s since the stone is launched horizontally, so that

\[ y = \frac{1}{2} a_y t^2 \] or \( t = \sqrt{\frac{2 y}{a_y}} \).

Using this result for \( t \) in the expression for \( \omega \) and assuming that upward is positive, we find

\[ \omega = 30 \sqrt{\frac{a_y}{2y}} = 30 \sqrt{-\frac{9.80 \text{ m/s}^2}{2(-20.0 \text{ m})}} = 14.8 \text{ rad/s} \]

45. **SSM REASONING** Since the car is traveling with a constant speed, its tangential acceleration must be zero. The radial or centripetal acceleration of the car can be found from Equation 5.2. Since the tangential acceleration is zero, the total acceleration of the car is equal to its radial acceleration.

**SOLUTION**

a. Using Equation 5.2, we find that the car’s radial acceleration, and therefore its total acceleration, is

\[ a = a_R = \frac{v_T^2}{r} = \frac{(75.0 \text{ m/s})^2}{625 \text{ m}} = 9.00 \text{ m/s}^2 \]

b. The direction of the car’s total acceleration is the same as the direction of its radial acceleration. That is, the direction is **radially inward**.

46. **REASONING** The magnitude \( \omega \) of each car’s angular speed can be evaluated from \( a_c = r \omega^2 \) (Equation 8.11), where \( r \) is the radius of the turn and \( a_c \) is the magnitude of the centripetal acceleration. We are given that the centripetal acceleration of each car is the same. In addition, the radius of each car’s turn is known. These facts will enable us to determine the ratio of the angular speeds.

**SOLUTION** Solving Equation 8.11 for the angular speed gives \( \omega = \sqrt{a_c / r} \). Applying this relation to each car yields:

**Car A:**

\[ \omega_A = \sqrt{a_{c,A} / r_A} \]

**Car B:**

\[ \omega_B = \sqrt{a_{c,B} / r_B} \]
Taking the ratio of these two angular speeds, and noting that \( a_{c, A} = a_{c, B} \), gives

\[
\frac{\omega_A}{\omega_B} = \frac{\sqrt{a_{c, A}}}{\sqrt{a_{c, B}}} = \frac{\sqrt{\frac{36 \text{ m}}{48 \text{ m}}}}{0.87}
\]

47. **REASONING**

a. According to Equation 8.2, the average angular speed is equal to the magnitude of the angular displacement divided by the elapsed time. The magnitude of the angular displacement is one revolution, or \( 2\pi \) rad. The elapsed time is one year, expressed in seconds.

b. The tangential speed of the earth in its orbit is equal to the product of its orbital radius and its orbital angular speed (Equation 8.9).

c. Since the earth is moving on a nearly circular orbit, it has a centripetal acceleration that is directed toward the center of the orbit. The magnitude \( a_c \) of the centripetal acceleration is given by Equation 8.11 as \( a_c = r\omega^2 \).

**SOLUTION**

a. The average angular speed is

\[
\omega = \bar{\omega} = \frac{\Delta \theta}{\Delta t} = \frac{2\pi \text{ rad}}{3.16 \times 10^7 \text{ s}} = 1.99 \times 10^{-7} \text{ rad/s}
\]

b. The tangential speed of the earth in its orbit is

\[v_T = r\omega = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s}) = 2.98 \times 10^4 \text{ m/s}\]

(8.9)

c. The centripetal acceleration of the earth due to its circular motion around the sun is

\[a_c = r\omega^2 = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s})^2 = 5.94 \times 10^{-3} \text{ m/s}^2\]

The acceleration is directed toward the center of the orbit.

48. **REASONING** As discussed in Section 8.5 and illustrated in Multiple-Concept Example 7, the total acceleration in a situation like that in this problem is the vector sum of the centripetal acceleration \( a_c \) and the tangential acceleration \( a_T \). Since these two accelerations are perpendicular, the Pythagorean theorem applies, and the total acceleration \( a \) is given by
\[ a = \sqrt{a_c^2 + a_T^2} \]. The total acceleration vector makes an angle \( \phi \) with respect to the radial direction, where \( \phi = \tan^{-1}\left(\frac{a_T}{a_c}\right) \).

**SOLUTION**

a. The centripetal acceleration of the ball is given by Equation 8.11 as
\[ a_c = r \omega^2 = (0.670 \text{ m})(16.0 \text{ rad/s})^2 = 172 \text{ m/s}^2 \]

The tangential acceleration of the ball is given by Equation 8.10 as
\[ a_T = r \alpha = (0.670 \text{ m})(64.0 \text{ rad/s}^2) = 42.9 \text{ m/s}^2 \]

Applying the Pythagorean theorem, we find that the magnitude of the total acceleration is
\[ a = \sqrt{a_c^2 + a_T^2} = \sqrt{(172 \text{ m/s}^2)^2 + (42.9 \text{ m/s}^2)^2} = 177 \text{ m/s}^2 \]

b. The total acceleration vector makes an angle relative to the radial acceleration of
\[ \phi = \tan^{-1}\left(\frac{a_T}{a_c}\right) = \tan^{-1}\left(\frac{42.9 \text{ m/s}^2}{172 \text{ m/s}^2}\right) = 14.0^\circ \]

49. **REASONING** The centripetal acceleration \( a_c \) at either corner is related to the angular speed \( \omega \) of the plate by \( a_c = r \omega^2 \) (Equation 8.11), where \( r \) is the radial distance of the corner from the rotation axis of the plate. The angular speed \( \omega \) is the same for all points on the plate, including both corners. But the radial distance \( r_A \) of corner \( A \) from the rotation axis of the plate is different from the radial distance \( r_B \) of corner \( B \). The fact that the centripetal acceleration at corner \( A \) is \( n \) times as great as the centripetal acceleration at corner \( B \) yields the relationship between the radial distances:

\[ r_A \omega^2 = n \left( r_B \omega^2 \right) \quad \text{or} \quad r_A = n r_B \quad (1) \]

The radial distance \( r_B \) at corner \( B \) is the length of the short side of the rectangular plate: \( r_B = L_1 \). The radial distance \( r_A \) at corner \( A \) is the length of a straight line from the rotation axis to corner \( A \). This line is the diagonal of the plate, so we obtain \( r_A \) from the Pythagorean theorem (Equation 1.7): \( r_A = \sqrt{L_1^2 + L_2^2} \).
SOLUTION Making the substitutions $r_A = \sqrt{L_1^2 + L_2^2}$ and $r_B = L_1$ in Equation (1) gives

$$\frac{\sqrt{L_1^2 + L_2^2}}{r_A} = n \frac{L_1}{r_B} \quad (2)$$

Squaring both sides of Equation (2) and solving for the ratio $L_1/L_2$ yields

$$n^2 L_1^2 = L_1^2 + L_2^2 \quad \text{or} \quad (n^2 - 1)L_1^2 = L_2^2 \quad \text{or} \quad \frac{L_1}{L_2} = \frac{1}{\sqrt{n^2 - 1}}$$

Thus, when $n = 2.00$, the ratio of the lengths of the sides of the rectangle is

$$\frac{L_1}{L_2} = \frac{1}{\sqrt{2^2 - 1}} = \frac{1}{\sqrt{3}} = 0.577$$

50. REASONING

a. Since the angular velocity of the fan blade is changing, there are simultaneously a tangential acceleration $a_T$ and a centripetal acceleration $a_c$ that are oriented at right angles to each other. The drawing shows these two accelerations for a point on the tip of one of the blades (for clarity, the blade itself is not shown). The blade is rotating in the counterclockwise (positive) direction.

The magnitude of the total acceleration is

$$a = \sqrt{a_c^2 + a_T^2},$$

given by the Pythagorean theorem.

The magnitude $a_c$ of the centripetal acceleration can be evaluated from $a_c = r\omega^2$ (Equation 8.11), where $\omega$ is the final angular velocity. The final angular velocity can be determined from Equation 8.4 as $\omega = \omega_0 + \alpha t$. The magnitude $a_T$ of the tangential acceleration follows from $a_T = r\alpha$ (Equation 8.10).

b. From the drawing we see that the angle $\phi$ can be obtained by using trigonometry, $\phi = \tan^{-1}\left(\frac{a_T}{a_c}\right)$.

SOLUTION

a. Substituting $a_c = r\omega^2$ (Equation 8.11) and $a_T = r\alpha$ (Equation 8.10) into $a = \sqrt{a_c^2 + a_T^2}$ gives

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(r\omega^2)^2 + (r\alpha)^2} = r\sqrt{\omega^4 + \alpha^2}$$
The final angular velocity $\omega$ is related to the initial angular velocity $\omega_0$ by $\omega = \omega_0 + \alpha t$ (see Equation 8.4). Thus, the magnitude of the total acceleration is

$$a = r \sqrt{\alpha^2 + \omega_0^2} = r \sqrt{(\omega_0 + \alpha t)^2 + \alpha^2}$$

$$= (0.380 \text{ m}) \sqrt{[1.50 \text{ rad/s} + (2.00 \text{ rad/s}^2)(0.500 \text{ s})]^2 + (2.00 \text{ rad/s}^2)^2} = 2.49 \text{ m/s}^2$$

b. The angle $\phi$ between the total acceleration $\mathbf{a}$ and the centripetal acceleration $\mathbf{a}_c$ is (see the drawing above)

$$\phi = \tan^{-1} \left( \frac{a_T}{a_c} \right) = \tan^{-1} \left( \frac{r \alpha}{r \omega^2} \right) = \tan^{-1} \left[ \frac{\alpha}{(\omega_0 + \alpha t)^2} \right]$$

$$= \tan^{-1} \left( \frac{2.00 \text{ rad/s}^2}{\left[1.50 \text{ rad/s} + (2.00 \text{ rad/s}^2)(0.500 \text{ s})\right]^2} \right) = 17.7^\circ$$

where we have used the same substitutions for $a_T$, $a_c$, and $\omega$ as in part (a).

### 51. **SSM REASONING**

a. The tangential speed $v_T$ of the sun as it orbits about the center of the Milky Way is related to the orbital radius $r$ and angular speed $\omega$ by Equation 8.9, $v_T = r \omega$. Before we use this relation, however, we must first convert $r$ to meters from light-years.

b. The centripetal force is the net force required to keep an object, such as the sun, moving on a circular path. According to Newton’s second law of motion, the magnitude $F_c$ of the centripetal force is equal to the product of the object’s mass $m$ and the magnitude $a_c$ of its centripetal acceleration (see Section 5.3): $F_c = ma_c$. The magnitude of the centripetal acceleration is expressed by Equation 8.11 as $a_c = r \omega^2$, where $r$ is the radius of the circular path and $\omega$ is the angular speed of the object.

### SOLUTION

a. The radius of the sun’s orbit about the center of the Milky Way is

$$r = (2.3 \times 10^4 \text{ light-years}) \left( \frac{9.5 \times 10^{15} \text{ m}}{1 \text{ light-year}} \right) = 2.2 \times 10^{20} \text{ m}$$

The tangential speed of the sun is

$$v_T = r \omega = (2.2 \times 10^{20} \text{ m})(1.1 \times 10^{-15} \text{ rad/s}) = 2.4 \times 10^5 \text{ m/s}$$  \hspace{1cm} (8.9)
b. The magnitude of the centripetal force that acts on the sun is

\[ F_c = m a_c = m r \omega^2 \]

Centripetal force

\[ = \left(1.99 \times 10^{30} \text{ kg}\right) \left(2.2 \times 10^{20} \text{ m}\right) \left(1.1 \times 10^{-15} \text{ rad/s}\right)^2 = 5.3 \times 10^{20} \text{ N} \]

52. **REASONING** The tangential acceleration and the centripetal acceleration of a point at a distance \( r \) from the rotation axis are given by Equations 8.10 and 8.11, respectively: \( a_T = r \alpha \) and \( a_c = r \omega^2 \). After the drill has rotated through the angle in question, \( a_c = 2a_T \), or

\[ r \omega^2 = 2r \alpha \]

This expression can be used to find the angular acceleration \( \alpha \). Once the angular acceleration is known, Equation 8.8 can be used to find the desired angle.

**SOLUTION** Solving the expression obtained above for \( \alpha \) gives

\[ \alpha = \frac{\omega^2}{2} \]

Solving Equation 8.8 for \( \theta \) (with \( \omega_0 = 0 \text{ rad/s} \) since the drill starts from rest), and using the expression above for the angular acceleration \( \alpha \) gives

\[ \theta = \frac{\omega^2}{2 \alpha} = \frac{\omega^2}{2 \left(\frac{\omega^2}{2}\right)} = \left(\frac{\omega^2}{2}\right) \left(\frac{2}{\omega^2}\right) = \frac{1.00 \text{ rad}}{} \]

Note that since both Equations 8.10 and 8.11 require that the angles be expressed in radians, the final result for \( \theta \) is in radians.

53. **SSM REASONING AND SOLUTION** From Equation 2.4, the linear acceleration of the motorcycle is

\[ a = \frac{v - v_0}{t} = \frac{22.0 \text{ m/s} - 0 \text{ m/s}}{9.00 \text{ s}} = 2.44 \text{ m/s}^2 \]

Since the tire rolls without slipping, the linear acceleration equals the tangential acceleration of a point on the outer edge of the tire: \( a = a_T \). Solving Equation 8.13 for \( \alpha \) gives

\[ \alpha = \frac{a_T}{r} = \frac{2.44 \text{ m/s}^2}{0.280 \text{ m}} = \frac{8.71 \text{ rad/s}^2}{\text{m}} \]
54. **REASONING AND SOLUTION**  
   a. If the wheel does not slip, a point on the rim rotates about the axle with a speed 
   
   \[ v_T = v = 15.0 \text{ m/s} \]
   
   For a point on the rim
   
   \[ \omega = \frac{v_T}{r} = \frac{15.0 \text{ m/s}}{0.330 \text{ m}} = 45.5 \text{ rad/s} \]
   
   b. \[ v_T = r \omega = (0.175 \text{ m})(45.5 \text{ rad/s}) = 7.96 \text{ m/s} \]

55. **REASONING**  
   The angular displacement \( \theta \) of each wheel is given by Equation 8.7  
   \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \), which is one of the equations of rotational kinematics. In this expression \( \omega_0 \) is the initial angular velocity, and \( \alpha \) is the angular acceleration, neither of which is given directly. Instead the initial linear velocity \( v_0 \) and the linear acceleration \( a \) are given. However, we can relate these linear quantities to their analogous angular counterparts by means of the assumption that the wheels are rolling and not slipping. Then, according to Equation 8.12 \( (v_0 = r \omega) \), we know that \( \omega_0 = \frac{v_0}{r} \), where \( r \) is the radius of the wheels. Likewise, according to Equation 8.13 \( (a = r \alpha) \), we know that \( \alpha = \frac{a}{r} \). Both Equations 8.12 and 8.13 are only valid if used with radian measure. Therefore, when we substitute the expressions for \( \omega_0 \) and \( \alpha \) into Equation 8.7, the resulting value for the angular displacement \( \theta \) will be in radians.

**SOLUTION**  
Substituting \( \omega_0 \) from Equation 8.12 and \( \alpha \) from Equation 8.13 into Equation 8.7, we find that

\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \left( \frac{v_0}{r} \right) t + \frac{1}{2} \left( \frac{a}{r} \right) t^2 \\
= \left( \frac{20.0 \text{ m/s}}{0.300 \text{ m}} \right)(8.00 \text{ s}) + \frac{1}{2} \left( \frac{1.50 \text{ m/s}^2}{0.300 \text{ m}} \right)(8.00 \text{ s})^2 = 693 \text{ rad}
\]

56. **REASONING AND SOLUTION**  
   The bike would travel with the same speed as a point on the wheel \( v = r \omega \). It would then travel a distance

\[
x = vt = r \omega t = (0.45 \text{ m})(9.1 \text{ rad/s}) \left( \frac{35 \text{ min}}{1 \text{ min}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 8.6 \times 10^3 \text{ m}
\]

57. **REASONING**  
   a. The constant angular acceleration \( \alpha \) of the wheel is defined by Equation 8.4 as the change in the angular velocity, \( \omega - \omega_0 \), divided by the elapsed time \( t \), or \( \alpha = (\omega - \omega_0)/t \). The time is
known. Since the wheel rotates in the positive direction, its angular velocity is the same as its angular speed. However, the angular speed is related to the linear speed $v$ of a wheel and its radius $r$ by $v = r\omega$ (Equation 8.12). Thus, $\omega = v/r$, and we can write for the angular acceleration that

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{v - v_0}{r t} = \frac{v - v_0}{r t}$$

b. The angular displacement $\theta$ of each wheel can be obtained from Equation 8.7 of the equations of kinematics: $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$, where $\omega_0 = v_0/r$ and $\alpha$ can be obtained as discussed in part (a).

**SOLUTION**
a. The angular acceleration of each wheel is

$$\alpha = \frac{v - v_0}{r t} = \frac{2.1 \text{ m/s} - 6.6 \text{ m/s}}{(0.65 \text{ m})(5.0 \text{ s})} = -1.4 \text{ rad/s}^2$$

b. The angular displacement of each wheel is

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \left(\frac{v_0}{r}\right) t + \frac{1}{2} \alpha t^2$$

$$= \left(\frac{6.6 \text{ m/s}}{0.65 \text{ m}}\right)(5.0 \text{ s}) + \frac{1}{2}(-1.4 \text{ rad/s}^2)(5.0 \text{ s})^2 = +33 \text{ rad}$$

58. *REASONING* For a wheel that rolls without slipping, the relationship between its linear speed $v$ and its angular speed $\omega$ is given by Equation 8.12 as $v = r\omega$, where $r$ is the radius of a wheel.

For a wheel that rolls without slipping, the relationship between the magnitude $a$ of its linear acceleration and the magnitude $\alpha$ of the angular acceleration is given by Equation 8.13 as $a = r\alpha$, where $r$ is the radius of a wheel. The linear acceleration can be obtained using the equations of kinematics for linear motion, in particular, Equation 2.9.

**SOLUTION**
a. From Equation 8.12 we have that

$$v = r\omega = (0.320 \text{ m})(288 \text{ rad/s}) = 92.2 \text{ m/s}$$

b. The magnitude of the angular acceleration is given by Equation 8.13 as $\alpha = a/r$. The linear acceleration $a$ is related to the initial and final linear speeds and the displacement $x$ by
Equation 2.9 from the equations of kinematics for linear motion; \( a = \frac{v^2 - v_0^2}{2x} \). Thus, the magnitude of the angular acceleration is

\[
\alpha = \frac{a}{r} = \frac{(v^2 - v_0^2)/(2x)}{r} = \frac{v^2 - v_0^2}{2xr} = \frac{(92.2 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(384 \text{ m})(0.320 \text{ m})} = 34.6 \text{ rad/s}^2
\]

59. **REASONING** The distance \( d \) traveled by the axle of a rolling wheel (radius = \( r \)) during one complete revolution is equal to the circumference \( (2\pi r) \) of the wheel: \( d = 2\pi r \). Therefore, when the bicycle travels a total distance \( D \) in a race, and the wheel makes \( N \) revolutions, the total distance \( D \) is \( N \) times the circumference of the wheel:

\[
D = Nd = N(2\pi r) \tag{1}
\]

We will apply Equation (1) first to the smaller bicycle wheel to determine its radius \( r_1 \). Equation (1) will then also determine the number of revolutions made by the larger bicycle wheel, which has a radius of \( r_2 = r_1 + 0.012 \text{ m} \).

**SOLUTION** Because \( 1 \text{ km} = 1000 \text{ m} \), the total distance traveled during the race is \( D = (4520 \text{ km})[(1000 \text{ m})/(1 \text{ km})] = 4520 \times 10^3 \text{ m} \). From Equation (1), then, the radius \( r_1 \) of the smaller bicycle wheel is

\[
r_1 = \frac{D}{2\pi N_1} = \frac{4520 \times 10^3 \text{ m}}{2\pi (2.18 \times 10^6)} = 0.330 \text{ m}
\]

The larger wheel, then, has a radius \( r_2 = 0.330 \text{ m} + 0.012 \text{ m} = 0.342 \text{ m} \). Over the same distance \( D \), this wheel would make \( N_2 \) revolutions, where, by Equation (1),

\[
N_2 = \frac{D}{2\pi r_2} = \frac{4520 \times 10^3 \text{ m}}{2\pi (0.342 \text{ m})} = 2.10 \times 10^6
\]

60. **REASONING** Our solution will be based on the relation \( \theta = \frac{s}{r} \) (Equation 8.1), where the angle \( \theta \) is expressed in radians and \( s \) denotes the arc length that the angle subtends on a circular arc of radius \( r \). We will take advantage of the following fact (see Section 8.6): the axle of a rolling wheel travels a distance \( s_{\text{axle}} \) that equals the circular arc length \( s_{\text{wheel}} \) along the outer edge of the rolling wheel.
**SOLUTION** We seek the angle $\theta_{\text{wheel}}$ through which each wheel rotates. Applying Equation 8.1 to one of the rolling wheels, we have

$$\theta_{\text{wheel}} = \frac{s_{\text{wheel}}}{r_{\text{wheel}}} \quad (1)$$

We also know that the wheel is rolling, so that $s_{\text{wheel}} = s_{\text{axle}}$ and Equation (1) becomes

$$\theta_{\text{wheel}} = \frac{s_{\text{axle}}}{r_{\text{wheel}}} \quad (2)$$

To evaluate the distance $s_{\text{axle}}$ we note that the axle moves on a circular arc subtended by the angle of 0.960 rad (see the drawing in the text). Using Equation 8.1 once again, we have

$$0.960 \text{ rad} = \frac{s_{\text{axle}}}{(9.00 \text{ m}) - (0.400 \text{ m})} \quad \text{or} \quad s_{\text{axle}} = (8.60 \text{ m})(0.960 \text{ rad})$$

In this result we have used the fact that the axle is 0.400 m closer to the apex of the 0.960-rad angle than the surface of the hill is. Using this value for $s_{\text{axle}}$ in Equation (2), we find that

$$\theta_{\text{wheel}} = \frac{s_{\text{axle}}}{r_{\text{wheel}}} = \frac{(8.60 \text{ m})(0.960 \text{ rad})}{0.400 \text{ m}} = 20.6 \text{ rad}$$

---

61. **Reasoning** As a penny-farthing moves, both of its wheels roll without slipping. This means that the axle for each wheel moves through a linear distance (the distance through which the bicycle moves) that equals the circular arc length measured along the outer edge of the wheel. Since both axles move through the same linear distance, the circular arc length measured along the outer edge of the large front wheel must equal the circular arc length measured along the outer edge of the small rear wheel. In each case the arc length $s$ is equal to the number $n$ of revolutions times the circumference $2\pi r$ of the wheel ($r = \text{radius}$).

**Solution** Since the circular arc length measured along the outer edge of the large front wheel must equal the circular arc length measured along the outer edge of the small rear wheel, we have

$$\frac{n_{\text{Rear}}2\pi r_{\text{Rear}}}{\text{Arc length for rear wheel}} = \frac{n_{\text{Front}}2\pi r_{\text{Front}}}{\text{Arc length for front wheel}}$$

Solving for $n_{\text{Rear}}$ gives

$$n_{\text{Rear}} = \frac{n_{\text{Front}}r_{\text{Front}}}{r_{\text{Rear}}} = \frac{276(1.20 \text{ m})}{0.340 \text{ m}} = 974 \text{ rev}$$

---

62. **Reasoning** While the ball is in the air, its angular speed $\omega$ is constant, and thus its angular displacement is given by $\theta = \omega t$ (Equation 8.2). The angular speed of the ball is
found by considering its rolling motion on the table top, because its angular speed does not change after it leaves the table. For rolling motion, the angular speed \( \omega \) is related to the linear speed \( v \) by \( \omega = \frac{v}{r} \) (Equation 8.12), where \( r \) is the radius of the ball. In order to determine the time \( t \) the ball spends in the air, we treat it as a projectile launched horizontally. The vertical displacement of the ball is then given by \( y = v_0y t + \frac{1}{2} a_y t^2 \) (Equation 3.5b), which we will use to determine the time \( t \) that elapses while the ball is in the air.

**SOLUTION** Since the ball is launched horizontally in the projectile motion, its initial velocity \( v_0 \) has no vertical component: \( v_{0y} = 0 \) m/s. Solving \( y = v_0y t + \frac{1}{2} a_y t^2 \) (Equation 3.5b) for the elapsed time \( t \), we obtain

\[
y = (0 \; \text{m/s})t + \frac{1}{2} a_y t^2 = \frac{1}{2} a_y t^2 \quad \text{or} \quad t = \sqrt{\frac{2y}{a_y}} \quad (1)
\]

Substituting Equation (1) and \( \omega = \frac{v}{r} \) (Equation 8.12) into \( \theta = \omega t \) (Equation 8.2) yields

\[
\theta = \left( \frac{v}{r} \right) \sqrt{\frac{2y}{a_y}} \quad (2)
\]

We choose the upward direction to be positive. Once the ball leaves the table, it is in free fall, so the vertical acceleration of the ball is that due to gravity: \( a_y = -9.80 \) m/s\(^2\). Further, the ball’s vertical displacement is negative as the ball falls to the floor: \( y = -2.10 \) m. The ball’s angular displacement while it is in the air is, from Equation (2),

\[
\theta = \frac{v}{r} \sqrt{\frac{2y}{a_y}} = \frac{3.60 \; \text{m/s}}{0.200 \; \text{m/s}} \sqrt{\frac{2(-2.10 \; \text{m})}{-9.80 \; \text{m/s}^2}} = 11.8 \; \text{rad}
\]

63. **REASONING** The wheels on both sides of the car have the same radius \( r = 0.350 \) m and undergo rolling motion, so we will use \( v = r\omega \) (Equation 8.12) to calculate their individual angular speeds:

\[
\omega_{\text{left}} = \frac{v_{\text{left}}}{r} \quad \text{and} \quad \omega_{\text{right}} = \frac{V_{\text{right}}}{r} \quad (1)
\]

In Equations (1) the linear speeds \( v_{\text{left}} \) and \( v_{\text{right}} \) at which the wheels on opposite sides of the car travel around the track differ. This is because the wheels on one side of the car are closer to the center of the track than are the wheels on the other side. As the car makes one complete lap of the track, therefore, both sets of wheels follow circular paths of different radii \( R_{\text{left}} \) and \( R_{\text{right}} \). The linear speed of each wheel is the circumference of its circular path divided by the elapsed time \( t \), which is the same for both sets of wheels:
Substituting Equations (2) into Equations (1), we obtain

\[ \omega_{\text{left}} = \frac{2\pi R_{\text{left}}}{t} = \frac{2\pi R_{\text{left}}}{r t} \quad \text{and} \quad \omega_{\text{right}} = \frac{2\pi R_{\text{right}}}{t} = \frac{2\pi R_{\text{right}}}{r t} \]  

\[ (3) \]

**SOLUTION** We will assume that the wheels on the left side of the car are closer to the center of the track than the wheels on the right side. We do not know the radii of the circular paths of either set of wheels, but the difference between them is \( R_{\text{right}} - R_{\text{left}} = 1.60 \text{ m} \). We can now calculate the difference between the angular speeds of the wheels on the left and right sides of the car by subtracting \( \omega_{\text{left}} \) from \( \omega_{\text{right}} \) [see Equations (3)]:

\[ \omega_{\text{right}} - \omega_{\text{left}} = \frac{2\pi R_{\text{right}}}{r t} - \frac{2\pi R_{\text{left}}}{r t} = \frac{2\pi \left( R_{\text{right}} - R_{\text{left}} \right)}{r t} = \frac{2\pi (1.60 \text{ m})}{(0.350 \text{ m})(19.5 \text{ s})} = 1.47 \text{ rad/s} \]

64. **REASONING** The average angular velocity of either mandible is given by \( \bar{\omega} = \Delta \theta / \Delta t \) (Equation 8.2), where \( \Delta \theta \) is the angular displacement of the mandible and \( \Delta t \) is the elapsed time. In order to calculate the average angular velocity in radians per second, we will first convert the angular displacement \( \Delta \theta \) from degrees to radians.

**SOLUTION** Converting an angular displacement of 90° into radians, we find that the angular displacement of the mandible is

\[ \Delta \theta = \left( 90 \text{ degrees} \right) \left( \frac{1 \text{ rev}}{360 \text{ degrees}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = \frac{\pi}{2} \text{ rad} \]

The average angular velocity of the mandible is

\[ \bar{\omega} = \frac{\Delta \theta}{\Delta t} = \frac{\frac{\pi}{2} \text{ rad}}{1.3 \times 10^{-4} \text{ s}} = 1.2 \times 10^4 \text{ rad/s} \]  

(8.2)

65. **SSM REASONING** The tangential acceleration \( a_T \) of the speedboat can be found by using Newton's second law, \( F_T = ma_T \), where \( F_T \) is the net tangential force. Once the tangential acceleration of the boat is known, Equation 2.4 can be used to find the tangential speed of the boat 2.0 s into the turn. With the tangential speed and the radius of the turn known, Equation 5.2 can then be used to find the centripetal acceleration of the boat.

**SOLUTION**

a. From Newton's second law, we obtain
\[ a_T = \frac{F_T}{m} = \frac{550 \text{ N}}{220 \text{ kg}} = 2.5 \text{ m/s}^2 \]

b. The tangential speed of the boat 2.0 s into the turn is, according to Equation 2.4,

\[ v_T = v_{0T} + a_T t = 5.0 \text{ m/s} + (2.5 \text{ m/s}^2)(2.0 \text{ s}) = 1.0 \times 10^1 \text{ m/s} \]

The centripetal acceleration of the boat is then

\[ a_c = \frac{v_T^2}{r} = \left( \frac{1.0 \times 10^1 \text{ m/s}}{32 \text{ m}} \right)^2 = 3.1 \text{ m/s}^2 \]

66. **REASONING** We know that the flywheel’s final angular velocity is \( \omega = 0.0 \text{ rad/s} \), since it comes to rest. We also that its initial angular velocity is \( \omega_0 = +220 \text{ rad/s} \) and that it has an angular acceleration of \( \alpha = -2.0 \text{ rad/s}^2 \). We have assumed that the initial angular velocity is positive, so the angular acceleration must be negative since it is a deceleration. With these three values, we can use \( \omega^2 = \omega_0^2 + 2\alpha \theta \) (Equation 8.8) to calculate the angular displacement \( \theta \). Moreover, we can use \( \omega = \omega_0 + \alpha t \) (Equation 8.4) to determine the time \( t \) during which the flywheel comes to rest.

**SOLUTION**

a. Solving Equation 8.8 for \( \theta \), we obtain

\[ \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(0 \text{ rad/s})^2 - (220 \text{ rad/s})^2}{2(-2.0 \text{ rad/s}^2)} = 1.2 \times 10^4 \text{ rad} \]

b. Solving Equation 8.4 for \( t \), we obtain

\[ t = \frac{\omega - \omega_0}{\alpha} = \frac{0 \text{ rad/s} - 220 \text{ rad/s}}{-2.0 \text{ rad/s}^2} = 1.1 \times 10^2 \text{ s} \]

67. **SSM REASONING AND SOLUTION** Since the angular speed of the fan decreases, the sign of the angular acceleration must be opposite to the sign for the angular velocity. Taking the angular velocity to be positive, the angular acceleration, therefore, must be a negative quantity. Using Equation 8.4 we obtain

\[ \omega_0 = \omega - \alpha t = 83.8 \text{ rad/s} - (-42.0 \text{ rad/s}^2)(1.75 \text{ s}) = 157.3 \text{ rad/s} \]

68. **REASONING** The top of the racket has both tangential and centripetal acceleration components given by Equations 8.10 and 8.11, respectively: \( a_T = r\alpha \) and \( a_c = r\omega^2 \). The total acceleration of the top of the racket is the resultant of these two components. Since
these acceleration components are mutually perpendicular, their resultant can be found by using the Pythagorean theorem.

**SOLUTION** Employing the Pythagorean theorem, we obtain

\[
a = \sqrt{a_T^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}
\]

Therefore,

\[
a = (1.5 \text{ m})\sqrt{(160 \text{ rad/s}^2)^2 + (14 \text{ rad/s})^4} = 380 \text{ m/s}^2
\]

69. **REASONING** The angular speed \( \omega \) of the sprocket can be calculated from the tangential speed \( v_T \) and the radius \( r \) using Equation 8.9 \( (v_T = r\omega) \). The radius is given as \( r = 4.0 \times 10^{-2} \text{ m} \). The tangential speed is identical to the linear speed given for a chain link at point A, so that \( v_T = 5.6 \text{ m/s} \). We need to remember, however, that Equation 8.9 is only valid if radian measure is used. Thus, the value calculated for \( \omega \) will be in rad/s, and we will have to convert to rev/s using the fact that \( 2\pi \) rad equals 1 rev.

**SOLUTION** Solving Equation 8.9 for the angular speed \( \omega \) gives

\[
\omega = \frac{v_T}{r} = \frac{5.6 \text{ m/s}}{4.0 \times 10^{-2} \text{ m}} = 140 \text{ rad/s}
\]

Using the fact that \( 2\pi \) rad equals 1 rev, we can convert this result as follows:

\[
\omega = (140 \text{ rad/s}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = \frac{1}{2\pi} \text{ rev/s}
\]

70. **REASONING** The centripetal acceleration of the trainee is given by \( a_c = r\omega^2 \) (Equation 8.11), where \( \omega \) is the angular speed (in rad/s) of the centrifuge, and \( r \) is the radius of the circular path the trainee follows. Because this radius is the length of the centrifuge arm, we will solve Equation 8.11 for \( r \). In the second exercise, the trainee’s total acceleration gains a tangential component \( a_T = r\alpha \) (Equation 8.10) due to the angular acceleration \( \alpha \) (in rad/s^2) of the centrifuge. The angular speed \( \omega \) and the length \( r \) of the centrifuge arm are both the same as in the first exercise, so the centripetal component of acceleration \( a_c = r\omega^2 \) is unchanged in the second exercise. The two components of the trainee’s acceleration are perpendicular and, thus, are related to the trainee’s total acceleration \( a \) by the Pythagorean theorem: \( a^2 = a_c^2 + a_T^2 \) (Equation 1.7). We will solve the Pythagorean theorem for the trainee’s tangential acceleration \( a_T \), and then use \( a_T = r\alpha \) to determine the angular acceleration \( \alpha \) of the centrifuge.

**SOLUTION**
a. Solving \( a_c = r\omega^2 \) (Equation 8.11) yields the length \( r \) of the centrifuge arm:
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b. Solving \( a^2 = a_c^2 + a_r^2 \) for the tangential component of the trainee’s total acceleration, we obtain \( a_T = \sqrt{a^2 - a_c^2} \). Then, using \( a_T = r\alpha \) (Equation 8.10), we find that the angular acceleration of the centrifuge is

\[
\alpha = \frac{a_T}{r} = \frac{\sqrt{(4.8)(9.80 \text{ m/s}^2)^2 - (3.2)(9.80 \text{ m/s}^2)^2}}{5.0 \text{ m}} = 7.0 \text{ rad/s}^2
\]

71. **REASONING AND SOLUTION**

a. From Equation 8.9, and the fact that 1 revolution = \( 2\pi \) radians, we obtain

\[
v_T = r\omega = (0.0568 \text{ m})(3.50 \text{ rev/s})(\frac{2\pi \text{ rad}}{1 \text{ rev}}) = 1.25 \text{ m/s}
\]

b. Since the disk rotates at constant tangential speed,

\[
v_{T1} = v_{T2} \quad \text{or} \quad \omega_1 r_1 = \omega_2 r_2
\]

Solving for \( \omega_2 \), we obtain

\[
\omega_2 = \frac{\omega_1 r_1}{r_2} = \frac{(3.50 \text{ rev/s})(0.0568 \text{ m})}{0.0249 \text{ m}} = 7.98 \text{ rev/s}
\]

72. **REASONING** Since the time \( t \) and angular acceleration \( \alpha \) are known, we will begin by using Equation 8.7 from the equations of kinematics to determine the angular displacement \( \theta \):

\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2
\]

However, the initial angular velocity \( \omega_0 \) is not given. We can determine it by resorting to another equation of kinematics, \( \omega = \omega_0 + \alpha t \) (Equation 8.4), which relates \( \omega_0 \) to the final angular velocity \( \omega \), the angular acceleration, and the time, all of which are known.

**SOLUTION** Solving Equation 8.4 for \( \omega_0 \) gives \( \omega_0 = \omega - \alpha t \). Substituting this result into \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \) gives
\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = (\omega - \alpha t) + \frac{1}{2} \alpha t^2 = \omega t - \frac{1}{2} \alpha t^2 \\
= (+1.88 \text{ rad/s})(10.0 \text{ s}) - \frac{1}{2}(-5.04 \text{ rad/s}^2)(10.0 \text{ s})^2 = +2.71 \times 10^2 \text{ rad}
\]

73. **REASONING** In addition to knowing the initial angular velocity \( \omega_0 \) and the acceleration \( \alpha \), we know that the final angular velocity \( \omega \) is 0 rev/s, because the wheel comes to a halt. With values available for these three variables, the unknown angular displacement \( \theta \) can be calculated from Equation 8.8 \( (\omega^2 = \omega_0^2 + 2 \alpha \theta) \).

When using any of the equations of rotational kinematics, it is not necessary to use radian measure. Any self-consistent set of units may be used to measure the angular quantities, such as revolutions for \( \theta \), rev/s for \( \omega_0 \) and \( \omega \), and rev/s\(^2 \) for \( \alpha \).

A greater initial angular velocity does not necessarily mean that the wheel will come to a halt on an angular section labeled with a greater number. It is certainly true that greater initial angular velocities lead to greater angular displacements for a given deceleration. However, remember that the angular displacement of the wheel in coming to a halt may consist of a number of complete revolutions plus a fraction of a revolution. In deciding on which number the wheel comes to a halt, the number of complete revolutions must be subtracted from the angular displacement, leaving only the fraction of a revolution remaining.

**SOLUTION** Solving Equation 8.8 for the angular displacement gives

\[
\theta = \frac{\omega^2 - \omega_0^2}{2 \alpha}.
\]

a. We know that \( \omega_0 = +1.20 \text{ rev/s} \), \( \omega = 0 \text{ rev/s} \), and \( \alpha = -0.200 \text{ rev/s}^2 \), where \( \omega_0 \) is positive since the rotation is counterclockwise and, therefore, \( \alpha \) is negative because the wheel decelerates. The value obtained for the displacement is

\[
\theta = \frac{\omega^2 - \omega_0^2}{2 \alpha} = \frac{(0 \text{ rev/s})^2 - (+1.20 \text{ rev/s})^2}{2(-0.200 \text{ rev/s}^2)} = +3.60 \text{ rev}
\]

To decide where the wheel comes to a halt, we subtract the three complete revolutions from this result, leaving 0.60 rev. Converting this value into degrees and noting that each angular section is 30.0º, we find the following number \( n \) for the section where the wheel comes to a halt:

\[
n = (0.60 \text{ rev}) \left( \frac{360^\circ}{1 \text{ rev}} \right) \left( \frac{1 \text{ angular section}}{30.0^\circ} \right) = 7.2
\]

A value of \( n = 7.2 \) means that the wheel comes to a halt in the section following number 7. Thus, it comes to a halt in section 8.

b. Following the same procedure as in part a, we find that
\[ \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(0 \text{ rev/s})^2 - (+1.47 \text{ rev/s})^2}{2(-0.200 \text{ rev/s}^2)} = +5.40 \text{ rev} \]

 Subtracting the five complete revolutions from this result leaves 0.40 rev. Converting this value into degrees and noting that each angular section is 30.0º, we find the following number \( n \) for the section where the wheel comes to a halt:

\[ n = (0.40 \text{ rev}) \left( \frac{360^\circ}{1 \text{ rev}} \right) \left( \frac{1 \text{ angular section}}{30.0^\circ} \right) = 4.8 \]

A value of \( n = 4.8 \) means that the wheel comes to a halt in the section following number 4. Thus, it comes to a halt in section 5.

74. **REASONING** The drawing shows a top view of the race car as it travels around the circular turn. Its acceleration \( \mathbf{a} \) has two perpendicular components: a centripetal acceleration \( \mathbf{a}_c \) that arises because the car is moving on a circular path and a tangential acceleration \( \mathbf{a}_T \) due to the fact that the car has an angular acceleration and its angular velocity is increasing. We can determine the magnitude of the centripetal acceleration from Equation 8.11 as \( a_c = r\omega^2 \), since both \( r \) and \( \omega \) are given in the statement of the problem. As the drawing shows, we can use trigonometry to determine the magnitude \( a \) of the total acceleration, since the angle (35.0º) between \( \mathbf{a} \) and \( \mathbf{a}_c \) is given.

**SOLUTION** Since the vectors \( \mathbf{a}_c \) and \( \mathbf{a} \) are one side and the hypotenuse of a right triangle, we have that

\[ a = \frac{a_c}{\cos 35.0^\circ} \]

The magnitude of the centripetal acceleration is given by Equation 8.11 as \( a_c = r\omega^2 \), so the magnitude of the total acceleration is

\[ a = \frac{a_c}{\cos 35.0^\circ} = \frac{r\omega^2}{\cos 35.0^\circ} = \frac{(23.5 \text{ m})(0.571 \text{ rad/s})^2}{\cos 35.0^\circ} = 9.35 \text{ m/s}^2 \]

75. **REASONING AND SOLUTION** Since the ball spins at 7.7 rev/s, it makes (7.7 rev/s)\( t \) revolutions while in flight, where \( t \) is the time of flight and must be determined. The ball’s vertical displacement is \( y = 0 \text{ m} \) since the ball returns to its launch point. The vertical
component of the ball’s initial velocity is \( v_{0y} = (19 \text{ m/s}) \sin 55^\circ \), assuming upward to be the positive direction. The acceleration due to gravity is \( a_y = -9.80 \text{ m/s}^2 \). With these three pieces of information at hand, we use \( y = v_{0y}t + \frac{1}{2}a_y t^2 \) (Equation 3.5b) to determine the time of flight. Noting that \( y = 0 \) m, we can solve this expression for \( t \) and find that

\[
t = \frac{-2v_{0y}}{a_y} = \frac{-2(19 \text{ m/s}) \sin 55^\circ}{-9.80 \text{ m/s}^2} = 3.2 \text{ s}
\]

and

Number of revolutions = \((7.7 \text{ rev/s})(3.2 \text{ s}) = 25 \text{ rev}\)

---

76. **REASONING AND SOLUTION** By inspection, the distance traveled by the "axle" or the center of the moving quarter is

\[
d = 2\pi(2r) = 4\pi r
\]

where \( r \) is the radius of the quarter. The distance \( d \) traveled by the "axle" of the moving quarter must be equal to the circular arc length \( s \) along the outer edge of the quarter. This arc length is \( s = r\theta \), where \( \theta \) is the angle through which the quarter rotates. Thus,

\[
4\pi r = r\theta
\]

so that \( \theta = 4\pi \text{ rad} \). This is equivalent to

\[
(4\pi \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 2 \text{ revolutions}
\]
1. (d) A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. A body, such as a bicycle wheel, can be moving, but the translational and angular accelerations must be zero \((a = 0 \text{ m/s}^2 \text{ and } \alpha = 0 \text{ rad/s}^2)\).

2. (e) As discussed in Section 9.2, a body is in equilibrium if the sum of the externally applied forces is zero and the sum of the externally applied torques is zero.

3. (b) The torque \(\tau_3\) is greater than \(\tau_2\), because the lever arm for the force \(F_3\) is greater than that for \(F_2\). The lines of action for the forces \(F_1\) and \(F_4\) pass through the axis of rotation. Therefore, the lever arms for these forces are zero, and the forces produce no torque.

4. (b) Since the counterclockwise direction is the positive direction for torque, the torque produced by the force \(F_1\) is \(\tau_1 = -(20.0 \text{ N})(0.500 \text{ m})\) and that produced by \(F_2\) is \(\tau_2 = +(35.0 \text{ N})[(1.10 \text{ m})(\cos 30.0^\circ)]\). The sum of these torques is the net torque.

5. (e) The clockwise torque produced by \(F_2\) is balanced by the counterclockwise torque produced by \(F\). The torque produced by \(F_2\) is (remembering that the counterclockwise direction is positive) \(\tau_2 = +F_2[(80.0 \text{ cm} - 20.0 \text{ cm})(\sin 55.0^\circ)]\), and the torque produced by \(F\) is \(\tau = -(175 \text{ N})(20.0 \text{ cm})\). Setting the sum of these torques equal to zero and solving for \(F_2\) gives the answer.

6. (d) The sum of the forces \((F - 2F_1 + F_2)\) equals zero. Select an axis that passes through the center of the puck and is perpendicular to the screen. The sum of the torques \([-FR + 2F(0) + FR]\) equals zero, where \(R\) is the radius of the puck. Thus, the puck is in equilibrium.

7. Magnitude of \(F_1\) = 12.0 N, Magnitude of \(F_2\) = 24.0 N

8. (c) The horizontal component of \(F_3\) is balanced by \(F_1\), and the vertical component of \(F_3\) is balanced by \(F_2\). Thus, the net force and, hence, the translational acceleration of the box, is zero. For an axis of rotation at the center of the box and perpendicular to the screen, the forces \(F_2\) and \(F_3\) produce no torque, because their lines of action pass through the axis. The force \(F_1\) does produce a torque about the axis, so the net torque is not zero and the box will have an angular acceleration.

9. Distance of center of gravity from support = 0.60 m.
10. (b) The moment of inertia of each particle is given by Equation 9.6 as \( I = mr^2 \), where \( m \) is its mass and \( r \) is the perpendicular distance of the particle from the axis. Using this equation, the moment of inertia of each particle is: A: \( 10m \cdot r_0^2 \), B: \( 8m \cdot r_0^2 \), C: \( 9m \cdot r_0^2 \).

11. \( I = 1.7 \text{ kg}\cdot\text{m}^2 \)

12. Magnitude \( \alpha \) of the angular acceleration = 1.3 rad/s\(^2\).

13. (a) According to Newton’s second law for rotational motion, Equation 9.7, the angular acceleration is equal to the torque exerted on the wheel (the product of the force magnitude and the lever arm) divided by the moment of inertia. Thus, the angular acceleration of the smaller wheel is

\[
\alpha = \frac{F(2R)}{M(2R)^2} = \frac{1}{2} \left( \frac{F}{MR} \right),
\]

so the smaller wheel has twice the angular acceleration.

14. Magnitude \( \alpha \) of the angular acceleration = 12.0 rad/s\(^2\)

15. (c) The translational kinetic energy is \( \frac{1}{2} Mv^2 \), where \( v \) is the speed of the center of mass of the wheel. The rotational kinetic energy is \( \frac{1}{2} I\omega^2 \), where \( I \) is the moment of inertia and \( \omega \) is the angular speed about the axis of rotation. Since \( I = MR^2 \) and \( \omega = \frac{v}{R} \) for rolling motion (See Equation 8.12), the rotational kinetic energy is

\[
\frac{1}{2} I\omega^2 = \frac{1}{2} \left( MR^2 \right) \left( \frac{v}{R} \right)^2 = \frac{1}{2} Mv^2,
\]

which is the same as the translational kinetic energy. Thus, the ratio of the two energies is 1.

16. (c) As each hoop rolls down the incline, the total mechanical energy is conserved. Thus, the loss in potential energy is equal to the gain in the total kinetic energy (translational plus rotational). Because the hoops have the same mass and fall through the same vertical distance, they lose the same amount of potential energy. Moreover, both start from rest. Therefore, their total kinetic energies at the bottom are the same.

17. (d) As discussed in Section 9.6, the angular momentum a system is conserved (remains constant) if the net external torque acting on the system is zero.

18. (b) The rotational kinetic energy of a rotating body is \( \text{KE}_r = \frac{1}{2} I\omega^2 \) (see Equation 9.9), where \( I \) is the moment of inertia and \( \omega \) is the angular speed. We also know that her angular momentum, \( L = I\omega \) (Equation 9.10), is conserved. Solving the last equation for \( I \) and substituting the result into the first equation gives

\[
\text{KE}_r = \frac{1}{2} I\omega^2 = \frac{1}{2} \left( \frac{L}{\omega} \right) \omega^2 = \frac{1}{2} L\omega.
\]

Since \( L \) is constant, the final rotational kinetic energy increases as \( \omega \) increases.

19. \( I = 0.60 \text{ kg}\cdot\text{m}^2 \)
Chapter 9 Problems

CHAPTER 9 | Rotational Dynamics

PROBLEMS

1. **REASONING** The drawing shows the wheel as it rolls to the right, so the torque applied by the engine is assumed to be clockwise about the axis of rotation. The force of static friction that the ground applies to the wheel is labeled as $f_s$. This force produces a counterclockwise torque $\tau$ about the axis of rotation, which is given by Equation 9.1 as $\tau = f_s \ell$, where $\ell$ is the lever arm. Using this relation we can find the magnitude $f_s$ of the static frictional force.

**SOLUTION** The countertorque is given as $\tau = f_s \ell$, where $f_s$ is the magnitude of the static frictional force and $\ell$ is the lever arm. The lever arm is the distance between the line of action of the force and the axis of rotation; in this case the lever arm is just the radius $r$ of the tire. Solving for $f_s$ gives

$$f_s = \frac{\tau}{\ell} = \frac{295 \text{ N} \cdot \text{m}}{0.350 \text{ m}} = 843 \text{ N}$$

2. **REASONING** The torque on either wheel is given by $\tau = F\ell$ (Equation 9.1), where $F$ is the magnitude of the force and $\ell$ is the lever arm. Regardless of how the force is applied, the lever arm will be proportional to the radius of the wheel.

**SOLUTION** The ratio of the torque produced by the force in the truck to the torque produced in the car is

$$\frac{\tau_{\text{truck}}}{\tau_{\text{car}}} = \frac{F_{\ell\text{truck}}}{F_{\ell\text{car}}} = \frac{F_{r\text{truck}}}{F_{r\text{car}}} = \frac{r_{\text{truck}}}{r_{\text{car}}} = \frac{0.25 \text{ m}}{0.19 \text{ m}} = 1.3$$

3. **REASONING** According to Equation 9.1, we have

$$\text{Magnitude of torque} = F\ell$$

where $F$ is the magnitude of the applied force and $\ell$ is the lever arm. From the figure in the text, the lever arm is given by $\ell = (0.28 \text{ m}) \sin 50.0^\circ$. Since both the magnitude of the torque and $\ell$ are known, Equation 9.1 can be solved for $F$.

**SOLUTION** Solving Equation 9.1 for $F$, we have
4. **REASONING** The net torque on the branch is the sum of the torques exerted by the children. Each individual torque $\tau$ is given by $\tau = F\ell$ (Equation 9.1), where $F$ is the magnitude of the force exerted on the branch by a child, and $\ell$ is the lever arm (see the diagram). The branch supports each child’s weight, so, by Newton's third law, the magnitude $F$ of the force exerted on the branch by each child has the same magnitude as the child’s weight: $F = mg$. Both forces are directed downwards. The lever arm for each force is the perpendicular distance between the axis and the force’s line of action, so we have $\ell = d \cos \theta$ (see the diagram).

**SOLUTION** The mass of the first child is $m_1 = 44.0$ kg. This child is a distance $d_1 = 1.30$ m from the tree trunk. The mass of the second child, hanging $d_2 = 2.10$ m from the tree trunk, is $m_2 = 35.0$ kg. Both children produce positive (counterclockwise) torques. The net torque exerted on the branch by the two children is then

$$\sum \tau = \tau_1 + \tau_2 = F_1\ell_1 + F_2\ell_2 = m_1 g d_1 \cos \theta + m_2 g d_2 \cos \theta = g \cos \theta (m_1 d_1 + m_2 d_2)$$

Substituting the given values, we obtain

$$\sum \tau = (9.80 \text{ m/s}^2)(\cos 27.0^\circ)\left[ (44.0 \text{ kg})(1.30 \text{ m}) + (35.0 \text{ kg})(2.10 \text{ m}) \right] = 1140 \text{ N} \cdot \text{m}$$

5. **SSM REASONING** To calculate the torques, we need to determine the lever arms for each of the forces. These lever arms are shown in the following drawings:
**Chapter 9  Problems**

**SOLUTION**

a. Using Equation 9.1, we find that the magnitude of the torque due to the weight $W$ is

$$\text{Magnitude of torque} = W\ell_W = (10\ 200\ \text{N})(2.5\ \text{m})\sin32^\circ = 13\ 500\ \text{N}\cdot\text{m}$$

b. Using Equation 9.1, we find that the magnitude of the torque due to the thrust $T$ is

$$\text{Magnitude of torque} = T\ell_T = (62\ 300\ \text{N})(2.5\ \text{m})\cos32^\circ = 132\ 000\ \text{N}\cdot\text{m}$$

---

**6. REASONING** The maximum torque will occur when the force is applied perpendicular to the diagonal of the square as shown. The lever arm $\ell$ is half the length of the diagonal. From the Pythagorean theorem, the lever arm is, therefore,

$$\ell = \frac{1}{2}\sqrt{(0.40\ \text{m})^2 + (0.40\ \text{m})^2} = 0.28\ \text{m}$$

Since the lever arm is now known, we can use Equation 9.1 to obtain the desired result directly.

**SOLUTION** Equation 9.1 gives

$$\tau = F\ell = (15\ \text{N})(0.28\ \text{m}) = 4.2\ \text{N}\cdot\text{m}$$

---

**7. SSM REASONING AND SOLUTION** The torque produced by each force of magnitude $F$ is given by Equation 9.1, $\tau = F\ell$, where $\ell$ is the lever arm and the torque is positive since each force causes a counterclockwise rotation. In each case, the torque produced by the couple is equal to the sum of the individual torques produced by each member of the couple.

a. When the axis passes through point $A$, the torque due to the force at $A$ is zero. The lever arm for the force at $C$ is $L$. Therefore, taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = 0 + FL = FL$$

b. Each force produces a counterclockwise rotation. The magnitude of each force is $F$ and each force has a lever arm of $L/2$. Taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = F\left(\frac{L}{2}\right) + F\left(\frac{L}{2}\right) = FL$$
c. When the axis passes through point $C$, the torque due to the force at $C$ is zero. The lever arm for the force at $A$ is $L$. Therefore, taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = FL + 0 = FL$$

Note that the value of the torque produced by the couple is the same in all three cases; in other words, when the couple acts on the tire wrench, the couple produces a torque that does not depend on the location of the axis.

---

8. **REASONING** We know that the torques generated by the two applied forces sum to zero. In order for this to be true, one of the forces must tend to produce a clockwise rotation about the pinned end of the meter stick, and the other must tend to produce a counterclockwise rotation about the pinned end. We will assume that the first force, applied perpendicular to the length of the meter stick at its free end, tends to produce a counterclockwise rotation (see the drawing). Counterclockwise is the positive direction.

With this assumption, we obtain

$$F_1\ell_1 - F_2\ell_2 = 0 \quad \text{or} \quad F_1\ell_1 = F_2\ell_2 \quad (1)$$

The drawing shows that the lever arm $\ell_1$ of the force $F_1$ is equal to the length of the meter stick: $\ell_1 = 1.00 \text{ m}$. The force $F_2$ is applied a distance $d$ from the pinned end of the meter stick. The lever arm $\ell_2$ of the second force is

$$\ell_2 = d \sin \theta \quad (2)$$

**SOLUTION** Substituting Equation (2) into Equation (1) and solving for the distance $d$, we obtain

$$F_1\ell_1 = F_2(d \sin \theta) \quad \text{or} \quad d = \frac{F_1\ell_1}{F_2 \sin \theta} = \frac{(2.00 \text{ N})(1.00 \text{ m})}{(6.00 \text{ N})\sin 30.0^\circ} = 0.667 \text{ m}$$
9. **REASONING** Each of the two forces produces a torque about the axis of rotation, one clockwise and the other counterclockwise. By setting the sum of the torques equal to zero, we will be able to determine the angle $\theta$ in the drawing.

**SOLUTION** The two forces act on the rod at a distance $x$ from the hinge. The torque $\tau_1$ produced by the force $F_1$ is given by $\tau_1 = +F_1 \ell_1$ (see Equation 9.1), where $F_1$ is the magnitude of the force and $\ell_1$ is the lever arm. It is a positive torque, since it tends to produce a counterclockwise rotation. Since $F_1$ is applied perpendicular to the rod, $\ell_1 = x$.

The torque $\tau_2$ produced by $F_2$ is $\tau_2 = -F_2 \ell_2$, where $\ell_2 = x \sin \theta$ (see the drawing). It is a negative torque, since it tends to produce a clockwise rotation. Setting the sum of the torques equal to zero, we have

$$+F_1 \ell_1 + (-F_2 \ell_2) = 0 \quad \text{or} \quad +F_1 x - F_2 \left( x \sin \theta \right) = 0$$

The distance $x$ in this relation can be eliminated algebraically. Solving for the angle $\theta$ gives

$$\sin \theta = \frac{F_1}{F_2} \quad \text{or} \quad \theta = \sin^{-1} \left( \frac{F_1}{F_2} \right) = \sin^{-1} \left( \frac{38.0 \text{ N}}{55.0 \text{ N}} \right) = 43.7^\circ$$

10. **REASONING AND SOLUTION** The net torque about the axis in text drawing (a) is

$$\Sigma \tau = \tau_1 + \tau_2 = F_1 b - F_2 a = 0$$

Considering that $F_2 = 3F_1$, we have $b - 3a = 0$. The net torque in drawing (b) is then

$$\Sigma \tau = F_1 (1.00 \text{ m} - a) - 3F_2 b = 0 \quad \text{or} \quad 1.00 \text{ m} - a - 3b = 0$$
Solving the first equation for \( b \), substituting into the second equation and rearranging, gives

\[
a = 0.100 \text{ m} \quad \text{and} \quad b = 0.300 \text{ m}
\]

11. **REASONING** Although this arrangement of body parts is vertical, we can apply Equation 9.3 to locate the overall center of gravity by simply replacing the horizontal position \( x \) by the vertical position \( y \), as measured relative to the floor.

**SOLUTION** Using Equation 9.3, we have

\[
y_{cg} = \frac{W_1y_1 + W_2y_2 + W_3y_3}{W_1 + W_2 + W_3}
\]

\[
= \frac{(438 \text{ N})(1.28 \text{ m}) + (144 \text{ N})(0.760 \text{ m}) + (87 \text{ N})(0.250 \text{ m})}{438 \text{ N} + 144 \text{ N} + 87 \text{ N}} = 1.03 \text{ m}
\]

12. **REASONING** The drawing shows the forces acting on the person. It also shows the lever arms for a rotational axis perpendicular to the plane of the paper at the place where the person’s toes touch the floor. Since the person is in equilibrium, the sum of the forces must be zero. Likewise, we know that the sum of the torques must be zero.

**SOLUTION** Taking upward to be the positive direction, we have

\[
F_{\text{FEET}} + F_{\text{HANDS}} - W = 0
\]

Remembering that counterclockwise torques are positive and using the axis and the lever arms shown in the drawing, we find

\[
W\ell_w - F_{\text{HANDS}}\ell_{\text{HANDS}} = 0
\]

\[
F_{\text{HANDS}} = \frac{W\ell_w}{\ell_{\text{HANDS}}} = \frac{(584 \text{ N})(0.840 \text{ m})}{1.250 \text{ m}} = 392 \text{ N}
\]

Substituting this value into the balance-of-forces equation, we find

\[
F_{\text{FEET}} = W - F_{\text{HANDS}} = 584 \text{ N} - 392 \text{ N} = 192 \text{ N}
\]

The force on each hand is half the value calculated above, or 196 N. Likewise, the force on each foot is half the value calculated above, or 96 N.
13. **REASONING** The drawing shows the bridge and the four forces that act on it: the upward force $F_1$ exerted on the left end by the support, the force due to the weight $W_h$ of the hiker, the weight $W_b$ of the bridge, and the upward force $F_2$ exerted on the right side by the support. Since the bridge is in equilibrium, the sum of the torques about any axis of rotation must be zero ($\Sigma \tau = 0$), and the sum of the forces in the vertical direction must be zero ($\Sigma F_y = 0$). These two conditions will allow us to determine the magnitudes of $F_1$ and $F_2$.

**SOLUTION**

a. We will begin by taking the axis of rotation about the right end of the bridge. The torque produced by $F_2$ is zero, since its lever arm is zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, $F_1$, in it. Setting the sum of the torques produced by the three forces equal to zero gives

$$\Sigma \tau = -F_1 L + W_h \left( \frac{4}{5} L \right) + W_b \left( \frac{1}{2} L \right) = 0$$

Algebraically eliminating the length $L$ of the bridge from this equation and solving for $F_1$ gives

$$F_1 = \frac{4}{5} W_h + \frac{1}{2} W_b = \frac{4}{5} (985 \text{ N}) + \frac{1}{2} (3610 \text{ N}) = 2590 \text{ N}$$

b. Since the bridge is in equilibrium, the sum of the forces in the vertical direction must be zero:

$$\Sigma F_y = F_1 - W_h - W_b + F_2 = 0$$

Solving for $F_2$ gives

$$F_2 = -F_1 + W_h + W_b = -2590 \text{ N} + 985 \text{ N} + 3610 \text{ N} = 2010 \text{ N}$$
14. **REASONING** The truck is subject to three vertical forces only (see the free-body diagram), and is in equilibrium. Therefore, the conditions \( \Sigma F_y = 0 \) (Equation 4.9b) and \( \Sigma \tau = 0 \) (Equation 9.2) apply to the forces and torques acting on it. The ground exerts upward forces \( F_R \) on the rear wheels and \( F_F \) on the front wheels, and the earth exerts a downward weight force \( W \) on the truck’s center of gravity. The net vertical force on the truck must be zero, so we find

\[
F_R + F_F - W = 0 \quad \text{(1)}
\]

where we have assumed upward to be the positive direction. To apply the zero net torque condition (Equation 9.2), we choose a rotation axis located on the ground, directly below the truck’s center of gravity (see the diagram). The weight force \( W \) has no lever arm about this axis, and so it generates no torque. The force \( F_F \) exerts a counterclockwise torque about this axis, and the force \( F_R \) exerts a clockwise torque about this axis. Since counterclockwise is the positive direction, we find from Equation 9.2 that

\[
\Sigma \tau = F_F \ell_F - F_R \ell_R = 0 \quad \text{or} \quad F_F \ell_F = F_R \ell_R \quad \text{(2)}
\]

We will use Equations (1) and (2) to find the two unknown forces.

**SOLUTION**

a. In order to find the force magnitude \( F_F \), we must eliminate the unknown force magnitude \( F_R \) from Equations (1) and (2). Solving Equation (1) for \( F_R \) yields \( F_R = W - F_F = mg - F_F \). Substituting this expression into Equation (2), we obtain

\[
F_F \ell_F = \left( mg - F_F \right) \ell_R = mg \ell_R - F_F \ell_R \quad \text{or} \quad F_F \left( \ell_F + \ell_R \right) = mg \ell_R \quad \text{(3)}
\]

Solving Equation (3) for the force magnitude \( F_F \) and noting from the drawing in the text that \( \ell_F = 2.30 \text{ m} \) and \( \ell_R = 0.63 \text{ m} \), we find that

\[
F_F = \frac{mg \ell_R}{\ell_F + \ell_R} = \frac{(7460 \text{ kg})(9.80 \text{ m/s}^2)(0.63 \text{ m})}{2.30 \text{ m} + 0.63 \text{ m}} = 1.57 \times 10^4 \text{ N}
\]

b. Returning to Equation (1), we obtain the magnitude of the force on the rear wheels:

\[
F_R = W - F_F = mg - F_F = (7460 \text{ kg})(9.80 \text{ m/s}^2) - 1.57 \times 10^4 \text{ N} = 5.74 \times 10^4 \text{ N}
\]
15. **REASONING** Since the forearm is in equilibrium, the sum of the torques about any axis of rotation must be zero \( \Sigma \tau = 0 \). For convenience, we will take the elbow joint to be the axis of rotation.

**SOLUTION** Let \( M \) and \( F \) be the magnitudes of the forces that the flexor muscle and test apparatus, respectively, exert on the forearm, and let \( \ell_M \) and \( \ell_F \) be the respective lever arms about the elbow joint. Setting the sum of the torques about the elbow joint equal to zero (with counterclockwise torques being taken as positive), we have

\[
\Sigma \tau = M\ell_M - F\ell_F = 0
\]

Solving for \( M \) yields

\[
M = \frac{F\ell_F}{\ell_M} = \frac{(190 \text{ N})(0.34 \text{ m})}{0.054 \text{ m}} = 1200 \text{ N}
\]

The direction of the force is to the left.

16. **REASONING** The net torque is the sum of the torques produced by the three forces: \( \Sigma \tau = \tau_A + \tau_B + \tau_D \). The magnitude of a torque is the magnitude of the force times the lever arm of the force, according to Equation 9.1. The lever arm is the perpendicular distance between the line of action of the force and the axis. A torque that tends to produce a counterclockwise rotation about the axis is a positive torque. Since the piece of wood is at equilibrium, the net torque is equal to zero.

**SOLUTION** Let \( L \) be the length of the short side of the rectangle, so that the length of the long side is \( 2L \). The counterclockwise torque produced by the force at corner \( B \) is \(+F\ell_B\), and the clockwise torque produced by the force at corner \( D \) is \(-F\ell_D\). Assuming that the force at \( A \) (directed along the short side of the rectangle) points toward corner \( B \), the counterclockwise torque produced by this force is \(+F_A\ell_A\). Setting the net torque equal to zero gives:

\[
\Sigma \tau = F_A\ell_A + F\ell_B - F\ell_D = 0
\]

\[
F_AL + (12 \text{ N})\left(\frac{1}{2}L\right) - (12 \text{ N})L = 0
\]

The length \( L \) can be eliminated algebraically from this result, which can then be solved for \( F_A \):

\[
F_A = -(12 \text{ N})\left(\frac{1}{2}\right) + 12 \text{ N} = 6.0 \text{ N (pointing toward corner B)}
\]

Since the value calculated for \( F_A \) is positive, our assumption that \( F_A \) points toward corner \( B \) must have been correct. Otherwise, the result for \( F_A \) would have been negative.
17. **REASONING** Multiple-Concept Example 8 discusses the static stability factor (SSF) and rollover. In that example, it is determined that the maximum speed \( v \) at which a vehicle can negotiate a curve of radius \( r \) is related to the SSF according to \( v = \sqrt{rg(SSF)} \). No value is given for the radius of the turn. However, by applying this result separately to the sport utility vehicle (SUV) and to the sports car (SC), we will be able to eliminate \( r \) algebraically and determine the maximum speed at which the sports car can negotiate the curve without rolling over.

**SOLUTION** Applying \( v = \sqrt{rg(SSF)} \) to each vehicle, we obtain

\[
\begin{align*}
v_{SUV} &= \sqrt{rg(SSF)_{SUV}} \\
v_{SC} &= \sqrt{rg(SSF)_{SC}}
\end{align*}
\]

Dividing these two expressions gives

\[
v_{SC} = \frac{v_{SUV}}{\sqrt{rg(SSF)_{SUV}}} \\text{or} \\
v_{SC} = v_{SUV} \sqrt{\frac{(SSF)_{SC}}{(SSF)_{SUV}}} = \left(18 \text{ m/s}\right) \sqrt{\frac{1.4}{0.80}} = 24 \text{ m/s}
\]

18. **REASONING** Since the wheelbarrow is in equilibrium, the net torque acting on it must be zero: \( \Sigma \tau = 0 \) (Equation 9.2). The magnitude of a torque is the magnitude of the force times the lever arm of the force (see Equation 9.1). The lever arm is the perpendicular distance between the line of action of the force and the axis. A torque that tends to produce a counterclockwise rotation about the axis is a positive torque.

**SOLUTION** The lever arms for the forces can be obtained from the distances shown in the text drawing for each design. Equation 9.1 can be used to obtain the magnitude of each torque. We will then write an expression for the zero net torque for each design. These expressions can be solved for the magnitude \( F \) of the man’s force in each case:

**Left design**

\[
\Sigma \tau = -(525 \text{ N})(0.400 \text{ m}) - (60.0 \text{ N})(0.600 \text{ m}) + F(1.300 \text{ m}) = 0
\]

\[
F = \frac{(525 \text{ N})(0.400 \text{ m}) + (60.0 \text{ N})(0.600 \text{ m})}{1.300 \text{ m}} = \boxed{189 \text{ N}}
\]

**Right design**

\[
\Sigma \tau = -(60.0 \text{ N})(0.600 \text{ m}) + F(1.300 \text{ m}) = 0
\]

\[
F = \frac{(60.0 \text{ N})(0.600 \text{ m})}{1.300 \text{ m}} = \boxed{27.7 \text{ N}}
\]

19. **REASONING** The jet is in equilibrium, so the sum of the external forces is zero, and the sum of the external torques is zero. We can use these two conditions to evaluate the forces exerted on the wheels.
SOLUTION

a. Let \( F_f \) be the magnitude of the normal force that the ground exerts on the front wheel. Since the net torque acting on the plane is zero, we have (using an axis through the points of contact between the rear wheels and the ground)

\[
\sum \tau = -W \ell_w + F_f \ell_f = 0
\]

where \( W \) is the weight of the plane, and \( \ell_w \) and \( \ell_f \) are the lever arms for the forces \( W \) and \( F_f \), respectively. Thus,

\[
\sum \tau = -(1.00 \times 10^6 \text{ N})(15.0 \text{ m} - 12.6 \text{ m}) + F_f (15.0 \text{ m}) = 0
\]

Solving for \( F_f \) gives \( F_f = 1.60 \times 10^5 \text{ N} \).

b. Setting the sum of the vertical forces equal to zero yields

\[
\sum F_y = F_f + 2F_r - W = 0
\]

where the factor of 2 arises because there are two rear wheels. Substituting the data gives

\[
\sum F_y = 1.60 \times 10^5 \text{ N} + 2F_r - 1.00 \times 10^6 \text{ N} = 0
\]

\[
F_r = 4.20 \times 10^5 \text{ N}
\]

20. **REASONING** The minimum value for the coefficient of static friction between the ladder and the ground, so that the ladder does not slip, is given by Equation 4.7:

\[
f_s^{\text{MAX}} = \mu_s N
\]

**SOLUTION** From Example 4, the magnitude of the force of static friction is \( G_x = 727 \text{ N} \). The magnitude of the normal force applied to the ladder by the ground is \( G_y = 1230 \text{ N} \). The minimum value for the coefficient of static friction between the ladder and the ground is

\[
\mu_s = \frac{f_s^{\text{MAX}}}{N} = \frac{G_x}{G_y} = \frac{727 \text{ N}}{1230 \text{ N}} = 0.591
\]

21. **REASONING** Since the beam is in equilibrium, the sum of the horizontal and vertical forces must be zero: \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \) (Equations 9.4a and b). In addition, the net torque about any axis of rotation must also be zero: \( \Sigma \tau = 0 \) (Equation 9.2).
**SOLUTION** The drawing shows the beam, as well as its weight $W$, the force $P$ that the pin exerts on the right end of the beam, and the horizontal and vertical forces, $H$ and $V$, applied to the left end of the beam by the hinge. Assuming that upward and to the right are the positive directions, we obtain the following expressions by setting the sum of the vertical and the sum of the horizontal forces equal to zero:

**Horizontal forces** \[ P \cos \theta - H = 0 \] (1)

**Vertical forces** \[ P \sin \theta + V - W = 0 \] (2)

Using a rotational axis perpendicular to the plane of the paper and passing through the pin, and remembering that counterclockwise torques are positive, we also set the sum of the torques equal to zero. In doing so, we use $L$ to denote the length of the beam and note that the lever arms for $W$ and $V$ are $\frac{1}{2}L$ and $L$, respectively. The forces $P$ and $H$ create no torques relative to this axis, because their lines of action pass directly through it.

**Torques** \[ W \left( \frac{1}{2}L \right) - VL = 0 \] (3)

Since $L$ can be eliminated algebraically, Equation (3) may be solved immediately for $V$:

\[ V = \frac{1}{2} W = \frac{1}{2} (340 \text{ N}) = 170 \text{ N} \]

Substituting this result into Equation (2) gives

\[ P \sin \theta + \frac{1}{2} W - W = 0 \]

\[ P = \frac{W}{2 \sin \theta} = \frac{340 \text{ N}}{2 \sin 39^\circ} = 270 \text{ N} \]

Substituting this result into Equation (1) yields

\[ \left( \frac{W}{2 \sin \theta} \right) \cos \theta - H = 0 \]

\[ H = \frac{W}{2 \tan \theta} = \frac{340 \text{ N}}{2 \tan 39^\circ} = 210 \text{ N} \]
22. **REASONING** Since the man holds the ball motionless, the ball and the arm are in equilibrium. Therefore, the net force, as well as the net torque about any axis, must be zero.

**SOLUTION** Using Equation 9.1, the net torque about an axis through the elbow joint is

\[ \sum \tau = M(0.0510 \text{ m}) - (22.0 \text{ N})(0.140 \text{ m}) - (178 \text{ N})(0.330 \text{ m}) = 0 \]

Solving this expression for \( M \) gives \( M = 1.21 \times 10^3 \text{ N} \).

The net torque about an axis through the center of gravity is

\[ \sum \tau = -(1210 \text{ N})(0.0890 \text{ m}) + F(0.140 \text{ m}) - (178 \text{ N})(0.190 \text{ m}) = 0 \]

Solving this expression for \( F \) gives \( F = 1.01 \times 10^3 \text{ N} \). Since the forces must add to give a net force of zero, we know that the direction of \( F \) is downward.

---

23. **SSM** **REASONING** The drawing shows the forces acting on the board, which has a length \( L \). The ground exerts the vertical normal force \( V \) on the lower end of the board. The maximum force of static friction has a magnitude of \( \mu_s V \) and acts horizontally on the lower end of the board. The weight \( W \) acts downward at the board's center. The vertical wall applies a force \( P \) to the upper end of the board, this force being perpendicular to the wall since the wall is smooth (i.e., there is no friction along the wall). We take upward and to the right as our positive directions. Then, since the horizontal forces balance to zero, we have

\[ \mu_s V - P = 0 \]

The vertical forces also balance to zero giving

\[ V - W = 0 \]

Using an axis through the lower end of the board, we express the fact that the torques balance to zero as

\[ PL \sin \theta - W \left( \frac{L}{2} \right) \cos \theta = 0 \]

Equations (1), (2), and (3) may then be combined to yield an expression for \( \theta \).

**SOLUTION** Rearranging Equation (3) gives

\[ \tan \theta = \frac{W}{2P} \]
But, \( P = \mu_b V \) according to Equation (1), and \( W = V \) according to Equation (2). Substituting these results into Equation (4) gives

\[
\tan \theta = \frac{V}{2\mu_b V} = \frac{1}{2\mu_b}
\]

Therefore,

\[
\theta = \tan^{-1}\left(\frac{1}{2\mu_b}\right) = \tan^{-1}\left(\frac{1}{2(0.65)}\right) = 37.6^\circ
\]

24. **REASONING** When the wheel is resting on the ground it is in equilibrium, so the sum of the torques about any axis of rotation is zero (\( \Sigma \tau = 0 \)). This equilibrium condition will provide us with a relation between the magnitude of \( \mathbf{F} \) and the normal force that the ground exerts on the wheel. When \( \mathbf{F} \) is large enough, the wheel will rise up off the ground, and the normal force will become zero. From our relation, we can determine the magnitude of \( \mathbf{F} \) when this happens.

**SOLUTION** The free body diagram shows the forces acting on the wheel: its weight \( \mathbf{W} \), the normal force \( \mathbf{F}_N \), the horizontal force \( \mathbf{F} \), and the force \( \mathbf{F}_E \) that the edge of the step exerts on the wheel. We select the axis of rotation to be at the edge of the step, so that the torque produced by \( \mathbf{F}_E \) is zero. Letting \( \ell_N, \ell_W, \) and \( \ell_F \) represent the lever arms for the forces \( \mathbf{F}_N, \mathbf{W}, \) and \( \mathbf{F} \), the sum of the torques is

\[
\Sigma \tau = -F_N \sqrt{r^2 -(r-h)^2} + W \sqrt{r^2 -(r-h)^2} - F (r-h) = 0
\]

Solving this equation for \( F \) gives

\[
F = \frac{(W-F_N)\sqrt{r^2 -(r-h)^2}}{r-h}
\]

When the bicycle wheel just begins to lift off the ground the normal force becomes zero (\( F_N = 0 \) N). When this happens, the magnitude of \( \mathbf{F} \) is

\[
F = \frac{(W-F_N)\sqrt{r^2 -(r-h)^2}}{r-h} = \frac{(25.0 \text{ N}-0 \text{ N})\sqrt{(0.340 \text{ m})^2 -(0.340 \text{ m}-0.120 \text{ m})^2}}{0.340 \text{ m}-0.120 \text{ m}} = 29 \text{ N}
\]
25. **REASONING** The following drawing shows the beam and the five forces that act on it: the horizontal and vertical components $S_x$ and $S_y$ that the wall exerts on the left end of the beam, the weight $W_b$ of the beam, the force due to the weight $W_c$ of the crate, and the tension $T$ in the cable. The beam is uniform, so its center of gravity is at the center of the beam, which is where its weight can be assumed to act. Since the beam is in equilibrium, the sum of the torques about any axis of rotation must be zero ($\Sigma \tau = 0$), and the sum of the forces in the horizontal and vertical directions must be zero ($\Sigma F_x = 0$, $\Sigma F_y = 0$). These three conditions will allow us to determine the magnitudes of $S_x$, $S_y$, and $T$.

**SOLUTION**

a. We will begin by taking the axis of rotation to be at the left end of the beam. Then the torques produced by $S_x$ and $S_y$ are zero, since their lever arms are zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, $T$, in it. Setting the sum of the torques produced by the three forces equal to zero gives (with $L$ equal to the length of the beam)

$$\Sigma \tau = -W_b \left( \frac{1}{2} L \cos 30.0^\circ \right) - W_c \left( L \cos 30.0^\circ \right) + T \left( L \sin 80.0^\circ \right) = 0$$

Algebraically eliminating $L$ from this equation and solving for $T$ gives

$$T = \frac{W_b \left( \frac{1}{2} \cos 30.0^\circ \right) + W_c (\cos 30.0^\circ)}{\sin 80.0^\circ}$$

$$= \frac{(1220 \text{ N}) \left( \frac{1}{2} \cos 30.0^\circ \right) + (1960 \text{ N}) (\cos 30.0^\circ)}{\sin 80.0^\circ} = 2260 \text{ N}$$
b. Since the beam is in equilibrium, the sum of the forces in the vertical direction is zero:

\[ \sum F_y = +S_y - W_b - W_c + T \sin 50.0^\circ = 0 \]

Solving for \( S_y \) gives

\[ S_y = W_b + W_c - T \sin 50.0^\circ = 1220 \text{ N} + 1960 \text{ N} - (2260 \text{ N}) \sin 50.0^\circ = 1450 \text{ N} \]

The sum of the forces in the horizontal direction must also be zero:

\[ \sum F_x = +S_x - T \cos 50.0^\circ = 0 \]

so that

\[ S_x = T \cos 50.0^\circ = (2260 \text{ N}) \cos 50.0^\circ = 1450 \text{ N} \]

26. **REASONING** Since the outstretched leg is stationary, it is in equilibrium. Thus, the net external torque acting on the leg must equal zero. This net torque arises because of the quadriceps force \( M \) and the weight \( W \) of the leg. The magnitude \( \tau \) of each torque can be expressed as \( \tau = F\ell \) (Equation 9.1), where \( F \) is the magnitude of the force and \( \ell \) is the lever arm for the force.

**SOLUTION** At equilibrium the net torque \( \Sigma \tau \) about an axis through the knee joint is zero, so that

\[ \Sigma \tau = \tau_{\text{muscle}} + \tau_{\text{weight}} = 0 \]

Using Equation 9.1 to express each torque, we have

\[ \tau_{\text{muscle}} = M\ell_{\text{muscle}} \quad \text{and} \quad \tau_{\text{weight}} = W\ell_{\text{weight}} \]

The torque \( \tau_{\text{muscle}} \) due to \( M \) acts counterclockwise and, therefore, is positive. The torque \( \tau_{\text{weight}} \) due to \( W \) acts clockwise and, therefore, is negative. Thus, Equation (1) becomes

\[ \Sigma \tau = (M\ell_{\text{muscle}}) + (-W\ell_{\text{weight}}) = 0 \quad \text{or} \quad M = W \left( \frac{\ell_{\text{weight}}}{\ell_{\text{muscle}}} \right) \]

The lever arm \( \ell \) is the distance between the line of action of the force and the axis of rotation, measured on a line that is perpendicular to both. With this in mind, we choose an axis for rotation that is at the knee joint (see the drawing that accompanies the problem statement) and can write the expression for \( M \) as follows:

\[ M = W \left( \frac{\ell_{\text{weight}}}{\ell_{\text{muscle}}} \right) = (44.5 \text{ N}) \left( \frac{0.250 \text{ m} \cos 30.0^\circ}{0.100 \text{ m} \sin 25.0^\circ} \right) = 228 \text{ N} \]
27. **REASONING** If we assume that the system is in equilibrium, we know that the vector sum of all the forces, as well as the vector sum of all the torques, that act on the system must be zero.

The figure below shows a free body diagram for the boom. Since the boom is assumed to be uniform, its weight $W_B$ is located at its center of gravity, which coincides with its geometrical center. There is a tension $T$ in the cable that acts at an angle $\theta$ to the horizontal, as shown. At the hinge pin $P$, there are two forces acting. The vertical force $V$ that acts on the end of the boom prevents the boom from falling down. The horizontal force $H$ that also acts at the hinge pin prevents the boom from sliding to the left. The weight $W_L$ of the wrecking ball (the "load") acts at the end of the boom.

By applying the equilibrium conditions to the boom, we can determine the desired forces.

**SOLUTION** The directions upward and to the right will be taken as the positive directions. In the $x$ direction we have

$$\sum F_x = H - T \cos \theta = 0$$  \hspace{1cm} (1)

while in the $y$ direction we have

$$\sum F_y = V - T \sin \theta - W_L - W_B = 0$$  \hspace{1cm} (2)

Equations (1) and (2) give us two equations in three unknown. We must, therefore, find a third equation that can be used to determine one of the unknowns. We can get the third equation from the torque equation.

In order to write the torque equation, we must first pick an axis of rotation and determine the lever arms for the forces involved. Since both $V$ and $H$ are unknown, we can eliminate them from the torque equation by picking the rotation axis through the point $P$ (then both $V$ and $H$ have zero lever arms). If we let the boom have a length $L$, then the lever arm for $W_L$ is $L \cos \phi$, while the lever arm for $W_B$ is $(L/2) \cos \phi$. From the figure, we see that the lever arm for $T$ is $L \sin(\phi - \theta)$. If we take counterclockwise torques as positive, then the torque equation is
\[ \sum \tau = -W_B \left( \frac{L \cos \phi}{2} \right) - W_L L \cos \phi + TL \sin (\phi - \theta) = 0 \]

Solving for \( T \), we have

\[ T = \frac{\frac{1}{2} W_B + W_L}{\sin (\phi - \theta)} \cos \phi \] (3)

a. From Equation (3) the tension in the support cable is

\[ T = \frac{\frac{1}{2} (3600 \text{ N}) + 4800 \text{ N}}{\sin (48^\circ - 32^\circ)} \cos 48^\circ = 1.6 \times 10^4 \text{ N} \]

b. The force exerted on the lower end of the hinge at the point \( P \) is the vector sum of the forces \( H \) and \( V \). According to Equation (1),

\[ H = T \cos \theta = \left(1.6 \times 10^4 \text{ N}\right) \cos 32^\circ = 1.4 \times 10^4 \text{ N} \]

and, from Equation (2)

\[ V = W_L + W_B + T \sin \theta = 4800 \text{ N} + 3600 \text{ N} + \left(1.6 \times 10^4 \text{ N}\right) \sin 32^\circ = 1.7 \times 10^4 \text{ N} \]

Since the forces \( H \) and \( V \) are at right angles to each other, the magnitude of their vector sum can be found from the Pythagorean theorem:

\[ F_P = \sqrt{H^2 + V^2} = \sqrt{(1.4 \times 10^4 \text{ N})^2 + (1.7 \times 10^4 \text{ N})^2} = 2.2 \times 10^4 \text{ N} \]

28. \textbf{REASONING} Although the crate is in translational motion, it undergoes no angular acceleration. Therefore, the net torque acting on the crate must be zero: \( \Sigma \tau = 0 \) (Equation 9.2). The four forces acting on the crate appear in the free-body diagram: its weight \( W \), the kinetic friction force \( f_k \), the normal force \( F_N \), and the tension \( T \) in the strap. We will take the edge of the crate sliding along the floor as the rotation axis for applying Equation 9.2. Both the friction force and the normal force act at this point. These two forces, therefore, generate no torque about the axis, so the clockwise torque of the weight \( W \) must balance the counterclockwise torque of the tension \( T \) in the strap:

\[ T \ell_T = W \ell_W \quad \text{or} \quad T = \frac{W \ell_W}{\ell_T} = \frac{mg \ell_W}{\ell_T} \] (1)
We will apply trigonometry to determine the lever arms $\ell_W$ and $\ell_T$ for the weight and the tension, respectively, and then calculate the magnitude $T$ of the tension in the strap.

**SOLUTION** The lever arm $\ell_W$ of the crate’s weight is shown in the diagram at the right, and is given by

$$\ell_W = d \cos\left(\theta + 25^\circ\right),$$

where $d$ is the distance between the axis of rotation (lower edge of the crate) and the crate’s center of gravity, and $\theta$ is the angle between that line and the bottom of the crate. The right triangle in the free-body diagram of the crate (drawing on the left) shows how we can use trigonometry to determine both the length $d$ and the angle $\theta$. The length $d$ is the hypotenuse of that right triangle, and the other sides are the half-height ($H/2 = 0.20$ m) and half-length ($L/2 = 0.45$ m) of the crate, so by the Pythagorean theorem (Equation 1.7) we find that

$$d = \sqrt{\left(\frac{H}{2}\right)^2 + \left(\frac{L}{2}\right)^2} = \sqrt{(0.20 \text{ m})^2 + (0.45 \text{ m})^2} = 0.49 \text{ m}$$

We can find the angle $\theta$ from the inverse tangent function:

$$\theta = \tan^{-1}\left(\frac{H/2}{L/2}\right) = \tan^{-1}\left(\frac{0.40 \text{ m}}{0.90 \text{ m}}\right) = 24^\circ$$

The lever arm $\ell_T$ of the tension force is illustrated in the drawing at the right, where we see that $\ell_T = L \sin\left(61^\circ - 25^\circ\right) = L \sin 36^\circ$.

Therefore, from Equation (1), the magnitude of the tension in the strap is

$$T = \frac{mg\ell_W}{\ell_T} = \frac{mgd \cos\left(24^\circ + 25^\circ\right)}{L \sin 36^\circ}$$

$$= \frac{(72 \text{ kg})(9.80 \text{ m/s}^2)(0.49 \text{ m}) \cos 49^\circ}{(0.90 \text{ m}) \sin 36^\circ}$$

$$= 430 \text{ N}$$
29. **REASONING** The drawing shows the forces acting on the board, which has a length \( L \). Wall 2 exerts a normal force \( P_2 \) on the lower end of the board. The maximum force of static friction that wall 2 can apply to the lower end of the board is \( \mu_s P_2 \) and is directed upward in the drawing. The weight \( W \) acts downward at the board's center. Wall 1 applies a normal force \( P_1 \) to the upper end of the board. We take upward and to the right as our positive directions.

**SOLUTION** Then, since the horizontal forces balance to zero, we have

\[
P_1 - P_2 = 0 \tag{1}
\]

The vertical forces also balance to zero:

\[
\mu_s P_2 - W = 0 \tag{2}
\]

Using an axis through the lower end of the board, we now balance the torques to zero:

\[
W\left(\frac{L}{2}\right)(\cos \theta) - P_1 L (\sin \theta) = 0 \tag{3}
\]

Rearranging Equation (3) gives

\[
\tan \theta = \frac{W}{2P_1} \tag{4}
\]

But \( W = \mu_s P_2 \) according to Equation (2), and \( P_2 = P_1 \) according to Equation (1). Therefore, \( W = \mu_s P_1 \), which can be substituted in Equation (4) to show that

\[
\tan \theta = \frac{\mu_s P_1}{2P_1} = \frac{\mu_s}{2} = \frac{0.98}{2}
\]

or

\[
\theta = \tan^{-1}(0.49) = 26^\circ
\]

From the drawing at the right,

\[
\cos \theta = \frac{d}{L}
\]

Therefore, the longest board that can be propped between the two walls is

\[
L = \frac{d}{\cos \theta} = \frac{1.5 \text{ m}}{\cos 26^\circ} = 1.7 \text{ m}
\]
30. **REASONING AND SOLUTION**  The weight \( W \) of the left side of the ladder, the normal force \( F_N \) of the floor on the left leg of the ladder, the tension \( T \) in the crossbar, and the reaction force \( R \) due to the right-hand side of the ladder, are shown in the following figure. In the vertical direction \( -W + F_N = 0 \), so that

\[
F_N = W = mg = (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 98.0 \text{ N}
\]

In the horizontal direction it is clear that \( R = T \). The net torque about the base of the ladder is

\[
\Sigma \tau = -T [(1.00 \text{ m}) \sin 75.0^\circ] - W [(2.00 \text{ m}) \cos 75.0^\circ] + R [(4.00 \text{ m}) \sin 75.0^\circ] = 0
\]

Substituting for \( W \) and using \( R = T \), we obtain

\[
T = \frac{(9.80 \text{ N})(2.00 \text{ m}) \cos 75.0^\circ}{(3.00 \text{ m}) \sin 75.0^\circ} = 17.5 \text{ N}
\]

31. **REASONING**  The net torque \( \Sigma \tau \) acting on the CD is given by Newton’s second law for rotational motion (Equation 9.7) as \( \Sigma \tau = I \alpha \), where \( I \) is the moment of inertia of the CD and \( \alpha \) is its angular acceleration. The moment of inertia can be obtained directly from Table 9.1, and the angular acceleration can be found from its definition (Equation 8.4) as the change in the CD’s angular velocity divided by the elapsed time.

**SOLUTION**  The net torque is \( \Sigma \tau = I \alpha \). Assuming that the CD is a solid disk, its moment of inertia can be found from Table 9.1 as \( I = \frac{1}{2} MR^2 \), where \( M \) and \( R \) are the mass and radius of the CD. Thus, the net torque is

\[
\Sigma \tau = I \alpha = \left( \frac{1}{2} MR^2 \right) \alpha
\]

The angular acceleration is given by Equation 8.4 as \( \alpha = (\omega - \omega_0)/t \), where \( \omega \) and \( \omega_0 \) are the final and initial angular velocities, respectively, and \( t \) is the elapsed time. Substituting this expression for \( \alpha \) into Newton’s second law yields

\[
\Sigma \tau = \left( \frac{1}{2} MR^2 \right) \alpha = \left( \frac{1}{2} MR^2 \right) \left( \frac{\omega - \omega_0}{t} \right)
\]

\[
= \left[ \frac{1}{2} \left( 17 \times 10^{-3} \text{ kg} \right) \left( 6.0 \times 10^{-2} \text{ m} \right)^2 \right] \left( \frac{21 \text{ rad/s} - 0 \text{ rad/s}}{0.80 \text{ s}} \right) = 8.0 \times 10^{-4} \text{ N} \cdot \text{m}
\]
32. **REASONING** For a rigid body rotating about a fixed axis, Newton's second law for rotational motion is given as \( \sum \tau = I \alpha \) (Equation 9.7), where \( I \) is the moment of inertia of the body and \( \alpha \) is the angular acceleration. In using this expression, we note that \( \alpha \) must be expressed in rad/s\(^2\).

**SOLUTION** Equation 9.7 gives
\[
I = \frac{\sum \tau}{\alpha} = \frac{10.0 \text{ N} \cdot \text{m}}{8.00 \text{ rad/s}^2} = 1.25 \text{ kg} \cdot \text{m}^2
\]

33. **REASONING** The moment of inertia of the stool is the sum of the individual moments of inertia of its parts. According to Table 9.1, a circular disk of radius \( R \) has a moment of inertia of \( I_{\text{disk}} = \frac{1}{2} M_{\text{disk}} R^2 \) with respect to an axis perpendicular to the disk center. Each thin rod is attached perpendicular to the disk at its outer edge. Therefore, each particle in a rod is located at a perpendicular distance from the axis that is equal to the radius of the disk. This means that each of the rods has a moment of inertia of \( I_{\text{rod}} = M_{\text{rod}} R^2 \).

**SOLUTION** Remembering that the stool has three legs, we find that its moment of inertia is
\[
I_{\text{stool}} = I_{\text{disk}} + 3I_{\text{rod}} = \frac{1}{2} M_{\text{disk}} R^2 + 3 M_{\text{rod}} R^2
= \frac{1}{2}(1.2 \text{ kg})(0.16 \text{ m})^2 + 3(0.15 \text{ kg})(0.16 \text{ m})^2 = 0.027 \text{ kg} \cdot \text{m}^2
\]

34. **REASONING** According to Newton’s second law for rotational motion, \( \Sigma \tau = I \alpha \), the angular acceleration \( \alpha \) of the blades is equal to the net torque \( \Sigma \tau \) applied to the blades divided by their total moment of inertia \( I \), both of which are known.

**SOLUTION** The angular acceleration of the fan blades is
\[
\alpha = \frac{\Sigma \tau}{I} = \frac{1.8 \text{ N} \cdot \text{m}}{0.22 \text{ kg} \cdot \text{m}^2} = 8.2 \text{ rad/s}^2
\]

35. **REASONING** Newton’s second law for rotational motion (Equation 9.2) indicates that the net external torque is equal to the moment of inertia times the angular acceleration. To determine the angular acceleration, we will use Equation 8.7 from the equations of rotational kinematics. This equation indicates that the angular displacement \( \theta \) is given by
\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2,
\]
where \( \omega_0 \) is the initial angular velocity, \( t \) is the time, and \( \alpha \) is the angular acceleration. Since both wheels start from rest, \( \omega_0 = 0 \text{ rad/s} \) for each. Furthermore, each wheel makes the same number of revolutions in the same time, so \( \theta \) and \( t \) are also the same for each. Therefore, the angular acceleration \( \alpha \) must be the same for each.
**SOLUTION** Using Equation 9.2, the net torque $\Sigma \tau$ that acts on each wheel is given by $\Sigma \tau = I \alpha$, where $I$ is the moment of inertia and $\alpha$ is the angular acceleration. Solving Equation 8.7 for the angular acceleration $\alpha = \frac{2(\theta - \omega_0 t)}{t^2}$ and substituting the result into Equation 9.2 gives

$$\Sigma \tau = I \alpha = I \left[ \frac{2(\theta - \omega_0 t)}{t^2} \right]$$

Table 9.1 indicates that the moment of inertia of a hoop is $I_{\text{hoop}} = MR^2$, while the moment of inertia of a disk is $I_{\text{disk}} = \frac{1}{2} MR^2$. The net external torques acting on the hoop and the disk are:

**Hoop**
$$\Sigma \tau = I_{\text{hoop}} \alpha = MR^2 \left[ \frac{2(\theta - \omega_0 t)}{t^2} \right]$$

$$= (4.0 \text{ kg})(0.35 \text{ m})^2 \left( \frac{2(13 \text{ rad}) - 2(0 \text{ rad/s})(8.0 \text{ s})}{(8.0 \text{ s})^2} \right) = 0.20 \text{ N}\cdot\text{m}$$

**Disk**
$$\Sigma \tau = I_{\text{disk}} \alpha = \frac{1}{2} MR^2 \left[ \frac{2(\theta - \omega_0 t)}{t^2} \right]$$

$$= \frac{1}{2} (4.0 \text{ kg})(0.35 \text{ m})^2 \left( \frac{2(13 \text{ rad}) - 2(0 \text{ rad/s})(8.0 \text{ s})}{(8.0 \text{ s})^2} \right) = 0.10 \text{ N}\cdot\text{m}$$

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**36. REASONING** The ladder is subject to three vertical forces: the upward pull $P$ of the painter on the top end of the ladder, the upward normal force $F_N$ that the ground exerts on the bottom end of the ladder, and the downward force $W$ of the ladder’s weight, which acts at the ladder’s center of gravity, halfway between the ends (see the free-body diagram of the ladder). The bottom end serves as the axis of rotation. The normal force $F_N$ is applied at the axis, so it has no lever arm. The net torque acting on the ladder can be obtained with the aid of Equation 9.1:
\[ \Sigma \tau = P\ell_P - W\ell_W + F_N(0) = PL - W\left(\frac{1}{2}L\right) = PL - \frac{1}{2}mgL \]  

(1)

where \( L \) is the length of the ladder and \( m \) is its mass.

Once we know the net torque \( \Sigma \tau \) acting on the ladder, we will use Newton’s second law for rotation, \( \Sigma \tau = I\alpha \) (Equation 9.7) to determine the moment of inertia \( I \) of the ladder.

**SOLUTION**

a. From Equation (1), the net torque acting on the ladder is

\[ \Sigma \tau = PL - \frac{1}{2}mgL = (245 \text{ N})(9.75 \text{ m}) - \frac{1}{2}(23.2 \text{ kg})(9.80 \text{ m/s}^2)(9.75 \text{ m}) = 1280 \text{ N} \cdot \text{m} \]

b. The ladder’s moment of inertia is found from \( \alpha = \Sigma \tau / I \) (Equation 9.7). Using the result found in part a, we obtain

\[ I = \frac{\Sigma \tau}{\alpha} = \frac{1280 \text{ N} \cdot \text{m}}{1.80 \text{ rad/s}^2} = 711 \text{ kg} \cdot \text{m}^2 \]

37. **SSM REASONING**

a. The angular acceleration \( \alpha \) is defined as the change, \( \omega - \omega_0 \), in the angular velocity divided by the elapsed time \( t \) (see Equation 8.4). Since all these variables are known, we can determine the angular acceleration directly from this definition.

b. The magnitude \( \tau \) of the torque is defined by Equation 9.1 as the product of the magnitude \( F \) of the force and the lever arm \( \ell \). The lever arm is the radius of the cylinder, which is known. Since there is only one torque acting on the cylinder, the magnitude of the force can be obtained by using Newton’s second law for rotational motion, \( \Sigma \tau = F\ell = I\alpha \).

**SOLUTION**

a. From Equation 8.4 we have that \( \alpha = (\omega - \omega_0)/t \). We are given that \( \omega_0 = 76.0 \text{ rad/s} \), \( \omega = \frac{1}{2}\omega_0 = 38.0 \text{ rad/s} \), and \( t = 6.40 \text{ s} \), so

\[ \alpha = \frac{\omega - \omega_0}{t} = \frac{38.0 \text{ rad/s} - 76.0 \text{ rad/s}}{6.40 \text{ s}} = -5.94 \text{ rad/s}^2 \]

The magnitude of the angular acceleration is \( -5.94 \text{ rad/s}^2 \).

b. Using Newton’s second law for rotational motion, we have that \( \Sigma \tau = F\ell = I\alpha \). Thus, the magnitude of the force is

\[ F = \frac{I\alpha}{\ell} = \frac{(0.615 \text{ kg} \cdot \text{m}^2)(5.94 \text{ rad/s}^2)}{0.0830 \text{ m}} = 44.0 \text{ N} \]
38. **REASONING** The angular acceleration $\alpha$ that results from the application of a net external torque $\Sigma \tau$ to a rigid object with a moment of inertia $I$ is given by Newton’s second law for rotational motion: $\alpha = \frac{\Sigma \tau}{I}$ (Equation 9.7). By applying this equation to each of the two situations described in the problem statement, we will be able to obtain the unknown angular acceleration.

**SOLUTION** The drawings at the right illustrate the two types of rotational motion of the object. In applying Newton’s second law for rotational motion, we need to keep in mind that the moment of inertia depends on where the axis is. For axis 1 (see top drawing), piece B is rotating about one of its ends, so according to Table 9.1, the moment of inertia is $I_{\text{for axis 1}} = \frac{1}{3} M_B L_B^2$, where $M_B$ and $L_B$ are the mass and length of piece B, respectively. For axis 2 (see bottom drawing), piece A is rotating about an axis through its midpoint. According to Table 9.1 the moment of inertia is $I_{\text{for axis 2}} = \frac{1}{12} M_A L_A^2$. Applying the second law for each of the axes, we obtain

$$\alpha_{\text{for axis 1}} = \frac{\Sigma \tau}{I_{\text{for axis 1}}} \quad \text{and} \quad \alpha_{\text{for axis 2}} = \frac{\Sigma \tau}{I_{\text{for axis 2}}}$$

As given, the net torque $\Sigma \tau$ is the same in both expressions. Dividing the expression for axis 2 by the expression for axis 1 gives

$$\frac{\alpha_{\text{for axis 2}}}{\alpha_{\text{for axis 1}}} = \frac{I_{\text{for axis 2}}}{I_{\text{for axis 1}}} = \frac{\frac{1}{3} M_B L_B^2}{\frac{1}{12} M_A L_A^2} = \frac{4 M_B L_B^2}{M_A L_A^2}$$

Using the expressions from Table 9.1 for the moments of inertia, this result becomes

$$\frac{\alpha_{\text{for axis 2}}}{\alpha_{\text{for axis 1}}} = \frac{I_{\text{for axis 1}}}{I_{\text{for axis 2}}} = \frac{\frac{1}{3} M_B L_B^2}{\frac{1}{12} M_A L_A^2} = \frac{4 M_B L_B^2}{M_A L_A^2}$$

Noting that $M_A = 2 M_B$ and $L_A = 2 L_B$, we find that

$$\frac{\alpha_{\text{for axis 2}}}{\alpha_{\text{for axis 1}}} = \frac{4 M_B L_B^2}{M_A L_A^2} = \frac{4 M_B L_B^2}{2 M_B (2 L_B)^2} = \frac{1}{2}$$

$$\alpha_{\text{for axis 2}} = \frac{1}{2} \alpha_{\text{for axis 1}} = \frac{4.6 \text{ rad/s}^2}{2} = 2.3 \text{ rad/s}^2$$
39. **REASONING** The figure below shows eight particles, each one located at a different corner of an imaginary cube. As shown, if we consider an axis that lies along one edge of the cube, two of the particles lie on the axis, and for these particles \( r = 0 \). The next four particles closest to the axis have \( r = \ell \), where \( \ell \) is the length of one edge of the cube. The remaining two particles have \( r = d \), where \( d \) is the length of the diagonal along any one of the faces. From the Pythagorean theorem, \( d = \sqrt{\ell^2 + \ell^2} = \ell \sqrt{2} \).

![Diagram of eight particles in an imaginary cube](image)

According to Equation 9.6, the moment of inertia of a system of particles is given by \( I = \sum mr^2 \).

**SOLUTION** Direct application of Equation 9.6 gives

\[
I = \sum mr^2 = 4(m\ell^2) + 2(md^2) = 4(m\ell^2) + 2(2m\ell^2) = 8m\ell^2
\]

or

\[
I = 8(0.12 \text{ kg})(0.25 \text{ m})^2 = 0.060 \text{ kg} \cdot \text{m}^2
\]

40. **REASONING AND SOLUTION** The final angular speed of the arm is \( \omega = \omega_f / r \), where \( r = 0.28 \text{ m} \). The angular acceleration needed to produce this angular speed is \( \alpha = (\omega - \omega_0) / t \). The net torque required is \( \Sigma \tau = I\alpha \). This torque is due solely to the force \( M \), so that \( \Sigma \tau = ML \). Thus,

\[
M = \frac{\Sigma \tau}{L} = \frac{\left( \frac{\omega - \omega_0}{t} \right)}{L}
\]

Setting \( \omega_0 = 0 \text{ rad/s} \) and \( \omega = \omega_f / r \), the force becomes

\[
M = \frac{I \left( \frac{\omega_f}{rt} \right)}{L} = \frac{I \omega_f}{Lrt} = \left( 0.065 \text{ kg} \cdot \text{m}^2 \right) \left( 5.0 \text{ m/s} \right) \left( 0.025 \text{ m} \right) \left( 0.28 \text{ m} \right) \left( 0.10 \text{ s} \right) = 460 \text{ N}
\]
41. **REASONING** The drawing shows the two identical sheets and the axis of rotation for each.

The time \( t \) it takes for each sheet to reach its final angular velocity depends on the angular acceleration \( \alpha \) of the sheet. This relation is given by Equation 8.4 as

\[
\frac{\omega - \omega_0}{\alpha} = \frac{I}{\tau}
\]

where \( \omega \) and \( \omega_0 \) are the final and initial angular velocities, respectively. We know that \( \omega_0 = 0 \) rad/s in each case and that the final angular velocities are the same. The angular acceleration can be determined by using Newton’s second law for rotational motion, Equation 9.7, as

\[
\alpha = \frac{\tau}{I}
\]

**SOLUTION** Substituting the relation \( \alpha = \frac{\tau}{I} \) into \( \frac{\omega - \omega_0}{\alpha} = \frac{I}{\tau} \) gives

\[
t = \frac{(\omega - \omega_0)}{\alpha} = \frac{(\omega - \omega_0)}{\frac{\tau}{I}} = \frac{I(\omega - \omega_0)}{\tau}
\]

The time it takes for each sheet to reach its final angular velocity is:

\[
t_{\text{Left}} = \frac{I_{\text{Left}}(\omega - \omega_0)}{\tau} \quad \text{and} \quad t_{\text{Right}} = \frac{I_{\text{Right}}(\omega - \omega_0)}{\tau}
\]

The moments of inertia \( I \) of the left and right sheets about the axes of rotation are given by the following relations, where \( M \) is the mass of each sheet (see Table 9.1 and the drawings above): \( I_{\text{Left}} = \frac{1}{3}MI_1^2 \) and \( I_{\text{Right}} = \frac{1}{3}MI_2^2 \). Note that the variables \( M, \omega, \omega_0, \) and \( \tau \) are the same for both sheets. Dividing the time-expression for the right sheet by that for the left sheet gives

\[
\frac{t_{\text{Right}}}{t_{\text{Left}}} = \frac{I_{\text{Right}}(\omega - \omega_0)}{I_{\text{Left}}(\omega - \omega_0)} = \frac{\frac{1}{3}MI_2^2}{\frac{1}{3}MI_1^2} = \frac{L_2^2}{L_1^2}
\]

Solving this expression for \( t_{\text{Right}} \) yields

\[
t_{\text{Right}} = t_{\text{Left}} \times \frac{L_2^2}{L_1^2} = (8.0 \text{ s}) \frac{(0.20 \text{ m})^2}{(0.40 \text{ m})^2} = 2.0 \text{ s}
\]
42. **REASONING** The time \( t \) it takes to completely unwind the hose from the reel is related to the reel’s angular displacement \( \theta \) and its angular acceleration \( \alpha \) by \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \) (Equation 8.7). The reel is initially at rest, so \( \omega_0 = 0 \text{ rad/s} \). Substituting this into \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \), and solving for the elapsed time \( t \), we obtain

\[
\theta = (0 \text{ rad/s}) t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2 \quad \text{or} \quad t = \sqrt{\frac{2\theta}{\alpha}}
\]

(Equation 1)

The hose unwinds from the reel without slipping, so the total arc length \( s \) traversed by a point on the reel’s rim is equal to the total length \( L \) of the hose. The reel’s angular displacement \( \theta \), therefore, is related to the length of the hose and the radius \( R \) of the reel by \( s = L = R\theta \) (Equation 8.1). Thus, the reel’s angular displacement is given by \( \theta = L/R \), so that Equation (1) becomes

\[
t = \sqrt{\frac{2L}{R\alpha}}
\]

(Equation 2)

The angular acceleration \( \alpha \) of the reel depends upon the net torque and the reel’s moment inertia \( I \) via \( \alpha = \Sigma \tau/I \) (Equation 9.7).

**SOLUTION** Assuming that the hose unwinds in the counterclockwise direction, the torque exerted on the reel by the tension in the hose is positive. Using Equation 9.1, we have \( \tau = TR \), where \( T \) is the magnitude of the tension in the hose and \( R \) is the radius of the reel. This torque is opposed by the frictional torque \( \tau_f \), so the net torque on the reel is \( \Sigma \tau = TR - \tau_f \). Thus, the angular acceleration of the reel is

\[
\alpha = \frac{\Sigma \tau}{I} = \frac{TR - \tau_f}{I}
\]

(Equation 9.7)

Substituting Equation 9.7 into Equation (2) now yields the elapsed time:

\[
t = \sqrt{\frac{2LI}{(TR - \tau_f)R}} = \sqrt{\frac{2LI}{TR(0.160 \text{ m})(25.0 \text{ N})(0.160 \text{ m}) - 3.40 \text{ N} \cdot \text{m}}} = 12 \text{ s}
\]

43. **REASONING**

a. The moment of inertia for the three-ball system is \( I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 \) (Equation 9.6), where \( m_1, m_2, \) and \( m_3 \) are the masses of the balls and \( r_1, r_2, \) and \( r_3 \) are the distances from the axis. In system A, the ball whose mass is \( m_1 \) does not contribute to the moment of inertia, because the ball is located on the axis and \( r_1 = 0 \text{ m} \). In system B, the ball whose mass is \( m_3 \) does not contribute to the moment of inertia, because it is located on the axis and \( r_3 = 0 \text{ m} \).
b. The magnitude of the torque is equal to the magnitude $F$ of the force times the lever arm $\ell$ (see Equation 9.1). In system A the lever arm is $\ell = 3.00$ m. In B the lever arm is $\ell = 0$ m, since the line of action of the force passes through the axis of rotation.

c. According to Newton’s second law for rotational motion, Equation 9.7, the angular acceleration $\alpha$ is given by $\alpha = (\Sigma \tau)/I$, where $\Sigma \tau$ is the net torque and $I$ is the moment of inertia. The angular velocity $\omega$ is given by Equation 8.4 as $\omega = \omega_0 + \alpha t$, where $\omega_0$ is the initial angular velocity and $t$ is the time.

**SOLUTION**

a. The moment of inertia for each system is

**System A** 

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (9.00 \text{ kg})(0 \text{ m})^2 + (6.00 \text{ kg})(3.00 \text{ m})^2 + (7.00 \text{ kg})(5.00 \text{ m})^2 = 229 \text{ kg} \cdot \text{m}^2$$

**System B** 

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (9.00 \text{ kg})(5.00 \text{ m})^2 + (6.00 \text{ kg})(4.00 \text{ m})^2 + (7.00 \text{ kg})(0 \text{ m})^2 = 321 \text{ kg} \cdot \text{m}^2$$

b. A torque that tends to produce a counterclockwise rotation about the axis is a positive torque. The torque produced by the force has a magnitude that is equal to the product of the force magnitude and the lever arm:

**System A** 

$$\tau = -F\ell = -(424 \text{ N})(3.00 \text{ m}) = -1270 \text{ N} \cdot \text{m}$$

The torque is negative because it produces a clockwise rotation about the axis.

**System B** 

$$\tau = F\ell = (424 \text{ N})(0 \text{ m}) = 0 \text{ N} \cdot \text{m}$$

c. The final angular velocity $\omega$ is related to the initial angular velocity $\omega_0$, the angular acceleration $\alpha$, and the time $t$ by $\omega = \omega_0 + \alpha t$ (Equation 8.4). The angular acceleration is given by Newton’s second law for rotational motion as $\alpha = (\Sigma \tau)/I$ (Equation 9.7), where $\Sigma \tau$ is the net torque and $I$ is the moment of inertia. Since there is only one torque acting on each system, it is the net torque, so $\Sigma \tau = \tau$. Substituting this expression for $\alpha$ into Equation 8.4 yields

$$\omega = \omega_0 + \alpha t = \omega_0 + \left(\frac{\tau}{I}\right)t$$

In both cases the initial angular velocity is $\omega_0 = 0$ rad/s, since the systems start from rest. The final angular velocities after 5.00 s are:
44. **REASONING** Each door starts from rest, so that it is initial angular velocity is \( \omega_0 = 0.00 \text{ rad/s} \). The angle \( \theta \) through which each door turns in a time \( t \) is the same. Due to the torque created by the applied force, each door has an angular acceleration \( \alpha \). The variables \( \omega_0, \theta, t, \) and \( \alpha \) are related according to \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \) (Equation 8.7), which is one of the equations of rotational kinematics. The angular acceleration \( \alpha \) is related to the torque \( \tau \) created by the applied force via Newton’s second law for rotation about a fixed axis \( \Sigma \tau = \tau = I \alpha \) (Equation 9.7), where \( I \) is the moment of inertia of the door. In addition, the magnitude \( \tau \) of the torque can be expressed as \( \tau = F \ell \) (Equation 9.1), where \( F \) is the magnitude of the applied force and \( \ell \) is the lever arm for the force.

**SOLUTION** Since \( \omega_0 = 0.00 \text{ rad/s} \) for each door, Equation 8.7 becomes

\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2
\]

(Equation 8.7)

Using \( \tau = F \ell \) in Newton’s second law for rotation, we obtain

\[
\tau = F \ell = I \alpha \quad \text{or} \quad \alpha = \frac{F \ell}{I}
\]

Substituting this result for \( \alpha \) into Equation 8.7, we have

\[
\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \left( \frac{F \ell}{I} \right) t^2 \quad \text{or} \quad t = \sqrt{\frac{2 \theta I}{F \ell}}
\]

We can now apply this result for the time \( t \) to both doors:

\[
t_A = \sqrt{\frac{2 \theta I_A}{F \ell_A}} \quad \text{and} \quad t_B = \sqrt{\frac{2 \theta I_B}{F \ell_B}}
\]

In these expressions for \( t_A \) and \( t_B \), the angle \( \theta \) and the force magnitude \( F \) are the same, so dividing \( t_B \) by \( t_A \) gives
According to Table 9.1 in the text, \( I_A = \frac{1}{3} ML^2 \) and \( I_B = \frac{1}{12} ML^2 \), where \( M \) and \( L \) are the mass and width of each door, respectively. In addition, the lever arms are \( \ell_A = L \) and \( \ell_B = \frac{1}{2} L \). Substituting these values into Equation (1) gives

\[
\frac{t_B}{t_A} = \sqrt{\frac{I_B \ell_A}{\ell_B I_A}} = \sqrt{\frac{1}{2}}
\]

\[
t_B = \frac{t_A}{\sqrt{2}} = 3.00 \text{ s} = 2.12 \text{ s}
\]

45. **Reasoning** The angular acceleration of the bicycle wheel can be calculated from Equation 8.4. Once the angular acceleration is known, Equation 9.7 can be used to find the net torque caused by the brake pads. The normal force can be calculated from the torque using Equation 9.1.

**Solution** The angular acceleration of the wheel is, according to Equation 8.4,

\[
\alpha = \frac{\omega - \omega_0}{t} = \frac{3.7 \text{ rad/s} - 13.1 \text{ rad/s}}{3.0 \text{ s}} = -3.1 \text{ rad/s}^2
\]

If we assume that all the mass of the wheel is concentrated in the rim, we may treat the wheel as a hollow cylinder. From Table 9.1, we know that the moment of inertia of a hollow cylinder of mass \( m \) and radius \( r \) about an axis through its center is \( I = mr^2 \). The net torque that acts on the wheel due to the brake pads is, therefore,

\[
\sum \tau = I \alpha = (mr^2) \alpha
\]

From Equation 9.1, the net torque that acts on the wheel due to the action of the two brake pads is

\[
\sum \tau = -2 f_k \ell
\]

where \( f_k \) is the kinetic frictional force applied to the wheel by each brake pad, and \( \ell = 0.33 \text{ m} \) is the lever arm between the axle of the wheel and the brake pad (see the drawing in the text). The factor of 2 accounts for the fact that there are two brake pads. The minus sign arises because the net torque must have the same sign as the angular acceleration. The kinetic frictional force can be written as (see Equation 4.8)

\[
f_k = \mu_k F_N
\]
where $\mu_k$ is the coefficient of kinetic friction and $F_N$ is the magnitude of the normal force applied to the wheel by each brake pad. Combining Equations (1), (2), and (3) gives

$$-2(\mu_k F_N)\ell = (mr^2)\alpha$$

$$F_N = \frac{-mr^2\alpha}{2\mu_k \ell} = \frac{-(1.3 \text{ kg})(0.33 \text{ m})^2(-3.1 \text{ rad/s}^2)}{2(0.85)(0.33 \text{ m})} = 0.78 \text{ N}$$

46. **REASONING** The parallel axis theorem states that the moment of inertia $I$ about an arbitrary axis is given by

$$I = I_{\text{cm}} + Mh^2 \quad (1)$$

where $I_{\text{cm}}$ is the moment of inertia relative to an axis that passes through the center of mass and is parallel to the axis of interest, $M$ is the total mass of the object (the solid cylinder in this case), and $h$ is the distance between the two axes. The axis of interest here is the axis that lies on the surface of the cylinder and is perpendicular to its circular ends. Thus, $I_{\text{cm}}$ is the moment of inertia relative to an axis that passes through the center of mass and is perpendicular to the cylinder’s circular ends, and referring to Table 9.1 in the text, we see that $I_{\text{cm}} = \frac{1}{2} MR^2$, where $R$ is the radius of the cylinder. Note that in Equation (1) the distance $h$ between these two axes is just $R$, the radius of the cylinder.

**SOLUTION** According to the parallel axis theorem as stated in Equation (1), the moment of inertia of the cylinder relative to an axis that lies on the surface of the cylinder and is perpendicular to its circular ends is

$$I = I_{\text{cm}} + Mh^2 = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$$

47. **REASONING** The following drawing shows the drum, pulley, and the crate, as well as the tensions in the cord
Let \( T_1 \) represent the magnitude of the tension in the cord between the drum and the pulley. Then, the net torque exerted on the drum must be, according to Equation 9.7, \( \Sigma \tau = I_1 \alpha_1 \), where \( I_1 \) is the moment of inertia of the drum, and \( \alpha_1 \) is its angular acceleration. If we assume that the cable does not slip, then Equation 9.7 can be written as

\[
\frac{-T_1 r_1 + \tau}{\Sigma \tau} = \left( \frac{m_1 r_1^2}{I_1} \right) \left( \frac{a}{r_1} \right)
\]

(1)

where \( \tau \) is the counterclockwise torque provided by the motor, and \( a \) is the acceleration of the cord \((a = 1.2 \text{ m/s}^2)\). This equation cannot be solved for \( \tau \) directly, because the tension \( T_1 \) is not known.

We next apply Newton’s second law for rotational motion to the pulley in the drawing:

\[
\frac{+T_1 r_2 - T_2 r_2}{\Sigma \tau} = \left( \frac{\frac{1}{2} m_2 r_2^2}{I_2} \right) \left( \frac{a}{r_2} \right)
\]

(2)

where \( T_2 \) is the magnitude of the tension in the cord between the pulley and the crate, and \( I_2 \) is the moment of inertia of the pulley.

Finally, Newton’s second law for translational motion \((\Sigma F_y = m a)\) is applied to the crate, yielding

\[
\frac{+T_2 - m_3 g}{\Sigma F_y} = m_3 a
\]

(3)

**SOLUTION** Solving Equation (1) for \( T_1 \) and substituting the result into Equation (2), then solving Equation (2) for \( T_2 \) and substituting the result into Equation (3), results in the following value for the torque

\[
\tau = r_1 \left[ a \left( m_1 + \frac{1}{2} m_2 + m_3 \right) + m_3 g \right]
\]

\[
= (0.76 \text{ m}) \left[ (1.2 \text{ m/s}^2) (150 \text{ kg} + \frac{1}{2} 130 \text{ kg} + 180 \text{ kg}) + (180 \text{ kg})(9.80 \text{ m/s}^2) \right] = 1700 \text{ N} \cdot \text{m}
\]

48. **REASONING**

a. The kinetic energy is given by Equation 9.9 as \( KE_R = \frac{1}{2} I \omega^2 \). Assuming the earth to be a uniform solid sphere, we find from Table 9.1 that the moment of inertia is \( I = \frac{2}{3} MR^2 \). The mass and radius of the earth are \( M = 5.98 \times 10^{24} \text{ kg} \) and \( R = 6.38 \times 10^6 \text{ m} \) (see the inside of
the text’s front cover). The angular speed $\omega$ must be expressed in rad/s, and we note that the earth turns once around its axis each day, which corresponds to $2\pi$ rad/day.

b. The kinetic energy for the earth’s motion around the sun can be obtained from Equation 9.9 as $KE_R = \frac{1}{2} I \omega^2$. Since the earth’s radius is small compared to the radius of the earth’s orbit ($R_{\text{orbit}} = 1.50 \times 10^{11}$ m, see the inside of the text’s front cover), the moment of inertia in this case is just $I = MR_{\text{orbit}}^2$. The angular speed $\omega$ of the earth as it goes around the sun can be obtained from the fact that it makes one revolution each year, which corresponds to $2\pi$ rad/year.

**SOLUTION**

a. According to Equation 9.9, we have

$$KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} \left( \frac{2}{3} MR^2 \right) \omega^2$$

$$= \frac{1}{2} \left( \frac{2}{3} \left( 5.98 \times 10^{24} \text{ kg} \right) \left( 6.38 \times 10^6 \text{ m} \right)^2 \right) \left[ \left( \frac{2\pi \text{ rad}}{1 \text{ day}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right]^2$$

$$= 2.57 \times 10^{29} \text{ J}$$

b. According to Equation 9.9, we have

$$KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} \left( MR_{\text{orbit}}^2 \right) \omega^2$$

$$= \frac{1}{2} \left( 5.98 \times 10^{24} \text{ kg} \right) \left( 1.50 \times 10^{11} \text{ m} \right)^2 \left[ \left( \frac{2\pi \text{ rad}}{1 \text{ yr}} \right) \left( \frac{1 \text{ yr}}{365 \text{ day}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right]^2$$

$$= 2.67 \times 10^{33} \text{ J}$$

49. **SSM REASONING AND SOLUTION**

a. The tangential speed of each object is given by Equation 8.9, $v_T = r\omega$. Therefore,

For object 1: $v_{T1} = (2.00 \text{ m})(6.00 \text{ rad/s}) = 12.0 \text{ m/s}$

For object 2: $v_{T2} = (1.50 \text{ m})(6.00 \text{ rad/s}) = 9.00 \text{ m/s}$

For object 3: $v_{T3} = (3.00 \text{ m})(6.00 \text{ rad/s}) = 18.0 \text{ m/s}$
b. The total kinetic energy of this system can be calculated by computing the sum of the kinetic energies of each object in the system. Therefore,

\[ KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 \]

\[ KE = \frac{1}{2} (6.00 \text{ kg})(12.0 \text{ m/s})^2 + (4.00 \text{ kg})(9.00 \text{ m/s})^2 + (3.00 \text{ kg})(18.0 \text{ m/s})^2 = 1.08 \times 10^3 \text{ J} \]

c. The total moment of inertia of this system can be calculated by computing the sum of the moments of inertia of each object in the system. Therefore,

\[ I = \sum mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \]

\[ I = (6.00 \text{ kg})(2.00 \text{ m})^2 + (4.00 \text{ kg})(1.50 \text{ m})^2 + (3.00 \text{ kg})(3.00 \text{ m})^2 = 60.0 \text{ kg} \cdot \text{m}^2 \]

d. The rotational kinetic energy of the system is, according to Equation 9.9,

\[ KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (60.0 \text{ kg} \cdot \text{m}^2)(6.00 \text{ rad/s})^2 = 1.08 \times 10^3 \text{ J} \]

This agrees, as it should, with the result for part (b).

50. **REASONING** The kinetic energy of a rotating object is expressed as \( KE_R = \frac{1}{2} I \omega^2 \) (Equation 9.9), where \( I \) is the object’s moment of inertia and \( \omega \) is its angular speed. According to Equation 9.6, the moment of inertia for rod A is just that of the attached particle, since the rod itself is massless. For rod A with its attached particle, then, the moment of inertia is \( I_A = ML^2 \). According to Table 9.1, the moment of inertia for rod B is \( I_B = \frac{1}{3} ML^2 \).

**SOLUTION** Using Equation 9.9 to calculate the kinetic energy, we find that

\[ \text{Rod A} \quad KE_R = \frac{1}{2} I_A \omega^2 = \frac{1}{2} (ML^2) \omega^2 \]

\[ = \frac{1}{2} (0.66 \text{ kg})(0.75 \text{ m})^2 (4.2 \text{ rad/s})^2 = 3.3 \text{ J} \]

\[ \text{Rod B} \quad KE_R = \frac{1}{2} I_B \omega^2 = \frac{1}{2} \left( \frac{1}{3} ML^2 \right) \omega^2 \]

\[ = \frac{1}{6} (0.66 \text{ kg})(0.75 \text{ m})^2 (4.2 \text{ rad/s})^2 = 1.1 \text{ J} \]
51. **REASONING** The kinetic energy of the flywheel is given by Equation 9.9. The moment of inertia of the flywheel is the same as that of a solid disk, and, according to Table 9.1 in the text, is given by \( I = \frac{1}{2} MR^2 \). Once the moment of inertia of the flywheel is known, Equation 9.9 can be solved for the angular speed \( \omega \) in rad/s. This quantity can then be converted to rev/min.

**SOLUTION** Solving Equation 9.9 for \( \omega \), we obtain,

\[
\omega = \sqrt{\frac{2(KE_R)}{I}} = \sqrt{\frac{2(KE_R)}{\frac{1}{2} MR^2}} = \sqrt{\frac{2(1.2 \times 10^5 \text{ J})}{(13 \text{ kg})(0.30 \text{ m})^2}} = 6.4 \times 10^4 \text{ rad/s}
\]

Converting this answer into rev/min, we find that

\[
\omega = (6.4 \times 10^4 \text{ rad/s}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 6.1 \times 10^5 \text{ rev/min}
\]

52. **REASONING** Each blade can be approximated as a thin rod rotating about an axis perpendicular to the rod and passing through one end. The moment of inertia of a blade is given in Table 9.1 as \( \frac{1}{3} ML^2 \), where \( M \) is the mass of the blade and \( L \) is its length. The total moment of inertia \( I \) of the two blades is just twice that of a single blade. The rotational kinetic energy \( KE_R \) of the blades is given by Equation 9.9 as \( KE_R = \frac{1}{2} I \omega^2 \), where \( \omega \) is the angular speed of the blades.

**SOLUTION**

a. The total moment of inertia of the two blades is

\[
I = \frac{1}{3} ML^2 + \frac{1}{3} ML^2 = \frac{2}{3} ML^2 = \frac{2}{3} (240 \text{ kg})(6.7 \text{ m})^2 = 7200 \text{ kg} \cdot \text{m}^2
\]

b. The rotational kinetic energy is

\[
KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (7200 \text{ kg} \cdot \text{m}^2)(44 \text{ rad/s})^2 = 7.0 \times 10^6 \text{ J}
\]

53. **REASONING** The rotational kinetic energy of a solid sphere is given by Equation 9.9 as \( KE_R = \frac{1}{2} I \omega^2 \), where \( I \) is its moment of inertia and \( \omega \) its angular speed. The sphere has translational motion in addition to rotational motion, and its translational kinetic energy is \( KE_T = \frac{1}{2} m v^2 \) (Equation 6.2), where \( m \) is the mass of the sphere and \( v \) is the speed of its center of mass. The fraction of the sphere’s total kinetic energy that is in the form of rotational kinetic energy is \( KE_R/(KE_R + KE_T) \).
**SOLUTION** The moment of inertia of a solid sphere about its center of mass is \( I = \frac{2}{5} mR^2 \), where \( R \) is the radius of the sphere (see Table 9.1). The fraction of the sphere’s total kinetic energy that is in the form of rotational kinetic energy is

\[
\frac{KE_R}{KE_R + KE_T} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} mv^2} = \frac{\frac{1}{2} \left( \frac{2}{5} mR^2 \right) \omega^2}{\frac{1}{2} \left( \frac{2}{5} mR^2 \right) \omega^2 + \frac{1}{2} mv^2} = \frac{\frac{2}{5} R^2 \omega^2}{\frac{2}{5} R^2 \omega^2 + v^2}
\]

Since the sphere is rolling without slipping on the surface, the translational speed \( v \) of the center of mass is related to the angular speed \( \omega \) about the center of mass by \( v = R \omega \) (see Equation 8.12). Substituting \( v = R \omega \) into the equation above gives

\[
\frac{KE_R}{KE_R + KE_T} = \frac{\frac{2}{5} R^2 \omega^2}{\frac{2}{5} R^2 \omega^2 + (R \omega)^2} = \frac{2}{7}
\]

54. **REASONING** Only the conservative force of gravity does work on the objects, so the total mechanical energy is conserved as they move down the ramp. The total mechanical energy \( E \) at any height \( h \) above the zero level is

\[
E = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 + mgh
\]

As the objects move down the ramp, potential energy is converted into kinetic energy. However, the kinetic energy for the marble is shared between the translational form \( \left( \frac{1}{2} mv^2 \right) \) and the rotational form \( \left( \frac{1}{2} I \omega^2 \right) \), whereas the kinetic energy for the cube is all translational. Therefore, at the bottom of the ramp, the marble will have the smaller center-of-mass speed, because it will arrive there with less translational kinetic energy than the cube has. We expect, then, that the ratio of the center-of-mass speeds \( v_\text{cube} / v_\text{marble} \) will be greater than one.

**SOLUTION** When applying energy conservation, we will assume that the zero level for measuring the height \( h \) is located at the bottom of the ramp. As applied to the marble, energy conservation gives

\[
\frac{\frac{1}{2} mv^2_{\text{marble}} + \frac{1}{2} I \omega^2_{\text{marble}}}{\text{Total mechanical energy at bottom of ramp}} = \frac{mgh}{\text{Total mechanical energy at top of ramp}}
\]

In Equation (1) we have used the fact that the marble starts at rest at the top of the ramp. Since the marble rolls without slipping, we know that \( \omega_{\text{marble}} = v_\text{marble} / r \) (Equation 8.12), where \( r \) is the radius of the marble. Referring to Table 9.1, we also know that the marble’s moment of inertia is \( I = \frac{2}{5} mr^2 \). Substituting these two expressions into Equation (1) gives
\[ \frac{1}{2}mv_{\text{marble}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{marble}}}{r}\right)^2 = mgh \quad \text{or} \quad v_{\text{marble}} = \sqrt{\frac{10gh}{7}} \]

As applied to the cube, energy conservation gives

\[ \frac{1}{2}mv_{\text{cube}}^2 = mgh \quad \text{or} \quad v_{\text{cube}} = \sqrt{2gh} \]

where we have used the fact that the cube starts at rest at the top of the ramp and does not rotate. The desired ratio of the center-of-mass speeds is

\[ \frac{v_{\text{cube}}}{v_{\text{marble}}} = \frac{\sqrt{2gh}}{\sqrt{10gh/7}} = \frac{14}{10} = 1.18 \]

55. **REASONING** Because we are ignoring frictional losses, the total mechanical energy \( E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh \) of both objects is conserved as they roll down the hill. We will apply this conservation principle twice: first, to determine the height \( h_0 - h_f \) of the hill, and second, to determine the translational speed \( v_f \) of the frozen juice can at the bottom of the hill. Both the basketball and the frozen juice can roll without slipping, so the translational speed \( v \) of either one is related to its radius \( r \) and angular speed \( \omega \) by \( v = r\omega \) (Equation 8.12).

**SOLUTION**

a. Applying the energy conservation principle to the basketball, we obtain

\[ \frac{\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f}{E_f} = \frac{\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0}{E_0} \quad (1) \]

The basketball starts from rest, so we have \( v_0 = 0 \text{ m/s} \) and \( \omega_0 = 0 \text{ rad/s} \). The basketball is a thin-walled spherical shell, so its moment of inertia is given by \( I = \frac{2}{3}mr^2 \) (see Table 9.1 in the text). Substituting these values along with \( \omega_t = v_t/r \) (Equation 8.12) into Equation (1) yields

\[ \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_t}{r}\right)^2 + mgh_f = 0 + 0 + \frac{2}{3}mgh_0 \quad \text{or} \quad \frac{1}{2}v_f^2 + \frac{2}{3}v_t^2 = g\left(h_0 - h_f\right) \]

Solving for the height \( h_0 - h_f \) of the hill, we obtain

\[ h_0 - h_f = \frac{5v_f^2}{6g} = \frac{5(6.6 \text{ m/s})^2}{6(9.80 \text{ m/s}^2)} = 3.7 \text{ m} \]
b. In Equation (1), we again substitute \( v_0 = 0 \) m/s and \( \omega_0 = 0 \) rad/s, but this time we use \( I = \frac{1}{2}mr^2 \) for the moment of inertia (see Table 9.1 in the text), because the frozen juice can is a solid cylinder:

\[
\frac{1}{2} \mu I v_f^2 + \frac{1}{2} \mu I \omega_f^2 \left( \frac{v_f}{L} \right)^2 + \mu gh_f = 0 + 0 + \mu gh_0 \quad \text{or} \quad \frac{1}{2} v_f^2 + \frac{1}{4} \omega_f^2 = \frac{3}{4} v_f^2 = g (h_0 - h_f)
\]

The final translational speed of the frozen juice can is

\[
v_f = \frac{4g(h_0 - h_f)}{3} \quad \text{or} \quad v_f = \sqrt{\frac{4g(h_0 - h_f)}{3}} = \sqrt{\frac{4(9.80 \text{ m/s}^2)(3.7 \text{ m})}{3}} = 7.0 \text{ m/s}
\]

56. **REASONING** The drawing shows the rod in its initial (dashed lines) and final (solid lines) orientations. Since both friction and air resistance are absent, the total mechanical energy is conserved. In this case, the total mechanical energy is

\[
E = \frac{1}{2} I \omega^2 + mgh
\]

In this expression, \( m \) is the rod’s mass, \( I \) is its moment of inertia, \( \omega \) is its angular speed, and \( h \) is the height of its center of mass above a reference level that we take to be the ground. Since the rod is uniform, its center of mass is located at the center of the rod. The energy-conservation principle will guide our solution.

**SOLUTION** Conservation of the total mechanical energy dictates that

\[
\frac{1}{2} I \omega_f^2 + mgh_f = \frac{1}{2} I \omega_0^2 + mgh_0 \quad \text{(1)}
\]

Since the rod is momentarily at rest in its final orientation, we know that \( \omega_f = 0 \) rad/s, so that Equation (1) becomes

\[
mgh_f = \frac{1}{2} I \omega_0^2 + mgh_0 \quad \text{or} \quad \frac{1}{2} I \omega_0^2 = mg(h_f - h_0) \quad \text{(2)}
\]

We can relate the initial angular speed \( \omega_0 \) to the initial linear speed \( v_0 \) by using Equation 8.9: \( \omega_0 = \frac{v_0}{L} \). With this substitution, the fact that \( h_f - h_0 = L \) (see the drawing), and the fact that \( I = \frac{1}{2} mL^2 \) (see Table 9.1 in the text), Equation (2) indicates that
\[ \frac{1}{2} I \omega_0^2 = mg (h_f - h_0) \quad \text{or} \quad \frac{1}{2} \left( \frac{1}{3} mL^2 \right) \left( \frac{v_0}{L} \right)^2 = mgL \quad \text{or} \quad \frac{1}{6} v_0^2 = gL \]

Solving for \( v_0 \) gives

\[ v_0 = \sqrt{6gL} = \sqrt{6 (9.80 \text{ m/s}^2)(0.80 \text{ m})} = 6.9 \text{ m/s} \]

57. **SSM REASONING AND SOLUTION** The conservation of energy gives

\[ mgh + \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega_0^2 \]

If the ball rolls without slipping, \( \omega = \frac{v}{R} \) and \( \omega_0 = \frac{v_0}{R} \). We also know that \( I = \frac{2}{5} mR^2 \). Substitution of the last two equations into the first and rearrangement gives

\[ v = \sqrt{v_0^2 - \frac{10}{7} gh} = \sqrt{(3.50 \text{ m/s})^2 - \frac{10}{7} (9.80 \text{ m/s}^2)(0.760 \text{ m})} = 1.3 \text{ m/s} \]

58. **REASONING** We first find the speed \( v_0 \) of the ball when it becomes airborne using the conservation of mechanical energy. Once \( v_0 \) is known, we can use the equations of kinematics to find its range \( x \).

**SOLUTION** When the tennis ball starts from rest, its total mechanical energy is in the form of gravitational potential energy. The gravitational potential energy is equal to \( mgh \) if we take \( h = 0 \text{ m} \) at the height where the ball becomes airborne. Just before the ball becomes airborne, its mechanical energy is in the form of rotational kinetic energy and translational kinetic energy. At this instant its total energy is \( \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega^2 \). If we treat the tennis ball as a thin-walled spherical shell of mass \( m \) and radius \( r \), and take into account that the ball rolls down the hill without slipping, its total kinetic energy can be written as

\[ \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv_0^2 + \frac{1}{2} (\frac{2}{3} m) r^2 \left( \frac{v_0}{r} \right)^2 = \frac{5}{6} mv_0^2 \]

Therefore, from conservation of mechanical energy, he have

\[ mgh = \frac{5}{6} mv_0^2 \quad \text{or} \quad v_0 = \sqrt{\frac{6gh}{5}} \]

The range of the tennis ball is given by \( x = v_0 t = v_0 (\cos \theta) t \), where \( t \) is the flight time of the ball. From Equation 3.3b, we find that the flight time \( t \) is given by
Therefore, the range of the tennis ball is

\[ x = v_{0x}t = v_0(\cos \theta)\left(-\frac{2v_0 \sin \theta}{a_y}\right) \]

If we take upward as the positive direction, then using the fact that \( a_y = -g \) and the expression for \( v_0 \) given above, we find

\[ x = \left(\frac{2\cos \theta \sin \theta}{g}\right) v_0^2 = \left(\frac{2\cos \theta \sin \theta}{g}\right) \left(\frac{6gh}{5}\right)^2 = \frac{12}{5} h \cos \theta \sin \theta \]

\[ = \frac{12}{5} (1.8 \text{ m})(\cos 35^\circ)(\sin 35^\circ) = 2.0 \text{ m} \]

59. **Reasoning** Let the two disks constitute the system. Since there are no external torques acting on the system, the principle of conservation of angular momentum applies. Therefore we have \( L_{\text{initial}} = L_{\text{final}} \), or

\[ I_A \omega_A + I_B \omega_B = (I_A + I_B) \omega_{\text{final}} \]

This expression can be solved for the moment of inertia of disk B.

**Solution** Solving the above expression for \( I_B \), we obtain

\[ I_B = I_A \left(\frac{\omega_{\text{final}} - \omega_A}{\omega_B - \omega_{\text{final}}}\right) = (3.4 \text{ kg} \cdot \text{m}^2) \left[\frac{-2.4 \text{ rad/s} - 7.2 \text{ rad/s}}{-9.8 \text{ rad/s} - (-2.4 \text{ rad/s})}\right] = 4.4 \text{ kg} \cdot \text{m}^2 \]

60. **Reasoning** The supernova explosion proceeds entirely under the influence of internal forces and torques. External forces and torques play no role. Therefore, the star’s angular momentum is conserved during the supernova and its subsequent transformation from a solid sphere into an expanding spherical shell: \( L_f = L_0 \). The star’s initial and final angular momenta are given by \( L = I\omega \) (Equation 9.10), where \( I \) is the star’s moment of inertia, and \( \omega \) is its angular velocity. Initially, the star is a uniform solid sphere with a moment of inertia given by \( I = \frac{2}{5} MR^2 \) (see Table 9.1 in the text). Following the supernova, the moment of inertia of the expanding spherical shell is \( I = \frac{2}{3} MR^2 \) (see Table 9.1 in the text). We will use
the angular-momentum-conservation principle to calculate the final angular velocity \( \omega_f \) of the expanding supernova shell.

**SOLUTION** Applying the angular-momentum-conservation principle yields

\[
I_f \omega_f = I_0 \omega_0 \quad \text{or} \quad \frac{2}{5} M R_f^2 \omega_f = \frac{2}{5} M R_0^2 \omega_0 \quad \text{or} \quad \omega_f = \frac{3R_0^2 \omega_0}{5R_f^2}
\]

Substituting \( R_0 = R \), \( R_f = 4.0R \), and \( \omega_0 = 2.0 \, \text{rev/d} \), we obtain the final angular velocity of the expanding shell:

\[
\omega_f = \frac{3R^2 \omega_0}{5(4.0R)^2} = 0.075 \, \text{rev/d}
\]

**REASONING** The carousel rotates on frictionless bearings and without air resistance, so no net external torque acts on the system comprised of the carousel and the person on it. Therefore, the total angular momentum \( L = I \omega \) (Equation 9.10) of the system is conserved \( (L_f = L_0) \). As the person moves closer to the center of the carousel, the person’s distance \( r \) from the rotation axis decreases from \( r_0 = 1.50 \, \text{m} \) to \( r_f = 0.750 \, \text{m} \). Therefore, the moment of inertia \( I_p = mr^2 \) (Equation 9.6) of the person also decreases, from \( I_{p0} = m r_0^2 \) to \( I_{pf} = m r_f^2 \). Consequently, the angular speed of the system increases from \( \omega_0 \) to \( \omega_f \), preserving the system’s total angular momentum. We will use the angular-momentum-conservation principle to find the mass \( m \) of the person.

**SOLUTION** The person and the carousel have the same initial angular velocity \( \omega_0 \), and the same final angular velocity \( \omega_f \). Therefore, the conservation of angular momentum principle can be expressed as

\[
\frac{I_{pf} \omega_f + I_C \omega_f}{I_f} = \frac{I_{p0} \omega_0 + I_C \omega_0}{I_0}
\]

where \( I_C \) is the moment of inertia of the carousel (without the person). Substituting \( I_{p0} = m r_0^2 \) and \( I_{pf} = m r_f^2 \) (Equation 9.6) for the person’s initial and final moments of inertia into Equation (1), we obtain

\[
m r_f^2 \omega_f + I_C \omega_f = m r_0^2 \omega_0 + I_C \omega_0 \quad \text{or} \quad I_C (\omega_f - \omega_0) = m \left( r_0^2 \omega_0 - r_f^2 \omega_f \right)
\]

Solving for the mass \( m \) of the person yields

\[
m = \frac{I_C (\omega_f - \omega_0)}{r_0^2 \omega_0 - r_f^2 \omega_f} = \frac{\left( 125 \, \text{kg} \cdot \text{m}^2 \right)(0.800 \, \text{rad/s} - 0.600 \, \text{rad/s})}{(1.50 \, \text{m})^2 (0.600 \, \text{rad/s}) - (0.750 \, \text{m})^2 (0.800 \, \text{rad/s})} = 28 \, \text{kg}
\]
62. **REASONING** Once the motorcycle is in the air, it is subject to no net external torque, since gravity and air resistance are being ignored. Therefore, its total angular momentum is conserved: \( L_t = L_0 \). The total angular momentum of the motorcycle is the sum of the angular momentum \( L_E = I_E \omega_E \) (Equation 9.10) of the engine and the angular momentum \( L_M = I_M \omega_M \) of the rest of the motorcycle (including the rider):

\[
\frac{I_E \omega_{Ef} + I_M \omega_{Mf}}{I_t} = \frac{I_E \omega_{E0} + I_M \omega_{M0}}{I_0}
\]  

(1)

We will use Equation (1) to find the ratio \( I_E/I_M \) of the moments of inertia of the engine and the rest of the motorcycle. We note that, so long as all angular velocities are expressed in rev/min, there is no need to convert to SI units (rad/s).

**SOLUTION** Initially, only the engine is rotating, so the rest of the motorcycle has no angular velocity: \( \omega_{M0} = 0 \) rad/s. Solving Equation (1) for the ratio \( I_E/I_M \), we obtain

\[
I_E \omega_{Ef} + I_M \omega_{Mf} = I_E \omega_{E0} + I_M (0) \quad \text{or} \quad I_E (\omega_{Ef} - \omega_{E0}) = -I_M \omega_{Mf} \quad \text{or} \quad \frac{I_E}{I_M} = \frac{-\omega_{Mf}}{\omega_{Ef} - \omega_{E0}}
\]

As usual, clockwise rotation is negative, and counterclockwise is positive. The ratio of the moments of inertia is, then,

\[
\frac{I_E}{I_M} = \frac{-\omega_{Mf}}{\omega_{Ef} - \omega_{E0}} = \frac{-(-3.8 \text{ rev/min})}{(+12500 \text{ rev/min}) - (+7700 \text{ rev/min})} = 7.9 \times 10^{-4}
\]

63. **REASONING** The rod and bug are taken to be the system of objects under consideration, and we note that there are no external torques acting on the system. As the bug crawls out to the end of the rod, each exerts a torque on the other. However, these torques are internal torques. The conservation of angular momentum states that the total angular momentum of a system remains constant (is conserved) if the net average external torque acting on the system is zero. We will use this principle to find the final angular velocity of the system.

**SOLUTION** The angular momentum \( L \) of the system (rod plus bug) is given by Equation 9.10 as the product of the system’s moment of inertia \( I \) and angular velocity \( \omega \), or \( L = I \omega \). The conservation of angular momentum can be written as

\[
\frac{I \omega}{\text{Final angular momentum}} = \frac{I_0 \omega_0}{\text{Initial angular momentum}}
\]

where \( \omega \) and \( \omega_0 \) are the final and initial angular velocities, respectively, and \( I \) and \( I_0 \) are the final and initial moments of inertia. The initial moment of inertia is given. The initial moment of inertia of the bug is zero, because it is located at the axis of rotation. The final
moment of inertia is the sum of the moment of inertia of the bug and that of the rod; 
\[ I = I_{\text{bug}} + I_0. \]
When the bug has reached the end of the rod, its moment of inertia is 
\[ I_{\text{bug}} = m L^2, \]
where \( m \) is its mass and \( L \) is the length of the rod. The final angular velocity of the system is, then, 
\[
\omega = \omega_0 \left( \frac{I_0}{I} \right) = \omega_0 \left( \frac{I_0}{I_{\text{bug}} + I_0} \right) = \omega_0 \left( \frac{I_0}{m L^2 + I_0} \right)
\]
\[
= (0.32 \text{ rad/s}) \left[ \frac{1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2}{4.2 \times 10^{-3} \text{ kg} \cdot (0.25 \text{ m})^2 + (1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2)} \right] = 0.26 \text{ rad/s}
\]

64. **REASONING** We consider a system consisting of the person and the carousel. Since the carousel rotates on frictionless bearings, no net external torque acts on this system. Therefore, its angular momentum is conserved, and we can set the total angular momentum of the system before the person hops on equal to the total angular momentum afterwards.

**SOLUTION** The initial angular momentum of the carousel is \( I_{\text{carousel}} \omega_0 \) (Equation 9.10), where \( I_{\text{carousel}} \) is the moment of inertia of the carousel and \( \omega_0 \) is its initial angular velocity. After the person climbs aboard, the total angular momentum of the carousel and person is 
\[
I_{\text{carousel}} \omega_1 + I_{\text{person}} \omega_1,
\]
where \( \omega_1 \) is the final angular velocity. According to Equation 9.6, the person’s moment of inertia is \( I_{\text{person}} = MR^2 \), since he is at the outer edge of the carousel, which has a radius \( R \). Applying the conservation of angular momentum, we have

\[
\frac{I_{\text{carousel}} \omega_1 + I_{\text{person}} \omega_1}{\text{Final total angular momentum}} = \frac{I_{\text{carousel}} \omega_0}{\text{Initial total angular momentum}}
\]
\[
\omega_1 = \frac{I_{\text{carousel}} \omega_0}{I_{\text{carousel}} + I_{\text{person}}} = \frac{I_{\text{carousel}} \omega_0}{I_{\text{carousel}} + MR^2}
\]
\[
= \frac{(125 \text{ kg} \cdot \text{m}^2)(3.14 \text{ rad/s})}{125 \text{ kg} \cdot \text{m}^2 + (40.0 \text{ kg})(1.50 \text{ m})^2} = 1.83 \text{ rad/s}
\]

65. **REASONING** Let the space station and the people within it constitute the system. Then as the people move radially from the outer surface of the cylinder toward the axis, any torques that occur are internal torques. Since there are no external torques acting on the system, the principle of conservation of angular momentum can be employed.
**SOLUTION**  Since angular momentum is conserved,

\[ I_{\text{final}} \omega_{\text{final}} = I_0 \omega_0 \]

Before the people move from the outer rim, the moment of inertia is

\[ I_0 = I_{\text{station}} + 500m_{\text{person}}r_{\text{person}}^2 \]

or

\[ I_0 = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2 + (500)(70.0 \text{ kg})(82.5 \text{ m})^2 = 3.24 \times 10^9 \text{ kg} \cdot \text{m}^2 \]

If the people all move to the center of the space station, the total moment of inertia is

\[ I_{\text{final}} = I_{\text{station}} = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2 \]

Therefore,

\[ \frac{\omega_{\text{final}}}{\omega_0} = \frac{I_0}{I_{\text{final}}} = \frac{3.24 \times 10^9 \text{ kg} \cdot \text{m}^2}{3.00 \times 10^9 \text{ kg} \cdot \text{m}^2} = 1.08 \]

This fraction represents a percentage increase of 8 percent.

66. **REASONING**  Since the rod (length = d, mass = m) changes shape without the aid of external torques, the principle of conservation of angular momentum applies. This principle states that the rod’s total angular momentum remains constant. Thus, the total angular momentum is the same in its final straight shape as it is in its initial bent shape.

**SOLUTION**  Angular momentum is the momentum of inertia \( I \) of a rotating object times the object’s angular velocity \( \omega \), according to Equation 9.10. Conservation of the total angular momentum indicates that

\[
\frac{I_{\text{straight}}\omega_{\text{straight}}}{I_{\text{final}}} = \frac{I_{\text{bent}}\omega_{\text{bent}}}{I_{\text{final}}} \quad \text{or} \quad \frac{\omega_{\text{straight}}}{\omega_{\text{bent}}} = \frac{I_{\text{bent}}\omega_{\text{bent}}}{I_{\text{straight}}} \quad (1)
\]

In its bent shape, the rod’s horizontal half (length = d/2, mass = m/2) rotates about the axis perpendicular to its left end (see text drawing), so that its mass is located at varying distances from the axis. According to Table 9.1, its moment of inertia is

\[ \frac{1}{3} \left( \frac{1}{2} m \right) \left( \frac{1}{2} d \right)^2 = \frac{1}{24} md^2 \]. As the vertical half rotates around the axis, all of its mass is located at the same distance of d/2 from the axis. Therefore, its moment of inertia is

\[ \left( \frac{1}{2} m \right) \left( \frac{1}{2} d \right)^2 = \frac{1}{8} md^2 \], according to Equation 9.4. The total moment of inertia of the bent rod, then is the sum of these two contributions:

\[ I_{\text{bent}} = \frac{1}{24} md^2 + \frac{1}{8} md^2 = \frac{1}{6} md^2 \quad (2) \]

According to Table 9.1 the momentum of inertia of the straight rod is
\[ I_{\text{straight}} = \frac{1}{3} md^2 \quad (3) \]

Substituting Equations (2) and (3) into Equation (1) gives

\[
\omega_{\text{straight}} = \frac{I_{\text{bent}} \omega_{\text{bent}}}{I_{\text{straight}}} = \frac{\left(\frac{1}{6} md^2\right)(9.0 \text{ rad/s})}{\frac{1}{3} md^2} = 4.5 \text{ rad/s}
\]

67. **REASONING AND SOLUTION** After the mass has moved inward to its final path the centripetal force acting on it is \( T = 105 \text{ N} \).

\[ \begin{diagram}
    \text{105 N}
\end{diagram} \]

Its centripetal acceleration is

\[ a_c = \frac{\omega^2 R}{m} = T/m \]

Now

\[ \omega = \omega R \quad \text{so} \quad R = T/(\omega^2 m) \]

The centripetal force is parallel to the line of action (the string), so the force produces no torque on the object. Hence, angular momentum is conserved.

\[ I\omega = I_0 \omega_0 \quad \text{so that} \quad \omega = (I_0/I)\omega_0 = (R_0^2/R^2)\omega_0 \]

Substituting and simplifying, we obtain

\[ R^3 = \frac{mR_0^4 \omega_0^2}{T} \quad \text{or} \quad R = \sqrt[3]{\frac{mR_0^4 \omega_0^2}{T}} = \sqrt[3]{\frac{(0.500 \text{ kg})(1.00 \text{ m})^4 (6.28 \text{ rad/s})^2}{105 \text{ N}}} = 0.573 \text{ m} \]

68. **REASONING AND SOLUTION** The block will just start to move when the centripetal force on the block just exceeds \( f_s^{\text{max}} \). Thus, if \( r_f \) is the smallest distance from the axis at which the block stays at rest when the angular speed of the block is \( \omega_0 \) then

\[ \mu_s F_N = mr_f \omega_0^2, \quad \text{or} \quad \mu_s mg = mr_f \omega_0^2 \]

Thus,
\[ \mu_s g = r_f \omega_f^2 \quad (1) \]

Since there are no external torques acting on the system, angular momentum will be conserved.

\[ I_0 \omega_0 = I_f \omega_f \]

where \( I_0 = mr_0^2 \), and \( I_f = mr_f^2 \). Making these substitutions yields

\[ r_0^2 \omega_0 = r_f^2 \omega_f \quad (2) \]

Solving Equation (2) for \( \omega_f \) and substituting into Equation (1) yields:

\[ \frac{\mu_s g}{2} = \frac{r_0^2 \omega_0}{r_f^2} \]

Solving for \( r_f \) gives

\[ r_f = \left( \frac{\omega_0^2 r_0^2}{\mu_s g} \right)^{1/3} = \left[ \frac{(2.2 \text{ rad/s})^2 (0.30 \text{ m})^4}{(0.75)(9.80 \text{ m/s}^2)} \right]^{1/3} = 0.17 \text{ m} \]

69. SSM **REASONING** In both parts of the problem, the magnitude of the torque is given by Equation 9.1 as the magnitude \( F \) of the force times the lever arm \( \ell \). In part (a), the lever arm is just the distance of 0.55 m given in the drawing. However, in part (b), the lever arm is less than the given distance and must be expressed using trigonometry as \( \ell = (0.55 \text{ m}) \sin \theta \). See the drawing at the right.

**SOLUTION**

a. Using Equation 9.1, we find that

Magnitude of torque = \( F \ell = (49 \text{ N})(0.55 \text{ m}) = 27 \text{ N} \cdot \text{m} \)

b. Again using Equation 9.1, this time with a lever arm of \( \ell = (0.55 \text{ m}) \sin \theta \), we obtain

Magnitude of torque = 15 N \cdot m = F \ell = (49 \text{ N})(0.55 \text{ m}) \sin \theta

\[ \sin \theta = \frac{15 \text{ N} \cdot \text{m}}{(49 \text{ N})(0.55 \text{ m})} \quad \text{or} \quad \theta = \sin^{-1} \left[ \frac{15 \text{ N} \cdot \text{m}}{(49 \text{ N})(0.55 \text{ m})} \right] = 34^\circ \]

70. **REASONING** Before any sand strikes the disk, only the disk is rotating. After the sand has landed on the disk, both the sand and the disk are rotating. If the sand and disk are taken to be the system of objects under consideration, we note that there are no external torques
acting on the system. As the sand strikes the disk, each exerts a torque on the other. However, these torques are exerted by members of the system, and, as such, are internal torques. The conservation of angular momentum states that the total angular momentum of a system remains constant (is conserved) if the net average external torque acting on the system is zero. We will use this principle to find the final angular velocity of the system.

**SOLUTION** The angular momentum of the system (sand plus disk) is given by Equation 9.10 as the product of the system’s moment of inertia \( I \) and angular velocity \( \omega \), or \( L = I \omega \). The conservation of angular momentum can be written as

\[
\frac{I_0 \omega_0}{\text{Initial angular momentum}} = \frac{I \omega}{\text{Final angular momentum}}
\]

where \( \omega \) and \( \omega_0 \) are the final and initial angular velocities, respectively, and \( I \) and \( I_0 \) are final and initial moments of inertia. The initial moment of inertia is given, while the final moment of inertia is the sum of the values for the rotating sand and disk, \( I = I_{\text{sand}} + I_0 \). We note that the sand forms a thin ring, so its moment of inertia is given by (see Table 9.1)

\[
I_{\text{sand}} = M_{\text{sand}} R_{\text{sand}}^2
\]

where \( M_{\text{sand}} \) is the mass of the sand and \( R_{\text{sand}} \) is the radius of the ring. Thus, the final angular velocity of the system is, then,

\[
\omega = \omega_0 \left( \frac{I_0}{I} \right) = \omega_0 \left( \frac{I_0}{I_{\text{sand}} + I_0} \right) = \omega_0 \left( \frac{I_0}{M_{\text{sand}} R_{\text{sand}}^2 + I_0} \right) = (0.067 \text{ rad/s}) \left[ \frac{0.10 \text{ kg} \cdot \text{m}^2}{(0.50 \text{ kg})(0.40 \text{ m})^2 + 0.10 \text{ kg} \cdot \text{m}^2} \right] = 0.037 \text{ rad/s}
\]

71. **REASONING** According to Table 9.1 in the text, the moment of inertia \( I \) of the disk is \( I = \frac{1}{2} MR^2 \), where \( M \) is the disk’s mass and \( R \) is its radius. Thus, we can determine the mass from this expression, provided we can obtain a value for \( I \). To obtain the value for \( I \), we will use Newton’s second law for rotational motion, \( \Sigma \tau = I \alpha \) (Equation 9.7), where \( \Sigma \tau \) is the net torque and \( \alpha \) is the angular acceleration.

**SOLUTION** From the moment of inertia of the disk, we have

\[
I = \frac{1}{2} MR^2 \quad \text{or} \quad M = \frac{2I}{R^2}
\]

Using Newton’s second law for rotational motion, we find for \( I \) that

\[
\Sigma \tau = I \alpha \quad \text{or} \quad I = \frac{\Sigma \tau}{\alpha}
\]

Substituting this expression for \( I \) into Equation (1) gives
\[ M = \frac{2I}{R^2} = \frac{2\Sigma \tau}{R^2 \alpha} \quad (2) \]

The net torque \( \Sigma \tau \) is due to the 45-N force alone. According to Equation 9.1, the magnitude of the torque that a force of magnitude \( F \) produces is \( F \ell \), where \( \ell \) is the lever arm of the force. In this case, we have \( \ell = R \), since the force is applied tangentially to the disk and perpendicular to the radius. Substituting \( \Sigma \tau = FR \) into Equation (2) gives

\[ M = \frac{2\Sigma \tau}{R^2 \alpha} = \frac{2FR}{R^2 \alpha} = \frac{2F}{R \alpha} = \frac{2(45 \text{ N})}{(0.15 \text{ m})(120 \text{ rad/s}^2)} = \frac{5.0 \text{ kg}}{.} \]

72. **REASONING** At every instant before the plank begins to tip, it is in equilibrium, and the net torque on it is zero: \( \Sigma \tau = 0 \) (Equation 9.2). When the person reaches the maximum distance \( x \) along the overhanging part, the plank is just about to rotate about the right support. At that instant, the plank loses contact with the left support, which consequently exerts no force on it. This leaves only three vertical forces acting on the plank: the weight \( W \) of the plank, the force \( F_R \) due to the right support, and the force \( P \) due to the person (see the free-body diagram of the plank). The force \( F_R \) acts at the right support, which we take as the axis, so its lever arm is zero. The lever arm for the force \( P \) is the distance \( x \). Since counterclockwise is the positive direction, Equation 9.2 gives

\[ \Sigma \tau = W\ell_w - Px = 0 \quad \text{or} \quad x = \frac{W\ell_w}{P} \quad (1) \]

\[\text{Free-body diagram of the plank}\]

\[\text{Axis}\]

\[\text{W}\]

\[\ell_w\]

\[d\]

\[L/2\]

\[F_R\]

\[P\]

\[x\]

**SOLUTION** The weight \( W = 225 \text{ N} \) of the plank is known, and the force \( P \) due to the person is equal to the person’s weight: \( P = 450 \text{ N} \). This is because the plank supports the person against the pull of gravity, and Newton’s third law tells us that the person and the plank exert forces of equal magnitude on each other. The plank’s weight \( W \) acts at the center of the uniform plank, so we have (see the drawing)

\[ \ell_w + d = \frac{1}{2} L \quad \text{or} \quad \ell_w = \frac{1}{2} L - d \quad (2) \]

where \( d = 1.1 \text{ m} \) is the length of the overhanging part of the plank, and \( L = 5.0 \text{ m} \) is the length of the entire plank. Substituting Equation (2) into Equation (1), we obtain
\[
x = \frac{W \left( \frac{1}{2} L - d \right)}{P} = \frac{(225 \text{ N}) \left[ \frac{1}{2} (5.0 \text{ m}) - 1.1 \text{ m} \right]}{450 \text{ N}} = \frac{0.70 \text{ m}}{}
\]

73. **REASONING** The rotational analog of Newton's second law is given by Equation 9.7, \( \sum \tau = I \alpha \). Since the person pushes on the outer edge of one section of the door with a force \( F \) that is directed perpendicular to the section, the torque exerted on the door has a magnitude of \( FL \), where the lever arm \( L \) is equal to the width of one section. Once the moment of inertia is known, Equation 9.7 can be solved for the angular acceleration \( \alpha \).

The moment of inertia of the door relative to the rotation axis is \( I = 4I_p \), where \( I_p \) is the moment of inertia for one section. According to Table 9.1, we find \( I_p = \frac{1}{3}ML^2 \), so that the rotational inertia of the door is \( I = \frac{4}{3}ML^2 \).

**SOLUTION** Solving Equation 9.7 for \( \alpha \), and using the expression for \( I \) determined above, we have

\[
\alpha = \frac{FL}{\frac{4}{3}ML^2} = \frac{F}{\frac{4}{3}ML} = \frac{68 \text{ N}}{\frac{4}{3}(85 \text{ kg})(1.2 \text{ m})} = 0.50 \text{ rad/s}^2
\]

74. **REASONING** To determine the angular acceleration of the pulley and the tension in the cord attached to the block, we will apply Newton’s second law to the pulley and the block separately. Only one external forces acts on the pulley, as its free-body diagram at the right shows. This force \( T \) is due to the tension in the cord. The torque that results from this force is the net torque acting on the pulley and obeys Newton’s second law for rotational motion (Equation 9.7). Two external forces act on the block, as its free-body diagram at the right indicates. These are (1) the force \( T' \) due to the tension in the cord and (2) the weight \( mg \) of the block. The net force that results from these forces obeys Newton’s second law for translational motion (Equation 4.2b).

**SOLUTION** Applying Newton’s second law for rotational motion to the pulley gives

\[
\sum \tau = +TR = I \alpha
\]
where we have written the torque as the magnitude of the tension force times the lever arm (the radius) as specified by Equation 9.1. In addition, we have assigned this torque a positive value, since it causes a counterclockwise rotation of the pulley. To obtain a value for $T$, we note that the tension has the same magnitude everywhere in the massless cord, so that $T = T'$. Thus, by applying Newton’s second law for translation (Equation 4.2b), we obtain

$$\Sigma F = T' - mg = -ma \quad \text{or} \quad T = T' = mg - ma$$

where we have used $a$ to denote the magnitude of the vertical acceleration of the block and included the minus sign to account for the fact that the block is accelerating downward. Substituting this result for $T$ into Equation (1) gives

$$mg - ma = I\alpha$$

To proceed further, we must deal with $a$. Note that the pulley rolls without slipping against the cord, so $a$ and $\alpha$ are related according to $a = R\alpha$ (Equation 8.13). With this substitution, Equation (2) becomes

$$\left( mg - m(R\alpha) \right) R = I\alpha \quad \text{or} \quad mgR = mR^2\alpha + I\alpha$$

Solving for $\alpha$, we find that

$$\alpha = \frac{mgR}{mR^2 + I} = \frac{(2.0 \text{ kg})(9.80 \text{ m/s}^2)(0.040 \text{ m})}{(2.0 \text{ kg})(0.040 \text{ m})^2 + 1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2} = 180 \text{ rad/s}^2$$

The value for the tension can be now obtained by substituting this value for $\alpha$ into Equation (1):

$$TR = I\alpha \quad \text{or} \quad T = \frac{I\alpha}{R} = \frac{\left(1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2\right)(180 \text{ rad/s}^2)}{0.040 \text{ m}} = 5.0 \text{ N}$$

75. **REASONING** The arm, being stationary, is in equilibrium, since it has no translational or angular acceleration. Therefore, the net external force and the net external torque acting on the arm are zero. Using the fact that the net external torque is zero will allow us to determine the magnitude of the force $M$. The drawing at the right shows three forces: $M$, the 47-N weight of the arm acting at the arm’s center of gravity (cg), and the 98-N force that acts upward on the right end of the arm. The 98-N force is applied to the arm by the ring. It is the reaction force that arises in response to the arm pulling downward on the ring. Its magnitude is 98 N, because it supports the 98-N weight hanging from the pulley system. Other forces also act on the arm
at the shoulder joint, but we can ignore them. The reason is that their lines of action pass directly through the axis at the shoulder joint, which is the axis that we will use to determine torques. Thus, these forces have zero lever arms and contribute no torque.

**SOLUTION** The magnitude of each individual torque is the magnitude of the force times the corresponding lever arm. The forces and their lever arms are as follows:

<table>
<thead>
<tr>
<th>Force</th>
<th>Lever Arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>98 N</td>
<td>0.61 m</td>
</tr>
<tr>
<td>47 N</td>
<td>0.28 m</td>
</tr>
<tr>
<td>( M )</td>
<td>((0.069 \text{ m}) \sin 29^\circ)</td>
</tr>
</tbody>
</table>

Each torque is positive if it causes a counterclockwise rotation and negative if it causes a clockwise rotation about the axis. Thus, since the net torque must be zero, we see that

\[
(98 \text{ N})(0.61 \text{ m}) - (47 \text{ N})(0.28 \text{ m}) - M \left[(0.069 \text{ m}) \sin 29^\circ\right] = 0
\]

Solving for \( M \) gives

\[
M = \frac{(98 \text{ N})(0.61 \text{ m}) - (47 \text{ N})(0.28 \text{ m})}{(0.069 \text{ m}) \sin 29^\circ} = 1400 \text{ N}
\]

76. **REASONING** The moment of inertia of a point particle is the mass \( m \) of the particle times the square of the perpendicular distance \( r \) of the particle from the axis about which the particle is rotating, according to Equation 9.4. For an object that is not a point particle, its mass is spread out over a region of space, and the range of distances where the mass is located must be taken into account. The moments of inertia given in Table 9.1 in the text illustrate how the shape of an object can influence its moment of inertia.

**SOLUTION**

a. In Table 9.1 the moment of inertia of a rod relative to an axis that is perpendicular to the rod at one end is given by

\[
I_{\text{rod}} = \frac{1}{3} ML^2 = \frac{1}{3} (2.00 \text{ kg})(2.00 \text{ m})^2 = 2.67 \text{ kg} \cdot \text{m}^2
\]

b. According to Equation 9.4, the moment of inertia of a point particle of mass \( m \) relative to a rotation axis located a perpendicular distance \( r \) from the particle is \( I = mr^2 \). Suppose that all the mass of the rod were located at a single point located at perpendicular distance \( r \) from the axis in part (a). If this point particle has the same moment of inertia as the rod in part (a), then the distance \( r \), which is called the radius of gyration, is given by

\[
r = \sqrt{\frac{I}{m}} = \sqrt{\frac{2.67 \text{ kg} \cdot \text{m}^2}{2.00 \text{ kg}}} = 1.16 \text{ m}
\]
77. **REASONING** When the modules pull together, they do so by means of forces that are internal. These pulling forces, therefore, do not create a net external torque, and the angular momentum of the system is conserved. In other words, it remains constant. We will use the conservation of angular momentum to obtain a relationship between the initial and final angular speeds. Then, we will use Equation 8.9 \( v = r\omega \) to relate the angular speeds \( \omega_0 \) and \( \omega_f \) to the tangential speeds \( v_0 \) and \( v_f \).

**SOLUTION** Let \( L \) be the initial length of the cable between the modules and \( \omega_0 \) be the initial angular speed. Relative to the center-of-mass axis, the initial momentum of inertia of the two-module system is \( I_0 = 2M(L/2)^2 \), according to Equation 9.6. After the modules pull together, the length of the cable is \( L/2 \), the final angular speed is \( \omega_f \), and the momentum of inertia is \( I_f = 2M(L/4)^2 \). The conservation of angular momentum indicates that

\[
\frac{I_f \omega_f}{\text{Final angular momentum}} = \frac{I_0 \omega_0}{\text{Initial angular momentum}}
\]

\[
\left[ 2M \left( \frac{L}{4} \right)^2 \right] \omega_f = \left[ 2M \left( \frac{L}{2} \right)^2 \right] \omega_0
\]

\[
\omega_f = 4 \omega_0
\]

According to Equation 8.9, \( \omega_f = v_f(L/4) \) and \( \omega_0 = v_0/(L/2) \). With these substitutions, the result that \( \omega_f = 4 \omega_0 \) becomes

\[
\frac{v_f}{L/4} = 4 \left( \frac{v_0}{L/2} \right)
\]

or

\[
v_f = 2v_0 = 2(17 \text{ m/s}) = 34 \text{ m/s}
\]

78. **REASONING**

a. The kinetic energy of the rolling wheel is the sum of its translational \( \left( \frac{1}{2}mv^2 \right) \) and rotational \( \left( \frac{1}{2}I\omega^2 \right) \) kinetic energies. In these expressions \( m \) and \( I \) are, respectively, the mass and moment of inertia of the wheel, and \( v \) and \( \omega \) are, respectively, its linear and angular speeds. The sliding wheel only has translational kinetic energy, since it does not rotate.

b. As the wheels move up the incline plane, the total mechanical energy is conserved, since only the conservative force of gravity does work on each wheel. Thus, the initial kinetic energy at the bottom of the incline is converted entirely into potential energy when the wheels come to a momentary halt. The potential energy PE is given by \( PE = mgh \) (Equation 6.5), where \( h \) is the height of the wheel above an arbitrary zero level.
**SOLUTION**

a. Since the rolling wheel is a disk, its moment of inertia is \( I = \frac{1}{2} mR^2 \) (see Table 9.1), where \( R \) is the radius of the disk. Furthermore, its angular speed \( \omega \) is related to the linear speed \( v \) of its center of mass by Equation 8.12 as \( \omega = \frac{v}{R} \). Thus, the total kinetic energy of the rolling wheel is

\[
KE = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} \left( \frac{1}{2} mR^2 \right) \left( \frac{v}{R} \right)^2
\]

\[
= \frac{3}{4} mv^2 = \frac{3}{4} (2.0 \text{ kg})(6.0 \text{ m/s})^2 = 54 \text{ J}
\]

The kinetic energy of the sliding wheel is

\[
KE = \frac{1}{2} mv^2 = \frac{1}{2} (2.0 \text{ kg})(6.0 \text{ m/s})^2 = 36 \text{ J}
\]

b. As each wheel rolls up the incline, its total mechanical energy is conserved. The initial kinetic energy \( KE \) at the bottom of the incline is converted entirely into potential energy \( PE \) when the wheels come to a momentary halt. Thus, the potential energies of the wheels have the values calculated in part a for the total kinetic energies.

The potential energy of a wheel is given by Equation 6.5 as \( PE = mgh \), where \( g \) is the acceleration due to gravity and \( h \) is the height relative to an arbitrary zero level. Therefore, the height reached by each wheel is as follows:

\[
Rolling \quad Wheel \quad h = \frac{PE}{mg} = \frac{54 \text{ J}}{(2.0 \text{ kg})(9.80 \text{ m/s}^2)} = 2.8 \text{ m}
\]

\[
Sliding \quad Wheel \quad h = \frac{PE}{mg} = \frac{36 \text{ J}}{(2.0 \text{ kg})(9.80 \text{ m/s}^2)} = 1.8 \text{ m}
\]

79. **REASONING AND SOLUTION** Consider the left board, which has a length \( L \) and a weight of \( (356 \text{ N})/2 = 178 \text{ N} \). Let \( F_V \) be the upward normal force exerted by the ground on the board. This force balances the weight, so \( F_V = 178 \text{ N} \). Let \( f_s \) be the force of static friction, which acts horizontally on the end of the board in contact with the ground. \( f_s \) points to the right. Since the board is in equilibrium, the net torque acting on the board through any axis must be zero. Measuring the torques with respect to an axis through the apex of the triangle formed by the boards, we have

\[
+ (178 \text{ N})(\sin 30.0^\circ) \left( \frac{L}{2} \right) + f_s (L \cos 30.0^\circ) - F_V (L \sin 30.0^\circ) = 0
\]
or

\[ 44.5 \, \text{N} + f_s \cos 30.0^\circ - F_V \sin 30.0^\circ = 0 \]

so that

\[ f_s = \frac{(178 \, \text{N})(\sin 30.0^\circ) - 44.5 \, \text{N}}{\cos 30.0^\circ} = 51.4 \, \text{N} \]

80. **REASONING AND SOLUTION** Newton's law applied to the 11.0-kg object gives

\[ T_2 - (11.0 \, \text{kg})(9.80 \, \text{m/s}^2) = (11.0 \, \text{kg})(4.90 \, \text{m/s}^2) \quad \text{or} \quad T_2 = 162 \, \text{N} \]

A similar treatment for the 44.0-kg object yields

\[ T_1 - (44.0 \, \text{kg})(9.80 \, \text{m/s}^2) = (44.0 \, \text{kg})(-4.90 \, \text{m/s}^2) \quad \text{or} \quad T_1 = 216 \, \text{N} \]

For an axis about the center of the pulley

\[ T_2 r - T_1 r = I(-\alpha) = (1/2) Mr^2 (-a/r) \]

Solving for the mass \( M \) we obtain

\[ M = \frac{-2/a(T_2 - T_1)} = \left[ -2/(4.90 \, \text{m/s}^2) \right] (162 \, \text{N} - 216 \, \text{N}) = 22.0 \, \text{kg} \]
ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. 0.12 m

2. (c) The restoring force is given by Equation 10.2 as $F = -kx$, where $k$ is the spring constant (positive). The graph of this equation is a straight line and indicates that the restoring force has a direction that is always opposite to the direction of the displacement. Thus, when $x$ is positive, $F$ is negative, and vice versa.

3. (b) According to Equations 10.4 and 10.11, the period $T$ is given by $T = \frac{1}{2\pi} \sqrt{\frac{m}{k}}$. Greater values for the mass $m$ and smaller values for the spring constant $k$ lead to greater values for the period.

4. (d) The maximum speed in simple harmonic motion is given by Equation 10.8 ($v_{\text{max}} = A\omega$). Thus, increases in both the amplitude $A$ and the angular frequency $\omega$ lead to an increase in the maximum speed.

5. (e) The maximum acceleration in simple harmonic motion is given by Equation 10.10 ($a_{\text{max}} = A\omega^2$). A decrease in the amplitude $A$ decreases the maximum acceleration, but this decrease is more than offset by the increase in the angular frequency $\omega$, which is squared in Equation 10.10.

6. 1.38 m/s

7. (b) The velocity has a maximum magnitude at point A, where the object passes through the position where the spring is unstrained. The acceleration at point A is zero, because the spring is unstrained there and is not applying a force to the object. The velocity is zero at point B, where the object comes to a momentary halt and reverses the direction of its travel. The magnitude of the acceleration at point B is a maximum, because the spring is maximally stretched there and, therefore, applies a force of maximum magnitude to the object.

8. 0.061 m

9. $+7.61 \text{ m/s}^2$

10. 0.050 m
11. (c) The principle of conservation of mechanical energy applies in the absence of nonconservative forces, so that \( KE + PE = \text{constant} \). Thus, the total energy is the same at all points of the oscillation cycle. At the equilibrium position, where the spring is unstrained, the potential energy is zero, and the kinetic energy is \( KE_{\text{max}} \); thus, the total energy is \( KE_{\text{max}} \). At the extreme ends of the cycle, where the object comes to a momentary halt, the kinetic energy is zero, and the potential energy is \( PE_{\text{max}} \); thus, the total energy is also \( PE_{\text{max}} \). Since both \( KE_{\text{max}} \) and \( PE_{\text{max}} \) equal the total energy, it must be true that \( KE_{\text{max}} = PE_{\text{max}} \).

12. (e) In simple harmonic motion the speed and, hence, \( KE \) has a maximum value as the object passes through its equilibrium position, which is position 2. \( EPE \) has a maximum value when the spring is maximally stretched at position 3. \( GPE \) has a maximum value when the object is at its highest point above the ground, that is, at position 1.

13. (a) At the instant the top block is removed, the total mechanical energy of the remaining system is all elastic potential energy and is \( \frac{1}{2} kA^2 \) (see Equation 10.13), where \( A \) is the amplitude of the previous simple harmonic motion. This total mechanical energy is conserved, because friction is absent. Therefore, the total mechanical energy of the ensuing simple harmonic motion is also \( \frac{1}{2} kA^2 \), and the amplitude remains the same as it was previously. The angular frequency \( \omega \) is given by Equation 10.11 as \( \omega = \sqrt{\frac{k}{m}} \). Thus, when the mass \( m \) attached to the spring decreases, the angular frequency increases.

14. (b) The angular frequency \( \omega \) of oscillation of a simple pendulum is given by Equation 10.16

\[
\omega = \sqrt{\frac{g}{L}}.
\]

It depends only on the magnitude \( g \) of the acceleration due to gravity and the length \( L \) of the pendulum. It does not depend on the mass. Therefore, the pendulum with the greatest length has the smallest frequency.

15. 1.7 s

16. (c) When the energy of the system is dissipated, the amplitude of the motion decreases. The motion is called damped harmonic motion.

17. (a) Resonance occurs when the frequency of the external force equals the frequency of oscillation of the object on the spring. The angular frequency of such a system is given by Equation 10.11

\[
\omega = \sqrt{\frac{k}{m}}.
\]

Since the frequency of the force is doubled, the new frequency must be \( 2\omega = 2\sqrt{\frac{k}{m}} \). The frequency of system A is, in fact, \( \omega = \sqrt{\frac{8k}{2m}} = 2\sqrt{\frac{k}{m}} \).
18. (c) According to Equation 10.17 \[ F = Y \left( \frac{\Delta L}{L_0} \right) A \], the force \( F \) required to stretch a piece of material is proportional to Young’s modulus \( Y \), the amount of stretch \( \Delta L \), and the cross-sectional area \( A \) of the material, but is inversely proportional to the initial length \( L_0 \) of the material. Solving this equation for the amount of stretch gives \( \Delta L = \frac{FL_0}{YA} \). Thus, the greater the cross-sectional area, the smaller is the amount of stretch, for given values of Young’s modulus, the initial length, and the stretching force. Thus, B stretches more than A, because B has the smaller cross-sectional area of solid material.

19. \( 0.50 \times 10^{-6} \) m

20. 0.0017
1. **REASONING AND SOLUTION** Using Equation 10.1, we first determine the spring constant:

\[
k = \frac{F^{\text{Applied}}}{x} = \frac{89.0 \text{ N}}{0.0191 \text{ m}} = 4660 \text{ N/m}
\]

Again using Equation 10.1, we find that the force needed to compress the spring by 0.0508 m is

\[
F^{\text{Applied}} = kx = (4660 \text{ N/m})(0.0508 \text{ m}) = 237 \text{ N}
\]

2. **REASONING** The weight of the block causes the spring to stretch. The amount \(x\) of stretching, according to Equation 10.1, depends on the magnitude \(F^{\text{Applied}}\) of the applied force and the spring constant \(k\).

**SOLUTION**

a. The applied force is equal to the weight of the block. The amount \(x\) that the spring stretches is equal to the length of the stretched spring minus the length of the unstretched spring. The spring constant is

\[
k = \frac{F^{\text{Applied}}}{x} = \frac{4.50 \text{ N}}{0.350 \text{ m} - 0.200 \text{ m}} = \boxed{30.0 \text{ N/m}} \quad (10.1)
\]

b. When a block of unknown weight is attached to the spring, it stretches it by 0.500 m – 0.200 m. The weight (or applied force) of the block is

\[
F^{\text{Applied}} = kx = (30.0 \text{ N/m})(0.500 \text{ m} - 0.200 \text{ m}) = \boxed{9.00 \text{ N}} \quad (10.1)
\]

3. **REASONING** The force required to stretch the spring is given by Equation 10.1 as \(F^{\text{Applied}}_x = kx\), where \(k\) is the spring constant and \(x\) is the displacement of the stretched spring from its unstrained length. Solving for the spring constant gives \(k = F^{\text{Applied}}_x / x\). The force applied to the spring has a magnitude equal to the weight \(W\) of the board, so \(F^{\text{Applied}}_x = W\). Since the board’s lower end just extends to the floor, the unstrained length \(x_0\) of the spring,
plus the length $L_0$ of the board, plus the displacement $x$ of the stretched spring must equal
the height $h_0$ of the room, or $x_0 + L_0 + x = h_0$. Thus, $x = h_0 - x_0 - L_0$.

**SOLUTION** Substituting $F_x^{\text{Applied}} = W$ and $x = h_0 - x_0 - L_0$ into Equation 10.1, we find

$$k = \frac{F_x^{\text{Applied}}}{x} = \frac{W}{h_0 - x_0 - L_0} = \frac{104 \text{ N}}{2.44 \text{ m} - 1.98 \text{ m} - 0.30 \text{ m}} = 650 \text{ N/m}$$

---

4. **REASONING** The restoring force of the spring and the static frictional force point in opposite directions. Since the box is in equilibrium just before it begins to move, the net force in the horizontal direction is zero at this instant. This condition, together with the expression for the restoring force (Equation 10.2) and the expression for the maximum static frictional force (Equation 4.7), will allow us to determine how far the spring can be stretched without the box moving upon release.

**SOLUTION** The drawing at the right shows the four forces that act on the box: its weight $mg$, the normal force $F_N$, the restoring force $F_x$ exerted by the spring, and the maximum static frictional force $F_{s,\text{MAX}}$. Since the box is not moving, it is in equilibrium. Let the $x$ axis be parallel to the table top. According to Equation 4.9a, the net force $\Sigma F_x$ in the $x$ direction must be zero, $\Sigma F_x = 0$.

The restoring force is $F_x = -kx$ (Equation 10.2), where $k$ is the spring constant and $x$ is the displacement of the spring (assumed to be in the $+x$ direction). The magnitude of the maximum static frictional force is $F_{s,\text{MAX}} = \mu s F_N$ (Equation 4.7), where $\mu_s$ is the coefficient of static friction and $F_N$ is the magnitude of the normal force. Thus, the condition for equilibrium can be written as

$$-kx + \mu_s F_N = 0 \quad \text{or} \quad x = \frac{\mu_s F_N}{k}$$

We can determine $F_N$ by noting that the box is also in vertical equilibrium, so that

$$-mg + F_N = 0 \quad \text{or} \quad F_N = mg$$

The distance that the spring is stretched from its unstrained position is

$$x = \frac{\mu_s F_N}{k} = \frac{\mu_s mg}{k} = \frac{(0.74)(0.80 \text{ kg})(9.80 \text{ m/s}^2)}{59 \text{ N/m}} = 9.8 \times 10^{-2} \text{ m}$$
5. **REASONING** The weight of the person causes the spring in the scale to compress. The amount $x$ of compression, according to Equation 10.1, depends on the magnitude $F_x^{\text{Applied}}$ of the applied force and the spring constant $k$.

**SOLUTION**

a. Since the applied force is equal to the person’s weight, the spring constant is

$$k = \frac{F_x^{\text{Applied}}}{x} = \frac{670 \text{ N}}{0.79 \times 10^{-2} \text{ m}} = \boxed{8.5 \times 10^4 \text{ N/m}}$$

(10.1)

b. When another person steps on the scale, it compresses by 0.34 cm. The weight (or applied force) that this person exerts on the scale is

$$F_x^{\text{Applied}} = k x = \left(8.5 \times 10^4 \text{ N/m}\right)\left(0.34 \times 10^{-2} \text{ m}\right) = \boxed{290 \text{ N}}$$

(10.1)

6. **REASONING** According to Equation 10.2, the displacement $x$ of the spring from its unstrained length is $x = -\frac{F}{k}$, where $F$ is the upward force that the spring applies to the 5.0-kg object and $k$ is the spring constant. The magnitude of $x$ is what we seek. The spring constant is given. To use Equation 10.2, however, we also need the value of $F$, which we can obtain by applying Newton’s second law of motion $\Sigma F = ma$ (Equation 4.1). $\Sigma F$ is the net force acting on the object, $m$ is the object’s mass, and $a$ is the object’s acceleration, which is the same as the elevator’s acceleration.

**SOLUTION** The displacement $x$ of the spring from its unstrained length is

$$x = -\frac{F}{k}$$

(10.2)

Along with the force of the spring, the other force that acts on the object is the downward-pointing weight of the object. Thus, assuming that upward is the positive direction, we can write the net force acting on the object as $\Sigma F = F - mg$. Newton’s second law becomes

$$\Sigma F = F - mg = ma \quad \text{or} \quad F = mg + ma$$

(1)

Substituting Equation (1) into Equation 10.2, we find that

$$x = -\frac{F}{k} = -\frac{m(g + a)}{k} = -\left(5.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2 + 0.60 \text{ m/s}^2\right) \frac{1}{830 \text{ N/m}} = -6.3 \times 10^{-2} \text{ m}$$

The amount that the spring stretches is the magnitude of this result or $\boxed{6.3 \times 10^{-2} \text{ m}}$. 
7. **REASONING** The block is at equilibrium as it hangs on the spring. Therefore, the downward-directed weight of the block is balanced by the upward-directed force applied to the block by the spring. The weight of the block is $mg$, where $m$ is the mass and $g$ is the acceleration due to gravity. The force exerted by the spring is given by Equation 10.2 ($F_x = -kx$), where $k$ is the spring constant and $x$ is the displacement of the spring from its unstrained length. We will apply this reasoning twice, once to the single block hanging, and again to the two blocks. Although the spring constant is unknown, we will be able to eliminate it algebraically from the resulting two equations and determine the mass of the second block.

**SOLUTION** Let upward be the positive direction. Setting the weight $mg$ equal to the force $-kx$ exerted by the spring in each case gives

\[
m_1g = -kx_1 \quad \text{and} \quad m_1g + m_2g = -kx_2
\]

One hanging block

Two hanging blocks

Dividing the equation on the right by the equation on the left, we can eliminate the unknown spring constant $k$ and find that

\[
\frac{m_1g + m_2g}{m_1g} = \frac{-kx_2}{-kx_1} \quad \text{or} \quad 1 + \frac{m_2}{m_1} = \frac{x_2}{x_1}
\]

It is given that $x_2/x_1 = 3.0$. Therefore, solving for the mass of the second block reveals that

\[
m_2 = m_1 \left( \frac{x_2}{x_1} - 1 \right) = (0.70 \text{ kg})(3.0 - 1) = 1.4 \text{ kg}
\]

8. **REASONING** The spring with the smaller spring constant ($k_1 = 33 \text{ N/m}$) is less stiff than the other spring ($k_2 = 59 \text{ N/m}$), and, therefore, stretches farther ($x_1 > x_2$). The end of the rod attached to the stiffer spring is higher than the other end by a vertical distance $d = x_1 - x_2$ (see the drawing). The angle $\theta$ that the rod makes with the horizontal is given by the inverse sine function:

\[
\theta = \sin^{-1} \left( \frac{d}{L} \right)
\]

where $L$ is the length of the rod.
The amount \( x \) that each spring is stretched is given by \( x = F/k \) (Equation 10.1). In order to use this relation, we will first need to find the amount of force \( F \) exerted on the rod by each spring. The rod is in equilibrium, so the net torque on it must be zero: \( \Sigma \tau = 0 \) (Equation 9.2). We will choose the rod’s center of gravity, located at the center of the uniform rod, as the axis of rotation. The rod’s weight acts at this point, and so generates no torque. This leaves only the torques due to the two spring forces, which must sum to zero. Both torques have the same lever arm about the center of gravity: \( \ell_1 = \ell_2 = L/2 \cos \theta \) (see the drawing at right) Both forces, therefore, must also have equal magnitudes, which can be seen as follows:

\[
\Sigma \tau = F_2 \left( \frac{1}{2} L \cos \theta \right) - F_1 \left( \frac{1}{2} L \cos \theta \right) = 0 \quad \text{or} \quad F_2 = F_1
\]

Together, the two vertical spring forces support the rod’s weight. Because their magnitudes are equal, each force must support half the rod’s weight: \( F_1 = F_2 = \frac{1}{2} mg \), where \( m \) is the mass of the rod, and \( g \) is the magnitude of the acceleration due to gravity.

**SOLUTION** As noted above, each spring stretches by an amount given by \( x = F/k \) (Equation 10.1). Therefore, the difference \( d \) in the heights between the high and low ends of the rod is

\[
d = x_1 - x_2 = \frac{F_1}{k_1} - \frac{F_2}{k_2}
\]

Now, using the fact that both force magnitudes are equal to half the weight of the rod \( (F_1 = F_2 = \frac{1}{2} mg) \), we obtain

\[
d = \frac{\frac{1}{2} mg}{k_1} - \frac{\frac{1}{2} mg}{k_2} = mg \left( \frac{1}{k_1} - \frac{1}{k_2} \right)
\]

Thus, using Equation 1.4 for the angle that the rod makes with the horizontal, we find

\[
\theta = \sin^{-1} \left( \frac{d}{L} \right) = \sin^{-1} \left[ \frac{mg}{2L} \left( \frac{1}{k_1} - \frac{1}{k_2} \right) \right]
\]

\[
= \sin^{-1} \left[ \frac{(1.4 \text{ kg})(9.80 \text{ m/s}^2)}{2(0.75 \text{ m})} \left( \frac{1}{33 \text{ N/m}} - \frac{1}{59 \text{ N/m}} \right) \right] = 7.0^\circ
\]
9. **REASONING AND SOLUTION** The force that acts on the block is given by Newton's Second law, \( F_x = ma_x \) (Equation 4.2a). Since the block has a constant acceleration, the acceleration is given by Equation 2.8 with \( v_0 = 0 \text{ m/s} \); that is, \( a_x = 2\frac{d}{t^2} \), where \( d \) is the distance through which the block is pulled. Therefore, the force that acts on the block is given by

\[
F_x = ma_x = \frac{2md}{t^2}
\]

The force acting on the block is the restoring force of the spring. Thus, according to Equation 10.2, \( F_x = -kx \), where \( k \) is the spring constant and \( x \) is the displacement. Solving Equation 10.2 for \( x \) and using the expression above for \( F_x \), we obtain

\[
x = -\frac{F_x}{k} = -\frac{2md}{kt^2} = -\frac{2(7.00 \text{ kg})(4.00 \text{ m})}{(415 \text{ N/m})(0.750 \text{ s})^2} = -0.240 \text{ m}
\]

The amount that the spring stretches is \( 0.240 \text{ m} \).

10. **REASONING AND SOLUTION** The figure at the right shows the original situation before the spring is cut. The weight \( W \) of the object stretches the string by an amount \( x \). Applying \( F_x^{\text{Applied}} = kx \) (Equation 10.1) to this situation, (in which \( F_x^{\text{Applied}} = W \)), gives

\[
W = kx \quad (1)
\]

The figure at the right shows the situation after the spring is cut into two segments of equal length.

Let \( k' \) represent the spring constant of each half of the spring after it is cut. Now the weight \( W \) of the object stretches each segment by an amount \( x' \).

Applying \( W = kx \) to this situation gives

\[
W = k'x' + k'x' = 2k'x' \quad (2)
\]

Combining Equations (1) and (2) yields

\[
kx = 2k'x'
\]

From Conceptual Example 2 we know that \( k' = 2k \), so that

\[
kx = 2(2k)x'
\]
Solving for $x'$ gives

$$x' = \frac{x}{4} = \frac{0.160 \text{ m}}{4} = 0.040 \text{ m}$$

11. **SSM Reasoning** When the ball is whirled in a horizontal circle of radius \( r \) at speed \( v \), the centripetal force is provided by the restoring force of the spring. From Equation 5.3, the magnitude of the centripetal force is \( \frac{mv^2}{r} \), while the magnitude of the restoring force is \( kx \) (see Equation 10.2). Thus,

$$\frac{mv^2}{r} = kx \quad (1)$$

The radius of the circle is equal to \( (L_0 + \Delta L) \), where \( L_0 \) is the unstretched length of the spring and \( \Delta L \) is the amount that the spring stretches. Equation (1) becomes

$$\frac{mv^2}{L_0 + \Delta L} = k \Delta L \quad (1')$$

If the spring were attached to the ceiling and the ball were allowed to hang straight down, motionless, the net force must be zero: \( mg - kx = 0 \), where \( -kx \) is the restoring force of the spring. If we let \( \Delta y \) be the displacement of the spring in the vertical direction, then

$$mg = k \Delta y$$

Solving for \( \Delta y \), we obtain

$$\Delta y = \frac{mg}{k} \quad (2)$$

**Solution** According to equation (1') above, the spring constant \( k \) is given by

$$k = \frac{mv^2}{\Delta L (L_0 + \Delta L)}$$

Substituting this expression for \( k \) into equation (2) gives

$$\Delta y = \frac{mg \Delta L (L_0 + \Delta L)}{mv^2} = \frac{g \Delta L (L_0 + \Delta L)}{v^2}$$

or

$$\Delta y = \frac{(9.80 \text{ m/s}^2)(0.010 \text{ m})(0.200 \text{ m} + 0.010 \text{ m})}{(3.00 \text{ m/s})^2} = 2.29 \times 10^{-3} \text{ m}$$
12. **REASONING** The free-body diagram shows the magnitudes and directions of the forces acting on the block. The weight \( mg \) acts downward. The maximum force of static friction \( f_{s}^{\text{max}} \) acts upward just before the block begins to slip. The force from the spring \( F_{x} \text{Applied} = kx \) (Equation 10.1) is directed to the right. The normal force \( F_{N} \) from the wall points to the left. The magnitude of the maximum force of static friction is related to the magnitude of the normal force according to Equation 4.7 \( f_{s}^{\text{max}} = \mu_{s} F_{N} \), where \( \mu_{s} \) is the coefficient of static friction. Since the block is at equilibrium just before it begins to slip, the forces in the \( x \) direction must balance and the forces in the \( y \) direction must balance. The balance of forces in the two directions will provide two equations, from which we will determine the coefficient of static friction.

**SOLUTION** Since the forces in the \( x \) direction and in the \( y \) direction must balance, we have

\[
F_{N} = kx \quad \text{and} \quad mg = \mu_{s} F_{N}
\]

Substituting the first equation into the second equation gives

\[
mg = \mu_{s} F_{N} = \mu_{s} (kx) \quad \text{or} \quad \mu_{s} = \frac{mg}{kx} = \frac{(1.6 \text{ kg})(9.80 \text{ m/s}^{2})}{(510 \text{ N/m})(0.039 \text{ m})} = 0.79
\]

13. **REASONING AND SOLUTION** From the drawing given with the problem statement, we see that the kinetic frictional force on the bottom block (#1) is given by

\[
f_{k1} = \mu_{k}(m_{1} + m_{2})g
\]

and the maximum static frictional force on the top block (#2) is

\[
f_{s2}^{\text{MAX}} = \mu_{s} m_{2}g
\]

Newton’s second law applied to the bottom block gives

\[
F - f_{k1} - kx = 0
\]

Newton’s second law applied to the top block gives

\[
f_{s2}^{\text{MAX}} - kx = 0
\]

a. To find the compression \( x \), we have from Equation (4) that

\[
x = \frac{f_{s2}^{\text{MAX}}}{k} = \frac{0.900}(15.0 \text{ kg})(9.80 \text{ m/s}^{2})/(325 \text{ N/m}) = 0.407 \text{ m}
\]
b. Solving Equation (3) for $F$ and then using Equation (1) to substitute for $f_{k1}$, we find that

$$F = kx + f_{k1} = kx + \mu_k(m_1 + m_2)g$$

$$F = (325 \text{ N/m})(0.407 \text{ m}) + (0.600)(45.0 \text{ kg})(9.80 \text{ m/s}^2) = 397 \text{ N}$$

14. **REASONING AND SOLUTION**  In phase 1 of the block's motion (uniform acceleration) we find that the net force on the block is $F_1 - f_k = ma$ where the force of friction is $f_k = \mu_k mg$. Therefore $F_1 = m(a + \mu_k g)$, which is just the force exerted by the spring on the block, i.e., $F_1 = kx_1$. So we have

$$kx_1 = ma + \mu_k g$$  (1)

We can find the acceleration using

$$a = \frac{v_f - v_0}{t} = \frac{5.00 \text{ m/s} - 0 \text{ m/s}}{0.500 \text{ s}} = 10.0 \text{ m/s}^2$$

In phase 2 of the block's motion (constant speed) $a = 0 \text{ m/s}^2$, so the force exerted by the spring is

$$kx_2 = m\mu_k g$$  (2)

so that

$$\mu_k = \frac{kx_2}{mg}$$

Using this expression for $\mu_k$ in Equation (1) we obtain

$$kx_1 = m \left[ a + \left( \frac{kx_2}{mg} \right) g \right]$$

a. Solving for $k$ gives

$$k = \frac{ma}{x_1 - x_2} = \frac{(15.0 \text{ kg})(10.0 \text{ m/s}^2)}{0.200 \text{ m} - 0.0500 \text{ m}} = 1.00 \times 10^3 \text{ N/m}$$

b. Substituting this value for $k$ into Equation (2), we have

$$\mu_k = \frac{kx_2}{mg} = \frac{(1.00 \times 10^3 \text{ N/m})(0.0500 \text{ m})}{(15.0 \text{ kg})(9.80 \text{ m/s}^2)} = 0.340$$

15. **SSM REASONING**  The frequency $f$ of the eardrum's vibration is related to its angular frequency $\omega$ via $\omega = 2\pi f$ (Equation 10.6). The maximum speed during vibration is given by $v_{\text{max}} = A\omega$ (Equation 10.8). We will find the frequency $f$ in part $a$ from Equations 10.6 and 10.8. In part $b$, we will find the maximum acceleration that the eardrum undergoes with the aid of $a_{\text{max}} = A\omega^2$ (Equation 10.10).
**SOLUTION**

a. Substituting \( \omega = 2\pi f \) (Equation 10.6) into \( v_{\text{max}} = A\omega \) (Equation 10.8) and solving for the frequency \( f \), we obtain

\[
v_{\text{max}} = A\omega = A(2\pi f) \quad \text{or} \quad f = \frac{v_{\text{max}}}{2\pi A} = \frac{2.9 \times 10^{-3} \text{ m/s}}{2\pi (6.3 \times 10^{-7} \text{ m})} = 730 \text{ Hz}
\]

b. Substituting \( \omega = 2\pi f \) (Equation 10.6) into \( a_{\text{max}} = A\omega^2 \) (Equation 10.10) yields the maximum acceleration of the vibrating eardrum:

\[
a_{\text{max}} = A\omega^2 = A(2\pi f)^2 = (6.3 \times 10^{-7} \text{ m})[2\pi (730 \text{ Hz})]^2 = 13 \text{ m/s}^2
\]

**16. REASONING** The period of the engine is the time for it to make one revolution. This time is the total time required to make one thousand revolutions divided by 1000. The frequency \( f \) of the rotational motion is given by Equation 10.5 as the reciprocal of the period. The angular frequency \( \omega \) is the frequency \( f \) times \( 2\pi \) (Equation 10.6).

**SOLUTION**

a. The period of the engine, which is the time to complete one revolution, is

\[
T = \frac{\text{Total time}}{\text{Number of revolutions}} = \frac{50.0 \times 10^{-3} \text{ s}}{1000} = 50.0 \times 10^{-6} \text{ s}
\]

b. The frequency \( f \) of the rotational motion is related to the period \( T \) by

\[
f = \frac{1}{T} = \frac{1}{50.0 \times 10^{-6} \text{ s}} = 2.00 \times 10^4 \text{ Hz} \quad (10.5)
\]

c. The angular frequency \( \omega \) of the rotational motion is related to the frequency \( f \) by

\[
\omega = 2\pi f = 2\pi (2.00 \times 10^4 \text{ Hz}) = 1.26 \times 10^5 \text{ rad/s} \quad (10.6)
\]

**17. REASONING** The force \( F_x \) that the spring exerts on the block just before it is released is equal to \(-kx\), according to Equation 10.2. Here \( k \) is the spring constant and \( x \) is the displacement of the spring from its equilibrium position. Once the block has been released, it oscillates back and forth with an angular frequency given by Equation 10.11 as \( \omega = \sqrt{k/m} \), where \( m \) is the mass of the block. The maximum speed that the block attains during the oscillatory motion is \( v_{\text{max}} = A\omega \) (Equation 10.8). The magnitude of the maximum acceleration that the block attains is \( a_{\text{max}} = A\omega^2 \) (Equation 10.10).
SOLUTION
a. The force $F_x$ exerted on the block by the spring is

$$F_x = -kx = -(82.0 \text{ N/m})(0.120 \text{ m}) = -9.84 \text{ N}$$

(10.2)

b. The angular frequency $\omega$ of the resulting oscillatory motion is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{82.0 \text{ N/m}}{0.750 \text{ kg}}} = 10.5 \text{ rad/s}$$

(10.11)

c. The maximum speed $v_{\text{max}}$ is the product of the amplitude and the angular frequency:

$$v_{\text{max}} = A\omega = (0.120 \text{ m})(10.5 \text{ rad/s}) = 1.26 \text{ m/s}$$

(10.8)

d. The magnitude $a_{\text{max}}$ of the maximum acceleration is

$$a_{\text{max}} = A\omega^2 = (0.120 \text{ m})(10.5 \text{ rad/s})^2 = 13.2 \text{ m/s}^2$$

(10.10)

18. REASONING AND SOLUTION
a. Since the object oscillates between $\pm 0.080 \text{ m}$, the motion’s amplitude of the motion is $0.080 \text{ m}$.

b. From the graph, the period is $T = 4.0 \text{ s}$. Therefore, according to Equation 10.4,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0 \text{ s}} = 1.6 \text{ rad/s}$$

c. Equation 10.11 relates the angular frequency to the spring constant: $\omega = \sqrt{k/m}$. Solving for $k$ we find

$$k = \omega^2m = (1.6 \text{ rad/s})^2(0.80 \text{ kg}) = 2.0 \text{ N/m}$$

d. At $t = 1.0 \text{ s}$, the graph shows that the spring has its maximum displacement. At this location, the object is momentarily at rest, so that its speed is $v = 0 \text{ m/s}$.

e. The acceleration of the object at $t = 1.0 \text{ s}$ is a maximum, and its magnitude is

$$a_{\text{max}} = A\omega^2 = (0.080 \text{ m})(1.6 \text{ rad/s})^2 = 0.20 \text{ m/s}^2$$

19. REASONING AND SOLUTION  From Conceptual Example 2, we know that when the spring is cut in half, the spring constant for each half is twice as large as that of the original spring. In this case, the spring is cut into four shorter springs. Thus, each of the four shorter springs with 25 coils has a spring constant of $4 \times 420 \text{ N/m} = 1680 \text{ N/m}$. 
The angular frequency of simple harmonic motion is given by Equation 10.11:

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1680 \text{ N/m}}{46 \text{ kg}}} = 6.0 \text{ rad/s} \]

20. **REASONING** The spring constant \( k \) of either spring is related to the mass \( m \) of the object attached to it and the angular frequency \( \omega \) of its oscillation by \( \omega = \sqrt{k/m} \) (Equation 10.11).

Squaring both sides of Equation 10.11 and solving for the mass \( m \), we find that \( m = k/\omega^2 \). The masses \( m \) of the two objects are unknown but identical, so we eliminate \( m \) and obtain

\[ m = \frac{k_1}{\omega_1^2} = \frac{k_2}{\omega_2^2} \quad \text{or} \quad k_2 = \frac{k_1 \omega_1^2}{\omega_2^2} \quad (1) \]

To deal with the angular frequencies, we turn to the maximum velocities. The magnitude \( v_{\text{max}} \) of the maximum velocity of either object is given by \( v_{\text{max}} = A\omega \) (Equation 10.8), where \( A \) is the amplitude of the object’s motion. Both objects have a maximum velocity of the same magnitude, so we see that

\[ v_{\text{max}} = A_1\omega_1 = A_2\omega_2 \quad \text{or} \quad \omega_2 = \frac{A_1\omega_1}{A_2} \quad (2) \]

**SOLUTION** The amplitude of the motion of the mass attached to spring 1 is twice that of the motion of the mass attached to spring 2: \( A_1 = 2A_2 \). With this substitution, Equation (2) becomes

\[ \omega_2 = \frac{2A_1\omega_1}{A_2} = 2\omega_1 \quad (3) \]

Substituting Equation (3) into Equation (1) yields

\[ k_2 = \frac{k_1 \omega_1^2}{\omega_2^2} = \frac{k_1 \left(2\omega_1^2\right)^2}{\omega_1^2} = 4k_1 = 4(174 \text{ N/m}) = 696 \text{ N/m} \]

21. **SSM REASONING** The frequency of vibration of the spring is related to the added mass \( m \) by Equations 10.6 and 10.11:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (1) \]

The spring constant can be determined from Equation 10.1.
**SOLUTION**  Since the spring stretches by 0.018 m when a 2.8-kg object is suspended from its end, the spring constant is, according to Equation 10.1,

\[ k = \frac{F_{\text{Applied}}}{x} = \frac{mg}{x} = \frac{(2.8 \text{ kg})(9.80 \text{ m/s}^2)}{0.018 \text{ m}} = 1.52 \times 10^3 \text{ N/m} \]

Solving Equation (1) for \( m \), we find that the mass required to make the spring vibrate at 3.0 Hz is

\[ m = \frac{k}{4\pi^2 f^2} = \frac{1.52 \times 10^3 \text{ N/m}}{4\pi^2 (3.0 \text{ Hz})^2} = 4.3 \text{ kg} \]

---

**REASONING**  The object’s maximum speed \( v_{\text{max}} \) occurs when it passes through the position where the spring is unstrained. The next instant when its maximum acceleration \( a_{\text{max}} \) occurs is when it stops momentarily at either \( x = +A \) or \( x = -A \), where \( A \) is the amplitude of the motion. The time \( t \) that elapses between these two instants is one fourth of the period \( T \), so the time we seek is

\[ t = \frac{1}{4} T \]  \hspace{1cm} (1)

The period is given by \( T = \frac{2\pi}{\omega} \) (Equation 10.4), where \( \omega \) is the angular frequency of the oscillations. To determine the angular frequency, we note that both the maximum speed \( v_{\text{max}} \) and the maximum acceleration \( a_{\text{max}} \) depend upon the angular frequency: \( v_{\text{max}} = A\omega \) (Equation 10.8) and \( a_{\text{max}} = A\omega^2 \) (Equation 10.10). Substituting Equation 10.8 into Equation 10.10, we eliminate the amplitude \( A \) and obtain a direct relationship between the maximum speed \( v_{\text{max}} \), the maximum acceleration \( a_{\text{max}} \), and the angular frequency \( \omega \):

\[ a_{\text{max}} = A\omega^2 = (A\omega)\omega = v_{\text{max}}\omega \quad \text{or} \quad \omega = \frac{a_{\text{max}}}{v_{\text{max}}} \]  \hspace{1cm} (2)

**SOLUTION**  Substituting Equation (2) into \( T = \frac{2\pi}{\omega} \) (Equation 10.4) yields

\[ T = \frac{2\pi}{\left(\frac{a_{\text{max}}}{v_{\text{max}}}\right)} = \frac{2\pi v_{\text{max}}}{a_{\text{max}}} \]  \hspace{1cm} (3)

Then, substituting Equation (3) into Equation (1), we obtain the time \( t \) between maximum speed and maximum acceleration:

\[ t = \frac{1}{4} \left( \frac{2\pi v_{\text{max}}}{a_{\text{max}}} \right) = \frac{\pi v_{\text{max}}}{2a_{\text{max}}} = \frac{\pi (1.25 \text{ m/s})}{2(6.89 \text{ m/s}^2)} = 0.285 \text{ s} \]
23. **REASONING**

a. The angular frequency \( \omega \) (in rad/s) is given by Equation 10.11

\[
\omega = \sqrt{\frac{k}{m}},
\]

where \( k \) is the spring constant and \( m \) is the mass of the object. The frequency \( f \) (in Hz) can be obtained from the angular frequency by using Equation 10.6 \((\omega = 2\pi f)\).

b. The block loses contact with the spring when the amplitude of the oscillation is sufficiently large. To understand why, consider the block at the very top of its oscillation cycle. There it is accelerating downward, with the maximum acceleration \( a_{\text{max}} \) of simple harmonic motion. Contact is maintained with the spring, as long as the magnitude of this acceleration is less than the magnitude \( g \) of the acceleration due to gravity. If \( a_{\text{max}} \) is greater than \( g \), the end of the spring falls away from under the block. \( a_{\text{max}} \) is given by Equation 10.10 \((a_{\text{max}} = A\omega^2)\), from which we can obtain the amplitude \( A \) when \( a_{\text{max}} = g \).

**SOLUTION**

a. From Equations 10.6 and 10.11 we have

\[
\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad \text{or} \quad f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{112 \text{ N/m}}{0.400 \text{ kg}}} = \frac{2.66 \text{ Hz}}{}
\]

b. Using Equation 10.10 with \( a_{\text{max}} = g \) gives

\[
a_{\text{max}} = g = A\omega^2 \quad \text{or} \quad A = \frac{g}{\omega^2}
\]

Substituting Equation 10.11 \((\omega = \sqrt{\frac{k}{m}})\) into this result gives

\[
A = \frac{g}{\omega^2} = \frac{g}{\left(\sqrt{\frac{k}{m}}\right)^2} = \frac{gm}{k} = \frac{(9.80 \text{ m/s}^2)(0.400 \text{ kg})}{112 \text{ N/m}} = 0.0350 \text{ m}
\]

24. **REASONING AND SOLUTION**

The cup slips when the force of static friction is overcome. So \( F = ma = \mu_s mg \), where the acceleration is the maximum value for the simple harmonic motion, i.e., so that

\[
a_{\text{max}} = A\omega^2 = A(2\pi f)^2 \quad \text{(10.10)}
\]

\[
\mu_s = A(2\pi f)^2/g = (0.0500 \text{ m})4\pi^2(2.00 \text{ Hz})^2/(9.80 \text{ m/s}^2) = 0.806
\]
25. **REASONING** We will find the work done by the spring force from \( W_{\text{elastic}} = \frac{1}{2} k x_0^2 - \frac{1}{2} k x_f^2 \) (Equation 10.12), where \( k \) is the spring constant and \( x_0 \) and \( x_f \) are the initial and final compressions of the spring, measured relative to its unstrained length. The distances given in the problem are measured in millimeters, and must be converted to meters before using in Equation 10.12 with \( k = 250 \text{ N/m} \).

**SOLUTION** The spring is initially compressed 5.0 mm, so we have \( x_0 = (5.0 \text{ mm}) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) = 5.0 \times 10^{-3} \text{ m} \).

To extrude the tip of the pen requires an additional compression of 6.0 mm, so the final compression of the spring is \( x_f = x_0 + 6.0 \text{ mm} = 5.0 \text{ mm} + 6.0 \text{ mm} = (11.0 \text{ mm}) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) = 11.0 \times 10^{-3} \text{ m} \).

Therefore, from Equation 10.12, the work done by the spring force is

\[
W_{\text{elastic}} = \frac{1}{2} k x_0^2 - \frac{1}{2} k x_f^2 = \frac{1}{2} k \left( x_0^2 - x_f^2 \right)
\]

\[
= \frac{1}{2} (250 \text{ N/m}) \left[ (5.0 \times 10^{-3} \text{ m})^2 - (11.0 \times 10^{-3} \text{ m})^2 \right] = -0.012 \text{ J}
\]

26. **REASONING** The work done in stretching or compressing a spring is given directly by Equation 10.12 as \( W = \frac{1}{2} k \left( x_0^2 - x_f^2 \right) \), where \( k \) is the spring constant and \( x_0 \) and \( x_f \) are, respectively, the initial and final displacements of the spring from its equilibrium position. The work is positive if the restoring force and the displacement have the same direction and negative if they have opposite directions.

**SOLUTION**

a. The work done in stretching the spring from +1.00 to +3.00 m is \( W = \frac{1}{2} k \left( x_0^2 - x_f^2 \right) = \frac{1}{2} (46.0 \text{ N/m}) \left[ (1.00 \text{ m})^2 - (3.00 \text{ m})^2 \right] = -1.84 \times 10^2 \text{ J} \)

b. The work done in stretching the spring from −3.00 m to +1.00 m is \( W = \frac{1}{2} k \left( x_0^2 - x_f^2 \right) = \frac{1}{2} (46.0 \text{ N/m}) \left[ (-3.00 \text{ m})^2 - (1.00 \text{ m})^2 \right] = +1.84 \times 10^2 \text{ J} \)

c. The work done in stretching the spring from −3.00 to +3.00 m is \( W = \frac{1}{2} k \left( x_0^2 - x_f^2 \right) = \frac{1}{2} (46.0 \text{ N/m}) \left[ (-3.00 \text{ m})^2 - (3.00 \text{ m})^2 \right] = 0 \text{ J} \)
27. **REASONING** As the block falls, only two forces act on it: its weight and the elastic force of the spring. Both of these forces are conservative forces, so the falling block obeys the principle of conservation of mechanical energy. We will use this conservation principle to determine the spring constant of the spring. Once the spring constant is known, Equation 10.11, \( \omega = \sqrt{\frac{k}{m}} \), may be used to find the angular frequency of the block’s vibrations.

**SOLUTION**

a. The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \), or \( E_f = E_0 \) (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

\[
\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + m g h_f + \frac{1}{2} k y_f^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + m g h_0 + \frac{1}{2} k y_0^2
\]

Before going any further, let’s simplify this equation by noting which variables are zero. Since the block starts and ends at rest, \( v_f = v_0 = 0 \) m/s. The block does not rotate, so its angular speed is zero, \( \omega_f = \omega_0 = 0 \) rad/s. Initially, the spring is unstretched, so that \( y_0 = 0 \) m. Setting these terms equal to zero in the equation above gives

\[
m g h_f + \frac{1}{2} k y_f^2 = m g h_0
\]

Solving this equation for the spring constant \( k \), we have that

\[
k = \frac{m g (h_0 - h_f)}{\frac{1}{2} y_f^2} = \frac{(0.450 \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m})}{\frac{1}{2} (0.150 \text{ m})^2} = 58.8 \text{ N/m}
\]

b. The angular frequency \( \omega \) of the block’s vibrations depends on the spring constant \( k \) and the mass \( m \) of the block:

\[
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{58.8 \text{ N/m}}{0.450 \text{ kg}}} = 11.4 \text{ rad/s} \quad (10.11)
\]

28. **REASONING** The elastic potential energy of an ideal spring is given by Equation 10.13

\[ PE_{\text{elastic}} = \frac{1}{2} k y^2 \]

where \( k \) is the spring constant and \( y \) is the amount of stretch or compression from the spring’s unstrained length. Thus, we need to determine \( y \) for an object of mass \( m \) hanging stationary from the spring. Since the object is stationary, it has no acceleration and is at equilibrium. The upward pull of the spring balances the downward-acting weight of the object. We can use this fact to determine \( y \).
**SOLUTION**  The magnitude of the pull of the spring is given by $ky$, according to Equation 10.2. The weight of the object is $mg$. Since these two forces must balance, we have $ky = mg$ or $y = mg/k$. Substituting this result into Equation 10.13 for the elastic potential energy gives

$$PE_{\text{elastic}} = \frac{1}{2} ky^2 = \frac{1}{2} k \left( \frac{mg}{k} \right)^2 = \frac{m^2 g^2}{2k}$$

Applying this expression to the two spring/object systems, we find

$$PE_1 = \frac{m_1^2 g^2}{2k} \quad \text{and} \quad PE_2 = \frac{m_2^2 g^2}{2k}$$

Here, we have omitted the subscript “elastic” for convenience. Dividing the right-hand equation by the left-hand equation gives

$$\frac{PE_2}{PE_1} = \frac{\frac{m_2^2 g^2}{2k}}{\frac{m_1^2 g^2}{2k}} = \frac{m_2^2}{m_1^2} \quad \text{or} \quad PE_2 = PE_1 \left( \frac{m_2^2}{m_1^2} \right) = (1.8 \text{ J}) \left(\frac{5.0 \text{ kg}}{3.2 \text{ kg}}\right)^2 = 4.4 \text{ J}$$

29. **SSM REASONING**  Since air resistance is negligible, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the block and the spring is the same at the instant it comes to a momentary halt on the spring and at the instant the block is dropped. Gravitational potential energy is one part of the total mechanical energy, and Equation 6.5 indicates that it is $mgh$ for a block of mass $m$ located at a height $h$ relative to an arbitrary zero level. This dependence on $h$ will allow us to determine the height at which the block was dropped.

**SOLUTION**  The conservation of mechanical energy states that the final total mechanical energy $E_f$ is equal to the initial total mechanical energy $E_0$. The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

$$\frac{1}{2} mv_i^2 + \frac{1}{2} I \omega_i^2 + mgh_i + \frac{1}{2} ky_i^2 = \frac{1}{2} mv_0^2 + \frac{1}{2} I \omega_0^2 + mgh_0 + \frac{1}{2} ky_0^2$$

The block does not rotate, so the angular speeds $\omega_i$ and $\omega_0$ are zero. Since the block comes to a momentary halt on the spring and is dropped from rest, the translational speeds $v_i$ and $v_0$ are also zero. Because the spring is initially unstrained, the initial displacement $y_0$ of the spring is likewise zero. Thus, the above expression can be simplified as follows:

$$mgh_i + \frac{1}{2} ky_i^2 = mgh_0$$

The block was dropped at a height of $h_0 - h_i$ above the compressed spring. Solving the simplified energy-conservation expression for this quantity gives
30. **REASONING** The forces applied to the ball by the spring and by gravity are conservative forces. Friction is nonconservative, but it is being ignored. The normal force exerted on the ball by the surface on which it slides is also conservative, but it acts perpendicular to the ball’s motion and does no work on the ball. Therefore, mechanical energy is conserved.

**SOLUTION** In applying the principle of conservation of mechanical energy, we must include kinetic energy \( \frac{1}{2}mv^2 \), gravitational potential energy \( mgh \), and elastic potential energy \( \frac{1}{2}kx^2 \). Thus, we have

\[
\frac{1}{2}mv_B^2 + mgh_B + \frac{1}{2}kx_B^2 = \frac{1}{2}mv_A^2 + mgh_A + \frac{1}{2}kx_A^2
\]

(1)

Initially the ball is at rest at point A, so that \( v_A = 0.00 \) m/s. At point B the ball is no longer against the spring, and the spring is unstrained, so that the displacement of the spring from its unstrained length is \( x_B = 0.00 \) m. Substituting these two zero values into Equation (1) gives

\[
\frac{1}{2}mv_B^2 + mgh_B = mgh_A + \frac{1}{2}kx_A^2 \quad \text{or} \quad \frac{1}{2}mv_B^2 = mg(h_A - h_B) + \frac{1}{2}kx_A^2
\]

(2)

In Equation (2), it is given that \( h_A - h_B = -0.300 \) m, since point B is higher than point A. Solving Equation (2) for the final speed \( v_B \) gives

\[
v_B = \sqrt{\frac{2g(h_A - h_B) + \frac{kx_A^2}{m}}{2}} = \sqrt{2(9.80 \text{ m/s}^2)(-0.300 \text{ m}) + \frac{(675 \text{ N/m})(0.0650 \text{ m})^2}{0.0585 \text{ kg}}} = 6.55 \text{ m/s}
\]
2 2 2 2 2 2
1 1 1 1 1 1
2 2 2 2 2 2
f f f f 0 0 0 0

E
0

\( \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + mgh_f + \frac{1}{2} ky_f^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + mgh_0 + \frac{1}{2} ky_0^2 \)

Since the ram does not rotate, the angular speeds \( \omega_f \) and \( \omega_0 \) are zero. Since the ram is initially at rest, the initial translational speed \( v_0 \) is also zero. Thus, the above expression can be simplified as follows:

\( \frac{1}{2} m v_f^2 + mgh_f + \frac{1}{2} ky_f^2 = mgh_0 + \frac{1}{2} ky_0^2 \)

In falling from its initial height of \( h_0 \) to its final height of \( h_f \), the ram falls through a distance of \( h_0 - h_f = 0.022 \text{ m} \), since the spring is compressed 0.030 m from its unstrained length to begin with and is still compressed 0.008 m when the ram makes contact with the staple. Solving the simplified energy-conservation expression for the final speed \( v_f \) gives

\[
\begin{align*}
  v_f &= \sqrt{\frac{k(y_f^2 - y_0^2)}{m}} + 2g(h_0 - h_f) \\
  &= \sqrt{ \frac{\left(32 \, 000 \text{ N/m}\right) \left((0.030 \text{ m})^2 - (0.008 \text{ m})^2\right)}{0.140 \text{ kg}}} + 2(9.80 \text{ m/s}^2)(0.022 \text{ m}) = 14 \text{ m/s}
\end{align*}
\]

32. **REASONING** Since air resistance is negligible, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the pellet/spring system is the same when the pellet comes to a momentary halt at the top of its trajectory as it is when the pellet is resting on the compressed spring. The fact that the total mechanical energy is conserved will allow us to determine the spring constant.

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \). The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

\[
\begin{align*}
  \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + mgh_f + \frac{1}{2} ky_f^2 &= \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + mgh_0 + \frac{1}{2} ky_0^2
\end{align*}
\]

The pellet does not rotate, so the angular speeds \( \omega_f \) and \( \omega_0 \) are zero. Since the pellet is at rest as it sits on the spring and since the pellet comes to a momentary halt at the top of its trajectory, the translational speeds \( v_0 \) and \( v_f \) are also zero. Because the spring is unstrained when the pellet reaches its maximum height, the final displacement \( y_f \) of the spring is likewise zero. Thus, Equation (1) simplifies to:

\[
mgh_f = mgh_0 + \frac{1}{2} ky_0^2
\]

Solving this simplified energy-conservation expression for the spring constant \( k \) and noting that the pellet rises to distance of \( h_f - h_i = 6.10 \text{ m} \) above its position on the compressed spring, we find that
\[
k = \frac{2mg (h_t - h_0)}{y_0^2} = \frac{2(2.10 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)(6.10 \text{ m})}{(9.10 \times 10^{-2} \text{ m})^2} = 303 \text{ N/m}
\]

33. \textbf{REASONING} The only force that acts on the block along the line of motion is the force due to the spring. Since the force due to the spring is a conservative force, the principle of conservation of mechanical energy applies. Initially, when the spring is unstrained, all of the mechanical energy is kinetic energy, \( \frac{1}{2}mv_0^2 \). When the spring is fully compressed, all of the mechanical energy is in the form of elastic potential energy, \( \frac{1}{2}kx_{\text{max}}^2 \), where \( x_{\text{max}} \) is the maximum compression of the spring, is the amplitude \( A \). Therefore, the statement of energy conservation can be written as

\[
\frac{1}{2}mv_0^2 = \frac{1}{2}kA^2
\]

This expression may be solved for the amplitude \( A \).

\textbf{SOLUTION} Solving for the amplitude \( A \), we obtain

\[
A = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(1.00 \times 10^{-2} \text{ kg})(8.00 \text{ m/s})^2}{124 \text{ N/m}}} = 7.18 \times 10^{-2} \text{ m}
\]

34. \textbf{REASONING} As the climber falls, only two forces act on him: his weight and the elastic force of the nylon rope. Both of these forces are conservative forces, so the falling climber obeys the conservation of mechanical energy. We will use this conservation law to determine how much the rope is stretched when it breaks his fall and momentarily brings him to rest.

\textbf{SOLUTION}

The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_i \) or \( E_f = E_i \) (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

\[
\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mg h_i + \frac{1}{2}k y_i^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mg h_0 + \frac{1}{2}k y_0^2
\]

Before going any further, let’s simplify this equation by noting which variables are zero. Since the climber starts and ends at rest, \( v_i = v_0 = 0 \text{ m/s} \). The climber does not rotate, so his angular speed is zero, \( \omega_i = \omega_0 = 0 \text{ rad/s} \). Initially, the spring is unstretched, so that \( y_0 = 0 \text{ m} \). Setting these terms to zero in the equation above gives

\[
mgh_i + \frac{1}{2}ky_i^2 = mgh_0
\]
Rearranging the terms in this equation, we have

\[ \frac{1}{2} k y_f^2 + mg \left( h_f - h_0 \right) = 0 \]

Note that the climber falls a distance of 0.750 m before the rope starts to stretch, and \( y_f \) is the displacement of the stretched rope. Since \( y_f \) points downward, it is considered to be negative. Thus, \( y_f - 0.750 \) m is the total downward displacement of the falling climber, which is also equal to \( h_f - h_0 \). With this substitution, the equation above becomes

\[ \frac{1}{2} k y_f^2 + mgy_f - mg(0.750) = 0 \]

This is a quadratic equation in the variable \( y_f \). Using the quadratic formula gives

\[
y_f = \frac{-mg \pm \sqrt{(mg)^2 - 4\left(\frac{1}{2} k\right) mg(0.750 \text{ m})}} {2\left(\frac{1}{2} k\right)}
\]

\[ y_f = \frac{-(86.0 \text{ kg})(9.80 \text{ m/s}^2)} {1.20 \times 10^3 \text{ N/m}} \]

\[ \pm \frac{\sqrt{[(86.0 \text{ kg})(9.80 \text{ m/s}^2)]^2 - 4\left(\frac{1}{2} k\right)(1.20 \times 10^3 \text{ N/m})[-(86.0 \text{ kg})(9.80 \text{ m/s}^2)(0.750 \text{ m})]}} {1.20 \times 10^3 \text{ N/m}} \]

There are two answers, \( y_f = +0.54 \) m and \(-1.95 \) m. Since the rope is stretched in the downward direction, which we have taken to be the negative direction, the displacement is \(-1.95 \) m. Thus, the amount that the rope is stretched is \( 1.95 \) m.

---

35. **REASONING** Since the surface is frictionless, we can apply the principle of conservation of mechanical energy, which indicates that the final total mechanical energy \( E_f \) of the object and spring is equal to their initial total mechanical energy \( E_0 \); \( E_f = E_0 \). This conservation equation, and the fact that the angular frequency \( \omega \) of the oscillation is related to the spring constant \( k \) and mass \( m \) by \( \omega = \sqrt{k/m} \) (Equation 10.11), will permit us to find the speed of the object at the instant when the spring is stretched by 0.048 m.

**SOLUTION** Equating the total mechanical energy of the system at the instant the spring is stretched by \( x_f = +0.048 \) m to the total mechanical energy when the spring is compressed by \( x_0 = -0.065 \) m, we have

\[ \frac{1}{2} mv_f^2 + \frac{1}{2} I \omega_f^2 + mgh_f + \frac{1}{2} k x_f^2 = \frac{1}{2} mv_0^2 + \frac{1}{2} I \omega_0^2 + mgh_0 + \frac{1}{2} k x_0^2 \]  

(1)
The object does not rotate, so the angular speeds $\omega_f$ and $\omega_0$ are zero. Since the object is initially at rest, $v_0 = 0 \text{ m/s}$. Finally, we note that the height of the object does not change during the motion, so $h_f = h_0$. Thus, Equation (1) simplifies to

$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}kx_0^2$$

Solving this expression for the final speed gives

$$v_f = \sqrt{\frac{k}{m} \sqrt{x_0^2 - x_f^2}}$$

We now recognize that the term $\sqrt{k/m}$ is the angular frequency $\omega$ of the motion (see Equation 10.11). With this substitution, the final speed becomes

$$v_f = \sqrt{\frac{k}{m} x_0^2 - x_f^2} = \omega \sqrt{x_0^2 - x_f^2} = (11.3 \text{ rad/s}) \sqrt{(-0.065 \text{ m})^2 - (0.048 \text{ m})^2} = 0.50 \text{ m/s}$$

36. **REASONING**

a. The acceleration of the box is zero when the net force acting on it is zero, in accord with Newton’s second law of motion. The net force includes the box’s weight (directed downward) and the restoring force of the spring (directed upward). The condition that the net force is zero will allow us to determine the magnitude of the spring’s displacement.

b. The speed of the box is zero when the spring is fully compressed, but the acceleration of the box is not zero at this instant. If the acceleration were zero, the box would be in equilibrium, and the net force on it would be zero. However, the box accelerates upward because the spring is exerting an upward force that is greater than the downward force due to the weight of the box. Thus, we cannot proceed as in part (a), so instead we will use energy conservation to determine the magnitude of the spring’s displacement.

**SOLUTION**

a. The drawing at the right shows the two forces acting on the box: its weight $mg$ and the restoring force $F_y$ exerted by the spring. At the instant the acceleration of the box is zero, it is in equilibrium. According to Equation 4.9b, the net force $\Sigma F_y$ in the $y$ direction must be zero, $\Sigma F_y = 0$.

The restoring force is given by Equation 10.2 as $F_y = -ky$, where $k$ is the spring constant and $y$ is the displacement of the spring (assumed to be in the downward, or negative, direction). Thus, the condition for equilibrium can be written as

$$\frac{-ky + mg}{\Sigma F_y} = 0 \quad \text{or} \quad y = -\frac{mg}{k}$$

Solving for the magnitude of the spring’s displacement gives
b. The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \), or \( E_f = E_0 \) (Equation 6.9a). The expression for the total mechanical energy of an object is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

\[
\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + m g h_f + \frac{1}{2} k y_f^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + m g h_0 + \frac{1}{2} k y_0^2
\]

We can simplify this equation by noting which variables are zero. Since the box comes to a momentary halt, \( v_f = 0 \) m/s. The box does not rotate, so its angular speed is zero, \( \omega_f = \omega_0 = 0 \) rad/s. Initially, the spring is unstretched, so that \( y_0 = 0 \) m. Setting these terms equal to zero in the equation above gives

\[
mgh_f + \frac{1}{2} ky_f^2 = \frac{1}{2} mv_0^2 + mgh_0
\]

The vertical displacement \( h_f - h_0 \) through which the box falls is equal to the displacement \( y_f \) of the spring, so \( y_f = h_f - h_0 \). Note that \( y_f \) is negative, because \( h_f \) is less than \( h_0 \). The downward-moving box compresses the spring in the downward direction, which, as usual, we take to be the negative direction. Substituting this expression for \( y_f \) into the equation above and rearranging terms, we find that

\[
\frac{1}{2} mv_0^2 - mg(h_f - h_0) - \frac{1}{2} k y_f^2 = 0
\]

or

\[
\frac{1}{2} ky_f^2 + mg y_f - \frac{1}{2} mv_0^2 = 0
\]

This is a quadratic equation in the variable \( y_f \). The solution is

\[
y_f = \frac{-mg \pm \sqrt{(mg)^2 - 4 \left( \frac{1}{2} k \right) \left( -\frac{1}{2} mv_0^2 \right)}}{2 \left( \frac{1}{2} k \right)} = \frac{-mg \pm \sqrt{(mg)^2 + \left( \frac{m}{k} \right) v_0^2}}{2 \left( \frac{1}{2} k \right)}
\]

Substituting in the numbers, we find that
The positive answer is discarded because the spring is compressed downward by the falling box, so the displacement of the spring is negative.

Therefore, the magnitude of the spring’s displacement is $7.6 \times 10^{-2}$ m.

37. **SSM REASONING** The angular frequency \( \omega \) (in rad/s) is given by 
\[
\omega = \sqrt{\frac{k}{m}}
\]
(Equation 10.11), where \( k \) is the spring constant and \( m \) is the mass of the object. However, we are given neither \( k \) nor \( m \). Instead, we are given information about how much the spring is compressed and the launch speed of the object. Once launched, the object has kinetic energy, which is related to its speed. Before launching, the spring/object system has elastic potential energy, which is related to the amount by which the spring is compressed. This suggests that we apply the principle of conservation of mechanical energy in order to use the given information. This principle indicates that the total mechanical energy of the system is the same after the object is launched as it is before the launch. The resulting equation will provide us with the value of \( k/m \) that we need in order to determine the angular frequency from \( \omega = \sqrt{\frac{k}{m}} \).

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \). The expression for the total mechanical energy for a spring/mass system is given by Equation 10.14, so that we have
\[
\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0 + \frac{1}{2}kx_0^2
\]
Since the object does not rotate, the angular speeds \( \omega_f \) and \( \omega_0 \) are zero. Since the object is initially at rest, the initial translational speed \( v_0 \) is also zero. Moreover, the motion takes place horizontally, so that the final height \( h_f \) is the same as the initial height \( h_0 \). Lastly, the spring is unstrained after the launch, so that \( x_f \) is zero. Thus, the above expression can be simplified as follows:
\[
\frac{1}{2}mv_f^2 = \frac{1}{2}kx_0^2 \quad \text{or} \quad \frac{k}{m} = \frac{v_f^2}{x_0^2}
\]
Substituting this result into Equation 10.11 shows that
\[
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{v_f^2}{x_0^2}} = \frac{v_f}{x_0} = \frac{1.50 \text{ m/s}}{0.0620 \text{ m}} = 24.2 \text{ rad/s}
\]
38. **REASONING** As the sphere oscillates vertically, it is subject to conservative forces only: gravity and the elastic spring force. Therefore, its total mechanical energy \( E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + \frac{1}{2}ky^2 \) (Equation 10.14) is conserved, and we use the variable \( y \) to denote the vertical stretching of the spring relative to its unstrained length. We will apply the energy conservation principle to find the spring constant \( k \).

**SOLUTION** In terms of the final and initial total mechanical energies \( E_f \) and \( E_0 \), the conservation principle gives us the following starting point:

\[
\frac{\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f + \frac{1}{2}ky_f^2}{E_f} = \frac{\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0 + \frac{1}{2}ky_0^2}{E_0}
\]

We note that the sphere does not rotate, so the angular speeds \( \omega_0 \) and \( \omega_f \) are zero. Equation (1) then becomes

\[
\frac{\frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}ky_f^2}{E_f} = \frac{\frac{1}{2}mv_0^2 + mgh_0 + \frac{1}{2}ky_0^2}{E_0}
\]

Solving Equation (2) for the spring constant \( k \), we obtain

\[
k = \frac{m\left(v_f^2 - v_0^2\right) + 2mg(h_0 - h_f)}{y_f^2 - y_0^2}
\]

As the spring stretches from \( y_0 = 0.12 \text{ m} \) to \( y_f = 0.23 \text{ m} \), the sphere moves downward by \( y_f - y_0 = 0.11 \text{ m} \), so the difference between the sphere’s initial and final heights is \( h_0 - h_f = 0.11 \text{ m} \), which is positive since \( h_0 \) is greater than \( h_f \). Therefore, from Equation (3), the spring constant \( k \) is

\[
k = \frac{(0.60 \text{ kg})\left[(5.70 \text{ m/s})^2 - (4.80 \text{ m/s})^2\right] + 2(0.60 \text{ kg})(9.80 \text{ m/s}^2)(0.11 \text{ m})}{(0.23 \text{ m})^2 - (0.12 \text{ m})^2} = 180 \text{ N/m}
\]

39. **REASONING AND SOLUTION**

a. Now look at conservation of energy before and after the split

**Before**

\[
\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2
\]

Solving for the amplitude \( A \) gives

\[
A = v_{\text{max}} \sqrt{\frac{m}{k}}
\]

**After**

\[
\frac{1}{2}\left(\frac{m}{2}\right)v_f^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_{\text{max}}^2 = \frac{1}{2}kA'^2
\]
Solving for the amplitude \( A' \) gives
\[
A' = v_{\text{max}} \sqrt{\frac{m}{2k}}
\]
Therefore, we find that
\[
A' = \frac{A}{\sqrt{2}} = \frac{5.08 \times 10^{-2} \text{ m}}{\sqrt{2}} = 3.59 \times 10^{-2} \text{ m}
\]
Similarly, for the frequency, we can show that
\[
f' = f \sqrt{2} = (3.00 \text{ Hz}) \sqrt{2} = 4.24 \text{ Hz}
\]

b. If the block splits at one of the extreme positions, the amplitude of the SHM would not change, so it would remain as \( 5.08 \times 10^{-2} \text{ m} \)
The frequency would be
\[
f' = f \sqrt{2} = (3.00 \text{ Hz}) \sqrt{2} = 4.24 \text{ Hz}
\]

40. **REASONING** The amount by which the spring stretches due to the weight of the 1.1-kg object can be calculated using Equation 10.1 (with the variable \( x \) replaced by \( y \)), where the force \( F_y^\text{Applied} \) is equal to the weight of the object. The position of the object when the spring is stretched is the equilibrium position for the vertical harmonic motion of the object-spring system.

**SOLUTION**

a. Solving Equation 10.1 for \( y \) with \( F_y^\text{Applied} \) equal to the weight of the object gives
\[
y = \frac{F_y^\text{Applied}}{k} = \frac{mg}{k} = \frac{(1.1 \text{ kg})(9.80 \text{ m/s}^2)}{120 \text{ N/m}} = 9.0 \times 10^{-2} \text{ m}
\]
b. The object is then pulled down another 0.20 m and released from rest (\( v_0 = 0 \text{ m/s} \)). At this point the spring is stretched by an amount of 0.090 m + 0.20 m = 0.29 m. We will let this point be the zero reference level (\( h = 0 \text{ m} \)) for the gravitational potential energy.

The kinetic energy, the gravitational potential energy, and the elastic potential energy at the point of release are:
\[
\text{KE} = \frac{1}{2} m v_0^2 = \frac{1}{2} m (0 \text{ m/s})^2 = 0 \text{ J}
\]
\[
\text{PE}_{\text{gravity}} = mgh = mg(0 \text{ m}) = 0 \text{ J}
\]
\[
\text{PE}_{\text{elastic}} = \frac{1}{2} k y_0^2 = \frac{1}{2} (120 \text{ N/m})(0.29 \text{ m})^2 = 5.0 \text{ J}
\]
The initial total mechanical energy \( E_0 \) is the sum of these three energies, so \( E_0 = 5.0 \text{ J} \). When the object has risen a distance of \( h = 0.20 \text{ m} \) above the release point, the spring is stretched by an amount \( 0.29 \text{ m} - 0.20 \text{ m} = 0.090 \text{ m} \). Since the total mechanical energy is conserved, its value at this point is still 5.0 J. Thus,

\[
E = KE + PE_{\text{gravity}} + PE_{\text{elastic}}
\]

\[
E = \frac{1}{2} mv^2 + mgh + \frac{1}{2} ky^2
\]

\[
5.0 \text{ J} = \frac{1}{2} (1.1 \text{ kg})v^2 + (1.1 \text{ kg})(9.80 \text{ m/s}^2)(0.20 \text{ m}) + \frac{1}{2} (120 \text{ N/m})(0.090 \text{ m})^2
\]

Solving for \( v \) yields \( v = 2.1 \text{ m/s} \).

41. **SSM REASONING** Using the principle of conservation of mechanical energy, the initial elastic potential energy stored in the elastic bands must be equal to the sum of the kinetic energy and the gravitational potential energy of the performer at the point of ejection:

\[
\frac{1}{2} k x^2 = \frac{1}{2} mv_0^2 + mg h
\]

where \( v_0 \) is the speed of the performer at the point of ejection and, from the figure at the right, \( h = x \sin \theta \).

Thus,

\[
\frac{1}{2} k x^2 = \frac{1}{2} m v_0^2 + mg x \sin \theta \tag{1}
\]

From the horizontal motion of the performer

\[
v_{0x} = v_0 \cos \theta \tag{2}
\]

where

\[
v_{0x} = \frac{s_x}{t} \tag{3}
\]

and \( s_x = 26.8 \text{ m} \). Combining equations (2) and (3) gives

\[
v_0 = \frac{s_x}{t \cos \theta}
\]

Equation (1) becomes:

\[
\frac{1}{2} k x^2 = \frac{1}{2} m \frac{s_x^2}{t^2 \cos^2 \theta} + mg x \sin \theta
\]
This expression can be solved for \( k \), the spring constant of the firing mechanism.

**SOLUTION** Solving for \( k \) yields:

\[
k = m \left( \frac{s_x}{x t \cos\theta} \right)^2 + \frac{2mg(\sin\theta)}{x}
\]

\[
k = (70.0 \text{ kg}) \left[ \frac{26.8 \text{ m}}{(3.00 \text{ m})(2.14 \text{ s})(\cos 40.0^\circ)} \right]^2
\]

\[
+ \frac{2(70.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 40.0^\circ)}{3.00 \text{ m}} = 2.37 \times 10^3 \text{ N/m}
\]

42. **REASONING AND SOLUTION** The bullet (mass \( m \)) moves with speed \( v \), strikes the block (mass \( M \)) in an inelastic collision and the two move together with a final speed \( V \). We first need to employ the conservation of linear momentum to the collision to obtain an expression for the final speed:

\[
mv = (m + M)V \quad \text{or} \quad V = \frac{mv}{m + M}
\]

The block/bullet system now compresses the spring by an amount \( x \). During the compression the total mechanical energy is conserved so that

\[
\frac{1}{2}(m + M)V^2 = \frac{1}{2}kx^2
\]

Substituting the expression for \( V \) into this equation, we obtain

\[
\frac{1}{2}(m + M)\left( \frac{mv}{m + M} \right)^2 = \frac{1}{2}kx^2
\]

Solving this expression for \( v \) gives

\[
v = \sqrt{\frac{k x^2 (m + M)}{m^2}} = \sqrt{\frac{(845 \text{ N/m})(0.200 \text{ m})^2 (2.51 \text{ kg})}{(0.0100 \text{ kg})^2}} = 921 \text{ m/s}
\]

43. **REASONING** As the ball swings down, it reaches its greatest speed at the lowest point in the motion. One complete cycle of the pendulum has four parts: the downward motion in which the ball attains its greatest speed at the lowest point, the subsequent upward motion in which the ball slows down and then momentarily comes to rest. The ball then retraces its motion, finally ending up where it originally began. The time it takes to reach the lowest point is one-quarter of the period of the pendulum, or \( t = (1/4)T \). The period is related to the
angular frequency $\omega$ of the pendulum by Equation 10.4, $T = 2\pi \omega$. Thus, the time for the ball to reach its lowest point is

$$t = \frac{1}{4} T = \frac{1}{4} \left(\frac{2\pi}{\omega}\right)$$

The angular frequency $\omega$ of the pendulum depends on its length $L$ and the acceleration $g$ due to gravity through the relation $\omega = \sqrt{g/L}$ (Equation 10.16). Thus, the time is

$$t = \frac{1}{4} \left(\frac{2\pi}{\omega}\right) = \frac{1}{4} \left(\frac{2\pi}{\sqrt{g/L}}\right) = \frac{\pi}{2} \sqrt{\frac{L}{g}}$$

**SOLUTION** After the ball is released, the time that has elapsed before it attains its greatest speed is

$$t = \frac{\pi}{2} \sqrt{\frac{L}{g}} = \frac{\pi}{2} \sqrt{\frac{0.65 \text{ m}}{9.80 \text{ m/s}^2}} = 0.40 \text{ s}$$

44. **REASONING** The magnitude $g$ of the acceleration due to gravity is related to the length $L$ and frequency $f$ of the simple pendulum by $2\pi f = \sqrt{g/L}$ (Equation 10.16). Squaring both sides of Equation 10.16 and solving for $g$, we obtain

$$4\pi^2 f^2 = \frac{g}{L} \quad \text{or} \quad g = 4\pi^2 f^2 L \quad (1)$$

The frequency $f$ is the number of complete vibrations made per second. In measuring the frequency of the simple pendulum, the astronauts recorded $N$ complete vibrations occurring over a total elapsed time $t$. The pendulum’s oscillation frequency is, then, given by

$$f = \frac{N}{t} \quad (2)$$

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

$$g = 4\pi^2 \left(\frac{N}{t}\right)^2 L = \frac{4\pi^2 N^2 L}{t^2} = \frac{4\pi^2 (100)^2 (1.2 \text{ m})}{(280 \text{ s})^2} = 6.0 \text{ m/s}^2$$

45. **REASONING**

a. The angular frequency $\omega$ of a simple pendulum can be found directly from Equation 10.16 as $\omega = \sqrt{g/L}$, where $g$ is the magnitude of the acceleration due to gravity and $L$ is the length of the pendulum.
b. The total mechanical energy of the pendulum as it swings back and forth is the gravitational potential energy it has just before it is released, since the pendulum is released from rest and has no initial kinetic energy. The reason is that friction is being neglected, and the tension in the cable is always perpendicular to the motion of the bob, so the tension does no work. Thus, the work done by nonconservative forces, such as friction and tension, is zero. This means that the total mechanical energy is conserved (see Equation 6.9b) and is the same at all points along the motion, including the initial point where the bob is released.

c. To find the speed of the bob as it passes through the lowest point of the swing, we will use the conservation of energy, which relates the total mechanical energy at the lowest point to that at the highest point.

**SOLUTION**

a. The angular frequency of the pendulum is

\[ \omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.80 \text{ m/s}^2}{0.79 \text{ m}}} = 3.5 \text{ rad/s} \]  

(10.16)

b. At the moment the pendulum is released, the only type of energy it has is its gravitational potential energy. Thus, its potential energy \( PE \) is equal to its initial total mechanical energy \( E_0 \), so \( PE = E_0 \). According to Equation 6.5, the potential energy of the pendulum is \( PE = mgh \), where \( m \) is the mass of the bob and \( h \) is its height above its equilibrium position (i.e., its position when the pendulum hangs straight down). The drawing shows that this height is related to the length \( L \) of the pendulum by \( h = L(1 - \cos 8.50^\circ) \). Thus, the total mechanical energy of the pendulum is

\[ E_0 = mgh = mgL(1 - \cos 8.50^\circ) \]

\[ = (0.24 \text{ kg})(9.80 \text{ m/s}^2)(0.79 \text{ m})(1 - \cos 8.50^\circ) = 2.0 \times 10^{-2} \text{ J} \]

c. As the bob passes through the lowest point of the swing, it has only kinetic energy, so its total mechanical energy is \( E_f = \frac{1}{2}mv_f^2 \). Since the total mechanical energy is conserved (\( E_f = E_0 \)), we have that

\[ \frac{1}{2}mv_f^2 = E_0 \]

Solving for the final speed gives

\[ v_f = \sqrt{\frac{2E_0}{m}} = \sqrt{\frac{2(2.0 \times 10^{-2} \text{ J})}{0.24 \text{ kg}}} = 0.41 \text{ m/s} \]
46. **REASONING AND SOLUTION** Applying Equation 10.16 and recalling that frequency and period are related by \( f = \frac{1}{T} \),

\[
2\pi f = \frac{2\pi}{T} = \frac{g}{L}
\]

where \( L \) is the length of the pendulum. Thus,

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

Solving for \( L \) gives

\[
L = g \left( \frac{T}{2\pi} \right)^2 = \left( 9.80 \text{ m/s}^2 \right) \left( \frac{9.2 \text{s}}{2\pi} \right)^2 = 21 \text{ m}
\]

47. **REASONING** According to Equation 10.15, the angular frequency \( \omega \) of a physical pendulum is \( \omega = \sqrt{\frac{mgL}{I}} \) and depends on the ratio of the mass \( m \) to the moment of inertia \( I \). Since the moment of inertia is directly proportional to the mass (see Equation 9.6), the mass algebraically cancels. Thus, the angular frequency is independent of the mass of the physical pendulum. According to Equation 10.4, the period is \( T = \frac{2\pi}{\omega} \). Since the angular frequency \( \omega \) is independent of the mass, so is the period. These two expressions will allow us to determine the periods of the wood and metal pendulums.

**SOLUTION**

a. The period \( T \) of a pendulum is given by Equation 10.4 as \( T = \frac{2\pi}{\omega} \), where \( \omega \) is its angular frequency. The angular frequency of a physical pendulum is given by Equation 10.15 as \( \omega = \sqrt{\frac{mgL}{I}} \), where \( m \) is its mass, \( L \) is the distance from the pivot to the center of mass, and \( I \) is the moment of inertia about the pivot. Combining these two relations yields

\[
T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{mgL}{I}}} = 2\pi \sqrt{\frac{I}{mgL}}
\]

The moment of inertia of a meter stick (a thin rod) that is oscillating about an axis at one end is given in Table 9.1 as \( I = \frac{1}{3} mL^2 \), where \( L_0 \) is the length of the stick. Since the meter stick is uniform, the distance \( L \) from one end to its center of mass is \( L = \frac{1}{2} L_0 \). Therefore, the period of oscillation of the wood pendulum is

\[
T = 2\pi \sqrt{\frac{I}{mgL}} = 2\pi \sqrt{\frac{\frac{1}{3} mL^2}{mg \left( \frac{1}{2} L_0 \right)}} = 2\pi \sqrt{\frac{2L_0}{3g}}
\]

\[
= 2\pi \sqrt{\frac{2(1.00 \text{ m})}{3(9.80 \text{ m/s}^2)}} = 1.64 \text{ s}
\]
b. The period is the same for the metal pendulum, since the mass has been eliminated algebraically in the expression for $T$.

48. **REASONING** For small-angle displacements, the frequency of simple harmonic motion for a physical pendulum is determined by $2\pi f = \sqrt{\frac{mgL}{I}}$, where $L$ is the distance between the axis of rotation and the center of gravity of the rigid body of moment of inertia $I$. Since the frequency $f$ and the period $T$ are related by $f = 1/T$, the period of pendulum A is given by

$$T_A = 2\pi \sqrt{\frac{I}{mgL}}$$

Since the pendulum is made from a thin, rigid, uniform rod, its moment of inertia is given by $I = \left(\frac{1}{3}\right)md^2$, where $d$ is the length of the rod. Since the rod is uniform, its center of gravity lies at its geometric center, and $L = d/2$. Therefore, the period of pendulum A is given by

$$T_A = 2\pi \sqrt{\frac{2d}{3g}}$$

For the simple pendulum we have

$$T_B = 2\pi \sqrt{\frac{d}{g}}$$

**SOLUTION** The ratio of the periods is, therefore,

$$\frac{T_A}{T_B} = \frac{2\pi \sqrt{2d/(3g)}}{2\pi \sqrt{d/g}} = \frac{\sqrt{2}}{\sqrt{3}} = 0.816$$

49. **REASONING** The relation between the period $T$ and angular frequency $\omega$ is $T = \frac{2\pi}{\omega}$ (Equation 10.6). The angular frequency of a physical pendulum is given by $\omega = \sqrt{\frac{mgL}{I}}$ (Equation 10.15), where $m$ is the mass of the pendulum, $g$ is the acceleration due to gravity, $L$ is the distance between the axis of rotation at the pivot point and the center of gravity of the rod, and $I$ is the moment of inertia of the rod. According to Table 9.1, the moment of inertia of a thin uniform rod of length $D$ is $I = \frac{1}{12}mD^2$. Combining these three equations algebraically will give us an expression for the period that we seek. However, the length $D$ of the rod is not given. Instead, the period of the simple pendulum is given. We will be able to use this information to eliminate the need for the missing length data.

**SOLUTION** Substituting Equation 10.15 for $\omega$ into Equation 10.6, shows that the period of the physical pendulum is
\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{mgL}{I}}} = 2\pi \sqrt{\frac{I}{mgL}} \]

Now we can use the expression \( I = \frac{1}{3} mD^2 \) for the moment of inertia of the rod. In addition, we recognize that the center of gravity of the uniform rod lies at the center of the rod, so that \( L = \frac{1}{2} D \). With these two substitutions the expression for the period becomes

\[ T = 2\pi \sqrt{\frac{I}{mgL}} = 2\pi \sqrt{\frac{\frac{1}{3} mD^2}{mg \left( \frac{1}{2} D \right)}} = 2\pi \sqrt{\frac{2D}{3g}} \]

(1)

At this point, we must deal with the unknown length \( D \) of the rod. To this end, we note that the period of the simple pendulum is given by Equations 10.6 and 10.16 as

\[ \omega_{\text{Simple}} = \frac{2\pi}{T_{\text{Simple}}} = \sqrt{\frac{g}{D}} \quad \text{or} \quad T_{\text{Simple}} = 2\pi \sqrt{\frac{D}{g}} \]

Solving this expression for \( D/g \) and substituting the result into Equation (1) gives

\[ T = \left( 2 \sqrt{\frac{2}{3}} \right) T_{\text{Simple}} = \left( 2 \sqrt{\frac{2}{3}} \right) (0.66 \, \text{s}) = [0.54 \, \text{s}] \]

50. **REASONING** As discussed in Section 10.4, a restoring torque \( \tau \) is the cause of the back-and-forth oscillatory motion. This restoring torque is given by \( \tau \approx -k'\theta \). This expression is analogous to Equation 10.2 for the Hooke’s law restoring force. We will derive this expression in the present context, so that we can identify the constant \( k' \), which is analogous to the spring constant \( k \) in Equation 10.2. Then, we will proceed by analogy with the simple harmonic motion that occurs for an object of mass \( m \) on a spring, in which case the angular frequency is given by \( \omega = \sqrt{k/m} \) (Equation 10.11).

**SOLUTION** As the object oscillates back and forth at the bottom of the bowl, it is subject to a restoring torque, which arises because of the object’s weight. According to Equation 9.1, the magnitude of the torque \( \tau \) is the magnitude \( mg \) of the weight times the lever arm \( \ell \), so that \( \tau = -mg\ell \). The minus sign is included because the torque is a restoring torque; it acts to reduce the angle \( \theta \) (see the drawing that accompanies the problem statement). The lever arm in the text drawing is \( \ell = R\sin \theta \). The restoring torque on the object, then, is given by \( \tau = -mgR\sin \theta \). When the angle \( \theta \) is small enough so that the object oscillates in simple harmonic motion, we can replace \( \sin \theta \) with the angle \( \theta \) itself, provided that the angle \( \theta \) is expressed in radians. Thus, we have for small angles that

\[ \tau \approx -mgR \frac{\theta}{k'} \]
The quantity $mgR$ has a constant value $k'$ and is analogous to the spring constant $k$. As usual in rotational motion, the moment of inertia $I$ appears in place of the mass $m$ in equations. Thus, by analogy with $\omega = \sqrt{k/m}$ (Equation 10.11), we find that

$$\omega = \sqrt{\frac{k'}{I}} = \sqrt{\frac{mgR}{I}} \quad (1)$$

The moment of inertia of the small object (assuming that all of its mass is located at the same distance $R$ from the axis of rotation) is $I = mR^2$ (Equation 9.6). Substituting this expression for $I$ into Equation (1) gives

$$\omega = \sqrt{\frac{mgR}{mR^2}} \quad \text{or} \quad \omega = \sqrt{\frac{g}{R}}$$

51. **SSM REASONING** When the tow truck pulls the car out of the ditch, the cable stretches and a tension exists in it. This tension is the force that acts on the car. The amount $\Delta L$ that the cable stretches depends on the tension $F$, the length $L_0$ and cross-sectional area $A$ of the cable, as well as Young’s modulus $Y$ for steel. All of these quantities are given in the statement of the problem, except for Young’s modulus, which can be found by consulting Table 10.1.

**SOLUTION** Solving Equation 10.17, $F = Y\left(\frac{\Delta L}{L_0}\right)A$, for the change in length, we have

$$\Delta L = \frac{FL_0}{AY} = \frac{(890 \text{ N})(9.1 \text{ m})}{\pi \left(0.50 \times 10^{-2} \text{ m}\right)^2 \left(2.0 \times 10^{11} \text{ N/m}^2\right)} = 5.2 \times 10^{-4} \text{ m}$$

52. **REASONING** The stress in either cable is the ratio $F/A$ of the magnitude $F$ of the stretching force to the cross-sectional area $A$ of the cable. The cables have circular cross-sections, so the area of each cable is given by $A = \pi r^2$. Therefore, the stress in either cable is given by

$$\text{Stress} = \frac{F}{\pi r^2} \quad (1)$$

We will use Equation (1) to solve for the stretching force $F_2$ acting on the second cable.

**SOLUTION** Setting the stresses in the cables equal via Equation (1), and solving for the stretching force in the second cable gives
\[
\frac{F_2}{r_2^2} = \frac{F_1}{r_1^2} \quad \text{or} \quad F_2 = \frac{F_1 r_2^2}{r_1^2} = \frac{(270 \text{ N})(5.1 \times 10^{-3} \text{ m})^2}{(3.5 \times 10^{-3} \text{ m})^2} = 570 \text{ N}
\]

53. \textbf{SSM REASONING AND SOLUTION} According to Equation 10.20, it follows that

\[
\Delta P = -\frac{B}{V_0} \Delta V = -\left(2.6 \times 10^{10} \text{ N/m}^2\right) \frac{-1 \times 10^{-10} \text{ m}^3}{1.0 \times 10^{-6} \text{ m}^3} = 2.6 \times 10^6 \text{ N/m}^2 \quad (10.20)
\]

where we have expressed the volume \( V_0 \) of the cube at the ocean’s surface as \( V_0 = \left(1.0 \times 10^{-2} \text{ m}\right)^3 = 1.0 \times 10^{-6} \text{ m}^3 \).

Since the pressure increases by \( 1.0 \times 10^4 \text{ N/m}^2 \) per meter of depth, the depth is

\[
\frac{2.6 \times 10^6 \text{ N/m}^2}{1.0 \times 10^4 \text{ N/m}^2} = 260 \text{ m}
\]

54. \textbf{REASONING} The amount \( \Delta L \) by which the bone changes length when a compression force or a tension force acts on it is specified by \( \Delta L = \frac{FL_0}{YA} \) (Equation 10.17), where \( F \) denotes the magnitude of either type of force, \( L_0 \) is the initial length of the bone, \( Y \) is the appropriate Young’s modulus, and \( A \) is the cross-sectional area of the bone. The values of Young’s modulus are given in Table 10.1 (\( Y_{\text{Compression}} = 9.4 \times 10^9 \text{ N/m}^2 \) and \( Y_{\text{Tension}} = 1.6 \times 10^{10} \text{ N/m}^2 \)). The values for \( F, L_0, \) and \( A \) are not given, but it is important to recognize that these variables have the \textit{same} values for both types of forces. We will apply Equation 10.17 twice, once for the compression force and once for the tension force. Since \( F, L_0, \) and \( A \) have the same values in both of the resulting equations, we will be able to eliminate them algebraically and determine the amount \( \Delta L_{\text{Tension}} \) by which the bone stretches.

\textbf{SOLUTION} Applying Equation 10.17 for both types of forces gives

\[
\Delta L_{\text{Tension}} = \frac{FL_0}{Y_{\text{Tension}}A} \quad \text{and} \quad \Delta L_{\text{Compression}} = \frac{FL_0}{Y_{\text{Compression}}A}
\]

Dividing the left-hand equation by the right-hand equation and eliminating the common variables algebraically shows that
55. **REASONING** The change $\Delta L$ in the length of the rope is given by $\Delta L = FL_0/(YA)$ (see Equation 10.17), where $F$ is the magnitude of the stretching force, $L_0$ is the unstretched length of the rope, $A$ is its cross-sectional area, and $Y$ is Young’s modulus for nylon (see Table 10.1). All the variables except $F$ are known. According to Newton’s third law, the action-reaction law, the force exerted on the rope by the skier is equal in magnitude to the force exerted on the skier by the rope. The force exerted on the skier by the rope can be obtained from Newton’s second law, since the mass and acceleration of the skier are known.

**SOLUTION** The change $\Delta L$ in the length of the rope is

$$\Delta L = \frac{FL_0}{YA} \quad (10.17)$$

Two horizontal forces act on the skier: (1) the towing force (magnitude $= F$) and (2) the resistive force (magnitude $= f$) due to the water. The net force acting on the skier has a magnitude of $F - f$. According to Newton’s second law (see Section 4.3), this net force is equal to the skier’s mass $m$ times the magnitude $a$ of her acceleration, or

$$F - f = ma$$

Solving this equation for $F$ and substituting the result into Equation 10.17, we find that

$$\Delta L = \frac{FL_0}{YA} = \frac{(f + ma)L_0}{YA} = \left[130 \text{ N} + (59 \text{ kg})(0.85 \text{ m/s}^2)\right](12 \text{ m}) = \frac{(3.7 \times 10^9 \text{ N/m}^2)(2.0 \times 10^{-5} \text{ m}^2)}{2.9 \times 10^{-2} \text{ m}} = 2.9 \times 10^{-2} \text{ m}$$

We have taken the value of $Y = 3.7 \times 10^9 \text{ N/m}^2$ for nylon from Table 10.1.

56. **REASONING** When the object is placed on top of the pipe, the pipe is compressed due to the weight of the object. The amount $\Delta L$ of compression depends on the weight $F$, the length $L_0$ and cross-sectional area $A$ of the pipe, as well as Young’s modulus $Y$ for steel. All of these quantities, except for the weight, are given in the statement of the problem. Young’s modulus can be found by consulting Table 10.1.
SOLUTION Using Equation 10.17, we find that the weight $F$ of the object is

$$F = Y \left( \frac{\Delta L}{L_0} \right) A = \left( 2.0 \times 10^{11} \text{ N/m}^2 \right) \left( \frac{5.7 \times 10^{-7} \text{ m}}{3.6 \text{ m}} \right) \pi \left( 65 \times 10^{-2} \text{ m} \right)^2 = 4.2 \times 10^4 \text{ N}$$

Note that in this calculation we have used the fact that the circular cross section of the pipe is a circle of area $A = \pi r^2$.

57. REASONING The shear stress is equal to the magnitude of the shearing force exerted on the bar divided by the cross sectional area of the bar. The vertical deflection $\Delta Y$ of the right end of the bar is given by Equation 10.18 [$F = S(\Delta Y / L_0)A$].

SOLUTION
a. The stress is

$$\frac{F}{A} = \frac{mg}{A} = \frac{(160 \text{ kg})(9.80 \text{ m/s}^2)}{3.2 \times 10^{-4} \text{ m}^2} = 4.9 \times 10^6 \text{ N/m}^2$$

b. Taking the value for the shear modulus $S$ of steel from Table 10.2, we find that the vertical deflection $\Delta Y$ of the right end of the bar is

$$\Delta Y = \left( \frac{F}{SA} \right) \frac{L_0}{S} = \left( 4.9 \times 10^6 \text{ N/m}^2 \right) \frac{0.10 \text{ m}}{8.1 \times 10^{10} \text{ N/m}^2} = 6.0 \times 10^{-6} \text{ m}$$

58. REASONING AND SOLUTION $F = S(\Delta Y / L_0)A$ for the shearing force. The shear modulus $S$ for copper is given in Table 10.2. From the figure we also see that $\tan \theta = (\Delta X / L_0)$ so that

$$\theta = \tan^{-1} \left( \frac{F}{SA} \right) = \tan^{-1} \left[ \frac{6.0 \times 10^6 \text{ N}}{4.2 \times 10^{10} \text{ N/m}^2 \left( 0.090 \text{ m}^2 \right)} \right] = 0.091^\circ$$

59. SSM REASONING AND SOLUTION The shearing stress is equal to the force per unit area applied to the rivet. Thus, when a shearing stress of $5.0 \times 10^8 \text{ Pa}$ is applied to each rivet, the force experienced by each rivet is

$$F = (\text{Stress})A = (\text{Stress})(\pi r^2) = (5.0 \times 10^8 \text{ Pa}) \left[ \pi \left( 5.0 \times 10^{-3} \text{ m} \right)^2 \right] = 3.9 \times 10^4 \text{ N}$$

Therefore, the maximum tension $T$ that can be applied to each beam, assuming that each rivet carries one-fourth of the total load, is $4F = 1.6 \times 10^5 \text{ N}$.
60. **REASONING** Both cylinders experience the same force $F$. The magnitude of this force is related to the change in length of each cylinder according to Equation 10.17: $F = Y(\Delta L / L_0)A$. See Table 10.1 for values of Young’s modulus $Y$. Each cylinder decreases in length; the total decrease being the sum of the decreases for each cylinder.

**SOLUTION** The length of the copper cylinder decreases by

$$\Delta L_{\text{copper}} = \frac{FL_0}{YA} = \frac{(6500 \text{ N})(3.0 \times 10^{-2} \text{ m})}{(1.1 \times 10^{11} \text{ N/m}^2)\pi(0.25 \times 10^{-2} \text{ m})^2} = 9.0 \times 10^{-5} \text{ m}$$

Similarly, the length of the brass decreases by

$$\Delta L_{\text{brass}} = \frac{(6500 \text{ N})(5.0 \times 10^{-2} \text{ m})}{(9.0 \times 10^{10} \text{ N/m}^2)\pi(0.25 \times 10^{-2} \text{ m})^2} = 1.8 \times 10^{-4} \text{ m}$$

Therefore, the amount by which the length of the stack decreases is $2.7 \times 10^{-4} \text{ m}$.

61. **REASONING AND SOLUTION** Equation 10.20 gives the desired result. Solving for $\Delta V/V_0$ and taking the value for the bulk modulus $B$ of aluminum from Table 10.3, we obtain

$$\frac{\Delta V}{V_0} = -\frac{\Delta P}{B} = -\frac{1.01 \times 10^5 \text{ Pa}}{7.1 \times 10^{10} \text{ N/m}^2} = 1.4 \times 10^{-6}$$

62. **REASONING** It takes force to stretch the wire. This force arises because the tuning peg at one end of the wire pulls on the fixed support at the other end. In accord with Newton’s action-reaction law, the fixed support pulls back. As a result of the oppositely-directed pulling forces at either end of the wire, the wire experiences an increased tension. For each turn, the change in length of the wire is equal to the circumference of the tuning peg, $\Delta L = 2\pi r_p$, where $r_p$ is the radius of the tuning peg. This change in length is related to the tension by virtue of Young’s modulus $Y$ for steel, which has a value of $Y = 2.0 \times 10^{11} \text{ N/m}^2$.

**SOLUTION** The tension is the force $F$ that is required to stretch the wire (unstrained length $= L_0$, cross-sectional area $= A$) by an amount $\Delta L$ and is determined by Young’s modulus according to Equation 10.17:

$$F = Y\left(\frac{\Delta L}{L_0}\right)A$$

Assuming that the wire has a circular cross-section, $A = \pi r_w^2$, where $r_w$ is the radius of the wire. When the tuning peg is turned through two revolutions, the length of the wire will increase by an amount equal to twice the circumference of the peg. Thus,
\( \Delta L = 2(2\pi r_p) \), where \( r_p \) is the radius of the tuning peg. With these substitutions, Equation 10.17 becomes:

\[
F = Y \left( \frac{4\pi r_p}{L_0} \right) r^2 = \frac{4Y r_p}{L_0} \left( \pi r_w \right)^2
\]

\[
F = \frac{4(2.0 \times 10^{11} \text{ N/m}^2)(1.8 \times 10^{-3} \text{ m})}{0.76 \text{ m}} \left[ \pi (0.80 \times 10^{-3} \text{ m}) \right]^2 = 1.2 \times 10^4 \text{ N}
\]

63. **REASONING AND SOLUTION** From the drawing we have \( \Delta x = 3.0 \times 10^{-3} \) m and

\[
A = 2\pi r \Delta x = 2\pi (1.00 \times 10^{-2} \text{ m})(3.0 \times 10^{-3} \text{ m})
\]

We now have Stress = \( F/A \). Therefore,

\[
F = (\text{Stress})A = (3.5 \times 10^8 \text{ Pa})[2\pi (1.00 \times 10^{-2} \text{ m})(3.0 \times 10^{-3} \text{ m})] = 6.6 \times 10^4 \text{ N}
\]

64. **REASONING** The person is to be suspended by \( N \) pieces of mohair, which collectively support the person against the downward force \( W = mg \) of gravity, where \( m = 75 \text{ kg} \) is the mass of the person and \( g \) is the magnitude of the acceleration due to gravity. Therefore, the tension \( T \) in each piece must be \( T = mg/N \), and the total number of pieces is

\[
N = \frac{mg}{T}
\]

The tension \( T \) in one piece of mohair is given by

\[
T = Y \left( \frac{\Delta L}{L_0} \right) A = Y \left( \frac{\Delta L}{L_0} \right) \pi r^2
\]  \hspace{1cm} (10.17)

where \( Y \) is Young’s modulus, \( \Delta L/L_0 \) is the strain, and \( A = \pi r^2 \) is the cross-sectional area of a single piece of mohair with a radius \( r \).

**SOLUTION** We take the value of Young’s modulus \( Y = 2.9 \times 10^9 \text{ N/m}^2 \) for mohair from Table 10.1 in the text. Substituting Equation 10.17 into Equation (1), we obtain

\[
N = \frac{mg}{Y \left( \frac{\Delta L}{L_0} \right) \pi r^2} = \frac{(75 \text{ kg})(9.80 \text{ m/s}^2)}{(2.9 \times 10^9 \text{ N/m}^2)(0.010)\pi (31 \times 10^{-6} \text{ m})^2} = 8400
\]
65. SSM REASONING Our approach is straightforward. We will begin by writing
Equation 10.17 \[ F = Y \left( \frac{\Delta L}{L_0} \right) A \] as it applies to the composite rod. In so doing, we will use subscripts for only those variables that have different values for the composite rod and the aluminum and tungsten sections. Thus, we note that the force applied to the end of the composite rod (see Figure 10.27) is also applied to each section of the rod, with the result that the magnitude \( F \) of the force has no subscript. Similarly, the cross-sectional area \( A \) is the same for the composite rod and for the aluminum and tungsten sections. Next, we will express the change \( \Delta L_{\text{Composite}} \) in the length of the composite rod as the sum of the changes in lengths of the aluminum and tungsten sections. Lastly, we will take into account that the initial length of the composite rod is twice the initial length of either of its two sections and thereby simply our equation algebraically to the point that we can solve it for the effective value of Young’s modulus that applies to the composite rod.

SOLUTION Applying Equation 10.17 to the composite rod, we obtain

\[ F = Y_{\text{Composite}} \left( \frac{\Delta L_{\text{Composite}}}{L_0, \text{Composite}} \right) A \]  

(1)

Since the change \( \Delta L_{\text{Composite}} \) in the length of the composite rod is the sum of the changes in lengths of the aluminum and tungsten sections, we have \( \Delta L_{\text{Composite}} = \Delta L_{\text{Aluminum}} + \Delta L_{\text{Tungsten}} \). Furthermore, the changes in length of each section can be expressed using Equation 10.17 \( \Delta L = \frac{FL_0}{YA} \), so that

\[ \Delta L_{\text{Composite}} = \Delta L_{\text{Aluminum}} + \Delta L_{\text{Tungsten}} = \frac{FL_0, \text{Aluminum}}{Y_{\text{Aluminum}} A} + \frac{FL_0, \text{Tungsten}}{Y_{\text{Tungsten}} A} \]

Substituting this result into Equation (1) gives

\[ F = \left( \frac{Y_{\text{Composite}} A}{L_0, \text{Composite}} \right) \Delta L_{\text{Composite}} = \left( \frac{Y_{\text{Composite}} A}{L_0, \text{Composite}} \right) \left( \frac{FL_0, \text{Aluminum}}{Y_{\text{Aluminum}} A} + \frac{FL_0, \text{Tungsten}}{Y_{\text{Tungsten}} A} \right) \]

\[ 1 = Y_{\text{Composite}} \left( \frac{L_0, \text{Aluminum}}{L_0, \text{Composite}} \frac{Y_{\text{Aluminum}}}{Y_{\text{Tungsten}}} + \frac{L_0, \text{Tungsten}}{L_0, \text{Composite}} \frac{Y_{\text{Tungsten}}}{Y_{\text{Aluminum}}} \right) \]

In this result we now use the fact that \( L_0, \text{Aluminum} / L_0, \text{Composite} = L_0, \text{Tungsten} / L_0, \text{Composite} = 1/2 \) and obtain
Solving for \( Y_{\text{Composite}} \) shows that

\[
Y_{\text{Composite}} = \frac{2Y_{\text{Tungsten}}}{Y_{\text{Tungsten}} + Y_{\text{Aluminum}}} = \frac{2(3.6 \times 10^{11} \text{ N/m}^2)(6.9 \times 10^{10} \text{ N/m}^2)}{(3.6 \times 10^{11} \text{ N/m}^2) + (6.9 \times 10^{10} \text{ N/m}^2)} = 1.2 \times 10^{11} \text{ N/m}^2
\]

The values for \( Y_{\text{Tungsten}} \) and \( Y_{\text{Aluminum}} \) have been taken from Table 10.1.

66. **REASONING** Table 10.4 indicates that the shear stress is given by \( F/A \), and the shear strain is given by \( \Delta X/L_0 \), where \( A \) is the area of the plate’s square face and \( L_0 \) is the plate’s thickness. The maximum amount of force \( F \) that can be applied as in the referenced text figure is determined by the static frictional force between the plate and the surface on which it rests. The maximum static frictional force is given by \( f_s\text{MAX} = \mu_s F_N \) (Equation 4.7), where \( \mu_s \) is the coefficient of static friction and \( F_N \) is the normal force that presses together the plate and the surface on which it rests. Since the plate does not accelerate in the direction perpendicular to the surface, the normal force and the plate’s weight balance, so that \( F_N = mg \). Thus, we know that

\[
F = f_s\text{MAX} = \mu_s F_N = \mu_s mg
\]

**SOLUTION**
a. Referring to Table 10.4 for the definition of shear stress, we find that

\[
\text{Maximum possible shear stress} = \frac{F}{A} = \frac{\mu_s mg}{A} = \frac{0.91(7.2 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)}{(3.0 \times 10^{-2} \text{ m})^2} = 710 \text{ N/m}^2
\]

b. Referring to Table 10.4 for the definition of shear strain and recognizing that the shear modulus \( S \) is given, we find that

\[
\text{Maximum possible shear strain} = \frac{\Delta X}{L_0} = \frac{\text{Maximum possible shear stress}}{S} = \frac{710 \text{ N/m}^2}{2.0 \times 10^{10} \text{ N/m}^2} = 3.6 \times 10^{-8}
\]

c. The maximum possible amount of shear deformation \( \Delta X \) can now be determined from the result in part b as follows:

\[
\Delta X = \left( \frac{\text{Maximum possible shear strain}}{L_0} \right) L_0 = \left( 3.6 \times 10^{-8} \right) \left( 1.0 \times 10^{-2} \text{ m} \right) = 3.6 \times 10^{-10} \text{ m}
\]
67. **REASONING AND SOLUTION**

a. Strain = \( \frac{\Delta L}{L_0} = \frac{F}{YA} \). In this case the area subjected to the compression is given by

\[
A = \pi \left( r_{\text{out}}^2 - r_{\text{in}}^2 \right) = \pi \left[ \left( 1.00 \times 10^{-2} \right)^2 - \left( 4.0 \times 10^{-3} \right)^2 \right] = 2.64 \times 10^{-4} \text{ m}^2
\]

and the force is \( F = mg \). Taking the value for Young’s modulus \( Y \) for bone compression from Table 10.1, we find that

\[
\text{Strain} = \frac{(63 \text{ kg})(9.80 \text{ m/s}^2)}{(9.4 \times 10^9 \text{ Pa})(2.64 \times 10^{-4} \text{ m}^2)} = 2.5 \times 10^{-4}
\]

b. \( \Delta L = \text{Strain} \times L_0 = (2.5 \times 10^{-4})(0.30 \text{ m}) = 7.5 \times 10^{-5} \text{ m} \)

---

68. **REASONING** We will use energy conservation. The person falls from rest and does not rotate, so initially he has only gravitational potential energy. Ignoring air resistance and friction, we may apply the conservation of mechanical energy. Since the person strikes the ground stiff-legged and comes to a halt without rotating, all of the energy he had to begin with must be absorbed by his leg as elastic potential energy. The height through which he falls determines the amount of his gravitational potential energy and, hence, the amount of energy his leg must absorb.

According to Equation 10.13, the elastic potential energy of an ideal spring is

\[
\text{PE}_{\text{Elastic}} = \frac{1}{2} k_{\text{Effective}} x^2,
\]

so we will need \( k_{\text{Effective}} \) for a thigh bone. To find it, we consider Equation 10.1 for the applied force needed to change the length of an ideal spring:

\[
F_x^{\text{Applied}} = k_{\text{Effective}} x,
\]

where \( k_{\text{Effective}} \) is the spring constant and \( x \) is the displacement. To change the length of a bone, in comparison, the necessary applied force is given by Equation 10.17 as follows:

\[
F_x^{\text{Applied}} = Y \left( \frac{\Delta L}{L_0} \right) A = \frac{YA}{L_0} \frac{\Delta L}{\text{The effective spring constant} \ k_{\text{Effective}} \ \text{The change in length or the displacement} \ x}
\]

In this expression \( Y \) is Young’s modulus, \( A \) is the effective cross-sectional area of the bone, \( L_0 \) is the initial length of the bone, and \( \Delta L \) is the change in length. Associating \( \Delta L \) with \( x \), we see that the effective spring constant of the bone is given by

\[
k_{\text{Effective}} = \frac{YA}{L_0} \tag{1}
\]

Equation 10.13, which specifies the elastic potential energy of an ideal spring as

\[
\text{PE}_{\text{Elastic}} = \frac{1}{2} k_{\text{Effective}} x^2,
\]

contains the displacement \( x \) of the spring. To eliminate it, we turn to
Equation 10.1, which gives the applied force as \( F^\text{Applied}_x = k_{\text{Effective}} x \). Solving Equation 10.1 for \( x \) and substituting the result into Equation 10.13 gives

\[
\text{PE}_{\text{Elastic}} = \frac{1}{2} k_{\text{Effective}} x^2 = \frac{1}{2} k_{\text{Effective}} \left( \frac{F^\text{Applied}_x}{k_{\text{Effective}}} \right)^2 = \left( \frac{F^\text{Applied}_x}{2k_{\text{Effective}}} \right)^2
\]

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \). The expression for the total mechanical energy for a spring/object system is given by Equation 10.14, so we have

\[
\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + m g h_f + \frac{1}{2} k x_f^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + m g h_0 + \frac{1}{2} k x_0^2
\]

Since the person does not rotate, the angular speeds \( \omega_f \) and \( \omega_0 \) are zero. The person is at rest both initially and finally, so the initial and final translational speeds \( v_0 \) and \( v_f \) are also zero. Moreover, the thighbone is initially unstrained, with the result that \( x_0 = 0 \). Thus, the above expression can be simplified to give

\[
mgh_f + \frac{1}{2} k x_f^2 = mgh_0
\]

Using Equation (2) to express the final elastic potential energy of the thighbone, we can write the simplified energy-conservation equation as follows:

\[
mgh_f + \left( \frac{F^\text{Applied}_x}{2k_{\text{Effective}}} \right)^2 = mgh_0 \quad \text{or} \quad h_f - h_0 = \frac{\left( \frac{F^\text{Applied}_x}{2k_{\text{Effective}}} \right)^2}{2mgk_{\text{Effective}}}
\]

As the man falls, his center of gravity moves from its initial height of \( h_0 \) to its final height of \( h_f \) which is a distance of \( h_0 - h_f \). Using Equation (1) for the effective spring constant of the bone, we find

\[
h_f - h_0 = \frac{\left( \frac{F^\text{Applied}_x}{2k_{\text{Effective}}} \right)^2}{2mgk_{\text{Effective}}} = \frac{\left( \frac{F^\text{Applied}_x}{2k_{\text{Effective}}} \right)^2 L_0}{2mgY_A}
\]

\[
= \frac{\left( 7.0 \times 10^4 \text{ N} \right)^2 (0.55 \text{ m})}{2 (65 \text{ kg}) (9.80 \text{ m/s}^2) (9.4 \times 10^9 \text{ N/m}^2) (4.0 \times 10^{-4} \text{ m}^2)} = 0.56 \text{ m}
\]

69. **REASONING AND SOLUTION** Strain = \( \Delta L/L_0 = F/(YA) \) where \( F = mg \) and \( A = \pi r^2 \). Setting the strain for the spider thread equal to the strain for the wire
\[
\frac{F}{YA} = \frac{F'}{Y'A'} \quad \text{so that} \quad \frac{F}{Yr^2} = \frac{F'}{Y'r'^2}
\]

Spider thread, Aluminum wire

Thus,

\[
r'^2 = \frac{F'Yr^2}{FY'}
\]

Taking the value for Young’s modulus \(Y'\) for aluminum from Table 10.1, we find that

\[
r' = \sqrt{\frac{(95 \text{ kg})(9.80 \text{ m/s}^2)(4.5 \times 10^9 \text{ Pa})(13 \times 10^{-6} \text{ m})^2}{(1.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(6.9 \times 10^{10} \text{ Pa})}} = 1.0 \times 10^{-3} \text{ m}
\]

70. **REASONING** We will find displacement \(\Delta X\) of the upper surface relative to the lower surface from \(F = S \left( \frac{\Delta X}{L_0} \right) A\) (Equation 10.18), where \(F\) is the applied horizontal force (see Figure 10.30), \(S\) is the shear modulus of brass, \(L_0\) is the distance between the upper and lower surfaces, and \(A\) is the cross-sectional area of either surface. Solving Equation 10.18 for \(\Delta X\) yields

\[
\Delta X = \frac{FL_0}{SA} = \left( \frac{F}{S} \right) \times \left( \frac{L_0}{A} \right)
\]

(1)

Note that the term \(L_0/A\) in Equation (1) depends on the orientation of the block. The block can rest on any one of its three unique surfaces. To make the displacement \(\Delta X\) of the top surface as large as possible, Equation (1) shows that we must orient the block so as to make the ratio \(L_0/A\) as large as possible. The largest possible value for the numerator of this ratio is the longest side of the block, so we choose \(L_0 = 0.040\) m. Gluing the block to the table so that the four longest edges are vertical means that the upper surface of the block is the surface with the smallest area: \(A = 0.010 \text{ m} \times 0.020 \text{ m}\) (see the drawing). This combination of the largest possible value of \(L_0\) and the smallest possible value of \(A\) makes the ratio \(L_0/A\) as large as possible. Therefore, this is the orientation with the greatest possible displacement \(\Delta X\) of the upper surface relative to the lower surface when the horizontal force is applied to the upper surface.
**SOLUTION** We obtain the shear modulus \( S = 3.5 \times 10^{10} \text{ N/m}^2 \) of brass from Table 10.2 in the text. From Equation (1), the maximum possible displacement of the upper surface relative to the lower surface is

\[
\Delta X = \frac{F L_0}{S A} = \frac{(770 \text{ N})(0.040 \text{ m})}{(3.5 \times 10^{10} \text{ N/m}^2)(0.010 \text{ m})(0.020 \text{ m})} = 4.4 \times 10^{-6} \text{ m}
\]

71. **REASONING** The unstretched length \( L_0 \) of the cable can be found from the relation

\[ L_0 = Y A (\Delta L) / F \] (Equation 10.17), where \( Y \) is Young’s modulus for steel (see Table 10.1), \( A \) is the cross-sectional area of the cable, \( \Delta L \) is the amount by which it stretches, and \( F \) is the magnitude of the stretching force. All the variables except \( F \) are known. According to Newton’s third law, the action-reaction law, the force exerted on the cable by the skier has the same magnitude as the force exerted on the skier by the cable. The force exerted on the skier by the cable can be obtained from Newton’s second law, since the mass and acceleration of the skier are known.

**SOLUTION** The unstrained length of the cable is

\[ L_0 = \frac{Y A (\Delta L)}{F} \] (10.17)

To determine \( F \), we examine the following free-body diagram of the skier. For convenience, the +x direction is taken to be parallel to the slope and to point upward (see the drawing).

Three forces act on the skier in the x direction: (1) the towing force (magnitude = \( F \)), (2) the frictional force (magnitude = \( f \)) exerted on the skis by the snow, and (3) the component of the skier’s weight that is parallel to the x axis (magnitude = \( W \sin 12^\circ = mg \sin 12^\circ \)). This component is shown to the right of the free-body diagram. The net force acting on the skier has a magnitude of \( F - f - mg \sin 12^\circ \). According to Newton’s second law (see Section 4.3), this net force is equal to the skier’s mass times the magnitude of her acceleration, or

\[ F - f - mg \sin 12^\circ = ma \]

Solving this equation for \( F \) and substituting the result into Equation 10.17, we find that
We have taken the value of $Y = 2.0 \times 10^{11} \text{ N/m}^2$ for steel from Table 10.1.

72. **REASONING** The unstrained length $L_0$ of the cord is related to the stretching force (tension) $F$ on the cord, the amount $\Delta L$ by which the cord is stretched, the cord’s cross-sectional area $A$, and Young’s modulus $Y = 3.7 \times 10^9 \text{ N/m}^2$ (see Table 10.1 in the text) by

$$F = Y \left( \frac{\Delta L}{L_0} \right) A$$

(10.17)

While swinging at a speed $v$ on the end of the cord, the bowling ball (mass = $m$) follows a circular path, and the tension $F$ in the cord must be sufficient to supply the necessary centripetal force $F_c = \frac{mv^2}{r}$ (Equation 5.3). Ignoring the stretch in the cord, the radius $r$ of the circle that the bowling ball traverses is equal to the length $L_0$ of the cord. Thus, Equation 5.3 becomes

$$F_c = \frac{mv^2}{L_0}$$

(5.3)

At the lowest point, the upward force $F$ on the ball is opposed by the downward weight force $mg$, so the difference between these two forces equals the centripetal force $F_c$:

$$F_c = \frac{mv^2}{L_0} = F - mg \quad \text{or} \quad F = \frac{mv^2}{L_0} + mg$$

(1)

To find the speed of the bowling ball, we take advantage of energy conservation. As the ball swings downward from release to its lowest point, it is subject to only one nonconservative force: the tension in the cord. This force is perpendicular to the bowling ball’s velocity at every instant, and, therefore, does no work. Consequently, the total mechanical energy $E = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 + mgh + \frac{1}{2} kx^2$ (Equation 10.14) of the bowling ball is conserved. The bowling ball starts from rest and undergoes no rotation ($\omega_0 = \omega_f = 0$ rad/s), so its initial translational kinetic energy is zero, as are its initial and final rotational kinetic energies. The cord is initially unstretched, and its final stretch $x = \Delta L$ is negligible, so the initial and final elastic potential energies are both zero. When we apply the energy conservation principle, then, we obtain

$$\frac{1}{2} mv_f^2 + mgh_f = mgh_0$$

(2)

Solving Equation (2) for the square of the speed $v_f^2$ of the bowling ball at its lowest point, we find that
\[ \frac{1}{2} m v_i^2 + \frac{1}{2} m g h_f = m v_0 h_0 \quad \text{or} \quad v_i^2 = 2g (h_0 - h_f) \quad (3) \]

We will use Equations (10.17), (1), and (3) to solve for the unstretched length \( L_0 \) of the cord.

**SOLUTION** Because Equation (1) refers to the instant when the bowling ball reaches its lowest point, the speed \( v \) in Equation (1) is identical with the final speed \( v_f \) in Equation (3): \( v = v_f \). Making this substitution in Equation (1), and setting Equation (10.17) and Equation (1) equal, we obtain

\[ Y \left( \frac{\Delta L}{L_0} \right) A = \frac{m v_i^2}{L_0} + mg \quad (4) \]

Multiplying both sides of Equation (4) by \( L_0 \) and solving, we find

\[ Y \left( \frac{\Delta L}{L_0} \right) A L_0 = \frac{m v_i^2}{L_0} L_0 + mg L_0 \quad \text{or} \quad L_0 = \frac{Y (\Delta L) A - m v_i^2}{mg} \quad (5) \]

Substituting Equation (3) for the square of the final speed into Equation (5) yields the unstrained length of the cord:

\[
L_0 = \frac{Y (\Delta L) A - 2mg (h_0 - h_f)}{mg} = \frac{Y (\Delta L) A}{mg} - 2(h_0 - h_f) = \frac{(3.7 \times 10^9 \text{ N/m}^2) (2.7 \times 10^{-3} \text{ m}) (3.4 \times 10^{-5} \text{ m}^2)}{(6.8 \text{ kg})(9.80 \text{ m/s}^2)} - 2(1.4 \text{ m}) = 2.3 \text{ m}
\]

---

73. **SSM REASONING** Equation 10.20 can be used to find the fractional change in volume of the brass sphere when it is exposed to the Venusian atmosphere. Once the fractional change in volume is known, it can be used to calculate the fractional change in radius.

**SOLUTION** According to Equation 10.20, the fractional change in volume is

\[
\frac{\Delta V}{V_0} = - \frac{\Delta P}{B} = - \frac{8.9 \times 10^6 \text{ Pa}}{6.7 \times 10^{10} \text{ Pa}} = -1.33 \times 10^{-4}
\]

Here, we have used the fact that \( \Delta P = 9.0 \times 10^6 \text{ Pa} - 1.0 \times 10^5 \text{ Pa} = 8.9 \times 10^6 \text{ Pa} \), and we have taken the value for the bulk modulus \( B \) of brass from Table 10.3. The initial volume of the sphere is \( V_0 = \frac{4}{3} \pi r_0^3 \). If we assume that the change in the radius of the sphere is very small relative to the initial radius, we can think of the sphere's change in volume as the addition or subtraction of a spherical shell of volume \( \Delta V \), whose radius is \( r_0 \) and whose thickness is...
\[ \Delta r \text{. Then, the change in volume of the sphere is equal to the volume of the shell and is given by } \Delta V = \left(4\pi r_0^2\right) \Delta r \text{. Combining the expressions for } V_0 \text{ and } \Delta V \text{, and solving for } \frac{\Delta r}{r_0} \text{, we have} \]

\[ \frac{\Delta r}{r_0} = \frac{1}{3} \frac{\Delta V}{V_0} \]

Therefore,

\[ \frac{\Delta r}{r_0} = \frac{1}{3} (-1.33 \times 10^{-4}) = -4.4 \times 10^{-5} \]

74. **REASONING** The number of times the diaphragm moves back and forth is the frequency \( f \) of the motion (in cycles/s or Hz) times the time interval \( t \). The frequency \( f \) is related to the angular frequency \( \omega \) (in rad/s) by Equation 10.6 (\( \omega = 2\pi f \)).

**SOLUTION** Solving Equation 10.6 (\( \omega = 2\pi f \)) for the frequency \( f \) gives

\[ f = \frac{\omega}{2\pi} \]

The number of times the diaphragm moves back and forth in 2.5 s is

\[
\text{Number of times} = ft = \left(\frac{\omega}{2\pi}\right) t = \left(\frac{7.54 \times 10^4 \text{ rad/s}}{2\pi}\right) (2.5 \text{ s}) = 3.00 \times 10^4
\]

75. **REASONING** The amplitude of simple harmonic motion is the distance from the equilibrium position to the point of maximum height. The angular frequency \( \omega \) is related to the period \( T \) of the motion by Equation 10.6. The maximum speed attained by the person is the product of the amplitude and the angular speed (Equation 10.8).

**SOLUTION**

a. Since the distance from the equilibrium position to the point of maximum height is the amplitude \( A \) of the motion, we have that \( A = 45.0 \text{ cm} = 0.450 \text{ m} \).

b. The angular frequency is inversely proportional to the period of the motion:

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{1.90 \text{ s}} = 3.31 \text{ rad/s}
\]

(10.6)

c. The maximum speed \( v_{\text{max}} \) attained by the person on the trampoline depends on the amplitude \( A \) and the angular frequency \( \omega \) of the motion:

\[
v_{\text{max}} = A\omega = (0.450 \text{ m})(3.31 \text{ rad/s}) = 1.49 \text{ m/s}
\]

(10.8)
76. **REASONING** The length \( L \) of a simple pendulum is related to its frequency \( f \) via
\[
2\pi f = \sqrt{\frac{g}{L}} \quad (\text{Equation 10.16}).
\]
In terms of its period \( T \), the frequency of a simple pendulum is \( f = \frac{1}{T} \) (Equation 10.5), so we have
\[
\frac{2\pi}{T} = \sqrt{\frac{g}{L}}.
\]
Solving for the length \( L \), we obtain
\[
\frac{T}{2\pi} = \sqrt{\frac{L}{g}} \quad \text{or} \quad \frac{T^2}{4\pi^2} = \frac{L}{g} \quad \text{or} \quad L = \frac{T^2g}{4\pi^2} \quad (1)
\]
We will use Equation (1) to calculate the difference \( L_2 - L_1 \) between the final and initial lengths of the pendulum.

**SOLUTION** The period of the original pendulum is \( T_1 = 1.25 \text{ s} \). When its length is increased from \( L_1 \) to \( L_2 \), its period increases by 0.20 s to \( T_2 = 1.25 \text{ s} + 0.20 \text{ s} = 1.45 \text{ s} \). From Equation (1), the difference \( L_2 - L_1 \) between the pendulum’s final and initial lengths is
\[
L_2 - L_1 = \frac{T_2^2g}{4\pi^2} - \frac{T_1^2g}{4\pi^2} = \frac{g}{4\pi^2}(T_2^2 - T_1^2) = \frac{9.80 \text{ m/s}^2}{4\pi^2}\left[(1.45 \text{ s})^2 - (1.25 \text{ s})^2\right] = 0.13 \text{ m}
\]

77. **SSM REASONING** Each spring supports one-quarter of the total mass \( m_{\text{total}} \) of the system (the empty car plus the four passengers), or \( \frac{1}{4} m_{\text{total}} \). The mass \( m_{\text{one passenger}} \) of one passenger is equal to \( \frac{1}{4} m_{\text{total}} \) minus one-quarter of the mass \( m_{\text{empty car}} \) of the empty car:
\[
m_{\text{one passenger}} = \frac{1}{4} m_{\text{total}} - \frac{1}{4} m_{\text{empty car}} \quad (1)
\]
The mass of the empty car is known. Since the car and its passengers oscillate up and down in simple harmonic motion, the angular frequency \( \omega \) of oscillation is related to the spring constant \( k \) and the mass \( \frac{1}{4} m_{\text{total}} \) supported by each spring by:
\[
\omega = \sqrt{\frac{k}{\frac{1}{4} m_{\text{total}}}} \quad (10.11)
\]
Solving this expression for \( \frac{1}{4} m_{\text{total}} \left( \frac{1}{4} m_{\text{total}} = k / \omega^2 \right) \) and substituting the result into Equation (1) gives
\[
m_{\text{one passenger}} = \frac{k}{\omega^2} - \frac{1}{4} m_{\text{empty car}} \quad (2)
\]
The angular frequency $\omega$ is inversely related to the period $T$ of oscillation by $\omega = \frac{2\pi}{T}$ (see Equation 10.4). Substituting this expression for $\omega$ into Equation (2) yields

$$m_{\text{one passenger}} = \frac{k}{\left(\frac{2\pi}{T}\right)^2} - \frac{1}{4}m_{\text{empty car}}$$

**SOLUTION** The mass of one of the passengers is

$$m_{\text{one passenger}} = \frac{k}{\left(\frac{2\pi}{T}\right)^2} - \frac{1}{4}m_{\text{empty car}} = \frac{1.30 \times 10^5 \text{ N/m}}{\left(\frac{2\pi}{0.370 \text{ s}}\right)^2} - \frac{1}{4}(1560 \text{ kg}) = 61 \text{ kg}$$

78. **REASONING**

a. According to the discussion in Section 10.8, the stress is the magnitude of the force per unit area required to cause an elastic deformation. We can determine the maximum stress that will fracture the femur by dividing the magnitude of the compressional force by the cross-sectional area of the femur.

b. The strain is defined in Section 10.8 as the change in length of the femur divided by its original length. Equation 10.17 shows how the strain $\Delta L/L_0$ is related to the stress $F/A$ and Young’s modulus $Y$ ($Y = 9.4 \times 10^9 \text{ N/m}^2$ for bone compression, according to Table 10.1).

**SOLUTION**

a. The maximum stress is equal to the maximum compressional force divided by the cross-sectional area of the femur:

$$\text{Maximum stress} = \frac{F}{A} = \frac{6.8 \times 10^4 \text{ N}}{4.0 \times 10^{-4} \text{ m}^2} = 1.7 \times 10^8 \text{ N/m}^2$$

b. The strain $\Delta L/L$ can be found by rearranging Equation 10.17:

$$\frac{\Delta L}{L_0} = \frac{1}{Y} \left(\frac{F}{A}\right) = \left(\frac{1}{9.4 \times 10^9 \text{ N/m}^2}\right)(1.7 \times 10^8 \text{ N/m}^2) = 1.8 \times 10^{-2}$$

79. **REASONING** The applied force required to stretch the bow string is given by Equation 10.1 as $F_x^{\text{Applied}} = kx$, where $k$ is the spring constant and $x$ is the displacement of the string. Solving for the displacement gives $x = F_x^{\text{Applied}} / k$. 
**Chapter 10 Problems**

**SOLUTION** The displacement of the bow string is

\[
x = \frac{F_{\text{Applied}}}{k} = \frac{240 \text{ N}}{480 \text{ N/m}} = 0.50 \text{ m}
\]

80. **REASONING** The distance \( \Delta X \) that the top surface of the disc moves relative to the bottom surface is given by \( \Delta X = F L_0 / (SA) \) (see Equation 10.18), where \( F \) is the magnitude of the shearing force, \( L_0 \) is the thickness of the cartilage, \( A \) is the cross-sectional area, and \( S \) is the shear modulus. Since the cross-section is circular, \( A = \pi r^2 \), where \( r \) is the radius.

**SOLUTION** The distance (shear deformation) \( \Delta X \) is

\[
\Delta X = \frac{F L_0}{SA} = \frac{(11 \text{ N})(7.0 \times 10^{-3} \text{ m})}{(1.2 \times 10^7 \text{ N/m}^2)(3.0 \times 10^{-2} \text{ m})^2} = 2.3 \times 10^{-6} \text{ m}
\] (10.18)

81. **REASONING** Since the surface is frictionless, we can apply the principle of conservation of mechanical energy, which indicates that the total mechanical energy of the spring/mass system is the same at the instant the block contacts the bottle (the final state of the system) and at the instant shown in the drawing (the initial state). Kinetic energy \( \frac{1}{2} m v^2 \) is one part of the total mechanical energy, and depends on the mass \( m \) and the speed \( v \) of the block. The dependence of the kinetic energy on speed is critical to our solution. In order for the block to knock over the bottle, it must at least reach the bottle. When launched with the minimum speed \( v_0 \) shown in the drawing, the block will reach the bottle with a final speed of \( v_f = 0 \text{ m/s} \). We will obtain the desired initial speed \( v_0 \) by solving the energy-conservation equation for this variable.

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \). The expression for the total mechanical energy for a spring/mass system is given by Equation 10.14, so that we have

\[
\frac{\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + mgh_f + \frac{1}{2} kx_f^2}{E_f} = \frac{\frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + mgh_0 + \frac{1}{2} kx_0^2}{E_0}
\]

Since the block does not rotate, the angular speeds \( \omega_f \) and \( \omega_0 \) are zero. Moreover, the block reaches the bottle with a final speed of \( v_f = 0 \text{ m/s} \) when the block is launched with the minimum initial speed \( v_0 \). In addition, the surface is horizontal, so that the final and initial heights, \( h_f \) and \( h_0 \), are the same. Thus, the above expression can be simplified as follows:
\[
\frac{1}{2} kx_1^2 = \frac{1}{2} kx_0^2 + \frac{1}{2} mv_0^2
\]

In this result, we are given no values for the spring constant \( k \) and the mass \( m \). However, we are given a value for the angular frequency \( \omega \). This frequency is given by Equation 10.11 \( \omega = \sqrt{\frac{k}{m}} \), which involves only the ratio \( k/m \). Therefore, in solving the simplified energy-conservation expression for the speed \( v_0 \), we will divide both sides by \( m \), so that the ratio \( k/m \) can be expressed using Equation 10.11.

\[
\frac{\frac{1}{2} kx_1^2}{m} = \frac{\frac{1}{2} kx_0^2 + \frac{1}{2} mv_0^2}{m} \quad \text{or} \quad v_0 = \sqrt{\left(\frac{k}{m}\right)\left(x_1^2 - x_0^2\right)}
\]

Substituting \( \omega = \sqrt{\frac{k}{m}} \) from Equation 10.11, we find

\[
v_0 = \omega \sqrt{(x_1^2 - x_0^2)} = (7.0 \text{ rad/s}) \sqrt{(0.080 \text{ m})^2 - (0.050 \text{ m})^2} = 0.44 \text{ m/s}
\]

82. **REASONING**

a. As the block rests stationary in its equilibrium position, it has no acceleration. According to Newton’s second law, the net force acting on the block is, therefore, zero. This means that the downward-directed weight of the block must be balanced by an upward-directed force. This upward force is the restoring force of the spring and is produced because the spring is compressed. The compression must be enough for the spring to exert on the block a restoring force that has a magnitude equal to the block’s weight. This balancing of forces will allow us to determine the magnitude of the spring’s compression.

b. As the block falls downward after being released, its speed is changing in the manner characteristic of simple harmonic motion. The block is not in equilibrium, and the forces acting on it do not balance to zero. Instead of thinking about forces, we may think about mechanical energy and its conservation. When the block is released from rest, the energy of the spring/block system is all in the form of gravitational potential energy. Being at rest, the block has no initial kinetic energy. It also has no initial elastic potential energy, since the spring is unstrained initially. When the block comes to a momentary halt at the lowest point in its fall, the energy is all in the form of elastic potential energy. Since the block is again at rest, it again has no kinetic energy. The spring has been compressed, and gravitational potential energy has been converted entirely into elastic potential energy. The amount by which the spring is compressed is determined by the amount of gravitational potential energy that must be converted into elastic potential energy. The amount must be enough that the elastic potential energy equals the gravitational potential energy. Thus, we will use energy conservation to determine the magnitude of the spring’s compression.

The compression of the spring is greater in the non-equilibrium case than in the equilibrium case. The reason is that in the non-equilibrium case, the block has been allowed to move, and its inertia carries it beyond its stationary equilibrium position on the spring. The
compression of the spring must increase beyond that for the stationary equilibrium position in order to produce the force that is needed to decelerate the block to a momentary halt.

**SOLUTION**

a. As the block rests stationary on the spring, the downward-directed weight balances the upward-directed restoring force from the spring. The magnitude of the weight is $mg$, and the magnitude of the restoring force is given by Equation 10.2 without the minus sign as $kx$. Thus, we have

$$mg = kx$$

or

$$x = \frac{mg}{k} = \frac{(0.64 \text{ kg})(9.80 \text{ m/s}^2)}{170 \text{ N/m}} = 0.037 \text{ m}$$

b. The conservation of mechanical energy states that the final total mechanical energy $E_f$ is equal to the initial total mechanical energy $E_0$. The expression for the total mechanical energy for an object on a spring is given by Equation 10.14, so that we have

$$\frac{1}{2}mv_i^2 + \frac{1}{2}I_0\omega_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I_0\omega_0^2 + mgh_0 + \frac{1}{2}kx_0^2$$

The block does not rotate, so the angular speeds $\omega_i$ and $\omega_0$ are zero. Since the block comes to a momentary halt on the spring and is released from rest, the translational speeds $v_i$ and $v_0$ are also zero. Because the spring is initially unstrained, the initial displacement $x_0$ of the spring is likewise zero. Thus, the above expression can be simplified as follows:

$$mgh_i + \frac{1}{2}kx_i^2 = mgh_0$$

or

$$\frac{1}{2}kx_i^2 = mg(h_0 - h_i)$$

The term $h_0 - h_i$ is the amount by which the spring has compressed, or $h_0 - h_i = x_i$. Making this substitution into the simplified energy-conservation equation gives

$$\frac{1}{2}kx_i^2 = mg(h_0 - h_i) = mgx_i$$

or

$$\frac{1}{2}kx_i = mg$$

Solving for $x$, we find

$$x = \frac{2mg}{k} = \frac{2(0.64 \text{ kg})(9.80 \text{ m/s}^2)}{170 \text{ N/m}} = 0.074 \text{ m}$$

As expected, the spring compresses more in the non-equilibrium situation.

83. **REASONING** Since air resistance is being ignored and the bungee cord is assumed to be an ideal spring, the bungee jumper oscillates up and down in simple harmonic motion. The angular frequency $\omega$ of oscillation is related to the spring constant $k$ and the mass $m$ of the jumper by Equation 10.11: $\omega = \sqrt{k/m}$. Solving for the spring constant gives

$$k = m\omega^2$$

(1)

The angular frequency $\omega$ is inversely proportional to the period $T$ of oscillation according to $\omega = 2\pi/T$ (see Equation 10.4). Substituting this expression for $\omega$ into Equation (1) yields
\[ k = m\omega^2 = m\left(\frac{2\pi}{T}\right)^2 \]

The mass of the jumper is known. The period of oscillation can be obtained from the fact that the jumper makes two complete oscillations in a time of 9.6 s.

**SOLUTION** Since the bungee jumper moves through 2 complete cycles in 9.6 s, the time to complete one cycle (the period of oscillation) is \( T = \frac{1}{2}(9.6 \text{ s}) = 4.8 \text{ s} \). The spring constant of the bungee cord is

\[ k = m\left(\frac{2\pi}{4.8}\right)^2 = 140 \text{ N/m} \]

84. **REASONING** It is assumed in Example 16 that the only forces acting on the jumper are the gravitational force (his weight) and, for the latter part of his descent, the elastic force of the bungee cord. Therefore, only conservative forces are present, and we may use energy conservation to guide our solution. He possesses gravitational potential energy with respect to the water and elastic potential energy. At the lowest point in his fall, he has no kinetic energy since he comes to a momentary halt and has zero speed.

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \), or \( E_f = E_0 \) (Equation 6.9a). The expression for the total mechanical energy of an object is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

\[ \frac{\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mg h_i + \frac{1}{2}ky_i^2}{E_f} = \frac{\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mg h_0 + \frac{1}{2}ky_0^2}{E_0} \]

We can simplify this equation by noting which variables are zero. The jumper starts from rest and momentarily comes to a halt at the bottom of the jump; thus, \( v_0 = v_f = 0 \text{ m/s} \). He does not rotate, so his angular speed is zero; \( \omega_i = \omega_0 = 0 \text{ rad/s} \). Initially, the bungee cord is unstretched, so that \( y_0 = 0 \text{ m} \). Setting these terms to zero in the equation above gives

\[ mgh_f + \frac{1}{2}k y_f^2 = mgh_0 \]

At his lowest point, the bungee cord is stretched in the downward direction, which is taken to be the negative direction. Thus, the stretch \( y_f \) is negative, and we note from Figure 10.35 that \( y_f = h_f - h_A \), where \( h_A \) is 37.0 m. Substituting this expression for \( y_f \) into the equation above and rearranging terms, we find that

\[ h_f^2 + \left(\frac{2mg}{k} - 2h_A\right)h_f + h_A^2 - \frac{2mgh_0}{k} = 0 \]
Thus, we have that
\[ h_t^2 - (53.8 \text{ m}) h_t + (440.1 \text{ m}^2) = 0 \]

This is a quadratic equation in the variable \( h_t \), and its solution is
\[ h_t = \frac{(-53.8 \text{ m}) \pm \sqrt{(-53.8 \text{ m})^2 - 4(440.1 \text{ m}^2)}}{2} = 45.2 \text{ m} \text{ or } 10.1 \text{ m} \]

The 45.2-m answer is discarded, because it implies that the jumper comes to a halt at a distance of only \( 46.0 \text{ m} - 45.2 \text{ m} = 0.81 \text{ m} \), which is above the point where the bungee cord is stretched. Thus, when he reaches the lowest point in his fall, his height above the water is \( 10.1 \text{ m} \).

85. **REASONING** The two blocks and the spring between them constitute the system in this problem. Since the surface is frictionless and the weights of the blocks are balanced by the normal forces from the surface, no net external force acts on the system. Thus, the system’s total mechanical energy and total linear momentum are each conserved. The conservation of these two quantities will give us two equations containing the two unknown speeds with which the blocks move away. Using these equations, we will be able to obtain the speeds.

**SOLUTION** The conservation of mechanical energy states that the final total mechanical energy \( E_f \) is equal to the initial total mechanical energy \( E_0 \). Only the translational kinetic energies of the blocks and the elastic potential energy of the spring are of interest here. There is no rotational kinetic energy since there is no rotation. Gravitational potential energy plays no role, because the surface is horizontal and the vertical height does not change. Thus, the expression for the conservation of the total mechanical energy is

\[
\frac{\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} kx_f^2}{E_f} = \frac{\frac{1}{2} m_1 v_{01}^2 + \frac{1}{2} m_2 v_{02}^2 + \frac{1}{2} kx_0^2}{E_0}
\]

Since the blocks are initially at rest, the initial translational speeds \( v_{01} \) and \( v_{02} \) are zero. In addition, the spring is neither compressed nor stretched after it is released, so that \( x_f = 0 \text{ m} \). Thus, the above expression can be simplified as follows:

\[
\frac{\frac{1}{2} m_1 v_{1f}^2}{E_f} + \frac{\frac{1}{2} m_2 v_{2f}^2}{E_f} = \frac{\frac{1}{2} kx_0^2}{E_0}
\]

(1)

Remembering that linear momentum is mass times velocity, we can express the conservation of the total linear momentum of the system as follows:
\[
\frac{m_1v_{1f} + m_2v_{2f}}{\text{Final total momentum}} = \frac{m_1v_{1f} + m_2v_{2f}}{\text{Initial total momentum}}
\]

Since the blocks are initially at rest, \(v_{01}\) and \(v_{02}\) are zero, so that the expression for the conservation of linear momentum becomes

\[
m_1v_{1f} + m_2v_{2f} = 0 \quad \text{or} \quad v_{2f} = -\frac{m_1v_{1f}}{m_2} \tag{2}
\]

Substituting this result into Equation (1) gives

\[
\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 \left( -\frac{m_1v_{1f}}{m_2} \right)^2 = \frac{1}{2} kx_0^2
\]

Solving for \(v_{1f}\), which we define to be the final speed of the 11.2-kg block, shows that

\[
v_{1f} = \sqrt{\frac{m_1 k x_0^2}{m_1(m_2 + m_1)}} = \sqrt{\frac{(21.7 \text{ kg})(1330 \text{ N/m})(0.141 \text{ m})^2}{(11.2 \text{ kg})(21.7 \text{ kg} + 11.2 \text{ kg})}} = 1.25 \text{ m/s}
\]

Substituting this value into Equation (2) gives

\[
v_{2f} = -\frac{m_1v_{1f}}{m_2} = -\frac{(11.2 \text{ kg})(1.25 \text{ m/s})}{21.7 \text{ kg}} = -0.645 \text{ m/s}
\]

The speed of the 21.7-kg block is the magnitude of this result or \(0.645 \text{ m/s}\).

86. **Reasoning** According to Equations 10.6 and 10.11, the frequency \(f\) at which an object of mass \(m\) oscillates on a spring with a spring constant \(k\) is \(f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}\). We will apply this expression to the case when only the first object is attached to the spring and then again to the case when both objects are attached.

**SOLUTION** For the two cases, we have

\[
\begin{align*}
    f_1 &= \frac{1}{2\pi} \sqrt{\frac{k}{m_1}} \quad \text{and} \quad f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{m_1 + m_2}} \\
    \text{Only first object} & \quad \text{attached to spring} \quad \text{Both objects} & \quad \text{attached to spring}
\end{align*}
\]

Dividing the expression for \(f_1\) by the expression for \(f_2\) gives

\[
\frac{f_1}{f_2} = \frac{1}{\frac{2\pi}{m_1 + m_2}} \sqrt{\frac{m_1 + m_2}{m_1}} = \sqrt{\frac{m_1 + m_2}{m_1}} \quad \text{or} \quad \frac{f_1^2}{f_2^2} = \frac{m_1 + m_2}{m_1} = 1 + \frac{m_2}{m_1}
\]
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Solving the result above on the right for the ratio \(m_2/m_1\) gives

\[
\frac{m_2}{m_1} = \frac{f_1^2}{f_2^2} - 1 = \left(\frac{12.0 \text{ Hz}}{4.00 \text{ Hz}}\right)^2 - 1 = 8.00
\]

87. **REASONING** The strain in the wire is given by \(\Delta L/L_0\). From Equation 10.17, the strain is therefore given by

\[
\frac{\Delta L}{L_0} = \frac{F}{YA}
\]

where \(F\) must be equal to the magnitude of the centripetal force that keeps the stone moving in the circular path of radius \(R\). Table 10.1 gives the value of \(Y\) for steel.

**SOLUTION** Combining Equation (1) with Equation 5.3 for the magnitude of the centripetal force, we obtain

\[
\frac{\Delta L}{L_0} = \frac{F}{YA} = \frac{(mv^2/R)}{Y(\pi r^2)} = \frac{(8.0 \text{ kg})(12 \text{ m/s})^2/(4.0 \text{ m})}{(2.0 \times 10^{11} \text{ Pa})\pi(1.0 \times 10^{-3} \text{ m})^2} = 4.6 \times 10^{-4}
\]

88. **REASONING AND SOLUTION** Use conservation of energy to find the speed of point A (take the pivot to have zero gravitational PE).

\[
E_{\text{up}} = mgh = E_{\text{down}} = \frac{1}{2}I\omega^2 + \frac{1}{2}ky^2
\]

where the moment of inertia of the bar is \(I = \frac{1}{3}mL^2\), \(L = \) bar length, and \(\omega = v/L\). Substituting these into the energy equation, noting from the drawing accompanying the problem statement that \(y = \sqrt{(0.100 \text{ m})^2 + (0.200 \text{ m})^2 - (0.100 \text{ m})} = 0.124 \text{ m}\) and that \(h = \frac{1}{2}L\), and solving for \(v\), we find that

\[
v = \sqrt{\frac{3(mgL - ky^2)}{m}} = \sqrt{\frac{3[(0.750 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m}) - (25.0 \text{ N/m})(0.124 \text{ m})^2]}{0.750 \text{ kg}}} = 2.08 \text{ m/s}
\]
89. **REASONING** The change in length of the wire is, According to Equation 10.17, \( \Delta L = \frac{FL_0}{YA} \), where the force \( F \) is equal to the tension \( T \) in the wire. The tension in the wire can be found by applying Newton's second law to the two crates.

**SOLUTION** The drawing shows the free-body diagrams for the two crates. Taking up as the positive direction, Newton's second law for each of the two crates gives

\[
T - m_1 g = m_1 a \\
T - m_2 g = -m_2 a
\]

Solving Equation (2) for \( a \), we find

\[
a = \frac{T - m_2 g}{m_2}.
\]

Substituting into Equation (1) gives

\[
T - 2m_1 g + \frac{m_1}{m_2} T = 0
\]

Solving for \( T \) we find

\[
T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2(3.0 \text{ kg})(5.0 \text{ kg})(9.80 \text{ m/s}^2)}{3.0 \text{ kg} + 5.0 \text{ kg}} = 37 \text{ N}
\]

Using the value given in Table 10.1 for Young’s modulus \( Y \) of steel, we find, therefore, that the change in length of the wire is given by Equation 10.17 as

\[
\Delta L = \frac{(37 \text{ N})(1.5 \text{ m})}{(2.0 \times 10^{11} \text{ N/m}^2)(1.3 \times 10^{-5} \text{ m}^2)} = 2.1 \times 10^{-5} \text{ m}
\]

90. **REASONING** If we compare Equation 10.17, which governs the stretching and compression of a solid cylinder, with Equation 10.1, we find that \( x \) is analogous to \( \Delta L \) and \( k \) is analogous to the term \( YA/L_0 \):

\[
F = \left( \frac{YA}{L_0} \right) \frac{\Delta L}{k}
\]

**SOLUTION**

a. Solving for \( k \) we have

\[
k = \frac{YA}{L_0} = \frac{Y(\pi r^2)}{L_0} = \frac{(3.1 \times 10^6 \text{ N/m}^2)(\pi)(0.091 \times 10^{-2} \text{ m})^2}{2.5 \times 10^{-2} \text{ m}} = 3.2 \times 10^2 \text{ N/m}
\]
b. The work done by the variable force is equal to the area under the \( F \)-versus-\( x \) curve. The amount \( x \) of stretch is

\[
x = \frac{F}{k} = \frac{3.0 \times 10^{-2} \text{ N}}{3.2 \times 10^{2} \text{ N/m}} = 9.4 \times 10^{-5} \text{ m}
\]

The work done is

\[
W = \frac{1}{2} F x = \frac{1}{2} \left( 3.0 \times 10^{-2} \text{ N} \right) \left( 9.4 \times 10^{-5} \text{ m} \right) = 1.4 \times 10^{-6} \text{ J}
\]

91. **SSM REASONING** The angular frequency for simple harmonic motion is given by Equation 10.11 as \( \omega = \sqrt{k/m} \). Since the frequency \( f \) is related to the angular frequency \( \omega \) by \( f = \omega/(2\pi) \) and \( f \) is related to the period \( T \) by \( f = 1/T \), the period of the motion is given by

\[
T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{k}{m}}
\]

**SOLUTION**

a. When \( m_1 = m_2 = 3.0 \text{ kg} \), we have that

\[
T_1 = T_2 = 2\pi \sqrt{\frac{3.0 \text{ kg}}{120 \text{ N/m}}} = 0.99 \text{ s}
\]

Both particles will pass through the position \( x = 0 \text{ m} \) for the first time one-quarter of the way through one cycle, or

\[
\Delta t = \frac{1}{4} T_1 = \frac{1}{4} T_2 = \frac{0.99 \text{ s}}{4} = 0.25 \text{ s}
\]

b. \( T_1 = 0.99 \text{ s} \), as in part (a) above, while

\[
T_2 = 2\pi \sqrt{\frac{27.0 \text{ kg}}{120 \text{ N/m}}} = 3.0 \text{ s}
\]

Each particle will pass through the position \( x = 0 \text{ m} \) every odd-quarter of a cycle, \( \frac{1}{4} T_1, \frac{3}{4} T_1, \frac{5}{4} T_1, \ldots \). Thus, the two particles will pass through \( x = 0 \text{ m} \) when

**3.0-kg particle**

\[
t = \frac{1}{4} T_1, \frac{3}{4} T_1, \frac{5}{4} T_1, \ldots
\]

**27.0-kg particle**

\[
t = \frac{1}{4} T_2, \frac{3}{4} T_2, \frac{5}{4} T_2, \ldots
\]

Since \( T_2 = 3T_1 \), we see that both particles will be at \( x = 0 \text{ m} \) simultaneously when \( t = \frac{3}{4} T_1 \), or \( t = \frac{1}{4} T_2 = \frac{3}{4} T_1 \). Thus,

\[
t = \frac{3}{4} T_1 = \frac{3}{4} \left( 0.99 \text{ s} \right) = 0.75 \text{ s}
\]
92. **REASONING AND SOLUTION** The frequency $f$ of the simple harmonic motion is given by Equations 10.6 and 10.11 as 

$$f = (1/2\pi) \sqrt{k/m}.$$ 

If we compare Equation 10.17, which governs the stretching and compression of a solid rod with Equation 10.1, we find that $x$ is analogous to $\Delta L$ and $k$ is analogous to the term $YA/L_0$:

$$F = \frac{YA}{L_0} \frac{\Delta L}{x}.$$ 

The value of Young’s modulus for copper is given in Table 10.1. Assuming that the rod has a circular cross-section, its area $A$ is equal to $\pi r^2$, and we have

$$f = \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \sqrt{\frac{YA}{L_0m}} = \frac{1}{2\pi} \sqrt{\frac{Y(\pi r^2)}{L_0m}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{(1.1 \times 10^{11} \text{ N/m}^2) \pi (3.0 \times 10^{-3} \text{ m})^2}{(2.0 \text{ m})(9.0 \text{ kg})}} = 66 \text{ Hz}$$
1. (b) According to the relation \( P_2 = P_1 + \rho gh \) (Equation 11.4), the pressure \( P_2 \) at the bottom of the container depends on the height \( h \) of the fluid above it. Since this height is the same for all three containers, the pressure at the bottom is the same for each container.

2. (d) According to the relation \( P_2 = P_1 + \rho gh \) (Equation 11.4), the pressure at any point depends on the height \( h \) of the fluid above it. Since this height is the same for the ceiling of chamber 1 and the floor of chamber 2, the pressure at these two locations is the same.

3. \( F = 3.0 \times 10^5 \text{ N} \)

4. (c) The pressure at the top of each liquid is the same, since the U-tube is open at both ends. Also, the pressure at the location of the dashed line is the same in both the left and right sides of the U tube, since these two locations are at the same level. Thus, the pressure increment \( \rho_1 gh_1 \) for liquid 1 must be equal to the pressure increment \( \rho_2 gh_2 \) for liquid 2, where \( h_1 \) and \( h_2 \) are the heights of the liquids above the dashed line. Since \( h_1 \) is greater than \( h_2 \), \( \rho_1 \) must be less than \( \rho_2 \).

5. (b) The pressure at the top of each liquid is the same, since the U-tube is open at both ends. Also, the pressure at the location of the dashed line is the same in both the left and right sides of the U tube, since these two locations are at the same level. Thus, the pressure increment \( \rho_1 gh_1 \) for liquid 1 must be equal to the pressure increment \( \rho_2 gh_2 \) for liquid 2, where \( h_1 \) and \( h_2 \) are the heights of the liquids above the dashed line. Since \( h_1 \) is 3 times as great as \( h_2 \), \( \rho_1 \) must be one-third that of \( \rho_2 \).

6. (b) According to the relation, \( P_2 = P_1 + \rho gh \) (Equation 11.4), a drop in the pressure \( P_1 \) at the top of the pool produces an identical drop in the pressure \( P_2 \) at the bottom of the pool.

7. \( W = 14\,000 \text{ N} \)

8. (a) According to Archimedes’ principle, the buoyant force equals the weight of the fluid that the object displaces. Both objects displace the same weight of fluid, since they have the same volume. The buoyant force does not depend on the depth of an object.

9. (d) The buoyant force (19.6 N) is less than the weight (29.4 N) of the object. Therefore, the object sinks.
10. (e) When an object floats, its weight \( \left( \rho_{\text{object}} V_{\text{object}} g \right) \) equals the buoyant force \( \left( \rho_{\text{fluid}} V_{\text{displaced}} g \right) \), where \( V_{\text{displaced}} \) is the volume of fluid displaced by the object. Thus, \( \rho_{\text{object}} V_{\text{object}} g = \rho_{\text{fluid}} V_{\text{displaced}} g \), so the density of the object is \( \rho_{\text{object}} = \rho_{\text{fluid}} \left( V_{\text{displaced}} / V_{\text{object}} \right) \). Thus, the density of the object is proportional to the ratio \( V_{\text{displaced}} / V_{\text{object}} \) of the volumes. This ratio is greatest for object C and least for B.

11. (a) The beaker with the ball contains less water, because part of the ball is below the water line. According to Archimedes’ principle, the weight of this “missing” (or displaced) water is equal to the magnitude of the buoyant force that acts on the ball. Since the ball is floating, the magnitude of the buoyant force equals the weight of the ball. Thus, the weight of the missing water is exactly equal to the weight of the ball, so the two beakers weigh the same.

12. (e) The volume flow rate is equal to the speed of the water times the cross-sectional area through which the water flows (see Equation 11.10). If the speed doubles and the cross-sectional area triples, the volume flow rate increases by a factor of six \((2 \times 3)\).

13. (c) Because water is incompressible and no water accumulates within the pipe, the volume of water per second flowing through the wide section is equal to that flowing through the narrow section. Thus, the volume flow rate is the same in both sections.

14. (b) Water is incompressible, so it cannot accumulate anywhere within the pipe. Thus, the volume flow rate is the same everywhere. Since the volume flow rate is equal to the speed of the water times the cross-sectional area of the pipe (see Equation 11.10), the speed is greatest where the cross-sectional area is smallest.

15. (b) The volume flow rate \( Q \) of the blood is the same everywhere. Since \( Q = A v \), we have that \( A_2 v_2 = A_1 v_1 \), where the subscript 2 denotes the unblocked region of the artery and 1 the partially blocked region. Since the area of a circle is \( A = \pi r^2 \), the speed \( v_1 \) of the blood in the partially blocked region is \( v_1 = v_2 \left( \frac{r_2^2}{r_1^2} \right) = \left( \frac{2.0 \text{ mm}}{1.0 \text{ mm}} \right) = 1.6 \text{ m/s} \).

16. (a) Since blood is an incompressible fluid, the volume flow rate \( Q \) is the same everywhere within the artery. \( Q \) is equal to the cross-sectional area of the artery times the speed of the blood, so the blood slows down, or decelerates, as it moves from the narrow region into the wider region. Because the hemoglobin molecule decelerates, the direction of the net force acting on it must be opposite to its velocity. Therefore, the pressure ahead of the molecule is greater than that behind it, so the pressure in the wider region is greater than that in the narrow region.

17. \( P_1 - P_2 = 207 \text{ Pa} \)
18. (b) The pressure at C is greater than that at B. These two points are at the same elevation, but the fluid is moving slower at C since it has a greater cross-sectional area. Since the fluid is moving slower at C, its pressure is greater. The pressure at B is greater than that at A. The speed of the fluid is the same at both points, since the pipe has the same cross-sectional area. However, B is at the lower elevation and, consequently, has more water above it than A. The greater the height of fluid above a given point, the greater is the pressure at that point, provided the cross-sectional area does not change.

19. \( P_B - P_A = 12\,000 \text{ Pa} \)

20. (d) A longer pipe offers a greater resistance to the flow of a viscous fluid than a shorter pipe does. The volume flow rate depends inversely on the length of the pipe. For a given pipe radius and pressure difference between the ends of the pipe, the volume flow rate is less in longer pipes. In this case the longer pipe is twice as long, so its volume flow rate \( Q_B \) is one-half that of \( Q_A \).

21. \( Q_B = 1.62 \times 10^4 \text{ m}^3/\text{s} \)
PROBLEMS

1. **SSM REASONING** According to Equation 4.5, the pillar’s weight is \( W = mg \). Equation 11.1 can be solved for the mass \( m \) to show that the pillar’s mass is \( m = \rho V \). The volume \( V \) of the cylindrical pillar is its height times its circular cross-sectional area.

**SOLUTION** Expressing the weight as \( W = mg \) (Equation 4.5) and substituting \( m = \rho V \) (Equation 11.1) for the mass give

\[
W = mg = (\rho V)g
\]

The volume of the pillar is \( V = h\pi r^2 \), where \( h \) is the height and \( r \) is the radius of the pillar. Substituting this expression for the volume into Equation (1), we find that the weight is

\[
W = (\rho V)g = \left[ \rho \left( h\pi r^2 \right) \right]g = \left( 2.2 \times 10^3 \text{ kg/m}^3 \right) \left( 2.2 \text{ m} \right) \pi \left( 0.50 \text{ m} \right)^2 \left( 9.80 \text{ m/s}^2 \right) = 3.7 \times 10^4 \text{ N}
\]

Converting newtons (N) to pounds (lb) gives

\[
W = \left( 3.7 \times 10^4 \text{ N} \right) \left( \frac{0.2248 \text{ lb}}{1 \text{ N}} \right) = 8.3 \times 10^3 \text{ lb}
\]

2. **REASONING** The density \( \rho \) of the solvent is given by \( \rho = \frac{m}{V} \) (Equation 11.1), where \( m \) is the mass of the solvent and \( V \) is its volume. The solvent occupies a cylindrical tank of radius \( r \) and height \( h \). Its volume \( V \), therefore, is the product of the circular cross-sectional area \( \pi r^2 \) of the tank and the height \( h \) of the solvent:

\[
V = \pi r^2 h
\]

**SOLUTION** Substituting Equation (1) into \( \rho = \frac{m}{V} \) (Equation 11.1), we obtain the density of the solvent:

\[
\rho = \frac{m}{V} = \frac{m}{\pi r^2 h} = \frac{14300 \text{ kg}}{\pi \left( 1.22 \text{ m} \right)^2 \left( 3.71 \text{ m} \right)} = 824 \text{ kg/m}^3
\]
3. **REASONING** Equation 11.1 can be used to find the volume occupied by 1.00 kg of silver. Once the volume is known, the area of a sheet of silver of thickness \( d \) can be found from the fact that the volume is equal to the area of the sheet times its thickness.

**SOLUTION** Solving Equation 11.1 for \( V \) and using a value of \( \rho = 10 \, 500 \, \text{kg/m}^3 \) for the density of silver (see Table 11.1), we find that the volume of 1.00 kg of silver is

\[
V = \frac{m}{\rho} = \frac{1.00 \, \text{kg}}{10 \, 500 \, \text{kg/m}^3} = 9.52 \times 10^{-5} \, \text{m}^3
\]

The area of the silver, is, therefore,

\[
A = \frac{V}{d} = \frac{9.52 \times 10^{-5} \, \text{m}^3}{3.00 \times 10^{-7} \, \text{m}} = 317 \, \text{m}^2
\]

4. **REASONING AND SOLUTION**

a. We will treat the neutron star as spherical in shape, so that its volume is given by the familiar formula, \( V = \frac{4}{3}\pi r^3 \). Then, according to Equation 11.1, the density of the neutron star described in the problem statement is

\[
\rho = \frac{m}{V} = \frac{3m}{4\pi r^3} = \frac{3(2.7 \times 10^{28} \, \text{kg})}{4\pi(1.2 \times 10^3 \, \text{m})^3} = 3.7 \times 10^{18} \, \text{kg/m}^3
\]

b. If a dime of volume \( 2.0 \times 10^{-7} \, \text{m}^3 \) were made of this material, it would weigh

\[
W = mg = \rho V g = (3.7 \times 10^{18} \, \text{kg/m}^3)(2.0 \times 10^{-7} \, \text{m}^3)(9.80 \, \text{m/s}^2) = 7.3 \times 10^{12} \, \text{N}
\]

This weight corresponds to

\[
7.3 \times 10^{12} \, \text{N} \left(\frac{1 \, \text{lb}}{4.448 \, \text{N}}\right) = 1.6 \times 10^{12} \, \text{lb}
\]

5. **REASONING** The density of the brass ball is given by Equation 11.1: \( \rho = m/V \). Since the ball is spherical, its volume is given by the expression \( V = \frac{4}{3}\pi r^3 \), so that the density may be written as

\[
\rho = \frac{m}{V} = \frac{3m}{4\pi r^3}
\]

This expression can be solved for the radius \( r \); but first, we must eliminate the mass \( m \) from the equation since its value is not specified explicitly in the problem. We can determine an expression for the mass \( m \) of the brass ball by analyzing the forces on the ball.

The only two forces that act on the ball are the upward tension \( T \) in the wire and the downward weight \( mg \) of the ball. If we take up as the positive direction, and we apply
Newton's second law, we find that \( T - mg = 0 \), or \( m = T / g \). Therefore, the density can be written as

\[
\rho = \frac{3m}{4\pi r^3} = \frac{3T}{4\pi r^3 g}
\]

This expression can now be solved for \( r \).

**SOLUTION** We find that

\[
r^3 = \frac{3T}{4\pi \rho g}
\]

Taking the cube root of both sides of this result and using a value of \( \rho = 8470 \text{ kg/m}^3 \) for the density of brass (see Table 11.1), we find that

\[
r = \sqrt[3]{\frac{3T}{4\pi \rho g}} = \sqrt[3]{\frac{3(120 \text{ N})}{4\pi (8470 \text{ kg/m}^3) (9.80 \text{ m/s}^2)}} = 7.0 \times 10^{-2} \text{ m}
\]

### 6. REASONING

The mass \( m \) of an amount of material is related to its density \( \rho \) and volume \( V \) according to \( m = \rho V \) (Equation 11.1). We will apply this expression separately to the helium and to the silver. In doing so, we will used the fact that the volume of a sphere is \( V = \frac{4}{3} \pi r^3 \), where \( r \) is the radius of the sphere.

**SOLUTION** Using Equation 11.1 to relate mass to density and volume and recognizing that the mass of the silver is equal to the mass of the helium, we have

\[
\mathbf{\rho}_H \mathbf{V}_H = \mathbf{\rho}_S \mathbf{V}_S
\]

Using the fact that the volume of a sphere is \( V = \frac{4}{3} \pi r^3 \), we write Equation (1) as follows:

\[
\rho_H \left[ \frac{4}{3} \pi (R - T)^3 \right] = \rho_S \left[ \frac{4}{3} \pi R^3 - \frac{4}{3} \pi (R - T)^3 \right]
\]

Note that the factor of \( \frac{4}{3} \pi \) appears in each term of this result and can be eliminated algebraically, so that we have

\[
\rho_H (R - T)^3 = \rho_S \left[ R^3 - (R - T)^3 \right] = \rho_S \left[ R^3 - R^3 + 3RT^2 - T^3 \right]
\]

Since \( T \) is assumed to be much less than \( R \), the left side of Equation (2) can be written simply as \( \rho_H R^3 \), with the result that
\[
\rho_H R^3 = \rho_S \left[3R^2 T - 3RT^2 + T^3\right] \quad \text{or} \quad \rho_H = \rho_S \left(\frac{3T}{R} - \frac{3T^2}{R^2} + \frac{T^3}{R^3}\right)
\]

In this result, since \( T \) is assumed to be much less than \( R \), the terms \( T^2 / R^2 \) and \( T^3 / R^3 \) are each much less than the term \( T/R \). Thus, with values for the densities of helium and silver taken from Table 11.1 in the text, we have

\[
\rho_H = \rho_S \left(\frac{3T}{R}\right) \quad \text{or} \quad \frac{T}{R} = \frac{\rho_H}{3\rho_S} = \frac{0.179 \text{ kg} / \text{m}^3}{3(10 \, 500 \text{ kg} / \text{m}^3)} = 5.68 \times 10^{-6}
\]

7. **SSM Reasoning** According to the definition of density \( \rho \) given in Equation 11.1, the mass \( m \) of a substance is \( m = \rho V \), where \( V \) is the volume. We will use this equation and the fact that the mass of the water and the gold are equal to find our answer. To convert from a volume in cubic meters to a volume in gallons, we refer to the inside of the front cover of the text to find that 1 gal = \( 3.785 \times 10^{-3} \) m\(^3\).

**Reasoning** Using Equation 11.1, we find that

\[
\rho_{\text{water}} V_{\text{water}} = \rho_{\text{gold}} V_{\text{gold}} \quad \text{or} \quad V_{\text{water}} = \frac{\rho_{\text{gold}} V_{\text{gold}}}{\rho_{\text{water}}}
\]

Using the fact that 1 gal = \( 3.785 \times 10^{-3} \) m\(^3\) and densities for gold and water from Table 11.1, we find

\[
V_{\text{water}} = \frac{\rho_{\text{gold}} V_{\text{gold}}}{\rho_{\text{water}}} = \frac{(19 \, 300 \text{ kg} / \text{m}^3)(0.15 \text{ m})(0.050 \text{ m})(0.050 \text{ m})}{(1000 \text{ kg} / \text{m}^3)} \left(\frac{1 \text{ gal}}{3.785 \times 10^{-3} \text{ m}^3}\right) = 1.9 \text{ gal}
\]

8. **Reasoning and Solution** We refer to Table 11.1 for values of \( \rho_{\text{soda}} = \rho_{\text{water}} = 1.000 \times 10^3 \text{ kg} / \text{m}^3 \) and \( \rho_{\text{Al}} = 2700 \text{ kg} / \text{m}^3 \) for the density of aluminum. The mass of aluminum required to make the can is

\[
m_{\text{Al}} = m_{\text{total}} - m_{\text{soda}}
\]

where the mass of the soda is

\[
m_{\text{soda}} = \rho_{\text{soda}} V_{\text{soda}} = (1.000 \times 10^3 \text{ kg} / \text{m}^3)(3.54 \times 10^{-4} \text{ m}^3) = 0.354 \text{ kg}
\]

Therefore, the mass of the aluminum used to make the can is

\[
m_{\text{Al}} = 0.416 \text{ kg} - 0.354 \text{ kg} = 0.062 \text{ kg}
\]
Using the definition of density, \( \rho = \frac{m}{V} \), we have

\[
V_{Al} = \frac{m_{Al}}{\rho_{Al}} = \frac{0.062 \text{ kg}}{2700 \text{ kg/m}^3} = 2.3 \times 10^{-5} \text{ m}^3
\]

9. **REASONING** The period \( T \) of a satellite is the time for it to make one complete revolution around the planet. The period is the circumference of the circular orbit \((2\pi R)\) divided by the speed \( v \) of the satellite, so that \( T = \frac{2\pi R}{v} \) (see Equation 5.1). In Section 5.5 we saw that the centripetal force required to keep a satellite moving in a circular orbit is provided by the gravitational force. This relationship tells us that the speed of the satellite must be \( v = \sqrt{\frac{GM}{R}} \) (Equation 5.5), where \( G \) is the universal gravitational constant and \( M \) is the mass of the planet. By combining this expression for the speed with that for the period, and using the definition of density, we can obtain the period of the satellite.

**SOLUTION** The period of the satellite is

\[
T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = 2\pi \sqrt{\frac{R^3}{GM}}
\]

According to Equation 11.1, the mass of the planet is equal to its density \( \rho \) times its volume \( V \). Since the planet is spherical, \( V = \frac{4}{3} \pi R^3 \). Thus, \( M = \rho V = \rho \left( \frac{4}{3} \pi R^3 \right) \). Substituting this expression for \( M \) into that for the period \( T \) gives

\[
T = 2\pi \sqrt{\frac{R^3}{GM}} = 2\pi \sqrt{\frac{R^3}{G\rho \left( \frac{4}{3} \pi R^3 \right)}} = \frac{3\pi}{\sqrt{G\rho}}
\]

The density of iron is \( \rho = 7860 \text{ kg/m}^3 \) (see Table 11.1), so the period of the satellite is

\[
T = \frac{3\pi}{\sqrt{G\rho}} = \frac{3\pi}{\sqrt{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \cdot 7860 \text{ kg/m}^3}} = 4240 \text{ s}
\]

10. **REASONING** The total mass of the solution is the sum of the masses of its constituents. Therefore,

\[
\rho_s V_s = \rho_w V_w + \rho_g V_g
\]

where the subscripts \( s, w, \) and \( g \) refer to the solution, the water, and the ethylene glycol, respectively. The volume of the water can be written as \( V_w = V_s - V_g \). Making this substitution for \( V_w \), Equation (1) above can be rearranged to give

\[
\frac{V_g}{V_s} = \frac{\rho_s - \rho_w}{\rho_g - \rho_w}
\]
Equation (2) can be used to calculate the relative volume of ethylene glycol in the solution.

**SOLUTION** The density of ethylene glycol is given in the problem. The density of water is given in Table 11.1 as 1.000×10³ kg/m³. The specific gravity of the solution is given as 1.0730. Therefore, the density of the solution is

\[ \rho_s = (\text{specific gravity of solution}) \times \rho_w \]

\[ = (1.0730)(1.000 \times 10^3 \text{ kg/m}^3) = 1.0730 \times 10^3 \text{ kg/m}^3 \]

Substituting the values for the densities into Equation (2), we obtain

\[ \frac{V_g}{V_s} = \frac{\rho_s - \rho_w}{\rho_g - \rho_w} = \frac{1.0730 \times 10^3 \text{ kg/m}^3 - 1.000 \times 10^3 \text{ kg/m}^3}{1116 \text{ kg/m}^3 - 1.000 \times 10^3 \text{ kg/m}^3} = 0.63 \]

Therefore, the volume percentage of ethylene glycol is 63%.

11. **REASONING** Since the inside of the box is completely evacuated; there is no air to exert an upward force on the lid from the inside. Furthermore, since the weight of the lid is negligible, there is only one force that acts on the lid; the downward force caused by the air pressure on the outside of the lid. In order to pull the lid off the box, one must supply a force that is at least equal in magnitude and opposite in direction to the force exerted on the lid by the outside air.

**SOLUTION** According to Equation 11.3, pressure is defined as \( P = F/A \); therefore, the magnitude of the force on the lid due to the air pressure is

\[ F = (0.85 \times 10^5 \text{ N/m}^2)(1.3 \times 10^{-2} \text{ m}^2) = 1.1 \times 10^3 \text{ N} \]

12. **REASONING** Since the weight is distributed uniformly, each tire exerts one-half of the weight of the rider and bike on the ground. According to the definition of pressure, Equation 11.3, the force that each tire exerts on the ground is equal to the pressure \( P \) inside the tire times the area \( A \) of contact between the tire and the ground. From this relation, the area of contact can be found.

**SOLUTION** The area of contact that each tire makes with the ground is

\[ A = \frac{F}{P} = \frac{\frac{1}{2}(W_{\text{person}} + W_{\text{bike}})}{P} = \frac{\frac{1}{2}(625 \text{ N} + 98 \text{ N})}{7.60 \times 10^5 \text{ Pa}} = 4.76 \times 10^{-4} \text{ m}^2 \]  

(11.3)

13. **REASONING** According to Equation 11.3, the pressure \( P \) exerted on the ground by the stack of blocks is equal to the force \( F \) exerted by the blocks (their combined weight) divided by the area \( A \) of the block’s surface in contact with the ground, or \( P = F/A \). Since the pressure is largest when the area is smallest, the least number of blocks is used when the surface area in contact with the ground is the smallest. This area is 0.200 m × 0.100 m.
**SOLUTION** The pressure exerted by \( N \) blocks stacked on top of one another is

\[
P = \frac{F}{A} = \frac{NW_{\text{one block}}}{A}
\]

where \( W_{\text{one block}} \) is the weight of one block. The least number of whole blocks required to produce a pressure of two atmospheres \((2.02 \times 10^5 \text{ Pa})\) is

\[
N = \frac{PA}{W_{\text{one block}}} = \left(2.02 \times 10^5 \text{ Pa}\right) \left(0.200 \text{ m} \times 0.100 \text{ m}\right) = \frac{169 \text{ N}}{169 \text{ N}} = 24
\]

14. **REASONING** Pressure is the magnitude of the force applied perpendicularly to a surface divided by the area of the surface, according to Equation 11.3. The force magnitude, therefore, is equal to the pressure times the area.

**SOLUTION** According to Equation 11.3, we have

\[
F = PA = \left(8.0 \times 10^4 \text{ lb/in.}^2\right) \left(6.1 \text{ in.}\right) \left(2.6 \text{ in.}\right) = 1.3 \times 10^6 \text{ lb}
\]

15. **SSM REASONING** The cap is in equilibrium, so the sum of all the forces acting on it must be zero. There are three forces in the vertical direction: the force \( F_{\text{inside}} \) due to the gas pressure inside the bottle, the force \( F_{\text{outside}} \) due to atmospheric pressure outside the bottle, and the force \( F_{\text{thread}} \) that the screw thread exerts on the cap. By setting the sum of these forces to zero, and using the relation \( F = PA \), where \( P \) is the pressure and \( A \) is the area of the cap, we can determine the magnitude of the force that the screw threads exert on the cap.

**SOLUTION** The drawing shows the free-body diagram of the cap and the three vertical forces that act on it. Since the cap is in equilibrium, the net force in the vertical direction must be zero.

\[
\sum F_y = -F_{\text{thread}} + F_{\text{inside}} - F_{\text{outside}} = 0
\]

Solving this equation for \( F_{\text{thread}} \), and using the fact that force equals pressure times area, \( F = PA \) (Equation 11.3), we have

\[
F_{\text{thread}} = F_{\text{inside}} - F_{\text{outside}} = P_{\text{inside}}A - P_{\text{outside}}A
\]

\[
= \left(P_{\text{inside}} - P_{\text{outside}}\right)A = \left(1.80 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}\right) \left(4.10 \times 10^{-4} \text{ m}^2\right) = 32 \text{ N}
\]
16. **REASONING** Pressure is defined in Equation 11.3 as the magnitude of the force acting perpendicular to a surface divided by the area over which the force acts. The force acting perpendicular to the slope is due to the component of the skier’s weight $W$ that is directed perpendicular to the slope. In the drawing at the right, this component is labeled $W_{\text{perpendicular}}$. Note that the fact that the skier is moving is of no importance.

**SOLUTION** We assume that each ski bears the same amount of force, namely $\frac{1}{2}W_{\text{perpendicular}}$ (see the drawing). According to Equation 11.3, the pressure that each ski applies to the snow is

$$P = \frac{\frac{1}{2}W_{\text{perpendicular}}}{A}$$

where $A$ is the area of each ski in contact with the snow. From the drawing, we see that $W_{\text{perpendicular}} = W \cos 35^\circ$, so that the pressure exerted by each ski on the snow is

$$P = \frac{\frac{1}{2}W_{\text{perpendicular}}}{A} = \frac{W \cos 35^\circ}{2A} = \frac{(58 \text{ kg})(9.80 \text{ m/s}^2) \cos 35^\circ}{2 \left(0.13 \text{ m}^2\right)} = 1.8 \times 10^3 \text{ Pa}$$

where we have used the fact that $W = mg$ (Equation 4.5).

17. **REASONING** The pressure $P$ due to the force $F_{\text{SonF}}$ that the suitcase exerts on the elevator floor is given by $P = \frac{F_{\text{SonF}}}{A}$ (Equation 11.3), where $A$ is the area of the elevator floor beneath the suitcase (equal to the product of the length and width of that region). According to Newton’s 3rd law, the magnitude $F_{\text{SonF}}$ of the downward force the suitcase exerts on the floor is equal to the magnitude $F_{\text{FonS}}$ of the upward force the floor exerts on the suitcase. We will use Newton’s 2nd law, $\sum F = ma$ (Equation 4.1), to determine the magnitude $F_{\text{FonS}}$ of the upward force on the suitcase, which has a mass $m$ and an upward acceleration of magnitude $a = 1.5 \text{ m/s}^2$, equal to that of the elevator.

**SOLUTION** There are only two forces acting on the suitcase, the upward force $F_{\text{FonS}}$ that the floor exerts on the suitcase, and the downward weight $W = mg$ (Equation 4.5) exerted by the earth, where $g$ is the magnitude of the acceleration due to gravity. Taking upwards as the positive direction, Newton’s 2nd law yields

$$\sum F = F_{\text{FonS}} - mg = ma$$

Solving Equation (1) for $F_{\text{FonS}}$, and noting that by Newton’s 3rd law, $F_{\text{FonS}} = F_{\text{SonF}}$, we obtain
\[ F_{\text{SonS}} = mg + ma = m(g + a) = F_{\text{SonF}} \]  

(2)

Substituting Equation (2) into \[ P = \frac{F_{\text{SonF}}}{A} \] (Equation 11.3), we find that

\[ P = \frac{F_{\text{SonF}}}{A} = \frac{m(g + a)}{A} = \frac{(16 \text{ kg})(9.80 \text{ m/s}^2 + 1.5 \text{ m/s}^2)}{(0.15 \text{ m})(0.50 \text{ m})} = 2400 \text{ Pa} \]

18. **REASONING** The spring is compressed because the piston applies a force to it. The displacement \( x \) of the spring under the influence of an applied force \( F_{\text{Applied}} \) can be obtained from \( F_{\text{Applied}} = kx \) (Equation 10.1), where \( k \) is the spring constant. The applied force arises because of the atmospheric pressure \( P \), according to \( F_{\text{Applied}} = PA \) (Equation 11.3), where \( A \) is the circular area of the piston. The work done by the atmospheric pressure in compressing the spring is equal to the elastic potential energy stored in the spring, since friction is absent. The elastic potential energy is \( PE_{\text{elastic}} = \frac{1}{2} kx^2 \) (Equation 10.13).

**SOLUTION**

a. According to Equation 10.1, the displacement of the spring is \( x = \frac{F_{\text{Applied}}}{k} \), where the applied force is given by Equation 11.3 as \( F_{\text{Applied}} = PA \). Therefore, we have

\[ x = \frac{F_{\text{Applied}}}{k} = \frac{PA}{k} \]

Since the area of a circle is \( A = \pi r^2 \), this result becomes

\[ x = \frac{PA}{k} = \frac{P \pi r^2}{k} = \frac{(1.013 \times 10^5 \text{ N/m}^2) \pi (0.024 \text{ m})^2}{3600 \text{ N/m}} = 0.051 \text{ m} \]

b. The work \( W \) done by the atmospheric pressure in compressing the spring is equal to the elastic potential energy stored in the spring, so that with the aid of Equation 10.13 we obtain

\[ W = PE_{\text{elastic}} = \frac{1}{2} kx^2 = \frac{1}{2} (3600 \text{ N/m})(0.051 \text{ m})^2 = 4.7 \text{ J} \]

19. **REASONING** The tension forces \( T \) acting on the edges of the square section of the bladder wall add up to give a total inward force \( F_{\text{in}} \). Because the section is in equilibrium, the net force acting on it must be zero. Therefore, the inward force \( F_{\text{in}} \) balances the outward force \( F_{\text{out}} \) due to the internal pressure \( P \) of the bladder: \( F_{\text{in}} = F_{\text{out}} \). The magnitude \( F_{\text{out}} \) of the outward force is found from \( F_{\text{out}} = PA \) (Equation 11.3), where \( A \) is the area of the outer surface of the bladder wall. Thus, we have that
\[ F_{\text{in}} = F_{\text{out}} = PA \]  

\[ F_{\text{in}} = 4T_{\text{in}} = 4T \sin \theta \quad \text{or} \quad T = \frac{F_{\text{in}}}{4 \sin \theta} \]  

Substituting \( F_{\text{in}} = PA \) [Equation (1)] into Equation (2) yields

\[ T = \frac{PA}{4 \sin \theta} \]  

The area \( A \) of the square is the product of the lengths \( l = 0.010 \text{ m} \) of two of its sides: \( A = l^2 \). Making this substitution into Equation (3), we obtain

\[ T = \frac{PA}{4 \sin \theta} = \frac{P l^2}{4 \sin \theta} = \frac{(3300 \text{ Pa})(0.010 \text{ m})^2}{4 \sin 5.0^\circ} = 0.95 \text{ N} \]

20. **REASONING** The magnitude of the force that would be exerted on the window is given by Equation 11.3, \( F = PA \), where the pressure can be found from Equation 11.4: \( P_2 = P_1 + \rho gh \). Since \( P_1 \) represents the pressure at the surface of the water, it is equal to atmospheric pressure, \( P_{\text{atm}} \). Therefore, the magnitude of the force is given by

\[ F = ( P_{\text{atm}} + \rho gh) A \]

where, if we assume that the window is circular with radius \( r \), its area \( A \) is given by \( A = \pi r^2 \).

**SOLUTION**

a. Thus, the magnitude of the force is

\[ F = 1.013 \times 10^5 \text{ Pa} + (1025 \text{ kg}/\text{m}^3)(9.80 \text{ m}/\text{s}^2)(11,000 \text{ m}) \pi (0.10 \text{ m})^2 = 3.5 \times 10^6 \text{ N} \]
b. The weight of a jetliner whose mass is \(1.2 \times 10^5\) kg is

\[
W = mg = (1.2 \times 10^5\ \text{kg})(9.80\ \text{m/s}^2) = 1.2 \times 10^6\ \text{N}
\]

Therefore, the force exerted on the window at a depth of 11 000 m is about three times greater than the weight of a jetliner!

21. **REASONING** The drawing at the right shows the situation. As discussed in Conceptual Example 6, the job of the pump is to draw air out of the pipe that dips down into the water. The atmospheric pressure in the well then pushes the water upward into the pipe. In the drawing, the best the pump can do is to remove all of the air, in which case, the pressure \(P_1\) at the top of the water in the pipe is zero. The pressure \(P_2\) at the bottom of the pipe at point \(A\) is the same as that at the point \(B\), namely, it is equal to atmospheric pressure \((1.013 \times 10^5\ \text{Pa})\), because the two points are at the same elevation, and point \(B\) is open to the atmosphere. Equation 11.4, \(P_2 = P_1 + \rho gh\) can be applied to obtain the maximum depth \(h\) of the well.

**SOLUTION** Setting \(P_1 = 0\ \text{Pa}\), and solving Equation 11.4 for \(h\), we have

\[
h = \frac{P_1}{\rho g} = \frac{1.013 \times 10^5\ \text{Pa}}{(1.000 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 10.3\ \text{m}
\]

22. **REASONING** The atmospheric pressure outside the tube pushes the sauce up the tube, to the extent that the smaller pressure in the bulb allows it. The smaller the pressure in the bulb, the higher the sauce will rise. The height \(h\) to which the sauce rises is related to the atmospheric pressure \(P_{\text{Atmospheric}}\) outside the tube, the pressure \(P_{\text{Bulb}}\) in the bulb, and the density \(\rho\) of the sauce by \(P_{\text{Atmospheric}} = P_{\text{Bulb}} + \rho gh\) (Equation 11.4).

**SOLUTION**

a. Solving Equation (11.4) for the absolute pressure in the bulb when the height of the sauce is 0.15 m, we find that
Chapter 11 Problems

P_{	ext{Bulb}} = P_{\text{Atmospheric}} - \rho gh

= 1.013 \times 10^5 \text{ Pa} - \left(1200 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) (0.15 \text{ m}) = 9.95 \times 10^4 \text{ Pa}

b. When the height of the sauce is 0.10 m, the absolute pressure in the bulb is

P_{\text{Bulb}} = P_{\text{Atmospheric}} - \rho gh

= 1.013 \times 10^5 \text{ Pa} - \left(1200 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) (0.10 \text{ m}) = 1.001 \times 10^5 \text{ Pa}

23. **SSM REASONING** Since the faucet is closed, the water in the pipe may be treated as a static fluid. The gauge pressure $P_2$ at the faucet on the first floor is related to the gauge pressure $P_1$ at the faucet on the second floor by Equation 11.4, $P_2 = P_1 + \rho gh$.

**SOLUTION**

a. Solving Equation 11.4 for $P_1$, we find the gauge pressure at the second-floor faucet is

$$P_1 = P_2 - \rho gh = 1.90 \times 10^5 \text{ Pa} - \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) (6.50 \text{ m}) = 1.26 \times 10^5 \text{ Pa}$$

b. If the second faucet were placed at a height $h$ above the first-floor faucet so that the gauge pressure $P_1$ at the second faucet were zero, then no water would flow from the second faucet, even if it were open. Solving Equation 11.4 for $h$ when $P_1$ equals zero, we obtain

$$h = \frac{P_2 - P_1}{\rho g} = \frac{1.90 \times 10^5 \text{ Pa} - 0}{\left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right)} = 19.4 \text{ m}$$

24. **REASONING AND SOLUTION** The gauge pressure of the solution at the location of the vein is

$$P = \rho gh = (1030 \text{ kg/m}^3) \left(9.80 \text{ m/s}^2\right) (0.610 \text{ m}) = 6.16 \times 10^3 \text{ Pa}$$

Now

$$1.013 \times 10^5 \text{ Pa} = 760 \text{ mm Hg} \quad \text{so} \quad 1 \text{ Pa} = 7.50 \times 10^{-3} \text{ mm Hg}$$

Then

$$P = \left(6.16 \times 10^3 \text{ Pa}\right) \left(\frac{7.50 \times 10^{-3} \text{ mm Hg}}{1 \text{ Pa}}\right) = 46.2 \text{ mm Hg}$$

25. **REASONING** Since the diver uses a snorkel, the pressure in her lungs is atmospheric pressure. If she is swimming at a depth $h$ below the surface, the pressure outside her lungs is
atmospheric pressure plus that due to the water. The water pressure \( P_2 \) at the depth \( h \) is related to the pressure \( P_1 \) at the surface by Equation 11.4, \( P_2 = P_1 + \rho gh \), where \( \rho \) is the density of the fluid, \( g \) is the magnitude of the acceleration due to gravity, and \( h \) is the depth. This relation can be used directly to find the depth.

**SOLUTION** We are given that the maximum difference in pressure between the outside and inside of the lungs is one-twentieth of an atmosphere, or \( P_2 - P_1 = \frac{1}{20} (1.01 \times 10^5 \text{ Pa}) \).

Solving Equation 11.4 for the depth \( h \) gives

\[
h = \frac{P_2 - P_1}{\rho g} = \frac{1}{20} (1.01 \times 10^5 \text{ Pa}) \left( \frac{1025 \text{ kg/m}^3}{9.80 \text{ m/s}^2} \right) = 0.50 \text{ m}
\]

---

26. **REASONING** The pressure \( P_2 \) at a lower point in a static fluid is related to the pressure \( P_1 \) at a higher point by Equation 11.4, \( P_2 = P_1 + \rho gh \), where \( \rho \) is the density of the fluid, \( g \) is the magnitude of the acceleration due to gravity, and \( h \) is the difference in heights between the two points. This relation can be used directly to find the pressure in the artery in the brain.

**SOLUTION** Solving Equation 11.4 for pressure \( P_1 \) in the brain (the higher point), gives

\[
P_1 = P_2 - \rho gh = 1.6 \times 10^4 \text{ Pa} - \left( \frac{1060 \text{ kg/m}^3}{9.80 \text{ m/s}^2} \right)(0.45 \text{ m}) = 1.1 \times 10^4 \text{ Pa}
\]

---

27. **REASONING** The pressure \( P \) at a distance \( h \) beneath the water surface at the vented top of the water tower is \( P = P_{\text{atm}} + \rho gh \) (Equation 11.4). We note that the value for \( h \) in this expression is different for the two houses and must take into account the diameter of the spherical reservoir in each case.

**SOLUTION**

a. The pressure at the level of house A is given by Equation 11.4 as \( P_A = P_{\text{atm}} + \rho gh_A \). Now the height \( h_A \) consists of the 15.0 m plus the diameter \( d \) of the tank. We first calculate the radius of the tank, from which we can infer \( d \). Since the tank is spherical, its full mass is given by \( M = \rho V = \rho \left( \frac{4}{3} \pi r^3 \right) \). Therefore,

\[
r^3 = \frac{3M}{4\pi \rho} \quad \text{or} \quad r = \left( \frac{3M}{4\pi \rho} \right)^{1/3} = \left[ \frac{3(5.25 \times 10^5 \text{ kg})}{4\pi(1.000 \times 10^3 \text{ kg/m}^3)} \right]^{1/3} = 5.00 \text{ m}
\]

Therefore, the diameter of the tank is 10.0 m, and the height \( h_A \) is given by
\[ h_A = 10.0 \text{ m} + 15.0 \text{ m} = 25.0 \text{ m} \]

According to Equation 11.4, the gauge pressure in house A is, therefore,

\[ P_A - P_{\text{atm}} = \rho g h_A = (1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(25.0 \text{ m}) = 2.45 \times 10^5 \text{ Pa} \]

b. The pressure at house B is \( P_B = P_{\text{atm}} + \rho g h_B \), where

\[ h_B = 15.0 \text{ m} + 10.0 \text{ m} - 7.30 \text{ m} = 17.7 \text{ m} \]

According to Equation 11.4, the gauge pressure in house B is

\[ P_B - P_{\text{atm}} = \rho g h_B = (1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(17.7 \text{ m}) = 1.73 \times 10^5 \text{ Pa} \]

28. **REASONING** If the pressure is \( P_1 \) at a certain point in a static fluid (density = \( \rho \)), the pressure \( P_2 \) in the fluid at a distance \( h \) beneath this point is \( P_2 = P_1 + \rho g h \) (Equation 11.4). As discussed in Section 11.4, this expression becomes \( P_{\text{atm}} = \rho_{\text{mercury}} g h_{\text{mercury}} \) when applied to the mercury barometer shown in Figure 11.11 of the text.

**SOLUTION** Using Equation 11.4 with \( P_2 = P_{\text{ground}} \) and \( P_1 = P_{\text{roof}} \), we have

\[ P_{\text{ground}} = P_{\text{roof}} + \rho_{\text{air}} g h_{\text{air}} \quad \text{or} \quad P_{\text{ground}} - P_{\text{roof}} = \rho_{\text{air}} g h_{\text{air}} \]

In this result, \( h_{\text{air}} \) is the height of a column of air that equals the height of the building. We can also express \( P_{\text{ground}} - P_{\text{roof}} \) using the mercury barometer, which indicates that \( P_{\text{ground}} - P_{\text{roof}} \) corresponds to a column of mercury that has a height of \( h_{\text{mercury}} = 760.0 - 747.0 = 13.0 \text{ mm} \). Thus, we have

\[ P_{\text{ground}} - P_{\text{roof}} = \rho_{\text{mercury}} g h_{\text{mercury}} \]

Equating the two expressions for \( P_{\text{ground}} - P_{\text{roof}} \), we obtain

\[ P_{\text{ground}} - P_{\text{roof}} = \rho_{\text{air}} g h_{\text{air}} = \rho_{\text{mercury}} g h_{\text{mercury}} \quad \text{or} \quad \rho_{\text{air}} h_{\text{air}} = \rho_{\text{mercury}} h_{\text{mercury}} \]

Taking the density of mercury from Table 11.1 in the text and solving for \( h_{\text{air}} \) gives

\[ h_{\text{air}} = \frac{\rho_{\text{mercury}} h_{\text{mercury}}}{\rho_{\text{air}}} = \left( \frac{13600 \text{ kg/m}^3}{1.29 \text{ kg/m}^3} \right) \left( 13.0 \times 10^{-3} \text{ m} \right) = 137 \text{ m} \]
29. **REASONING AND SOLUTION** The pressure at the bottom of the container is

\[ P = P_{\text{atm}} + \rho_w g h_w + \rho_m g h_m \]

We want \( P = 2P_{\text{atm}} \), and we know \( h = h_w + h_m = 1.00 \text{ m} \). Using the above and rearranging gives

\[
h_m = \frac{P_{\text{atm}} - \rho_w g h}{(\rho_m - \rho_w) g} = \frac{1.01 \times 10^5 \text{ Pa} - (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.00 \text{ m})}{(13.6 \times 10^3 \text{ kg/m}^3 - 1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.74 \text{ m}
\]

30. **REASONING AND SOLUTION** The mercury, being more dense, will flow from the right container into the left container until the pressure is equalized. Then the pressure at the bottom of the left container will be \( P = P_{\text{atm}} + \rho_w g h_w + \rho_m g h_{mL} \) and the pressure at the bottom of the right container will be \( P = P_{\text{atm}} + \rho_m g h_{mR} \). Equating gives

\[
\rho_w g h_w + \rho_m g (h_{mL} - h_{mR}) = 0 \tag{1}
\]

Both liquids are incompressible and immiscible so

\[
h_w = 1.00 \text{ m} \text{ and } h_{mL} + h_{mR} = 1.00 \text{ m}
\]

Using these in (1) and solving for \( h_{mL} \) gives, \( h_{mL} = (1/2)(1.00 - \rho_w/\rho_m) = 0.46 \text{ m} \).

So the fluid level in the left container is \( 1.00 \text{ m} + 0.46 \text{ m} = 1.46 \text{ m} \) from the bottom.

31. **REASONING** Pressure is defined as the magnitude of the force acting perpendicular to a surface divided by the area of the surface. Thus, the magnitude of the total force acting on the vertical dam surface is the pressure times the area \( A \) of the surface. But exactly what pressure should we use? According to Equation 11.4, the pressure at any depth \( h \) under the water is \( P = P_{\text{atm}} + \rho g h \), where \( P_{\text{atm}} \) is the pressure of the atmosphere acting on the water at the top of the reservoir. Clearly, \( P \) has different values at different depths. As a result, we need to use an average pressure. In Equation 11.4, it is only the term \( \rho g h \) that depends on depth, and the dependence is linear. In other words, the value of \( \rho g h \) is proportional to \( h \). Therefore, the average value of this term is \( \rho g \left( \frac{1}{2} H \right) \), where \( H \) is the total depth of the water in the full reservoir and \( \frac{1}{2} H \) is the average depth. The average pressure acting on the vertical surface of the dam in contact with the water is, then,

\[
\bar{P} = P_{\text{atm}} + \rho g \left( \frac{1}{2} H \right) \tag{1}
\]
According to the definition of pressure given in Equation 11.3, the magnitude \( F_{\text{total}} \) of the total force acting on the vertical surface of the dam is \( F_{\text{total}} = \bar{P}A \), where \( \bar{P} \) is given by Equation (1). Using Equation (1) to substitute for \( \bar{P} \) gives

\[
F_{\text{total}} = \bar{P}A = \left[ P_{\text{atm}} + \rho g \left( \frac{1}{2} H \right) \right] A
\]

The area is \( A = (120 \text{ m})(12 \text{ m}) \), and \( P_{\text{atm}} = 1.01 \times 10^5 \text{ Pa} \). Table 11.1 gives the density of water as \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \). With these values, we find that

\[
F_{\text{total}} = \left[ 1.01 \times 10^5 \text{ Pa} + \left( 1.00 \times 10^3 \text{ kg/m}^3 \right) \left( 9.80 \text{ m/s}^2 \right) \frac{1}{2} (12 \text{ m}) \right] (120 \text{ m})(12 \text{ m}) = 2.3 \times 10^8 \text{ N}
\]

32. **REASONING** Both the outside air and the attic air exert a force perpendicular to the roof surfaces. The outside air pushes inward toward the attic, and the attic air pushes outward. When the inside and the outside air pressures are equal, the forces on the roof balance to a net force of zero. However, when the pressure of the attic air exceeds the pressure of the outside air, the excess pressure creates a net force acting perpendicular to each half of the roof and directed from the inside toward the outside. The magnitude \( F \) of this force is \( F = PA \) (Equation 11.3), with \( P \) being the excess pressure and \( A \) being the area of each half of the roof. The drawing at the right shows the force on the right half of the roof.

**SOLUTION**

a. Since we seek the force in newtons (N), we begin by converting the excess pressure in the attic from millimeters of mercury to N/m\(^2\) or pascals (Pa).

\[
P = \left( 75.0 \text{ mm mercury} \right) \left( \frac{1.013 \times 10^5 \text{ Pa}}{760.0 \text{ mm mercury}} \right) = 1.00 \times 10^4 \text{ Pa}
\]

The drawing in the **REASONING** shows the force acting on the right half of the roof. The components of this force are

**Horizontal (+x) direction** \( F^x_{\text{right}} = F \sin \theta \)

**Vertical (+y) direction** \( F^y_{\text{right}} = F \cos \theta \)

For the left half of the roof (not shown in the drawing) the results are
Horizontal (+x) direction \( F_x^{\text{left}} = -F \sin \theta \)

Vertical (+y) direction \( F_y^{\text{left}} = F \cos \theta \)

The components combine separately to give the components of the net force acting on the entire roof:

Horizontal (+x) direction \( F_x = F \sin \theta - F \sin \theta = 0 \)

Vertical (+y) direction \( F_y = F \cos \theta + F \cos \theta = 2F \cos \theta \)

Since the horizontal components balance to zero, the net force acting on the entire roof has only a vertical component and points vertically upward. Therefore, using \( F = PA \) (Equation 11.3), we obtain

Net force \( = 2F \cos \theta = 2PA \cos \theta \)

\[
= 2 \left( 1.00 \times 10^4 \text{ Pa} \right) \left( 14.5 \text{ m} \right) \left( 4.21 \text{ m} \cos 30.0^\circ \right) = 1.06 \times 10^6 \text{ N, vertically upward}
\]

b. To determine the net force in pounds, we convert the answer from part a using the fact that \( 1 \text{ N} = 0.2248 \text{ lb} \) (see page facing the inside of the front cover of the text):

\[
\text{Net force} = \left( 1.06 \times 10^6 \text{ N} \right) \left( \frac{0.2248 \text{ lb}}{1 \text{ N}} \right) = 2.38 \times 10^5 \text{ lb}
\]

33. **REASONING** According to Equation 11.4, the initial pressure at the bottom of the pool is \( P_0 = (P_{\text{atm}})_0 + \rho gh \), while the final pressure is \( P_f = (P_{\text{atm}})_f + \rho gh \). Therefore, the change in pressure at the bottom of the pool is

\[
\Delta P = P_f - P_0 = \left( P_{\text{atm}} \right)_f + \rho gh - \left( P_{\text{atm}} \right)_0 - \rho gh = \left( P_{\text{atm}} \right)_f - \left( P_{\text{atm}} \right)_0
\]

According to Equation 11.3, \( F = PA \), the change in the force at the bottom of the pool is

\[
\Delta F = (\Delta P) A = \left( P_{\text{atm}} \right)_f - \left( P_{\text{atm}} \right)_0 \ A
\]

**SOLUTION** Direct substitution of the data given in the problem into the expression above yields

\[
\Delta F = \left( 765 \text{ mm Hg} - 755 \text{ mm Hg} \right) \left( 12 \text{ m} \right) \left( 24 \text{ m} \right) \left( \frac{133 \text{ Pa}}{1.0 \text{ mm Hg}} \right) = 3.8 \times 10^5 \text{ N}
\]

Note that the conversion factor \( 133 \text{ Pa} = 1.0 \text{ mm Hg} \) is used to convert mm Hg to Pa.
34. **REASONING** Equation 11.5 gives the force \( F_2 \) of the output plunger in terms of the force \( F_1 \) applied to the input piston as \( F_2 = F_1 \left( \frac{A_2}{A_1} \right) \), where \( A_2 \) and \( A_1 \) are the corresponding areas. In this problem the chair begins to rise when the output force just barely exceeds the weight, so \( F_2 = 2100 \text{ N} \). We are given the input force as 55 N. We seek the ratio of the radii, so we will express the area of each circular cross section as \( \pi r^2 \) when we apply Equation 11.5.

**SOLUTION** According to Equation 11.5, we have

\[
\frac{A_2}{A_1} = \frac{F_2}{F_1} \quad \text{or} \quad \frac{\pi r_2^2}{\pi r_1^2} = \frac{F_2}{F_1}
\]

Solving for the ratio of the radii yields

\[
\frac{r_2}{r_1} = \sqrt{\frac{F_2}{F_1}} = \sqrt{\frac{2100 \text{ N}}{55 \text{ N}}} = 6.2
\]

35. **REASONING** We label the input piston as “2” and the output plunger as “1.” When the bottom surfaces of the input piston and output plunger are at the same level, Equation 11.5, \( F_2 = F_1 \left( \frac{A_2}{A_1} \right) \), applies. However, this equation is not applicable when the bottom surface of the output plunger is \( h = 1.50 \text{ m} \) above the input piston. In this case we must use Equation 11.4, \( P_2 = P_1 + \rho gh \), to account for the difference in heights. In either case, we will see that the input force is less than the combined weight of the output plunger and car.

**SOLUTION**

a. Using \( A = \pi r^2 \) for the circular areas of the piston and plunger, the input force required to support the 24 500-N weight is

\[
F_2 = F_1 \left( \frac{A_2}{A_1} \right) = (24 \text{ 500 N}) \left[ \frac{\pi \left( 7.70 \times 10^{-3} \text{ m} \right)^2}{\pi \left( 0.125 \text{ m} \right)^2} \right] = 93.0 \text{ N} \quad (11.5)
\]

b. The pressure \( P_2 \) at the input piston is related to the pressure \( P_1 \) at the bottom of the output plunger by Equation 11.4, \( P_2 = P_1 + \rho gh \), where \( h \) is the difference in heights. Setting \( P_2 = F_2 / A_2 = F_2 / \left( \pi r_2^2 \right) \), \( P_1 = F_1 / \left( \pi r_1^2 \right) \), and solving for \( F_2 \), we have
\[ F_2 = F_1 \left( \frac{\pi r_2^2}{\pi r_1^2} \right) + \rho g h \left( \pi r_2^2 \right) \tag{11.4} \]

\[ = (24 \ 500 \text{ N}) \left[ \frac{\pi \left( 7.70 \times 10^{-3} \text{ m} \right)^2}{\pi \left( 0.125 \text{ m} \right)^2} \right] \]

\[ + \left( 8.30 \times 10^2 \text{ kg/m}^3 \right) \left( 9.80 \text{ m/s}^2 \right) \left( 7.70 \times 10^{-3} \text{ m} \right) \pi \left( 7.70 \times 10^{-3} \text{ m} \right)^2 = 94.9 \text{ N} \]

36. **REASONING** We designate \( F_1 \) and \( A_1 \), respectively, as the magnitude of the force applied to the input piston and the circular cross-sectional area of the piston. The analogous quantities for the output plunger are \( F_2 \) and \( A_2 \). Since the difference in height between the input piston and output plunger is being neglected, the relation between the force magnitudes and areas is \( F_2 / F_1 = A_2 / A_1 \) (Equation 11.5). Using the fact that the area of each circular cross-section is \( \pi r^2 \), we can utilize the data given for the ratio of the radii of the input piston and output plunger and determine the force \( F_2 \) applied to the car.

**SOLUTION** According to Equation 11.5, we have

\[ \frac{F_2}{F_1} = \frac{A_2}{A_1} \]

Substituting \( A_1 = \pi r_1^2 \) and \( A_2 = \pi r_2^2 \), we find that

\[ \frac{F_2}{F_1} = \frac{\pi r_2^2}{\pi r_1^2} \quad \text{or} \quad F_2 = F_1 \left( \frac{r_2}{r_1} \right)^2 = (45 \text{ N})(8.3)^2 = 3100 \text{ N} \]

37. **REASONING** The pressure \( P' \) exerted on the bed of the truck by the plunger is \( P' = P - P_{\text{atm}} \). According to Equation 11.3, \( F = P'A \), so the force exerted on the bed of the truck can be expressed as \( F = (P - P_{\text{atm}})A \). If we assume that the plunger remains perpendicular to the floor of the load bed, the torque that the plunger creates about the axis shown in the figure in the text is

\[ \tau = F\ell = (P - P_{\text{atm}})A\ell = (P - P_{\text{atm}})(\pi r^2)\ell \]

**SOLUTION** Direct substitution of the numerical data into the expression above gives

\[ \tau = (3.54 \times 10^6 \text{ Pa} - 1.01 \times 10^5 \text{ Pa})\pi(0.150 \text{ m})^2 (3.50 \text{ m}) = 8.50 \times 10^5 \text{ N} \cdot \text{m} \]
38. REASONING The magnitude $F_1$ of the force the spring exerts on the piston is found from $F_1 = kx$ (Equation 10.3, without the minus sign), where $k$ is the spring constant of the spring, and $x$ is the amount by which the spring is compressed from its unstrained position. The piston and the plunger are at the same height, so the fluid pressure $P = \frac{F}{A}$ (Equation 11.3) at the piston is equal to the fluid pressure at the plunger. Therefore, the magnitude of the force $F_2$ that the rock exerts on the plunger is given by $F_2 = F_1 \left( \frac{A_2}{A_1} \right)$ (Equation 11.5) where $A_1$ is the area of the piston and $A_2$ is the area of the plunger. The magnitude $F_2$ of the force the rock exerts on the plunger is equal to the magnitude $W = mg$ (Equation 4.5) of the rock’s weight, where $m$ is the rock’s mass and $g$ is the magnitude of the acceleration due to gravity.

SOLUTION Solving $F_1 = kx$ (Equation 10.3, without the minus sign) for $x$, we obtain

$$x = \frac{F_1}{k} \quad (1)$$

Solving $F_2 = F_1 \left( \frac{A_2}{A_1} \right)$ (Equation 11.5) for $F_1$ and substituting $F_2 = mg$ yields

$$F_1 = F_2 \left( \frac{A_1}{A_2} \right) = mg \left( \frac{A_1}{A_2} \right) \quad (2)$$

Substituting Equation (2) into Equation (1), we find that

$$x = \frac{F_1}{k} = \frac{mg \left( \frac{A_1}{A_2} \right)}{k} = \frac{(40.0 \text{ kg}) \left(9.80 \text{ m/s}^2 \right) \left( \frac{15 \text{ cm}^2}{65 \text{ cm}^2} \right)}{1600 \text{ N/m}} = 5.7 \times 10^{-2} \text{ m}$$

39. REASONING Since the input piston and the output plungers are assumed to be at the same vertical level, we can apply Pascal’s principle in the form given in Equation 11.5:

$$F_{PL} = F_{MC} \frac{A_{PL}}{A_{MC}}$$

In this expression $F_{PL}$ is the force applied by the output plunger to the rotating disc, and $A_{PL}$ is the area of the output plunger; $F_{MC}$ is the force applied to the input piston in the master cylinder, and $A_{MC}$ is the area of the input piston. The circular areas of the output plungers and the input piston are given by $A = \pi r^2$, where $r$ is the radius. To determine the force $F_{MC}$ applied to the input piston, we must consider what happens to the brake pedal under the influence of the force $F$ that is applied to it. We will assume that it is at equilibrium, so that the net torque applied to the pedal is zero.
**SOLUTION** Using Equation 11.5 and the fact that the areas are given by \( A = \pi r^2 \), we have

\[
F_{PL} = F_{MC} \frac{A_{PL}}{A_{MC}} = F_{MC} \frac{r_{PL}^2}{\pi r_{MC}^2} = F_{MC} \left( \frac{r_{PL}}{r_{MC}} \right)^2
\]  

(1)

To find the force \( F_{MC} \) applied to the input piston, we assume that the pedal is at equilibrium. Therefore, we know that the net torque applied to the pedal is zero. According to Equation 9.1, the magnitude of a torque is the product of the magnitude of the applied force \( F \) and the lever arm \( \ell \) of the force. Thus, we have

\[
\left( F \ell \right)_{\text{positive}} + \left( -F_{MC} \ell_{MC} \right)_{\text{negative}} = 0
\]  

(2)

In this equation, we note that the torque due to force \( F \) is positive because it acts counterclockwise. Note also that the pedal pushes against the input piston with a force of magnitude \( F_{MC} \) and the piston pushes back with a reaction force of the same magnitude (Newton’s third law). This reaction force creates the negative clockwise torque in Equation (2). Solving Equation (2) for \( F_{MC} \) gives

\[
F_{MC} = F \frac{\ell}{\ell_{MC}}
\]  

(3)

Substituting Equation (3) into Equation (1) reveals that

\[
F_{PL} = F \frac{\ell}{\ell_{MC}} \left( \frac{r_{PL}}{r_{MC}} \right)^2 = (9.00 \text{ N}) \left( \frac{0.150 \text{ m}}{0.0500 \text{ m}} \right) \left( \frac{1.90 \times 10^{-2} \text{ m}}{9.50 \times 10^{-3} \text{ m}} \right)^2 = 108 \text{ N}
\]

40. **REASONING** The ice with the bear on it is floating, so that the upward-acting buoyant force balances the downward-acting weight \( W_{\text{ice}} \) of the ice and weight \( W_{\text{bear}} \) of the bear. The magnitude \( F_B \) of the buoyant force is the weight \( W_{\text{H}_2\text{O}} \) of the displaced water, according to Archimedes’ principle. Thus, we have \( F_B = W_{\text{H}_2\text{O}} = W_{\text{ice}} + W_{\text{bear}} \), the expression with which we will obtain \( W_{\text{bear}} \). We can express each of the weights \( W_{\text{H}_2\text{O}} \) and \( W_{\text{ice}} \) as mass times the magnitude of the acceleration due to gravity (Equation 4.5) and then relate the mass to the density and the displaced volume by using Equation 11.1.

**SOLUTION** Since the ice with the bear on it is floating, the upward-acting buoyant force \( F_B \) balances the downward-acting weight \( W_{\text{ice}} \) of the ice and the weight \( W_{\text{bear}} \) of the bear. The buoyant force has a magnitude that equals the weight \( W_{\text{H}_2\text{O}} \) of the displaced water, as stated by Archimedes’ principle. Thus, we have
\[ F_B = W_{H_2O} = W_{\text{ice}} + W_{\text{bear}} \quad \text{or} \quad W_{\text{bear}} = W_{H_2O} - W_{\text{ice}} \quad (1) \]

In Equation (1), we can use Equation 4.5 to express the weights \( W_{H_2O} \) and \( W_{\text{ice}} \) as mass \( m \) times the magnitude \( g \) of the acceleration due to gravity. Then, the each mass can be expressed as \( m = \rho V \) (Equation 11.1). With these substitutions, Equation (1) becomes

\[ W_{\text{bear}} = m_{H_2O}g - m_{\text{ice}}g = (\rho_{H_2O}V_{H_2O})g - (\rho_{\text{ice}}V_{\text{ice}})g \quad (2) \]

When the heaviest possible bear is on the ice, the ice is just below the water surface and displaces a volume of water that is \( V_{H_2O} = V_{\text{ice}} \). Substituting this result into Equation (2), we find that

\[
W_{\text{bear}} = (\rho_{H_2O}V_{\text{ice}})g - (\rho_{\text{ice}}V_{\text{ice}})g = (\rho_{H_2O} - \rho_{\text{ice}})V_{\text{ice}}g
\]

\[
= \left(1025 \text{ kg/m}^3 - 917 \text{ kg/m}^3\right)\left(5.2 \text{ m}^3\right)\left(9.80 \text{ m/s}^2\right) = 5500 \text{ N}
\]

41. **SSM REASONING**  According to Archimedes principle, the buoyant force that acts on the block is equal to the weight of the water that is displaced by the block. The block displaces an amount of water \( V \), where \( V \) is the volume of the block. Therefore, the weight of the water displaced by the block is \( W = mg = \left(\rho_{\text{water}}V\right)g \).

**SOLUTION**  The buoyant force that acts on the block is, therefore,

\[
F = \rho_{\text{water}}Vg = \left(1.00 \times 10^3 \text{ kg/m}^3\right)\left(0.10 \text{ m} \times 0.20 \text{ m} \times 0.30 \text{ m}\right)\left(9.80 \text{ m/s}^2\right) = 59 \text{ N}
\]

42. **REASONING**  When the cylindrical tube is floating, it is in equilibrium, and there is no net force acting on it. Therefore, the upward-directed buoyant force must have a magnitude that equals the magnitude of the tube’s weight, which acts downward. Since the magnitude \( F_B \) of the buoyant force equals the weight \( W \) of the tube, we have \( F_B = W \). This fact, together with Archimedes’ principle, will guide our solution.

**SOLUTION**  According to Archimedes’ principle, the magnitude of the buoyant force equals the weight of the displaced fluid, which is the mass \( m \) of the displaced fluid times the magnitude \( g \) of the acceleration due to gravity, or \( W = mg \) (Equation 4.5). But the mass is equal to the density \( \rho \) of the fluid times the displaced volume \( V \), or \( m = \rho V \) (Equation 11.1). The result is that the weight of the displaced fluid is \( \rho Vg \). Therefore, \( F_B = W \) becomes

\[
\rho Vg = W
\]
The volume $V$ of the displaced fluid equals the cross-sectional area $A$ of the cylindrical tube times the height $h$ beneath the fluid surface, or $V = Ah$. With this substitution, our previous result becomes $\rho Ah g = W$. Thus, the height $h$ to which the fluid rises is

$$h = \frac{W}{\rho A g}$$

a. The height $h_{\text{acid}}$ that the acid rises is

$$h_{\text{acid}} = \frac{W}{\rho_{\text{acid}} A g} = \frac{5.88 \times 10^{-2} \text{ N}}{(1280 \text{ kg/m}^3)(7.85 \times 10^{-5} \text{ m}^2)(9.80 \text{ m/s}^2)} = 5.97 \times 10^{-2} \text{ m}$$

b. The antifreeze rises to a height of

$$h_{\text{antifreeze}} = \frac{W}{\rho_{\text{antifreeze}} A g} = \frac{5.88 \times 10^{-2} \text{ N}}{(1073 \text{ kg/m}^3)(7.85 \times 10^{-5} \text{ m}^2)(9.80 \text{ m/s}^2)} = 7.12 \times 10^{-2} \text{ m}$$

43. **REASONING** Since the duck is in equilibrium, its downward-acting weight is balanced by the upward-acting buoyant force. According to Archimedes’ principle, the magnitude of the buoyant force is equal to the weight of the water displaced by the duck. Setting the weight of the duck equal to the magnitude of the buoyant force will allow us to find the average density of the duck.

**SOLUTION** Since the weight $W_{\text{duck}}$ of the duck is balanced by the magnitude $F_B$ of the buoyant force, we have that $W_{\text{duck}} = F_B$. The duck’s weight is $W_{\text{duck}} = mg = (\rho_{\text{duck}} V_{\text{duck}})g$, where $\rho_{\text{duck}}$ is the average density of the duck and $V_{\text{duck}}$ is its volume. The magnitude of the buoyant force, on the other hand, equals the weight of the water displaced by the duck, or $F_B = m_{\text{water}}g$, where $m_{\text{water}}$ is the mass of the displaced water. But $m_{\text{water}} = \rho_{\text{water}}(\frac{1}{4}V_{\text{duck}})$, since one-quarter of the duck’s volume is beneath the water. Thus,

$$\frac{\rho_{\text{duck}} V_{\text{duck}} g}{\text{Weight of duck}} = \rho_{\text{water}} \left(\frac{1}{4}V_{\text{duck}}\right) g$$

Solving this equation for the average density of the duck (and taking the density of water from Table 11.1) gives

$$\rho_{\text{duck}} = \frac{1}{4} \rho_{\text{water}} = \frac{1}{4} \left(1.00 \times 10^3 \text{ kg/m}^3\right) = 250 \text{ kg/m}^3$$
44. **REASONING** The paperweight weighs less in water than in air, because of the buoyant force $F_B$ of the water. The buoyant force points upward, while the weight points downward, leading to an effective weight in water of $W_{\text{in water}} = W - F_B$. There is also a buoyant force when the paperweight is weighed in air, but it is negligibly small. Thus, from the given weights, we can obtain the buoyant force, which is the weight of the displaced water, according to Archimedes’ principle. From the weight of the displaced water and the density of water, we can obtain the volume of the water, which is also the volume of the completely immersed paperweight.

**SOLUTION** We have

$$W_{\text{in water}} = W - F_B \quad \text{or} \quad F_B = W - W_{\text{in water}}$$

According to Archimedes’ principle, the buoyant force is the weight of the displaced water, which is $mg$, where $m$ is the mass of the displaced water. Using Equation 11.1, we can write the mass as the density times the volume or $m = \rho V$. Thus, for the buoyant force, we have

$$F_B = W - W_{\text{in water}} = \rho V g$$

Solving for the volume and using $\rho = 1.00 \times 10^3 \text{ kg/m}^3$ for the density of water (see Table 11.1), we find

$$V = \frac{W - W_{\text{in water}}}{\rho g} = \frac{6.9 \text{ N} - 4.3 \text{ N}}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 2.7 \times 10^{-4} \text{ m}^3$$

45. **SSM REASONING** According to Equation 11.1, the density of the life jacket is its mass divided by its volume. The volume is given. To obtain the mass, we note that the person wearing the life jacket is floating, so that the upward-acting buoyant force balances the downward-acting weight of the person and life jacket. The magnitude of the buoyant force is the weight of the displaced water, according to Archimedes’ principle. We can express each of the weights as $mg$ (Equation 4.5) and then relate the mass of the displaced water to the density of water and the displaced volume by using Equation 11.1.

**SOLUTION** According to Equation 11.1, the density of the life jacket is

$$\rho_J = \frac{m_J}{V_J} \quad (1)$$

Since the person wearing the life jacket is floating, the upward-acting buoyant force $F_B$ balances the downward-acting weight $W_p$ of the person and the weight $W_J$ of the life jacket. The buoyant force has a magnitude that equals the weight $W_{H_2O}$ of the displaced water, as stated by Archimedes’ principle. Thus, we have

$$F_B = W_{H_2O} = W_p + W_J \quad (2)$$
In Equation (2), we can use Equation 4.5 to express each weight as mass \( m \) times the magnitude \( g \) of the acceleration due to gravity. Then, the mass of the water can be expressed as \( m_{H_2O} = \rho_{H_2O} V_{H_2O} \) (Equation 11.1). With these substitutions, Equation (2) becomes

\[
m_{H_2O} g = m_p g + m_j g \quad \text{or} \quad (\rho_{H_2O} V_{H_2O}) g = m_p g + m_j g
\]

Solving this result for \( m_j \) shows that

\[
m_j = \rho_{H_2O} V_{H_2O} - m_p
\]

Substituting this result into Equation (1) and noting that the volume of the displaced water is

\[
V_{H_2O} = 3.1 \times 10^{-2} \, \text{m}^3 + 6.2 \times 10^{-2} \, \text{m}^3 \quad \text{gives}
\]

\[
\rho_j = \frac{\rho_{H_2O} V_{H_2O} - m_p}{V_j} = \frac{(1.00 \times 10^3 \, \text{kg/m}^3) \left(3.1 \times 10^{-2} \, \text{m}^3 + 6.2 \times 10^{-2} \, \text{m}^3\right) - 81 \, \text{kg}}{3.1 \times 10^{-2} \, \text{m}^3} = 390 \, \text{kg/m}^3
\]

46. **REASONING** The mass \( m \) of the shipping container is related to its weight by \( W = mg \) (Equation 4.5). We neglect the mass of the balloon and the air contained in it. When it just begins to rise off the ocean floor, the shipping container's weight \( W \) is balanced by the upward buoyant force \( F_B \) on the shipping container and the balloon:

\[
mg = F_B \quad (1)
\]

Both the shipping container and the balloon are fully submerged in the water. Therefore, Archimedes' principle holds that the magnitude \( F_B \) of the buoyant force is equal to the weight \( W_{water} = \rho_{water} g V \) of the water displaced by the total volume \( V = V_{container} + V_{balloon} \) of the container and the balloon:

\[
F_B = W_{water} = \rho_{water} g \left( V_{container} + V_{balloon} \right) \quad (11.6)
\]

The volume of the container is the product of its length \( l \), width \( w \), and height \( h \):

\[
V_{container} = lwh \quad (2)
\]

The volume of the spherical balloon depends on its radius \( r \) via

\[
V_{balloon} = \frac{4}{3} \pi r^3 \quad (3)
\]

**SOLUTION** Substituting Equation 11.6 into Equation (1), we obtain

\[
m g = \rho_{water} g \left( V_{container} + V_{balloon} \right) \quad \text{or} \quad m = \rho_{water} \left( V_{container} + V_{balloon} \right) \quad (4)
\]

Replacing the volumes of the container and the balloon in Equation (4) with Equations (2) and (3) yields the mass \( m \) of the shipping container:
\[ m = \rho_{\text{water}} \left( \text{Iwh} + \frac{4}{3} \pi r^3 \right) \]
\[ = \left(1025 \text{ kg/m}^3\right) \left[ (6.1 \text{ m})(2.4 \text{ m})(2.6 \text{ m}) + \frac{4}{3} \pi (1.5 \text{ m})^3 \right] = 5.4 \times 10^4 \text{ kg} \]

47. **REASONING AND SOLUTION** The buoyant force exerted by the water must at least equal the weight of the logs plus the weight of the people,

\[ F_B = W_L + W_P \]
\[ \rho_w g V = \rho_L g V + W_P \]

Now the volume of logs needed is

\[ V = \frac{M_p}{\rho_w - \rho_L} = \frac{4(80.0 \text{ kg})}{1.00 \times 10^3 \text{ kg/m}^3 - 725 \text{ kg/m}^3} = 1.16 \text{ m}^3 \]

The volume of one log is

\[ V_L = \pi (8.00 \times 10^{-2} \text{ m})^2 (3.00 \text{ m}) = 6.03 \times 10^{-2} \text{ m}^3 \]

The number of logs needed is

\[ N = \frac{V}{V_L} = (1.16)/(6.03 \times 10^{-2}) = 19.2 \]

Therefore, at least 20 logs are needed.

48. **REASONING** The free-body diagram shows the two forces acting on the balloon, its weight \( W \) and the buoyant force \( F_B \). Newton’s second law, Equation 4.2b, states that the net force \( \Sigma F_y \) in the \( y \) direction is equal to the mass \( m \) of the balloon times its acceleration \( a_y \) in that direction:

\[ \frac{F_B - W}{\Sigma F_y} = ma_y \quad \text{(4.2b)} \]

Our solution is based on this statement.
**SOLUTION** Solving Equation 4.2b for the acceleration $a_y$ gives

$$a_y = \frac{F_B - W}{m} \quad (1)$$

The weight $W$ of an object is equal to its mass $m$ times the magnitude $g$ of the acceleration due to gravity, or $W = mg$ (Equation 4.5). The mass, in turn, is equal to the product of an object’s density $\rho$ and its volume $V$, so $m = \rho V$ (Equation 11.1). Combining these two relations, the weight can be expressed as $W = \rho V g$.

According to Archimedes’ principle, the magnitude $F_B$ of the buoyant force is equal to the weight of the cool air that the balloon displaces, so $F_B = m_{\text{cool air}} g = (\rho_{\text{cool air}} V) g$. Since we are neglecting the weight of the balloon fabric and the basket, the weight of the balloon is just that of the hot air inside the balloon. Thus, $m = m_{\text{hot air}} = \rho_{\text{hot air}} V$ and $W = m_{\text{hot air}} g = (\rho_{\text{hot air}} V) g$.

Substituting the expressions $F_B = (\rho_{\text{cool air}} V) g$, $m = \rho_{\text{hot air}} V$, and $W = (\rho_{\text{hot air}} V) g$ into Equation (1) gives

$$a_y = \frac{F_B - W}{m} = \frac{\rho_{\text{cool air}} V g - \rho_{\text{hot air}} V g}{\rho_{\text{hot air}} V} = \frac{(\rho_{\text{cool air}} - \rho_{\text{hot air}}) g}{\rho_{\text{hot air}} V}$$

$$= \frac{(1.29 \text{ kg/m}^3 - 0.93 \text{ kg/m}^3)(9.80 \text{ m/s}^2)}{0.93 \text{ kg/m}^3} = 3.8 \text{ m/s}^2$$

---

**49. REASONING AND SOLUTION** The figure at the right shows the two forces that initially act on the box. Since the box is not accelerated, the two forces must have zero resultant: $F_{B0} - W_{\text{box}} = 0$. Therefore,

$$F_{B0} = W_{\text{box}} \quad (1)$$

From Archimedes' principle, the buoyant force on the box is equal to the weight of the water that is displaced by the box:

$$F_{B0} = W_{\text{disp}} \quad (2)$$

Combining (1) and (2) we have $W_{\text{box}} = W_{\text{disp}}$, or $m_{\text{box}} g = \rho_{\text{water}} V_{\text{disp}}$. Therefore,

$$m_{\text{box}} = \rho_{\text{water}} V_{\text{disp}}$$

Since the box floats with one-third of its height beneath the water, $V_{\text{disp}} = (1/3)V_{\text{box}}$, or $V_{\text{disp}} = (1/3)L^3$. Therefore,

$$m_{\text{box}} = \frac{\rho_{\text{water}} L^3}{3} \quad (3)$$
The figure at the right shows the three forces that act on the box after water is poured into the box. The box begins to sink when

\[ W_{\text{box}} + W_{\text{water}} \geq F_B \]  \hspace{1cm} (4)

The box just begins to sink when the equality is satisfied. From Archimedes' principle, the buoyant force on the system is equal to the weight of the water that is displaced by the system: \( F_B = W_{\text{displaced}} \).

The equality in Equation (4) can be written as

\[ m_{\text{box}}g + m_{\text{water}}g = m_{\text{displaced}}g \]  \hspace{1cm} (5)

When the box begins to sink, the volume of the water displaced is equal to the volume of the box; Equation (5) then becomes

\[ m_{\text{box}} + \rho_{\text{water}}V_{\text{water}} = \rho_{\text{water}}V_{\text{box}}. \]

The volume of water in the box at this instant is \( V_{\text{water}} = L^2h \), where \( h \) is the depth of the water in the box. Thus, the equation above becomes

\[ m_{\text{box}} + \rho_{\text{water}}L^2h = \rho_{\text{water}}L^3. \]

Using Equation (3) for the mass of the box, we obtain

\[ \frac{\rho_{\text{water}}L^3}{3} + \rho_{\text{water}}L^2h = \rho_{\text{water}}L^3. \]

Solving for \( h \) gives

\[ h = \frac{2}{3}L = \frac{2}{3}(0.30 \text{ m}) = 0.20 \text{ m} \]

50. **REASONING** The fraction of the specimen’s apparent volume that is solid will be given by

\[ \frac{V_{\text{solid}}}{V_{\text{total}}} = \frac{V_{\text{solid}}}{V_{\text{solid}} + V_{\text{hollow}}} \]  \hspace{1cm} (1)

where \( V_{\text{solid}} \) is the volume of the solid part of the specimen and \( V_{\text{hollow}} \) is the volume of the hollow part. Since the specimen weighs twice as much in air as it does in water, \( W_{\text{air}} = 2W_{\text{water}} \). Furthermore, we are told that the density of the solid part of the specimen is \( 5.0 \times 10^3 \text{ kg/m}^3 \); this is five times greater than the density of water \( \rho_{\text{fluid}} \), so that \( \rho_{\text{solid}} = 5\rho_{\text{fluid}} \). Using this information and Archimedes’ principle, we will obtain a value for the fraction expressed in Equation (1).

**SOLUTION** Let \( M \) represent the mass of the rock. Then, according to Equation 11.1, the volume of the solid part of the rock is \( V_{\text{solid}} = \frac{M}{\rho_{\text{solid}}} \).
From Archimedes’ principle, we know that when then rock is submerged in water, the buoyant force on the rock will be equal to the weight of the water that is displaced by the rock so that

\[ F_B = W_{\text{fluid displaced}} = \rho_{\text{fluid}} g V_{\text{fluid displaced}} \]

We know that the rock will displace a volume of water that is equal to the total volume of the rock, so the volume of water displaced is equal to \( V_{\text{total}} \), and we have

\[ W_{\text{fluid displaced}} = \rho_{\text{fluid}} g V_{\text{total}} \]

Since the weight of the rock in air is twice that in water, we can write

\[ 2 \left( \frac{\rho_{\text{fluid}} g V_{\text{total}}}{\rho_{\text{solid}} V_{\text{solid}}} \right) = M_g \]

or \( M_g = 2 \rho_{\text{fluid}} g V_{\text{total}} \). Substituting in the relation \( M = \rho_{\text{solid}} V_{\text{solid}} \) and solving for the ratio of the volumes, we obtain

\[ \frac{V_{\text{solid}}}{V_{\text{total}}} = 2 \frac{\rho_{\text{fluid}}}{\rho_{\text{solid}}} = 2 \left( \frac{1.00 \times 10^3 \text{ kg/m}^3}{5.0 \times 10^3 \text{ kg/m}^3} \right) = 0.40 \]

51. **SSM REASONING** The height of the cylinder that is in the oil is given by

\[ h_{\text{oil}} = \frac{V_{\text{oil}}}{\pi r^2} \]

where \( V_{\text{oil}} \) is the volume of oil displaced by the cylinder and \( r \) is the radius of the cylinder. We must, therefore, find the volume of oil displaced by the cylinder. After the oil is poured in, the buoyant force that acts on the cylinder is equal to the sum of the weight of the water displaced by the cylinder and the weight of the oil displaced by the cylinder. Therefore, the magnitude of the buoyant force is given by

\[ F = \rho_{\text{water}} g V_{\text{water}} + \rho_{\text{oil}} g V_{\text{oil}} \]

Since the cylinder floats in the fluid, the net force that acts on the cylinder must be zero. Therefore, the buoyant force that supports the cylinder must be equal to the weight of the cylinder, or

\[ \rho_{\text{water}} g V_{\text{water}} + \rho_{\text{oil}} g V_{\text{oil}} = mg \]

where \( m \) is the mass of the cylinder. Substituting values into the expression above leads to

\[ V_{\text{water}} + (0.725) V_{\text{oil}} = 7.00 \times 10^{-3} \text{ m}^3 \quad (1) \]

From the figure in the text, \( V_{\text{cylinder}} = V_{\text{water}} + V_{\text{oil}} \). Substituting values into the expression for \( V_{\text{cylinder}} \) gives

\[ V_{\text{water}} + V_{\text{oil}} = 8.48 \times 10^{-3} \text{ m}^3 \quad (2) \]
Subtracting Equation (1) from Equation (2) yields \( V_{\text{oil}} = 5.38 \times 10^{-3} \text{ m}^3 \).

**SOLUTION** The height of the cylinder that is in the oil is, therefore,

\[
h_{\text{oil}} = \frac{V_{\text{oil}}}{\pi r^2} = \frac{5.38 \times 10^{-3} \text{ m}^3}{\pi (0.150 \text{ m})^2} = 7.6 \times 10^{-2} \text{ m}
\]

52. **REASONING AND SOLUTION** Only the weight of the block compresses the spring. Applying Hooke’s law gives \( W = kx \). The spring is stretched by the buoyant force acting on the block minus the weight of the block. Hooke’s law again gives \( F_B - W = 2kx \). Eliminating \( kx \) gives \( F_B = 3W \). Now \( F_B = \rho_w g V \), so that the volume of the block is

\[
V = 3M/\rho_w = 3(8.00 \text{ kg})/(1.00 \times 10^3 \text{ kg/m}^3) = 2.40 \times 10^{-2} \text{ m}^3
\]

The volume of wood in the block is

\[
V_w = M/\rho_b = (8.00 \text{ kg})/(840 \text{ kg/m}^3) = 9.52 \times 10^{-3} \text{ m}^3
\]

The volume of the block that is hollow is \( V - V_w = 1.45 \times 10^{-2} \text{ m}^3 \). The percentage of the block that is hollow is then

\[
100(1.45 \times 10^{-2})/(2.40 \times 10^{-2}) = 60.3\%
\]

53. **REASONING AND SOLUTION** The upward buoyant force must equal the weight of the shell if it is floating, \( \rho_w g V = W \). The submerged volume of the shell is \( V = (4/3)\pi R_2^3 \), where \( R_2 \) is its outer radius. Now \( R_2^3 = (3/4)m/(\rho_w \pi) \) gives

\[
R_2 = \sqrt[3]{\frac{3m}{4\pi \rho_w}} = \sqrt[3]{\frac{3(1.00 \text{ kg})}{4\pi (1.00 \times 10^3 \text{ kg/m}^3)}} = 6.20 \times 10^{-2} \text{ m}
\]

The weight of the shell is \( W = \rho_g g(V_2 - V_1) \) so \( R_1^3 = R_2^3 - (3/4)m/(\pi \rho_g) \), and

\[
R_1 = \sqrt[3]{\frac{3m}{4\pi} \left( \frac{1}{\rho_w} - \frac{1}{\rho_g} \right)} = \sqrt[3]{\frac{3(1.00 \text{ kg})}{4\pi} \left( \frac{1}{1.00 \times 10^3 \text{ kg/m}^3} - \frac{1}{2.60 \times 10^3 \text{ kg/m}^3} \right)} = 5.28 \times 10^{-2} \text{ m}
\]
54. **REASONING** The speed $v$ of the gasoline in the fuel line is related to its mass flow rate, the density $\rho$ of the gasoline, and the cross-sectional area $A$ of the fuel line by

$$v = \frac{\text{Mass flow rate}}{\rho A}$$

(Equation 11.7). Solving Equation 11.7 for the speed $v$, we obtain

$$v = \frac{\text{Mass flow rate}}{\rho A} = \frac{5.88 \times 10^{-2} \text{ kg/s}}{(735 \text{ kg/m}^3)\pi (3.18 \times 10^{-3} \text{ m})^2} = 2.52 \text{ m/s}$$

**SOLUTION** The fuel line has a circular cross-section with a radius $r$. Therefore, its cross-sectional area is $A = \pi r^2$. Equation (1) then yields the speed of the gasoline in the fuel line:

$$v = \frac{\text{Mass flow rate}}{\rho \pi r^2} = \frac{5.88 \times 10^{-2} \text{ kg/s}}{(735 \text{ kg/m}^3)\pi (3.18 \times 10^{-3} \text{ m})^2} = 2.52 \text{ m/s}$$

55. **SSM REASONING AND SOLUTION** The mass flow rate $Q_{\text{mass}}$ is the amount of fluid mass that flows per unit time. Therefore,

$$Q_{\text{mass}} = \frac{m}{t} = \frac{\rho V}{t} = \frac{(1030 \text{ kg/m}^3)(9.5 \times 10^{-4} \text{ m}^3)}{6.0 \text{ h}} = \frac{1.0 \text{ h}}{3600 \text{ s}} = 4.5 \times 10^{-5} \text{ kg/s}$$

56. **REASONING**

a. According to Equation 11.10, the volume flow rate $Q$ is equal to the product of the cross-sectional area $A$ of the artery and the speed $v$ of the blood, $Q = Av$. Since $Q$ and $A$ are known, we can determine $v$.

b. Since the volume flow rate $Q_2$ through the constriction is the same as the volume flow rate $Q_1$ in the normal part of the artery, $Q_2 = Q_1$. We can use this relation to find the blood speed in the constricted region.

**SOLUTION**

a. Since the artery is assumed to have a circular cross-section, its cross-sectional area is $A_1 = \pi r_1^2$, where $r_1$ is the radius. Thus, the speed of the blood is

$$v_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\pi r_1^2} = \frac{3.6 \times 10^{-6} \text{ m}^3/\text{s}}{\pi (5.2 \times 10^{-3} \text{ m})^2} = 4.2 \times 10^{-2} \text{ m/s}$$

(11.10)

b. The volume flow rate is the same in the normal and constricted parts of the artery, so $Q_2 = Q_1$. Since $Q_2 = A_2 v_2$, the blood speed is $v_2 = Q_2/A_2 = Q_1/A_2$. We are given that the radius of the constricted part of the artery is one-third that of the normal artery, so $r_2 = \frac{1}{3} r_1$. Thus, the speed of the blood at the constriction is
57. **REASONING** The length $L$ of the side of the square can be obtained, if we can find a value for the cross-sectional area $A$ of the ducts. The area is related to the volume flow rate $Q$ and the air speed $v$ by Equation 11.10 ($Q = Av$). The volume flow rate can be obtained from the volume $V$ of the room and the replacement time $t$ as $Q = V/t$.

**SOLUTION** For a square cross section with sides of length $L$, we have $A = L^2$. And we know that the volume flow rate is $Q = V/t$. Therefore, using Equation 11.10 gives

$$Q = Av \quad \text{or} \quad \frac{V}{t} = L^2 v$$

Solving for $L$ shows that

(a) Air speed $= 3.0 \text{ m} / \text{s}$

$$L = \sqrt{\frac{V}{tv}} = \sqrt{\frac{120 \text{ m}^3}{(1200 \text{ s})(3.0 \text{ m} / \text{s})}} = 0.18 \text{ m}$$

(b) Air speed $= 5.0 \text{ m} / \text{s}$

$$L = \sqrt{\frac{V}{tv}} = \sqrt{\frac{120 \text{ m}^3}{(1200 \text{ s})(5.0 \text{ m} / \text{s})}} = 0.14 \text{ m}$$

58. **REASONING** The volume flow rate (in cubic meters per second) of the falling water is the same as it was when it left the faucet. This is because no water is added to or taken out of the stream after the water leaves the faucet. With the volume flow rate unchanging, the equation of continuity applies in the form $A_1v_1 = A_2v_2$ (Equation 11.9). We will assign $A_1$ and $v_1$ to be the cross-sectional area and speed of the water at any point below the faucet, and $A_2$ and $v_2$ to be the cross-sectional area and speed of the water at the faucet.

**SOLUTION** Using the equation of continuity as given in Equation 11.9, we have

$$A_1v_1 = A_2v_2 \quad \text{or} \quad A_1 = \frac{A_2v_2}{v_1} \quad (11.9)$$

Since the effects of air resistance are being ignored, the water can be treated as a freely-falling object, as Chapter 2 discusses. Thus, the acceleration of the water is that due to gravity. To find the speed $v_1$ of the water, given its initial speed $v_2$ as it leaves the faucet, we use the relation $v_1^2 = v_2^2 + 2ay$ (or $v_1 = \sqrt{v_2^2 + 2ay}$) from Equation 2.9 of the equations of kinematics. Substituting Equation 2.9 into Equation 11.9 gives

$$A_1 = \frac{A_2v_2}{v_1} = \frac{A_2v_2}{\sqrt{v_2^2 + 2ay}}$$
Choosing downward as the positive direction, so \( y = +0.10 \text{ m} \) and \( a = +9.80 \text{ m/s}^2 \), the cross-sectional area of the stream at a distance of 0.10 m below the faucet is

\[
A_1 = \frac{A_2v_2}{\sqrt{v_2^2 + 2ay}} = \frac{(1.8 \times 10^{-4} \text{ m}^2)(0.85 \text{ m/s})}{\sqrt{(0.85 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.10 \text{ m})}} = 9.3 \times 10^{-5} \text{ m}^2
\]

59. **REASONING** The number \( N \) of capillaries can be obtained by dividing the total cross-sectional area \( A_{\text{cap}} \) of all the capillaries by the cross-sectional area \( a_{\text{cap}} \) of a single capillary. We know the radius \( r_{\text{cap}} \) of a single capillary, so \( a_{\text{cap}} \) can be calculated as \( a_{\text{cap}} = \pi r_{\text{cap}}^2 \). To find \( A_{\text{cap}} \), we will use the equation of continuity.

**SOLUTION** The number \( N \) of capillaries is

\[
N = \frac{A_{\text{cap}}}{a_{\text{cap}}} = \frac{A_{\text{cap}}}{\pi r_{\text{cap}}^2}
\]

(1)

where the cross-sectional area \( a_{\text{cap}} \) of a single capillary has been replaced by \( a_{\text{cap}} = \pi r_{\text{cap}}^2 \).

To obtain the total cross-sectional area \( A_{\text{cap}} \) of all the capillaries, we use the equation of continuity for an incompressible fluid (see Equation 11.9). For present purposes, this equation is

\[
A_{v\text{aorta}}v_{\text{aorta}} = A_{\text{cap}}v_{\text{cap}} \quad \text{or} \quad A_{\text{cap}} = \frac{A_{v\text{aorta}}v_{\text{aorta}}}{v_{\text{cap}}}
\]

(2)

The cross-sectional area of the aorta is \( A_{\text{aorta}} = \pi r_{\text{aorta}}^2 \), and with this substitution, Equation (2) becomes

\[
A_{\text{cap}} = \frac{A_{v\text{aorta}}v_{\text{aorta}}}{v_{\text{cap}}} = \frac{\pi r_{\text{aorta}}^2 v_{\text{aorta}}}{v_{\text{cap}}}
\]

Substituting this result into Equation (1), we find that

\[
N = \frac{A_{\text{cap}}}{\pi r_{\text{cap}}^2} = \frac{(\pi r_{\text{aorta}}^2/v_{\text{cap}}) v_{\text{aorta}}}{\pi r_{\text{cap}}^2} = \frac{r_{\text{aorta}}^2 v_{\text{aorta}}}{r_{\text{cap}}^2 v_{\text{cap}}}
\]

\[
= \frac{(1.1 \text{ cm})^2 (40 \text{ cm/s})}{(6 \times 10^{-4} \text{ cm})^2 (0.07 \text{ cm/s})} = 2 \times 10^9
\]
60. **REASONING** The mass flow rate is given in kg/s by $\rho A v$, where $\rho$ is the density of the fluid in kg/m$^3$, $A$ is the cross-sectional area of the pipe in m$^2$, and $v$ is the speed of the fluid in m/s. Multiplying the mass flow rate by the time $t$ of the flow gives the total mass of the fluid that leaves the pipe in that time period. Since all the water that the underground pipe delivers to the hydrant is poured onto the fire via the three hoses, we can determine the answer for part a of the problem by considering only the underground pipe. To find the speed of the water in each of the hoses, we will explicitly use the fact that the mass flow rate of the water delivered by the pipe equals the total mass flow rate of the three hoses.

**SOLUTION**

a. There are $t = 3600$ s in one hour. Therefore, using $\rho A_{\text{pipe}} v_{\text{pipe}}$ as the mass flow rate and $A_{\text{pipe}} = \pi r_{\text{pipe}}^2$ as the circular cross-sectional area of the pipe, we find that the total mass $M$ of the water ($\rho = 1.00 \times 10^3$ kg/m$^3$) poured onto a fire in one hour is

$$M = \rho A_{\text{pipe}} v_{\text{pipe}} t = \rho \pi r_{\text{pipe}}^2 v_{\text{pipe}} t$$

$$= \left(1.00 \times 10^3 \text{ kg/m}^3\right) \pi (0.080 \text{ m})^2 (3.0 \text{ m/s})(3600 \text{ s}) = 2.2 \times 10^5 \text{ kg}$$

b. The mass flow rate of the underground pipe is $\rho A_{\text{pipe}} v_{\text{pipe}}$, whereas the mass flow rate of a single hose is $\rho A_{\text{hose}} v_{\text{hose}}$. Since there are three identical hoses and the mass flow rate of the pipe equals the total mass flow rate of the hoses, we have

$$\rho A_{\text{pipe}} v_{\text{pipe}} = 3 \rho A_{\text{hose}} v_{\text{hose}} \quad \text{or} \quad \rho A_{\text{pipe}} v_{\text{pipe}} = 3 \rho \pi r_{\text{hose}}^2 v_{\text{hose}} \quad \text{or} \quad r_{\text{pipe}} v_{\text{pipe}} = 3 r_{\text{hose}} v_{\text{hose}}$$

Solving for $v_{\text{hose}}$ gives

$$v_{\text{hose}} = \frac{r_{\text{pipe}}^2 v_{\text{pipe}}}{3 r_{\text{hose}}^2} = \frac{(0.080 \text{ m})^2 (3.0 \text{ m/s})}{3 (0.020 \text{ m})^2} = 16 \text{ m/s}$$

61. **SSM REASONING AND SOLUTION** Using Bernoulli’s equation, we have

$$\Delta P = P_1 - P_2 = (1/2) \rho v_2^2 - (1/2) \rho v_1^2 = (1/2)(1.29 \text{ kg/m}^3)[(8.5 \text{ m/s})^2 - (1.1 \text{ m/s})^2]$$

$$\Delta P = 46 \text{ Pa}$$

The air flows from high pressure to low pressure (from lower to higher velocity), so it enters at $B$ and exits at $A$.

62. **REASONING** We will use Bernoulli’s equation to determine the amount by which the pressure $P_{\text{in}}$ inside the cargo area exceeds the outside pressure $P_{\text{out}}$. In comparing these two
pressures we will ignore any difference in elevation between the cargo area and the outside air just above the tarpaulin.

**SOLUTION** As applied to this problem, Bernoulli’s equation given in Equation 11.11 becomes

\[
P_{\text{in}} + \frac{1}{2} \rho v_{\text{in}}^2 + \rho g y_{\text{in}} = P_{\text{out}} + \frac{1}{2} \rho v_{\text{out}}^2 + \rho g y_{\text{out}}
\]

The air inside the cargo area is stationary with respect to the truck, so that \( v_{\text{in}} = 0 \) m/s. Moreover, we are ignoring any difference in elevation between the cargo area and the outside air just above the tarpaulin, so \( y_{\text{in}} = y_{\text{out}} \). With these facts in mind, we see that Bernoulli’s equation reduces to

\[
P_{\text{in}} = P_{\text{out}} + \frac{1}{2} \rho v_{\text{out}}^2 \quad \text{or} \quad P_{\text{in}} - P_{\text{out}} = \frac{1}{2} \rho v_{\text{out}}^2 = \frac{1}{2} \left(1.29 \text{ kg/m}^3\right) \left(27 \text{ m/s}\right)^2 = 470 \text{ Pa}
\]

63. **SSM REASONING AND SOLUTION** Let the speed of the air below and above the wing be given by \( v_1 \) and \( v_2 \), respectively. According to Equation 11.12, the form of Bernoulli’s equation with \( y_1 = y_2 \), we have

\[
P_1 - P_2 = \frac{1}{2} \rho \left(v_2^2 - v_1^2\right) = \frac{1.29 \text{ kg/m}^3}{2} \left(251 \text{ m/s}\right)^2 - \left(225 \text{ m/s}\right)^2 = 7.98 \times 10^3 \text{ Pa}
\]

From Equation 11.3, the lifting force is, therefore,

\[
F = (P_1 - P_2)A = (7.98 \times 10^3 \text{ Pa})(24.0 \text{ m}^2) = 1.92 \times 10^5 \text{ N}
\]

64. **REASONING** The absolute pressure in the pipe must be greater than atmospheric pressure. Our solution proceeds in two steps. We will begin with Bernoulli’s equation. Then we will incorporate the equation of continuity.

**SOLUTION** According to Bernoulli’s equation, as given by Equation 11.11, we have

\[
\frac{P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1}{\text{Pipe}} = \frac{P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2}{\text{Nozzle}}
\]

The pipe and nozzle are horizontal, so that \( y_1 = y_2 \) and Bernoulli’s equation simplifies to

\[
P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \quad \text{or} \quad P_1 = P_2 + \frac{1}{2} \rho \left(v_2^2 - v_1^2\right)
\]

where \( P_1 \) is the absolute pressure of the water in the pipe. We have values for the pressure \( P_2 \) (atmospheric pressure) at the nozzle opening and the speed \( v_1 \) in the pipe. However, to solve this expression we also need a value for the speed \( v_2 \) at the nozzle opening. We obtain this value by using the equation of continuity, as given by Equation 11.9:
\[A_1v_1 = A_2v_2 \quad \text{or} \quad \pi r_1^2v_1 = \pi r_2^2v_2 \quad \text{or} \quad v_2 = \frac{r_1^2v_1}{r_2^2}\]

Here, we have used that fact that the area of a circle is \(A = \pi r^2\). Substituting this result for \(v_2\) into Bernoulli’s equation, we find that

\[P_1 = P_2 + \frac{1}{2} \rho \left(\frac{r_1^4}{r_2^4} - 1\right) v_1^2\]

Taking the density of water to be \(\rho = 1.00 \times 10^3\) kg/m\(^3\) (see Table 11.1), we find that the absolute pressure of the water in the pipe is

\[P_1 = P_2 + \frac{1}{2} \rho \left(\frac{r_1^4}{r_2^4} - 1\right) v_1^2\]

\[= 1.01 \times 10^5 \text{ Pa} + \frac{1}{2} \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left[\frac{(1.9 \times 10^{-2} \text{ m})^4}{(4.8 \times 10^{-3} \text{ m})^4} - 1\right] (0.62 \text{ m/s})^2 = 1.48 \times 10^5 \text{ Pa}\]

65. **REASONING** We assume that region 1 contains the constriction and region 2 is the normal region. The difference in blood pressures between the two points in the horizontal artery is given by Bernoulli’s equation (Equation 11.12) as \(P_2 - P_1 = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2\), where \(v_1\) and \(v_2\) are the speeds at the two points. Since the volume flow rate is the same at the two points, the speed at 1 is related to the speed at 2 by Equation 11.9, the equation of continuity: \(A_1v_1 = A_2v_2\), where \(A_1\) and \(A_2\) are the cross-sectional areas of the artery. By combining these two relations, we will be able to determine the pressure difference.

**SOLUTION** Solving the equation of continuity for the blood speed in region 1 gives \(v_1 = v_2 A_2/A_1\). Substituting this result into Bernoulli’s equation yields

\[P_2 - P_1 = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 = \frac{1}{2} \rho \left(\frac{v_2 A_2}{A_1}\right)^2 - \frac{1}{2} \rho v_2^2\]

Since \(A_1 = \frac{1}{4} A_2\), the pressure difference is

\[P_2 - P_1 = \frac{1}{2} \rho \left(\frac{v_2 A_2}{\frac{1}{4} A_2}\right)^2 - \frac{1}{2} \rho v_2^2 = \frac{1}{2} \rho v_2^2 (16 - 1)\]

\[= \frac{1}{2} \left(1060 \text{ kg/m}^3\right) (0.11 \text{ m/s})^2 (15) = 96 \text{ Pa}\]

We have taken the density \(\rho\) of blood from Table 11.1.
66. **REASONING**  
   a. The drawing shows two points, labeled 1 and 2, in the fluid. Point 1 is at the top of the water, and point 2 is where it flows out of the dam at the bottom. Bernoulli’s equation, Equation 11.11, can be used to determine the speed \( v_2 \) of the water exiting the dam. 
   
   ![Diagram of fluid flow](image)

   b. The number of cubic meters per second of water that leaves the dam is the volume flow rate \( Q \). According to Equation 11.10, the volume flow rate is the product of the cross-sectional area \( A_2 \) of the crack and the speed \( v_2 \) of the water; \( Q = A_2 v_2 \).

**SOLUTION**  
   a. According to Bernoulli’s equation, as given in Equation 11.11, we have

   \[
P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2
   \]

   Setting \( P_1 = P_2 \), \( v_1 = 0 \text{ m/s} \), and solving for \( v_2 \), we obtain

   \[
v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80 \text{ m/s}^2)(15.0 \text{ m})} = 17.1 \text{ m/s}
   \]

   b. The volume flow rate of the water leaving the dam is

   \[
   Q = A_2 v_2 = (1.30 \times 10^{-3} \text{ m}^2)(17.1 \text{ m/s}) = 2.22 \times 10^{-2} \text{ m}^3/\text{s}
   \]

(11.10)

67. **REASONING** The pressure \( P \), the fluid speed \( v \), and the elevation \( y \) at any two points in an ideal fluid of density \( \rho \) are related by Bernoulli’s equation:

   \[
P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2
   \]  

   (Equation 11.11), where 1 and 2 denote, respectively, the first and second floors. With the given data and a density of \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \) for water (see Table 11.1), we can solve Bernoulli’s equation for the desired pressure \( P_2 \).

**SOLUTION**  
   Solving Bernoulli’s equation for \( P_2 \) and taking the elevation at the first floor to be \( y_1 = 0 \text{ m} \), we have
$$P_2 = P_1 + \frac{1}{2} \rho \left(v_1^2 - v_2^2\right) + \rho g \left(y_1 - y_2\right)$$

$$= 3.4 \times 10^5 \text{ Pa} + \frac{1}{2} \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left[\left(2.1 \text{ m/s}\right)^2 - \left(3.7 \text{ m/s}\right)^2\right]$$

$$+ \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) (0 \text{ m} - 4.0 \text{ m}) = 3.0 \times 10^5 \text{ Pa}$$

68. **REASONING** The volume of water per second leaking into the hold is the volume flow rate $Q$. The volume flow rate is the product of the effective area $A$ of the hole and the speed $v_1$ of the water entering the hold, $Q = Av_1$ (Equation 11.10). We can find the speed $v_1$ with the aid of Bernoulli’s equation.

**SOLUTION** According to Bernoulli’s equation, which relates the pressure $P$, water speed $v$, and elevation $y$ of two points in the water:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2$$

(11.11)

In this equation, the subscript “1” refers to the point below the surface where the water enters the hold, and the subscript “2” refers to a point on the surface of the lake. Since the amount of water in the lake is large, the water level at the surface drops very, very slowly as water enters the hold of the ship. Thus, to a very good approximation, the speed of the water at the surface is zero, so $v_2 = 0$ m/s. Setting $P_1 = P_2$ (since the empty hold is open to the atmosphere) and $v_2 = 0$ m/s, and then solving for $v_1$, we obtain $v_1 = \sqrt{2g \left(y_2 - y_1\right)}$.

Substituting $v_1 = \sqrt{2g \left(y_2 - y_1\right)}$ into Equation 11.10, we find that the volume flow rate $Q$ of the water entering the hold is

$$Q = Av_1 = A \sqrt{2g \left(y_2 - y_1\right)} = \left(8.0 \times 10^{-3} \text{ m}^2\right) \sqrt{2 \left(9.80 \text{ m/s}^2\right) (2.0 \text{ m})} = 5.0 \times 10^{-2} \text{ m}^3/\text{s}$$

69. **SSM REASONING** Since the pressure difference is known, Bernoulli’s equation can be used to find the speed $v_2$ of the gas in the pipe. Bernoulli’s equation also contains the unknown speed $v_1$ of the gas in the Venturi meter; therefore, we must first express $v_1$ in terms of $v_2$. This can be done by using Equation 11.9, the equation of continuity.

**SOLUTION**

a. From the equation of continuity (Equation 11.9) it follows that $v_1 = \left(A_2 / A_1\right)v_2$.

Therefore,

$$v_1 = \frac{0.0700 \text{ m}^2}{0.0500 \text{ m}^2} v_2 = (1.40) v_2$$

Substituting this expression into Bernoulli’s equation (Equation 11.12), we have
\[ P_1 + \frac{1}{2} \rho (1.40 v_2)^2 = P_2 + \frac{1}{2} \rho v_2^2 \]

Solving for \( v_2 \), we obtain
\[ v_2 = \sqrt{\frac{2(P_2 - P_1)}{\rho (1.40)^2 - 1}} = \sqrt{\frac{2(120 \text{ Pa})}{(1.30 \text{ kg/m}^3)(1.40)^2 - 1}} = 14 \text{ m/s} \]

b. According to Equation 11.10, the volume flow rate is
\[ Q = A_2 v_2 = (0.0700 \text{ m}^2)(14 \text{ m/s}) = 0.98 \text{ m}^3/\text{s} \]

70. **REASONING** The gauge pressure in the reservoir is the pressure difference \( P_2 - P_1 \) between the reservoir \( (P_2) \) and the atmosphere \( (P_1) \). The muzzle is open to the atmosphere, so the pressure there is atmospheric pressure. Because we are ignoring the height difference between the reservoir and the muzzle, this is an example of fluid flow in a horizontal pipe, for which Bernoulli’s equation is
\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \]  
where we have used the fact that the speed \( v_2 \) of the water in the reservoir is zero. We will determine the speed \( v_1 \) of the water stream at the muzzle by considering its subsequent projectile motion. The horizontal displacement of the water stream after leaving the muzzle is given by \( x = v_{0x} t + \frac{1}{2} a_x t^2 \) (Equation 3.5a). With air resistance neglected, the water stream undergoes no horizontal acceleration \( (a_x = 0 \text{ m/s}^2) \). The horizontal component \( v_{0x} \) of the initial velocity of the water stream is identical with the velocity \( v_1 \) in Equation (1), so Equation 3.5a becomes
\[ x = v_{0x} t + \frac{1}{2} (0 \text{ m/s}^2) t^2 \quad \text{or} \quad v_{0x} = \frac{x}{t} \]  
The vertical displacement \( y \) of the water stream is given by \( y = v_{0y} t + \frac{1}{2} a_y t^2 \) (Equation 3.5b), where \( a_y = -9.8 \text{ m/s}^2 \) is the acceleration due to gravity, with upward taken as the positive direction. The velocity of the water at the instant it leaves the muzzle is horizontal, so the vertical component of its velocity is zero. This means that we have \( v_{0y} = 0 \text{ m/s} \) in Equation 3.5b. Solving Equation 3.5b for the elapsed time \( t \), we obtain
\[ y = (0 \text{ m/s}) t + \frac{1}{2} a_y t^2 \quad \text{or} \quad t = \frac{\sqrt{2y}}{a_y} \]
SOLUTION  Substituting Equation (3) into Equation (2) gives

\[ v_1 = \frac{x}{\sqrt{\frac{2y}{a_y}}} = x \sqrt{\frac{a_y}{2y}} \]  

(4)

Substituting Equation (4) for \( v_1 \) into Equation (1), we obtain the gauge pressure \( P_2 - P_1 \):

\[ P_2 - P_1 = \frac{1}{2} \rho \left( x \sqrt{\frac{a_y}{2y}} \right)^2 = \frac{1}{2} \rho x^2 \left( \frac{a_y}{2y} \right) = \frac{\rho x^2 a_y}{4y} \]

\[ = \left( \frac{1.000 \times 10^3 \text{ kg/m}^3 \times (7.3 \text{ m})^2 \times (-9.80 \text{ m/s}^2)}{4 \times (-0.75 \text{ m})} \right) = 1.7 \times 10^5 \text{ Pa} \]

71. **REASONING AND SOLUTION**  As seen in the drawing at the right, the lower pipe is at the level of zero potential energy. If the flow rate is uniform in both pipes, we have \( (1/2) \rho v_1^2 = (1/2) \rho v_2^2 + \rho gh \) (since \( P_1 = P_2 \)) and \( A_1 v_1 = A_2 v_2 \). We can solve for \( v_1 \), i.e., \( v_1 = v_2 \frac{A_2}{A_1} \), and plug into the previous expression to find \( v_2 \). In so doing, we find that

\[ v_2 = \sqrt{\frac{2gh}{\left( \frac{A_2}{A_1} \right)^2 - 1}} = \sqrt{\left[ \frac{\frac{2(9.80 \text{ m/s}^2)(10.0 \text{ m})}{\pi (0.0400 \text{ m})^2}}{\pi (0.0200 \text{ m})^2} \right] - 1} = 3.61 \text{ m/s} \]

The volume flow rate is then given by

\[ Q = A_2 v_2 = \pi r_2^2 v_2 = \pi (0.0400 \text{ m})^2 (3.61 \text{ m/s}) = 1.81 \times 10^{-2} \text{ m}^3/\text{s} \]

72. **REASONING**  In level flight the lift force must balance the plane’s weight \( W \), so its magnitude is also \( W \). The lift force arises because the pressure \( P_B \) beneath the wings is greater than the pressure \( P_T \) on top of the wings. The lift force, then, is the pressure difference times the effective wing surface area \( A \), so that \( W = (P_B - P_T)A \). The area is given, and we can determine the pressure difference by using Bernoulli’s equation.
**SOLUTION** According to Bernoulli’s equation, we have

\[ P_B + \frac{1}{2} \rho v_B^2 + \rho g y_B = P_T + \frac{1}{2} \rho v_T^2 + \rho g y_T \]

Since the flight is level, the height is constant and \( y_B = y_T \), where we assume that the wing thickness may be ignored. Then, Bernoulli’s equation simplifies and may be rearranged as follows:

\[ P_B + \frac{1}{2} \rho v_B^2 = P_T + \frac{1}{2} \rho v_T^2 \quad \text{or} \quad P_B - P_T = \frac{1}{2} \rho (v_T^2 - v_B^2) \]

Recognizing that \( W = (P_B - P_T)A \), we can substitute for the pressure difference from Bernoulli’s equation to show that

\[ W = \frac{1}{2} \rho (v_T^2 - v_B^2) A \]

\[ = \frac{1}{2} \left( 1.29 \text{ kg/m}^3 \right) \left[ (62.0 \text{ m/s})^2 - (54.0 \text{ m/s})^2 \right] (16 \text{ m}^2) = 9600 \text{ N} \]

We have used a value of 1.29 kg/m\(^3\) from Table 11.1 for the density of air. This is an approximation, since the density of air decreases with increasing altitude above sea level.

73. **REASONING** The top and bottom surfaces of the roof are at the same height, so we can use Bernoulli’s equation in the form of Equation 11.12, \( P_i + \frac{1}{2} \rho v_i^2 = P_j + \frac{1}{2} \rho v_j^2 \), to determine the wind speed. We take point 1 to be inside the roof and point 2 to be outside the roof. Since the air inside the roof is not moving, \( v_1 = 0 \text{ m/s} \). The net outward force \( \Sigma F \) acting on the roof is the difference in pressure \( P_1 - P_2 \) times the area \( A \) of the roof, so \( \Sigma F = (P_1 - P_2)A \).

**SOLUTION** Setting \( v_1 = 0 \text{ m/s} \) in Bernoulli’s equation and solving it for the speed \( v_2 \) of the wind, we obtain

\[ v_2 = \sqrt{\frac{2(\Sigma F)}{\rho}} \]

Since the pressure difference is equal to the net outward force divided by the area of the roof, \( (P_1 - P_2) = \Sigma F / A \), the speed of the wind is

\[ v_2 = \sqrt{\frac{2(\Sigma F)}{\rho A}} = \sqrt{\frac{2(22000 \text{ N})}{(1.29 \text{ kg/m}^3)(5.0 \text{ m} \times 6.3 \text{ m})}} = 33 \text{ m/s} \]
74. **REASONING** We will apply Bernoulli’s equation to this problem. The pump located beneath the surface draws water into the intake pipe, and the pressure of the moving water decreases. However, the pressure of the water moving in the intake pipe can only decrease to $P_{\text{intake}} = 0 \text{ Pa}$, at which instant the speed of the water in the intake pipe would have its maximum value. In contrast, the pressure at the surface of the reservoir is the normal atmospheric pressure, so that $P_{\text{surface}} = 1.01 \times 10^5 \text{ Pa}$. Moreover, the reservoir is large, so that we can assume that the speed of the water at the surface is $v_{\text{surface}} = 0 \text{ m/s}$.

**SOLUTION** As applied to this problem, Bernoulli’s equation given in Equation 11.11 becomes

$$P_{\text{intake}} + \frac{1}{2} \rho v_{\text{intake}}^2 + \rho g y_{\text{intake}} = P_{\text{surface}} + \frac{1}{2} \rho v_{\text{surface}}^2 + \rho g y_{\text{surface}}$$

Using the facts that $P_{\text{intake}} = 0 \text{ Pa}$ and $v_{\text{surface}} = 0 \text{ m/s}$, we see that Bernoulli’s equation reduces to

$$\frac{1}{2} \rho v_{\text{intake}}^2 + \rho g y_{\text{intake}} = P_{\text{surface}} + \rho g (y_{\text{surface}} - y_{\text{intake}})$$

Solving for $v_{\text{intake}}$ and using $\rho = 1.00 \times 10^3 \text{ kg/m}^3$ for the density of water (see Table 11.1) gives

$$v_{\text{intake}} = \sqrt{\frac{2P_{\text{surface}}}{\rho} + 2g (y_{\text{surface}} - y_{\text{intake}})}$$

$$= \sqrt{\frac{2(1.01 \times 10^5 \text{ Pa})}{1.00 \times 10^3 \text{ kg/m}^3} + 2(9.80 \text{ m/s}^2)(12 \text{ m})} = 21 \text{ m/s}$$

75. **SSM REASONING AND SOLUTION** Bernoulli’s equation (Equation 11.12) is

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

If we let the right hand side refer to the air above the plate, and the left hand side refer to the air below the plate, then $v_1 = 0 \text{ m/s}$, since the air below the plate is stationary. We wish to find $v_2$ for the situation illustrated in part $b$ of the figure shown in the text. Solving the equation above for $v_2$ (with $v_2 = v_{2b}$ and $v_1 = 0$) gives

$$v_{2b} = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$  \hspace{1cm} (1)

In Equation (1), $P_1$ is atmospheric pressure and $P_2$ must be determined. We must first consider the situation in part $a$ of the text figure.
The figure at the right shows the forces that act on the rectangular plate in part a of the text drawing. \( F_1 \) is the force exerted on the plate from the air below the plate, and \( F_2 \) is the force exerted on the plate from the air above the plate. Applying Newton’s second law, we have (taking “up” to be the positive direction),

\[
F_1 - F_2 - mg = 0
\]

\[
F_1 - F_2 = mg
\]

Thus, the difference in pressures exerted by the air on the plate in part a of the drawing is

\[
P_1 - P_2 = \frac{F_1 - F_2}{A} = \frac{mg}{A}
\]

(2)

where \( A \) is the area of the plate. From Bernoulli’s equation (Equation 11.12) we have, with \( v_2 = v_{2a} \) and \( v_{1a} = 0 \) m/s,

\[
P_1 - P_2 = \frac{1}{2} \rho v_{2a}^2
\]

(3)

where \( v_{2a} \) is the speed of the air along the top of the plate in part a of the text drawing. Combining Equations (2) and (3) we have

\[
\frac{mg}{A} = \frac{1}{2} \rho v_{2a}^2
\]

(4)

The following figure, on the left, shows the forces that act on the plate in part b of the text drawing. The notation is the same as that used when the plate was horizontal (part a of the text figure). The figure at the right below shows the same forces resolved into components along the plate and perpendicular to the plate.

Applying Newton’s second law we have \( F_1 - F_2 - mg \sin \theta = 0 \), or

\[
F_1 - F_2 = mg \sin \theta
\]

Thus, the difference in pressures exerted by the air on the plate in part b of the text figure is
\[ P_1 - P_2 = \frac{F_1 - F_2}{A} = \frac{mg \sin \theta}{A} \]

Using Equation (4) above,

\[ P_1 - P_2 = \frac{1}{2} \rho v_{2a}^2 \sin \theta \]

Thus, Equation (1) becomes

\[ v_{2b} = \sqrt{\frac{2(\frac{1}{2} \rho v_{2a}^2 \sin \theta)}{\rho}} \]

Therefore,

\[ v_{2b} = \sqrt{v_{2a}^2 \sin \theta} = \sqrt{(11.0 \text{ m/s})^2 \sin 30.0^\circ} = 7.78 \text{ m/s} \]

76. **REASONING AND SOLUTION**

a. Since the volume flow rate, \( Q = Av \), is the same at each point, and since \( v \) is greater at the lower point, the upper hole must have the larger area.

b. Call the upper hole number 1 and the lower hole number 2 (the surface of the water is position 0). Take the zero level of potential energy at the bottom hole and then write Bernoulli’s equation as

\[ P_1 + (1/2)\rho v_1^2 + \rho gh = P_2 + (1/2)\rho v_2^2 + P_0 + \rho g(2h) \]

in which \( P_1 = P_2 = P_0 \) and from which we obtain

\[ v_1 = \sqrt{2gh} \quad \text{and} \quad v_2 = \sqrt{4gh} \quad \text{or} \quad v_2/v_1 = \sqrt{2} \]

Using the fact that \( Q_1 = A_1 v_1 = Q_2 = A_2 v_2 \), we have \( v_2/v_1 = A_1/A_2 \). But since the ratio of the areas is \( A_1/A_2 = r_1^2/r_2^2 \), we can write that

\[ \frac{r_1}{r_2} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{v_2^2}{v_1}} = \sqrt{\frac{4}{2}} = 1.19 \]

77. **SSM REASONING AND SOLUTION** The Reynold’s number, Re, can be written as

\[ \text{Re} = \frac{2\nu \rho R}{\eta} \]

To find the average speed \( \bar{v} \),
\[ v = \frac{(\text{Re}) \eta}{2 \rho R} = \frac{2000 (4.0 \times 10^{-3} \text{ Pa} \cdot \text{s})}{2 (1060 \text{ kg/m}^3) (8.0 \times 10^{-3} \text{ m})} = 0.5 \text{ m/s} \]

78. **REASONING** Bernoulli’s equation does not apply to this problem, because the fluid is not ideal. It has a viscosity, and Poiseuille’s law, as given in Equation 11.14, must be used instead.

**SOLUTION** According to Equation 11.14, we have

\[ Q = \frac{\pi R^4 (P_{\text{Input}} - P_{\text{Output}})}{8 \eta L} \quad \text{or} \quad P_{\text{Input}} - P_{\text{Output}} = \frac{Q 8 \eta L}{\pi R^4} \]

Solving for the input pressure gives

\[ P_{\text{Input}} = P_{\text{Output}} + \frac{Q 8 \eta L}{\pi R^4} \]

\[ = 1.013 \times 10^5 \text{ Pa} + \frac{(5.3 \times 10^{-5} \text{ m}^3 / \text{s}) (0.14 \text{ Pa} \cdot \text{s}) (37 \text{ m})}{\pi (6.0 \times 10^{-3} \text{ m})^4} = 6.4 \times 10^5 \text{ Pa} \]

79. **REASONING** The volume flow rate \( Q \) is governed by Poiseuille’s law: \( Q = \frac{\pi R^4 (P_2 - P_1)}{8 \eta L} \) (Equation 11.14). We will obtain the factor \( R_{\text{dilated}}/R_{\text{normal}} \) by applying this law to the dilated and to the normal blood vessel.

**SOLUTION** Solving Poiseuille’s law for the radius \( R \) gives \( R = \sqrt[4]{\frac{8 \eta L Q}{\pi (P_2 - P_1)}} \). When the vessel dilates, the viscosity \( \eta \), the length \( L \) of the vessel, and the pressures \( P_1 \) and \( P_2 \) at the ends of the vessel do not change. Thus, applying this result to the dilated and normal vessel, we find that

\[ \frac{R_{\text{dilated}}}{R_{\text{normal}}} = \sqrt[4]{\frac{8 \eta L Q_{\text{dilated}}}{\pi (P_2 - P_1)}} \quad \sqrt[4]{\frac{8 \eta L Q_{\text{normal}}}{\pi (P_2 - P_1)}} = \sqrt[4]{\frac{Q_{\text{dilated}}}{Q_{\text{normal}}}} = \sqrt[4]{2} = 1.19 \]

where \( \frac{Q_{\text{dilated}}}{Q_{\text{normal}}} = 2 \), since the effect of the dilation is to double the volume flow rate.
Chapter 11 Problems

80. **REASONING** Because the level of the blood in the transfusion bottle is a height \( h \) above the input end of the needle, the pressure \( P_2 \) at the input end of the needle is greater than one atmosphere, as we see from \( P_2 = P_{\text{atm}} + \rho gh \) (Equation 11.4), where \( P_{\text{atm}} \) is the pressure at the level of the blood in the bottle, \( \rho \) is the density of blood, and \( g \) is the magnitude of the acceleration due to gravity. The pressure at the output end of the needle is atmospheric \( (P_1 = P_{\text{atm}}) \), so that there is the pressure difference \( P_2 - P_1 \) necessary to push blood, a viscous fluid, through the needle. The volume flow rate \( Q \) of blood through the needle is given by Poiseuille’s law as

\[
Q = \frac{4}{8\eta L} \pi R^4 \left( \frac{P_2 - P_1}{P_{\text{atm}}} \right)
\]

(Equation 11.14), where \( R \) and \( L \) are, respectively, the radius and length of the needle, and \( \eta \) is the viscosity of blood.

**SOLUTION** Solving \( P_2 = P_{\text{atm}} + \rho gh \) (Equation 11.4) for \( h \), we find that

\[
h = \frac{P_2 - P_{\text{atm}}}{\rho g}
\]

Because the pressure \( P_1 \) at the output end of the needle is equal to \( P_{\text{atm}} \), Equation 11.4 becomes \( Q = \frac{4}{8\eta L} \pi R^4 \left( P_2 - P_{\text{atm}} \right) \). Solving for the pressure difference, we obtain

\[
P_2 - P_{\text{atm}} = \frac{8\eta LQ}{\pi R^4}
\]

(Equation 2)

Substituting Equation (2) into Equation (1) yields

\[
h = \frac{\left( \frac{8\eta LQ}{\pi R^4} \right)}{\frac{8\eta LQ}{\pi R^4 \rho g}} = \frac{8(4.0 \times 10^{-3} \text{ Pa} \cdot \text{s})(0.030 \text{ m})(4.5 \times 10^{-8} \text{ m}^3/\text{s})}{\pi \left( 2.5 \times 10^{-4} \text{ m} \right)^4 \left( 1060 \text{ kg/m}^3 \right)(9.80 \text{ m/s}^2)} = 0.34 \text{ m}
\]

81. **SSM** **REASONING** The volume flow rate \( Q \) of a viscous fluid flowing through a pipe of radius \( R \) is given by Equation 11.14 as

\[
Q = \frac{\pi R^4 \left( P_2 - P_1 \right)}{8\eta L}
\]

where \( P_2 - P_1 \) is the pressure difference between the ends of the pipe, \( L \) is the length of the pipe, and \( \eta \) is the viscosity of the fluid. Since all the variables are known except \( L \), we can use this relation to find it.

**SOLUTION** Solving Equation 11.14 for the pipe length, we have

\[
L = \frac{\frac{\pi R^4 \left( P_2 - P_1 \right)}{8\eta Q}}{8(1.0 \times 10^{-3} \text{ Pa} \cdot \text{s})(2.8 \times 10^{-4} \text{ m}^3/\text{s})} = 1.7 \text{ m}
\]
82. **REASONING** In this problem, we are treating air as a viscous fluid. According to Poiseuille’s law, a fluid with viscosity $\eta$ flowing through a pipe of radius $R$ and length $L$ has a volume flow rate $Q$ given by Equation 11.14: 
\[ Q = \frac{\pi R^4 (P_2 - P_1)}{8 \eta L} \]
This expression can be solved for the quantity $P_2 - P_1$, the difference in pressure between the ends of the air duct. First, however, we must determine the volume flow rate $Q$ of the air.

**SOLUTION** Since the fan forces air through the duct such that 280 m$^3$ of air is replenished every ten minutes, the volume flow rate is
\[ Q = \frac{280 \text{ m}^3}{10.0 \text{ min}} \cdot \frac{1.0 \text{ min}}{60 \text{ s}} = 0.467 \text{ m}^3 / \text{s} \]
The difference in pressure between the ends of the air duct is, according to Poiseuille’s law,
\[ P_2 - P_1 = \frac{8 \eta L Q}{\pi R^4} = \frac{8(1.8 \times 10^{-5} \text{ Pa} \cdot \text{s})(5.5 \text{ m})(0.467 \text{ m}^3 / \text{s})}{\pi(7.2 \times 10^{-2} \text{ m})^4} = 4.4 \text{ Pa} \]

83. **REASONING** The speed $v$ of the water in the hose is related to the volume flow rate $Q$ by $v = Q/A$ (Equation 11.10), where $A$ is the cross-sectional area of the hose. Since the hose is cylindrical (radius = $R$), $A = \pi R^2$. Thus, we have that
\[ v = \frac{Q}{A} = \frac{Q}{\pi R^2} \] (1)
We turn to Poiseuille’s law to express $Q$ in terms of the upstream pressure $P_2$ and the downstream pressure $P_1$
\[ Q = \frac{\pi R^4 (P_2 - P_1)}{8 \eta L} \] (11.14)
In Poiseuille’s law, the upstream pressure $P_2$ is the same for both hoses, because it is the pressure at the outlet. The downstream pressure $P_1$ is also the same for both hoses, since each is open to the atmosphere at the exit end. Therefore, the term $P_2 - P_1$ is the same for both hoses.

**SOLUTION** Substituting Equation 11.14 into Equation (1) gives
\[ v = \frac{Q}{\pi R^2} = \frac{\frac{\pi R^4 (P_2 - P_1)}{8 \eta L}}{\pi R^2} = \frac{R^2 (P_2 - P_1)}{8 \eta L} \]
To find the ratio $v_B/v_A$ of the speeds, we apply this result to each hose, recognizing that the pressure difference $P_2 - P_1$, the length $L$, and the viscosity $\eta$ of the water are the same for both hoses:
84. **REASONING AND SOLUTION**

a. Using Stoke’s law, the viscous force is

\[
F = 6\pi \eta R v = 6\pi (1.00 \times 10^{-3} \text{ Pa} \cdot \text{s})(5.0 \times 10^{-4} \text{ m})(3.0 \text{ m/s}) = 2.8 \times 10^{-5} \text{ N}
\]

b. When the sphere reaches its terminal speed, the net force on the sphere is zero, so that the magnitude of \( F \) must be equal to the magnitude of \( mg \), or \( F = mg \). Therefore, \( 6\pi \eta R v_T = mg \), where \( v_T \) is the terminal speed of the sphere. Solving for \( v_T \), we have

\[
v_T = \frac{mg}{6\pi \eta R} = \frac{(1.0 \times 10^{-5} \text{ kg})(9.80 \text{ m/s}^2)}{6\pi (1.00 \times 10^{-3} \text{ Pa} \cdot \text{s})(5.0 \times 10^{-4} \text{ m})} = 1.0 \times 10^4 \text{ m/s}
\]

85. **SSM REASONING** As the depth \( h \) increases, the pressure increases according to Equation 11.4 \( (P_2 = P_1 + \rho gh) \). In this equation, \( P_1 \) is the pressure at the shallow end, \( P_2 \) is the pressure at the deep end, and \( \rho \) is the density of water \( (1.00 \times 10^3 \text{ kg/m}^3 \), see Table 11.1). We seek a value for the pressure at the deep end minus the pressure at the shallow end.

**SOLUTION** Using Equation 11.4, we find

\[
P_{\text{Deep}} = P_{\text{Shallow}} + \rho gh \quad \text{or} \quad P_{\text{Deep}} - P_{\text{Shallow}} = \rho gh
\]

The drawing at the right shows that a value for \( h \) can be obtained from the 15-m length of the pool by using the tangent of the 11° angle:

\[
\tan 11^\circ = \frac{h}{15 \text{ m}} \quad \text{or} \quad h = (15 \text{ m}) \tan 11^\circ
\]

\[
P_{\text{Deep}} - P_{\text{Shallow}} = \rho g (15 \text{ m}) \tan 11^\circ
\]

\[
= \left(1.00 \times 10^3 \text{ kg/m}^3\right)(9.80 \text{ m/s}^2)(15 \text{ m}) \tan 11^\circ = 2.9 \times 10^4 \text{ Pa}
\]
86. **REASONING** We apply Bernoulli’s equation as follows:

\[
\frac{P_S + \frac{1}{2} \rho v_S^2 + \rho g y_S}{\text{At surface of vaccine in reservoir}} = \frac{P_O + \frac{1}{2} \rho v_O^2 + \rho g y_O}{\text{At opening}}
\]

**SOLUTION** The vaccine’s surface in the reservoir is stationary during the inoculation, so that \( v_S = 0 \) m/s. The vertical height between the vaccine’s surface in reservoir and the opening can be ignored, so \( y_S = y_O \). With these simplifications Bernoulli’s equation becomes

\[
P_S = P_O + \frac{1}{2} \rho v_O^2
\]

Solving for the speed at the opening gives

\[
v_O = \sqrt{\frac{2(P_S - P_O)}{\rho}} = \sqrt{\frac{2(4.1 \times 10^6 \text{ Pa})}{1100 \text{ kg/m}^3}} = 86 \text{ m/s}
\]

87. **SSM REASONING** The buoyant force exerted on the balloon by the air must be equal in magnitude to the weight of the balloon and its contents (load and hydrogen). The magnitude of the buoyant force is given by \( \rho_{\text{air}} V g \). Therefore,

\[
\rho_{\text{air}} V g = W_{\text{load}} + \rho_{\text{hydrogen}} V g
\]

where, since the balloon is spherical, \( V = (4/3)\pi r^3 \). Making this substitution for \( V \) and solving for \( r \), we obtain

\[
r = \left[ \frac{3W_{\text{load}}}{4\pi g(\rho_{\text{air}} - \rho_{\text{hydrogen}})} \right]^{1/3}
\]

**SOLUTION** Direct substitution of the data given in the problem yields

\[
r = \left[ \frac{3(5750 \text{ N})}{4\pi(9.80 \text{ m/s}^2)(1.29 \text{ kg/m}^3 - 0.0899 \text{ kg/m}^3)} \right]^{1/3} = 4.89 \text{ m}
\]

88. **REASONING** The pressure difference across the diver’s eardrum is \( P_{\text{ext}} - P_{\text{int}} \), where \( P_{\text{ext}} \) is the external pressure and \( P_{\text{int}} \) is the internal pressure. The diver’s eardrum ruptures when \( P_{\text{ext}} - P_{\text{int}} = 35 \text{ kPa} \). The exterior pressure \( P_{\text{ext}} \) on the diver’s eardrum increases with the depth \( h \) of the dive according to \( P_{\text{ext}} = P_{\text{atm}} + \rho g h \) (Equation 11.4), where \( P_{\text{atm}} \) is the atmospheric pressure at the surface of the water, \( \rho \) is the density of seawater, and \( g \) is the magnitude of the acceleration due to gravity. Solving Equation 11.4 for the depth \( h \) yields
\[ h = \frac{P_{\text{ext}} - P_{\text{atm}}}{\rho g} \] (1)

**SOLUTION** When the diver is at the surface, the internal pressure is equalized with the external pressure: \( P_{\text{int}} = P_{\text{atm}} \). If the diver descends slowly from the surface, and takes steps to equalize the exterior and interior pressures, there is no pressure difference across the eardrum, regardless of depth: \( P_{\text{ext}} = P_{\text{int}} \). But if the diver sinks rapidly to a depth \( h \) without equalizing the pressures, then the internal pressure is still atmospheric pressure. In this case, \( P_{\text{atm}} \) in Equation (1) equals \( P_{\text{int}} \), so the eardrum is at risk for rupture at the depth \( h \) given by

\[
h = \frac{35 \, \text{kPa}}{(1025 \, \text{kg/m}^3)(9.80 \, \text{m/s}^2)} = \frac{35 \times 10^3 \, \text{Pa}}{(1025 \, \text{kg/m}^3)(9.80 \, \text{m/s}^2)} = 3.5 \, \text{m}
\]

89. **REASONING** The weight \( W \) of the water bed is equal to the mass \( m \) of water times the acceleration \( g \) due to gravity: \( W = mg \) (Equation 4.5). The mass, on the other hand, is equal to the density \( \rho \) of the water times its volume \( V \), or \( m = \rho V \) (Equation 11.1).

**SOLUTION** Substituting \( m = \rho V \) into the relation \( W = mg \) gives

\[
W = mg = (\rho V) g
\]

\[
= (1.00 \times 10^3 \, \text{kg/m}^3)(1.83 \, \text{m} \times 2.13 \, \text{m} \times 0.229 \, \text{m})(9.80 \, \text{m/s}^2) = 8750 \, \text{N}
\]

We have taken the density of water from Table 11.1. Since the weight of the water bed is greater than the additional weight that the floor can tolerate, the bed should not be purchased.

90. **REASONING** The flow rate \( Q \) of water in the pipe is given by Poiseuille’s law

\[
Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L} \] (Equation 11.14), where \( R \) is the radius of the pipe, \( P_2 - P_1 \) is the difference in pressure between the ends of the pipe, \( \eta \) is the viscosity of water, and \( L \) is the length of the pipe. As the radius \( R \) of the pipe gets smaller, its length \( L \) does not change, nor does the viscosity \( \eta \) of the water. We are told that the pressure difference \( P_2 - P_1 \) also remains constant. Moving all of the constant quantities to one side of Equation 11.14, we obtain

\[
\frac{\pi (P_2 - P_1)}{8\eta L} = \frac{Q}{R^4}
\]

(1)

We will use Equation (1) to find the final flow rate \( Q_f \) in terms of the initial flow rate \( Q_0 \), the initial radius \( R_0 \) and the final radius \( R_f \) of the pipe.
**SOLUTION**  Equation (1) shows that the ratio \( \frac{Q}{R^4} \) of the flow rate to the fourth power of the pipe’s inner radius remains constant as the inner radius decreases to \( R_f \) from \( R_0 \). Therefore, we have that

\[
\frac{Q_f}{R_f^4} = \frac{Q_0}{R_0^4} \quad \text{or} \quad Q_f = \frac{Q_0 R_f^4}{R_0^4} = Q_0 \left( \frac{R_f}{R_0} \right)^4
\]

(2)

Note that the ratio \( \left( \frac{R_f}{R_0} \right)^4 \) multiplying the initial flow rate \( Q_0 \) in Equation (2) is unitless, so that \( Q_f \) will have the same units as \( Q_0 \). Thus, to obtain the final flow rate in gallons per minute, there is no need to convert \( Q_0 \) to SI units (m³/s). Substituting the given values into Equation (2) yields the final flow rate:

\[
Q_f = (740 \text{ gal/min}) \left( \frac{0.19 \text{ m}}{0.24 \text{ m}} \right)^4 = 290 \text{ gal/min}
\]

91. **REASONING AND SOLUTION**  14.0 karat gold is \((14.0)/(24.0)\) gold or 58.3%. The weight of the gold in the necklace is then \((1.27 \text{ N})(0.583) = 0.740 \text{ N}\). This corresponds to a volume given by \( V = \frac{M}{\rho} = \frac{W}{(\rho g)} \). Thus,

\[
V = \frac{0.740 \text{ N}}{\left( 19 \text{ 300 kg/m}^3 \right)(9.80 \text{ m/s}^2)} = 3.91 \times 10^{-6} \text{ m}^3
\]

92. **REASONING**  The radius \( R \) of the pipe is related to the volume flow rate \( Q \), the length \( L \) of the pipe, the viscosity \( \eta \) of the glycerol, and the pressure difference \( P_2 - P_1 \) between the input and output ends by Poiseuille’s law: \( Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L} \) (Equation 11.14). Solving Equation 11.14 for the radius \( R \), we obtain

\[
R^4 = \frac{8\eta L Q}{\pi (P_2 - P_1)} \quad \text{or} \quad R = \sqrt[4]{\frac{8\eta L Q}{\pi (P_2 - P_1)}}
\]

(1)

The volume flow rate \( Q \) is the ratio of the volume \( V \) of glycerol that flows through the pipe to the total elapsed time \( t \):

\[
Q = \frac{V}{t}
\]

(2)

Substituting Equation (2) into Equation (1) yields
\[ R = 4 \sqrt{\frac{8\eta LV}{\pi (P_2 - P_1) t}} \]  

**SOLUTION** Because the output end of the pipe is at atmospheric pressure, \( P_1 = 1.013 \times 10^5 \) Pa. The elapsed time \( t \) must be converted to SI units (seconds):

\[ t = \left( \frac{55 \text{ min}}{1 \text{ min}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 3300 \text{ s} . \]

Therefore, the radius of the pipe is

\[ R = 4 \sqrt{\frac{8(0.934 \text{ Pa} \cdot \text{s})(15 \text{ m})(7.2 \text{ m}^3)}{\pi \left[ (8.6 \times 10^5 \text{ Pa}) - (1.013 \times 10^5 \text{ Pa}) \right] (3300 \text{ s})}} = 1.8 \times 10^{-2} \text{ m} \]

93. **REASONING AND SOLUTION** The pump must generate an upward force to counteract the weight of the column of water above it. Therefore, \( F = mg = (\rho hV)g \). The required pressure is then

\[ P = \frac{F}{A} = \rho gh = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(71 \text{ m}) = 7.0 \times 10^5 \text{ Pa} \]

94. **REASONING** The density \( \rho \) of an object is equal to its mass \( m \) divided by its volume \( V \), or \( \rho = \frac{m}{V} \) (Equation 11.1). The volume of a sphere is \( V = \frac{4}{3} \pi r^3 \), where \( r \) is the radius. According to the discussion of Archimedes’ principle in Section 11.6, any object that is solid throughout will float in a liquid if the density of the object is less than or equal to the density of the liquid. If not, the object will sink.

**SOLUTION**

a. The average density of the sun is

\[ \rho = \frac{m}{V} = \frac{m}{\frac{4}{3} \pi r^3} = \frac{1.99 \times 10^{30} \text{ kg}}{\frac{4}{3} \pi \left( 6.96 \times 10^8 \text{ m} \right)^3} = 1.41 \times 10^3 \text{ kg/m}^3 \]

b. Since the average density of the solid object \((1.41 \times 10^3 \text{ kg/m}^3)\) is greater than that of water \((1.00 \times 10^3 \text{ kg/m}^3\), see Table 11.1), the object will [sink].

c. The average density of Saturn is

\[ \rho = \frac{m}{V} = \frac{m}{\frac{4}{3} \pi r^3} = \frac{5.7 \times 10^{26} \text{ kg}}{\frac{4}{3} \pi \left( 6.0 \times 10^7 \text{ m} \right)^3} = 0.63 \times 10^3 \text{ kg/m}^3 \]

The average density of this solid object \((0.63 \times 10^3 \text{ kg/m}^3)\) is less than that of water \((1.00 \times 10^3 \text{ kg/m}^3)\), so the object will [float].
95. **REASONING AND SOLUTION**

a. The volume flow rate is given by Equation 11.10. Assuming that the line has a circular cross section, \( A = \pi r^2 \), we have

\[
Q = Av = (\pi r^2)v = \pi (0.0065 \text{ m})^2 (1.2 \text{ m/s}) = 1.6 \times 10^{-4} \text{ m}^3/\text{s}
\]

b. The volume flow rate calculated in part (a) above is the flow rate for all twelve holes. Therefore, the volume flow rate through one of the twelve holes is

\[
Q_{\text{hole}} = \frac{Q}{12} = (\pi r_{\text{hole}}^2) v_{\text{hole}}
\]

Solving for \( v_{\text{hole}} \) we have

\[
v_{\text{hole}} = \frac{Q}{12\pi r_{\text{hole}}^2} = \frac{1.6 \times 10^{-4} \text{ m}^3/\text{s}}{12\pi (4.6 \times 10^{-4} \text{ m})^2} = 2.0 \times 10^1 \text{ m/s}
\]

96. **REASONING** The power generated by the log splitter pump is the ratio of the work \( W \) done on the piston to the elapsed time \( t \):

\[
\text{Power} = \frac{W}{t} \quad (6.10a)
\]

The work done on the piston by the pump is equal to the magnitude \( F \) of the force exerted on the piston by the hydraulic oil, multiplied by the distance \( s \) through which the piston moves:

\[
W = (F \cos 0^\circ)s = Fs \quad (6.1)
\]

We have used \( \theta = 0^\circ \) in Equation 6.1 because the piston moves in the same direction as the force acting on it. The magnitude \( F \) of the force applied to the piston is given by

\[
F = PA \quad (11.3)
\]

where \( A \) is the cross-sectional area of the piston and \( P \) is the pressure of the hydraulic oil.

**SOLUTION** The head of the piston is circular with a radius \( r \), so its cross-sectional area is given by \( A = \pi r^2 \). Substituting Equation 11.3 into Equation 6.1, therefore, yields

\[
W = PA_s = P\pi r^2s \quad (1)
\]

Substituting Equation (1) into Equation 6.10a gives the power required to operate the pump:

\[
\text{Power} = \frac{W}{t} = \frac{P\pi r^2s}{t} = \frac{(2.0 \times 10^7 \text{ Pa})\pi (0.050 \text{ m})^2 (0.60 \text{ m})}{25 \text{ s}} = 3.8 \times 10^3 \text{ W}
\]
97. **REASONING** When an object is completely submerged within a fluid, its apparent weight in the fluid is equal to its true weight \( mg \) minus the upward-acting buoyant force. According to Archimedes’ principle, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object. The weight of the displaced fluid depends on the volume of the object. We will apply this principle twice, once for the object submerged in each fluid, to find the volume of the object.

**SOLUTION** The apparent weights of the object in ethyl alcohol and in water are:

- **Ethyl alcohol**
  \[
  \text{Weight in alcohol} = \frac{15.2 \text{ N}}{mg} - \rho_{\text{alcohol}} g V
  \]
  (1)

- **Water**
  \[
  \text{Weight in water} = \frac{13.7 \text{ N}}{mg} - \rho_{\text{water}} g V
  \]
  (2)

These equations contain two unknowns, the volume \( V \) of the object and its mass \( m \). By subtracting Equation (2) from Equation (1), we can eliminate the mass algebraically. The result is

\[
15.2 \text{ N} - 13.7 \text{ N} = g V \left( \rho_{\text{water}} - \rho_{\text{alcohol}} \right)
\]

Solving this equation for the volume, and using the densities from Table 11.1, we have

\[
V = \frac{15.2 \text{ N} - 13.7 \text{ N}}{g \left( \rho_{\text{water}} - \rho_{\text{alcohol}} \right)} = \frac{1.5 \text{ N}}{(9.80 \text{ m/s}^2)\left(1.00 \times 10^3 \text{ kg/m}^3 - 806 \text{ kg/m}^3\right)} = 7.9 \times 10^{-4} \text{ m}^3
\]

98. **REASONING** According to Equation 11.4, the pressure \( P_{\text{mercury}} \) at a point 7.10 cm below the ethyl alcohol-mercury interface is

\[
P_{\text{mercury}} = P_{\text{interface}} + \rho_{\text{mercury}} gh_{\text{mercury}}
\]

where \( P_{\text{interface}} \) is the pressure at the alcohol-mercury interface, and \( h_{\text{mercury}} = 0.0710 \text{ m} \). The pressure at the interface is

\[
P_{\text{interface}} = P_{\text{atm}} + \rho_{\text{ethyl}} gh_{\text{ethyl}}
\]

Equation (2) can be used to find the pressure at the interface. This value can then be used in Equation (1) to determine the pressure 7.10 cm below the interface.

**SOLUTION** Direct substitution of the numerical data into Equation (2) yields

\[
P_{\text{interface}} = 1.01 \times 10^5 \text{ Pa} + (806 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.10 \text{ m}) = 1.10 \times 10^5 \text{ Pa}
\]

Therefore, the pressure 7.10 cm below the ethyl alcohol-mercury interface is

\[
P_{\text{mercury}} = 1.10 \times 10^5 \text{ Pa} + (13600 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0710 \text{ m}) = 1.19 \times 10^5 \text{ Pa}
\]
99. SSM REASONING Let the length of the tube be denoted by \( L \), and let the length of the liquid be denoted by \( \ell \). When the tube is whirled in a circle at a constant angular speed about an axis through one end, the liquid collects at the other end and experiences a centripetal force given by (see Equation 8.11, and use \( F = ma \) ) \( F = mL\omega^2 = mL\omega^2 \).

Since there is no air in the tube, this is the only radial force experienced by the liquid, and it results in a pressure of

\[
P = \frac{F}{A} = \frac{mL\omega^2}{A}
\]

where \( A \) is the cross-sectional area of the tube. The mass of the liquid can be expressed in terms of its density \( \rho \) and volume \( V \) : \( m = \rho V = \rho A\ell \). The pressure may then be written as

\[
P = \frac{\rho A(L\omega^2)}{A} = \rho\ell\omega^2 \tag{1}
\]

If the tube were completely filled with liquid and allowed to hang vertically, the pressure at the bottom of the tube (that is, where \( h = L \) ) would be given by

\[
P = \rho gL \tag{2}
\]

SOLUTION According to the statement of the problem, the quantities calculated by Equations (1) and (2) are equal, so that \( \rho\ell\omega^2 = \rho gL \). Solving for \( \omega \) gives

\[
\omega = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.80 \text{ m/s}^2}{0.0100 \text{ m}}} = 31.3 \text{ rad/s}
\]

100. REASONING The total mass \( m_T \) of the rock is the sum of the mass \( m_G \) of the gold and the mass \( m_Q \) of the quartz: \( m_T = m_G + m_Q \). Thus, the mass of the gold is

\[
m_G = m_T - m_Q \tag{1}
\]

The total volume \( V_T \) of the rock is the sum of the volume \( V_G \) of the gold and volume \( V_Q \) of the quartz:

\[
V_T = V_G + V_Q \tag{2}
\]

The volume of a substance is related to the mass and the density of the substance according to the definition of density: \( \rho = m/V \) (Equation 11.1). These relations will enable us to find the mass of the gold in the rock.
**SOLUTION** Substituting \( m_Q = \rho_Q V_Q \) from Equation 11.1 into Equation (1) gives

\[
m_G = m_T - m_Q = m_T - \rho_Q V_Q
\]

Solving Equation (2) for the volume \( V_Q \) of the quartz and substituting the result into Equation (3) yields

\[
m_G = m_T - \rho_Q (V_T - V_G) = m_T - \rho_Q \left( V_T - \frac{m_G}{\rho_G} \right)
\]

Substituting \( V_G = \frac{m_G}{\rho_G} \) from Equation 11.1 into Equation (4) gives

\[
m_G = m_T - \rho_Q \left( V_T - \frac{m_G}{\rho_G} \right) = m_T - \rho_Q \left( \frac{m_T}{\rho_G} - \frac{m_G}{\rho_G} \right)
\]

Solving Equation (5) for the mass of the gold, and using the densities for gold and quartz given in Table 11.1, gives

\[
m_G = \frac{V_T - \frac{m_T}{\rho_Q}}{\rho_G} = \frac{4.00 \times 10^{-3} \text{ m}^3 - 12.0 \text{ kg}}{2660 \text{ kg/m}^3} = 1.6 \text{ kg}
\]

101. **REASONING** The number of gallons per minute of water that the fountain uses is the volume flow rate \( Q \) of the water. According to Equation 11.10, the volume flow rate is

\[
Q = Av
\]

where \( A \) is the cross-sectional area of the pipe and \( v \) is the speed at which the water leaves the pipe. The area is given. To find a value for the speed, we will use Bernoulli’s equation. This equation states that the pressure \( P \), the fluid speed \( v \), and the elevation \( y \) at any two points (1 and 2) in an ideal fluid of density \( \rho \) are related according to

\[
P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2
\]

(Equation 11.11). Point 1 is where the water leaves the pipe, so we are seeking \( v_1 \). We must now select point 2. Note that the problem specifies the maximum height to which the water rises. The maximum height occurs when the water comes to a momentary halt at the top of its path in the air. Therefore, we choose this point as point 2, so that we can take advantage of the fact that \( v_2 = 0 \) m/s.

**SOLUTION** We know that \( P_1 = P_2 = P_{\text{atmospheric}} \) and \( v_2 = 0 \) m/s. Substituting this information into Bernoulli’s equation, we obtain

\[
P_{\text{atmospheric}} + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_{\text{atmospheric}} + \frac{1}{2} \rho (0 \text{ m/s})^2 + \rho gy_2
\]

Solving for \( v_1 \) gives \( v_1 = \sqrt{2g \left( y_2 - y_1 \right)} \), which we now substitute for the speed \( v \) in the expression \( Q = Av \) for the volume flow rate:
\[ Q = A v_1 = A \sqrt{2g(y_2 - y_1)} \]
\[ = (5.00 \times 10^{-4} \text{ m}^2) \sqrt{2(9.80 \text{ m/s}^2)(5.00 \text{ m} - 0 \text{ m})} = 4.95 \times 10^{-3} \text{ m}^3/\text{s} \]

To convert this result to gal/min, we use the facts that 1 gal = 3.79 \times 10^{-3} \text{ m}^3 and 1 min = 60 s:

\[ Q = \left(4.95 \times 10^{-3} \text{ m}^3/\text{s}\right) \left(\frac{1 \text{ gal}}{3.79 \times 10^{-3} \text{ m}^3}\right) \left(60 \text{ s}/1 \text{ min}\right) = 78.4 \text{ gal/min} \]

102. **REASONING AND SOLUTION** The force exerted on the top surface is

\[ F_2 = P_2 A_2 = P_{\text{atm}} \pi R_2^2 \]

The force exerted on the bottom surface is \( F_1 = P_1 A_1 = (P_{\text{atm}} + \rho gh) \pi R_1^2 \). Equating and rearranging yields

\[ R_2^2 = R_1^2 (1 + \rho gh/P_{\text{atm}}) \]

or

\[ R_2^2 = 1.485 R_1^2 \] (1)

Consider a right triangle formed by drawing a vertical line from a point on the circumference of the bottom circle to the plane of the top circle so that two sides are equal to \( R_2 - R_1 \) and \( h \). Then \( \tan 30.0^\circ = (R_2 - R_1)/h \).

a. Now,

\[ R_1 = R_2 - h \tan 30.0^\circ = R_2 - 2.887 \text{ m} \] (2)

Substituting (2) into (1) and rearranging yields

\[ 0.485 R_2^2 - 8.57 R_2 + 12.38 = 0 \]

which has two roots, namely, \( R_2 = 16.1 \text{ m} \) and \( 1.59 \text{ m} \). The value \( R_2 = 1.59 \text{ m} \) leads to a negative value for \( R_1 \). Clearly, a radius cannot be negative, so we can eliminate the root \( R_2 = 1.59 \text{ m} \), and we conclude that \( R_2 = 16.1 \text{ m} \).

b. Now that \( R_2 \) is known, Equation (2) gives \( R_1 = 13.2 \text{ m} \).
103. **REASONING AND SOLUTION**  The figure at the right shows the forces that act on the balloon as it holds the passengers and the ballast stationary above the earth. $W_0$ is the combined weight of the balloon, the load of passengers, and the ballast. The quantity $F_B$ is the buoyant force provided by the air outside the balloon and is given by

$$F_B = \rho_{\text{air}} g V_{\text{balloon}}$$  

(1)

Since the balloon is stationary, it follows that $F_B - W_0 = 0$, or

$$F_B = W_0$$  

(2)

Then the ballast is dropped overboard, the balloon accelerates upward through a distance $y$ in a time $t$ with acceleration $a_y$, where (from kinematics) $a_y = \frac{2y}{t^2}$.

The figure at the right shows the forces that act on the balloon while it accelerates upward.

Applying Newton's second law, we have $F_B - W = ma_y$, where $W$ is the weight of the balloon and the load of passengers. Replacing $m$ by $(W/g)$ we have

$$F_B - W = \frac{W}{g} a_y$$

Thus,

$$F_B = W + \frac{W}{g} \left( \frac{2y}{t^2} \right) = W \left( 1 + \frac{2y}{gt^2} \right)$$

Solving for $W$ gives

$$W = \frac{F_B}{1 + \frac{2y}{gt^2}} = F_B \left( \frac{gt^2}{gt^2 + 2y} \right)$$

The amount of ballast that must be thrown overboard is therefore [using Equations (1) and (2)]

$$\Delta W = W_0 - W = F_B - F_B \left( \frac{gt^2}{gt^2 + 2y} \right) = \left( 1 - \frac{gt^2}{gt^2 + 2y} \right) \rho_{\text{air}} g V_{\text{balloon}}$$

$$\Delta W = r_{\text{air}} g \left( \frac{4}{3} \rho_{\text{balloon}} r_{\text{balloon}}^3 \right) \left( 1 - \frac{gt^2}{gt^2 + 2y} \right)$$
\[ \Delta W = (1.29 \text{ kg/m}^3)(9.80 \text{ m/s}^2) \left[ \frac{4}{3} \pi (6.25 \text{ m})^3 \right] \left[ 1 - \frac{(9.80 \text{ m/s}^2)(15.0 \text{ s})^2}{(9.80 \text{ m/s}^2)(15.0 \text{ s})^2 + 2(105 \text{ m})} \right] \]

= \boxed{1120 \text{ N}}

---

104. **REASONING AND SOLUTION**

a. At the surface of the water (position 1) and at the exit of the hose (position 2) the pressures are equal \( P_1 = P_2 \) to the atmospheric pressure. If the hose exit defines \( y = 0 \text{ m} \), we have from Bernoulli's equation \( (1/2) \rho v_1^2 + \rho gy = (1/2) \rho v_2^2 \). If we take the speed at the surface of the water to be zero (i.e., \( v_1 = 0 \text{ m/s} \)) we find that

\[ v_2 = \sqrt{2gy} \]

b. The siphon will stop working when \( v_2 = 0 \text{ m/s} \), or \( y = \boxed{0 \text{ m}} \), i.e., when the end of the hose is at the water level in the tank.

c. At point A we have

\[ P_A + \rho g(h + y) + (1/2) \rho v_A^2 = P_0 + (1/2) \rho v_2^2 \]

But \( v_A = v_2 \) so that

\[ P_A = P_0 - \rho g(y + h) \]
1. (e) On each scale there are 100 degrees between the ice and steam points, so the size of the degree is the same on each scale.

2. (b) According to Equation 12.2, the change in length $\Delta L$ of each rod is given by $\Delta L = \alpha L_0 \Delta T$, where $\alpha$ is the coefficient of linear expansion, $L_0$ is the initial length, and $\Delta T$ is the change in temperature. Since the initial length and the change in temperature are the same for each rod, the rod with the larger coefficient of linear expansion has the greater increase in length as the temperature rises. Thus, the aluminum rod lengthens more than the steel rod, so the rods will meet to the right of the midpoint.

3. $1.2 \times 10^{-3}$ cm

4. (d) In Arrangement I cooling allows the ball to pass through the hole. Therefore, the ball must shrink more than the hole, and the coefficient of linear thermal expansion of metal A must be greater than that of metal B. In Arrangement II heating allows the ball to pass through the hole. Therefore, the coefficient of linear thermal expansion of metal C must be greater than that of metal A.

5. (c) The gap expands as the temperature is increased, in a way similar to that of a hole when it expands according to the coefficient of linear thermal expansion of the surrounding material. In this case the surrounding material is copper.

6. (b) The change in volume $\Delta V$ is given by Equation 12.3 as $\Delta V = \beta V_0 \Delta T$, where $\beta$ is the coefficient of volume thermal expansion, $V_0$ is the initial volume, and $\Delta T$ is the change in temperature. Since the sphere and the cube are made from the same material, the coefficient of volume expansion is the same for each. Moreover, the temperature change is the same for each. Therefore, the change in volume is proportional to the initial volume. The initial volume of the cube is greater, since the sphere would fit within the cube. Thus, the change in volume of the cube is greater.

7. (a) To keep the overflow to a minimum, the container should be made from a material that has the greatest coefficient of volume thermal expansion and filled with a liquid that has the smallest coefficient of volume thermal expansion. That way, when the full container is heated, the cavity holding the liquid will expand more and the liquid will expand less, both effects leading to a reduced amount of overflow.

8. 77 $^\circ$C
9. (e) The heat \( Q \) required to raise the temperature of a mass \( m \) of material by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of the material. Since the material is the same in all cases, the specific heat capacity is the same. What matters is the product of \( m \) and \( \Delta T \). Since this product is the same in all cases, the amount of heat needed is also the same.

10. (c) The samples cool as heat is removed from each one. However, the temperature change that results as heat is removed is different. The heat \( Q \) that must be removed to lower the temperature of a mass \( m \) of material by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of the material. Solving for \( \Delta T \) gives \( \Delta T = Q/(cm) \). For a given amount of heat removed, the fall in temperature is inversely proportional to the product \( cm \). The sample (sample A) with the largest value of \( cm \) will experience the smallest drop in temperature. The sample (sample B) with the smallest value of \( cm \) will experience the largest drop in temperature.

11. 49 °C

12. 21.6 °C

13. 6.75 kg

14. 0.38 kg

15. (d) Technique A is the way water normally freezes at normal atmospheric pressure. Technique B applies at pressures that are above normal, since the freezing point is then lower. Technique C works also, because the heat of vaporization is carried away by each kilogram of vapor that is removed. This heat comes from the remaining liquid, and since no heat can flow in from the surroundings to replenish this loss of heat, the remaining liquid will freeze.

16. \( 5.54 \times 10^{-4} \) kg/s

17. (a) Relative humidity is defined by Equation 12.6 as the ratio (expressed as a percentage) of the partial pressure of water vapor in the air to the equilibrium vapor pressure of water at the existing temperature. The greater the temperature, the greater is the equilibrium vapor pressure of water. It is the equilibrium vapor pressure of water at the existing temperature, then, that determines the maximum amount of water that the air can contain. Warmer air can contain a greater maximum amount of water vapor than cooler air. Statement A is certainly true if both humidity values are quoted at the same temperature. Statement B could be true if the 30% humidity referred to a higher temperature than the 40% value. Likewise, statement C could be true if the 30% humidity referred to a higher temperature than the 40% value.

18. (b) The dew point is the temperature at which the existing partial pressure of water vapor in the air would be the equilibrium vapor pressure of water. The equilibrium vapor pressure of water always increases with increasing temperature, and only this answer is consistent with such a relationship.
CHAPTER 12 | TEMPERATURE AND HEAT

PROBLEMS

1. **SSM REASONING**
   a. According to the discussion in Section 12.1, the size of a Fahrenheit degree is smaller than that of a Celsius degree by a factor of \( \frac{5}{9} \); thus, \( 1 \text{ F}^\circ = \frac{5}{9} \text{ C}^\circ \). This factor will be used to find the temperature difference in Fahrenheit degrees.

   b. The size of one kelvin is identical to that of one Celsius degree (see Section 12.2), \( 1 \text{ K} = 1 \text{ C}^\circ \). Thus, the temperature difference, expressed in kelvins, is the same as that expressed in Celsius degrees.

   **SOLUTION**
   a. The difference in the two temperatures is \( 34 \text{ C}^\circ - 3 \text{ C}^\circ = 31 \text{ C}^\circ \). This difference, expressed in Fahrenheit degrees, is

   \[
   \text{Temperature difference} = 31 \text{ C}^\circ = \left(31 \text{ C}^\circ\right)\left(\frac{1 \text{ F}^\circ}{\frac{5}{9} \text{ C}^\circ}\right) = 56 \text{ F}^\circ
   \]

   b. Since \( 1 \text{ K} = 1 \text{ C}^\circ \), the temperature difference, expressed in kelvins, is

   \[
   \text{Temperature difference} = 31 \text{ C}^\circ = \left(31 \text{ C}^\circ\right)\left(\frac{1 \text{ K}}{1 \text{ C}^\circ}\right) = 31 \text{ K}
   \]

2. **REASONING** We will first convert the temperature from the Kelvin scale to the Celsius scale by using Equation 12.1, \( T_c = T - 273.15 \), where \( T_c \) is the Celsius temperature and \( T \) is the Kelvin temperature. Then, following the approach discussed in Section 12.1, we will convert from the Celsius scale to the Fahrenheit scale by multiplying the Celsius temperature by a factor of \( \frac{9}{5} \) and adding 32 to the result.

   **SOLUTION** The temperature on the Celsius scale is

   \[
   T_c = T - 273.15 = (312.0 - 273.15) \text{ C} = 38.9 \text{ C}^\circ
   \]

   (12.1)

   The temperature of 38.9 °C is equivalent to a Fahrenheit temperature of

   \[
   \text{Temperature} = \left(38.9\right)\left(\frac{9}{5}\right) + 32.0 = 102 \text{ F}^\circ
   \]
3. **REASONING**
   
a. The relationship between the Kelvin temperature \( T \) and the Celsius temperature \( T_C \) is given by \( T = T_C + 273.15 \) (Equation 12.1).

b. The relationship between the Kelvin temperature \( T \) and the Fahrenheit temperature \( T_F \) can be obtained by following the procedure outlined in Examples 1 and 2 in the text. On the Kelvin scale the ice point is 273.15 K. Therefore, a Kelvin temperature \( T \) is \( T - 273.15 \) kelvins above the ice point. The size of the kelvin is larger than the size of a Fahrenheit degree by a factor of \( \frac{9}{5} \). As a result a temperature that is \( T - 273.15 \) kelvins above the ice point on the Kelvin scale is \( \frac{9}{5} (T - 273.15) \) °F above the ice point on the Fahrenheit scale. This amount must be added to the ice point of 32.0 °F. The relationship between the Kelvin and Fahrenheit temperatures, then, is given by

\[
T_F = \frac{9}{5} (T - 273.15) + 32.0
\]

**SOLUTION**
   
a. Solving Equation 12.1 for \( T_C \), we find that

   - **Day** \( T_C = T - 273.15 = 375 - 273.15 = 102 \) °C
   - **Night** \( T_C = T - 273.15 = 1.00 \times 10^2 - 273.15 = -173 \) °C

b. Using the equation developed in the **REASONING**, we find

   - **Day** \( T_F = \frac{9}{5} (T - 273.15) + 32.0 = \frac{9}{5} (375 - 273.15) + 32.0 = 215 \) °F
   - **Night** \( T_F = \frac{9}{5} (T - 273.15) + 32.0 = \frac{9}{5} (1.00 \times 10^2 - 273.15) + 32.0 = -2.80 \times 10^2 \) °F

4. **REASONING AND SOLUTION**  The difference between these two averages, expressed in Fahrenheit degrees, is

\[
98.6 \, ^\circ F - 98.2 \, ^\circ F = 0.4 \, ^\circ F
\]

Since 1 °C is equal to \( \frac{9}{5} \) °F, we can make the following conversion

\[
0.4 \, ^\circ F \left( \frac{1 \, ^\circ C}{\frac{9}{5} \, ^\circ F} \right) = 0.2 \, ^\circ C
\]
5. **REASONING AND SOLUTION**

a. The Kelvin temperature and the temperature on the Celsius scale are related by Equation 12.1: \( T = T_c + 273.15 \), where \( T \) is the Kelvin temperature and \( T_c \) is the Celsius temperature. Therefore, a temperature of 77 K on the Celsius scale is

\[
T_c = T - 273.15 = 77 \text{ K} - 273.15 \text{ K} = -196 \degree \text{C}
\]

b. The temperature of \(-196 \degree \text{C}\) is 196 Celsius degrees below the ice point of 0 \degree \text{C}. Since \( 1 \degree \text{C} = \frac{9}{5} \degree \text{F} \), this number of Celsius degrees corresponds to

\[
196 \degree \text{C} \left( \frac{9}{5} \degree \text{F} \right) = 353 \degree \text{F}
\]

Subtracting 353 Fahrenheit degrees from the ice point of 32.0 \degree \text{F} on the Fahrenheit scale gives a Fahrenheit temperature of \(-321 \degree \text{F}\).

6. **REASONING** 1 A\degree is larger than 1 B\degree, because there are 90 A\degree between the ice and boiling points of water, while there are 110 B\degree between these points. Moreover, a given temperature on the A scale is hotter than the same reading on the B scale. For example, +20 \degree A is hotter than +20 \degree B, because +20 \degree A is 50 A\degree above the ice point of water while +20 \degree B is at the ice point. Thus, we expect +40 \degree A to correspond to more than +40 \degree B.

**SOLUTION**

a. Since there are 90.0 A\degree and 110.0 B\degree between the ice and boiling points of water, we have that

\[
1 \text{ A}\degree = \left( \frac{110.0}{90.0} \right) \text{ B}\degree = 1.22 \text{ B}\degree
\]

b. +40.0 \degree A is 70.0 A\degree above the ice point of water. On the B thermometer, this is

\[
70.0 \text{ A}\degree \left( \frac{1.22 \text{ B}\degree}{1 \text{ A}\degree} \right) = 85.4 \text{ B}\degree \text{ above the ice point.}
\]

The temperature on the B scale is, therefore,

\[
T = +20.0 \text{ B} + 85.4 \text{ B} = 105.4 \text{ B}
\]

7. **REASONING AND SOLUTION** If the voltage is proportional to the temperature difference between the junctions, then

\[
\frac{V_1}{\Delta T_1} = \frac{V_2}{\Delta T_2} \quad \text{or} \quad \Delta T_2 = \frac{V_2}{V_1} \Delta T_1
\]
Thus,

\[ T_2 - 0.0 \, \degree C = \frac{1.90 \times 10^{-3}}{4.75 \times 10^{-3}} (110.0 \, \degree C - 0.0 \, \degree C) \]

Solving for \( T_2 \) yields \( T_2 = 44.0 \, \degree C \).

8. **REASONING** The absolute zero point on the Kelvin scale occurs at \(-273.15 \, \degree C\). This temperature is below the methane freezing point of \(-182.6 \, \degree C\) by an amount that is \(90.6 \, \degree C\) \([-182.6 \, \degree C - (-273.15 \, \degree C)]\). Before we can locate the absolute zero point on the Methane scale, we must convert \(90.6 \, \degree C\) into an equivalent number of Methane degrees (\(M \degree\)). To determine the equivalence between Methane degrees and Celsius degrees, we will calculate the temperature difference between the freezing and boiling points of methane on Titan in Celsius degrees, and equate this to 100 \(M \degree\), which is the difference between the two points on the Methane scale.

**SOLUTION** The temperature difference between the freezing and boiling points of methane on Titan is, by definition, 100 \(M \degree\). The freezing point and boiling point, in degrees Celsius, are \(-182.6 \, \degree C\) and \(-155.2 \, \degree C\), respectively. Thus, we have the equivalence between Celsius degrees and Methane degrees:

\[
100.0 \, M \degree = \left[ -155.2 \, \degree C - (-182.6 \, \degree C) \right] = 27.4 \, \degree C
\]

Therefore, if the absolute zero point on the Kelvin scale is \(90.6 \, \degree C\) below methane’s freezing point on the Methane scale, the difference in Methane degrees is

\[
\left( 90.6 \, \degree C \right) \left( \frac{100.0 \, M \degree}{27.4 \, \degree C} \right) = 331 \, M \degree
\]

If the freezing point of methane is \(0 \, M \degree\), and the absolute zero point of the Kelvin scale is \(331 \, M \degree\) below this, then \(0 \, K\) corresponds to \(-331 \, M \degree\)

9. **SSM REASONING AND SOLUTION** We begin by noting that on the Rankine scale the difference between the steam point and the ice point temperatures is

\[ 671.67 \, \degree R - 491.67 \, \degree R = 180.00 \, \degree R \]

Thus, the Rankine and Fahrenheit degrees are the same size, since the difference between the steam point and ice point temperatures is also 180 degrees on the Fahrenheit scale. The difference in the ice points of the two scales is \(491.67 - 32.00 = 459.67\). To get a Rankine from a Fahrenheit temperature, this amount must be added, so \(T_R = T_F + 459.67\).
10. **REASONING** The change in length $\Delta L$ of the pipe is proportional to the coefficient of linear expansion $\alpha$ for steel, the original length $L_0$ of the pipe, and the change in temperature $\Delta T$. The coefficient of linear expansion for steel can be found in Table 12.1.

**SOLUTION** The change in length of the pipe is

$$\Delta L = \alpha L_0 \Delta T = \left[ 1.2 \times 10^{-5} \text{ (C$^{\circ}$)}^{-1} \right] (65 \text{ m}) \left[ 18^\circ \text{C} - (-45^\circ \text{C}) \right] = 4.9 \times 10^{-2} \text{ m} \quad (12.2)$$

11. **SSM REASONING AND SOLUTION** Using Equation 12.2 and the value for the coefficient of thermal expansion of steel given in Table 12.1, we find that the linear expansion of the aircraft carrier is

$$\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6} \text{ C$^{\circ}$}^{-1})(370 \text{ m})(21^\circ \text{C} - 2.0^\circ \text{C}) = 0.084 \text{ m} \quad (12.2)$$

12. **REASONING** The height $L_0$ of the Eiffel Tower at the lower temperature can be determined from $L_0 = \Delta L / (\alpha \Delta T)$ (Equation 12.2), where $\Delta L$ is the increase in the height, $\alpha$ is the coefficient of linear expansion for steel, and $\Delta T$ is the change in temperature. The coefficient of linear expansion for steel can be found in Table 12.1.

**SOLUTION** The height of the Eiffel Tower at the lower temperature is

$$L_0 = \frac{\Delta L}{\alpha \Delta T} = \frac{19.4 \times 10^{-2} \text{ m}}{1.2 \times 10^{-5} \text{ (C$^{\circ}$)}^{-1} \left[ 41^\circ \text{C} - (-9^\circ \text{C}) \right]} = 3.2 \times 10^2 \text{ m} \quad (12.2)$$

13. **REASONING AND SOLUTION**

a. The radius of the hole will be **larger** when the plate is heated, because the hole expands as if it were made of copper.

b. According to Equation 12.2, the expansion of the radius is $\Delta r = \alpha r_0 \Delta T$. Using the value for the coefficient of thermal expansion of copper given in Table 12.1, we find that the fractional change in the radius is

$$\Delta r / r_0 = \alpha \Delta T = [17 \times 10^{-6} \text{ (C$^{\circ}$)}^{-1}](110^\circ \text{C} - 11^\circ \text{C}) = 0.0017$$
14. **REASONING AND SOLUTION** The value for the coefficient of thermal expansion of steel is given in Table 12.1. The relation, $\Delta L = \alpha L_0 \Delta T$, written in terms of the diameter $d$ of the rod, is

$$\Delta T = \frac{\Delta d}{\alpha d_0} = \frac{0.0026 \text{ cm}}{\left[12 \times 10^{-6} \text{ (C$^\circ$)$^{-1}$}\right](2.0026 \text{ cm})} = \frac{110 \text{ C$^\circ$}}{(12.2)}$$

15. **SSM REASONING AND SOLUTION** The change in the coin’s diameter is $\Delta d = \alpha d_0 \Delta T$, according to Equation 12.2. Solving for $\alpha$ gives

$$\alpha = \frac{\Delta d}{d_0 \Delta T} = \frac{2.3 \times 10^{-5} \text{ m}}{(1.8 \times 10^{-2} \text{ m})(75 \text{ C$^\circ$})} = \frac{1.7 \times 10^{-5} \text{ (C$^\circ$)$^{-1}$}}{(12.2)}$$

16. **REASONING** The average speed $\bar{v}$ of the flagpole’s contraction is given by $\bar{v} = \frac{\Delta L}{\Delta t}$ (Equation 2.1), where $\Delta L$ is the amount by which it contracts, and $\Delta t$ is the elapsed time. The amount of contraction the pole undergoes is found from $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), where $\alpha$ is the coefficient of thermal expansion for aluminum (see Table 12.1 in the text), $L_0$ is the length of the pole before it begins contracting, and $\Delta T$ is the difference between the higher and lower temperatures of the pole.

**SOLUTION** Substituting $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2) into $\bar{v} = \frac{\Delta L}{\Delta t}$ (Equation 2.1) yields

$$\bar{v} = \frac{\alpha L_0 \Delta T}{\Delta t}$$

The elapsed time $\Delta t$ is given in minutes, which must be converted to SI units (seconds) before employing Equation (1):

$$\Delta t = \left(27.0 \text{ min} \right) \left(60 \text{ s} \text{ min}^{-1} \right) = 1620 \text{ s}$$

Substituting this result and the given values into Equation (1), we obtain

$$\bar{v} = \frac{\left[23 \times 10^{-6} \text{ (C$^\circ$)$^{-1}$}\right](19 \text{ m})\left[12.0 \text{ C$^\circ$} - (-20.0 \text{ C$^\circ$})\right]}{1620 \text{ s}} = \frac{8.6 \times 10^{-6} \text{ m/s}}{}$$

17. **REASONING** According to $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), the factors that determine the amount $\Delta L$ by which the length of a rod changes are the coefficient of thermal expansion $\alpha$, its initial length $L_0$, and the change $\Delta T$ in temperature. The materials from which the rods are made have different coefficients of thermal expansion (see Table 12.1 in the text). Also,
the change in length is the same for each rod when the change in temperature is the same. Therefore, the initial lengths must be different to compensate for the fact that the expansion coefficients are different.

**SOLUTION**  The change in length of the lead rod is (from Equation 12.2)

\[
\Delta L_L = \alpha_L L_{0,L} \Delta T
\]  

(1)

Similarly, the change in length of the quartz rod is

\[
\Delta L_Q = \alpha_Q L_{0,Q} \Delta T
\]  

(2)

In Equations (1) and (2) the temperature change \( \Delta T \) is the same for both rods. Since each changes length by the same amount, \( \Delta L_L = \Delta L_Q \). Equating Equations (1) and (2) and solving for \( L_{0,Q} \) yields

\[
L_{0,Q} = \left( \frac{\alpha_L}{\alpha_Q} \right) L_{0,L} = \left[ \frac{29 \times 10^{-6} (\text{C}^{-1})}{0.50 \times 10^{-6} (\text{C}^{-1})} \right] (0.10 \text{ m}) = 5.8 \text{ m}
\]

The values for the coefficients of thermal expansion for lead and quartz have been taken from Table 12.1.

18. **REASONING**  To determine the fractional decrease in length \( \frac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{Gold}}} \), we need the decrease \( \Delta L \) in the rod’s length. It is the sum of the decreases in the silver part and the gold part of the rod, or \( \Delta L = \Delta L_{\text{Silver}} + \Delta L_{\text{Gold}} \). Each of the decreases can be expressed in terms of the coefficient of linear expansion \( \alpha \), the initial length \( L_0 \), and the change in temperature \( \Delta T \), according to Equation 12.2.

**SOLUTION**  Using Equation 12.2 to express the decrease in length of each part of the rod, we find the total decrease in the rod’s length to be

\[
\Delta L = \alpha_{\text{Silver}} L_{0,\text{Silver}} \Delta T + \alpha_{\text{Gold}} L_{0,\text{Gold}} \Delta T
\]

The fractional decrease in the rod’s length is, then,

\[
\frac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{Gold}}} = \frac{\alpha_{\text{Silver}} L_{0,\text{Silver}} \Delta T + \alpha_{\text{Gold}} L_{0,\text{Gold}} \Delta T}{L_{0,\text{Silver}} + L_{0,\text{Gold}}}
\]

\[
= \alpha_{\text{Silver}} \left( \frac{L_{0,\text{Silver}}}{L_{0,\text{Silver}} + L_{0,\text{Gold}}} \right) \Delta T + \alpha_{\text{Gold}} \left( \frac{L_{0,\text{Gold}}}{L_{0,\text{Silver}} + L_{0,\text{Gold}}} \right) \Delta T
\]

Silver fraction = \( \frac{1}{3} \)  
Gold fraction = \( \frac{2}{3} \)
Recognizing that one third of the rod is silver and two thirds is gold and taking values for the coefficients of linear expansion for silver and gold from Table 12.1, we have

\[
\frac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{Gold}}} = \alpha_{\text{Silver}} \left( \frac{1}{3} \right) \Delta T + \alpha_{\text{Gold}} \left( \frac{2}{3} \right) \Delta T = \left[ \alpha_{\text{Silver}} \left( \frac{1}{3} \right) + \alpha_{\text{Gold}} \left( \frac{2}{3} \right) \right] \Delta T
\]

\[
= \left[ 19 \times 10^{-6} \, (\text{C}^{-1}) \right] \left( \frac{1}{3} \right) + \left[ 14 \times 10^{-6} \, (\text{C}^{-1}) \right] \left( \frac{2}{3} \right) (26 \, \text{C}^\circ) = 4.1 \times 10^{-4}
\]

19. **REASONING AND SOLUTION** \( \Delta L = \alpha L_0 \Delta T \) gives for the expansion of the aluminum

\[
\Delta L_A = \alpha_A L_A \Delta T
\]

and for the expansion of the brass

\[
\Delta L_B = \alpha_B L_B \Delta T
\]

The air gap will be closed when \( \Delta L_A + \Delta L_B = 1.3 \times 10^{-3} \, \text{m} \). Thus, taking the coefficients of thermal expansion for aluminum and brass from Table 12.1, adding Equations (1) and (2), and solving for \( \Delta T \), we find that

\[
\Delta T = \frac{\Delta L_A + \Delta L_B}{\alpha_A L_A + \alpha_B L_B} = \frac{1.3 \times 10^{-3} \, \text{m}}{23 \times 10^{-6} \, (\text{C}^{-1}) (1.0 \, \text{m}) + 19 \times 10^{-6} \, (\text{C}^{-1}) (2.0 \, \text{m})} = 21 \, \text{C}^\circ
\]

The desired temperature is then

\[
T = 28 \, \text{C}^\circ + 21 \, \text{C}^\circ = 49 \, \text{C}^\circ
\]

20. **REASONING** Young’s modulus \( Y \) can be obtained from \( F = Y \left( \Delta L / L_0 \right) A \) (Equation 10.17), where \( F \) is the magnitude of the stretching force applied to the ruler, \( \Delta L \) and \( L_0 \) are the change in length and original length, respectively, and \( A \) is the cross-sectional area. Solving for \( Y \) gives

\[
Y = \frac{FL_0}{A \Delta L}
\]

The change in the length of the ruler is given by \( \Delta L = \alpha L_0 \Delta T \) (Equation 12.2), where \( \alpha \) is the coefficient of linear expansion and \( \Delta T \) is the amount by which the temperature changes. Substituting this expression for \( \Delta L \) into the equation above for \( Y \) gives the desired result.

**SOLUTION** Substituting \( \Delta L = \alpha L_0 \Delta T \) into \( Y = FL_0 / (A \Delta L) \) and using the fact that \( \Delta T = 39 \, \text{C}^\circ \), we find that
21. **SSM REASONING AND SOLUTION**

Recall that $\omega = \frac{2\pi}{T}$ (Equation 10.6), where $\omega$ is the angular frequency of the pendulum and $T$ is the period. Using this fact and Equation 10.16, we know that the period of the pendulum before the temperature rise is given by

$$T_1 = 2\pi\sqrt{\frac{L_0}{g}}$$

where $L_0$ is the length of the pendulum. After the temperature has risen, the period becomes (using Equation 12.2),

$$T_2 = 2\pi\sqrt{\frac{L_0 + \alpha L_0 \Delta T}{g}}$$

Dividing these expressions, solving for $T_2$, and taking the coefficient of thermal expansion of brass from Table 12.1, we find that

$$T_2 = T_1 \sqrt{1 + \alpha \Delta T} = (2.0000 \text{ s}) \sqrt{1 + (19 \times 10^{-6} / \text{C°} \cdot 140 \text{ C°})} = 2.0027 \text{ s}$$

22. **REASONING**

The length of either heated strip is $L_0 + \Delta L$, where $L_0$ is the initial length and $\Delta L$ is the amount by which it expands. The expansion $\Delta L$ can be expressed in terms of the coefficient of linear expansion $\alpha$, the initial length $L_0$, and the change in temperature $\Delta T$, according to Equation 12.2. To find the change in temperature, we will set the length of the heated steel strip equal to the length of the heated aluminum strip and solve the resulting equation for $\Delta T$.

**SOLUTION**

According to Equation 12.2, the expansion is $\Delta L = \alpha L_0 \Delta T$. Using this equation we have

$$L_{0, \text{Steel}} + \alpha_{\text{Steel}} L_{0, \text{Steel}} \Delta T = L_{0, \text{Aluminum}} + \alpha_{\text{Aluminum}} L_{0, \text{Aluminum}} \Delta T$$

$$\frac{\Delta L_{\text{Steel}}}{L_{0, \text{Steel}}} = \frac{\Delta L_{\text{Aluminum}}}{L_{0, \text{Aluminum}}}$$

We know that the steel strip is 0.10% longer than the aluminum strip, so that $L_{0, \text{Steel}} = (1.0010) L_{0, \text{Aluminum}}$. Substituting this result into the equation above, solving for $\Delta T$, and taking values for the coefficients of linear expansion for aluminum and steel from Table 12.1 give
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\[(1.0010)L_0, \text{Aluminum} (1 + \alpha_{\text{Steel}} \Delta T) = L_0, \text{Aluminum} (1 + \alpha_{\text{Aluminum}} \Delta T)\]

\[0.0010 + (1.0010)\alpha_{\text{Steel}} \Delta T = \alpha_{\text{Aluminum}} \Delta T\]

\[\Delta T = \frac{0.0010}{\alpha_{\text{Aluminum}} - (1.0010)\alpha_{\text{Steel}}} = \frac{0.0010}{23 \times 10^{-6} \text{ (C°)}^{-1} - (1.0010)[12 \times 10^{-6} \text{ (C°)}^{-1}]} = 91 \text{ C°}\]

23. REASONING AND SOLUTION The initial diameter of the sphere, \(d_s\), is

\[d_s = (5.0 \times 10^{-4})d_r + d_r\]  \hspace{1cm} (1)

where \(d_r\) is the initial diameter of the ring. Applying \(\Delta L = \alpha L_0 \Delta T\) to the diameter of the sphere gives

\[\Delta d_s = \alpha_s d_s \Delta T\]  \hspace{1cm} (2)

and to the ring gives

\[\Delta d_r = \alpha_r d_r \Delta T\]  \hspace{1cm} (3)

If the sphere is just to fit inside the ring, we must have

\[d_s + \Delta d_s = d_r + \Delta d_r\]

Using Equations (2) and (3) in this expression and solving for \(\Delta T\) give

\[\Delta T = \frac{d_r - d_s}{\alpha_s d_s - \alpha_r d_r}\]

Substituting Equation (1) into this result and taking values for the coefficients of thermal expansion of steel and lead from Table 10.1 yield

\[\Delta T = \frac{-5.0 \times 10^{-4}}{29 \times 10^{-6} \text{ (C°)}^{-1}[5.0 \times 10^{-4} + 1] - 12 \times 10^{-6} \text{ (C°)}^{-1}} = -29 \text{ C°}\]

The final temperature is

\[T_f = 70.0 \text{ °C} - 29 \text{ C°} = 41 \text{ °C}\]

24. REASONING From Equation 10.17 and the discussion in Section 10.8, the stress, \(F/A\), required to change the length \(L_0\) of a rod by an amount \(\Delta L\) is

\[\text{Stress} = Y \frac{\Delta L}{L_0}\]  \hspace{1cm} (1)
where \( Y \) is Young’s modulus of the rod. If the rod were free to contract as it cooled, its length would change by an amount
\[
|\Delta L| = \alpha L_0 |\Delta T|
\]
, as predicted by Equation 12.2, where \( \alpha \) is the coefficient of linear expansion. (We have included vertical bars around \( \Delta L \) and \( \Delta T \) in Equation 12.2 to indicate the fact that we seek only the magnitude of the change in temperature.) Since the rod is fastened securely at both ends to immovable supports, the rod will stretch by this amount as it is cooled, due to the stress applied by the supports. Thus, Equation (1) becomes
\[
\text{Stress} = \frac{\alpha L_0 |\Delta T|}{L_0} = Y \alpha |\Delta T|
\]

**SOLUTION** When the tensile stress applied at the ends reaches \( 2.3 \times 10^7 \) N/m², the rod will rupture. The corresponding temperature change can be found by solving Equation (2) for \( |\Delta T| \):
\[
|\Delta T| = \frac{\text{Stress}}{Y \alpha} = \frac{2.3 \times 10^7 \text{ N/m}^2}{(1.1 \times 10^{11} \text{ N/m}^2)(17 \times 10^{-6} \text{ (Cº)}^{-1})} = 12 \text{ Cº}
\]

Values for \( Y \) and \( \alpha \) for copper have been taken from Tables 10.1 and 12.1, respectively.

25. **REASONING** When the ball and the plate are both heated to a higher common temperature, the ball passes through the hole. Since the ball’s diameter is greater than the hole’s diameter to start with, this must mean that the hole expands more than the ball for the same temperature change. The hole expands as if it were filled with the material that surrounds it. We conclude, therefore, that the coefficient of linear expansion for the plate is greater than that for the ball.

In each arrangement, the ball’s diameter exceeds the hole’s diameter by the same amount, the diameters of the holes are the same, the diameters of the balls are virtually the same, and the initial temperatures are the same. The only significant difference between the various arrangements, then, is in the coefficients of linear expansion. The ball and the hole are both expanding. However, the hole is expanding more than the ball, to the extent that the coefficient of linear expansion of the material of the plate exceeds that of the ball. Thus, if we examine the difference between the two coefficients, we can anticipate the order in which the balls fall through the holes as the temperature increases. Referring to Table 12.1, we find the following differences for each of the arrangements:

**Arrangement A**
\[
\alpha_{\text{Lead}} - \alpha_{\text{Gold}} = 29 \times 10^{-6} \text{ (Cº)}^{-1} - 14 \times 10^{-6} \text{ (Cº)}^{-1} = 15 \times 10^{-6} \text{ (Cº)}^{-1}
\]

**Arrangement B**
\[
\alpha_{\text{Aluminum}} - \alpha_{\text{Steel}} = 23 \times 10^{-6} \text{ (Cº)}^{-1} - 12 \times 10^{-6} \text{ (Cº)}^{-1} = 11 \times 10^{-6} \text{ (Cº)}^{-1}
\]

**Arrangement C**
\[
\alpha_{\text{Silver}} - \alpha_{\text{Quartz}} = 19 \times 10^{-6} \text{ (Cº)}^{-1} - 0.50 \times 10^{-6} \text{ (Cº)}^{-1} = 18.5 \times 10^{-6} \text{ (Cº)}^{-1}
\]

In Arrangement C the coefficient of linear expansion of the plate-material exceeds that of the ball-material by the greatest amount. Therefore, the quartz ball will fall through the hole first.
Next in sequence is Arrangement A, so the gold ball will fall second. Last is Arrangement B, so the steel ball will be the last to fall.

**SOLUTION** According to Equation 12.2, the diameter increases by an amount \( \Delta D = \alpha D_0 \Delta T \) when the temperature increases by an amount \( \Delta T \), where \( D_0 \) is the initial diameter and \( \alpha \) is the coefficient of linear expansion. Thus, we can write the final diameter as \( D = D_0 + \alpha D_0 \Delta T \). Since the diameters of the ball and the hole are the same when the ball falls through the hole, we have

\[
\frac{D_{\text{Ball}} + \alpha_{\text{Ball}} D_0, \text{Ball} \Delta T}{\text{Final diameter of ball}} = \frac{D_{\text{Hole}} + \alpha_{\text{Hole}} D_0, \text{Hole} \Delta T}{\text{Final diameter of hole}}
\]

Solving for the change in temperature, we obtain

\[
\Delta T = \frac{D_{0, \text{Ball}} - D_{0, \text{Hole}}}{\alpha_{\text{Hole}} D_0, \text{Hole} - \alpha_{\text{Ball}} D_0, \text{Ball}}
\]

**Arrangement A**

\[
\Delta T = \frac{D_{0, \text{Gold}} - D_{0, \text{Lead}}}{\alpha_{\text{Lead}} D_0, \text{Lead} - \alpha_{\text{Gold}} D_0, \text{Gold}} = \frac{1.0 \times 10^{-5} \text{ m}}{29 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m}) - 14 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m} + 1.0 \times 10^{-5} \text{ m})} = 6.7 \text{ C°}
\]

**Arrangement B**

\[
\Delta T = \frac{D_{0, \text{Steel}} - D_{0, \text{Aluminum}}}{\alpha_{\text{Aluminum}} D_0, \text{Aluminum} - \alpha_{\text{Steel}} D_0, \text{Steel}} = \frac{1.0 \times 10^{-5} \text{ m}}{23 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m}) - 12 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m} + 1.0 \times 10^{-5} \text{ m})} = 9.1 \text{ C°}
\]

**Arrangement C**

\[
\Delta T = \frac{D_{0, \text{Quartz}} - D_{0, \text{Silver}}}{\alpha_{\text{Silver}} D_0, \text{Silver} - \alpha_{\text{Quartz}} D_0, \text{Quartz}} = \frac{1.0 \times 10^{-5} \text{ m}}{19 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m}) - 0.50 \times 10^{-6} \text{ (C°)}^{-1} (0.10 \text{ m} + 1.0 \times 10^{-5} \text{ m})} = 5.4 \text{ C°}
\]

Since each arrangement has an initial temperature of 25.0 °C, the temperatures at which the balls fall through the holes are as follows:
Arrangement A \[ T = 25.0 \, ^\circ C + 6.7 \, ^\circ C = 31.7 \, ^\circ C \]

Arrangement B \[ T = 25.0 \, ^\circ C + 9.1 \, ^\circ C = 34.1 \, ^\circ C \]

Arrangement C \[ T = 25.0 \, ^\circ C + 5.4 \, ^\circ C = 30.4 \, ^\circ C \]

The order in which the balls fall through the hole as the temperature increases is C, A, B, as we anticipated.

26. **REASONING AND SOLUTION** The figure below (at the left) shows the forces that act on the middle of the aluminum wire for any value of the angle \( \theta \). The figure below (at the right) shows the same forces after they have been resolved into \( x \) and \( y \) components.

![Diagram of forces on aluminum wire](image)

Applying Newton's second law to the vertical forces in figure on the right gives \( 2T \sin \theta - W = 0 \). Solving for \( T \) gives:

\[
T = \frac{W}{2\sin \theta}
\]

(1)

The following figures show how the angle is related to the initial length \( L_0 \) of the wire and to the final length \( L \) after the temperature drops.

![Angle-related figures](image)

If the distance between the supports does not change, then the distance \( x \) is the same in both figures. Thus, at the original temperature,

\[
x = L_0 \cos \theta_0
\]

(2)

while at the lower temperature

\[
x = L \cos \theta
\]

(3)

Equating the right hand sides of Equations (2) and (3) leads to

\[
\cos \theta = \frac{L_0 \cos \theta_0}{L}
\]

(4)
Now, \( L = L_0 + \Delta L \), and from Equation 12.2, it follows that

\[
\frac{\Delta L}{L_0} = \alpha \Delta T
\]

Thus, Equation (4) becomes

\[
\cos \theta = \frac{L_0 \cos \theta_0}{L_0 + \Delta L} = \frac{\cos \theta_0}{1 + (\Delta L / L_0)} = \frac{\cos \theta_0}{1 + \alpha \Delta T}
\]

From the figure in the text \( \theta_0 = 3.00^\circ \). Noting that the temperature of the wire drops by 20.0 °C \( (\Delta T = -20.0 \, \text{°C}) \) and taking the coefficient of thermal expansion of aluminum from Table 12.1, we find that the wire makes an angle \( \theta \) with the horizontal that is

\[
\theta = \cos^{-1}\left( \frac{\cos \theta_0}{1 + \alpha \Delta T} \right) = \cos^{-1}\left( \frac{\cos 3.00^\circ}{1 + \left[ 23 \times 10^{-6} \, \text{(°C)}^{-1} \right] (-20.0 \, \text{°C})} \right) = 2.446^\circ
\]

Using this value for \( \theta \) in Equation (1) gives

\[
T = \frac{W}{2 \sin \theta} = \frac{85.0 \, \text{N}}{2 \sin 2.446^\circ} = \left[ 996 \, \text{N} \right]
\]

27. **REASONING AND SOLUTION**  Let \( L_0 = 0.50 \, \text{m} \) and \( L \) be the true length of the line at 40.0 °C. The ruler has expanded an amount

\[
\Delta L_r = L - L_0 = \alpha_r L_0 \Delta T_r
\]  \hspace{1cm} (1)

The copper plate must shrink by an amount

\[
\Delta L_p = L_0 - L = \alpha_p L \Delta T_p
\]  \hspace{1cm} (2)

Eliminating \( L \) from Equations (1) and (2), solving for \( \Delta T_p \), and using values of the coefficients of thermal expansion for copper and steel from Table 12.1, we find that

\[
\Delta T_p = \frac{\alpha_r \Delta T_r}{\alpha_p \left( 1 + \alpha_r \Delta T_r \right)}
\]

\[
= \frac{-12 \times 10^{-6} \, \text{(°C)}^{-1} \left( 40.0 \, \text{°C} - 20.0 \, \text{°C} \right)}{17 \times 10^{-6} \, \text{(°C)}^{-1} \left[ 1 + \left[ 12 \times 10^{-6} \, \text{(°C)}^{-1} \left( 40.0 \, \text{°C} - 20.0 \, \text{°C} \right) \right] \right] \} = -14 \, \text{°C}
\]

Therefore, \( T_p = 40.0 \, \text{°C} - 14 \, \text{°C} = \left[ 26 \, \text{°C} \right] \).
28. **REASONING** We can identify the liquid by computing the coefficient of volume expansion $\beta$ and then comparing the result with the values of $\beta$ given in Table 12.1. The relation $\Delta V = \beta V_0 \Delta T$ (Equation 12.3) can be used to calculate $\beta$.

**SOLUTION** Solving Equation 12.3 for $\beta$, we have that $\beta = \Delta V / (V_0 \Delta T)$. The change in the volume of the liquid is $\Delta V = 1.500 \text{ L} - 1.383 \text{ L} = 0.117 \text{ L}$. Therefore, the coefficient of volume expansion for the unknown liquid is

$$
\beta = \frac{\Delta V}{V_0 \Delta T} = \frac{0.117 \text{ L}}{(1.500 \text{ L})(97.1 ^\circ \text{C} - 15.0 ^\circ \text{C})} = 9.50 \times 10^{-4} (\text{C}^\circ)^{-1} = 950 \times 10^{-6} (\text{C}^\circ)^{-1}
$$

A comparison with the values of $\beta$ in Table 12.1 indicates that the liquid is gasoline.

29. **REASONING** The change $\Delta V$ in the interior volume of the shell is given by Equation 12.3 as $\Delta V = \beta V_0 \Delta T$, where $\beta$ is the coefficient of volume expansion, $V_0$ is the initial volume, and $\Delta T$ is the increase in temperature. The interior volume behaves as if it were filled with the surrounding silver. The interior volume is spherical, and the volume of a sphere is $\frac{4}{3} \pi r^3$, where $r$ is the radius of the sphere.

**SOLUTION** In applying Equation 12.3, we note that the initial spherical volume of the interior space is $V_0 = \frac{4}{3} \pi r^3$, so that we have

$$
\Delta V = \beta V_0 \Delta T = \beta \left( \frac{4}{3} \pi r^3 \right) \Delta T
$$

$$
= \left[ 57 \times 10^{-6} (\text{C}^\circ)^{-1} \right] \frac{4}{3} \pi \left( 2.0 \times 10^{-2} \text{ m} \right)^3 (147 ^\circ \text{C} - 18 ^\circ \text{C}) = \boxed{2.5 \times 10^{-7} \text{ m}^3}
$$

We have taken the coefficient of volume expansion for silver from Table 12.1.

30. **REASONING** The carbon tetrachloride contracts from its volume $V_0 = 2.54 \times 10^{-4} \text{ m}^3$ at the higher temperature $T_0 = 75.0 ^\circ \text{C}$ to a volume $V$ at the lower temperature $T = -13.0 ^\circ \text{C}$.

The smaller volume $V$ is the larger volume $V_0$ minus the difference $\Delta V$ in volume:

$$
V = V_0 - \Delta V \quad \text{(1)}
$$

We will find the difference in volume from $\Delta V = \beta V_0 \Delta T$ (Equation 12.3), where $\beta$ is the coefficient of volume expansion for carbon tetrachloride and $\Delta T$ is the difference between the higher and lower temperatures.

**SOLUTION** Substituting $\Delta V = \beta V_0 \Delta T$ (Equation 12.3) into Equation (1), we obtain

$$
V = V_0 - \beta V_0 \Delta T \quad \text{or} \quad V = V_0 \left( 1 - \beta \Delta T \right)
$$
Therefore, the volume \( V \) of the carbon tetrachloride at \(-13.0 \, ^\circ \text{C} \) is
\[
V = (2.54 \times 10^{-4} \, \text{m}^3) \left[ 1 - 1240 \times 10^{-6} \, (\text{C}^{-1}) \left( 75.0 \, ^\circ \text{C} - (-13.0 \, ^\circ \text{C}) \right) \right] = 2.26 \times 10^{-4} \, \text{m}^3
\]

We have taken the value for \( \beta \), the coefficient of volume expansion for carbon tetrachloride, from Table 12.1 in the text.

31. **SSM REASONING AND SOLUTION** The volume \( V_0 \) of an object changes by an amount \( \Delta V \) when its temperature changes by an amount \( \Delta T \); the mathematical relationship is given by Equation 12.3: \( \Delta V = \beta V_0 \Delta T \). Thus, the volume of the kettle at 24 \(^\circ\text{C}\) can be found by solving Equation 12.3 for \( V_0 \). According to Table 12.1, the coefficient of volumetric expansion for copper is \( 51 \times 10^{-6} \, (\text{C}^{-1}) \). Solving Equation 12.3 for \( V_0 \), we have
\[
V_0 = \frac{\Delta V}{\beta \Delta T} = \frac{1.2 \times 10^{-5} \, \text{m}^3}{[51 \times 10^{-6} \, (\text{C}^{-1})][100 \, ^\circ \text{C} - 24 \, ^\circ \text{C}]} = 3.1 \times 10^{-3} \, \text{m}^3
\]

32. **REASONING** Consider one gallon of cider on a day when the temperature is 4.0 \(^\circ\text{C}\). On a day when the temperature is 26.0 \(^\circ\text{C}\) the volume of this cider will be greater than one gallon due to volume thermal expansion. It will be greater by an amount \( \Delta V = \beta V_0 \Delta T \) (Equation 12.3), where \( \beta \) is the coefficient of volume expansion, \( V_0 \) is the initial volume of one gallon, and \( \Delta T \) is the change in temperature. You can sell this extra volume of cider for \( $2.00 \) per gallon and earn a bit of extra cash.

**SOLUTION** According to Equation 12.3, the change in volume of the cider is
\[
\Delta V = \beta V_0 \Delta T = \left[ 280 \times 10^{-6} \, (\text{C}^{-1}) \right] (1.0 \, \text{gal}) \left[ (26 \, ^\circ \text{C}) - (4.0 \, ^\circ \text{C}) \right] = 6.2 \times 10^{-3} \, \text{gal}
\]
At a cost of two dollars per gallon, this amounts to
\[
(6.2 \times 10^{-3} \, \text{gal}) \left( \frac{\$2.00}{1 \, \text{gal}} \right) = $0.01 \quad \text{or} \quad 1 \, \text{penny}
\]

33. **REASONING AND SOLUTION** Both the coffee and beaker expand as the temperature increases. For the expansion of the coffee (water)
\[
\Delta V_W = \beta_W V_0 \Delta T
\]
For the expansion of the beaker (Pyrex glass)
\[
\Delta V_G = \beta_G V_0 \Delta T
\]
The excess expansion of the coffee, hence the amount which spills, is
\[ \Delta V = \Delta V_W - \Delta V_G = (\beta_W - \beta_G) V_0 \Delta T \]

Taking the coefficients of volumetric expansion \( \beta_W \) and coffee (water) and \( \beta_G \) for the beaker (Pyrex glass) from Table 12.1, we find

\[
\begin{align*}
\Delta V &= (\beta_W - \beta_G) V_0 \Delta T \\
&= \left[ 207 \times 10^{-6} \left( \text{C}^{-1} \right) - 9.9 \times 10^{-6} \left( \text{C}^{-1} \right) \right] (0.50 \times 10^{-3} \text{ m}^3)(92 \, ^{\circ}\text{C} - 18 \, ^{\circ}\text{C}) = 7.3 \times 10^{-6} \text{ m}^3
\end{align*}
\]

34. **REASONING AND SOLUTION** Both the water and pipe expand as the temperature increases. For the expansion of the water

\[ \Delta V_W = \beta_W V_0 \Delta T \]

For the expansion of the copper pipe

\[ \Delta V_c = \beta_c V_0 \Delta T \]

The initial volume of the pipe and water is \( V_0 = \pi r^2 L \) The reservoir then needs a capacity of

\[ \Delta V = \Delta V_W - \Delta V_c = (\beta_W - \beta_c) V_0 \Delta T \]

Taking the coefficients of volumetric expansion \( \beta_W \) and \( \beta_c \) for water and copper from Table 12.1, we find

\[
\begin{align*}
\Delta V &= (\beta_W - \beta_c) V_0 \Delta T \\
&= (207 \times 10^{-6} \text{ C}^{-1} - 51 \times 10^{-6} \text{ C}^{-1}) \pi (9.5 \times 10^{-3} \text{ m})^2 (76 \text{ m})(54 \text{ C}) = 1.8 \times 10^{-4} \text{ m}^3
\end{align*}
\]

35. **SSM REASONING** When the temperature increases, both the gasoline and the tank cavity expand. If they were to expand by the same amount, there would be no overflow. However, the gasoline expands more than the tank cavity, and the overflow volume is the amount of gasoline expansion minus the amount of the tank cavity expansion. The tank cavity expands as if it were solid steel.

**SOLUTION** The coefficients of volumetric expansion \( \beta_g \) and \( \beta_s \) for gasoline and steel are available in Table 12.1. According to Equation 12.3, the volume expansion of the gasoline is

\[ \Delta V_g = \beta_g V_0 \Delta T = \left[ 950 \times 10^{-6} \left( \text{C}^{-1} \right) \right] (20.0 \text{ gal}) \left[ (35 \text{ C}) - (17 \text{ C}) \right] = 0.34 \text{ gal} \]

while the volume of the steel tank expands by an amount
\[ \Delta V_s = \beta_s V_0 \Delta T = \left[ 36 \times 10^{-6} \left( \text{C}^{-1} \right) \right] (20.0 \text{ gal}) \left[ (35 \text{ C}) - (17 \text{ C}) \right] = 0.013 \text{ gal} \]

The amount of gasoline that spills out is

\[ \Delta V_g - \Delta V_s = 0.33 \text{ gal} \]

36. **REASONING** According to \( \Delta V = \beta V_0 \Delta T \) (Equation 12.3), the change \( \Delta V \) in volume depends on the coefficient of volume expansion \( \beta \), the initial volume \( V_0 \), and the change \( \Delta T \) in temperature. The liquid expands more because its coefficient of volume expansion is larger, and the change in volume is directly proportional to that coefficient. The volume of liquid that spills over is equal to the change in volume of the liquid minus the change in volume of the can.

**SOLUTION** The volume of liquid that spills over the can is the difference between the increase in the volume of the liquid and that of the aluminum can:

\[ \Delta V = \beta_L V_0 \Delta T - \beta_A V_0 \Delta T \]

Therefore,

\[ \beta_L = \frac{\Delta V}{V_0 \Delta T} + \beta_A \]

\[ = \frac{3.6 \times 10^{-6} \text{ m}^3}{\left(3.5 \times 10^{-4} \text{ m}^3\right) (78 \text{ C} - 5 \text{ C})} + 69 \times 10^{-6} \left( \text{C}^{-1} \right) = 2.1 \times 10^{-4} \left( \text{C}^{-1} \right) \]

37. **REASONING** The change \( \Delta V \) in volume is given by Equation 12.3 as \( \Delta V = \beta V_0 \Delta T \), where \( \beta \) is the coefficient of volume expansion, \( V_0 \) is the initial volume, and \( \Delta T \) is the increase in temperature. The increase in volume of the mercury is given directly by this equation, with \( V_0 \) being the initial volume of the interior space of the brass shell minus the initial volume of the steel ball. If the space occupied by the mercury did not change with temperature, the spillage would simply be the increase in volume of the mercury. However, the space occupied by the mercury does change with temperature. Both the brass shell and the steel ball expand. The interior volume of the brass shell expands as if it were solid brass, and this expansion provides more space for the mercury to occupy, thereby reducing the amount of spillage. The expansion of the steel ball, in contrast, takes up space that would otherwise be occupied by mercury, thereby increasing the amount of spillage. The total spillage, therefore, is \( \Delta V_{\text{Mercury}} - \Delta V_{\text{Brass}} + \Delta V_{\text{Steel}} \).

**SOLUTION** Table 12.1 gives the coefficients of volume expansion for mercury, brass, and steel. Applying Equation 12.3 to the mercury, the brass cavity, and the steel ball, we have
Spillage = $\Delta V_{\text{Mercury}} - \Delta V_{\text{Brass}} + \Delta V_{\text{Steel}}$

$= \beta_{\text{Mercury}} V_0,\text{Mercury} \Delta T - \beta_{\text{Brass}} V_0,\text{Brass} \Delta T + \beta_{\text{Steel}} V_0,\text{Steel} \Delta T$

$= \left[182 \times 10^{-6} \text{ (C°)}^{-1}\right] \left[(1.60 \times 10^{-3} \text{ m}^3) - (0.70 \times 10^{-3} \text{ m}^3)\right] (12 \text{ C°})$

Mercury

$- \left[57 \times 10^{-6} \text{ (C°)}^{-1}\right] (1.60 \times 10^{-3} \text{ m}^3) (12 \text{ C°})$

Brass

$+ \left[36 \times 10^{-6} \text{ (C°)}^{-1}\right] (0.70 \times 10^{-3} \text{ m}^3) (12 \text{ C°}) = 1.2 \times 10^{-6} \text{ m}^3$

Steel

38. **REASONING** When the mercury is heated, it expands a distance $\Delta L$ into the capillary, creating a small cylinder of mercury in the capillary with a volume $\Delta V = \pi r^2 \Delta L$. The radius $r$ of the capillary is assumed constant, because we are ignoring the thermal expansion of the glass. Because the increase $\Delta V$ in the volume of the mercury is due to thermal expansion, we can also express it as $\Delta V = \beta V_0 \Delta T$ (Equation 12.3), where $\beta = 182 \times 10^{-6} \text{ (C°)}^{-1}$ is the coefficient of volume expansion for mercury (see Table 12.1), $V_0$ is the volume of the mercury before heating, and $\Delta T = 1.0 \text{ C°}$ is the increase in the temperature of the mercury.

**SOLUTION** Solving $\Delta V = \pi r^2 \Delta L$ for $\Delta L$ yields

$$\Delta L = \frac{\Delta V}{\pi r^2} \quad (1)$$

Substituting $\Delta V = \beta V_0 \Delta T$ (Equation 12.3) into Equation (1), we obtain

$$\Delta L = \frac{\Delta V}{\pi r^2} = \frac{\beta V_0 \Delta T}{\pi r^2} = \left[182 \times 10^{-6} \text{ (C°)}^{-1}\right] (45 \text{ mm}^3) (1.0 \text{ C°}) = 9.0 \text{ mm}$$

39. **SSM REASONING** In order to keep the water from expanding as its temperature increases from 15 to 25 °C, the atmospheric pressure must be increased to compress the water as it tries to expand. The magnitude of the pressure change $\Delta P$ needed to compress a substance by an amount $\Delta V$ is, according to Equation 10.20, $\Delta P = B(\Delta V/V_0)$. The ratio
\[ \Delta V / V_0 \] is, according to Equation 12.3, \[ \Delta V / V_0 = \beta \Delta T \]. Combining these two equations yields

\[ \Delta P = B \beta \Delta T \]

**SOLUTION** Taking the value for the coefficient of volumetric expansion \( \beta \) for water from Table 12.1, we find that the change in atmospheric pressure that is required to keep the water from expanding is

\[ \Delta P = (2.2 \times 10^9 \text{ N/m}^2) \left[ 207 \times 10^{-6} \left( \text{C}^\circ \right)^{-1} \right] (25 \text{ C} - 15 \text{ C}) \]

\[ = (4.6 \times 10^6 \text{ Pa}) \left( \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \right) = 45 \text{ atm} \]

---

40. **REASONING** The density of mercury is given by \( \rho = m / V \) (Equation 11.1), where \( m \) is the mass of a volume \( V \) of mercury. As the temperature increases, the mass does not change, but the volume increases because of volume thermal expansion. Therefore, we expect the density at the higher temperature to be less than the density at the lower temperature.

**SOLUTION** If \( \rho_0 \) represents the density of mercury at 0 °C, and \( \rho \) represents its density at 166 °C, then the ratio of the density of mercury at 166 °C to that at 0 °C is

\[ \frac{\rho}{\rho_0} = \frac{m/V}{m/V_0} = \frac{V_0}{V} \]

and the density at 166 °C is given by

\[ \rho = \rho_0 \frac{V_0}{V} \quad (1) \]

The volume \( V_0 \) of an object changes by an amount \( \Delta V \) when its temperature changes by an amount \( \Delta T \); the mathematical relationship is \( \Delta V = \beta V_0 \Delta T \) (Equation 12.3), where \( \beta = 182 \times 10^{-6} \left( \text{C}^\circ \right)^{-1} \) is the coefficient of volume expansion for mercury (see Table 12.1). In Equation 12.3, we have \( \Delta V = V - V_0 \), so that \( V = V_0 + \Delta V = V_0 + \beta V_0 \Delta T \). Therefore, Equation (1) becomes

\[ \rho = \rho_0 \left( \frac{V_0}{V_0 + \beta V_0 \Delta T} \right) = \rho_0 \left( \frac{1}{1 + \beta \Delta T} \right) \]

and we find that

\[ \rho = \frac{\rho_0}{1 + \beta \Delta T} = \frac{13600 \text{ kg/m}^3}{1 + \left[ 182 \times 10^{-6} \left( \text{C}^\circ \right)^{-1} \right] \left[ (166 \text{ C}) - (0 \text{ C}) \right]} = 13200 \text{ kg/m}^3 \]
41. **REASONING**  The cavity that contains the liquid in either Pyrex thermometer expands according to Equation 12.3, \( \Delta V_g = \beta_g V_0 \Delta T \). On the other hand, the volume of mercury expands by an amount \( \Delta V_m = \beta_m V_0 \Delta T \), while the volume of alcohol expands by an amount \( \Delta V_a = \beta_a V_0 \Delta T \). Therefore, the net change in volume for the mercury thermometer is
\[
\Delta V_m - \Delta V_g = (\beta_m - \beta_g)V_0\Delta T
\]
while the net change in volume for the alcohol thermometer is
\[
\Delta V_a - \Delta V_g = (\beta_a - \beta_g)V_0\Delta T
\]
In each case, this volume change is related to a movement of the liquid into a cylindrical region of the thermometer with volume \( \pi r^2 h \), where \( r \) is the radius of the region and \( h \) is the height of the region. For the mercury thermometer, therefore,
\[
h_m = \frac{(\beta_m - \beta_g)V_0\Delta T}{\pi r^2}
\]
Similarly, for the alcohol thermometer
\[
h_a = \frac{(\beta_a - \beta_g)V_0\Delta T}{\pi r^2}
\]
These two expressions can be combined to give the ratio of the heights, \( h_a/h_m \).

**SOLUTION**  Taking the values for the coefficients of volumetric expansion for methyl alcohol, Pyrex glass, and mercury from Table 12.1, we divide the two expressions for the heights of the liquids in the thermometers and find that
\[
\frac{h_a}{h_m} = \frac{\beta_a - \beta_g}{\beta_m - \beta_g} = \frac{1200 \times 10^{-6} \ (C^\circ)^{-1} - 9.9 \times 10^{-6} \ (C^\circ)^{-1}}{182 \times 10^{-6} \ (C^\circ)^{-1} - 9.9 \times 10^{-6} \ (C^\circ)^{-1}} = 6.9
\]
Therefore, the degree marks are 6.9 times further apart on the alcohol thermometer than on the mercury thermometer.

42. **REASONING AND SOLUTION**  When the temperature is 0.0 °C, \( P = \rho_0 gh_0 \), and when the temperature is 38.0 °C, \( P = \rho gh \). Equating and solving for \( h \) gives \( h = (\rho_0/\rho)h_0 \). Now \( \rho_0/\rho = V/V_0 \), since the mass of mercury in the tube remains constant. Thus, we have that \( h = (V/V_0)h_0 \). Now,
\[ \Delta V = V - V_0 = \beta V_0 \Delta T \quad \text{or} \quad \frac{V}{V_0} = 1 + \beta \Delta T \]

Therefore,
\[ h = (1 + \beta \Delta T) h_0 = \{1 + [182 \times 10^{-6} \text{ (C}^\circ\text{)}^{-1}](38.0 \text{ C}^\circ - 0.0 \text{ C}^\circ)\}(0.760 \text{ m}) = 0.765 \text{ m} \]

where we have taken the value for the coefficient of volumetric expansion \( \beta \) for mercury from Table 12.1.

43. **SSM REASONING** Since there is no heat lost or gained by the system, the heat lost by the water in cooling down must be equal to the heat gained by the thermometer in warming up. The heat \( Q \) lost or gained by a substance is given by Equation 12.4 as \( Q = cm \Delta T \), where \( c \) is the specific heat capacity, \( m \) is the mass, and \( \Delta T \) is the change in temperature. Thus, we have that
\[
\frac{c_{ \text{H}_2\text{O}} m_{ \text{H}_2\text{O}} \Delta T_{ \text{H}_2\text{O}}}{\text{Heat lost by water}} = \frac{c_{\text{therm}} m_{\text{therm}} \Delta T_{\text{therm}}}{\text{Heat gained by thermometer}}
\]

We can use this equation to find the temperature of the water before the insertion of the thermometer.

**SOLUTION** Solving the equation above for \( \Delta T_{\text{H}_2\text{O}} \), and using the value of \( c_{\text{H}_2\text{O}} \) from Table 12.2, we have
\[
\Delta T_{\text{H}_2\text{O}} = \frac{c_{\text{therm}} m_{\text{therm}} \Delta T_{\text{therm}}}{c_{\text{H}_2\text{O}} m_{\text{H}_2\text{O}}}
\]
\[
= \frac{[815 \text{ J/(kg} \cdot \text{C}^\circ)](31.0 \text{ g})(41.5 \text{ C}^\circ - 12.0 \text{ C}^\circ)}{[4186 \text{ J/(kg} \cdot \text{C}^\circ)](119 \text{ g})} = 1.50 \text{ C}^\circ
\]

The temperature of the water before the insertion of the thermometer was
\[ T = 41.5 \text{ C}^\circ + 1.50 \text{ C}^\circ = 43.0 \text{ C}^\circ \]

44. **REASONING** When the blood reaches the body's surface, the blood has a temperature of \( T_{\text{initial}} = 37.0 \text{ C}^\circ \). When it leaves the surface after releasing energy in the form of heat, the blood has a temperature \( T_{\text{final}} \). Because heat is released, the blood temperature decreases by an amount \( |\Delta T| \), the vertical bars denoting the magnitude or absolute value of \( \Delta T \). To determine \( T_{\text{final}} \), we will subtract \( |\Delta T| \) from \( T_{\text{initial}} \). To find \( |\Delta T| \) we will use the fact that the heat released at the surface has a magnitude of \( |Q| = cm|\Delta T| \) (Equation 12.4), where
\( c = 4186 \, J/(\text{kg} \cdot \text{C}^\circ) \) is the specific heat capacity of the blood (see the entry for water in Table 12.2) and \( m \) is the mass of the blood.

**SOLUTION** The final temperature of the blood is

\[
T_{\text{final}} = T_{\text{initial}} - |\Delta T| \tag{1}
\]

Solving \( |Q| = cm|\Delta T| \) (Equation 12.4) for \( |\Delta T| = \frac{|Q|}{cm} \). Substituting this result into Equation (1) gives

\[
T_{\text{final}} = T_{\text{initial}} - \frac{|Q|}{cm} = 37.0 \, ^\circ\text{C} - \frac{2000 \, J}{\left[ 4186 \, J/(\text{kg} \cdot \text{C}^\circ) \right] \left( 0.6 \, \text{kg} \right)} = 36.2 \, ^\circ\text{C}
\]

45. **REASONING** We assume that no heat is lost through the chest to the outside. Then, energy conservation dictates that the heat gained by the soda is equal to the heat lost by the watermelon in reaching the final temperature \( T_f \). Each quantity of heat is given by Equation 12.4, \( Q = cm\Delta T \), where we write the change in temperature \( \Delta T \) as the higher temperature minus the lower temperature.

**SOLUTION** Starting with the statement of energy conservation, we have

Heat gained by soda = Heat lost by watermelon

\[(cm\Delta T)_{\text{soda}} = (cm\Delta T)_{\text{watermelon}}\]

Since the watermelon is being treated like water, we take the specific heat capacity of water from Table 12.2. Thus, the above equation becomes

\[
\left[ 3800 \, J/(\text{kg} \cdot \text{C}^\circ) \right] 12 \times 0.35 \, \text{kg} \left( T_f - 5.0 \, ^\circ\text{C} \right) = \left[ 4186 \, J/(\text{kg} \cdot \text{C}^\circ) \right] 6.5 \, \text{kg} \left( 27 \, ^\circ\text{C} - T_f \right)
\]

Suppressing units for convenience and algebraically simplifying, we have

\[
1.6 \times 10^4 \, T_f - 8.0 \times 10^4 = 7.3 \times 10^5 - 2.7 \times 10^4 T_f
\]

Solving for \( T_f \), we obtain

\[
T_f = \frac{8.1 \times 10^5}{4.3 \times 10^4} = 19 \, ^\circ\text{C}
\]

46. **REASONING** Since the container of the glass and the liquid is being ignored and since we are assuming negligible heat exchange with the environment, the principle of conservation of energy applies. In reaching equilibrium the cooler liquid gains heat, and the hotter glass loses heat. We will apply this principle by equating the heat gained to the heat lost. The heat \( Q \) that must be supplied or removed to change the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. In using this equation as we apply the energy-conservation principle, we must remember to express the change in temperature \( \Delta T \) as the higher minus the lower temperature.
**SOLUTION** Applying the energy-conservation principle and using Equation 12.4 give

\[
c_{\text{Liquid}}m\Delta T_{\text{Liquid}} = c_{\text{Glass}}m\Delta T_{\text{Glass}}
\]

Since it is the same for both the glass and the liquid, the mass \(m\) can be eliminated algebraically from this equation. Solving for \(c_{\text{Liquid}}\) and taking the specific heat capacity for glass from Table 12.2, we find

\[
c_{\text{Liquid}} = \frac{c_{\text{Glass}}\Delta T_{\text{Glass}}}{\Delta T_{\text{Liquid}}} = \frac{[840 \text{ J/(kg} \cdot \text{C})][83.0 \text{ C} - 53.0 \text{ C}]}{(53.0 \text{ C} - 43.0 \text{ C})} = 2500 \text{ J/(kg} \cdot \text{C})
\]

47. **REASONING** The metabolic processes occurring in the person’s body produce the heat that is added to the water. As a result, the temperature of the water increases. The heat \(Q\) that must be supplied to increase the temperature of a substance of mass \(m\) by an amount \(\Delta T\) is given by Equation 12.4 as \(Q = cm\Delta T\), where \(c\) is the specific heat capacity. The increase \(\Delta T\) in temperature is the final higher temperature \(T_f\) minus the initial lower temperature \(T_0\). Hence, we will solve Equation 12.4 for the desired final temperature.

**SOLUTION** From Equation 12.4, we have

\[
Q = cm\Delta T = cm(T_f - T_0)
\]

Solving for the final temperature, noting that the heat is \(Q = (3.0 \times 10^5 \text{ J/h})(0.50 \text{ h})\) and taking the specific heat capacity of water from Table 12.2, we obtain

\[
T_f = T_0 + \frac{Q}{cm} = 21.00 \text{ } ^\circ\text{C} + \frac{(3.0 \times 10^5 \text{ J/h})(0.50 \text{ h})}{[4186 \text{ J/(kg} \cdot \text{C})][1.2 \times 10^3 \text{ kg}]} = 21.03 \text{ } ^\circ\text{C}
\]

48. **REASONING** The change \(\Delta T\) in temperature is determined by the amount \(Q\) of heat added, the specific heat capacity \(c\) and mass \(m\) of the material, according to \(Q = cm\Delta T\) (Equation 12.4). The heat supplied to each bar and the mass of each bar are the same, but the changes in temperature are different. The only factor that can account for the different temperature changes is the specific heat capacities, which must be different. We will apply Equation 12.4 to each bar and thereby determine the unknown specific heat capacity.

**SOLUTION** The heat supplied to each bar is given by \(Q = cm\Delta T\) (Equation 12.4). The amount of heat \(Q_G\) supplied to the glass is equal to the heat \(Q_S\) supplied to the other substance. Thus,

\[
Q_G = Q_S \quad \text{or} \quad c_G m \Delta T_G = c_S m \Delta T_S
\]  \hspace{1cm} (1)
We know that $c_G = 840 \text{ J/(kg·C°)}$ from Table 12.2. Solving Equation (1) for $c_S$ and eliminating the mass $m$ algebraically, we obtain

$$c_S = c_G \left( \frac{\Delta T_G}{\Delta T_S} \right) = \left[ 840 \text{ J/(kg·C°)} \right] \left( \frac{88 \text{ °C} - 25 \text{ °C}}{250.0 \text{ °C} - 25 \text{ °C}} \right) = 235 \text{ J/(kg·C°)}$$

49. **SSM REASONING** Let the system be comprised only of the metal forging and the oil. Then, according to the principle of energy conservation, the heat lost by the forging equals the heat gained by the oil, or $Q_{\text{metal}} = Q_{\text{oil}}$. According to Equation 12.4, the heat lost by the forging is $Q_{\text{metal}} = c_{\text{metal}} m_{\text{metal}} (T_{0\text{metal}} - T_{\text{eq}})$, where $T_{\text{eq}}$ is the final temperature of the system at thermal equilibrium. Similarly, the heat gained by the oil is given by $Q_{\text{oil}} = c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})$.

**SOLUTION**

$$Q_{\text{metal}} = Q_{\text{oil}}$$

$$c_{\text{metal}} m_{\text{metal}} (T_{0\text{metal}} - T_{\text{eq}}) = c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})$$

Solving for $T_{0\text{metal}}$, we have

$$T_{0\text{metal}} = \frac{c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})}{c_{\text{metal}} m_{\text{metal}}} + T_{\text{eq}}$$

or

$$T_{0\text{metal}} = \left[ \frac{2700 \text{ J/(kg·C°)}}{430 \text{ J/(kg·C°)}} \right] \frac{(710 \text{ kg})(47 \text{ °C} - 32 \text{ °C})}{(75 \text{ kg})} + 47 \text{ °C} = 940 \text{ °C}$$

50. **REASONING** The volume $V$ of a mass $m$ of water is given by $V = m/\rho$ (Equation 11.1), where $\rho$ is the mass density of water (see Table 11.1). In order to warm a mass $m$ of ice water to body temperature, the body must provide an amount $Q$ of heat given by $Q = cm\Delta T$ (Equation 12.4), where $c$ is the specific heat of water (see Table 12.2), and $\Delta T$ is 37 C°, the difference between body temperature (37 °C) and the temperature of ice water (0 °C). We will use Equation 12.4 to calculate the required mass $m$ of ice water, and Equation 11.1 to find the corresponding volume $V$.

**SOLUTION** The volume $V$ of the water is

$$V = \frac{m}{\rho} \quad (11.1)$$

Solving $Q = cm\Delta T$ (Equation 12.4) for the mass $m$, we obtain

$$m = \frac{Q}{c\Delta T} \quad (1)$$
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Substituting Equation (1) into Equation 11.1 yields

\[ V = \frac{Q}{c\Delta T} = \frac{Q}{\rho c\Delta T} \quad (2) \]

The amount \( Q \) of heat is given as 430 kcal, which must be converted to joules via the equivalence 1 kcal = 4186 J:

\[ Q = \left( 430 \text{ kcal} \right) \left( \frac{4186 \text{ J}}{1 \text{ kcal}} \right) = 1.8 \times 10^6 \text{ J} \]

Equation (2), then, gives the volume of ice water in m\(^3\):

\[ V = \frac{Q}{\rho c\Delta T} = \frac{1.8 \times 10^6 \text{ J}}{\left( 1.0 \times 10^3 \text{ kg/m}^3 \right) \left[ 4186 \text{ J/}(\text{kg} \cdot \text{C}^\circ) \right] \left( 37 \text{ C}^\circ \right)} = 0.012 \text{ m}^3 \]

To convert this result to liters, we use the equivalence 1 liter = 1.0 x 10\(^{-3}\) m\(^3\):

\[ V = \left( 0.012 \text{ m}^3 \right) \left( \frac{1 \text{ liter}}{1.0 \times 10^{-3} \text{ m}^3} \right) = 12 \text{ liters} \]

51. **REASONING** According to Equation 6.10b, the average power is the change in energy divided by the time. The change in energy in this problem is the heat supplied to the water and the coffee mug to raise their temperature from 15 to 100 °C, which is the boiling point of water. The time is given as three minutes (180 s). The heat \( Q \) that must be added to raise the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. This equation will be used for the water and the material of which the mug is made.

**SOLUTION** Using Equation 6.10b, we write the average power \( \bar{P} \) as

\[ \bar{P} = \frac{\text{Change in energy}}{\text{Time}} = \frac{Q_{\text{Water}} + Q_{\text{Mug}}}{\text{Time}} \]

The heats \( Q_{\text{Water}} \) and \( Q_{\text{Mug}} \) each can be expressed with the aid of Equation 12.4, so that we obtain

\[ \bar{P} = \frac{Q_{\text{Water}} + Q_{\text{Mug}}}{\text{Time}} = \frac{c_{\text{Water}}m_{\text{Water}}\Delta T + c_{\text{Mug}}m_{\text{Mug}}\Delta T}{\text{Time}} \]

\[ = \frac{\left[ 4186 \text{ J/}(\text{kg} \cdot \text{C}^\circ) \right] (0.25 \text{ kg}) (100.0 \text{ °C} - 15 \text{ °C})}{180 \text{ s}} + \frac{\left[ 920 \text{ J/}(\text{kg} \cdot \text{C}^\circ) \right] (0.35 \text{ kg}) (100.0 \text{ °C} - 15 \text{ °C})}{180 \text{ s}} = 650 \text{ W} \]
The specific heat of water has been taken from Table 12.2.

52. **REASONING** Two portions of the same liquid that have the same mass, but different initial temperatures, when mixed together, will yield an equilibrium mixture that has a temperature lying exactly midway between the two initial temperatures. That is, the final temperature is 

\[ \frac{1}{2}(T_{0A} + T_{0B}) \]

This occurs only when the mixing occurs without any exchange of heat with the surroundings. Then, all of the heat lost from the warmer portion is gained by the cooler portion. Since each portion is identical except for temperature, the warmer portion cools down by the same number of degrees that the cooler portion warms up, yielding a mixture whose final equilibrium temperature is midway between the two initial temperatures.

There are two ways to apply the logic above to the problem at hand. Portions A and B have the same mass \( m \), so they will yield a combined mass of \( 2m \). Thus, we can imagine portions A and B mixed together to yield an equilibrium temperature of

\[ \frac{1}{2}(T_{0A} + T_{0B}) = \frac{1}{2}(94.0 \, ^\circ C + 78.0 \, ^\circ C) = 86.0 \, ^\circ C \]

This mixture has a mass \( 2m \), just like the mass of portion C, so when it is mixed with portion C, the final equilibrium temperature will be 

\[ \frac{1}{2}(86.0 \, ^\circ C + 34.0 \, ^\circ C) = 60.0 \, ^\circ C \]

The other way to apply the logic is to mix half of portion C with portion A and half with portion B. This will produce two mixtures with different temperatures and each having a mass \( 2m \), which can then be combined. The final equilibrium temperature of this combination is again 60.0 \( ^\circ C \). Either way, the guiding principle is that the heat gained by the cooler liquid is equal to the heat lost by the warmer liquid.

**SOLUTION** Since we are assuming negligible heat exchange with the surroundings, the principle of conservation of energy applies in the following form: **heat lost equals heat gained.** In reaching equilibrium the warmer portions lose heat and the cooler portions gain heat. In either case, the heat \( Q \) that must be supplied or removed to change the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by 

\[ Q = cm\Delta T \] (Equation 12.4),

where \( c \) is the specific heat capacity. The final temperature is 50.0 \( ^\circ C \). We have then,

\[ \frac{c_cm_C(50.0 \, ^\circ C - 34.0 \, ^\circ C)}{Heat \ gained} = \frac{c_A m_A (94.0 \, ^\circ C - 50.0 \, ^\circ C) + c_B m_B (78.0 \, ^\circ C - 50.0 \, ^\circ C)}{Heat \ lost} \]

The specific heat capacities for each portion have the same value \( c \), while \( m_A = m_B = m \). With these substitutions, we find

\[ cm_c(50.0 \, ^\circ C - 34.0 \, ^\circ C) = cm(94.0 \, ^\circ C - 50.0 \, ^\circ C) + cm(78.0 \, ^\circ C - 50.0 \, ^\circ C) \]

\[ m_c(50.0 \, ^\circ C - 34.0 \, ^\circ C) = m(94.0 \, ^\circ C - 50.0 \, ^\circ C) + m(78.0 \, ^\circ C - 50.0 \, ^\circ C) \]

Solving this equation for the mass \( m_C \) of portion C gives

\[ m_c = \frac{m(94.0 \, ^\circ C - 50.0 \, ^\circ C) + m(78.0 \, ^\circ C - 50.0 \, ^\circ C)}{50.0 \, ^\circ C - 34.0 \, ^\circ C} = 4.50m \]
53. **REASONING AND SOLUTION** We wish to convert 2.0% of the heat $Q$ into gravitational potential energy, i.e., $(0.020)Q = mgh$. Thus,

$$mg = \frac{(0.020)Q}{h} = \frac{(0.020)(110 \text{ Calories})}{2.1 \text{ m}} = 4.4 \times 10^3 \text{ N}$$

54. **REASONING** Each second, the water heater increases the temperature of the water passing through from $T_0 = 11 ^\circ C$ to a final temperature $T$ by transferring an amount of heat $Q = cm\Delta T$ (Equation 12.4), where $m$ is the mass of the water heated in one second, $c = 4186 \text{ J/(kg} \cdot ^\circ \text{C})$ is the specific heat of water (see Table 12.2 in the text), and $\Delta T = T - T_0$ is the difference between the higher and lower temperatures of the water. Solving $Q = cm\Delta T$ for the temperature difference $\Delta T = \frac{Q}{cm}$, we have that the final temperature of the water is

$$T = T_0 + \Delta T = T_0 + \frac{Q}{cm} \quad (1)$$

The power $P$ of the water heater is 28 kW, or $28 \times 10^3 \text{ J/s}$. This means that the water heater increases the internal energy of the water by $28 \times 10^3 \text{ J}$ each second. Therefore, the maximum amount of heat the water can absorb in one second is $Q = 28 \times 10^3 \text{ J}$. The last quantity in Equation (1) that we must find is the mass $m$ of water passing through the heater in one second. We will determine the value of $m$ from the volume flow rate, the number of showers in simultaneous use, and the density of water.

**SOLUTION** There are four showers in operation simultaneously, so the total volume flow rate of water through the water heater is

$$\text{Volume flow rate} = 4\left(14 \times 10^{-5} \text{ m}^3/\text{s}\right) = 56 \times 10^{-5} \text{ m}^3/\text{s} \quad (2)$$

Therefore, the heater must heat a volume $V = 56 \times 10^{-5} \text{ m}^3$ of water each second. The corresponding mass $m$ of the water can be found from $m = \rho V$ (Equation 11.1) by using a value of $\rho = 1.00 \times 10^3 \text{ kg/m}^3$ for the density of water (see Table 11.1 in the text). Substituting Equation 11.1 into Equation (1), then, yields the maximum possible hot water temperature:

$$T = T_0 + \frac{Q}{c\rho V} = 11 ^\circ C + \frac{28 \times 10^3 \text{ J}}{\left[4186 \text{ J/(kg} \cdot ^\circ \text{C})\right] \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(56 \times 10^{-5} \text{ m}^3\right)} = 23 ^\circ C$$

55. **SSM REASONING AND SOLUTION** As the rock falls through a distance $h$, its initial potential energy $m_{\text{rock}} gh$ is converted into kinetic energy. This kinetic energy is then
converted into heat when the rock is brought to rest in the pail. If we ignore the heat absorbed by the pail, the principle of conservation of energy indicates that

\[ m_{\text{rock}} gh = c_{\text{rock}} m_{\text{rock}} \Delta T + c_{\text{water}} m_{\text{water}} \Delta T \]

where we have used Equation 12.4 to express the heat absorbed by the rock and the water. Table 12.2 gives the specific heat capacity of the water. Solving for \( \Delta T \) yields

\[ \Delta T = \frac{m_{\text{rock}} gh}{c_{\text{rock}} m_{\text{rock}} + c_{\text{water}} m_{\text{water}}} \]

Substituting values yields

\[ \Delta T = \frac{(0.20 \text{ kg})(9.80 \text{ m/s}^2)(15 \text{ m})}{[1840 \text{ J/(kg C\(^\circ\))}](0.20 \text{ kg}) + [4186 \text{ J/(kg C\(^\circ\))}](0.35 \text{ kg})} = 0.016 \text{ C\(^\circ\)} \]

56. **REASONING AND SOLUTION** The heat lost by the steel rod is \( Q = cm\Delta T = c \rho V_0 \Delta T \). Table 12.2 gives the specific heat capacity \( c \) of steel. The rod contracts according to the equation \( \Delta L = \alpha L_0 \Delta T \). Table 12.1 gives the coefficient of thermal expansion of steel. We also know that \( F = AY(\Delta L/L_0) \), Equation 10.17, so that \( F = AY\alpha \Delta T \). Table 10.1 gives Young’s modulus \( Y \) for steel.

Combining expressions yields,

\[ F = \frac{\alpha Q Y}{c \rho L_0} = \left[ \frac{12 \times 10^{-6} \text{ (C\(^\circ\))}^{-1}}{452 \text{ J/(kg C\(^\circ\))}(7860 \text{ kg/m}^3)(2.0 \text{ m})} \right] \times \left(3300 \text{ J}(2.0 \times 10^{11} \text{ Pa}) \right) = 1.1 \times 10^3 \text{ N} \]

57. **SSM REASONING** Heat \( Q_1 \) must be added to raise the temperature of the aluminum in its solid phase from 130 \(^\circ\)C to its melting point at 660 \(^\circ\)C. According to Equation 12.4, \( Q_1 = cm\Delta T \). The specific heat \( c \) of aluminum is given in Table 12.2. Once the solid aluminum is at its melting point, additional heat \( Q_2 \) must be supplied to change its phase from solid to liquid. The additional heat required to melt or liquify the aluminum is \( Q_2 = mL_1 \), where \( L_1 \) is the latent heat of fusion of aluminum. Therefore, the total amount of heat which must be added to the aluminum in its solid phase to liquify it is

\[ Q_{\text{total}} = Q_1 + Q_2 = m(c\Delta T + L_1) \]

**SOLUTION** Substituting values, we obtain
\[ Q_{\text{total}} = (0.45 \text{ kg}) \left( [9.00 \times 10^2 \text{ J/(kg} \cdot \text{C}^\circ)](660 \text{ } ^\circ\text{C} - 130 \text{ } ^\circ\text{C}) + 4.0 \times 10^5 \text{ J/kg} \right) = 3.9 \times 10^5 \text{ J} \]

58. **REASONING** The amount of heat removed when a liquid freezes into a solid is determined by its latent heat of fusion \( L_f \). The amount of heat removed is \( Q = mL_f \) (Equation 12.5), where \( m \) is the mass of the material that freezes. The latent heat of fusion for water is \( 33.5 \times 10^4 \text{ J/kg} \), whereas the value for ethyl alcohol is \( 10.8 \times 10^4 \text{ J/kg} \), as given in Table 12.3.

**SOLUTION** The same amount of heat is removed from the water as from the ethyl alcohol. Therefore, applying Equation 12.5 to each material, we have

\[ Q_{\text{water}} = Q_{\text{alcohol}} \quad \text{or} \quad m_{\text{water}}L_{f, \text{water}} = m_{\text{alcohol}}L_{f, \text{alcohol}} \]

Solving this expression for \( m_{\text{alcohol}} \) gives

\[ m_{\text{alcohol}} = m_{\text{water}} \frac{L_{f, \text{water}}}{L_{f, \text{alcohol}}} = (3.0 \text{ kg}) \left( \frac{33.5 \times 10^4 \text{ J/kg}}{10.8 \times 10^4 \text{ J/kg}} \right) = 9.3 \text{ kg} \]

59. **REASONING**
   a. When water changes from the liquid to the ice phase at 0 °C, the amount of heat released is given by \( Q = mL_f \) (Equation 12.5), where \( m \) is the mass of the water and \( L_f \) is its latent heat of fusion.

   b. When heat \( Q \) is supplied to the tree, its temperature changes by an amount \( \Delta T \). The relation between \( Q \) and \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity and \( m \) is the mass. This equation can be used to find the change in the tree’s temperature.

**SOLUTION**
   a. Taking the latent heat of fusion for water as \( L_f = 33.5 \times 10^4 \text{ J/kg} \) (see Table 12.3), we find that the heat released by the water when it freezes is

\[ Q = mL_f = (7.2 \text{ kg}) (33.5 \times 10^4 \text{ J/kg}) = 2.4 \times 10^6 \text{ J} \]

   b. Solving Equation 12.4 for the change in temperature, we have

\[ \Delta T = \frac{Q}{cm} = \frac{2.4 \times 10^6 \text{ J}}{2.5 \times 10^3 \text{ J/(kg} \cdot \text{C}^\circ)(180 \text{ kg})} = 5.3 \text{ C}^\circ \]
a. The amount of heat $Q$ required to melt a mass $m$ of a substance is directly proportional to the latent heat of fusion $L_f$ according to $Q = mL_f$ (Equation 12.5). Since each object has the same mass and more heat is required to melt B, the substance from which B is made has the larger latent heat of fusion. We will use Equation 12.5 to determine the latent heat.

b. The amount of heat required to melt a substance is also directly proportional to its mass, according to Equation 12.5. If the mass is doubled, the heat required to melt the substance also doubles.

**SOLUTION**

a. According to Equation 12.5, the latent heats of fusion for A and B are

$$L_{f,A} = \frac{Q_A}{m} = \frac{3.0 \times 10^4 \text{ J}}{3.0 \text{ kg}} = \boxed{1.0 \times 10^4 \text{ J/kg}}$$

$$L_{f,B} = \frac{Q_B}{m} = \frac{9.0 \times 10^4 \text{ J}}{3.0 \text{ kg}} = \boxed{3.0 \times 10^4 \text{ J/kg}}$$

b. The amount of heat required to melt object A when its mass is 6.0 kg is

$$Q = m_A L_{f,A} = (6.0 \text{ kg})(1.0 \times 10^4 \text{ J/kg}) = \boxed{6.0 \times 10^4 \text{ J}}$$ (12.5)

**61. SSM REASONING** From the conservation of energy, the heat lost by the mercury is equal to the heat gained by the water. As the mercury loses heat, its temperature decreases; as the water gains heat, its temperature rises to its boiling point. Any remaining heat gained by the water will then be used to vaporize the water.

According to Equation 12.4, the heat lost by the mercury is $Q_{\text{mercury}} = (cm\Delta T)_{\text{mercury}}$. The heat required to vaporize the water is, from Equation 12.5, $Q_{\text{vap}} = (m_{\text{vap}}L_v)_{\text{water}}$. Thus, the total amount of heat gained by the water is $Q_{\text{water}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_v)_{\text{water}}$.

**SOLUTION**

$$Q_{\text{lost by mercury}} = Q_{\text{gained by water}}$$

$$(cm\Delta T)_{\text{mercury}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_v)_{\text{water}}$$

where $\Delta T_{\text{mercury}} = (205^\circ\text{C} - 100.0^\circ\text{C})$ and $\Delta T_{\text{water}} = (100.0^\circ\text{C} - 80.0^\circ\text{C})$. The specific heats of mercury and water are given in Table 12.2, and the latent heat of vaporization of water is given in Table 12.3. Solving for the mass of the water that vaporizes gives
**TEMPERATURE AND HEAT**

\[ m_{\text{vap}} = \frac{c_{\text{mercury}} m_{\text{mercury}} \Delta T_{\text{mercury}} - c_{\text{water}} m_{\text{water}} \Delta T_{\text{water}}}{(L_v)_{\text{water}}} \]

\[ = \frac{[139 \text{ J/(kg} \cdot \text{C^°})](2.10 \text{ kg})(105 \text{ C^°}) - [4186 \text{ J/(kg} \cdot \text{C^°})](0.110 \text{ kg})(20.0 \text{ C^°})}{22.6 \times 10^5 \text{ J/kg}} \]

\[ = 9.49 \times 10^{-3} \text{ kg} \]

62. **REASONING**  The minimum required mass \( m_w \) of water is the mass that just reaches equilibrium with the condensed benzene at the boiling point of benzene (80.1 °C; see Table 12.3 in the text). If less water is used, not all of the benzene will condense, and if more water is used, then the benzene will be cooled below its boiling point before reaching equilibrium. Therefore, the final temperature of the water is 80.1 °C, the boiling point of benzene.

The mass \( m_w \) of the water needed to condense the benzene vapor depends upon the amount of heat \( Q \) that the water must absorb from the benzene. The benzene vapor is at its boiling point, so the heat \( Q \) it must give up is only the amount of heat associated with the phase change from vapor to liquid. This amount of heat is found from \( Q = m_b L_v \) (Equation 12.5), where \( m_b \) is the mass of the benzene vapor and \( L_v \) is the heat of vaporization for benzene (see Table 12.3 in the text). As the water absorbs heat from the condensing benzene, its temperature will rise by an amount \( \Delta T \). We will use \( Q = cm_w \Delta T \) (Equation 12.4), where \( c \) is the specific heat of water and \( m_w \) is the mass of water mixed with the benzene vapor, to determine the amount of heat \( Q \) absorbed by the water. Assuming that heat is exchanged only between the water and the benzene, this is the same as the amount of heat given up by the benzene.

**SOLUTION**  Solving \( Q = cm_w \Delta T \) (Equation 12.4) for the required mass \( m_w \) of water, we obtain

\[ m_w = \frac{Q}{c \Delta T} \quad (1) \]

Substituting \( Q = m_b L_v \) (Equation 12.5) into Equation (1) yields the required mass of water:

\[ m_w = \frac{m_b L_v}{c \Delta T} = \frac{(0.054 \text{ kg})(3.94 \times 10^5 \text{ J/kg})}{4186 \text{ J/(kg} \cdot \text{C^°})}(80.1^\circ \text{C} - 41^\circ \text{C}) = 0.13 \text{ kg} \]

63. **REASONING AND SOLUTION**  The heat required to evaporate the water is \( Q = mL_v \), and to lower the temperature of the jogger we have \( Q = m_j c \Delta T \). Equating these two expressions and solving for the mass \( m \) of the water, we have
To determine the linear speed \( v \) that ice (mass \( m \)) acquires upon being accelerated, we utilize the principle of conservation of energy. This principle indicates that the energy used to accelerate the ice equals the kinetic energy of the moving ice, since the ice starts from rest. Kinetic energy is \( \frac{1}{2}mv^2 \). The energy is the total amount of heat that is removed from the steam and consists of the following three parts:

**First**, the steam at 100.0 °C must be condensed into liquid water at 100.0 °C. The amount of heat removed in the condensation process is given by \( Q = mL_v \) (Equation 12.5), where \( m \) is the mass of the steam and \( L_v = 22.6 \times 10^5 \) J/kg is the latent heat of vaporization of water (see Table 12.3).

**Second**, the hot water at 100.0 °C must be cooled to 0.0 °C. The amount of heat removed in the cooling process is \( |Q| = cm|\Delta T| \) (Equation 12.4), where \( c = 4186 \) J/(kg·C°) is the specific heat capacity of liquid water (see Table 12.2). The vertical bars denote the absolute magnitude or magnitude of the quantity involved.

**Third**, the liquid water at 0.0 °C must be frozen into ice at 0.0 °C. The amount of heat removed in the freezing process is given by \( Q = mL_f \) (Equation 12.5), where \( L_f = 33.5 \times 10^4 \) J/kg is the latent heat of fusion of water (see Table 12.3).

**SOLUTION** The principle of conservation of energy indicates that

\[
\frac{mL_v}{c} + \frac{cm|\Delta T|}{c} + \frac{mL_f}{c} = \frac{1}{2}mv^2
\]

Note that the same mass \( m \) appears in each term of this equation and can be eliminated algebraically. Solving for the speed \( v \) reveals that

\[
v = \sqrt{2 \left(L_v + c|\Delta T| + L_f \right)}
\]

\[
= \sqrt{2 \left(22.6 \times 10^5 \frac{J}{kg} + 4186 \frac{J}{kg \cdot C°}(100.0 \ C°) + 33.5 \times 10^4 \frac{J}{kg}\right)}
\]

\[
= 2.46 \times 10^3 \ m/s
\]
65. **REASONING** Since there is no heat lost or gained by the system, the heat lost by the coffee in cooling down must be equal to the heat gained by the ice as it melts plus the heat gained by the melted water as it subsequently heats up. The heat $Q$ lost or gained by a substance is given by Equation 12.4 as $Q = cm\Delta T$, where $c$ is the specific heat capacity (see Table 12.2), $m$ is the mass, and $\Delta T$ is the change in temperature. The heat that is required to change ice at 0 °C into liquid water at 0 °C is given by Equation 12.5 as $Q = m_{\text{ice}}L_f$, where $m_{\text{ice}}$ is the mass of ice and $L_f$ is the latent heat of fusion for water (see Table 12.3).

**SOLUTION** We have that

\[
\frac{m_{\text{coffee}}c_{\text{coffee}}\Delta T_{\text{coffee}}}{\text{Heat lost by coffee}} = \frac{m_{\text{ice}}L_f + m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}}}{\text{Heat gained by ice and liquid water}}
\]

The mass of the coffee can be expressed in terms of its density as $m_{\text{coffee}} = \rho_{\text{coffee}}V_{\text{coffee}}$ (Equation 11.1). The change in temperature of the coffee is $\Delta T_{\text{coffee}} = 85 \, ^\circ\text{C} - T$, where $T$ is the final temperature of the coffee. The change in temperature of the water is $\Delta T_{\text{water}} = T - 0 \, ^\circ\text{C}$. With these substitutions, the equation above becomes

\[
\rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}}(85 \, ^\circ\text{C} - T) = m_{\text{ice}}L_f + m_{\text{water}}c_{\text{water}}(T - 0 \, ^\circ\text{C})
\]

Solving this equation for the final temperature gives

\[
T = \frac{-m_{\text{ice}}L_f + \rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}}(85 \, ^\circ\text{C})}{\rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}} + m_{\text{ice}}c_{\text{water}}}
\]

\[
= \frac{-2(11 \times 10^{-3} \, \text{kg})(3.35 \times 10^5 \, \text{J/kg}) + (1.0 \times 10^3 \, \text{kg/m}^3)(150 \times 10^{-6} \, \text{m}^3)[4186 \, \text{J/(kg \cdot ^\circ\text{C})}](85 \, ^\circ\text{C})}{(1.0 \times 10^3 \, \text{kg/m}^3)(150 \times 10^{-6} \, \text{m}^3)[4186 \, \text{J/(kg \cdot ^\circ\text{C})}] + (2)(11 \times 10^{-3} \, \text{kg})[4186 \, \text{J/(kg \cdot ^\circ\text{C})}]}
\]

\[
= 64 \, ^\circ\text{C}
\]

66. **REASONING** The droplets of water in the air give up an amount of heat $Q_w$ while cooling to the freezing point, and an amount $Q_t$ while freezing into snow. The snow loses a further amount of heat $Q_s$ as it cools from the freezing point to the air temperature. The total amount of heat transferred to the air as the lake water is transformed into a blanket of snow is the sum of the heat transferred during all three processes:

\[Q_{\text{total}} = Q_w + Q_t + Q_s \quad (1)\]

The heat lost by the cooling water droplets is found from $Q_w = c_w m\Delta T_w$ (Equation 12.4), where $m$ is the total mass of water sprayed into the air in a minute, $c_w$ is the specific heat capacity of water, and $\Delta T_w = 12.0 \, ^\circ\text{C}$ is the difference between the temperature of the lake water (12.0 °C) and the freezing point (0 °C). For the freezing process, the heat loss is given
by \( Q_f = mL_f \) (Equation 12.5), where \( L_f \) is the latent heat of fusion of water (see Table 12.3 in the text). Lastly, the heat lost by the snow as it cools to the air temperature is given by \( Q_s = c_s m\Delta T_s \) (Equation 12.4), where \( \Delta T_s = 7.0 \) °C is the higher initial temperature of the snow (0.0 °C, the freezing point) minus the lower final temperature (−7.0 °C, air temperature), and \( c_s \) is the specific heat capacity of snow.

**SOLUTION** Substituting \( Q_w = c_w m\Delta T_w \) (Equation 12.4), \( Q_f = mL_f \) (Equation 12.5), and \( Q_s = c_s m\Delta T_s \) (Equation 12.4) into Equation (1) yields

\[
Q_{\text{total}} = Q_w + Q_f + Q_s = c_w m\Delta T_w + mL_f + c_s m\Delta T_s = m\left( c_w \Delta T_w + L_f + c_s \Delta T_s \right)
\]

The snow maker pumps and sprays lake water at a rate of 130 kg per minute, so the mass \( m \) of the water turned to snow in one minute is \( m = 130 \) kg. Equation (2) then yields the total amount of heat transferred to the air each minute:

\[
Q = (130 \text{ kg}) \left[ 4186 \text{ J/} (\text{kg} \cdot \text{C}^\circ) \right] (12.0 \text{ C}^\circ) + 33.5 \times 10^4 \text{ J/kg} + \left[ 2.00 \times 10^3 \text{ J/} (\text{kg} \cdot \text{C}^\circ) \right] (7.0 \text{ C}^\circ)
\]

\[
= 5.2 \times 10^7 \text{ J}
\]

67. [SSM] **REASONING** All of the heat generated by friction goes into the ice, so this heat provides the heat needed for melting to occur. Since the surface on which the block slides is horizontal, the gravitational potential energy does not change, and energy conservation dictates that the heat generated by friction equals the amount by which the kinetic energy decreases or \( Q_{\text{Friction}} = \frac{1}{2} Mv_0^2 - \frac{1}{2} Mv^2 \), where \( v_0 \) and \( v \) are, respectively, the initial and final speeds and \( M \) is the mass of the block. In reality, the mass of the block decreases as the melting proceeds. However, only a very small amount of ice melts, so we may consider \( M \) to be essentially constant at its initial value. The heat \( Q \) needed to melt a mass \( m \) of water is given by Equation 12.5 as \( Q = mL_f \), where \( L_f \) is the latent heat of fusion. Thus, by equating \( Q_{\text{Friction}} \) to \( mL_f \) and solving for \( m \), we can determine the mass of ice that melts.

**SOLUTION** Equating \( Q_{\text{Friction}} \) to \( mL_f \) and solving for \( m \) gives

\[
Q_{\text{Friction}} = \frac{1}{2} Mv_0^2 - \frac{1}{2} Mv^2 = mL_f
\]

\[
m = \frac{M \left( v_0^2 - v^2 \right)}{2L_f} = \frac{(42 \text{ kg}) \left[ (7.3 \text{ m/s})^2 - (3.5 \text{ m/s})^2 \right]}{2 \left( 33.5 \times 10^4 \text{ J/kg} \right)} = 2.6 \times 10^{-3} \text{ kg}
\]

We have taken the value for the latent heat of fusion for water from Table 12.3.
68. **REASONING** When the minimum mass $m_w$ of water is used to solidify the molten gold, the heat $Q_g$ lost by the gold is exactly the amount needed to raise the temperature of the water to the boiling point and then convert the water into steam. If the mass of the water is smaller than $m_w$, all of the water will be converted to steam before the gold has entirely solidified. The water first gains an amount of heat $Q_1 = c_w m_w \Delta T$ (Equation 12.4) when it is raised to the boiling point ($100.0 \, ^\circ \text{C}$), where $c_w = 4186 \, \text{J/(kg} \cdot \text{C}^\circ)$ is the specific heat capacity of water (see Table 12.2) and $\Delta T$ is the change in its temperature. When the boiling water is converted to steam, it absorbs a further amount of heat $Q_2 = m_w L_w$ (Equation 12.5), where $L_w = 22.6 \times 10^5 \, \text{J/kg}$ is the latent heat of vaporization of water (see Table 12.3). The total amount of heat given off by the molten gold as it solidifies is found from $Q_g = m_g L_g$ (Equation 12.5), where $m_g$ is the mass of the gold and $L_g = 6.28 \times 10^4 \, \text{J/kg}$ is the latent heat of fusion of gold (see Table 12.3). The total amount of heat $Q_w$ gained by the water is equal to the heat lost by the gold, so we have that

$$Q_w = Q_1 + Q_2 = Q_g$$

**(SOLUTION)** Substituting $Q_1 = c_w m_w \Delta T$ (Equation 12.4), $Q_2 = m_w L_w$ (Equation 12.5), and $Q_g = m_g L_g$ (Equation 12.5) into Equation (1), we obtain

$$c_w m_w \Delta T + m_w L_w = m_g L_g$$

Solving Equation (2) for $m_w$ yields

$$m_w \left( c_w \Delta T + L_w \right) = m_g L_g \quad \text{or} \quad m_w = \frac{m_g L_g}{c_w \Delta T + L_w}$$

Therefore,

$$m_w = \frac{(0.180 \, \text{kg}) \left(6.28 \times 10^4 \, \text{J/kg} \right)}{4186 \, \text{J/(kg} \cdot \text{C}^\circ) \left(100.0 \, ^\circ \text{C} - 23.0 \, ^\circ \text{C} \right) + 22.6 \times 10^5 \, \text{J/kg}} = 4.38 \times 10^{-3} \, \text{kg}$$

69. **SSM REASONING** The system is comprised of the unknown material, the glycerin, and the aluminum calorimeter. From the principle of energy conservation, the heat gained by the unknown material is equal to the heat lost by the glycerin and the calorimeter. The heat gained by the unknown material is used to melt the material and then raise its temperature from the initial value of $-25.0 \, ^\circ \text{C}$ to the final equilibrium temperature of $T_{eq} = 20.0 \, ^\circ \text{C}$.

**SOLUTION**
Taking values for the specific heat capacities of glycerin and aluminum from Table 12.2, we have

\[
(0.10 \text{ kg})L_f + [160 \text{ J/(kg·C°)}](0.10 \text{ kg})(45.0 \text{ C°}) = [2410 \text{ J/(kg·C°)}](0.100 \text{ kg})(7.0 \text{ C°})
\]
\[+ [9.0 \times 10^2 \text{ J/(kg·C°)}](0.150 \text{ kg})(7.0 \text{ C°})
\]

Solving for \( L_f \) yields,

\[
L_f = \frac{1.9 \times 10^4 \text{ J/kg}}{}
\]

70. **REASONING** The amount of heat released in the condensation process is given by \( Q = mL_v \) (Equation 12.5), where \( m \) is the mass of the water vapor and \( L_v = 22.6 \times 10^5 \text{ J/kg} \) is the latent heat of vaporization of water (see Table 12.3). We note that the mass of the water vapor is not given. However, \( m \) is also the mass of the liquid water that falls as rain. Therefore, it can be calculated from \( m = \rho V \) (Equation 11.1), where \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \) (see Table 11.1) is the density of liquid water and \( V \) is the volume of the liquid water.

**SOLUTION**

a. Using \( Q = mL_v \) (Equation 12.5) and the fact that \( m = \rho V \) (Equation 11.1), where \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \) (see Table 11.1) is the density of liquid water and \( V \) is the volume of the liquid water, we find that

\[
Q = \rho V L_v
\]

In this result, the volume of the rain water is the depth of the rainfall times the area over which the rain occurs. Thus, we have

\[
Q = \rho V L_v
= \left(1.00 \times 10^3 \text{ kg/m}^3\right)
\left(0.0254 \text{ m}\right)
\left(2.59 \times 10^6 \text{ m}^2\right)
\left(22.6 \times 10^5 \text{ J/kg}\right)
= 1.49 \times 10^{14} \text{ J}
\]

b. The number of homes that could by heated for a year with this energy is

\[
\text{Number of homes} = \frac{1.49 \times 10^{14} \text{ J}}{1.50 \times 10^{11} \text{ J/home}} = 993 \text{ homes}
\]
71. REASONING  In order to melt, the bullet must first heat up to 327.3 °C (its melting point) and then undergo a phase change. According to Equation 12.4, the amount of heat necessary to raise the temperature of the bullet to 327.3 °C is \( Q = cm(327.3 \, ^\circ C - 30.0 \, ^\circ C) \), where \( m \) is the mass of the bullet. The amount of heat required to melt the bullet is given by \( Q_{\text{melt}} = mL_f \), where \( L_f \) is the latent heat of fusion of lead.

The lead bullet melts completely when it comes to a sudden halt; all of the kinetic energy of the bullet is converted into heat; therefore,

\[
\frac{1}{2}mv^2 = cm(327.3 \, ^\circ C - 30.0 \, ^\circ C) + mL_f
\]

The value for the specific heat \( c \) of lead is given in Table 12.2, and the value for the latent heat of fusion \( L_f \) of lead is given in Table 12.3. This expression can be solved for \( v \), the minimum speed of the bullet for such an event to occur. Note that the mass \( m \) of the bullet appears in each term on both sides of the expression and, therefore, is eliminated algebraically from the solution.

**SOLUTION**  Solving for \( v \), we find that the minimum speed of the lead bullet is

\[
v = \sqrt{2L_f + 2c (327.3 \, ^\circ C - 30.0 \, ^\circ C)}
\]

\[
v = \sqrt{2(2.32 \times 10^4 \, J/kg) + 2[128 \, J/(kg \cdot ^\circ C)](327.3 \, ^\circ C - 30.0 \, ^\circ C)} = 3.50 \times 10^2 \, m/s
\]

72. REASONING  To freeze either liquid, heat must be removed to cool the liquid to its freezing point. In either case, the heat \( Q \) that must be removed to lower the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. The amount \( \Delta T \) by which the temperature is lowered is the initial temperature \( T_0 \) minus the freezing point temperature \( T \). Once the liquid has been cooled to its freezing point, additional heat must be removed to convert the liquid into a solid at the freezing point. The heat \( Q \) that must be removed to freeze a mass \( m \) of liquid into a solid is given by Equation 12.5 as \( Q = mL_f \) where \( L_f \) is the latent heat of fusion. The total heat to be removed, then, is the sum of that specified by Equation 12.4 and that specified by Equation 12.5, or \( Q_{\text{Total}} = cm \, (T_0 - T) + mL_f \). Since we know that same amount of heat is removed from each liquid, we can set \( Q_{\text{Total}} \) for liquid A equal to \( Q_{\text{Total}} \) for liquid B and solve the resulting equation for \( L_{f, A} - L_{f, B} \).

**SOLUTION**  Setting \( Q_{\text{Total}} \) for liquid A equal to \( Q_{\text{Total}} \) for liquid B gives
\[ c_A m (T_0 - T_A) + mL_{f, A} = c_B m (T_0 - T_B) + mL_{f, B} \]

Noting that the mass \( m \) can be eliminated algebraically from this result and solving for \( L_{f, A} - L_{f, B} \), we find

\[
L_{f, A} - L_{f, B} = c_B (T_0 - T_B) - c_A (T_0 - T_A)
\]

\[
= \left[ 2670 \text{ J/(kg} \cdot \text{C}) \right] \left[ 25.0 \text{ °C} - (-96.0 \text{ °C}) \right]
\]

\[
- \left[ 1850 \text{ J/(kg} \cdot \text{C}) \right] \left[ 25.0 \text{ °C} - (-68.0 \text{ °C}) \right] = 1.51 \times 10^5 \text{ J/kg}
\]

73. **REASONING** The amount \( Q \) of heat required to melt an iceberg at \( 0 \text{ °C} \) is equal to \( mL_f \), where \( m \) is its mass and \( L_f \) is the latent heat of fusion for water (see Table 12.3). The mass is related to the density \( \rho \) and the volume \( V \) of the ice by Equation 11.1, \( m = \rho V \).

**SOLUTION**

a. The amount of heat required to melt the iceberg is

\[ Q = mL_f = \rho V L_f \]  

(12.5)

\[
= \left( \frac{917 \text{ kg/m}^3}{120 \times 10^3 \text{ m}} \right) \left( \frac{35 \times 10^3 \text{ m}}{230 \text{ m}} \right) \left( 3.35 \times 10^5 \text{ J/kg} \right) 
\]

\[ = 3.0 \times 10^{20} \text{ J} \]

b. The number of years it would take to melt the iceberg is equal to the energy required to melt it divided by the energy consumed per year by the U.S.

\[ \text{Number of years} = \frac{3.0 \times 10^{20} \text{ J}}{1.1 \times 10^{30} \text{ J/y}} = 2.7 \text{ years} \]

74. **REASONING** Boiling occurs when the vapor pressure of the substance at a particular temperature equals the atmospheric pressure at that temperature.

**SOLUTION** According to the vapor pressure curve for carbon dioxide that accompanies the problem statement, a temperature of \( 20 \text{ °C} \) corresponds to a vapor pressure of \( 5.5 \times 10^6 \text{ Pa} \). Therefore, we can conclude that an atmospheric pressure of \( 5.5 \times 10^6 \text{ Pa} \) would be required for carbon dioxide to boil at a temperature of \( 20 \text{ °C} \).

75. **SSM** **REASONING** The definition of percent relative humidity is given by Equation 12.6 as follows:
Percent relative humidity = \( \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temperature}} \) \times 100

Using \( R \) to denote the percent relative humidity, \( P \) to denote the partial pressure of water vapor, and \( P_V \) to denote the equilibrium vapor pressure of water at the existing temperature, we can write Equation 12.6 as

\[ R = \frac{P}{P_V} \times 100 \]

The partial pressure of water vapor \( P \) is the same at the two given temperatures. The relative humidity is not the same at the two temperatures, however, because the equilibrium vapor pressure \( P_V \) is different at each temperature, with values that are available from the vapor pressure curve given with the problem statement. To determine the ratio \( R_{10}/R_{40} \), we will apply Equation 12.6 at each temperature.

**SOLUTION** Using Equation 12.6 and reading the values of \( P_{V,10} \) and \( P_{V,40} \) from the vapor pressure curve given with the problem statement, we find

\[
\frac{R_{10}}{R_{40}} = \frac{P/P_{V,10}}{P/P_{V,40}} = \frac{P_{V,40}}{P_{V,10}} = \frac{7200 \text{ Pa}}{1300 \text{ Pa}} = \frac{5.5}{1}
\]

76. **REASONING AND SOLUTION** From the vapor pressure curve that accompanies Problem 75, it is seen that the partial pressure of water vapor in the atmosphere at 10 °C is about 1400 Pa, and that the equilibrium vapor pressure at 30 °C is about 4200 Pa. The relative humidity is, from Equation 12.6,

\[
\text{Percent relative humidity} = \left( \frac{1400 \text{ Pa}}{4200 \text{ Pa}} \right) \times 100 = 33\%
\]

77. **REASONING** The definition of relative humidity is given by Equation 12.6 as:

\[
\text{Percent relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temperature}} \times 100
\]

The partial pressure of water vapor is given in the problem statement. The equilibrium vapor pressure can be found by consulting the vapor pressure curve for water that accompanies Problem 75.

**SOLUTION** By using the vapor pressure curve for water given in Problem 75, we estimate that at a temperature of 37 °C the water vapor in the lungs has an equilibrium vapor pressure of \( 6.3 \times 10^3 \) Pa. The relative humidity is, then,
Percent relative humidity = \( \frac{5.5 \times 10^3 \text{ Pa}}{6.3 \times 10^3 \text{ Pa}} \times 100 = 87\% \)

78. **REASONING** We will rely on Equation 12.6, which defines what is meant by “percent relative humidity:”

\[
\text{Percent relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temperature}} \times 100 \quad (12.6)
\]

Note especially that the denominator on the right in this equation is *not* the total atmospheric air pressure. Thus, if the partial pressure of water vapor in the air remains the same and an increase in temperature causes the equilibrium vapor pressure of water to increase, the percent relative humidity will decrease.

**SOLUTION**

a. The percentage of atmospheric pressure is

\[
\left( \frac{1300 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) \times 100\% = 1.3\%
\]

b. The percentage is not 100%. The percentage is that determined in part a, namely, 1.3%.

c. The relative humidity at 35 °C is

\[
\left( \frac{1300 \text{ Pa}}{5500 \text{ Pa}} \right) \times 100\% = 24\%
\quad (12.6)
\]

As expected, this value is less than the 100% value at 10 °C.

79. **REASONING** To bring the water to the point where it just begins to boil, its temperature must be increased to the boiling point. The heat \( Q \) that must be added to raise the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. The amount \( \Delta T \) by which the temperature changes is the boiling temperature minus the initial temperature of 100.0 °C. The boiling temperature is the temperature at which the vapor pressure of the water equals the external pressure of \( 3.0 \times 10^5 \text{ Pa} \) and can be read from the vapor pressure curve for water given in Figure 12.32.

**SOLUTION** Using Equation 12.4, with \( T_{BP} \) being the boiling temperature and \( T_0 \) being the initial temperature, we have
According to Figure 12.32, an external pressure of \(3.0 \times 10^5\) Pa corresponds to a boiling point temperature of \(T_{BP} = 134\) °C. Using this value in Equation 12.4 and taking the specific heat capacity for water from Table 12.2, we determine the heat to be

\[
Q = cm(T_{BP} - T_0) = [4186 \text{ J/(kg·C°)}](2.0 \text{ kg})(134 \text{ °C} - 100.0 \text{ °C}) = 2.8 \times 10^5 \text{ J}
\]

80. **REASONING AND SOLUTION** Equation 12.6 defines the relative humidity as

\[
\text{Percent relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temperature}} \times 100
\]

According to the vapor pressure curve that accompanies Problem 75, the equilibrium vapor pressure of water at 36°C is \(5.8 \times 10^3\) Pa. Since the water condenses on the coils when the temperature of the coils is 30 °C, the relative humidity at 30 °C is 100 percent. From Equation 12.6 this implies that the partial pressure of water vapor in the air must be equal to the equilibrium vapor pressure of water at 30 °C. From the vapor pressure curve, this pressure is \(4.4 \times 10^3\) Pa. Thus, the relative humidity in the room is

\[
\text{Percent relative humidity} = \frac{4.4 \times 10^3 \text{ Pa}}{5.8 \times 10^3 \text{ Pa}} \times 100 = 76\%
\]

81. **SSM REASONING** Since her glasses (at 10 °C) steam up when she enters the room, the partial pressure of water vapor in the air in the room must be greater or equal to the vapor pressure of water at 10 °C. At 10 °C the equilibrium vapor pressure is 1250 Pa (see the vapor pressure curve for water that accompanies Problem 75). The problem asks for the smallest possible value of the relative humidity of the room. Therefore, we take the partial pressure of water vapor in the air in the room to be 1250 Pa.

**SOLUTION** According to Equation 12.6, the relative humidity is

\[
\text{Percent relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temperature}} \times 100
\]

At 25 °C, the equilibrium vapor pressure is 3200 Pa (see the vapor pressure curve for water that accompanies Problem 75). Using Equation 12.6 we find for the smallest possible relative humidity that
Partial pressure of water vapor 1250 Pa
Percent relative humidity 100 100 39%
Equilibrium vapor pressure of 3200 Pa
water at the existing temperature

82. **REASONING AND SOLUTION** The pressure inside the container is due to the weight on the piston in addition to the pressure of the atmosphere. The 120-kg mass produces a pressure of

\[ P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(120 \text{ kg})(9.80 \text{ m/s}^2)}{\pi (0.061 \text{ m})^2} = 1.0 \times 10^5 \text{ Pa} \]

If we include atmospheric pressure (1.01 \times 10^5 \text{ Pa}), the total pressure inside the container is

\[ P_{\text{total}} = P + P_{\text{atm}} = 2.0 \times 10^5 \text{ Pa} \]

Examination of the vaporization curve for water in Figure 12.32 shows that the temperature corresponding to this pressure at equilibrium is \( T = 120 \degree \text{C} \).

83. **REASONING** We must first find the equilibrium temperature \( T_{\text{eq}} \) of the iced tea. Once this is known, we can use the vapor pressure curve that accompanies Problem 75 to find the partial pressure of water vapor at that temperature and then estimate the relative humidity using Equation 12.6.

According to the principle of energy conservation, when the ice is mixed with the tea, the heat lost by the tea is gained by the ice, or \( Q_{\text{tea}} = Q_{\text{ice}} \). The heat gained by the ice is used to melt the ice at 0.0 \degree \text{C}; the remainder of the heat is used to bring the water at 0.0 \degree \text{C} up to the final equilibrium temperature \( T_{\text{eq}} \).

**SOLUTION**

\[ Q_{\text{tea}} = Q_{\text{ice}} \]

\[ c_{\text{water}} m_{\text{tea}} (30.0 \degree \text{C} - T_{\text{eq}}) = m_{\text{ice}} L_f + c_{\text{water}} m_{\text{ice}} (T_{\text{eq}} - 0.00 \degree \text{C}) \]

The specific heat capacity of water is given in Table 12.2, and the latent heat of fusion \( L_f \) of water is given in Table 12.3. Solving for \( T_{\text{eq}} \), we have

\[ T_{\text{eq}} = \frac{c_{\text{water}} m_{\text{tea}} (30.0 \degree \text{C}) - m_{\text{ice}} L_f}{c_{\text{water}} (m_{\text{tea}} + m_{\text{ice}})} = \frac{[4186 \text{ J/(kg}\cdot\degree\text{C})] (0.300 \text{ kg})(30.0\degree\text{C}) - (0.0670 \text{ kg})(33.5 \times 10^4 \text{ J/kg})}{[4186 \text{ J/(kg}\cdot\degree\text{C})] (0.300 \text{ kg} + 0.0670 \text{ kg})} = 9.91 \degree \text{C} \]

According to the vapor pressure curve that accompanies Problem 75, at a temperature of 9.91 \degree \text{C}, the equilibrium vapor pressure is approximately 1250 Pa. At 30 \degree \text{C}, the
equilibrium vapor pressure is approximately 4400 Pa. Therefore, according to Equation 12.6, the percent relative humidity is approximately

\[
\text{Percent relative humidity} = \left( \frac{1250 \text{ Pa}}{4400 \text{ Pa}} \right) \times 100 = 28\%
\]

84. **REASONING AND SOLUTION** The water will boil if the vapor pressure of the water is equal to the ambient pressure. The pressure at a depth \( h \) in the water can be determined from Equation 11.4:

\[
P_2 = P_1 + \rho gh.
\]

When \( h = 10.3 \text{ m} \),

\[
P_2 = (1.01 \times 10^5 \text{ Pa}) + \left[ (1.000 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (10.3 \text{ m}) \right] = 2.02 \times 10^5 \text{ Pa}
\]

The vapor pressure curve in Figure 12.32 shows that the vapor pressure of water is equal to \( 2.02 \times 10^5 \text{ Pa} \) at a temperature of 123 °C. Thus, the water at that depth has a temperature of \( T = 123 \text{ °C} \).

85. **REASONING** The change in temperature is the final temperature \( T \) minus the initial temperature \( T_0 \), or \( \Delta T = T - T_0 \). Thus, \( T = T_0 + \Delta T \). When the bat is heated, its length changes by an amount given by Equation 12.2 as \( \Delta L = \alpha L_0 \Delta T \), where \( \alpha \) is the coefficient of linear expansion, and \( L_0 \) is the bat’s initial length. Solving this expression for \( \Delta T \) and substituting the result into \( T = T_0 + \Delta T \) will allow us to find the final temperature of the bat.

**SOLUTION** Solving \( \Delta L = \alpha L_0 \Delta T \) for the change in temperature and substituting the result into \( T = T_0 + \Delta T \) gives

\[
T = T_0 + \Delta T = T_0 + \frac{\Delta L}{\alpha L_0} = 17 \text{ °C} + \frac{0.00016 \text{ m}}{23 \times 10^{-6} \text{ (°C)}^{-1}} (0.86 \text{ m}) = 25 \text{ °C}
\]

86. **REASONING** As the body perspires, heat \( Q \) must be added to change the water from the liquid to the gaseous state. The amount of heat depends on the mass \( m \) of the water and the latent heat of vaporization \( L_v \), according to \( Q = m L_v \) (Equation 12.5).

**SOLUTION** The mass of water lost to perspiration is

\[
m = \frac{Q}{L_v} = \frac{(240 \text{ Calories}) \left( \frac{4186 \text{ J}}{1 \text{ Calorie}} \right)}{2.42 \times 10^6 \text{ J/kg}} = 0.42 \text{ kg}
\]
87. **REASONING** From the principle of conservation of energy, the heat lost by the coin must be equal to the heat gained by the liquid nitrogen. The heat lost by the silver coin is, from Equation 12.4, 

\[ Q = c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}} \]  

(see Table 12.2 for the specific heat capacity of silver). If the liquid nitrogen is at its boiling point, \(-195.8 \, ^\circ C\), then the heat gained by the nitrogen will cause it to change phase from a liquid to a vapor. The heat gained by the liquid nitrogen is \( Q = m_{\text{nitrogen}} L_v \), where \( m_{\text{nitrogen}} \) is the mass of liquid nitrogen that vaporizes, and \( L_v \) is the latent heat of vaporization for nitrogen (see Table 12.3).

**SOLUTION**

\[ Q_{\text{lost by coin}} = Q_{\text{gained by nitrogen}} \]

\[ c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}} = m_{\text{nitrogen}} L_v \]

Solving for the mass of the nitrogen that vaporizes

\[ m_{\text{nitrogen}} = \frac{c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}}}{L_v} \]

\[ = \frac{[235 \, \text{J/(kg} \cdot {^\circ} \text{C})](1.5 \times 10^{-2} \, \text{kg})[25 \, ^\circ C - (-195.8 \, ^\circ C)]}{2.00 \times 10^5 \, \text{J/kg}} = 3.9 \times 10^{-3} \, \text{kg} \]

88. **REASONING** Since the container is being ignored and since we are assuming negligible heat exchange with the environment, the principle of conservation of energy applies in the following form: **heat gained equals heat lost.** In reaching equilibrium the colder aluminum gains heat in warming to 0.0 \(^\circ C\), and the warmer water loses heat in cooling to 0.0 \(^\circ C\). In either case, the heat \( Q \) that must be supplied or removed to change the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. In using this equation as we apply the energy-conservation principle, we must remember to express the change in temperature \( \Delta T \) as the higher minus the lower temperature. The water that freezes into ice also loses heat. The heat \( Q \) lost when a mass \( m \) of water freezes is given by Equation 12.5 as \( Q = mL_f \), where \( L_f \) is the latent heat of fusion. By including this amount of lost heat in the energy-conservation equation, we will be able to calculate the mass of water that is frozen.

**SOLUTION** Using the energy-conservation principle and Equations 12.4 and 12.5 gives

\[ \frac{c_{\text{Aluminum}} m_{\text{Aluminum}} \Delta T_{\text{Aluminum}}}{\text{Heat gained by aluminum}} = \frac{c_{\text{Water}} m_{\text{Water}} \Delta T_{\text{Water}}}{\text{Heat lost by water}} + \frac{m_{\text{Ice}} L_f, \text{Water}}{\text{Heat lost by water that freezes}} \]

Solving for \( m_{\text{Ice}} \), taking values for the specific heat capacities from Table 12.2, and taking the latent heat for water from Table 12.3, we find that
\[ m_{\text{ice}} = \frac{c_{\text{Aluminum}} m_{\text{Aluminum}} \Delta T_{\text{Aluminum}} - c_{\text{Water}} m_{\text{Water}} \Delta T_{\text{Water}}}{L_{c, \text{Water}}} \]

\[ = \frac{[9.00 \times 10^2 \text{ J/(kg} \cdot \text{C})](0.200 \text{ kg})(0.0 \text{ C} - (-155 \text{ C}))}{33.5 \times 10^4 \text{ J/kg}} \]

\[ - \frac{[4186\text{ J/(kg} \cdot \text{C})](1.5 \text{ kg})(3.0 \text{ C} - 0.0 \text{ C})}{33.5 \times 10^4 \text{ J/kg}} = 0.027 \text{ kg} \]

89. **REASONING** The disk will fall into the pipe when it reaches a temperature \( T_0 \) at which its diameter is equal to that of the inner diameter of the pipe. At the higher temperature of \( T = 85 \text{ C} \), the diameter of the disk is \( \Delta L = 3.9 \times 10^{-5} \text{ m} \) greater than the diameter of the pipe. We will use \( \Delta L = \alpha L_0 \Delta T \) (Equation 12.2) to determine the required change in temperature \( \Delta T \) for this to occur. The lower temperature \( T_0 \) will then be the higher temperature minus the difference \( \Delta T \) between the higher and lower temperatures:

\[ T_0 = T - \Delta T \]  

(1)

**SOLUTION** Solving Equation 12.2 for the change \( \Delta T \) in temperature, we obtain

\[ \Delta T = \frac{\Delta L}{\alpha L_0} \]  

(2)

Substituting Equation (2) into Equation (1) yields the final temperature of the aluminum disk:

\[ T_0 = T - \frac{\Delta L}{\alpha L_0} = 85 \text{ C} - \frac{3.9 \times 10^{-5} \text{ m}}{23 \times 10^{-6} (\text{C}^{-1})(0.065 \text{ m} + 3.9 \times 10^{-5} \text{ m})} = 59 \text{ C} \]

We have taken the value for \( \alpha \) (the coefficient of thermal expansion) for aluminum from Table 12.1 in the text.

90. **REASONING** In order to find the change in the temperature, we must first determine the higher temperature \( T_1 \) and the lower temperature \( T_2 \). Each temperature corresponds to a different equilibrium vapor pressure \( P_V \) of water on the curve given in Problem 75. We will employ the definition of percent relative humidity (denoted by \( R \)) in order to find the equilibrium vapor pressures that will allow us to locate the temperatures \( T_1 \) and \( T_2 \) on this curve:

\[ \text{Percent relative humidity} = R = \frac{P}{P_V} \times 100 \]  

(12.6)
In Equation 12.6, the quantity \( P \) denotes the actual partial pressure of water vapor in the atmosphere. The dew point is the temperature at which the equilibrium water vapor pressure equals \( P \). The dew point does not change, so the actual partial pressure \( P \) of water vapor in the atmosphere also remains unchanged. The vapor pressure curve of Problem 75 shows that the dew point is 14 °C when \( P = 1800 \text{ Pa} \). Solving Equation 12.6 for the equilibrium vapor pressure \( P_V \) of water at the existing temperature, we obtain

\[
P_V = \frac{P}{R} \times 100
\]  

**SOLUTION** When the relative humidity is \( R_1 = 50.0\% \), then, Equation (1) gives the equilibrium vapor pressure of water \( P_{V1} \):

\[
P_{V1} = \frac{P}{R_1} \times 100 = \frac{1800 \text{ Pa}}{50.0} \times 100 = 3600 \text{ Pa}
\]

On the vapor pressure curve of Problem 75, this corresponds to a temperature of approximately \( T_1 = 26 \degree \text{C} \). Similarly, when the percent relative humidity is 69\%, the equilibrium vapor pressure of water is

\[
P_{V2} = \frac{P}{R_2} \times 100 = \frac{1800 \text{ Pa}}{69} \times 100 = 2600 \text{ Pa}
\]

Again making use of the vapor pressure curve of Problem 75, we find that the equilibrium vapor pressure of water is 2600 Pa at a temperature of approximately \( T_2 = 21 \degree \text{C} \). Therefore, the relative humidity increases from 50.0\% to 69\% when the temperature drops by

\[
T_1 - T_2 = 26 \degree \text{C} - 21 \degree \text{C} = 5 \degree \text{C}
\]

91. **SSM REASONING** The increase \( \Delta V \) in volume is given by Equation 12.3 as \( \Delta V = \beta V_0 \Delta T \), where \( \beta \) is the coefficient of volume expansion, \( V_0 \) is the initial volume, and \( \Delta T \) is the increase in temperature. The lead and quartz objects experience the same change in volume. Therefore, we can use Equation 12.3 to express the two volume changes and set them equal. We will solve the resulting equation for \( \Delta T_{Quartz} \):

**SOLUTION** Recognizing that the lead and quartz objects experience the same change in volume and expressing that change with Equation 12.3, we have

\[
\frac{\beta_{\text{Lead}} V_0 \Delta T_{\text{Lead}}}{\Delta V_{\text{Lead}}} = \frac{\beta_{\text{Quartz}} V_0 \Delta T_{\text{Quartz}}}{\Delta V_{\text{Quartz}}}
\]

In this result \( V_0 \) is the initial volume of each object. Solving for \( \Delta T_{\text{Quartz}} \) and taking values for the coefficients of volume expansion for lead and quartz from Table 12.1 gives
92. **REASONING**  According to Equation 12.4, the heat required to warm the pool can be calculated from \( Q = cm\Delta T \). The specific heat capacity \( c \) of water is given in Table 12.2. In order to use Equation 12.4, we must first determine the mass of the water in the pool. Equation 11.1 indicates that the mass can be calculated from \( m = \rho V \), where \( \rho \) is the density of water and \( V \) is the volume of water in the pool.

**SOLUTION**  Combining these two expressions, we have \( Q = cm\Delta T \), or

\[
Q = \frac{4186 \text{ J/(kg} \cdot \text{C})}{1.00 \times 10^3 \text{ kg/m}^3} (12.0 \text{ m} \times 9.00 \text{ m} \times 1.5 \text{ m}) (27 \text{ °C} - 15 \text{ °C}) = 8.14 \times 10^9 \text{ J}
\]

Using the fact that 1 kWh = 3.6 \times 10^6 J, the cost of using electrical energy to heat the water in the pool at a cost of $0.10 per kWh is

\[
(8.14 \times 10^9 \text{ J}) \left( \frac{\$0.10}{3.6 \times 10^6 \text{ J}} \right) = \$230
\]

93. **REASONING**  Each section of concrete expands as the temperature increases by an amount \( \Delta T \). The amount of the expansion \( \Delta L \) is proportional to the initial length of the section, as indicated by Equation 12.2. Thus, to find the total expansion of the three sections, we can apply this expression to the total length of concrete, which is \( L_0 = 3(2.4 \text{ m}) \). Since the two gaps in the drawing are identical, each must have a minimum width that is one half the total expansion.

**SOLUTION**  Using Equation 12.2 and taking the value for the coefficient of thermal expansion for concrete from Table 12.1, we find

\[
\Delta L = \alpha L_0 \Delta T = \left[ 12 \times 10^{-6} \text{ (C}^{-1}) \right] \left[ 3(2.4 \text{ m}) \right] (32 \text{ C}^o)
\]

The minimum necessary gap width is one half this value or

\[
\frac{1}{2} \left[ 12 \times 10^{-6} \text{ (C}^{-1}) \right] \left[ 3(2.4 \text{ m}) \right] (32 \text{ C}^o) = 1.4 \times 10^{-3} \text{ m}
\]

94. **REASONING**  The relationship between the pressure \( P \) and temperature \( T \) of a gas kept at a constant volume is illustrated by the pressure-versus-temperature graph in Figure 12.4. The pressures and temperatures at any two points on this graph can be used to calculate the slope \( \Delta P/\Delta T \) of the graph. If we choose one of the points to be the absolute zero of temperature measurement, then the slope can be expressed as
\[
\frac{\Delta P}{\Delta T} = \frac{P - P_0}{T - T_0}
\]

(1)

where \(P\) and \(T\) are the pressure and temperature, respectively, of any given point on the graph, and \(P_0 = 0\) Pa, \(T_0 = -273.15\)° correspond to the pressure and temperature of gas at the absolute zero of temperature measurement. We will use the given data in Equation (1) to determine the slope of the graph from the temperature \(T_1 = 0.00\) °C at the higher pressure \(P_1 = 5.00 \times 10^3\) Pa. Once the slope \(\Delta P/\Delta T\) is known, we can use Equation (1) to calculate the temperature \(T_2\) when the gas is at the lower pressure \(P_2 = 2.00 \times 10^3\) Pa.

**SOLUTION** Substituting \(P_0 = 0\) Pa into Equation (1) yields

\[
\frac{\Delta P}{\Delta T} = \frac{P - (0\text{ Pa})}{T - T_0} = \frac{P}{T - T_0}
\]

(2)

The slope is the same, no matter which pair of points is used to calculate it, so from Equation (2) we have that

\[
\frac{\Delta P}{\Delta T} = \frac{P_1}{T_1 - T_0} = \frac{P_2}{T_2 - T_0}
\]

(3)

Solving Equation (3) for \(T_2\), we obtain

\[P_1(T_2 - T_0) = P_2(T_1 - T_0) \quad \text{or} \quad T_2 - T_0 = \frac{P_2}{P_1}(T_1 - T_0) \quad \text{or} \quad T_2 = \frac{P_2}{P_1}(T_1 - T_0) + T_0\]

Therefore, the temperature \(T_2\) is

\[T_2 = \left(\frac{2.00 \times 10^3\text{ Pa}}{5.00 \times 10^3\text{ Pa}}\right)[0.00\text{ °C} - (-273.15\text{ °C})] + (-273.15\text{ °C}) = -164\text{ °C}\]

95. **SSM REASONING** According to the statement of the problem, the initial state of the system is comprised of the ice and the steam. From the principle of energy conservation, the heat lost by the steam equals the heat gained by the ice, or \(Q_{\text{steam}} = Q_{\text{ice}}\). When the ice and the steam are brought together, the steam immediately begins losing heat to the ice. An amount \(Q_{1(\text{lost})}\) is released as the temperature of the steam drops from 130 °C to 100 °C, the boiling point of water. Then an amount of heat \(Q_{2(\text{lost})}\) is released as the steam condenses into liquid water at 100 °C. The remainder of the heat lost by the "steam" \(Q_{3(\text{lost})}\) is the heat
that is released as the water at 100 °C cools to the equilibrium temperature of $T_{eq} = 50.0 \, ^\circ C$.

According to Equation 12.4, $Q_{1(\text{lost})}$ and $Q_{3(\text{lost})}$ are given by

$$Q_{1(\text{lost})} = c_{\text{steam}} m_{\text{steam}} (T_{\text{steam}} - 100.0 \, ^\circ C) \quad \text{and} \quad Q_{3(\text{lost})} = c_{\text{water}} m_{\text{steam}} (100.0 \, ^\circ C - T_{eq})$$

$Q_{2(\text{lost})}$ is given by $Q_{2(\text{lost})} = m_{\text{steam}} L_v$, where $L_v$ is the latent heat of vaporization of water.

The total heat lost by the steam has three effects on the ice. First, a portion of this heat $Q_{1(\text{gained})}$ is used to raise the temperature of the ice to its melting point at 0.00 °C. Then, an amount of heat $Q_{2(\text{gained})}$ is used to melt the ice completely (we know this because the problem states that after thermal equilibrium is reached the liquid phase is present at 50.0 °C). The remainder of the heat $Q_{3(\text{gained})}$ gained by the "ice" is used to raise the temperature of the resulting liquid at 0.0 °C to the final equilibrium temperature. According to Equation 12.4, $Q_{1(\text{gained})}$ and $Q_{3(\text{gained})}$ are given by

$$Q_{1(\text{gained})} = c_{\text{ice}} m_{\text{ice}} (0.00 \, ^\circ C - T_{\text{ice}}) \quad \text{and} \quad Q_{3(\text{gained})} = c_{\text{water}} m_{\text{ice}} (T_{eq} - 0.0 \, ^\circ C)$$

$Q_{2(\text{gained})}$ is given by $Q_{2(\text{gained})} = m_{\text{ice}} L_T$, where $L_T$ is the latent heat of fusion of ice.

**SOLUTION** According to the principle of energy conservation, we have

$$Q_{\text{steam}} = Q_{\text{ice}}$$

$$Q_{1(\text{lost})} + Q_{2(\text{lost})} + Q_{3(\text{lost})} = Q_{1(\text{gained})} + Q_{2(\text{gained})} + Q_{3(\text{gained})}$$

or

$$c_{\text{steam}} m_{\text{steam}} (T_{\text{steam}} - 100.0 \, ^\circ C) + m_{\text{steam}} L_v + c_{\text{water}} m_{\text{steam}} (100.0 \, ^\circ C - T_{eq})$$

$$= c_{\text{ice}} m_{\text{ice}} (0.00 \, ^\circ C - T_{\text{ice}}) + m_{\text{ice}} L_T + c_{\text{water}} m_{\text{ice}} (T_{eq} - 0.0 \, ^\circ C)$$

Values for specific heats are given in Table 12.2, and values for the latent heats are given in Table 12.3. Solving for the ratio of the masses gives

$$\frac{m_{\text{steam}}}{m_{\text{ice}}} = \frac{c_{\text{ice}} (0.00 \, ^\circ C - T_{\text{ice}}) + L_T + c_{\text{water}} (T_{eq} - 0.0 \, ^\circ C)}{c_{\text{steam}} (T_{\text{steam}} - 100.0 \, ^\circ C) + L_v + c_{\text{water}} (100.0 \, ^\circ C - T_{eq})}$$

$$= \frac{2.00 \times 10^3 \, \text{J/(kg} \cdot ^\circ \text{C})}{[0.0 \, ^\circ \text{C} - (10.0 \, ^\circ \text{C})] + 33.5 \times 10^4 \, \text{J/kg} + [4186 \, \text{J/(kg} \cdot ^\circ \text{C})]} (50.0 \, ^\circ \text{C} - 0.0 \, ^\circ \text{C})$$

$$\left[ \frac{2020 \, \text{J/(kg} \cdot ^\circ \text{C})}{(130 \, ^\circ \text{C} - 100.0 \, ^\circ \text{C}) + 22.6 \times 10^5 \, \text{J/kg} + [4186 \, \text{J/(kg} \cdot ^\circ \text{C})]} (100.0 \, ^\circ \text{C} - 50.0 \, ^\circ \text{C}) \right]$$

or
96. **REASONING** The mass \( m_{\text{remaining}} \) of the liquid water that remains at 100 °C is equal to the original mass \( m \) minus the mass \( m_{\text{vaporized}} \) of the liquid water that has been vaporized. The heat \( Q \) required to vaporize this mass of liquid is given by Equation 12.5 as \( Q = m L_v \), where \( L_v \) is the latent heat of vaporization for water. Thus, we have

\[
m_{\text{remaining}} = m - m_{\text{vaporized}} = m - \frac{Q}{L_v}
\]

The heat required to vaporize the water comes from the heat that is removed from the water at 0 °C when it changes phase from the liquid state to ice. This heat is also given by Equation 12.5 as \( Q = mL_f \) where \( L_f \) is the latent heat of fusion for water. Thus, the remaining mass of liquid water can be written as

\[
m_{\text{remaining}} = m - \frac{Q}{L_v} = m - \frac{mL_f}{L_v} = m \left(1 - \frac{L_f}{L_v}\right)
\]

**SOLUTION** Using the values of \( L_f \) and \( L_v \) from Table 12.3, we find that the mass of liquid water that remains at 100 °C is

\[
m_{\text{remaining}} = m \left(1 - \frac{L_f}{L_v}\right) = (2.00 \text{ g}) \left(1 - \frac{3.35 \times 10^5 \text{ J/kg}}{2.26 \times 10^6 \text{ J/kg}}\right) = 1.70 \text{ g}
\]

97. **REASONING** When heat \( Q \) is supplied to the silver bar, its temperature changes by an amount \( \Delta T \). The relation between \( Q \) and \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of silver and \( m \) is the mass. Solving this equation for \( m \) yields

\[
m = \frac{Q}{c \Delta T} \tag{1}
\]

When the temperature of the bar changes by an amount \( \Delta T \), the change \( \Delta L \) in its length is given by Equation 12.2 as \( \Delta L = \alpha L_0 \Delta T \). Solving this equation for \( \Delta T \) gives

\[
\Delta T = \frac{\Delta L}{\alpha L_0} \tag{2}
\]

where \( L_0 \) is the initial length and \( \alpha \) is the coefficient of linear expansion for silver. Substituting the expression for \( \Delta T \) in Equation (2) into Equation (1) gives

\[
m = \frac{Q}{c \Delta T} = \frac{Q}{c \left(\frac{\Delta L}{\alpha L_0}\right)} = \frac{\alpha Q L_0}{c \Delta L}
\]
**TEMPERATURE AND HEAT**

**SOLUTION** Taking the values for $\alpha$ and $c$ from Tables 12.1 and 12.2, respectively, the mass of the silver bar is

$$m = \frac{\alpha Q L_0}{c \Delta L} = \frac{[19 \times 10^{-6} \text{ (C$^{-1}$)}][4200 \text{ J}(0.15 \text{ m})]}{[235 \text{ J/(kg$\cdot$C)}][4.3 \times 10^{-3} \text{ m}]} = 1.2 \times 10^{-2} \text{ kg}$$

---

98. **REASONING** The rod contracts upon cooling, and we can use Equation 12.2 to express the change $\Delta T$ in temperature as

$$\Delta T = \frac{\Delta L}{\alpha L_0} \quad (1)$$

where $\Delta L$ and $L_0$ are the change in length and original length, respectively, and $\alpha$ is the coefficient of linear expansion for brass. Originally, the rod was stretched by the 860-N block that hangs from the lower end of the rod. This change in length can be found from Equation 10.17 as

$$\Delta L = \frac{F L_0}{Y A} \quad (2)$$

where $F$ is the magnitude of the stretching force, $Y$ is Young’s modulus for brass, and $A$ is the cross-sectional area of the rod. Substituting Equation (2) into Equation (1) and noting that $L_0$ is algebraically eliminated from the final result, we have

$$\Delta T = \frac{F L_0}{\alpha Y A} = \frac{F}{\alpha Y A}$$

(Note: The length $L_0$ that appears in Equation (1) is slightly larger than the length $L_0$ that appears in Equation (2). This difference is extremely small, however, so we assume that they are the same. Thus, the two lengths appearing in the equation above can be algebraically eliminated, as shown above.)

**SOLUTION** The stretching force $F$ is the weight of the block (860 N). Young’s modulus $Y$ for brass can be obtained from Table 10.1, and the coefficient of linear expansion $\alpha$ can be found in Table 12.1. Thus, the change in temperature of the brass rod is

$$\Delta T = \frac{F}{\alpha Y A} = \frac{860 \text{ N}}{[19 \times 10^{-6} \text{ (C$^{-1}$)}][9.0 \times 10^{10} \text{ N/m}^2][1.3 \times 10^{-5} \text{ m}^2]} = 39 \text{ C$^\circ$}$$

---

99. **SSM REASONING** When heat $Q$ is supplied to the block, its temperature changes by an amount $\Delta T$. The relation between $Q$ and $\Delta T$ is given by

$$Q = cm\Delta T \quad (12.4)$$
where \( c \) is the specific heat capacity and \( m \) is the mass. When the temperature of the block changes by an amount \( \Delta T \), the change \( \Delta V \) in its volume is given by Equation 12.3 as \( \Delta V = \beta V_0 \Delta T \), where \( \beta \) is the coefficient of volume expansion and \( V_0 \) is the initial volume of the block. Solving for \( \Delta T \) gives

\[
\Delta T = \frac{\Delta V}{\beta V_0}
\]

Substituting this expression for \( \Delta T \) into Equation 12.4 gives

\[
Q = cm\Delta T = cm\left(\frac{\Delta V}{\beta V_0}\right)
\]

**SOLUTION** The heat supplied to the block is

\[
Q = cm\left(\frac{\Delta V}{\beta V_0}\right) = \left[\frac{750 \, J/(kg \cdot ^\circ C)}{}\right](130 \, kg)(1.2 \times 10^{-5} \, m^3) = 4.0 \times 10^5 \, J
\]

100. **REASONING** The change in length of the wire is the sum of the change in length of each of the two segments: \( \Delta L = \Delta L_{al} + \Delta L_{st} \). Using Equation 12.2 to express the changes in length, we have

\[
\alpha L_0 \Delta T = \alpha_{al} L_0 \Delta T + \alpha_{st} L_{0st} \Delta T
\]

Dividing both sides by \( L_0 \) and algebraically canceling \( \Delta T \) gives

\[
\alpha = \alpha_{al} \left(\frac{L_{0al}}{L_0}\right) + \alpha_{st} \left(\frac{L_{0st}}{L_0}\right)
\]

The length of the steel segment of the wire is given by \( L_{0st} = L_0 - L_{0al} \). Making this substitution leads to

\[
\alpha = \alpha_{al} \left(\frac{L_{0al}}{L_0}\right) + \alpha_{st} \left(\frac{L_0 - L_{0al}}{L_0}\right)
\]

\[
= \alpha_{al} \left(\frac{L_{0al}}{L_0}\right) + \alpha_{st} \left(\frac{L_0}{L_0}\right) - \alpha_{st} \left(\frac{L_{0al}}{L_0}\right)
\]

This expression can be solved for the desired quantity, \( L_{0al} / L_0 \).
**SOLUTION** Solving for the ratio \( \frac{L_{0\text{al}}}{L_0} \) and taking values for the coefficients of thermal expansion for aluminum and steel from Table 12.1 gives

\[
\frac{L_{0\text{al}}}{L_0} = \frac{\alpha_{\text{al}} - \alpha_{\text{st}}}{\alpha_{\text{al}} - \alpha_{\text{st}}} = \frac{19 \times 10^{-6} (\text{C}^\circ)^{-1} - 12 \times 10^{-6} (\text{C}^\circ)^{-1}}{23 \times 10^{-6} (\text{C}^\circ)^{-1} - 12 \times 10^{-6} (\text{C}^\circ)^{-1}} = 0.6
\]

101. **REASONING** Because the container is insulated, no heat is transferred to the surroundings. Therefore, in order to reach equilibrium at temperature \( T_{\text{eq}} \), the oil must absorb an amount of heat \( Q_{\text{oil}} \) equal to the heat \( Q_{\text{water}} \) given up by the water. Neither the water nor the oil undergoes a phase change, so we will use \( Q = cm\Delta T \) (Equation 12.4) to determine the amount of heat exchanged between the liquids. In Equation 12.4, \( c \) is the specific heat, \( m \) is the mass and \( \Delta T \) is the temperature difference that each liquid undergoes. Thus, we have

\[
Q_{\text{oil}} = c_{\text{oil}}m_{\text{oil}}\Delta T_{\text{oil}} \quad \text{and} \quad Q_{\text{water}} = c_{\text{water}}m_{\text{water}}\Delta T_{\text{water}}
\]

(1)

For the water, the difference between the higher and lower temperatures is \( \Delta T_{\text{water}} = 90.0 \text{ °C} - T_{\text{eq}} \). With this substitution, the expression for \( Q_{\text{water}} \) becomes

\[
Q_{\text{water}} = c_{\text{water}}m_{\text{water}}\left(90.0 \text{ °C} - T_{\text{eq}}\right)
\]

(2)

For the oil, the temperature change \( \Delta T_{\text{oil}} \) is related to the increase \( \Delta V \) in its volume by \( \Delta V = \beta V_0 \Delta T_{\text{oil}} \) (Equation 12.3), where \( \beta \) is the coefficient of volume expansion of the oil, and \( V_0 \) is the volume of the oil before the water is added. Solving Equation 12.3 for \( \Delta T_{\text{oil}} \), we obtain

\[
\Delta T_{\text{oil}} = \frac{\Delta V}{\beta V_0}
\]

(3)

Substituting Equation (3) into the first of Equations (1) yields

\[
Q_{\text{oil}} = \frac{c_{\text{oil}}m_{\text{oil}}\Delta V}{\beta V_0}
\]

(4)

The mass \( m_{\text{oil}} \) of the oil in the container is related to its density \( \rho \) and volume \( V_0 \) according to \( \rho = m_{\text{oil}}/V_0 \) (Equation 11.1). Solving Equation 11.1 for the mass of the oil yields \( m_{\text{oil}} = \rho V_0 \), which we substitute into Equation (4):

\[
Q_{\text{oil}} = \frac{c_{\text{oil}}\rho V_0 \Delta V}{\beta V_0} = \frac{c_{\text{oil}}\rho \Delta V}{\beta}
\]

(5)
**SOLUTION**  Equation (5) gives the amount of heat absorbed by the oil, so it must be equal to Equation (2), which gives the amount of heat lost by the water. Therefore, we have

\[
\frac{c_{oil} \rho \Delta V}{\beta} = c_{water} m_{water} (90.0 \, ^\circ C - T_{eq}) \quad \text{or} \quad 90.0 \, ^\circ C - T_{eq} = \frac{c_{oil} \rho \Delta V}{\beta c_{water} m_{water}} \tag{6}
\]

Solving Equation (6) for the equilibrium temperature, we obtain

\[
T_{eq} = 90.0 \, ^\circ C - \frac{c_{oil} \rho \Delta V}{\beta c_{water} m_{water}} = 90.0 \, ^\circ C - \frac{\left[ 1970 \, J/(kg \cdot ^\circ C) \right] \left[ 924 \, kg/m^3 \right] \left[ 1.20 \times 10^{-5} \, m^3 \right]}{\left[ 721 \times 10^{-6} \, (^\circ C)^{-1} \right] \left[ 4186 \, J/(kg \cdot ^\circ C) \right] \left( 0.125 \, kg \right)} = 32.1 \, ^\circ C
\]

---

**102. REASONING AND SOLUTION**

a. As the wheel heats up, it will expand. Its radius, and therefore, its moment of inertia, will increase. Since no net external torque acts on the wheel, conservation of angular momentum applies where, according to Equation 9.10, the angular momentum is given by:

\[
L = I \omega,
\]

where \( I \) is the moment of inertia and \( \omega \) is the angular velocity. When the moment of inertia increases at the higher temperature, the angular speed must decrease in order for the angular momentum to remain the same.

Thus, the angular speed of the wheel decreases as the wheel heats up.

b. According to the principle of conservation of angular momentum,

\[
I_0 \omega_0 = I_f \omega_f
\]

Solving for \( \omega_f \), we have, treating the bicycle wheel as a thin-walled hollow hoop (\( I = MR^2 \), see Table 9.1)

\[
\omega_f = \omega_0 \left( \frac{I_0}{I_f} \right) = \omega_0 \left( \frac{MR_0^2}{MR_f^2} \right) = \omega_0 \left( \frac{R_0}{R_f} \right)^2
\]

According to Equation 12.2, \( \Delta R = \alpha R_0 \Delta T \), and the final radius of the wheel at the higher temperature is,

\[
R_f = R_0 + \Delta R = R_0 + \alpha R_0 \Delta T = R_0(1 + \alpha \Delta T)
\]
Therefore, taking the coefficient of thermal expansion $\alpha$ for steel from Table 12.1, we find that the angular speed of the wheel at the higher temperature is

$$\omega_f = \omega_0 \left( \frac{R_0}{R_0(1 + \alpha \Delta T)} \right)^2 = \omega_0 \left( \frac{1}{1 + \alpha \Delta T} \right)^2$$

$$= (18.00 \text{ rad/s}) \left( \frac{1}{1 + [12 \times 10^{-6} \text{ (C)}^{-1}][300.0 \text{ °C} - (-100.0 \text{ °C})]} \right)^2 = 17.83 \text{ rad/s}$$
CHAPTER 13 | THE TRANSFER OF HEAT

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (d) The heat conducted during a time $t$ through a bar is given by $Q = \frac{(kA\Delta T)t}{L}$, where $k$ is the thermal conductivity, and $A$ and $L$ are the cross-sectional area and length of the bar.

2. (b) This arrangement conducts more heat for two reasons. First, the temperature difference $\Delta T$ between the ends of each bar is greater in A than in B. Second, the cross-sectional area available for heat conduction is twice as large in A as in B. A greater cross-sectional area means more heat is conducted, everything else remaining the same.

3. (e) This arrangement conducts more heat for two reasons. First, the cross-sectional area $A$ available for heat flow is twice as large in B than in A. Twice the cross-sectional area means twice the heat that is conducted. Second, the length of the bars in B is one-half the combined length in A, which also means that twice the heat is conducted in B as in A. Thus, the heat conducted in B is $2 \times 2 = 4$ times greater than that in A.

4. (e) The heat conducted through a material is given by $Q = \frac{(kA\Delta T)t}{L}$. The heat $Q$, cross-sectional area $A$, thickness $L$, and time $t$ are the same for the two smaller bars. Thus, the product $k\Delta T$ must also be the same for each. Since the thermal conductivity $k_1$ is greater than $k_2$, the temperature difference $\Delta T$ across material 3 is less than that across 2, $k_3$ must be greater than $k_2$. Likewise, since the temperature difference across material 2 is less than that across 1, $k_2$ must be greater than $k_1$.

5. $k_2 = 170 \text{ J/(s·m·C°)}$

6. (b) The heat conducted through a material is given by $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1). The heat $Q$, cross-sectional area $A$, length $L$, and time $t$ are the same for the two smaller bars. Thus, the product $k\Delta T$ must also be the same for each. Since the thermal conductivity $k_1$ is greater than $k_2$, the temperature difference $\Delta T$ across the left bar is smaller than that across the right bar. Thus, the temperature where the two bars are joined together ($400 \text{ °C} - \Delta T$) is greater than $300 \text{ °C}$.

7. (b) The eagle is being lifted upward by rising warm air. Convection is the method of heat transfer that utilizes the bulk movement of a fluid, such as air.
8. (c) The radiant energy emitted per second is given by \( Q/t = \varepsilon \sigma T^4 A \) (Equation 13.2). Note that it depends on the product of \( T^4 \) and the surface area \( A \) of the cube. The product \( T^4 A \) is equal to \( 1944 T_0^4 L_0^2 \), \( 1536 T_0^4 L_0^2 \), and \( 864 T_0^4 L_0^2 \) for B, A, and C, respectively.

9. Energy emitted per second = 128 J/s

10. (a) The radiant energy emitted per second is given by \( Q/t = \varepsilon \sigma T^4 A \) (Equation 13.2). The energy emitted per second depends on the emissivity \( \varepsilon \) of the surface. Since a black surface has a greater emissivity than a silver surface, the black-painted object emits energy at a greater rate and, therefore, cools down faster.

11. (d) The radiant energy emitted per second is given by \( Q/t = \varepsilon \sigma T^4 A \) (Equation 13.2), and it depends on the product \( \varepsilon T^4 \). Since the energy emitted per second is the same for both objects, the product \( \varepsilon T^4 \) is the same for both. Since the emissivity of B is 16 times smaller than the emissivity of A, the temperature of B must be \( \sqrt[4]{16} = 2 \) times greater than A.

12. Difference in net powers = 127 W
1. **REASONING** Since heat \( Q \) is conducted from the blood capillaries to the skin, we can use the relation \( Q = \frac{(kA\Delta T)t}{L} \) (Equation 13.1) to describe how the conduction process depends on the various factors. We can determine the temperature difference between the capillaries and the skin by solving this equation for \( \Delta T \) and noting that the heat conducted per second is \( Q/lt \).

**SOLUTION** Solving Equation 13.1 for the temperature difference, and using the fact that \( Q/lt = 240\, \text{J/s} \), yields

\[
\Delta T = \frac{Q/lt}{kA} = \frac{(240\, \text{J/s})(2.0 \times 10^{-3}\, \text{m})}{0.20\, \text{J/}(\text{s} \cdot \text{m} \cdot ^\circ\text{C})}(1.6\, \text{m}^2) = 1.5\, ^\circ\text{C}
\]

We have taken the thermal conductivity of body fat from Table 13.1.

2. **REASONING** The energy \( Q \) conducted through thickness \( L \) of concrete with cross-sectional area \( A \) in a time \( t \) is \( Q = \frac{(kA\Delta T)t}{L} \) (Equation 13.1), where \( \Delta T \) is the temperature difference between the outside and inside surface of the wall and \( k = 1.1\, \text{J/}(\text{s} \cdot \text{m} \cdot ^\circ\text{C}) \) is the thermal conductivity of concrete (see Table 13.1). This equation can be solved for the desired time \( t \).

**SOLUTION** Since one kilowatt-hour of energy costs $0.10, one dollar’s worth of energy is ten kilowatt-hours, or 10.0 kWh. This amount of energy must be converted into watt-hours (Wh) for use in Equation 13.1, because the thermal conductivity \( k \) uses energy units of joules (J) and one watt (W) is one joule per second (J/s):

\[
Q = (10.0\, \text{kWh})\left(\frac{10^3\, \text{Wh}}{1\, \text{kWh}}\right) = 10.0 \times 10^3\, \text{Wh}
\]

Solving Equation 13.1 for the time \( t \), we find that

\[
t = \frac{QL}{kA\Delta T} = \frac{(10.0 \times 10^3\, \text{Wh})(0.10\, \text{m})}{[1.1\, \text{J/}(\text{s} \cdot \text{m} \cdot ^\circ\text{C})](9.0\, \text{m}^2)(20.0\, ^\circ\text{C} - 12.8\, ^\circ\text{C})} = 14\, \text{h}
\]
3. **REASONING AND SOLUTION** According to Equation 13.1, the heat per second lost is

\[
\frac{Q}{t} = \frac{kA\Delta T}{L} = \left[0.040 \text{ J/(s m C)}\right] (1.6 \text{ m}^2) (25 \text{ C}) = 8.0 \times 10^2 \text{ J/s}
\]

where the value for the thermal conductivity \( k \) of wool has been taken from Table 13.1.

4. **REASONING** The amount of heat \( Q \) conducted in a time \( t \) is given by

\[
Q = \frac{(kA\Delta T)t}{L} \tag{13.1}
\]

where \( k \) is the thermal conductivity, \( A \) is the area, \( \Delta T \) is the temperature difference, and \( L \) is the thickness. We will apply this relation to each arrangement to obtain the ratio of the heat conducted when the bars are placed end-to-end to the heat conducted when one bar is placed on top of the other.

**SOLUTION** Applying Equation 13.1 for the conduction of heat to both arrangements gives

\[
Q_a = \frac{kA_a(\Delta T)t}{L_a} \quad \text{and} \quad Q_b = \frac{kA_b(\Delta T)t}{L_b}
\]

Note that the thermal conductivity \( k \), the temperature difference \( \Delta T \), and the time \( t \) are the same in both arrangements. Dividing \( Q_a \) by \( Q_b \) gives

\[
\frac{Q_a}{Q_b} = \frac{kA_a(\Delta T)t}{kA_b(\Delta T)t} \cdot \frac{L_a}{L_b} = \frac{A_a}{A_b} \cdot \frac{L_a}{L_b}
\]

From the text drawing we see that \( A_b = 2A_a \) and \( L_a = 2L_b \). Thus, the ratio is

\[
\frac{Q_a}{Q_b} = \frac{A_aL_b}{A_bL_a} = \frac{A_aL_b}{(2A_a)(2L_b)} = \frac{1}{4}
\]

5. **REASONING** The heat conducted through the iron poker is given by Equation 13.1, \( Q = (kA\Delta T)t/L \). If we assume that the poker has a circular cross-section, then its cross-sectional area is \( A = \pi r^2 \). Table 13.1 gives the thermal conductivity of iron as \( 79 \text{ J/(s m C)} \).
6. **REASONING** The heat $Q$ conducted during a time $t$ through a block of length $L$ and cross-sectional area $A$ is $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where $k$ is the thermal conductivity, and $\Delta T$ is the temperature difference.

**SOLUTION** The cross-sectional area and length of each block are: $A_A = 2L_0^2$ and $L_A = 3L_0$, $A_B = 3L_0^2$ and $L_B = 2L_0$, $A_C = 6L_0^2$ and $L_C = L_0$. The heat conducted through each block is:

**Case A**

$$Q_A = \frac{A_A}{L_A} k\Delta T t = \frac{2L_0^2}{3L_0} k\Delta T t = \left(\frac{2}{3} L_0\right) k\Delta T t$$

$$= \frac{2}{3} (0.30 \text{ m}) \left[ 250 \text{ J/(s \cdot m \cdot °C)} \right] (35 \text{ °C} - 19 \text{ °C}) (5.0 \text{ s}) = 4.0 \times 10^3 \text{ J}$$

**Case B**

$$Q_B = \frac{A_B}{L_B} k\Delta T t = \frac{3L_0^2}{2L_0} k\Delta T t = \left(\frac{3}{2} L_0\right) k\Delta T t$$

$$= \frac{3}{2} (0.30 \text{ m}) \left[ 250 \text{ J/(s \cdot m \cdot °C)} \right] (35 \text{ °C} - 19 \text{ °C}) (5.0 \text{ s}) = 9.0 \times 10^3 \text{ J}$$

**Case C**

$$Q_C = \frac{A_C}{L_C} k\Delta T t = \frac{6L_0^2}{L_0} k\Delta T t = (6L_0) k\Delta T t$$

$$= 6 (0.30 \text{ m}) \left[ 250 \text{ J/(s \cdot m \cdot °C)} \right] (35 \text{ °C} - 19 \text{ °C}) (5.0 \text{ s}) = 3.6 \times 10^4 \text{ J}$$

7. **SSM REASONING AND SOLUTION** Values for the thermal conductivities of Styrofoam and air are given in Table 11.1. The conductance of an 0.080 mm thick sample of Styrofoam of cross-sectional area $A$ is

$$\frac{k_s A}{L_s} = \frac{[0.010 \text{ J/(s \cdot m \cdot °C)}]}{0.080 \times 10^{-3} \text{ m}} = \left[ 125 \text{ J/(s \cdot m^2 \cdot °C)} \right] A$$
The conductance of a 3.5 mm thick sample of air of cross-sectional area $A$ is

$$k_a A = \frac{[0.0256 \text{ J/(s·m·C)}] A}{3.5 \times 10^{-3} \text{ m}} = [7.3 \text{ J/(s·m²·C)}] A$$

Dividing the conductance of Styrofoam by the conductance of air for samples of the same cross-sectional area $A$, gives

$$\frac{[125 \text{ J/(s·m²·C)}] A}{[7.3 \text{ J/(s·m²·C)}] A} = 17$$

Therefore, the body can adjust the conductance of the tissues beneath the skin by a factor of 17.

8. **REASONING** The energy $Q$ conducted through one face (thickness $L$ and surface area $A$) of the cubical box in a time $t$ is $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where $\Delta T$ is the temperature difference between the outside and inside surface of the box and $k$ is the thermal conductivity of the material from which the box is made. With the aid of this equation, we can determine $k$.

**SOLUTION** Since the cube has six faces, the total heat conducted through all six faces is $Q_{\text{total}} = 6Q$. Using Equation 13.1 for $Q$, we have

$$Q_{\text{total}} = \frac{6(kA\Delta T)t}{L}$$

Noting that the number of seconds in one day is $(24 \text{ h})(\frac{3600 \text{ s}}{1 \text{ h}}) = 8.64 \times 10^4 \text{ s}$ and solving for the thermal conductivity $k$, we find that

$$k = \frac{Q_{\text{total}}L}{6A\Delta Ti}$$

$$= \frac{(3.10 \times 10^6 \text{ J})(3.00 \times 10^{-2} \text{ m})}{6(0.350 \text{ m})^2[21.0^\circ\text{C} - (-78.5^\circ\text{C})](8.64 \times 10^4 \text{ s})} = 1.47 \times 10^{-2} \text{ J/(s·m·C)}$$

9. **REASONING** The heat $Q$ conducted along the bar is given by the relation $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1). We can determine the temperature difference between the hot end of the bar and a point 0.15 m from that end by solving this equation for $\Delta T$ and noting that the heat conducted per second is $Q/t$ and that $L = 0.15 \text{ m}$. 
SOLUTION Solving Equation 13.1 for the temperature difference, using the fact that \(Q/t = 3.6 \text{ J/s}\), and taking the thermal conductivity of brass from Table 13.1, yield

\[
\Delta T = \frac{(Q/t)L}{kA} = \frac{(3.6 \text{ J/s})(0.15 \text{ m})}{[110 \text{ J/}(\text{s} \cdot \text{m} \cdot \text{C})](2.6 \times 10^{-4} \text{ m}^2)} = 19 \text{ C}^\circ
\]

The temperature at a distance of 0.15 m from the hot end of the bar is

\[
T = 306 \text{ } ^\circ\text{C} - 19 \text{ } ^\circ\text{C} = 287 \text{ } ^\circ\text{C}
\]

10. REASONING The heat lost by conduction through the wall is \(Q_{\text{wall}}\) and that lost through the window is \(Q_{\text{window}}\). The total heat lost through the wall and window is \(Q_{\text{wall}} + Q_{\text{window}}\). The percentage of the total heat lost by the window is

\[
\text{Percentage} = \left(\frac{Q_{\text{window}}}{Q_{\text{wall}} + Q_{\text{window}}} \right) \times 100\%
\]

The amount of heat \(Q\) conducted in a time \(t\) is given by

\[
Q = \frac{(kA\Delta T)t}{L}
\]

where \(k\) is the thermal conductivity, \(A\) is the area, \(\Delta T\) is the temperature difference, and \(L\) is the thickness.

SOLUTION Substituting Equation (13.1) into Equation (1), and letting the symbols “S” denote the Styrofoam wall and “G” the glass window, we have that

\[
\text{Percentage} = \left(\frac{Q_{\text{window}}}{Q_{\text{wall}} + Q_{\text{window}}} \right) \times 100\%
\]

\[
= \left[\frac{k_GA_G(\Delta T)t}{L_G} + \frac{k_SA_S(\Delta T)t}{L_S} \right] \times 100\% = \left(\frac{k_GA_G}{L_G} + \frac{k_SA_S}{L_S} \right) \times 100\%
\]

Here we algebraically eliminated the temperature difference \(\Delta T\) and the time \(t\), since they are the same in each term. According to Table 13.1 the thermal conductivity of glass is \(k_G = 0.80 \text{ J/}(\text{s} \cdot \text{m} \cdot \text{C}^\circ)\), while the value for Styrofoam is \(k_S = 0.010 \text{ J/}(\text{s} \cdot \text{m} \cdot \text{C}^\circ)\). The percentage of the total heat lost by the window is
Percentage = \left( \frac{k_G A_G}{L_G} \right) \times \frac{L}{k_S A_S + \frac{k_G A_G}{L_G}} \times 100\% \\
= \left\{ \frac{0.80 \text{ J/(s} \cdot \text{m} \cdot ^\circ \text{C)}(0.16 \text{ m}^2)}{[0.010 \text{ J/(s} \cdot \text{m} \cdot ^\circ \text{C)}(18 \text{ m}^2)] + \frac{0.80 \text{ J/(s} \cdot \text{m} \cdot ^\circ \text{C)}(0.16 \text{ m}^2)}{2.0 \times 10^{-3} \text{ m}}} \right\} \times 100\% = 97\%

11. **REASONING** To find the total heat conducted, we will apply Equation 13.1 to the steel portion and the iron portion of the rod. In so doing, we use the area of a square for the cross section of the steel. The area of the iron is the area of the circle minus the area of the square. The radius of the circle is one half the length of the diagonal of the square.

**SOLUTION** In preparation for applying Equation 13.1, we need the area of the steel and the area of the iron. For the steel, the area is simply \( A_{\text{steel}} = a^2 \), where \( a \) is the length of a side of the square. For the iron, the area is \( A_{\text{iron}} = \pi R^2 - a^2 \). To find the radius \( R \), we use the Pythagorean theorem, which indicates that the length \( D \) of the diagonal is related to the length of the sides according to \( D^2 = a^2 + a^2 \). Therefore, the radius of the circle is \( R = \frac{1}{2} D = \frac{1}{2} \sqrt{2} a \). For the iron, then, the area is

\[ A_{\text{iron}} = \pi R^2 - a^2 = \pi \left( \frac{\sqrt{2} a}{2} \right)^2 - a^2 = \left( \frac{\pi}{2} - 1 \right) a^2 \]

Taking values for the thermal conductivities of steel and iron from Table 13.1 and applying Equation 13.1, we find

\[ Q_{\text{total}} = Q_{\text{steel}} + Q_{\text{iron}} \]

\[ = \left[ \frac{(kA\Delta T)t}{L} \right]_{\text{steel}} + \left[ \frac{(kA\Delta T)t}{L} \right]_{\text{iron}} = \left[ k_{\text{steel}} a^2 + k_{\text{iron}} \left( \frac{\pi}{2} - 1 \right) a^2 \right] \frac{(\Delta T)t}{L} \]

\[ = \left[ \left( 14 \frac{\text{J}}{\text{s} \cdot \text{m} \cdot ^\circ \text{C}} \right)(0.010 \text{ m})^2 + \left( 79 \frac{\text{J}}{\text{s} \cdot \text{m} \cdot ^\circ \text{C}} \right) \left( \frac{\pi}{2} - 1 \right)(0.010 \text{ m})^2 \right] \times \frac{(78 ^\circ \text{C} - 18 ^\circ \text{C})(120 \text{ s})}{0.50 \text{ m}} = 85 \text{ J} \]
12. **REASONING** The energy $Q$ conducted through a layer of material (thickness $L$ and surface area $A$) in a time $t$ is $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where $\Delta T$ is the temperature difference between the two surfaces of area $A$ and $k$ is the thermal conductivity of the material. The heat conducted per second per square meter of area is

$$\frac{Q}{At} = \frac{k\Delta T}{L} \quad (1)$$

**SOLUTION**

a. Before Equation (1) can be applied to the ice-aluminum combination, the temperature $T$ at the interface must be determined. We find the temperature at the interface by noting that the heat conducted through the ice must be equal to the heat conducted through the aluminum: $Q_{\text{ice}} = Q_{\text{aluminum}}$. Applying Equation 13.1 to this condition, we have

$$\left( \frac{kA\Delta T}{L} \right)_{\text{ice}} = \left( \frac{kA\Delta T}{L} \right)_{\text{aluminum}}$$

or

$$\frac{[2.2 \text{ J/(s m C)}] A[(-10.0 \: ^\circ \text{C}) - T]}{0.0050 \: \text{m}} = \frac{[240 \text{ J/(s m C)}] A[T - (-25.0 \: ^\circ \text{C})]}{0.0015 \: \text{m}}$$

The factors $A$ and $t$ can be eliminated algebraically, and the thermal conductivities are given in Table 13.1. Solving for $T$ gives $T = -24.959 \: ^\circ \text{C}$ for the temperature at the interface.

Applying Equation (1) to the ice leads to

$$\left( \frac{Q}{At} \right)_{\text{ice}} = \frac{[2.2 \text{ J/(s m C)}][-10.0 \: ^\circ \text{C} - (-24.959 \: ^\circ \text{C})]}{0.0050 \: \text{m}} = 6.58 \times 10^3 \: \text{J/(s m)^2}$$

Since heat is not building up in the materials, the rate of heat transfer per unit area is the same throughout the ice-aluminum combination. Thus, this must be the heat per second per square meter that is conducted through the ice-aluminum combination.

b. Applying Equation (1) to the aluminum in the absence of any ice gives:

$$\left( \frac{Q}{At} \right)_{\text{Al}} = \frac{[240 \: \text{J/(s m C)}][-10.0 \: ^\circ \text{C} - (-25.0 \: ^\circ \text{C})]}{0.0015 \: \text{m}} = 2.40 \times 10^6 \: \text{J/(s m)^2}$$

13. **REASONING**

a. The heat $Q$ conducted through the tile in a time $t$ is given by $Q = \frac{(kA \Delta T)t}{L}$ (Equation 13.1), where $k$ is the thermal conductivity of the tile, $A$ is its cross-sectional area, $L$ is the distance between the outer and inner surfaces, and $\Delta T$ is the temperature difference between the outer and inner surfaces.
b. We will use \( Q = cm\Delta T \) (Equation 12.4) to find the increase \( \Delta T \) in the temperature of a mass \( m = 2.0 \) kg of water when an amount of heat \( Q \) is transferred to it.

**SOLUTION**

a. The time \( t = 5.0 \) min must be converted to SI units (seconds):

\[
t = (5.0 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 3.0 \times 10^2 \text{ s}
\]

Because the tile is cubical, its thickness is equal to the length \( L \) of one of its sides, and its cross-sectional area \( A \) is the product of two of its side lengths \( A = (L)(L) = L^2 \). Applying \( Q = \frac{(kA \Delta T)t}{L} \) (Equation 13.1), we obtain the amount of heat conducted by the tile in five minutes:

\[
Q = \frac{(kA \Delta T)t}{L} = \frac{(kL^2 \Delta T)t}{L} = (kL \Delta T)t
\]

\[
= \left[ 0.065 \frac{\text{J}}{(\text{s} \cdot \text{m} \cdot \text{C}^\circ)} \right] (0.10 \text{ m}) (1150 \ \text{C} - 20.0 \ \text{C}) (3.0 \times 10^2 \text{ s}) = \boxed{2200 \text{ J}}
\]

b. Solving \( Q = cm\Delta T \) (Equation 12.4) for the increase \( \Delta T \) in temperature, we obtain

\[
\Delta T = \frac{Q}{cm}
\]  

(1)

In Equation (1), we will use the value of \( Q \) found in part (a), and the specific heat capacity \( c \) of water given in Table 12.2 in the text. With these values, the increase in temperature of two liters of water is

\[
\Delta T = \frac{2200 \text{ J}}{4186 \frac{\text{J}}{\text{kg} \cdot \text{C}^\circ} (2.0 \text{ kg})} = \boxed{0.26 \text{ C}^\circ}
\]

14. **REASONING** The flow of heat from the heating elements through the pot bottoms and into the boiling water occurs because the temperature \( T \) of each burner is greater than the temperature \( T_0 = 100.0 \) °C of the boiling water. The temperature difference \( \Delta T = T - T_0 \) drives heat flow at a rate that is given by \( \frac{Q}{t} = \frac{kA \Delta T}{L} \) (Equation 13.1), where the bottom of a pot has a thermal conductivity \( k \), a cross-sectional area \( A \), and a thickness \( L \). The thermal conductivities of copper and aluminum \((k_{Cu}, k_{Al})\) are different (see Table 13.1), but the two pot bottoms are identical in every other respect. Further, because both pots are boiling away at the same rate, the flow of heat must occur at the same rate \( \frac{Q}{t} \) through both bottoms. We will use Equation 13.1 and the temperature \( T_{Al} \) of the heating element under the aluminum-
bottomed pot to determine the temperature \( T_{Cu} \) of the heating element under the copper-bottomed pot.

**SOLUTION**  Both pot bottoms have identical rates \( \frac{Q}{t} \) of heat flow, as well as equal cross-sectional areas \( A \) and thicknesses \( L \). Therefore, \( \frac{Q}{t} = \frac{kA \Delta T}{L} \) (Equation 13.1) yields

\[
\frac{Q}{t} = \frac{k_{Cu} (\Delta T)_{Cu}}{L} = \frac{k_{Al} (\Delta T)_{Al}}{L} \quad \text{or} \quad k_{Cu} (\Delta T)_{Cu} = k_{Al} (\Delta T)_{Al} \quad (1)
\]

Solving Equation (1) for \((\Delta T)_{Cu}\) yields

\[
(\Delta T)_{Cu} = \frac{k_{Al} (\Delta T)_{Al}}{k_{Cu}} \quad (2)
\]

Substituting \((\Delta T)_{Cu} = T_{Cu} - T_0\) into Equation (2) and solving for \( T_{Cu} \), we obtain

\[
T_{Cu} - T_0 = \frac{k_{Al} (\Delta T)_{Al}}{k_{Cu}} \quad \text{or} \quad T_{Cu} = \frac{k_{Al} (\Delta T)_{Al}}{k_{Cu}} + T_0
\]

Therefore, the temperature of the heating element underneath the copper-bottomed pot is

\[
T_{Cu} = \left[ \frac{240 \text{ J/(s} \cdot \text{m} \cdot \text{C})}{390 \text{ J/(s} \cdot \text{m} \cdot \text{C})} \right] \left(155.0 \, ^\circ\text{C} - 100.0 \, ^\circ\text{C}\right) + 100.0 \, ^\circ\text{C} = 134 \, ^\circ\text{C}
\]

15. **REASONING**  Heat is delivered from the heating element to the water via conduction. The amount of heat \( Q \) conducted in a time \( t \) is given by

\[
Q = \left( \frac{k_{copper} A \Delta T}{L} \right) t \quad (13.1)
\]

where \( k_{copper} \) is the thermal conductivity of copper, \( A \) is the area of the bottom of the pot, \( \Delta T \) is the temperature difference, and \( L \) is the thickness of the bottom of the pot. Since the water is boiling under one atmosphere of pressure, the temperature difference is \( \Delta T = T_E - 100.0 \, ^\circ\text{C} \), where \( T_E \) is the temperature of the heating element. Substituting this expression for \( \Delta T \) into Equation (13.1) and solving for \( T_E \), we have

\[
T_E = 100.0 \, ^\circ\text{C} + \frac{LQ}{k_{copper} At} \quad (1)
\]

When water boils, it changes from the liquid to the vapor phase. The heat required to make the water change phase is \( Q = mL_v \), according to Equation 12.5, where \( m \) is the mass and \( L_v \) is the latent heat of vaporization of water.
**SOLUTION** Substituting \( Q = mL_v \) into Equation (1), and noting that the bottom of the pot is circular so that its area is \( A = \pi R^2 \), we have that

\[
T_E = 100.0 \, ^\circ C + \frac{L Q}{k_{\text{Copper}} A t} = 100.0 \, ^\circ C + \frac{L (mL_v)}{k_{\text{Copper}} \left(\pi r^2\right) t} \tag{2}
\]

The thermal conductivity of copper can be found in Table 13.1 \( k_{\text{copper}} = 390 \, \text{J/(s \cdot m \cdot ^\circ C)} \), and the latent heat of vaporization for water can be found in Table 12.3 \((L_v = 22.6 \times 10^5 \, \text{J/kg})\). The temperature of the heating element is

\[
T_E = 100.0 \, ^\circ C + \left(2.0 \times 10^{-3} \, \text{m}\right)\left(0.45 \, \text{kg}\right)\left(22.6 \times 10^5 \, \text{J/kg}\right) \left[390 \, \text{J/(s \cdot m \cdot ^\circ C)}\right] \pi (0.065 \, \text{m})^2 (120 \, \text{s}) = 103.3 \, ^\circ C
\]

16. **REASONING** The heat lost per second due to conduction through the glass is given by Equation 13.1 as \( Q/t = (kA\Delta T)/L \). In this expression, we have no information for the thermal conductivity \( k \), the cross-sectional area \( A \), or the length \( L \). Nevertheless, we can apply the equation to the initial situation and again to the situation where the outside temperature has fallen. This will allow us to eliminate the unknown variables from the calculation.

**SOLUTION** Applying Equation 13.1 to the initial situation and to the situation after the outside temperature has fallen, we obtain

\[
\left(\frac{Q}{t}\right)_{\text{Initial}} = \frac{kA\left(T_{\text{in}} - T_{\text{out, initial}}\right)}{L} \quad \text{and} \quad \left(\frac{Q}{t}\right)_{\text{Colder}} = \frac{kA\left(T_{\text{in}} - T_{\text{out, colder}}\right)}{L}
\]

Dividing these two equations to eliminate the common variables gives

\[
\frac{(Q/t)_{\text{Colder}}}{(Q/t)_{\text{Initial}}} = \frac{kA\left(T_{\text{in}} - T_{\text{out, colder}}\right)}{L} \quad \frac{T_{\text{in}} - T_{\text{out, colder}}}{T_{\text{in}} - T_{\text{out, initial}}}
\]

Remembering that twice as much heat is lost per second when the outside is colder, we find

\[
\frac{2(Q/t)_{\text{Initial}}}{(Q/t)_{\text{Initial}}} = 2 = \frac{T_{\text{in}} - T_{\text{out, colder}}}{T_{\text{in}} - T_{\text{out, initial}}}
\]

Solving for the colder outside temperature gives

\[
T_{\text{out, colder}} = 2T_{\text{out, initial}} - T_{\text{in}} = 2(5.0 \, ^\circ C) - (25 \, ^\circ C) = -15 \, ^\circ C
\]
17. **REASONING** The heat $Q$ required to change liquid water at 100.0 °C into steam at 100.0 °C is given by the relation $Q = mL_v$ (Equation 12.5), where $m$ is the mass of the water and $L_v$ is the latent heat of vaporization. The heat required to vaporize the water is conducted through the bottom of the pot and the stainless steel plate. The amount of heat conducted in a time $t$ is given by $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where $k$ is the thermal conductivity, $A$ and $L$ are the cross-sectional area and length, and $\Delta T$ is the temperature difference. We will use these two relations to find the temperatures at the aluminum-steel interface and at the steel surface in contact with the heating element.

**SOLUTION**

a. Substituting Equation 12.5 into Equation 13.1 and solving for $\Delta T$, we have

$$\Delta T = \frac{QL}{kAt} = \frac{(mL_v)L}{kAt}$$

The thermal conductivity $k_{Al}$ of aluminum can be found in Table 13.1, and the latent heat of vaporization for water can be found in Table 12.3. The temperature difference $\Delta T_{Al}$ between the aluminum surfaces is

$$\Delta T_{Al} = \frac{(mL_v)L}{k_{Al}At} = \frac{(0.15 \text{ kg})(22.6 \times 10^5 \text{ J/kg})(3.1 \times 10^{-3} \text{ m})}{[240 \text{ J/(s·m·C°)}](0.015 \text{ m}^2)(240 \text{ s})} = 1.2 \text{ C°}$$

The temperature at the aluminum-steel interface is $T_{Al-Steel} = 100.0 \text{ °C} + \Delta T_{Al} = 101.2 \text{ °C}$.

b. Using the thermal conductivity $k_{ss}$ of stainless steel from Table 13.1, we find that the temperature difference $\Delta T_{ss}$ between the stainless steel surfaces is

$$\Delta T_{ss} = \frac{(mL_v)L}{k_{ss}At} = \frac{(0.15 \text{ kg})(22.6 \times 10^5 \text{ J/kg})(1.4 \times 10^{-3} \text{ m})}{[14 \text{ J/(s·m·C°)}](0.015 \text{ m}^2)(240 \text{ s})} = 9.4 \text{ C°}$$

The temperature at the steel-burner interface is $T = 101.2 \text{ °C} + \Delta T_{ss} = 110.6 \text{ °C}$.

18. **REASONING** If the cylindrical rod were made of solid copper, the amount of heat it would conduct in a time $t$ is, according to Equation 13.1, $Q_{copper} = (k_{copper}A_2 \Delta T/L)t$. Similarly, the amount of heat conducted by the lead-copper combination is the sum of the heat conducted through the copper portion of the rod and the heat conducted through the lead portion:
\[ Q_{\text{combination}} = \left[ k_{\text{copper}}(A_2 - A_1)\Delta T / L + k_{\text{lead}} A_1 \Delta T / L \right] t. \]

Since the lead-copper combination conducts one-half the amount of heat than does the solid copper rod, \( Q_{\text{combination}} = \frac{1}{2} Q_{\text{copper}}, \) or

\[
\frac{k_{\text{copper}}(A_2 - A_1)\Delta T}{L} + \frac{k_{\text{lead}} A_1 \Delta T}{L} = \frac{1}{2} \left( \frac{k_{\text{copper}} A_2 \Delta T}{L} \right)
\]

This expression can be solved for \( A_1 / A_2 \), the ratio of the cross-sectional areas. Since the cross-sectional area of a cylinder is circular, \( A = \pi r^2 \). Thus, once the ratio of the areas is known, the ratio of the radii can be determined.

**SOLUTION** Solving for the ratio of the areas, we have

\[
\frac{A_1}{A_2} = \frac{k_{\text{copper}}}{2\left(k_{\text{copper}} - k_{\text{lead}}\right)}
\]

The cross-sectional areas are circular so that \( A_1 / A_2 = (\pi r_1^2) / (\pi r_2^2) = (r_1 / r_2)^2 \); therefore,

\[
\frac{r_1}{r_2} = \sqrt{\frac{k_{\text{copper}}}{2\left(k_{\text{copper}} - k_{\text{lead}}\right)}} = \sqrt{\frac{390 \text{ J/(s} \cdot \text{m} \cdot ^{\circ}\text{C})}{2\left[390 \text{ J/(s} \cdot \text{m} \cdot ^{\circ}\text{C}) - 35 \text{ J/(s} \cdot \text{m} \cdot ^{\circ}\text{C})\right]}} = 0.74
\]

where we have taken the thermal conductivities of copper and lead from Table 13.1.

19. **SSM** **REASONING** Heat flows along the rods via conduction, so that Equation 13.1 applies: \( Q = \frac{(kA\Delta T)t}{L} \), where \( Q \) is the amount of heat that flows in a time \( t \), \( k \) is the thermal conductivity of the material from which a rod is made, \( A \) is the cross-sectional area of the rod, and \( \Delta T \) is the difference in temperature between the ends of a rod. In arrangement \( a \), this expression applies to each rod and \( \Delta T \) has the same value of \( \Delta T = T_W - T_C \). The total heat \( Q' \) is the sum of the heats through each rod. In arrangement \( b \), the situation is more complicated. We will use the fact that the same heat flows through each rod to determine the temperature at the interface between the rods and then use this temperature to determine \( \Delta T \) and the heat flow through either rod.

**SOLUTION** For arrangement \( a \), we apply Equation 13.1 to each rod and obtain for the total heat that

\[
Q' = Q_1 + Q_2 = \frac{k_1 A (T_W - T_C) t}{L} + \frac{k_2 A (T_W - T_C) t}{L} = \left( \frac{k_1 + k_2}{2} \right) A (T_W - T_C) t
\]  (1)
For arrangement \( b \), we use \( T \) to denote the temperature at the interface between the rods and note that the same heat flows through each rod. Thus, using Equation 13.1 to express the heat flowing in each rod, we have

\[
\begin{align*}
\frac{k_1 A(T_w - T)}{L} &= \frac{k_2 A(T - T_C)}{L} \quad \text{or} \quad k_1 (T_w - T) = k_2 (T - T_C)
\end{align*}
\]

Solving this expression for the temperature \( T \) gives

\[
T = \frac{k_1 T_w + k_2 T_C}{k_1 + k_2} \quad (2)
\]

Applying Equation 13.1 to either rod in arrangement \( b \) and using Equation (2) for the interface temperature, we can determine the heat \( Q \) that is flowing. Choosing rod 2, we find that

\[
Q = \frac{k_2 A(T - T_C)}{L} = \frac{k_2 A \left( \frac{k_1 T_w + k_2 T_C}{k_1 + k_2} - T_C \right)}{L}
\]

\[
= \frac{k_2 A \left( \frac{k_1 T_w - k_1 T_C}{k_1 + k_2} \right)}{L} = \frac{k_2 A k_1 (T_w - T_C)}{L(k_1 + k_2)} \quad (3)
\]

Using Equations (1) and (3), we obtain for the desired ratio that

\[
\frac{Q'}{Q} = \frac{(k_1 + k_2) A(T_w - T_C)}{L} = \frac{(k_1 + k_2) A(T_w - T_C)}{L k_2 A k_1 (T_w - T_C)} = \left( \frac{k_1 + k_2}{k_2 k_1} \right)^2
\]

Using the fact that \( k_2 = 2k_1 \), we obtain

\[
\frac{Q'}{Q} = \left( \frac{k_1 + k_2}{k_2 k_1} \right)^2 = \left( \frac{k_1 + 2k_1}{2k_1 k_1} \right)^2 = 4.5
\]

20. **REASONING** According to Equation 6.10b, power \( P \) is the change in energy \( Q \) divided by the time \( t \) during which the change occurs, or \( P = Q/t \). The power radiated by a filament is given by the Stefan-Boltzmann law as
where \( e \) is the emissivity, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is the temperature (in kelvins), and \( A \) is the surface area. This expression will be used to find the ratio of the filament areas of the bulbs.

**SOLUTION** Solving Equation (13.2) for the area, we have

\[
A = \frac{P}{e \sigma T^4}
\]

Taking the ratio of the areas gives

\[
\frac{A_1}{A_2} = \frac{\frac{P_1}{e_1 \sigma T_1^4}}{\frac{P_2}{e_2 \sigma T_2^4}}
\]

Setting \( e_2 = e_1 \), and \( P_2 = P_1 \), we have that

\[
\frac{A_1}{A_2} = \frac{\frac{P_1}{e_1 \sigma T_1^4}}{\frac{P_1}{e_2 \sigma T_2^4}} = \left( \frac{T_2}{T_1} \right)^4 = \frac{(2100 \, \text{K})^4}{(2700 \, \text{K})^4} = 0.37
\]

21. **REASONING** The radiant energy \( Q \) radiated by the sun is given by \( Q = e \sigma T^4 A t \) (Equation 13.2), where \( e \) is the emissivity, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is its temperature (in Kelvins), \( A \) is the surface area of the sun, and \( t \) is the time. The radiant energy emitted per second is \( Q/t = e \sigma T^4 A \). Solving this equation for \( T \) gives the surface temperature of the sun.

**SOLUTION** The radiant power produced by the sun is \( Q/t = 3.9 \times 10^{26} \, \text{W} \). The surface area of a sphere of radius \( r \) is \( A = 4\pi r^2 \). Since the sun is a perfect blackbody, \( e = 1 \). Solving Equation 13.2 for the surface temperature of the sun gives

\[
T = \sqrt[4]{\frac{Q/t}{e \sigma 4\pi r^2}} = \sqrt[4]{\frac{3.9 \times 10^{26} \, \text{W}}{(1) \left[ 5.67 \times 10^{-8} \, \text{J/(s \cdot m^2 \cdot K^4)} \right] 4\pi (6.96 \times 10^8 \, \text{m})^2}} = 5800 \, \text{K}
\]
22. **REASONING** According to the Stefan-Boltzmann law, the radiant power emitted by the “radiator” is \( \frac{Q}{t} = e\sigma T^4 A \) (Equation 13.2), where \( Q \) is the energy radiated in a time \( t \), \( e \) is the emissivity of the surface, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is the temperature in Kelvins, and \( A \) is the area of the surface from which the radiant energy is emitted. We will apply this law to the “radiator” before and after it is painted. In either case, the same radiant power is emitted.

**SOLUTION** Applying the Stefan-Boltzmann law, we obtain the following:

\[
\left( \frac{Q}{t} \right)_{\text{after}} = e_{\text{after}} \sigma T_{\text{after}}^4 A \quad \text{and} \quad \left( \frac{Q}{t} \right)_{\text{before}} = e_{\text{before}} \sigma T_{\text{before}}^4 A
\]

Since the same radiant power is emitted before and after the “radiator” is painted, we have

\[
\left( \frac{Q}{t} \right)_{\text{after}} = \left( \frac{Q}{t} \right)_{\text{before}} \quad \text{or} \quad e_{\text{after}} \sigma T_{\text{after}}^4 A = e_{\text{before}} \sigma T_{\text{before}}^4 A
\]

The terms \( \sigma \) and \( A \) can be eliminated algebraically, so this result becomes

\[
e_{\text{after}} \sigma T_{\text{after}}^4 A = e_{\text{before}} \sigma T_{\text{before}}^4 A \quad \text{or} \quad e_{\text{after}} T_{\text{after}}^4 = e_{\text{before}} T_{\text{before}}^4
\]

Remembering that the temperature in the Stefan-Boltzmann law must be expressed in Kelvins, so that \( T_{\text{before}} = 62 \, ^\circ\text{C} + 273 = 335 \, \text{K} \) (see Section 12.2), we find that

\[
T_{\text{after}}^4 = \frac{e_{\text{before}}}{e_{\text{after}}} \left( \frac{T_{\text{before}}}{T_{\text{after}}} \right)^4 = \frac{0.75}{0.50} (335 \, \text{K}) = 371 \, \text{K}
\]

On the Celsius scale, this temperature is 371 K - 273 = 98 \, ^\circ\text{C}.

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23. **SSM REASONING** The radiant energy \( Q \) absorbed by the person’s head is given by \( Q = e\sigma T^4 At \) (Equation 13.2), where \( e \) is the emissivity, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is the Kelvin temperature of the environment surrounding the person \( (T = 28 \, ^\circ\text{C} + 273 = 301 \, \text{K}) \), \( A \) is the area of the head that is absorbing the energy, and \( t \) is the time. The radiant energy absorbed per second is \( \frac{Q}{t} = e\sigma T^4 A \).

**SOLUTION**

a. The radiant energy absorbed per second by the person’s head when it is covered with hair \((e = 0.85)\) is

\[
\frac{Q}{t} = e\sigma T^4 A = (0.85) \left[ 5.67 \times 10^{-8} \, \text{J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4) \right] (301 \, \text{K})^4 (160 \times 10^{-4} \, \text{m}^2) = 6.3 \, \text{J/s}
\]

b. The radiant energy absorbed per second by a bald person’s head \((e = 0.65)\) is

\[
\frac{Q}{t} = e\sigma T^4 A = (0.65) \left[ 5.67 \times 10^{-8} \, \text{J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4) \right] (301 \, \text{K})^4 (160 \times 10^{-4} \, \text{m}^2) = 4.8 \, \text{J/s}
\]
24. **REASONING** The Stefan-Boltzmann law of radiation states that the energy $Q$ radiated by an object in a time $t$ is $Q = e \sigma T^4 A t$ (Equation 13.2), where $e$ is the object’s emissivity, $\sigma$ is the Stefan-Boltzmann constant, $T$ is the Kelvin temperature, and $A$ is the object’s surface area. We can rearrange this equation as follows to give the energy per second $Q/t$:

$$\frac{Q}{t} = e \sigma T^4 A$$  \hspace{1cm} (1)

We will proceed by applying this result at the two temperatures specified in the problem statement.

**SOLUTION** At the two temperatures $T_1$ and $T_2$ we have from Equation (1) that

$$\frac{Q}{t} = e \sigma T_1^4 A \quad \text{and} \quad 2 \left( \frac{Q}{t} \right) = e \sigma T_2^4 A$$

Dividing the equation on the right by the equation on the left gives

$$\frac{2Q/t}{Q/t} = \frac{e \sigma T_2^4 A}{e \sigma T_1^4 A} \quad \text{or} \quad 2 = \frac{T_2^4}{T_1^4} \quad \text{or} \quad \frac{T_2}{T_1} = 2^{1/4} = 1.19$$

25. **SSM REASONING** According to the discussion in Section 13.3, the net power $P_{\text{net}}$ radiated by the person is $P_{\text{net}} = e \sigma A \left( T^4 - T_0^4 \right)$, where $e$ is the emissivity, $\sigma$ is the Stefan-Boltzmann constant, $A$ is the surface area, and $T$ and $T_0$ are the temperatures of the person and the environment, respectively. Since power is the change in energy per unit time (see Equation 6.10b), the time $t$ required for the person to emit the energy $Q$ contained in the dessert is $t = Q/P_{\text{net}}$.

**SOLUTION** The time required to emit the energy from the dessert is

$$t = \frac{Q}{P_{\text{net}}} = \frac{Q}{e \sigma A \left( T^4 - T_0^4 \right)}$$

The energy is $Q = (260 \text{ Calories}) \left( \frac{4186 \text{ J}}{1 \text{ Calorie}} \right)$, and the Kelvin temperatures are

$T = 36 \degree \text{C} + 273 = 309 \text{ K}$ and $T_0 = 21 \degree \text{C} + 273 = 294 \text{ K}$. The time is

$$t = \frac{(260 \text{ Calories}) \left( \frac{4186 \text{ J}}{1 \text{ Calorie}} \right)}{(0.75) \left[ 5.67 \times 10^{-8} \text{ J/(s \cdot m^2 \cdot K^4)} \right] (1.3 \text{ m}^2) \left[ (309 \text{ K})^4 - (294 \text{ K})^4 \right]} = 1.2 \times 10^4 \text{ s}$$
26. **REASONING** The net rate at which energy is being lost via radiation cannot exceed the production rate of 115 J/s, if the body temperature is to remain constant. The net rate at which an object at temperature $T$ radiates energy in a room where the temperature is $T_0$ is given by Equation 13.3 as $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$. $P_{\text{net}}$ is the net energy per second radiated. We need only set $P_{\text{net}}$ equal to 115 J/s and solve for $T_0$. We note that the temperatures in this equation must be expressed in Kelvins, not degrees Celsius.

**SOLUTION** According to Equation 13.3, we have

$$P_{\text{net}} = e\sigma A(T^4 - T_0^4) \quad \text{or} \quad T_0^4 = T^4 - \frac{P_{\text{net}}}{e\sigma A}$$

Using Equation 12.1 to convert from degrees Celsius to Kelvins, we have $T = 34 + 273 = 307$ K. Using this value, it follows that

$$T_0 = \sqrt[4]{T^4 - \frac{P_{\text{net}}}{e\sigma A}}$$

$$= \sqrt[4]{(307 \text{ K})^4 - \frac{115 \text{ J/s}}{0.700 \times 5.67 \times 10^{-8} \text{ J/(s \cdot m^2 \cdot K)}} \times (1.40 \text{ m}^2)} = 287 \text{ K (14 °C)}$$

27. **REASONING** The net radiant power of the baking dish is given by $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$, (Equation 13.3), where $e$ is the emissivity of the dish, $\sigma$ is the Stefan-Boltzmann constant, $A$ is the total surface area of the dish, $T$ is the Kelvin temperature of the dish, and $T_0$ is the temperature of the kitchen. As the dish cools down, both its net radiant power $P_{\text{net}}$ and temperature $T$ decrease, but its emissivity $e$ and surface area $A$ remain constant. We will therefore solve Equation 13.3 for the product of these constant quantities and the Stefan-Boltzmann constant:

$$e\sigma A = \frac{P_{\text{net}}}{(T^4 - T_0^4)} \quad \text{(1)}$$

Because the left side of Equation (1) is constant, the ratio on the right hand side cannot change as the temperature $T$ decreases. We will use this fact to determine the radiant power $P_{\text{net},1}$ of the baking dish when its temperature is 175 °C.

**SOLUTION** From Equation (1), we have that

$$\frac{P_{\text{net},1}}{(T_1^4 - T_0^4)} = \frac{P_{\text{net},2}}{(T_2^4 - T_0^4)} \quad \text{or} \quad P_{\text{net},1} = P_{\text{net},2} \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} \quad \text{(2)}$$
In Equation (2), $P_{\text{net,2}} = 12.0 \text{ W}$ is the net radiant power when the temperature is 35 °C. We first convert temperatures to the Kelvin scale by adding 273 to each temperature given in degrees Celsius (see Equation 12.1). The initial temperature of the baking dish is $T_1 = 175 \text{ °C} + 273 = 448 \text{ K}$, its final temperature is $T_2 = 35 \text{ °C} + 273 = 308 \text{ K}$, and the room temperature is $T_0 = 22 \text{ °C} + 273 = 295 \text{ K}$. Equation (2), then, gives the net radiant power when the dish is first brought out of the oven:

$$P_{\text{net,1}} = (12.0 \text{ W}) \left[ \frac{(448 \text{ K})^4 - (295 \text{ K})^4}{(308 \text{ K})^4 - (295 \text{ K})^4} \right] = 275 \text{ W}$$

28. **REASONING** According to the Stefan-Boltzmann law, the power radiated by an object is $Q/t = e \sigma T^4 A$ (Equation 13.2), where $e$ is the emissivity of the object, $\sigma$ is the Stefan-Boltzmann constant, $T$ is the temperature (in kelvins) of the object, and $A$ is the surface area of the object. The surface area of a sphere is $A = 4\pi R^2$, where $R$ is the radius. Substituting this expression for $A$ into Equation 13.2, and solving for the radius yields

$$R = \sqrt[4]{\frac{Q}{4\pi e \sigma T^4}} \quad (1)$$

This expression will be used to find the radius of Sirius B.

**SOLUTION** Writing Equation (1) for both stars, we have

$$R_{\text{Sirius}} = \sqrt[4]{\frac{(Q/t)_{\text{Sirius}}}{4\pi e_{\text{Sirius}} \sigma T_{\text{Sirius}}^4}} \quad \text{and} \quad R_{\text{Sun}} = \sqrt[4]{\frac{(Q/t)_{\text{Sun}}}{4\pi e_{\text{Sun}} \sigma T_{\text{Sun}}^4}}$$

Dividing $R_{\text{Sirius}}$ by $R_{\text{Sun}}$ and remembering that $(Q/t)_{\text{Sirius}} = 0.040 (Q/t)_{\text{Sun}}$ and $e_{\text{Sirius}} = e_{\text{Sun}}$, we obtain

$$\frac{R_{\text{Sirius}}}{R_{\text{Sun}}} = \sqrt[4]{\frac{(Q/t)_{\text{Sirius}}}{4\pi e_{\text{Sirius}} \sigma T_{\text{Sirius}}^4}} = \sqrt[4]{\frac{0.040 (Q/t)_{\text{Sun}}}{4\pi e_{\text{Sun}} \sigma T_{\text{Sun}}^4}} = \sqrt[4]{\frac{0.040 T_{\text{Sirius}}^4}{T_{\text{Sun}}^4}}$$

Solving for the radius of Sirius B, and noting that $T_{\text{Sirius}} = 4T_{\text{Sun}}$, gives

$$R_{\text{Sirius}} = \sqrt[4]{0.040 \left( \frac{T_{\text{Sun}}}{T_{\text{Sirius}}} \right)^2} R_{\text{Sun}} = \sqrt[4]{0.040 \left( \frac{T_{\text{Sun}}}{4T_{\text{Sun}}} \right)^2} \left( 6.96 \times 10^8 \text{ m} \right) = 8.7 \times 10^6 \text{ m}$$
29. **REASONING AND SOLUTION**

a. The radiant power lost by the body is

\[ P_L = e\sigma T^4 A = (0.80)[5.67 \times 10^{-8} \text{ J/(s·m}^2\cdot\text{K}^4)](307 \text{ K})^4(1.5 \text{ m}^2) = 604 \text{ W} \]

The radiant power gained by the body from the room is

\[ P_g = (0.80)[5.67 \times 10^{-8} \text{ J/(s·m}^2\cdot\text{K}^4)](298 \text{ K})^4(1.5 \text{ m}^2) = 537 \text{ W} \]

The net loss of radiant power is

\[ P = P_L - P_g = 67 \text{ W} \]

b. The net energy lost by the body is

\[ Q = Pt = (67 \text{ W})(3600 \text{ s}) = 58 \text{ Calories} \]

30. **REASONING** At equilibrium the temperature of the solar collector is not changing. Since the only energy loss is due to the emission of radiation, we conclude that the temperature is not changing because the collector is emitting the same amount of energy as it is absorbing. Thus, it is emitting 880 J/s for each 1.0 m² of its surface. The Stefan-Boltzmann law of radiation specifies that the energy \( Q \) radiated by an object in a time \( t \) is

\[ Q = e\sigma T^4 A \]

(Equation 13.2), where \( e \) is the object’s emissivity, \( \sigma = 5.67 \times 10^{-8} \text{ J/(s·m}^2\cdot\text{K}^4) \) is the Stefan-Boltzmann constant, \( T \) is the Kelvin temperature, and \( A \) is the object’s surface area. We can rearrange this equation as follows to give the energy per second \( Q/t \):

\[ \frac{Q}{t} = e\sigma T^4 A \]

This equation can be solved for the temperature \( T \).

**SOLUTION** Solving Equation (1) for the temperature \( T \), we find that

\[ T = \left( \frac{Q}{e\sigma A} \right)^{1/4} = \left[ \frac{880 \text{ J/s}}{0.75[5.67 \times 10^{-8} \text{ J/(s·m}^2\cdot\text{K}^4)](1.0 \text{ m}^2)} \right]^{1/4} = 380 \text{ K} \]

31. **REASONING** The liquid helium is at its boiling point, so its temperature does not rise as it absorbs heat from the radiating shield. Instead, the net heat \( Q \) absorbed in a time \( t \) converts a mass \( m \) of liquid helium into helium gas, according to

\[ Q = mL_v \]

(Equation 12.5), where \( L_v = 2.1 \times 10^4 \text{ J/kg} \) is the latent heat of vaporization of helium. The ratio of the net heat \( Q \) absorbed by the helium to the elapsed time \( t \) is equal to the net power \( P_{net} \) absorbed by helium:

\[ P_{net} = \frac{Q}{t} \]

(Equation 6.10b). The net power absorbed by the helium depends upon the temperature \( T = 77 \text{ K} \) maintained by the radiating shield and the temperature \( T_0 = 4.2 \text{ K} \) of the boiling helium, as we see from

\[ P_{net} = e\sigma A(T^4 - T_0^4) \]

(Equation 13.3), where \( \sigma = 5.67 \times 10^{-8} \text{ J/(s·m}^2\cdot\text{K}^4) \) is the Stefan-Boltzmann constant, \( e = 1 \) is the emissivity of the
container (a perfect blackbody radiator), and \( A \) is the surface area of the container. Because the container is a sphere of radius \( R \), its surface area is given by \( A = 4\pi R^2 \). Therefore, the net power absorbed by the helium can be expressed as

\[
P_{\text{net}} = e\sigma A \left( T^4 - T_0^4 \right) = 4\pi R^2 e\sigma \left( T^4 - T_0^4 \right)
\]  

(1)

**SOLUTION** Solving \( Q = mL_v \) (Equation 12.5) for \( m \), we obtain

\[
m = \frac{Q}{L_v}
\]  

(2)

Solving \( P_{\text{net}} = \frac{Q}{t} \) (Equation 6.10b) for \( Q \) yields \( Q = P_{\text{net}}t \). Substituting this into Equation (2), we find that

\[
m = \frac{Q}{L_v} = \frac{P_{\text{net}}t}{L_v}
\]  

(3)

Substituting Equation (1) into Equation (3) gives

\[
m = \frac{P_{\text{net}}t}{L_v} = \frac{4\pi R^2 e\sigma \left( T^4 - T_0^4 \right)t}{L_v}
\]

Since 1 hour is equivalent to 3600 seconds, the mass of helium that boils away in one hour is

\[
m = 4\pi (0.30 \text{ m})^2 (1) \left[ 5.67 \times 10^{-8} \text{ J/(s m}^2 \cdot \text{K}^4) \right] \left[ (77 \text{ K})^4 - (4.2 \text{ K})^4 \right](3600 \text{ s}) = 0.39 \text{ kg}
\]

32. **REASONING AND SOLUTION** According to Equation 13.2, for the sphere we have \( Q/t = e\sigma A_s T_s^4 \), and for the cube \( Q/t = e\sigma A_c T_c^4 \). Equating and solving we get

\[
T_c^4 = (A_s/A_c)T_s^4
\]

Now

\[
A_s/A_c = (4\pi R^2)/(6L^2)
\]

The volume of the sphere and the cube are the same, \((4/3)\pi R^3 = L^3\), so \( R = \left( \frac{3}{4\pi} \right)^{1/3} L \).

The ratio of the areas is

\[
\frac{A_s}{A_c} = \frac{4\pi R^2}{6L^2} = \frac{4\pi}{6} \left( \frac{3}{4\pi} \right)^{2/3} = 0.806
\]

The temperature of the cube is, then

\[
T_c = \left( \frac{A_s}{A_c} \right)^{1/4} T_s = (0.806)^{1/4} (773 \text{ K}) = 732 \text{ K}
\]
33. **REASONING** The total radiant power emitted by an object that has a Kelvin temperature $T$, surface area $A$, and emissivity $e$ can be found by rearranging Equation 13.2, the Stefan-Boltzmann law: $Q = e\sigma T^4 A t$. The emitted power is $P = Q/t = e\sigma T^4 A$. Therefore, when the original cylinder is cut perpendicular to its axis into $N$ smaller cylinders, the ratio of the power radiated by the pieces to that radiated by the original cylinder is

$$\frac{P_{\text{pieces}}}{P_{\text{original}}} = \frac{e\sigma T^4 A_2}{e\sigma T^4 A_1}$$

where $A_1$ is the surface area of the original cylinder, and $A_2$ is the sum of the surface areas of all $N$ smaller cylinders. The surface area of the original cylinder is the sum of the surface area of the ends and the surface area of the cylinder body; therefore, if $L$ and $r$ represent the length and cross-sectional radius of the original cylinder, with $L = 10r$,

$$A_1 = (\text{area of ends}) + (\text{area of cylinder body}) = 2(\pi r^2) + (2\pi r)L = 2(\pi r^2) + (2\pi r)(10r) = 22\pi r^2$$

When the original cylinder is cut perpendicular to its axis into $N$ smaller cylinders, the total surface area $A_2$ is

$$A_2 = N2(\pi r^2) + (2\pi r)L = N2(\pi r^2) + (2\pi r)(10r) = (2N + 20)\pi r^2$$

Substituting the expressions for $A_1$ and $A_2$ into Equation (1), we obtain the following expression for the ratio of the power radiated by the $N$ pieces to that radiated by the original cylinder

$$\frac{P_{\text{pieces}}}{P_{\text{original}}} = \frac{e\sigma T^4 A_2}{e\sigma T^4 A_1} = \frac{(2N + 20)\pi r^2}{22\pi r^2} = \frac{N + 10}{11}$$

**SOLUTION** Since the total radiant power emitted by the $N$ pieces is twice that emitted by the original cylinder, $P_{\text{pieces}}/P_{\text{original}} = 2$, we have $(N + 10)/11 = 2$. Solving this expression for $N$ gives $N = 12$. Therefore, there are 12 smaller cylinders.

34. **REASONING** The drawing shows a cross-sectional view of the small sphere inside the larger spherical asbestos shell. The small sphere produces a net radiant energy, because its temperature (800.0 °C) is greater than that of its environment (600.0 °C). This energy is then conducted through the thin asbestos shell (thickness = $L$). By setting the net radiant energy produced by the small sphere equal to the energy conducted through the asbestos shell, we will be able to obtain the temperature $T_2$ of the outer surface of the shell.
**SOLUTION** The heat $Q$ conducted during a time through the thin asbestos shell is given by Equation 13.1 as $Q = \frac{(k_{\text{asbestos}} A_2 \Delta T)}{L} t$, where $k_{\text{asbestos}}$ is the thermal conductivity of asbestos (see Table 13.1), $A_2$ is the surface area of the spherical shell ($A_2 = 4\pi r_2^2$), $\Delta T$ is the temperature difference between the inner and outer surfaces of the shell ($\Delta T = 600.0 \, ^\circ C - T_2$), and $L$ is the thickness of the shell. Solving this equation for $T_2$ yields

$$T_2 = 600.0 \, ^\circ C - \frac{QL}{k_{\text{asbestos}} (4\pi r_2^2) t}$$

The heat $Q$ is produced by the net radiant energy generated by the small sphere inside the asbestos shell. According to Equation 13.3, the net radiant energy is $Q = P_{\text{net}} t = e\sigma A_1 (T^4 - T_0^4) t$, where $e$ is the emissivity, $\sigma$ is the Stefan-Boltzmann constant, $A_1$ is the surface area of the sphere ($A_1 = 4\pi r_1^2$), $T$ is the temperature of the sphere ($T = 800.0 \, ^\circ C = 1073.2 \, K$) and $T_0$ is the temperature of the environment that surrounds the sphere ($T_0 = 600.0 \, ^\circ C = 873.2 \, K$). Substituting this expression for $Q$ into the expression above for $T_2$, and algebraically eliminating the time $t$ and the $4\pi$ factors, gives

$$T_2 = 600.0 \, ^\circ C - \frac{\left[ e\sigma (T^4 - T_0^4) \right] L}{k_{\text{asbestos}} \left( \frac{r_2}{r_1} \right)^2}$$

$$= 600.0 \, ^\circ C - \frac{\left( 0.90 \right) \left( 5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K} \right) \left( (1073.2 \, K)^4 - (873.2 \, K)^4 \right)}{\left( 0.090 \frac{J}{s \cdot m \cdot C^0} \right) (10.0)^2}$$

$$= [558 \, ^\circ C]$$

35. **REASONING** The heat transferred in a time $t$ is given by Equation 13.1, $Q = (kA \Delta T) t / L$. If the same amount of heat per second is conducted through the two plates, then $(Q/t)_{\text{al}} = (Q/t)_{\text{st}}$. Using Equation 13.1, this becomes

$$\frac{k_{\text{al}} A \Delta T}{L_{\text{al}}} = \frac{k_{\text{st}} A \Delta T}{L_{\text{st}}}$$

This expression can be solved for $L_{\text{st}}$. 
SOLUTION  Solving for \( L_{st} \) gives

\[
L_{st} = \frac{k_{st}}{k_{al}} L_{al} = \frac{14 \text{ J/(s m C)}^\circ}{240 \text{ J/(s m C)}^\circ} (0.035 \text{ m}) = 2.0 \times 10^{-3} \text{ m}
\]

36. REASONING  The inner radius \( r_{in} \) and outer radius \( r_{out} \) of the pipe determine the cross-sectional area \( A \) (copper only) of the pipe, which is the difference between the area \( A_{out} \) of a circle with radius \( r_{out} \) and the area \( A_{in} \) of a circle with a radius \( r_{in} \):

\[
A = A_{out} - A_{in} = \pi r_{out}^2 - \pi r_{in}^2
\]  

(Equation 1)

The heat \( Q \) that flows along the pipe in a time \( t = 15 \text{ min} \) is related to the cross-sectional area \( A \) by \( Q = \frac{(kA \Delta T)t}{L} \) (Equation 13.1), where \( k \) is the thermal conductivity of copper (see Table 13.1 in the text), \( L \) is the length of the pipe, and \( \Delta T \) is the temperature difference between the faucet, where the temperature is 4.0 \( ^\circ \text{C} \), and the point on the pipe 3.0 m from the faucet where the temperature is 25 \( ^\circ \text{C} \): \( \Delta T = 25 ^\circ \text{C} - 4.0 ^\circ \text{C} = 21 ^\circ \text{C} \). We will use Equation (1) and Equation 13.1 to find the inner radius of the pipe.

SOLUTION  Solving Equation (1) for the inner radius \( r_{in} \) of the pipe yields

\[
\pi r_{in}^2 = \pi r_{out}^2 - A \quad \text{or} \quad r_{in}^2 = r_{out}^2 - \frac{A}{\pi} \quad \text{or} \quad r_{in} = \sqrt{r_{out}^2 - \frac{A}{\pi}}
\]  

(Equation 2)

An expression for the cross-sectional area \( A \) of the pipe may be obtained by solving \( Q = \frac{(kA \Delta T)t}{L} \) (Equation 13.1):

\[
A = \frac{QL}{(kA \Delta T)t}
\]  

(Equation 3)

Substituting Equation (3) into Equation (2) yields

\[
r_{in} = \sqrt{r_{out}^2 - \frac{QL}{\pi(kA \Delta T)t}}
\]  

(Equation 4)

Before using Equation (4), we convert the time \( t \) from minutes to seconds:

\[
t = (15 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 9.0 \times 10^2 \text{ s}
\]

The inner radius of the pipe is, therefore,

\[
r_{in} = \sqrt{(0.013 \text{ m})^2 - \frac{(270 \text{ J})(3.0 \text{ m})}{\pi \left[ 390 \text{ J/(s m C)}^\circ \right] \left[ (25 ^\circ \text{C} - 4.0 ^\circ \text{C}) \right] \left( 9.0 \times 10^2 \text{ s} \right)}} = 0.012 \text{ m}
\]
37. **REASONING AND SOLUTION** Solving the Stefan-Boltzmann law, Equation 13.2, for the time \( t \), and using the fact that \( Q_{\text{blackbody}} = Q_{\text{bulb}} \), we have

\[
    t_{\text{blackbody}} = \frac{Q_{\text{blackbody}}}{\sigma T^4 A} = \frac{Q_{\text{bulb}}}{\sigma T^4 A} = \frac{P_{\text{bulb}} t_{\text{bulb}}}{\sigma T^4 A}
\]

where \( P_{\text{bulb}} \) is the power rating of the light bulb. Therefore,

\[
    t_{\text{blackbody}} = \frac{(100.0 \text{ J/s}) (3600 \text{ s})}{5.67 \times 10^{-8} \text{ J/(s m}^2 \cdot \text{K}^4)} (303 \text{ K})^4 \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ d}}{24 \text{ h}} \right) = 14.5 \text{ d}
\]

38. **REASONING** The power radiated by an object is given by \( Q/t = e \sigma T^4 A \) (Equation 13.2), where \( e \) is the emissivity of the object, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is the temperature (in kelvins) of the object, and \( A \) is its surface area.

The power that the object absorbs from the room is given by \( Q/lt = e \sigma T_0^4 A \). Except for the temperature \( T_0 \) of the room, this expression has the same form as that for the power radiated by the object. Note especially that the area \( A \) is the surface area of the object, not the room. Review Example 8 in the text to understand this important point.

**SOLUTION** The object emits three times as much power as it absorbs from the room, so it follows that \( (Q/t)_{\text{emitted}} = 3(Q/t)_{\text{absorbed}} \). Using the Stefan-Boltzmann law for each of the powers, we find

\[
    \frac{e \sigma T^4 A}{\text{Power emitted}} = \frac{3e \sigma T_0^4 A}{\text{Power absorbed}}
\]

Solving for the temperature \( T \) of the object gives

\[
    T = \sqrt[4]{3} T_0 = \sqrt[3]{3} (293 \text{ K}) = 386 \text{ K}
\]

39. **REASONING AND SOLUTION** The heat \( Q \) conducted during a time \( t \) through a wall of thickness \( L \) and cross sectional area \( A \) is given by Equation 13.1:

\[
    Q = \frac{k A \Delta T t}{L}
\]

The radiant energy \( Q \), emitted in a time \( t \) by a wall that has a Kelvin temperature \( T \), surface area \( A \), and emissivity \( e \) is given by Equation (13.2):
If the amount of radiant energy emitted per second per square meter at 0 °C is the same as the heat lost per second per square meter due to conduction, then

\[
\frac{Q}{tA}_{\text{conduction}} = \frac{Q}{tA}_{\text{radiation}}
\]

Making use of Equations 13.1 and 13.2, the equation above becomes

\[
\frac{k\Delta T}{L} = e\sigma T^4
\]

Solving for the emissivity \(e\) gives:

\[
e = \frac{k\Delta T}{L\sigma T^4} = \frac{[1.1 \text{ J/(s·m·K)}](293.0 \text{ K} - 273.0 \text{ K})}{(0.10 \text{ m})[5.67 \times 10^{-8} \text{ J/(s·m²·K⁴)}](273.0 \text{ K})^4} = 0.70
\]

Remark on units: Notice that the units for the thermal conductivity were expressed as J/(s·m·K) even though they are given in Table 13.1 as J/(s·m·C°). The two units are equivalent since the "size" of a Celsius degree is the same as the "size" of a Kelvin; that is, 1 C° = 1 K. Kelvins were used, rather than Celsius degrees, to ensure consistency of units. However, Kelvins must be used in Equation 13.2 or any equation that is derived from it.

40. REASONING

a. According to Equation 13.2, the radiant power \(P\) (or energy per unit time) emitted by an object is \(P = \frac{Q}{t} = e\sigma T^4A\), where \(e\) is the emissivity of the object, \(\sigma\) is the Stefan-Boltzmann constant, \(T\) is the temperature (in kelvins) of the object, and \(A\) is its surface area. This expression will allow us to find the ratio of the two absorbed powers. The reason is that the object and the room have the same constant temperature. Since the object’s temperature is constant, it must be absorbing the same power that it is emitting.

b. The temperature of the two bars in part (b) of the text drawing can be obtained directly from Equation 13.2: \(T = \sqrt[4]{\frac{P}{e\sigma A}}\).

SOLUTION

a. The power absorbed by the two bars in part (b) of the text drawing is given by \(Q/t = e\sigma T^4A_2\) (Equation 13.2), where \(A_2\) is the total surface area of the two bars that is exposed to the room: \(A_2 = 28L_0^2\). The power absorbed by the single bar in part (a) of the text drawing is \(Q/t = e\sigma T^4A_1\), where \(A_1\) is the total surface area of the single bar: \(A_1 = 22L_0^2\). The ratio of the power \(P_2\) absorbed by the two bars to the power \(P_1\) absorbed by the single bar is
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\[ \frac{P_2}{P_1} = \frac{e\sigma T^4}{e\sigma T^4} \left( \frac{28 L_2^2}{22 L_0^2} \right) = \frac{1.27}{1} \]

b. The temperature \( T_2 \) of the two bars in part (b) of the text drawing and the temperature \( T_1 \) of the single bar in part (a) are

\[ T_2 = 4\sqrt{\frac{P_2}{e\sigma A_2}} \quad \text{and} \quad T_1 = 4\sqrt{\frac{P_1}{e\sigma A_1}} \]

Dividing \( T_2 \) by \( T_1 \), and noting that \( P_2 = P_1 \), gives

\[ \frac{T_2}{T_1} = 4\sqrt{\frac{A_1}{e\sigma A_1}} = \frac{A_1}{A_2} \]

Solving for \( T_2 \) gives

\[ T_2 = T_1 \sqrt{\frac{A_1}{A_2}} = (450.0 \text{ K}) \sqrt{\frac{22L_2^2}{28L_0^2}} = 424 \text{ K} \]

41. **REASONING**  Heat \( Q \) flows along the length \( L \) of the bar via conduction, so that Equation 13.1 applies: \( Q = \frac{(kA\Delta T)t}{L} \), where \( k \) is the thermal conductivity of the material from which the bar is made, \( A \) is the cross-sectional area of the bar, \( \Delta T \) is the difference in temperature between the ends of the bar, and \( t \) is the time during which the heat flows. We will apply this expression twice in determining the length of the bar.

**SOLUTION**  Solving Equation 13.1 for the length \( L \) of the bar gives

\[ L = \frac{(kA\Delta T)t}{Q} = \frac{kA(T_W - T_C)t}{Q} \quad \text{(1)} \]

where \( T_W \) and \( T_C \), respectively are the temperatures at the warmer and cooler ends of the bar. In this result, we do not know the terms \( k, A, t, \) or \( Q \). However, we can evaluate the heat \( Q \) by recognizing that it flows through the entire length of the bar. This means that we can also apply Equation 13.1 to the 0.13 m of the bar at its cooler end and thereby obtain an expression for \( Q \):

\[ Q = \frac{kA(T - T_C)t}{D} \]

where the length of the bar through which the heat flows is \( D = 0.13 \text{ m} \) and the temperature at the 0.13-m point is \( T = 23 ^\circ\text{C} \), so that \( \Delta T = T - T_C \). Substituting this result into Equation (1) and noting that the terms \( k, A, \) and \( t \) can be eliminated algebraically, we find
\[
L = \frac{kA(T_w - T_C) t}{Q} = \frac{kA(T_w - T_C) t}{kA(T - T_C) t} = \frac{kA(T_w - T_C) fD}{kA(T - T_C) fD} = \frac{\left( T_w - T_C \right) D}{\left( T - T_C \right) } = \frac{(48 \, ^\circ C - 11 \, ^\circ C)(0.13 \, m)}{23 \, ^\circ C - 11 \, ^\circ C} = 0.40 \, m
\]

42. **REASONING** If \( m \) kilograms of ice melt in \( t \) seconds, then \( Q = mL_f \) (Equation 12.5) joules of heat must be delivered to the ice through the copper rod in \( t \) seconds, where \( L_f = 33.5 \times 10^4 \, J/kg \) is the latent heat of fusion of water. The mass of ice per second that melts, then, is given by the ratio \( \frac{m}{t} \). The rate \( \frac{Q}{t} \) of heat flow through the copper rod is found from \( \frac{Q}{t} = \frac{kA\Delta T}{L} \) (Equation 13.1), where \( k \) is the thermal conductivity of copper, \( A \) and \( L \) are, respectively, the cross-sectional area and length of the rod, and \( \Delta T = 100.0 \, ^\circ C \) is the difference in temperature between the boiling water and the ice-water mixture.

**SOLUTION** Solving \( Q = mL_f \) (Equation 12.5) for \( m = \frac{Q}{L_f} \). Dividing this by the elapsed time \( t \), we obtain an expression for the mass of ice per second that melts:

\[
\frac{m}{t} = \frac{\left( \frac{Q}{t} \right)}{L_f}
\]

Substituting \( \frac{Q}{t} = \frac{kA\Delta T}{L} \) (Equation 13.1) into Equation (1), we find that

\[
\frac{m}{t} = \frac{kA\Delta T}{L_fL} = \frac{\left[ 390 \, J/(s \cdot m \cdot ^\circ C) \right] \left( 4.0 \times 10^{-4} \, m^2 \right) \left( 100.0 \, ^\circ C \right)}{(33.5 \times 10^4 \, J/kg)(1.5 \, m)} = \frac{3.1 \times 10^{-5}}{\, kg/s}
\]

43. **SSM REASONING** The rate at which heat is conducted along either rod is given by Equation 13.1, \( \frac{Q}{t} = \frac{kA\Delta T}{L} \). Since both rods conduct the same amount of heat per second, we have

\[
\frac{k_sA_s \Delta T}{L_s} = \frac{k_iA_i \Delta T}{L_i}
\]

Since the same temperature difference is maintained across both rods, we can algebraically cancel the \( \Delta T \) terms. Because both rods have the same mass, \( m_s = m_i \); in terms of the densities of silver and iron, the statement about the equality of the masses becomes \( \rho_s(L_sA_s) = \rho_i(L_iA_i) \), or
\[
\frac{A_s}{A_i} = \frac{\rho_1 L_1}{\rho_s L_s}
\]  

(2)

Equations (1) and (2) may be combined to find the ratio of the lengths of the rods. Once the ratio of the lengths is known, Equation (2) can be used to find the ratio of the cross-sectional areas of the rods. If we assume that the rods have circular cross sections, then each has an area of \( A = \pi r^2 \). Hence, the ratio of the cross-sectional areas can be used to find the ratio of the radii of the rods.

**SOLUTION**

a. Solving Equation (1) for the ratio of the lengths and substituting the right hand side of Equation (2) for the ratio of the areas, we have

\[
\frac{L_s}{L_1} = \frac{k_s A_s}{k_1 A_1} = \frac{k_s (\rho_1 L_1)}{k_1 (\rho_s L_s)} \quad \text{or} \quad \left( \frac{L_s}{L_1} \right)^2 = \frac{k_s \rho_1}{k_1 \rho_s}
\]

Solving for the ratio of the lengths, we have

\[
\frac{L_s}{L_1} = \sqrt{\frac{k_s \rho_1}{k_1 \rho_s}} = \sqrt{\frac{[420 \text{ J/(s·m·C°)]}(7860 \text{ kg/m}^3)}{[79 \text{ J/(s·m·C°)]}(10 500 \text{ kg/m}^3)}} = 2.0
\]

b. From Equation (2) we have

\[
\frac{\pi r_s^2}{\pi r_i^2} = \frac{\rho_1 L_1}{\rho_s L_s} \quad \text{or} \quad \left( \frac{r_s}{r_i} \right)^2 = \frac{\rho_1 L_1}{\rho_s L_s}
\]

Solving for the ratio of the radii, we have

\[
\frac{r_s}{r_i} = \sqrt{\frac{\rho_1}{\rho_s} \left( \frac{L_1}{L_s} \right)} = \sqrt{\frac{7860 \text{ kg/m}^3}{10 500 \text{ kg/m}^3} \left( \frac{1}{2.0} \right)} = 0.61
\]

44. **REASONING** Heat per second is an energy change per unit time, which is power (see Equation 6.10b). Therefore, the heat per second \( Q_{\text{con}}/t \) gained by the sphere due to conduction is given by Equation 13.1 as

\[
P_{\text{con}} = \frac{Q_{\text{con}}}{t} = \frac{(kA_{\text{rod}} \Delta T)}{L}
\]  

(13.1)

where \( k \) is the thermal conductivity of copper (see Table 13.1 in the text), \( A_{\text{rod}} \) is the cross-sectional area of the rod, \( \Delta T \) is the temperature difference between the wall (24 °C) and the ice (0 °C), and \( L \) is the length of the rod *outside* the sphere. The reason that \( L \) in
Equation 13.1 is not equal to the total length of the rod is that the portion of the rod that is embedded in the ice is at the same temperature as the ice. As there is no temperature difference across this portion of the rod, there is no heat conduction along it.

The net radiant power gained by the sphere is found from

\[ P_{\text{rad}} = e\sigma A_{\text{sphere}} \left( T^4 - T_0^4 \right) \]  

(13.3)

where \( e \) is the emissivity of the ice, \( \sigma \) is the Stefan-Boltzmann constant, \( A_{\text{sphere}} \) is the surface area of the ice sphere, \( T \) is the Kelvin temperature of the room, and \( T_0 \) is the Kelvin temperature of the sphere.

**SOLUTION** One end of the rod is at the center of the ice sphere, so the length \( L \) of the rod outside of the sphere is equal to the total length of the rod minus the sphere’s radius \( R \):

\[ L = 0.25 \text{ m} - 0.15 \text{ m} = 0.10 \text{ m}. \]

With this substitution, Equation 13.1 gives the sphere’s conductive power gain:

\[
P_{\text{con}} = \frac{(kA_{\text{rod}} \Delta T)}{L} = \frac{\left[ 390 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}) \right] \left( 1.2 \times 10^{-4} \text{ m}^2 \right) \left(24 \text{ °C} - 0 \text{ °C} \right)}{0.10 \text{ m}} = 11 \text{ W}
\]

The surface area of a sphere is given by \( A_{\text{sphere}} = 4\pi R^2 \), so that the net radiant power gain given by Equation 13.3 becomes

\[
P_{\text{rad}} = e\sigma A_{\text{sphere}} \left( T^4 - T_0^4 \right) = 4\pi e\sigma R^2 \left( T^4 - T_0^4 \right)
\]

(1)

To find the net radiant power gain \( P_{\text{rad}} \) from Equation (1), we first convert the temperatures to the Kelvin scale (see Equation 12.1): the temperature of the sphere is \( T_0 = 0 \text{ °C} + 273 = 273 \text{ K} \), and the temperature of the room is \( T = 24 \text{ °C} + 273 = 297 \text{ K} \).

Substituting the Kelvin temperatures into Equation (1) yields the net radiant power gain of the sphere:

\[
P_{\text{rad}} = 4\pi e\sigma R^2 \left( T^4 - T_0^4 \right)
\]

\[
= 4\pi (0.90) \left[ 5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4) \right] (0.15 \text{ m})^2 \left[ (297 \text{ K})^4 - (273 \text{ K})^4 \right]
\]

\[
= 32 \text{ W}
\]

Thus, the ratio of the heat gain per second due to conduction to the net heat gain per second due to radiation is

\[
\frac{P_{\text{con}}}{P_{\text{rad}}} = \frac{11 \text{ W}}{32 \text{ W}} = 0.34
\]
CHAPTER 14  THE IDEAL GAS LAW AND KINETIC THEORY

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (d) This statement is not true, because the number of molecules that have a mass of ten grams depends on the molecular mass of the substance, which is different for different substances.

2. 10.2 g

3. (b) For an ideal gas, we know that \( P = \frac{nRT}{V} \). Increasing \( n \), \( T \), and \( V \) together may or may not cause \( P \) to increase. Increasing \( n \) and \( T \) certainly causes \( P \) to increase. However, increasing \( V \) causes \( P \) to decrease. Therefore, the result of increasing all three variables together depends on the relative amounts by which they are increased.

4. 2.25 mol

5. 1.25 kg/m³

6. (a) If the speed of every atom in a monatomic ideal gas were doubled, the root-mean-square speed of the atoms would be doubled. According to the kinetic theory of gases, the Kelvin temperature is proportional to the square of the root-mean-square speed (see Equation 14.6). Therefore, the Kelvin temperature would be multiplied by a factor of \( 2^2 = 4 \).

7. (d) According to the kinetic theory of gases, the average translational kinetic energy per molecule is proportional to the Kelvin temperature (see Equation 14.6). Since each gas has the same temperature, each has the same average translational kinetic energy. However, this kinetic energy is \( \frac{1}{2} m v_{\text{rms}}^2 \), and depends on the mass \( m \). Since each type of molecule has the same kinetic energy, the molecules with the larger masses have the smaller translational rms speeds \( v_{\text{rms}} \).

8. (c) According to the kinetic theory of gases, the internal energy \( U \) of a monatomic ideal gas is \( U = \frac{3}{2} nRT \) (Equation 14.7). However, the ideal gas law indicates that \( PV = nRT \), so that \( U = \frac{3}{2} PV \). Since \( P \) is doubled and \( V \) is reduced by one-half, the product \( PV \) and the internal energy are unchanged.
9. (e) Increasing the cross-sectional area of the diffusion channel and increasing the difference in solute concentrations between its ends certainly would increase the diffusion rate. However, increasing its length would decrease the rate (see Equation 14.8). Therefore, increasing all three factors simultaneously could lead to a decrease in the diffusion rate, depending on the relative amounts of the change in the factors.

10. $6.0 \times 10^{-11}$ kg/s
CHAPTER 14

THE IDEAL GAS LAW
AND KINETIC THEORY

PROBLEMS

1. **SSM REASONING AND SOLUTION** Since hemoglobin has a molecular mass of 64 500 u, the mass per mole of hemoglobin is 64 500 g/mol. The number of hemoglobin molecules per mol is Avogadro's number, or $6.022 \times 10^{23}$ mol$^{-1}$. Therefore, one molecule of hemoglobin has a mass (in kg) of

$$
\left( \frac{64500 \text{ g/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} \right) \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) = 1.07 \times 10^{-22} \text{ kg}
$$

2. **REASONING** The number of molecules in a known mass of material is the number $n$ of moles of the material times the number $N_A$ of molecules per mole (Avogadro's number). We can find the number of moles by dividing the known mass $m$ by the mass per mole.

**SOLUTION** Using the periodic table on the inside of the text’s back cover, we find that the molecular mass of Tylenol ($C_8H_9NO_2$) is

Molecular mass of Tylenol $= 8(12.011 \text{ u}) + 9(1.00794 \text{ u})$ Mass of 8 carbon atoms Mass of 9 hydrogen atoms
+ $14.0067 \text{ u} + 2(15.9994) = 151.165 \text{ u}$ Mass of 1 nitrogen atom Mass of 2 oxygen atoms

The molecular mass of Advil ($C_{13}H_{18}O_2$) is

Molecular mass of Advil $= 13(12.011 \text{ u}) + 18(1.00794 \text{ u}) + 2(15.9994) = 206.285 \text{ u}$ Mass of 13 carbon atoms Mass of 18 hydrogen atoms Mass of 2 oxygen atoms

a. Therefore, the number of molecules of pain reliever in the standard dose of Tylenol is

$$
\text{Number of molecules} = n N_A = \left( \frac{m}{\text{Mass per mole}} \right) N_A
$$

$$
= \left( \frac{325 \times 10^{-3} \text{ g}}{151.165 \text{ g/mol}} \right) \left( 6.022 \times 10^{23} \text{ mol}^{-1} \right) = 1.29 \times 10^{21}
$$
b. Similarly, the number of molecules of pain reliever in the standard dose of Advil is

\[
\text{Number of molecules} = n N_A = \left( \frac{m}{\text{Mass per mole}} \right) N_A
\]

\[
= \left( \frac{2.00 \times 10^{-1}}{206.285 \text{ g/mol}} \right) (6.022 \times 10^{23} \text{ mol}^{-1}) = 5.84 \times 10^{20}
\]

3. **REASONING AND SOLUTION** The number \( n \) of moles contained in a sample is equal to the number \( N \) of atoms in the sample divided by the number \( N_A \) of atoms per mole (Avogadro’s number):

\[
n = \frac{N}{N_A} = \frac{30.1 \times 10^{23}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 5.00 \text{ mol}
\]

Since the sample has a mass of 135 g, the mass per mole is

\[
\frac{135 \text{ g}}{5.00 \text{ mol}} = 27.0 \text{ g/mol}
\]

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic mass of the substance. Therefore, the atomic mass is 27.0 u. The periodic table of the elements reveals that the unknown element is **aluminum**.

4. **REASONING** The mass of one of its atoms (in atomic mass units) has the same numerical value as the element’s mass per mole (in units of g/mol). Atomic mass units can be converted into kilograms using the fact that 1 u = 1.6605 \times 10^{-27} \text{ kg}. Dividing the mass of the sample by the mass per mole gives the number of moles of atoms in the sample.

**SOLUTION**

a. The mass per mole is 196.967 g/mol. Since the mass of one of its atoms (in atomic mass units) has the same numerical value as the mass per mole, the mass of a single atom is

\[
m = 196.967 \text{ u}
\]

b. To convert the mass from atomic mass units to kilograms, we use the conversion factor of 1 u = 1.6605 \times 10^{-27} \text{ kg}:

\[
m = (196.967 \text{ u}) \left( \frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) = 3.2706 \times 10^{-25} \text{ kg}
\]

c. The number \( n \) of moles of atoms is equal to the mass \( m \) divided by the mass per mole:

\[
n = \frac{m}{\text{Mass per mole}} = \frac{285 \text{ g}}{196.967 \text{ g/mol}} = 1.45 \text{ mol}
\]
5. **SSM REASONING** The mass (in grams) of the active ingredient in the standard dosage is the number of molecules in the dosage times the mass per molecule (in grams per molecule). The mass per molecule can be obtained by dividing the molecular mass (in grams per mole) by Avogadro’s number. The molecular mass is the sum of the atomic masses of the molecule’s atomic constituents.

**SOLUTION** Using $N$ to denote the number of molecules in the standard dosage and $m_{\text{molecule}}$ to denote the mass of one molecule, the mass (in grams) of the active ingredient in the standard dosage can be written as follows:

$$m = N m_{\text{molecule}}$$

Using $M$ to denote the molecular mass (in grams per mole) and recognizing that

$$m_{\text{molecule}} = \frac{M}{N_A},$$

where $N_A$ is Avogadro’s number and is the number of molecules per mole, we have

$$m = N m_{\text{molecule}} = N \left( \frac{M}{N_A} \right)$$

$M$ (in grams per mole) is equal to the molecular mass in atomic mass units. We can obtain this quantity by referring to the periodic table on the inside of the back cover of the text to find the molecular masses of the constituent atoms in the active ingredient. Thus, we have

Molecular mass = $22(12.011 \text{ u}) + 23(1.00794 \text{ u}) + 1(35.453 \text{ u}) + 2(14.0067 \text{ u}) + 2(15.9994 \text{ u})$

= 382.89 u

The mass of the active ingredient in the standard dosage is

\[
 m = N \left( \frac{M}{N_A} \right) = \left( 1.572 \times 10^{19} \text{ molecules} \right) \left( \frac{382.89 \text{ g/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} \right) = 1.00 \times 10^{-2} \text{ g}
\]

6. **REASONING** To find the molecular mass of chlorophyll-$a$ ($C_{55}H_{72}MgN_4O_5$) in atomic mass units, we use the number and atomic mass of each constituent atom in the molecule (see the periodic table of the elements on the inside of the back cover of the text). The molecular mass obtained in this fashion is the mass per mole in g/mol, which we can then use to obtain the mass (in grams) of 3.00 moles of chlorophyll-$a$.

**SOLUTION**

a. The molecular mass of $C_{55}H_{72}MgN_4O_5$ is
b. The molecular mass indicates that the mass per mol is 893.505 g/mol. Therefore, the mass of 3.00 mol is

\[
(893.505 \text{ g/mol})(3.00 \text{ mol}) = 2680 \text{ g}
\]

7. **REASONING**

a. The number \( n \) of moles of water is equal to the mass \( m \) of water divided by the mass per mole: \( n = m/(\text{Mass per mole}) \). The mass per mole (in g/mol) of water has the same numerical value as its molecular mass (which we can determine). According to Equation 4.5, the total mass of the runner is equal to her weight \( W \) divided by the magnitude \( g \) of the acceleration due to gravity, or \( W/g \). Since 71% of the runner’s total mass is water, the mass of water is \( m = (0.71)W/g \).

b. The number \( N \) of water molecules is the product of the number \( n \) of moles and Avogadro’s number \( N_A \) (which is the number of molecules per mole), or \( N = nN_A \).

**SOLUTION**

a. Starting with \( n = m/(\text{Mass per mole}) \) and substituting in the relation \( m = (0.71)W/g \), we have

\[
n = \frac{m}{\text{Mass per mole}} = \frac{(0.71)W}{g} \quad \text{(Mass per mole)}
\]

The molecular mass of water (H\(_2\)O) is 2 (1.00794 u) + 15.9994 u = 18.0153 u. The mass per mole is then 18.0153 g/mol. However, we need to convert this value into kilograms per mole for use in Equation (1). This is because the value for the weight \( W \) in Equation (1) is given in newtons (N), which is a SI unit. The SI unit for mass is the kilogram (kg), not the gram (g). Converting the mass per mole value to kilograms per mole gives

\[
\text{Mass per mole} = 18.0153 \frac{\text{g}}{\text{mol}} = \left(18.0153 \frac{\text{g}}{\text{mol}}\right) \left(1 \text{ kg} \frac{10^3 \text{ g}}{1 \text{ kg}}\right)
\]

Substituting this relation into Equation 1 gives

\[
n = \frac{(0.71)W}{g} = \frac{(0.71)(580 \text{ N})}{9.80 \text{ m/s}^2} = \frac{2.3 \times 10^3 \text{ mol}}{18.0153 \frac{\text{g}}{\text{mol}}} = 2.3 \times 10^3 \text{ mol}
\]
b. The number of water molecules in the runner’s body is

\[ N = nN_A = (2.3 \times 10^3 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1}) = 1.4 \times 10^{27} \]

8. **REASONING** The number \( n \) of moles of a species can be calculated from the mass \( m \) (in grams) of the species and its molecular mass, or mass per mole \( M \) (in grams per mole), according to \( n = \frac{m}{M} \). To calculate the percentage, we divide the number \( n \) of moles of that species by the total number \( n_{\text{Total}} \) of moles in the mixture and multiply that fraction by 100%. The total number of moles is the sum of the numbers of moles for each component.

The component with the greatest number of moles has the greatest percentage. For the three components described, this would be helium, because it has the greatest mass and the smallest mass per mole. The component with the smallest number of moles has the smallest percentage. For the three components described, this would be argon, because it has the smallest mass and the greatest mass per mole.

**SOLUTION** The percentage \( p_{\text{Argon}} \) of argon is

\[
p_{\text{Argon}} = \frac{n_{\text{Argon}}}{n_{\text{Argon}} + n_{\text{Neon}} + n_{\text{Helium}}} \times 100 = \frac{m_{\text{Argon}}}{M_{\text{Argon}}} \times 100
\]

\[
= \frac{1.20 \text{ g}}{39.948 \text{ g/mol}} \times 100 = 3.1\%
\]

The percentage of neon is

\[
p_{\text{Neon}} = \frac{n_{\text{Neon}}}{n_{\text{Argon}} + n_{\text{Neon}} + n_{\text{Helium}}} \times 100 = \frac{m_{\text{Neon}}}{M_{\text{Neon}}} \times 100
\]

\[
= \frac{2.60 \text{ g}}{20.180 \text{ g/mol}} \times 100 = 13.4\%
\]
The percentage of helium is

\[ p_{\text{Helium}} = \frac{n_{\text{Helium}}}{n_{\text{Argon}} + n_{\text{Neon}} + n_{\text{Helium}}} \times 100 = \frac{\frac{m_{\text{Helium}}}{M_{\text{Helium}}}}{\frac{m_{\text{Argon}}}{M_{\text{Argon}}} + \frac{m_{\text{Neon}}}{M_{\text{Neon}}} + \frac{m_{\text{Helium}}}{M_{\text{Helium}}}} \times 100 \]

\[ = \frac{3.20 \text{ g}}{4.0026 \text{ g/mol}} + \frac{2.60 \text{ g}}{20.180 \text{ g/mol}} + \frac{3.20 \text{ g}}{39.948 \text{ g/mol}} \times 100 = 83.4\% \]

9. **SSM REASONING** The initial solution contains \( N_0 \) molecules of arsenic trioxide, and this number is reduced by a factor of 100 with each dilution. After one dilution, in other words, the number of arsenic trioxide molecules in the solution is \( N_1 = \frac{N_0}{100} \). After 2 dilutions, the number remaining is \( N_2 = \frac{N_1}{100} = N_0/100^2 \). Therefore, after \( d \) dilutions, the number \( N \) of arsenic trioxide molecules that remain in the solution is given by

\[ N = \frac{N_0}{100^d} \]  

(1)

We seek the maximum value of \( d \) for which at least one molecule of arsenic trioxide remains. Setting \( N = 1 \) in Equation (1), we obtain \( 100^d = N_0 \). Taking the common logarithm of both sides yields \( \log(100^d) = \log N_0 \), which we solve for \( d \):

\[ d \log 100 = \log N_0 \quad \text{or} \quad 2d = \log N_0 \quad \text{or} \quad d = \frac{1}{2} \log N_0 \]  

(2)

The initial number \( N_0 \) of molecules of arsenic trioxide in the undiluted solution depends upon the number \( n_0 \) of moles of the substance present via \( N_0 = n_0 N_A \), where \( N_A \) is Avogadro’s number. Because we know the original mass \( m \) of the arsenic trioxide, we can find the number \( n_0 \) of moles from the relation \( n_0 = \frac{m}{\text{Mass per mole}} \). We will use the chemical formula \( \text{As}_2\text{O}_3 \) for arsenic trioxide, and the periodic table found on the inside of the back cover of the textbook to determine the mass per mole of arsenic trioxide.

**SOLUTION** The mass per mole of arsenic trioxide \( (\text{As}_2\text{O}_3) \) is the sum of the atomic masses of its constituent atoms. There are two atoms of arsenic and three atoms of oxygen per molecule, so we have
Chapter 14 Problems

**Mass of arsenic atoms**

\[
\text{Molecular mass} = 2\left(74.9216 \text{ u}\right) + 3\left(15.9994 \text{ u}\right) = 197.8414 \text{ u}
\]

Therefore, the mass per mole of arsenic trioxide is 197.8414 g/mol. From the relations \(N_0 = n_0N_A\) and \(n_0 = \frac{m}{\text{Mass per mole}}\), we see that the initial number of arsenic trioxide molecules in the sample is

\[
N_0 = n_0N_A = \left(\frac{m}{\text{Mass per mole}}\right)N_A = \frac{mN_A}{\text{Mass per mole}}
\]

(3)

Substituting Equation (3) into Equation (2), we obtain

\[
d = \frac{1}{2}\log N_0 = \frac{1}{2}\log \left(\frac{mN_A}{\text{Mass per mole}}\right) = \frac{1}{2}\log \left[\frac{\left(18.0 \text{ g}\right)\left(6.022 \times 10^{23} \text{ mol}^{-1}\right)}{197.8414 \text{ g/mol}}\right] = 11.4
\]

It is only possible to conduct integral numbers of dilutions, so we conclude from this result that \(d = 11\) dilutions result in a solution with at least one molecule of arsenic trioxide remaining, while \(d = 12\) dilutions yield a solution in which there may or may not be any molecules of arsenic trioxide present. Thus, the original solution may undergo at most 11 dilutions.

**Reasoning** The number \(n\) of moles of water molecules in the glass is equal to the mass \(m\) of water divided by the mass per mole. According to Equation 11.1, the mass of water is equal to its density \(\rho\) times its volume \(V\). Thus, we have

\[
n = \frac{m}{\text{Mass per mole}} = \frac{\rho V}{\text{Mass per mole}}
\]

The volume of the cylindrical glass is \(V = \pi r^2h\), where \(r\) is the radius of the cylinder and \(h\) is its height. The number of moles of water can be written as

\[
n = \frac{\rho V}{\text{Mass per mole}} = \frac{\rho \left(\pi r^2h\right)}{\text{Mass per mole}}
\]

**Solution** The molecular mass of water (H\(_2\)O) is \(2(1.00794 \text{ u}) + (15.9994 \text{ u}) = 18.0 \text{ u}\). The mass per mole of H\(_2\)O is 18.0 g/mol. The density of water (see Table 11.1) is \(1.00 \times 10^3 \text{ kg/m}^3\) or \(1.00 \text{ g/cm}^3\).

\[
n = \frac{\rho \left(\pi r^2h\right)}{\text{Mass per mole}} = \frac{\left(1.00 \text{ g/cm}^3\right)\left(\pi\right)\left(4.50 \text{ cm}\right)^2\left(12.0 \text{ cm}\right)}{18.0 \text{ g/mol}} = 42.4 \text{ mol}
\]
11. **REASONING** Both gases fill the balloon to the same pressure $P$, volume $V$, and temperature $T$. Assuming that both gases are ideal, we can apply the ideal gas law $PV = nRT$ to each and conclude that the same number of moles $n$ of each gas is needed to fill the balloon. Furthermore, the number of moles can be calculated from the mass $m$ (in grams) and the mass per mole $M$ (in grams per mole), according to $n = \frac{m}{M}$. Using this expression in the equation $n_{\text{Helium}} = n_{\text{Nitrogen}}$ will allow us to obtain the desired mass of nitrogen.

**SOLUTION** Since the number of moles of helium equals the number of moles of nitrogen, we have

\[
\frac{m_{\text{Helium}}}{M_{\text{Helium}}} = \frac{m_{\text{Nitrogen}}}{M_{\text{Nitrogen}}}
\]

Solving for $m_{\text{Nitrogen}}$ and taking the values of mass per mole for helium (He) and nitrogen ($N_2$) from the periodic table on the inside of the back cover of the text, we find

\[
m_{\text{Nitrogen}} = \frac{M_{\text{Nitrogen}}m_{\text{Helium}}}{M_{\text{Helium}}} = \frac{(28.0 \text{ g/mol})(0.16 \text{ g})}{4.00 \text{ g/mol}} = 1.1 \text{ g}
\]

12. **REASONING** The pressure $P$ of the water vapor in the container can be found from the ideal gas law, Equation 14.1, as $P = \frac{nRT}{V}$, where $n$ is the number of moles of water, $R$ is the universal gas constant, $T$ is the Kelvin temperature, and $V$ is the volume. The variables $R$, $T$, and $V$ are known, and the number of moles can be obtained by noting that it is equal to the mass $m$ of the water divided by its mass per mole.

**SOLUTION** Substituting $n = \frac{m}{(\text{Mass per mole})}$ into the ideal gas law, we have

\[
P = \frac{nRT}{V} = \frac{\left(\frac{m}{\text{Mass per mole}}\right)RT}{V}
\]

The mass per mole (in g/mol) of water (H$_2$O) has the same numerical value as its molecular mass. The molecular mass of water is $2(1.00794 \text{ u}) + 15.9994 \text{ u} = 18.0153 \text{ u}$. The mass per mole of water is $18.0153 \text{ g/mol}$. Thus, the pressure of the water vapor is

\[
P = \frac{\left(\frac{m}{\text{Mass per mole}}\right)RT}{V} = \frac{\left(\frac{4.0 \text{ g}}{18.0153 \text{ g/mol}}\right)[8.31 \text{ J/(mol·K)}](388 \text{ K})}{0.030 \text{ m}^3} = 2.4 \times 10^4 \text{ Pa}
\]
13. **REASONING** Since the absolute pressure, volume, and temperature are known, we may use the ideal gas law in the form of Equation 14.1 to find the number of moles of gas. When the volume and temperature are raised, the new pressure can also be determined by using the ideal gas law.

**SOLUTION**

a. The number of moles of gas is

\[
n = \frac{PV}{RT} = \frac{\left(1.72 \times 10^5 \text{ Pa}\right)\left(2.81 \text{ m}^3\right)}{8.31 \text{ J/}(\text{mol} \cdot \text{K})\left[(273.15 + 15.5) \text{ K}\right]} = 201 \text{ mol}
\]

b. When the volume is raised to 4.16 m³ and the temperature raised to 28.2 °C, the pressure of the gas is

\[
P = \frac{nRT}{V} = \frac{(201 \text{ mol})\left[8.31 \text{ J/}(\text{mol} \cdot \text{K})\right]\left[(273.15 + 28.2) \text{ K}\right]}{(4.16 \text{ m}^3)} = 1.21 \times 10^5 \text{ Pa}
\]

14. **REASONING** According to the ideal gas law, \( PV = nRT \), the temperature \( T \) is directly proportional to the product \( PV \), for a fixed number \( n \) of moles. Therefore, tanks with equal values of \( PV \) have the same temperature. Using the data in the table given with the problem statement, we see that the values of \( PV \) for each tank are (starting with tank A): 100 Pa · m³, 150 Pa · m³, 100 Pa · m³, and 150 Pa · m³. Tanks A and C have the same temperature, while B and D have the same temperature.

**SOLUTION** The temperature of each gas can be found from the ideal gas law, Equation 14.1:

\[
T_A = \frac{P_AV_A}{nR} = \frac{(25.0 \text{ Pa})\left(4.0 \text{ m}^3\right)}{(0.10 \text{ mol})\left[8.31 \text{ J/}(\text{mol} \cdot \text{K})\right]} = 120 \text{ K}
\]

\[
T_B = \frac{P_BV_B}{nR} = \frac{(30.0 \text{ Pa})\left(5.0 \text{ m}^3\right)}{(0.10 \text{ mol})\left[8.31 \text{ J/}(\text{mol} \cdot \text{K})\right]} = 180 \text{ K}
\]

\[
T_C = \frac{P_CV_C}{nR} = \frac{(20.0 \text{ Pa})\left(5.0 \text{ m}^3\right)}{(0.10 \text{ mol})\left[8.31 \text{ J/}(\text{mol} \cdot \text{K})\right]} = 120 \text{ K}
\]

\[
T_D = \frac{P_DV_D}{nR} = \frac{(2.0 \text{ Pa})\left(75 \text{ m}^3\right)}{(0.10 \text{ mol})\left[8.31 \text{ J/}(\text{mol} \cdot \text{K})\right]} = 180 \text{ K}
\]
15. **REASONING AND SOLUTION** According to the ideal gas law (Equation 14.1), the total number of moles \( n \) of fresh air in a normal breath is

\[
 n = \frac{PV}{RT} = \frac{(1.0 \times 10^5 \text{ Pa})(5.0 \times 10^{-4} \text{ m}^3)}{[8.31 \text{ J/(mol} \cdot \text{K})](310 \text{ K})} = 1.94 \times 10^{-2} \text{ mol}
\]

The total number of molecules in a normal breath is \( nN_A \), where \( N_A \) is Avogadro’s number. Since fresh air contains approximately 21% oxygen, the total number of oxygen molecules in a normal breath is \((0.21)nN_A\) or

\[(0.21)(1.94 \times 10^{-2} \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1}) = 2.5 \times 10^{21} \]

16. **REASONING** We can use the ideal gas law, Equation 14.1 \((PV = nRT)\) to find the number of moles of helium in the Goodyear blimp, since the pressure, volume, and temperature are known. Once the number of moles is known, we can find the mass of helium in the blimp.

**SOLUTION** The number \( n \) of moles of helium in the blimp is, according to Equation 14.1,

\[
 n = \frac{PV}{RT} = \frac{(1.1 \times 10^5 \text{ Pa})(5400 \text{ m}^3)}{[8.31 \text{ J/(mol} \cdot \text{K})](280 \text{ K})} = 2.55 \times 10^5 \text{ mol}
\]

According to the periodic table on the inside of the text’s back cover, the atomic mass of helium is 4.002 60 u. Therefore, the mass per mole is 4.002 60 g/mol. The mass \( m \) of helium in the blimp is, then,

\[
m = \left(2.55 \times 10^5 \text{ mol}\right)(4.002 60 \text{ g/mol})\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 1.0 \times 10^3 \text{ kg}
\]

17. **REASONING** The maximum number of balloons that can be filled is the volume of helium available at the pressure in the balloons divided by the volume per balloon. The volume of helium available at the pressure in the balloons can be determined using the ideal gas law. Since the temperature remains constant, the ideal gas law indicates that \( PV = nRT \) = constant, and we can apply it in the form of Boyle’s law, \( P_iV_i = P_fV_f \). In this expression \( V_f \) is the final volume at the pressure in the balloons, \( V_i \) is the volume of the cylinder, \( P_i \) is the initial pressure in the cylinder, and \( P_f \) is the pressure in the balloons. However, we need to remember that a volume of helium equal to the volume of the cylinder will remain in the cylinder when its pressure is reduced to atmospheric pressure at the point when balloons can no longer be filled.

**SOLUTION** Using Boyle’s law we find that

\[
 V_f = \frac{PV_i}{P_f}
\]
The volume of helium available for filling balloons is

\[ V_f = 0.0031 \text{ m}^3 = \frac{P V_1}{P_f} - 0.0031 \text{ m}^3 \]

The maximum number of balloons that can be filled is

\[ N_{\text{Balloon}} = \frac{P V_1}{P_f} - 0.0031 \text{ m}^3 \times \frac{1.6 \times 10^7 \text{ Pa}}{1.2 \times 10^5 \text{ Pa} / 0.034 \text{ m}^3} = 12 \]

18. **REASONING** According to the ideal gas law, \( PV = nRT \), the absolute pressure \( P \) is directly proportional to the temperature \( T \) when the volume is held constant, provided that the temperature is measured on the Kelvin scale. The pressure is not proportional to the Celsius temperature.

**SOLUTION**

a. According to Equation 14.1, the pressures at the two temperatures are

\[ P_1 = \frac{n R T_1}{V} \quad \text{and} \quad P_2 = \frac{n R T_2}{V} \]

Taking the ratio \( P_2/P_1 \) of the final pressure to the initial pressure gives

\[ \frac{P_2}{P_1} = \frac{n R T_2}{n R T_1} = \frac{T_2}{T_1} = \frac{70.0 \text{ K}}{35.0 \text{ K}} = 2.00 \]

b. Before determining the ratio of the pressures, we must convert the Celsius temperature scale to the Kelvin temperature scale. Thus, the ratio of the pressures at the temperatures of 35.0 °C and 70.0 °C is

\[ \frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{273.15 + 70.0}{273.15 + 35.0} = 1.11 \]

19. **SSM REASONING** According to Equation 11.1, the mass density \( \rho \) of a substance is defined as its mass \( m \) divided by its volume \( V \): \( \rho = m/V \). The mass of nitrogen is equal to the number \( n \) of moles of nitrogen times its mass per mole: \( m = n \text{(Mass per mole)} \). The number of moles can be obtained from the ideal gas law (see Equation 14.1) as \( n = (PV)/(RT) \). The mass per mole (in g/mol) of nitrogen has the same numerical value as its molecular mass (which we know).

**SOLUTION** Substituting \( m = n \text{(Mass per mole)} \) into \( \rho = m/V \), we obtain
\[ \rho = \frac{m}{V} = \frac{n (\text{Mass per mole})}{V} \]  

(1)

Substituting \( n = \frac{PV}{RT} \) from the ideal gas law into Equation 1 gives the following result:

\[ \rho = \frac{n (\text{Mass per mole})}{V} = \frac{\left( \frac{P}{RT} \right) (\text{Mass per mole})}{\dot{\lambda}} = \frac{P (\text{Mass per mole})}{RT} \]

The pressure is 2.0 atmospheres, or \( P = 2 \times 1.013 \times 10^5 \text{ Pa} \). The molecular mass of nitrogen is given as 28 u, which means that its mass per mole is 28 g/mol. Expressed in terms of kilograms per mol, the mass per mole is

\[ \text{Mass per mole} = \left( 28 \frac{\text{g}}{\text{mol}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \]

The density of the nitrogen gas is

\[ \rho = \frac{P (\text{Mass per mole})}{RT} = \frac{2 \times 1.013 \times 10^5 \text{ Pa} \left( 28 \frac{\text{g}}{\text{mol}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right)}{[8.31 \text{ J}/(\text{mol} \cdot \text{K})](310 \text{ K})} = 2.2 \text{ kg/m}^3 \]

20. **REASONING** The ideal gas law is \( PV = nRT \) (Equation 14.1). We need to put this in terms of the mass density \( \rho = \frac{m}{V} \) (Equation 11.1). We can then set \( P_{\text{He}} = P_{\text{Ne}} \), and solve the resulting expression for \( T_{\text{Ne}} \), the temperature of the neon.

**SOLUTION** We begin by writing Equation 14.1 in terms of the mass density \( \rho \) for an ideal gas. Recall that the number \( n \) of moles of a substance is equal to its mass \( m \) in grams, divided by its mass per mole \( M \). Therefore, \( PV = mRT / M \), and we have \( P = (m/V)RT / M = \rho RT / M \), where \( \rho \) is the same for each gas. The two gases have the same absolute pressures, so that \( P_{\text{He}} = P_{\text{Ne}} \), and it follows that

\[ \frac{\rho RT_{\text{He}}}{M_{\text{He}}} = \frac{\rho RT_{\text{Ne}}}{M_{\text{Ne}}} \]

The term \( \rho R \) can be eliminated algebraically from this result. Solving for the temperature of the neon \( T_{\text{Ne}} \) and using the mass per mole for helium (4.0026 g/mol) and neon (20.180 g/mol) from the periodic table on the inside of the back cover of the text, we find

\[ T_{\text{Ne}} = T_{\text{He}} \left( \frac{M_{\text{Ne}}}{M_{\text{He}}} \right) = (175 \text{ K}) \left( \frac{20.180 \text{ g/mol}}{4.0026 \text{ g/mol}} \right) = 882 \text{ K} \]
21. **REASONING** According to Equation 14.2, \( PV = NkT \), where \( P \) is the pressure, \( V \) is the volume, \( N \) is the number of molecules in the sample, \( k \) is Boltzmann's constant, and \( T \) is the Kelvin temperature. The number of gas molecules per unit volume in the atmosphere is \( N/V = P/(kT) \). This can be used to find the desired ratio for the two planets.

**SOLUTION** We have

\[
\frac{(N/V)_{Venus}}{(N/V)_{Earth}} = \frac{P_{Venus}}{P_{Earth}} \left( \frac{kT_{Venus}}{kT_{Earth}} \right) = \frac{9.0 \times 10^6 \text{ Pa}}{1.0 \times 10^5 \text{ Pa}} \left( \frac{320 \text{ K}}{740 \text{ K}} \right) = 39
\]

Thus, we can conclude that the atmosphere of Venus is 39 times "thicker" than that of Earth.

22. **REASONING** The mass \( m \) of nitrogen that must be removed from the tank is equal to the number of moles withdrawn times the mass per mole. The number of moles withdrawn is the initial number \( n_i \) minus the final number of moles \( n_f \), so we can write

\[
m = \left( n_i - n_f \right) \text{(Mass per mole)}
\]

The final number of moles is related to the initial number by the ideal gas law.

**SOLUTION** From the ideal gas law (Equation 14.1), we have

\[
n_f = \frac{P_f V}{RT} \quad \text{and} \quad n_i = \frac{P_i V}{RT}
\]

Note that the volume \( V \) and temperature \( T \) do not change. Dividing the first by the second equation gives

\[
\frac{n_f}{n_i} = \frac{P_f V}{RT} \left( \frac{P_i}{P_f} \right) \quad \text{or} \quad n_f = n_i \left( \frac{P_f}{P_i} \right)
\]

Substituting this expression for \( n_f \) into Equation 1 gives

\[
m = \left( n_i - n_f \right) \text{(Mass per mole)} = \left[ n_i - n_i \left( \frac{P_f}{P_i} \right) \right] \text{(Mass per mole)}
\]

The molecular mass of nitrogen (\( \text{N}_2 \)) is 2 (14.0067 u) = 28.0134 u. Therefore, the mass per mole is 28.0134 g/mol. The mass of nitrogen that must be removed is

\[
m = \left[ n_i - n_i \left( \frac{P_f}{P_i} \right) \right] \text{(Mass per mole)} = \left[ 0.85 \text{ mol} - (0.85 \text{ mol}) \left( \frac{25 \text{ atm}}{38 \text{ atm}} \right) \right] \left( 28.0134 \text{ g/mol} \right) = 8.1 \text{ g}
\]
23. **REASONING** Since the temperature of the confined air is constant, Boyle's law applies, and $P_{\text{surface}} V_{\text{surface}} = P_h V_h$, where $P_{\text{surface}}$ and $V_{\text{surface}}$ are the pressure and volume of the air in the tank when the tank is at the surface of the water, and $P_h$ and $V_h$ are the pressure and volume of the trapped air after the tank has been lowered a distance $h$ below the surface of the water. Since the tank is completely filled with air at the surface, $V_{\text{surface}}$ is equal to the volume $V_{\text{tank}}$ of the tank. Therefore, the fraction of the tank's volume that is filled with air when the tank is a distance $h$ below the water's surface is

$$\frac{V_h}{V_{\text{tank}}} = \frac{V_h}{V_{\text{surface}}} = \frac{P_{\text{surface}}}{P_h}.$$

We can find the absolute pressure at a depth $h$ using Equation 11.4. Once the absolute pressure is known at a depth $h$, we can determine the ratio of the pressure at the surface to the pressure at the depth $h$.

**SOLUTION** According to Equation 11.4, the trapped air pressure at a depth $h = 40.0$ m is

$$P_h = P_{\text{surface}} + \rho gh = (1.01 \times 10^5 \text{ Pa}) + \left[ (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(40.0 \text{ m}) \right] = 4.93 \times 10^5 \text{ Pa},$$

where we have used a value of $\rho = 1.00 \times 10^3 \text{ kg/m}^3$ for the density of water. The desired volume fraction is

$$\frac{V_h}{V_{\text{tank}}} = \frac{P_{\text{surface}}}{P_h} = \frac{1.01 \times 10^5 \text{ Pa}}{4.93 \times 10^5 \text{ Pa}} = 0.205.$$

24. **REASONING**

a. The volume $V$ of gas in the tank is related to the Kelvin temperature $T$ and absolute pressure $P$ of the gas by $V = nRT / P$ (Equation 14.1), where $n$ is the number of moles of gas and $R$ is the universal gas constant.

b. The mass of chlorine gas that has leaked out of the tank is equal to the initial mass (11.0 g) minus the mass remaining in the tank at the later time. The mass at the later time is equal to the mass per mole (in g/mol) of chlorine times the number $n$ of moles remaining in the tank. The number of moles of gas remaining can be found from the ideal gas law, $n = PV / (RT)$ (Equation 14.1).

**SOLUTION**

a. The number $n$ of moles of chlorine originally in the tank is equal to the mass of chlorine gas (11.0 g) divided by the mass per mole of chlorine gas (70.9 g/mol), so $n = (11.0 \text{ g}) / (70.9 \text{ g/mol})$. The temperature of the gas is $82 \ ^\circ\text{C}$, which, when converted to the Kelvin temperature scale, is $(82 + 273) \text{ K}$. The volume of the tank is
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\[ V = \frac{nRT}{P} = \frac{\left( \frac{11.0 \text{ g}}{70.9 \text{ g/mol}} \right) \left[ \frac{8.31 \text{ J/(mol \cdot K)}}{82 + 273} \right]}{5.60 \times 10^5 \text{ Pa}} = 8.17 \times 10^{-4} \text{ m}^3 \]  

(14.1)

b. The mass \( m_{\text{Cl}} \) of chlorine gas that has leaked out of the tank is \( m_{\text{Cl}} = 11.0 \text{ g} - m_{\text{R}} \), where \( m_{\text{R}} \) is the mass of the gas remaining in the tank. The remaining mass, in turn, is equal to the number \( n_{\text{R}} \) of moles of gas remaining in the tank times the mass per mole (70.9 g/mol) of chlorine gas. Thus,

\[ m_{\text{Cl}} = 11.0 \text{ g} - m_{\text{R}} = 11.0 \text{ g} - n_{\text{R}} \left( 70.9 \text{ g/mol} \right) \]

The number of remaining moles of gas can be found from the ideal gas law, \( n_{\text{R}} = \frac{PV}{RT} \), so that

\[ m_{\text{Cl}} = 11.0 \text{ g} - \left( \frac{PV}{RT} \right) \left( 70.9 \text{ g/mol} \right) \]

\[ = 11.0 \text{ g} - \left( \frac{3.80 \times 10^5 \text{ Pa}}{8.31 \text{ J/(mol \cdot K)}} \right) \left( 8.17 \times 10^{-4} \text{ m}^3 \right) \left( 70.9 \text{ g/mol} \right) = 2.3 \text{ g} \]

25. **REASONING** The mass (in grams) of the air in the room is the mass of the nitrogen plus the mass of the oxygen. The mass of the nitrogen is the number of moles of nitrogen times the molecular mass (in grams/mol) of nitrogen. The mass of the oxygen can be obtained in a similar way. The number of moles of each species can be found by using the given percentages and the total number of moles. To obtain the total number of moles, we will apply the ideal gas law. If we substitute the Kelvin temperature \( T \), the pressure \( P \), and the volume \( V \) (length \( \times \) width \( \times \) height) of the room in the ideal gas law, we can obtain the total number \( n_{\text{Total}} \) of moles of gas as \( n_{\text{Total}} = \frac{PV}{RT} \), because the ideal gas law does not distinguish between types of ideal gases.

**SOLUTION** Using \( m \) to denote the mass (in grams), \( n \) to denote the number of moles, and \( M \) to denote the molecular mass (in grams/mol), we can write the mass of the air in the room as follows:

\[ m = m_{\text{Nitrogen}} + m_{\text{Oxygen}} = n_{\text{Nitrogen}} M_{\text{Nitrogen}} + n_{\text{Oxygen}} M_{\text{Oxygen}} \]

We can now express this result using \( f \) to denote the fraction of a species that is present and \( n_{\text{Total}} \) to denote the total number of moles:

\[ m = n_{\text{Nitrogen}} M_{\text{Nitrogen}} + n_{\text{Oxygen}} M_{\text{Oxygen}} = f_{\text{Nitrogen}} n_{\text{Total}} M_{\text{Nitrogen}} + f_{\text{Oxygen}} n_{\text{Total}} M_{\text{Oxygen}} \]

According to the ideal gas law, we have \( n_{\text{Total}} = \frac{PV}{RT} \). With this substitution, the mass of the air becomes
The mass per mole for nitrogen (N\textsubscript{2}) and for oxygen (O\textsubscript{2}) can be obtained from the periodic table on the inside of the back cover of the text. They are, respectively, 28.0 and 32.0 g/mol. The temperature of 22 ºC must be expressed on the Kelvin scale as 295 K (see Equation 12.1). The mass of the air is, then,

\[
m = \left( f_{\text{Nitrogen}}M_{\text{Nitrogen}} + f_{\text{Oxygen}}M_{\text{Oxygen}} \right) \frac{PV}{RT}
\]

\[
= \left[ 0.79 \left( 28.0 \ \text{g/mol} \right) + 0.21 \left( 32.0 \ \text{g/mol} \right) \right] \left[ \frac{1.01 \times 10^5 \text{ Pa}}{8.31 \text{ J/(mol·K)}} \right] \left[ (2.5 \text{ m})(4.0 \text{ m})(5.0 \text{ m}) \right] = 5.9 \times 10^4 \text{ g}
\]

26. **REASONING** The ideal gas law is \( PV = nRT \). Since the number of moles is constant, this equation can be written as \( \frac{PV}{T} = nR = \text{constant} \). Thus, the value of \( \frac{PV}{T} \) is the same initially and finally, and we can write

\[
\frac{P_0V_0}{T_0} = \frac{P_fV_f}{T_f}
\]

This expression can be solved for the final temperature \( T_f \).

We have no direct data for the initial and final pressures. However, we can deal with this by realizing that pressure is the magnitude of the force applied perpendicularly to a surface divided by the area of the surface. Thus, the magnitudes of the forces that the initial and final pressures apply to the piston (and, therefore, to the spring) are given by Equation 11.3 as

\[
\begin{align*}
F_0 &= P_0A \\
F_f &= P_fA
\end{align*}
\]

The forces in Equations (2) are applied to the spring, and the force \( F_{x}^{\text{Applied}} \) that must be applied to stretch an ideal spring by an amount \( x \) with respect to its unstrained length is given by Equation 10.1 as

\[
F_{x}^{\text{Applied}} = kx
\]

where \( k \) is the spring constant.

Lastly, we note that the final volume is the initial volume plus the amount by which the volume increases as the spring stretches. The increased volume due to the additional stretching is \( A(x_f - x_0) \). Therefore, we have
\[ V_f = V_0 + A(x_f - x_0) \]  \hspace{1cm} (4)

**SOLUTION**  The final temperature can be obtained by rearranging Equation (1) to show that

\[ T_f = \left( \frac{P V_f}{P_0 V_0} \right) T_0 \]  \hspace{1cm} (5)

Into this result we can now substitute expressions for \( P_0 \) and \( P_f \). These expressions can be obtained by using Equations (2) in Equation (3) as follows (and recognizing that, for the initial and final forces, \( P_0 A = F_{x \text{Applied}} \) and \( P_f A = F_{x \text{Applied}} \)):

\[ \frac{P_0 A}{V_0} = kx_0 \quad \text{and} \quad \frac{P_f A}{V_f} = kx_f \]  \hspace{1cm} (6)

Thus, we find that

\[ P_0 = \frac{kx_0}{A} \quad \text{and} \quad P_f = \frac{kx_f}{A} \]  \hspace{1cm} (7)

Finally, we substitute Equations (7) for the initial and final pressure and Equation (4) for the final volume into Equation (5). With these substitutions Equation (5) becomes

\[ T_f = \left( \frac{P V_f}{P_0 V_0} \right) T_0 = \left( \frac{kx_f}{A} \right) \left[ \frac{V_0 + A(x_f - x_0)}{V_0} \right] T_0 = \frac{x_f [V_0 + A(x_f - x_0)] T_0}{x_0 V_0} = \frac{kx_f}{kx_0} = \frac{P_f}{P_0} \]

\[ = \frac{(0.1000 \text{ m}) \left[ 6.00 \times 10^{-4} \text{ m}^3 + (2.50 \times 10^{-3} \text{ m}^2)(0.1000 \text{ m} - 0.0800 \text{ m}) \right](273 \text{ K})}{(0.0800 \text{ m})(6.00 \times 10^{-4} \text{ m}^3)} = 3.70 \times 10^2 \text{ K} \]

27. **SSM REASONING AND SOLUTION**  If the pressure at the surface is \( P_1 \) and the pressure at a depth \( h \) is \( P_2 \), we have that \( P_2 = P_1 + \rho gh \). We also know that \( P_1 V_1 = P_2 V_2 \). Then,

\[ \frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{P_1 + \rho gh}{P_1} = 1 + \frac{\rho gh}{P_1} \]

Therefore,

\[ \frac{V_1}{V_2} = 1 + \frac{(1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.200 \text{ m})}{1.01 \times 10^5 \text{ Pa}} = 1.02 \]
28. **REASONING** The graph that accompanies Problem 75 in Chapter 12 can be used to determine the equilibrium vapor pressure of water in the air when the temperature is 30.0 °C (303 K). Equation 12.6 can then be used to find the partial pressure of water in the air at this temperature. Using this pressure, the ideal gas law can then be used to find the number of moles of water vapor per cubic meter.

**SOLUTION** According to the graph that accompanies Problem 75 in Chapter 12, the equilibrium vapor pressure of water vapor at 30.0 °C is approximately 4250 Pa. According to Equation 12.6,

\[
\text{Partial pressure of water vapor} = \left( \frac{\text{Percent relative humidity}}{100} \right) \times \left( \frac{\text{Equilibrium vapor pressure of water at the existing temperature}}{100} \right)
\]

\[
= \frac{(55)(4250 \text{ Pa})}{100} = 2.34 \times 10^3 \text{ Pa}
\]

The ideal gas law then gives the number of moles of water vapor per cubic meter of air as

\[
\frac{n}{V} = \frac{P}{RT} = \frac{(2.34 \times 10^3 \text{ Pa})}{[8.31 \text{ J/(mol} \cdot \text{K})](303 \text{ K})} = \frac{0.93 \text{ mol/m}^3}{14.1}
\]

29. **REASONING** The desired percentage is the volume the atoms themselves occupy divided by the total volume that the gas occupies, multiplied by the usual factor of 100. The volume \( V_{\text{Atoms}} \) that the atoms themselves occupy is the volume of an atomic sphere \( \left( \frac{4}{3} \pi r^3 \right) \), where \( r \) is the atomic radius, times the number of atoms present, which is the number \( n \) of moles times Avogadro’s number \( N_A \). The total volume \( V_{\text{Gas}} \) that the gas occupies can be taken to be that calculated from the ideal gas law, because the atoms themselves occupy such a small volume.

**SOLUTION** The total volume \( V_{\text{Gas}} \) that the gas occupies is given by the ideal gas law as

\[
V_{\text{Gas}} = \frac{nRT}{P}
\]

where the temperature and pressure at STP conditions are 273 K and 1.01 \times 10^5 \text{ Pa}. Thus, we can write the desired percentage as

\[
\text{Percentage} = \frac{V_{\text{Atoms}}}{V_{\text{Gas}}} \times 100 = \frac{\left( \frac{4}{3} \pi r^3 \right) nN_A}{nRT} \times 100 = \frac{\left( \frac{4}{3} \pi r^3 \right) N_A P}{RT} \times 100
\]

\[
= \frac{\frac{4}{3} \pi \left( 2.0 \times 10^{-10} \text{ m} \right)^3 \left( 6.022 \times 10^{23} \text{ mol}^{-1} \right) \left( 1.01 \times 10^5 \text{ Pa} \right)}{[8.31 \text{ J/(mol} \cdot \text{K})](273 \text{ K})} \times 100 = 0.090 \%
\]
30. **REASONING AND SOLUTION** We need to determine the amount of He inside the balloon. We begin by using Archimedes’ principle; the balloon is being buoyed up by a force equal to the weight of the air displaced. The buoyant force, $F_b$, therefore, is equal to

$$F_b = mg = \rho V g = (1.19 \text{ kg/m}^3) \left( \frac{4}{3} \pi (1.50 \text{ m})^3 \right) (9.80 \text{ m/s}^2) = 164.9 \text{ N}$$

Since the material from which the balloon is made has a mass of 3.00 kg (weight = 29.4 N), the He inside the balloon weighs $164.9 \text{ N} - 29.4 \text{ N} = 135.5 \text{ N}$. Hence, the mass of the helium present in the balloon is $m = \frac{135.5 \text{ N}}{9.80 \text{ m/s}^2} = 13.8 \text{ kg}$. Now we can determine the number of moles of He present in the balloon:

$$n = \frac{m}{M} = \frac{13.8 \text{ kg}}{4.0026 \times 10^{-3} \text{ kg/mol}} = 3450 \text{ mol}$$

Using the ideal gas law to find the pressure, we have

$$P = \frac{nRT}{V} = \frac{(3450 \text{ mol})(8.31 \text{ J/(mol \cdot K)}) (305 \text{ K})}{\frac{4}{3} \pi (1.50 \text{ m})^3} = 6.19 \times 10^5 \text{ Pa}$$

31. **SSM REASONING** According to the ideal gas law (Equation 14.1), $PV = nRT$. Since $n$, the number of moles of the gas, is constant, $n_1R = n_2R$. Therefore, $P_1V_1/T_1 = P_2V_2/T_2$, where $T_1 = 273 \text{ K}$ and $T_2$ is the temperature we seek. Since the beaker is cylindrical, the volume $V$ of the gas is equal to $Ad$, where $A$ is the cross-sectional area of the cylindrical volume and $d$ is the height of the region occupied by the gas, as measured from the bottom of the beaker. With this substitution for the volume, the expression obtained from the ideal gas law becomes

$$\frac{P_1d_1}{T_1} = \frac{P_2d_2}{T_2}$$

where the pressures $P_1$ and $P_2$ are equal to the sum of the atmospheric pressure and the pressure caused by the mercury in each case. These pressures can be determined using Equation 11.4. Once the pressures are known, Equation (1) can be solved for $T_2$.

**SOLUTION** Using Equation 11.4, we obtain the following values for the pressures $P_1$ and $P_2$. Note that the initial height of the mercury is $h_1 = \frac{1}{2} (1.520 \text{ m}) = 0.760 \text{ m}$, while the final height of the mercury is $h_2 = \frac{1}{4} (1.520 \text{ m}) = 0.380 \text{ m}$.

$$P_1 = P_0 + \rho gh_1 = (1.01 \times 10^5 \text{ Pa}) + \left[ (1.36 \times 10^4 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.760 \text{ m}) \right] = 2.02 \times 10^5 \text{ Pa}$$
THE IDEAL GAS LAW AND KINETIC THEORY

\[ P_2 = P_0 + \rho gh_2 = (1.01 \times 10^5 \text{ Pa}) + \left[ (1.36 \times 10^4 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (0.380 \text{ m}) \right] = 1.52 \times 10^5 \text{ Pa} \]

In these pressure calculations, the density of mercury is \( \rho = 1.36 \times 10^4 \text{ kg/m}^3 \). In Equation (1) we note that \( d_1 = 0.760 \text{ m} \) and \( d_2 = 1.14 \text{ m} \). Solving Equation (1) for \( T_2 \) and substituting values, we obtain

\[ T_2 = \left( \frac{P_2 d_2}{P_1 d_1} \right) T_1 = \left[ \frac{(1.52 \times 10^5 \text{ Pa})(1.14 \text{ m})}{(2.02 \times 10^5 \text{ Pa})(0.760 \text{ m})} \right] (273 \text{ K}) = 308 \text{ K} \]

32. **REASONING AND SOLUTION** At the instant just before the balloon lifts off, the buoyant force from the outside air has a magnitude that equals the magnitude of the total weight. According to Archimedes’ principle, the buoyant force is the weight of the displaced outside air (density \( \rho_0 = 1.29 \text{ kg/m}^3 \)). The mass of the displaced outside air is \( \rho_0 V \), where \( V = 650 \text{ m}^3 \). The corresponding weight is the mass times the magnitude \( g \) of the acceleration due to gravity. Thus, we have

\[
\frac{(\rho_0 V)g}{\text{Buoyant force}} = \frac{m_{\text{total}}g}{\text{Total weight of balloon}}
\]

The total mass of the balloon is \( m_{\text{total}} = m_{\text{load}} + m_{\text{air}} \), where \( m_{\text{load}} = 320 \text{ kg} \) and \( m_{\text{air}} \) is the mass of the hot air within the balloon. The mass of the hot air can be calculated from the ideal gas law by using it to obtain the number of moles \( n \) of air and multiplying \( n \) by the mass per mole of air, \( M = 29 \times 10^{-3} \text{ kg/mol} \):

\[
m_{\text{air}} = n M = \left( \frac{PV}{RT} \right) M
\]

Thus, the total mass of the balloon is \( m_{\text{total}} = m_{\text{load}} + PVM/(RT) \) and Equation (1) becomes

\[
\rho_0 V = m_{\text{load}} + \left( \frac{PV}{RT} \right) M
\]

Solving for \( T \) gives

\[
T = \frac{PVM}{(\rho_0 V - m_{\text{load}}) R} = \frac{(1.01 \times 10^5 \text{ Pa})(650 \text{ m}^3)(29 \times 10^{-3} \text{ kg/mol})}{\left[ 1.29 \text{ kg/m}^3(650 \text{ m}^3) - 320 \text{ kg} \right] \left[ 8.31 \text{ J/(mol} \cdot \text{K}) \right]} = 440 \text{ K}
\]

33. **SSM REASONING** The smoke particles have the same average translational kinetic energy as the air molecules, namely, \( \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT \), according to Equation 14.6. In this expression \( m \) is the mass of a smoke particle, \( v_{\text{rms}} \) is the rms speed of a particle, \( k \) is
Boltzmann’s constant, and \( T \) is the Kelvin temperature. We can obtain the mass directly from this equation.

**SOLUTION** Solving Equation 14.6 for the mass \( m \), we find

\[
m = \frac{3kT}{v_{\text{rms}}^2} = \frac{3(1.38 \times 10^{-23} \text{ J/K})(301 \text{ K})}{(2.8 \times 10^{-3} \text{ m/s})^2} = 1.6 \times 10^{-15} \text{ kg}
\]

34. **REASONING** According to the kinetic theory of gases, the average kinetic energy of an atom is related to the temperature of the gas by \( \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT \) (Equation 14.6). We see that the temperature is proportional to the product of the mass and the square of the rms-speed. Therefore, the tank with the greatest value of \( mv_{\text{rms}}^2 \) has the greatest temperature. Using the information from the table given with the problem statement, we see that the values of \( mv_{\text{rms}}^2 \) for each tank are:

<table>
<thead>
<tr>
<th>Tank</th>
<th>Product of the mass and the square of the rms-speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( mv_{\text{rms}}^2 )</td>
</tr>
<tr>
<td>B</td>
<td>( m(2v_{\text{rms}})^2 = 4mv_{\text{rms}}^2 )</td>
</tr>
<tr>
<td>C</td>
<td>( (2m)v_{\text{rms}}^2 = 2mv_{\text{rms}}^2 )</td>
</tr>
<tr>
<td>D</td>
<td>( 2m(2v_{\text{rms}})^2 = 8mv_{\text{rms}}^2 )</td>
</tr>
</tbody>
</table>

Thus, tank D has the greatest temperature, followed by tanks B, C, and A.

**SOLUTION** The temperature of the gas in each tank can be determined from Equation 14.6:

\[
T_A = \frac{mv_{\text{rms}}^2}{3k} = \frac{3(3.32 \times 10^{-26} \text{ kg})(1223 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 1200 \text{ K}
\]

\[
T_B = \frac{m(2v_{\text{rms}})^2}{3k} = \frac{3(3.32 \times 10^{-26} \text{ kg})(2 \times 1223 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 4800 \text{ K}
\]

\[
T_C = \frac{(2m)v_{\text{rms}}^2}{3k} = \frac{2(3.32 \times 10^{-26} \text{ kg})(1223 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 2400 \text{ K}
\]

\[
T_D = \frac{2m(2v_{\text{rms}})^2}{3k} = \frac{2(3.32 \times 10^{-26} \text{ kg})(2 \times 1223 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 9600 \text{ K}
\]

These results confirm the conclusion reached in the **REASONING**.
35. **REASONING** Assuming that neon (a monatomic gas) behaves as an ideal gas, its internal energy $U$ can be found from $U = \frac{3}{2} nRT$ (Equation 14.7), where $n$ is the number of moles of neon, $R = 8.31 \text{ J/(mol} \cdot \text{K)}$ is the universal gas constant, and $T$ is the Kelvin temperature. We seek the increase $\Delta U = U_f - U_i$ in the internal energy of the neon as its temperature increases from $T_i$ to $T_f$. Because the gas is in a closed, rigid container, neither the number $n$ of moles nor the volume $V$ of the neon change when its temperature is raised. Consequently, we will employ the ideal gas law, $PV = nRT$ (Equation 14.1), to determine the value of the quantity $nR$ from the volume $V$, the initial pressure $P_i$, and the initial Kelvin temperature $T_i$ of the neon.

**SOLUTION** From $U = \frac{3}{2} nRT$ (Equation 14.7), the increase in the internal energy of the gas is given by

$$\Delta U = U_f - U_i = \frac{3}{2} nR(T_f - T_i)$$

Solving $P_iV = nRT_i$ (Equation 14.1) for the quantity $nR$ yields $nR = \frac{PV}{T_i}$. Substituting this result into Equation (1), we find that

$$\Delta U = \frac{3}{2} \left( \frac{P_iV}{T_i} \right) (T_f - T_i) = \frac{3}{2} \left( \frac{1.01 \times 10^5 \text{ Pa}}{293.2 \text{ K}} \right) (294.3 \text{ K} - 293.2 \text{ K}) = 3.9 \times 10^5 \text{ J}$$

36. **REASONING** The average translational kinetic energy of a molecule of the gas is $\overline{KE} = \frac{3}{2} kT$ (Equation 14.6), where $k$ is Boltzmann’s constant and $T$ is the Kelvin temperature of the gas. We do not know the temperature. However, we do know the volume $V$ and the absolute pressure $P$ of the gas, and we do know that for an ideal gas $PV = NkT$ (Equation 14.2), where $N$ is the number of gas molecules. We also know that the number of gas molecules is the number $n$ of moles times Avogadro’s number $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$.

**SOLUTION** Using Equation 14.6, we have $\overline{KE} = \frac{3}{2} kT$. Solving Equation 14.2 for $kT$, we obtain $kT = \frac{PV}{N}$. With this substitution, Equation 14.6 becomes

$$\overline{KE} = \frac{3}{2} kT = \frac{3PV}{2N}$$

Since $N = nN_A$ in the above result, the average translational kinetic energy of a molecule is

$$\overline{KE} = \frac{3PV}{2nN_A} = \frac{3(4.5 \times 10^5 \text{ Pa})(8.5 \times 10^{-3} \text{ m}^3)}{2[(2.0 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1})]} = 4.8 \times 10^{-21} \text{ J}$$
37. **SSM REASONING AND SOLUTION** Using the expressions for $v^2$ and $(\bar{v})^2$ given in the statement of the problem, we obtain:

a. \[ v^2 = \frac{1}{3}(v_1^2 + v_2^2 + v_3^2) = \frac{1}{3}[(3.0 \text{ m/s})^2 + (7.0 \text{ m/s})^2 + (9.0 \text{ m/s})^2] = 46.3 \text{ m}^2/\text{s}^2 \]

b. \[ (\bar{v})^2 = \left[\frac{1}{3}(v_1 + v_2 + v_3)\right]^2 = \left[\frac{1}{3}(3.0 \text{ m/s} + 7.0 \text{ m/s} + 9.0 \text{ m/s})\right]^2 = 40.1 \text{ m}^2/\text{s}^2 \]

$v^2$ and $(\bar{v})^2$ are not equal, because they are two different physical quantities.

38. **REASONING** According to the kinetic theory of gases, the average kinetic energy of an atom is $\overline{KE} = \frac{1}{2}kT$ (Equation 14.6), where $k$ is Boltzmann’s constant and $T$ is the Kelvin temperature. Therefore, the ratio of the average kinetic energies is equal to the ratio of the Kelvin temperatures of the gases. We are given no direct information about the temperatures. However, we do know that the temperature of an ideal gas is related to the pressure $P$, the volume $V$, and the number $n$ of moles of the gas via the ideal gas law, $PV = nRT$. Thus, the ideal gas law can be solved for the temperature, and the ratio of the temperatures can be related to the other properties of the gas. In this way, we will obtain the desired kinetic-energy ratio.

**SOLUTION** Using Equation 14.6 and the ideal gas law in the form $T = \frac{PV}{nR}$, we find that the desired ratio is

\[
\frac{\overline{KE}_{\text{Krypton}}}{\overline{KE}_{\text{Argon}}} = \frac{\frac{3}{2}kT_{\text{Krypton}}}{\frac{3}{2}kT_{\text{Argon}}} = \frac{\frac{3}{2}k \left( \frac{PV}{n_{\text{Krypton}}R} \right)}{\frac{3}{2}k \left( \frac{PV}{n_{\text{Argon}}R} \right)} = \frac{n_{\text{Argon}}}{n_{\text{Krypton}}} \]

Here, we have taken advantage of the fact that the pressure and volume of each gas are the same. While we are not given direct information about the number of moles of each gas, we do know that their masses are the same. Furthermore, the number of moles can be calculated from the mass $m$ (in grams) and the mass per mole $M$ (in grams per mole), according to $n = \frac{m}{M}$. Substituting this expression into our result for the kinetic-energy ratio gives

\[
\frac{\overline{KE}_{\text{Krypton}}}{\overline{KE}_{\text{Argon}}} = \frac{n_{\text{Argon}}}{n_{\text{Krypton}}} = \frac{\frac{M_{\text{Argon}}}{M_{\text{Krypton}}} \cdot \frac{m}{M_{\text{Krypton}}}}{\frac{M_{\text{Argon}}}{M_{\text{Argon}}} \cdot \frac{m}{M_{\text{Argon}}}} = \frac{M_{\text{Krypton}}}{M_{\text{Argon}}} \]
Taking the masses per mole from the periodic table on the inside of the back cover of the text, we find
\[
\frac{\overline{K}E_{\text{Krypton}}}{\overline{K}E_{\text{Argon}}} = \frac{M_{\text{Krypton}}}{M_{\text{Argon}}} = \frac{83.80 \, \text{g/mol}}{39.948 \, \text{g/mol}} = 2.098
\]

39. **REASONING** The translational rms-speed \( v_{\text{rms}} \) is related to the Kelvin temperature \( T \) by \( \frac{1}{2} mv_{\text{rms}}^2 = \frac{3}{2} kT \) (Equation 14.6), where \( m \) is the mass of the oxygen molecule and \( k \) is Boltzmann’s constant. Solving this equation for the rms-speed gives \( v_{\text{rms}} = \sqrt{3kT/m} \). This relation will be used to find the ratio of the speeds.

**SOLUTION** The rms-speeds in the ionosphere and near the earth’s surface are
\[
\left( v_{\text{rms}} \right)_{\text{ionosphere}} = \sqrt{\frac{3kT_{\text{ionosphere}}}{m}} \quad \text{and} \quad \left( v_{\text{rms}} \right)_{\text{earth's surface}} = \sqrt{\frac{3kT_{\text{earth's surface}}}{m}}
\]

Dividing the first equation by the second and using the fact that \( T_{\text{ionosphere}} = 3T_{\text{earth's surface}} \), we find that
\[
\frac{\left( v_{\text{rms}} \right)_{\text{ionosphere}}}{\left( v_{\text{rms}} \right)_{\text{earth's surface}}} = \sqrt{\frac{3kT_{\text{ionosphere}}}{3kT_{\text{earth's surface}}}} = \sqrt{\frac{T_{\text{ionosphere}}}{T_{\text{earth's surface}}}} = \sqrt{3} = 1.73
\]

40. **REASONING** The average kinetic energy \( \overline{K}E \) of each molecule in the gas is directly proportional to the Kelvin temperature \( T \), according to \( \overline{K}E = \frac{3}{2} kT \) (Equation 14.6), where \( k \) is the Boltzmann constant. Solving for the temperature, we obtain
\[
T = \frac{2\left( \overline{K}E \right)}{3k} \tag{1}
\]

The average kinetic energy \( \overline{K}E \) of one molecule is the total average kinetic energy \( \left( \overline{K}E \right)_{\text{total}} \) of all the molecules divided by the total number \( N \) of molecules:
\[
\overline{K}E = \frac{\left( \overline{K}E \right)_{\text{total}}}{N} \tag{2}
\]

We will determine the number \( N \) of molecules in the gas by multiplying the number \( n \) of moles by Avogadro’s number \( N_A \):
\[
N = nN_A \tag{3}
\]
The total average kinetic energy \( \left( \overline{KE} \right)_{\text{total}} \) of all the molecules is equal to the kinetic energy \( KE_{\text{bullet}} \) of the bullet, which has a mass \( m \) and a speed \( v \). Thus, according to Equation 6.2,

\[
\left( \overline{KE} \right)_{\text{total}} = KE_{\text{bullet}} = \frac{1}{2} mv^2
\]

**SOLUTION** Substituting Equation (2) into Equation (1) yields

\[
T = \frac{2(KE)}{3k} = \frac{2 \left( \frac{\overline{KE}}{N} \right)}{3k} = \frac{2(KE)_{\text{total}}}{3kN}
\]

Substituting Equation (3) and Equation 6.2 into Equation (4), we obtain

\[
T = \frac{2 \left( \frac{KE}{N} \right)}{3k} = \frac{2 \left( \frac{1}{2} mv^2 \right)}{3k (nN_A)} = \frac{mv^2}{3knN_A}
\]

The Boltzmann constant \( k \) is equal to the ratio of the universal gas constant \( R \) to Avogadro’s number: \( k = R/N_A \). Making this substitution into Equation (5) yields the Kelvin temperature of the gas:

\[
T = \frac{mv^2}{3 \left( \frac{R}{nN_A} \right) N_A} = \frac{mv^2}{3 \left( \frac{8.0 \times 10^{-3} \text{ kg}}{3770 \text{ m/s}} \right)(2.0 \text{ mol})} = 95 \text{ K}
\]

41. **REASONING** Since the xenon atom does not interact with any other atoms or molecules on its way up, we can apply the principle of conservation of mechanical energy (see Section 6.5) and set the final kinetic plus potential energy equal to the initial kinetic plus potential energy. Thus, during the rise, the atom’s initial kinetic energy is converted entirely into gravitational potential energy, because the atom comes to a momentary halt at the top of its trajectory. The initial kinetic energy \( \frac{1}{2} mv_0^2 \) is equal to the average translational kinetic energy. Therefore, \( \frac{1}{2} mv_0^2 = \overline{KE} = \frac{3}{2} kT \), according to Equation 14.6, where \( k \) is Boltzmann’s constant and \( T \) is the Kelvin temperature. The gravitational potential energy is \( mgh \), according to Equation 6.5.

**SOLUTION** Equation 6.9b gives the principle of conservation of mechanical energy:

\[
\frac{1}{2} m v_f^2 + mgh_f = \frac{1}{2} m v_0^2 + mgh_0
\]

Final mechanical energy  Initial mechanical energy

In this expression, we know that \( \frac{1}{2} mv_0^2 = \overline{KE} = \frac{3}{2} kT \) and that \( \frac{1}{2} mv_f^2 = 0 \text{ J} \) (since the atom comes to a halt at the top of its trajectory). Furthermore, we can take the height at the
earth’s surface to be \( h_0 = 0 \) m. Taking this information into account, we can write the energy-conservation equation as follows:

\[
mgh_f = \frac{3}{2} kT \quad \text{or} \quad h_f = \frac{3kT}{2mg}
\]

Using \( M \) to denote the molecular mass (in kilograms per mole) and recognizing that \( m = \frac{M}{N_A} \), where \( N_A \) is Avogadro’s number and is the number of xenon atoms per mole, we have

\[
h_f = \frac{3kN_A T}{2Mg} = \frac{3RT}{2Mg} = \frac{3\left[8.31 \text{ J/(mol} \cdot \text{K}\right] \left(291 \text{ K}\right)}{2 \left(131.29 \times 10^{-3} \text{ kg/mol}\right) \left(9.80 \text{ m/s}^2\right)} = 2820 \text{ m}
\]

42. **REASONING AND SOLUTION** Since we are treating the air as a diatomic ideal gas \((PV = nRT)\), it follows that

\[
U = \frac{5}{2} nRT = \frac{5}{2} PV = \frac{5}{2} \left(7.7 \times 10^6 \text{ Pa}\right) \left(5.6 \times 10^5 \text{ m}^3\right) = 1.1 \times 10^{13} \text{ J}
\]

The number of joules of energy consumed per day by one house is

\[
30.0 \text{ kW} \cdot \text{h} = \left(30.0 \times 10^3 \frac{\text{J}}{\text{s}}\right) \left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 1.08 \times 10^8 \text{ J}
\]

The number of homes that could be served for one day by \( 1.1 \times 10^{13} \text{ J} \) of energy is

\[
\left(1.1 \times 10^{13} \text{ J}\right) \left(\frac{1 \text{ home}}{1.08 \times 10^8 \text{ J}}\right) = 1.0 \times 10^5 \text{ homes}
\]

43. **SSM REASONING AND SOLUTION**

a. Assuming that the direction of travel of the bullets is positive, the average change in momentum per second is

\[
\frac{\Delta p}{\Delta t} = m \Delta v/\Delta t = (200)(0.0050 \text{ kg})[(0 \text{ m/s}) - (1200 \text{ m/s})]/(10.0 \text{ s}) = -120 \text{ N}
\]

b. The average force exerted on the bullets is \( \bar{F} = \Delta p/\Delta t \). According to Newton’s third law, the average force exerted on the wall is \( -\bar{F} = 120 \text{ N} \).
c. The pressure $P$ is the magnitude of the force on the wall per unit area, so

$$P = \frac{120 \text{ N}}{3.0 \times 10^{-4} \text{ m}^2} = 4.0 \times 10^5 \text{ Pa}$$

44. **REASONING** Because the container holds 1.000 mol of neon, we know that the number of neon atoms inside is equal to Avogadro’s number: $N = N_A$. These $N_A$ atoms are continually bouncing back and forth between the walls of the cubical container, with an rms speed $v_{\text{rms}}$ found from $\frac{1}{2}m v_{\text{rms}}^2 = \frac{3}{2}kT$ (Equation 14.6), where $m$ is the mass of a single neon atom, $k$ is the Boltzmann constant, and $T$ is the Kelvin temperature. Following the development of kinetic theory given in Section 14.3 of the text, the time $t$ that elapses between successive collisions of one atom with one wall of the container is

$$t = \frac{2L}{v_{\text{rms}}} \quad (1)$$

This is because we assume that, between successive collisions with a given wall, the atom travels to the opposite wall and back, which is a total distance of $2L$. We will use $\frac{1}{2}m v_{\text{rms}}^2 = \frac{3}{2}kT$ (Equation 14.6) to determine the rms speed of the neon atoms for use in Equation (1).

There are three identical pairs of walls, so on average 1/3 of the $N_A$ atoms in the container collide with a given wall in the time $t$. Therefore, the rate at which atoms collide with a given wall is

$$\text{Collision rate} = \frac{N_A}{3t} \quad (2)$$

**SOLUTION** Substituting Equation (1) into Equation (2) yields

$$\text{Collision rate} = \frac{N_A}{3t} = \frac{N_A}{3 \left( \frac{2L}{v_{\text{rms}}} \right)} = \frac{N_A v_{\text{rms}}}{6L} \quad (3)$$

Solving $\frac{1}{2}m v_{\text{rms}}^2 = \frac{3}{2}kT$ (Equation 14.6) for the rms speed of the atoms, we obtain

$$\frac{1}{2}m v_{\text{rms}}^2 = \frac{3}{2}kT \quad \text{or} \quad v_{\text{rms}}^2 = \frac{3kT}{m} \quad \text{or} \quad v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (4)$$

Substituting Equation (4) into Equation (3) yields

$$\text{Collision rate} = \frac{N_A v_{\text{rms}}}{6L} = \frac{N_A}{6L} \sqrt{\frac{3kT}{m}} \quad (5)$$

Therefore, the rate at which neon atoms collide with each wall of the container is
Collision rate = \( \frac{6.022 \times 10^{23 \text{ atoms}}}{6(0.300 \text{ m})} \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{3.35 \times 10^{-26} \text{ kg}}} = 2.01 \times 10^{26 \text{ atoms/s}} \)

45. **REASONING** The mass \( m \) of oxygen that diffuses in a time \( t \) through a trachea is given by Equation 14.8 as \( m = (DA\Delta C)t/L \), where \( \Delta C \) is the concentration difference between the two ends of the trachea, \( A \) and \( L \) are, respectively, its cross-sectional area and length, and \( D \) is the diffusion constant. The concentration difference \( \Delta C \) is the higher concentration \( C_2 \) minus the lower concentration \( C_1 \), or \( \Delta C = C_2 - C_1 \).

**SOLUTION** Substituting the relation \( \Delta C = C_2 - C_1 \) into Equation 14.8 and solving for \( C_1 \) gives

\[
C_1 = C_2 - \left( \frac{L}{DA} \right) \left( \frac{m}{t} \right)
\]

We recognize that \( m/t \) is the mass per second of oxygen diffusing through the trachea. Thus, the oxygen concentration at the interior end of the trachea is

\[
C_1 = 0.28 \text{ kg/m}^3 - \left[ \frac{1.9 \times 10^{-3} \text{ m}}{(1.1 \times 10^{-5} \text{ m}^2/\text{s})(2.1 \times 10^{-9} \text{ m}^2)} \right] (1.7 \times 10^{-12} \text{ kg/s}) = 0.14 \text{ kg/m}^3
\]

46. **REASONING** According to Fick’s law of diffusion the mass \( m \) of sucrose that diffuses in a time \( t \) through the water contained in a channel of length \( L \) and cross-sectional area \( A \) is \( m = (DA\Delta C)t/L \) (Equation 14.8), where \( \Delta C \) is the concentration difference between the ends of the channel and \( D \) is the diffusion constant. This equation can be solved directly for the desired time.

**SOLUTION** Solving Fick’s law for the time \( t \), we obtain

\[
t = \frac{mL}{DA\Delta C} = \frac{\left(8.0 \times 10^{-13} \text{ kg}\right)(0.015 \text{ m})}{\left(5.0 \times 10^{-10} \text{ m}^2/\text{s}\right)(7.0 \times 10^{-4} \text{ m}^2)(3.0 \times 10^{-3} \text{ kg/m}^3)} = 11 \text{ s}
\]

47. **REASONING AND SOLUTION**

a. As stated, the time required for the first solute molecule to traverse a channel of length \( L \) is \( t = L^2/(2D) \). Therefore, for water vapor in air at 293 K, where the diffusion constant is \( D = 2.4 \times 10^{-5} \text{ m}^2/\text{s} \), the time \( t \) required for the first water molecule to travel \( L = 0.010 \text{ m} \) is
\[
t = \frac{L^2}{2D} = \frac{(0.010 \text{ m})^2}{2 \left( 2.4 \times 10^{-5} \text{ m}^2/\text{s} \right)} = 2.1 \text{ s}
\]

b. If a water molecule were traveling at the translational rms speed \( v_{\text{rms}} \) for water, the time \( t \) it would take to travel the distance \( L = 0.010 \text{ m} \) would be given by \( t = L/v_{\text{rms}} \), where, according to Equation 14.6 \((\overline{KE} = \frac{1}{2} m v_{\text{rms}}^2)\), \( v_{\text{rms}} = \sqrt{2(\overline{KE})/m} \). Before we can use the last expression for the translation rms speed \( v_{\text{rms}} \), we must determine the mass \( m \) of a water molecule and the average translational kinetic energy \( \overline{KE} \). Using the periodic table on the inside of the text’s back cover, we find that the molecular mass of a water molecule is

\[
\text{Mass of one oxygen atom} + 2 \times \text{Mass of two hydrogen atoms} = 18.0153 \text{ u}
\]

The mass of a single molecule is

\[
m = \frac{18.0153 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 2.99 \times 10^{-26} \text{ kg}
\]

The average translational kinetic energy of water molecules at 293 K is, according to Equation 14.6,

\[
\overline{KE} = \frac{3}{2} kT = \frac{3}{2} \left( 1.38 \times 10^{-23} \text{ J/K} \right) (293 \text{ K}) = 6.07 \times 10^{-21} \text{ J}
\]

Therefore, the translational rms speed of water molecules is

\[
 v_{\text{rms}} = \sqrt{\frac{2(\overline{KE})}{m}} = \sqrt{\frac{2(6.07 \times 10^{-21} \text{ J})}{2.99 \times 10^{-26} \text{ kg}}} = 637 \text{ m/s}
\]

Thus, the time \( t \) required for a water molecule to travel the distance \( L = 0.010 \text{ m} \) at this speed is

\[
t = \frac{L}{v_{\text{rms}}} = \frac{0.010 \text{ m}}{637 \text{ m/s}} = \frac{1.6 \times 10^{-5} \text{ s}}{}
\]

Thus, the time \( t \) required for a water molecule to travel the distance \( L = 0.010 \text{ m} \) at this speed is

\[
t = \frac{L}{v_{\text{rms}}} = \frac{0.010 \text{ m}}{637 \text{ m/s}} = \frac{1.6 \times 10^{-5} \text{ s}}{}
\]

c. In part (a), when a water molecule diffuses through air, it makes millions of collisions each second with air molecules. The speed and direction change abruptly as a result of each collision. Between collisions, the water molecules move in a straight line at constant speed. Although a water molecule does move very quickly between collisions, it wanders only very slowly in a zigzag path from one end of the channel to the other. In contrast, a water molecule traveling unobstructed at its translational rms speed [as in part (b)], will have a larger displacement over a much shorter time. Therefore, the answer to part (a) is much longer than the answer to part (b).

48. **REASONING** The diffusion of the glycine is driven by the concentration difference \( \Delta C = C_2 - C_1 \) between the ends of the tube, where \( C_2 \) is the higher concentration and
C₁ = C₂ − ΔC is the lower concentration. The concentration difference is related to the mass rate of diffusion by Fick’s law: \( \frac{m}{t} = \frac{DA \Delta C}{L} \) (Equation 14.8), where \( D \) is the diffusion constant of glycine in water, and \( A \) and \( L \) are, respectively, the cross-sectional area and length of the tube.

**SOLUTION** Solving \( \frac{m}{t} = \frac{DA \Delta C}{L} \) (Equation 14.8) for \( \Delta C \), we obtain

\[
\Delta C = \frac{L \left( \frac{m}{t} \right)}{DA}
\]

(1)

Substituting Equation (1) into \( C₁ = C₂ − ΔC \) yields

\[
C₁ = C₂ − ΔC = C₂ − \frac{L \left( \frac{m}{t} \right)}{DA}
\]

(2)

We note that the length of the tube is \( L = 2.0 \text{ cm} = 0.020 \text{ m} \). Substituting this and the other given values into Equation (2), we find that the lower concentration is

\[
C₁ = 8.3 \times 10^{-3} \text{ kg/m}^3 - \frac{(0.020 \text{ m}) \left( 4.2 \times 10^{-14} \text{ kg/s} \right)}{(1.06 \times 10^{-9} \text{ m}^2/\text{s})(1.5 \times 10^{-4} \text{ m}^2)} = 3.0 \times 10^{-3} \text{ kg/m}^3
\]

49. **SSM REASONING** The mass \( m \) of methane that diffuses out of the tank in a time \( t \) is given by \( m = \frac{(DA \Delta C)t}{L} \) (Equation 14.8), where \( D \) is the diffusion constant for methane, \( A \) is the cross-sectional area of the pipe, \( L \) is the length of the pipe, and \( ΔC = C₁ − C₂ \) is the difference in the concentration of methane at the two ends of the pipe. The higher concentration \( C₁ \) at the tank-end of the pipe is given, and the concentration \( C₂ \) at the end of the pipe that is open to the atmosphere is zero.

**SOLUTION** Solving \( m = \frac{(DA \Delta C)t}{L} \) (Equation 14.8) for the cross-sectional area \( A \), we obtain

\[
A = \frac{mL}{(D \Delta C)t}
\]

(1)

The elapsed time \( t \) must be converted from hours to seconds:

\[
t = \left( 12 \text{ hr} \right) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right) = 4.32 \times 10^4 \text{ s}
\]
Therefore, the cross-sectional area $A$ of the pipe is

$$A = \frac{\left(9.00 \times 10^{-4} \text{ kg}\right)(1.50 \text{ m})}{\left(2.10 \times 10^{-5} \text{ m}^2/\text{s}\right)(0.650 \text{ kg/m}^3 - 0 \text{ kg/m}^3)(4.32 \times 10^4 \text{ s})} = 2.29 \times 10^{-3} \text{ m}^2$$

50. **REASONING** Since mass is conserved, the mass flow rate is the same at all points, as described by the equation of continuity (Equation 11.8). Therefore, the mass flow rate at which CCl$_4$ enters the tube is the same as that at point A. The concentration difference of CCl$_4$ between point A and the left end of the tube, $\Delta C$, can be calculated by using Fick's law of diffusion (Equation 14.8). The concentration of CCl$_4$ at point A can be found from $C_A = C_{\text{left end}} - \Delta C$.

**SOLUTION**

a. As discussed above in the reasoning, the mass flow rate of CCl$_4$ as it passes point A is the same as the mass flow rate at which CCl$_4$ enters the left end of the tube; therefore, the mass flow rate of CCl$_4$ at point A is $5.00 \times 10^{-13} \text{ kg/s}$.

b. Solving Fick's law for $\Delta C$, we obtain

$$\Delta C = \frac{mL}{DA t} = \frac{(m/ t)L}{DA}$$

$$= \frac{(5.00 \times 10^{-13} \text{ kg/s})(5.00 \times 10^{-3} \text{ m})}{(20.0 \times 10^{-10} \text{ m}^2/\text{s})(3.00 \times 10^{-4} \text{ m}^2)} = 4.2 \times 10^{-3} \text{ kg/m}^3$$

Then,

$$C_A = C_{\text{left end}} - \Delta C = (1.00 \times 10^{-2} \text{ kg/m}^3) - (4.2 \times 10^{-3} \text{ kg/m}^3) = 5.8 \times 10^{-3} \text{ kg/m}^3$$

51. **SSM** **REASONING AND SOLUTION**

a. The average concentration is $C_{\text{av}} = (1/2) (C_1 + C_2) = (1/2)C_2 = m/V = m/(AL)$, so that $C_2 = 2m/(AL)$. Fick's law then becomes $m = DAC_2 t/L = DA(2m/AL)t/L = 2Dmt/L^2$. Solving for $t$ yields

$$t = L^2 / (2D)$$

b. Substituting the given data into this expression yields

$$t = (2.5 \times 10^{-2} \text{ m})^2 /[2(1.0 \times 10^{-5} \text{ m}^2/\text{s})] = 31 \text{ s}$$
52. **REASONING AND SOLUTION** Equation 14.8, gives Fick's law of diffusion:

\[ m = \frac{DA \Delta C \cdot t}{L} \]

Solving for the time \( t \) gives

\[ t = \frac{mL}{DA \Delta C} \]  (1)

The time required for the water to evaporate is equal to the time it takes for 2.0 grams of water vapor to traverse the tube and can be calculated from Equation (1) above. Since air in the tube is completely dry at the right end, the concentration of water vapor is zero, \( C_1 = 0 \text{ kg/m}^3 \), and \( \Delta C = C_2 - C_1 = C_2 \). The concentration at the left end of the tube, \( C_2 \), is equal to the density of the water vapor above the water. This can be found from the ideal gas law:

\[ PV = nRT \quad \Rightarrow \quad P = \frac{\rho RT}{M} \]

where \( M = 0.0180152 \text{ kg/mol} \) is the mass per mole for water (H\(_2\)O). From the figure that accompanies Problem 75 in Chapter 12, the equilibrium vapor pressure of water at 20 °C is \( 2.4 \times 10^3 \text{ Pa} \). Therefore,

\[ C_2 = \rho = \frac{PM}{RT} = \frac{(2.4 \times 10^3 \text{ Pa})(0.0180 \text{ kg/mol})}{[8.31 \text{ J/(mol·K)}](293 \text{ K})} = 1.8 \times 10^{-2} \text{ kg/m}^3 \]

Substituting values into Equation (1) gives

\[ t = \frac{(2.0 \times 10^{-3} \text{ kg})(0.15 \text{ m})}{(2.4 \times 10^{-5} \text{ m}^2/\text{s})(3.0 \times 10^{-4} \text{ m}^2)(1.8 \times 10^{-2} \text{ kg/m}^3)} = 2.3 \times 10^6 \text{ s} \]

This is about 27 days!

53. **REASONING AND SOLUTION** To find the temperature \( T_2 \), use the ideal gas law with \( n \) and \( V \) constant. Thus, \( P_1/T_1 = P_2/T_2 \). Then,

\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right) = (284 \text{ K}) \left( \frac{3.01 \times 10^5 \text{ Pa}}{2.81 \times 10^5 \text{ Pa}} \right) = 304 \text{ K} \]

54. **REASONING** Pushing down on the pump handle lowers the piston from its initial height \( h_i = 0.55 \text{ m} \) above the bottom of the pump cylinder to a final height \( h_f \). The distance \( d \) the biker must push the handle down is the difference between these two heights: \( d = h_i - h_f \).

The initial and final heights of the piston determine the initial and final volumes of air inside the pump cylinder, both of which are the product of the piston height \( h \) and its cross-sectional area \( A \):
We will assume that the air in the cylinder may be treated as an ideal gas, and that compressing it produces no change in temperature. Since none of the air escapes from the cylinder up to the point where the pressure inside equals the pressure in the inner tube, we can apply Boyle’s law:

\[ P_i V_i = P_f V_f \]  

(14.3)

We will use Equations (1) and Equation 14.3 to determine the distance \( d = h_1 - h_f \) through which the pump handle moves before air begins to flow. We note that the final pressure \( P_f \) is equal to the pressure of air in the inner tube.

**SOLUTION** Since the cross-sectional area \( A \) of the piston does not change, substituting Equations (1) into Equation 14.3 yields

\[ P_i A h_i = P_f A h_f \quad \text{or} \quad P_i h_i = P_f h_f \]  

(2)

Solving Equation (2) for the unknown final height \( h_f \) we obtain

\[ h_f = \frac{P_f h_i}{P_i} \]  

(3)

With Equation (3), we can now calculate the distance \( d = h_1 - h_f \) that the biker pushes down on the handle before air begins to flow from the pump to the inner tube:

\[ d = h_1 - h_f = h_1 - \frac{P_f h_i}{P_i} = h_1 \left( 1 - \frac{P_f}{P_i} \right) \]

\[ = (0.55 \text{ m}) \left( 1 - \frac{1.0 \times 10^5 \text{ Pa}}{2.4 \times 10^5 \text{ Pa}} \right) = 0.32 \text{ m} \]

55. **SSM REASONING** According to the ideal gas law (Equation 14.1), \( PV = nRT \). Since \( n \), the number of moles, is constant, \( n_1 R = n_2 R \). Thus, according to Equation 14.1, we have

\[ \frac{P V_1}{T_1} = \frac{P V_2}{T_2} \]

**SOLUTION** Solving for \( T_2 \), we have

\[ T_2 = \left( \frac{P_2}{P_1} \right) \left( \frac{V_2}{V_1} \right) T_1 = \left( \frac{48.5 P_1}{P_1} \right) \left[ \frac{V_1/16}{V_1} \right] (305 \text{ K}) = 925 \text{ K} \]
56. **REASONING** According to the kinetic theory of gases, the average kinetic energy $\overline{KE}$ of an atom in an ideal gas is related to the Kelvin temperature of the gas by $\overline{KE} = \frac{1}{2} kT$ (Equation 14.6). Since the temperature is the same for both gases, the A and B atoms have the same average kinetic energy. The average kinetic energy is related to the rms-speed by $\overline{KE} = \frac{1}{2} m v_{\text{rms}}^2$. Since both gases have the same average kinetic energy and gas A has the smaller mass, it has the greater rms-speed. Therefore, gas A has the greater diffusion rate.

**SOLUTION** For a fixed temperature, the ratio $R_A/R_B$ of the diffusion rates for the two types of atoms is equal to the ratio $v_{\text{rms, A}}/v_{\text{rms, B}}$ of the rms-speeds. According to Equation 14.6, it follows that $v_{\text{rms}} = \sqrt{2\overline{KE}/m}$, so that

$$\frac{R_A}{R_B} = \frac{v_{\text{rms, A}}}{v_{\text{rms, B}}} = \sqrt{\frac{2\overline{KE}}{m_B}} = \sqrt{\frac{2\overline{KE}}{m_A}} = \sqrt{\frac{2.0 \text{ u}}{1.0 \text{ u}}} = [1.4]$$

57. **SSM** **REASONING** The behavior of the molecules is described by Equation 14.5: $PV = \frac{2}{3} N (\frac{1}{2} m v_{\text{rms}}^2)$. Since the pressure and volume of the gas are kept constant, while the number of molecules is doubled, we can write $P_1 V_2 = P_2 V_1$, where the subscript 1 refers to the initial condition, and the subscript 2 refers to the conditions after the number of molecules is doubled. Thus,

$$\frac{2}{3} N_1 \left[ \frac{1}{2} m (v_{\text{rms, 1}})^2 \right] = \frac{2}{3} N_2 \left[ \frac{1}{2} m (v_{\text{rms, 2}})^2 \right] \quad \text{or} \quad N_1 (v_{\text{rms, 1}})^2 = N_2 (v_{\text{rms, 2}})^2$$

The last expression can be solved for $(v_{\text{rms}})_2$, the final translational rms speed.

**SOLUTION** Since the number of molecules is doubled, $N_2 = 2N_1$. Solving the last expression above for $(v_{\text{rms}})_2$, we find

$$(v_{\text{rms}})_2 = (v_{\text{rms}})_1 \sqrt{\frac{N_1}{N_2}} = (463 \text{ m/s}) \sqrt{\frac{N_1}{2N_1}} = \frac{463 \text{ m/s}}{\sqrt{2}} = 327 \text{ m/s}$$

58. **REASONING** The average kinetic energy per molecule is proportional to the Kelvin temperature of the carbon dioxide gas. This relation is expressed by Equation 14.6 as
\[ \frac{1}{2} m v_{\text{rms}}^2 = \frac{1}{2} kT, \] where \( m \) is the mass of a carbon dioxide molecule. The mass \( m \) is equal to the molecular mass of carbon dioxide (44.0 u), expressed in kilograms.

**SOLUTION** Solving Equation 14.6 for the temperature of the gas, we have

\[
T = \frac{m v_{\text{rms}}^2}{3k} = \frac{(44.0 \text{ u}) \left( \frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) (650 \text{ m/s})^2}{3 \left( 1.38 \times 10^{-23} \text{ J/K} \right)} = 750 \text{ K}
\]

59. **REASONING** The ideal gas law specifies that the pressure \( P \), the volume \( V \), the number of moles \( n \), and the Kelvin temperature \( T \) are related according to \( PV = nRT \) (Equation 14.1), where \( R \) is the universal gas constant. We will apply this expression to the situation where the oxygen gas is stored in the tank (pressure \( P_{\text{tank}} \), the volume \( V_{\text{tank}} \), the number of moles \( n \), and the Kelvin temperature \( T_{\text{tank}} \)) and to the situation where the oxygen is being administered to a patient (pressure \( P_{\text{patient}} \), the volume \( V_{\text{patient}} \), the number of moles \( n \), and the Kelvin temperature \( T_{\text{patient}} \)). The number of moles is the same in both situations.

**SOLUTION** The ideal gas law gives

\[
\frac{P_{\text{patient}} V_{\text{patient}}}{T_{\text{patient}}} = \frac{P_{\text{tank}} V_{\text{tank}}}{T_{\text{tank}}} = nR
\]

Solving for \( V_{\text{patient}} \) shows that

\[
V_{\text{patient}} = \frac{T_{\text{patient}} P_{\text{tank}} V_{\text{tank}}}{P_{\text{patient}} T_{\text{tank}}} = \frac{(297 \text{ K})(65.0 \text{ atm})(1.00 \text{ m}^3)}{(1.00 \text{ atm})(288 \text{ K})} = 67.0 \text{ m}^3
\]

60. **REASONING** We assume that the volume \( V \) of a material is filled with \( N \) cubes each with a volume \( d^3 \), where \( d \) is the length of each edge of a cube. At the center of each cube is one molecule. Thus, we have that \( V = N d^3 \), which can be solved for \( d \):

\[
d = \left( \frac{V}{N} \right)^{1/3}
\]

We can express the volume \( V \) in terms of the material’s density \( \rho = m/V \) (Equation 11.1), where \( m \) is the total mass of the volume \( V \). Therefore, \( V = m/\rho \), which we can substitute into Equation (1) and obtain
\[ d = \left( \frac{V}{N} \right)^{1/3} = \left( \frac{m}{N \rho} \right)^{1/3} = \left( \frac{m_{\text{molecule}}}{\rho} \right)^{1/3} \]  

(2)

Note in Equation (2) that the quantity \( m/N \) is the mass of an individual molecule \( m_{\text{molecule}} \) of the material, whether that molecule is in the vapor phase or in the liquid phase. We will apply Equation (2) to the vapor phase and to the liquid phase in order to determine the desired ratio.

**SOLUTION** Applying Equation (2) to both the vapor and the liquid phase, we obtain

\[
\frac{d_{\text{vapor}}}{d_{\text{liquid}}} = \left( \frac{m_{\text{molecule}} / \rho_{\text{vapor}}}{m_{\text{molecule}} / \rho_{\text{liquid}}} \right)^{1/3} = \left( \frac{\rho_{\text{liquid}}}{\rho_{\text{vapor}}} \right)^{1/3} = \left( \frac{958 \text{ kg/m}^3}{0.598 \text{ kg/m}^3} \right)^{1/3} = \frac{11.7}{1}
\]

---

61. **REASONING** The rms-speed \( v_{\text{rms}} \) of the sulfur dioxide molecules is related to the Kelvin temperature \( T \) by \( \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT \) (Equation 14.6), where \( m \) is the mass of a \( \text{SO}_2 \) molecule and \( k \) is Boltzmann’s constant. Solving this equation for the rms-speed gives

\[ v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \]

(1)

The temperature can be found from the ideal gas law, Equation 14.1, as \( T = PV / (nR) \), where \( P \) is the pressure, \( V \) is the volume, \( n \) is the number of moles, and \( R \) is the universal gas constant. All the variables in this relation are known. Substituting this expression for \( T \) into Equation (1) yields

\[ v_{\text{rms}} = \sqrt{\frac{3k}{m} \left( \frac{PV}{nR} \right)} = \sqrt{\frac{3kPV}{nmR}} \]

The mass \( m \) of a single \( \text{SO}_2 \) molecule will be calculated in the Solutions section.

**SOLUTION** Using the periodic table on the inside of the text’s back cover, we find the molecular mass of a sulfur dioxide molecule (\( \text{SO}_2 \)) to be

\[
\frac{32.07 \text{ u}}{\text{Mass of a single sulfur atom}} + 2 \left( \frac{15.9994 \text{ u}}{\text{Mass of two oxygen atoms}} \right) = 64.07 \text{ u}
\]

Since \( 1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} \) (see Section 14.1), the mass of a sulfur dioxide molecule is

\[
m = \left( 64.07 \text{ u} \right) \left( \frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) = 1.064 \times 10^{-25} \text{ kg}
\]

The translational rms-speed of the sulfur dioxide molecules is
\[ v_{\text{rms}} = \sqrt{\frac{3kPV}{nmR}} \]
\[ = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(2.12 \times 10^4 \text{ Pa})(50.0 \text{ m}^3)}{(421 \text{ mol})(1.064 \times 10^{-25} \text{ kg})[8.31 \text{ J/(mol \cdot K)}]} = 343 \text{ m/s} \]

62. **REASONING**

Power is the rate at which energy is produced, according to \( \text{Power} = \frac{\text{Energy}}{t} \) (Equation 6.10b), so the time required for the engine to produce a certain amount of energy is given by

\[ t = \frac{\text{Energy}}{\text{Power}} \quad (1) \]

Assuming that helium (a monatomic gas) behaves as an ideal gas, its internal energy \( U \) can be found from \( U = \frac{3}{2} nRT \) (Equation 14.7), where \( n \) is the number of moles of helium in the container, \( R = 8.31 \text{ J/(mol \cdot K)} \) is the universal gas constant, and \( T \) is the Kelvin temperature. We will determine the quantity \( nRT \) in Equation 14.7 from the pressure \( P \) and volume \( V \) of the helium via the ideal gas law \( PV = nRT \) (Equation 14.1).

**SOLUTION**

The engine must produce an amount of energy equal to the internal energy \( U = \frac{3}{2} nRT \) (Equation 14.7) of the helium, so from Equation (1) we have that

\[ t = \frac{\text{Energy}}{\text{Power}} = \frac{U}{\text{Power}} = \frac{\frac{3}{2} nRT}{\text{Power}} \quad (2) \]

Substituting \( PV = nRT \) (Equation 14.1) into Equation (2) yields

\[ t = \frac{\frac{3}{2} nRT}{\text{Power}} = \frac{3PV}{2(\text{Power})} \quad (3) \]

Using the equivalence 1 hp = 746 W in Equation (3), we obtain

\[ t = \frac{3PV}{2(\text{Power})} = \frac{3(6.2 \times 10^5 \text{ Pa})(0.010 \text{ m}^3)}{2(0.25 \text{ hp})(746 \text{ W/1 hp})} = 5.0 \times 10^1 \text{ s} \]

63. **REASONING**

When perspiration absorbs heat from the body, the perspiration vaporizes. The amount \( Q \) of heat required to vaporize a mass \( m_{\text{perspiration}} \) of perspiration is given by Equation 12.5 as \( Q = m_{\text{perspiration}} L_v \), where \( L_v \) is the latent heat of vaporization for water at body temperature. The average energy \( \overline{E} \) given to a single water molecule is equal to the heat \( Q \) divided by the number \( N \) of water molecules.
**SOLUTION** Since $E = Q/N$ and $Q = m_{\text{perspiration}}L_v$, we have

$$E = \frac{Q}{N} = \frac{m_{\text{perspiration}}L_v}{N}$$

But the mass of perspiration is equal to the mass $m_{\text{H}_2\text{O molecule}}$ of a single water molecule times the number $N$ of water molecules. The mass of a single water molecule is equal to its molecular mass (18.0 u), converted into kilograms. The average energy given to a single water molecule is

$$E = m_{\text{H}_2\text{O molecule}}L_v = \frac{m_{\text{H}_2\text{O molecule}}N L_v}{N}$$

$$E = m_{\text{H}_2\text{O molecule}}L_v = (18.0 \text{ u})\left(1.66 \times 10^{-27} \text{ kg} \frac{1}{1 \text{ u}}\right)(2.42 \times 10^6 \text{ J/kg}) = 7.23 \times 10^{-20} \text{ J}$$

64. **REASONING AND SOLUTION** The volume of the cylinder is $V = AL$ where $A$ is the cross-sectional area of the piston and $L$ is the length. We know $P_1V_1 = P_2V_2$ so that the new pressure $P_2$ can be found. We have

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right) = P_1 \left(\frac{A_1 L_1}{A_2 L_2}\right) = P_1 \left(\frac{L_1}{L_2}\right) \quad (\text{since } A_1 = A_2)$$

$$= (1.01 \times 10^5 \text{ Pa})\left(\frac{L}{2L}\right) = 5.05 \times 10^4 \text{ Pa}$$

The force on the piston and spring is, therefore,

$$F = P_2A = (5.05 \times 10^4 \text{ Pa})\pi (0.0500 \text{ m})^2 = 397 \text{ N}$$

The spring constant is $k = F/x$ (Equation 10.1), so

$$k = \frac{F}{x} = \frac{397 \text{ N}}{0.200 \text{ m}} = 1.98 \times 10^3 \text{ N/m}$$
1. (c) This sign convention for $Q$ and $W$ is discussed in Section 15.3 (see, in particular, Equation 15.1).

2. $\Delta U = -9.3 \times 10^5$ J

3. (c) According to the discussion in Section 15.4, the area under a pressure–volume graph is the work $W$ for any kind of process. Since the graph shows the gas being compressed, work is done on the gas.

4. (b) The work done by a gas is the area under the pressure-volume graph. The areas under curves A and B are the same (4 ‘squares’ each), and each is greater than that under curve C (3 ‘squares’).

5. $W = +1.2 \times 10^5$ J

6. (a) The first law of thermodynamics states that the heat $Q$ is related to the change $\Delta U$ in the internal energy and the work $W$ by $Q = \Delta U + W$. Since all three paths start at A and end at B, the change in the internal energy of each gas is the same. Therefore, the path that involves the greatest amount of work is the one that has greatest amount of heat added to the gas. The drawing shows that the work (which is the area under the pressure-volume graph) is greatest for path 1 and smallest for path 3.

7. (e) The first law of thermodynamics, $Q = (U_B - U_A) + W$, can be used to find the heat $Q$, since $U_B$ and $U_A$ are given and $W$ is the area under the pressure-volume graph.

8. (e) The internal energy of a monatomic ideal gas depends directly on its temperature (see Equation 14.7). In an isothermal process the temperature does not change. Therefore, the internal energy of the gas does not change.

9. $W = -1.81 \times 10^4$ J

10. (c) The temperature of the gas increases when its internal energy increases. According to the first law of thermodynamics, $\Delta U = Q - W$. Both heat and work can change the internal energy and, hence, the temperature of the gas. If $Q = 0$ J, $\Delta U$ can still increase if $W$ is a negative number, which means that work is done on the gas.

11. (d) The work done on a monatomic gas during an adiabatic compression is given by (See Equation 15.4) $W = \frac{3}{2} nR \left( T_i - T_f \right)$. 
12. (a) The change in temperature is greatest when the change $\Delta U$ in the internal energy of the gas is greatest. According to the first law of thermodynamics, $\Delta U = Q - W$, the change in the internal energy is greatest when heat $Q$ is added and no work ($W = 0$ J) is done by the gas. The gas does no work when its volume remains constant, so its change in temperature is greater than if the volume had changed.

13. (c) A more-efficient engine produces more work from the same amount of input heat, as expressed by $W = eQ_H$ (Equation 15.11). Part of the heat $Q_H$ from the hot reservoir is used to perform work $W$, and the remainder $Q_C$ is rejected to the cold reservoir. The conservation of energy states that $Q_H = W + Q_C$ (Equation 15.12). If $Q_H$ is constant and $W$ increases, the heat $Q_C$ must decrease.

14. (a) The efficiency $e$ of a heat engine depends on the ratio $Q_C/Q_H$ through the relation $e = 1 - Q_C/Q_H$ (Equation 15.13). Doubling $Q_C$ and $Q_H$ does not change this ratio.

15. (d) The efficiency of a Carnot engine is given by $e = 1 - (T_C/T_H)$, Equation 15.15, where $T_C$ and $T_H$ are the Kelvin temperatures of the cold and hot reservoirs. The efficiencies of engines C, B, and A, are, respectively, 0.50, 0.20, and 0.11.

16. $W_2/W_1 = 1.5$

17. (c) The refrigerator uses the work $W$ done by the electrical energy to remove heat $Q_C$ from its interior and deposit heat $Q_H$ into the room. In accordance with the conservation of energy, $Q_H = Q_C + W$. Therefore, the heat delivered into the room is greater than the electrical energy used to produce the work.

18. (e) According the conservation of energy, the heat $Q_H$ delivered to the room equals the heat $Q_C$ removed from the interior of the refrigerator plus the work $W$: $Q_H = Q_C + W$. The coefficient of performance is the heat $Q_C$ removed from the refrigerator divided by the work $W$ done by the refrigerator; $\text{COP} = Q_C/W$ (Equation 15.16).

19. (b) The sun loses heat, so its entropy decreases. The earth, on the other hand, gains heat so its entropy increases. The transfer of heat from the sun to the earth, like the flow of heat from a hot reservoir to a cold reservoir, is irreversible. Therefore, the entropy of the sun-earth system increases.

20. (e) The change $\Delta S$ in entropy of the gas is given by $\Delta S = (Q/T)_R$ (Equation 15.18), where $Q$ is the heat added and $T$ is its temperature (which remains constant since the expansion is isothermal).
Chapter 15 Problems

1. **Reasoning** Since the student does work, \( W \) is positive, according to our convention. Since his internal energy decreases, the change \( \Delta U \) in the internal energy is negative. The first law of thermodynamics will allow us to determine the heat \( Q \).

**Solution**

a. The work is \( W = +1.6 \times 10^4 \text{ J} \).

b. The change in internal energy is \( \Delta U = -4.2 \times 10^4 \text{ J} \).

c. Applying the first law of thermodynamics from Equation 15.1, we find that

\[
Q = \Delta U + W = (-4.2 \times 10^4 \text{ J}) + (1.6 \times 10^4 \text{ J}) = -2.6 \times 10^4 \text{ J}
\]

2. **Reasoning** The system undergoes two processes. For the first or outgoing process, we know both the heat and the work. For the second or return process, we know only the heat and wish to determine the work. We will apply the first law of thermodynamics to both processes. To determine the work in the return process, we will use the fact that the return process brings the system back to its initial state.

**Solution**

a. As applied to the return process, the first law of thermodynamics (Equation 15.1) indicates that

\[
(\Delta U)_{\text{return}} = Q_{\text{return}} - W_{\text{return}} \quad \text{or} \quad W_{\text{return}} = Q_{\text{return}} - (\Delta U)_{\text{return}} \quad (1)
\]

We know that \( Q_{\text{return}} = -114 \text{ J} \). However, to use Equation (1) to calculate the \( W_{\text{return}} \), we also need a value for \( (\Delta U)_{\text{return}} \). To find this value, we begin by applying the first law of thermodynamics to the outgoing process:

\[
(\Delta U)_{\text{outgoing}} = Q_{\text{outgoing}} - W_{\text{outgoing}} = (165 \text{ J}) - (312 \text{ J}) = -147 \text{ J}
\]

Now we use the fact that the return process brings the system back to its initial state and express the overall change in the internal energy due to the outgoing and the return process as follows:

\[
(\Delta U)_{\text{overall}} = (\Delta U)_{\text{outgoing}} + (\Delta U)_{\text{return}} = 0 \quad \text{or} \quad (\Delta U)_{\text{return}} = -(\Delta U)_{\text{outgoing}} \quad (2)
\]
In obtaining Equation (2), we have recognized that internal energy is a function only of the state of the system, so that \((\Delta U)_{\text{overall}} = 0\), because the system begins and ends in the same state. We can now substitute Equation (2) into Equation (1) and obtain that

\[
W_{\text{return}} = Q_{\text{return}} - (\Delta U)_{\text{return}} = Q_{\text{return}} + (\Delta U)_{\text{outgoing}} = (-114 \text{ J}) + (-147 \text{ J}) = -261 \text{ J}
\]

b. Since \(W_{\text{return}}\) is negative, work is done on the system.

3. **REASONING** Energy in the form of work leaves the system, while energy in the form of heat enters. More energy leaves than enters, so we expect the internal energy of the system to decrease, that is, we expect the change \(\Delta U\) in the internal energy to be negative. The first law of thermodynamics will confirm our expectation. As far as the environment is concerned, we note that when the system loses energy, the environment gains it, and when the system gains energy the environment loses it. Therefore, the change in the internal energy of the environment must be opposite to that of the system.

**SOLUTION**

a. The system gains heat so \(Q\) is positive, according to our convention. The system does work, so \(W\) is also positive, according to our convention. Applying the first law of thermodynamics from Equation 15.1, we find for the system that

\[
\Delta U = Q - W = (77 \text{ J}) - (164 \text{ J}) = -87 \text{ J}
\]

As expected, this value is negative, indicating a decrease.

b. The change in the internal energy of the environment is opposite to that of the system, so that \(\Delta U_{\text{environment}} = +87 \text{ J}\).

4. **REASONING** The first law of thermodynamics, which is a statement of the conservation of energy, states that, due to heat \(Q\) and work \(W\), the internal energy of the system changes by an amount \(\Delta U\) according to \(\Delta U = Q - W\) (Equation 15.1). This law can be used directly to find \(\Delta U\).

**SOLUTION** \(Q\) is positive \((+7.6 \times 10^4 \text{ J})\) since heat flows into the system; \(W\) is also positive \((+7.6 \times 10^4 \text{ J})\) since work is done by the system. Using the first law of thermodynamics from Equation 15.1, we obtain

\[
\Delta U = Q - W = (+7.6 \times 10^4 \text{ J}) - (+4.8 \times 10^4 \text{ J}) = +2.8 \times 10^4 \text{ J}
\]

The plus sign indicates that the internal energy of the system increases.
5. **REASONING** In both cases, the internal energy $\Delta U$ that a player loses before becoming exhausted and leaving the game is given by the first law of thermodynamics: $\Delta U = Q - W$ (Equation 15.1), where $Q$ is the heat lost and $W$ is the work done while playing the game. The algebraic signs of the internal energy change $\Delta U$ and the heat loss $Q$ are negative, while that of the work $W$ is positive.

**SOLUTION**

a. From the first law of thermodynamics, the work $W$ that the first player does before leaving the game is equal to the heat lost minus the internal energy loss:

$$W = Q - \Delta U = -6.8 \times 10^5 \text{ J} - (-8.0 \times 10^5 \text{ J}) = +1.2 \times 10^5 \text{ J}$$

b. Again employing the first law of thermodynamics, we find that the heat loss $Q$ experienced by the more-warmly-dressed player is the sum of the work done and the internal energy change:

$$Q = W + \Delta U = 2.1 \times 10^5 \text{ J} + (-8.0 \times 10^5 \text{ J}) = -5.9 \times 10^5 \text{ J}$$

As expected, the algebraic sign of the heat loss is negative. The magnitude of the heat loss is, therefore, $5.9 \times 10^5 \text{ J}$.

---

6. **REASONING** According to the discussion in Section 14.3, the internal energy $U$ of a monatomic ideal gas is given by $U = \frac{3}{2} nRT$ (Equation 14.7), where $n$ is the number of moles, $R$ is the universal gas constant, and $T$ is the Kelvin temperature. When the temperature changes to a final value of $T_f$ from an initial value of $T_i$, the internal energy changes by an amount

$$\Delta U = \frac{3}{2} n R (T_f - T_i)$$

Solving this equation for the final temperature yields $T_f = \left(\frac{2}{3nR}\right) \Delta U + T_i$. We are given $n$ and $T_i$, but must determine $\Delta U$. The change $\Delta U$ in the internal energy of the gas is related to the heat $Q$ and the work $W$ by the first law of thermodynamics, $\Delta U = Q - W$ (Equation 15.1). Using these two relations will allow us to find the final temperature of the gas.

**SOLUTION** Substituting $\Delta U = Q - W$ into the expression for the final temperature gives

$$T_f = \left(\frac{2}{3nR}\right) (Q - W) + T_i$$

$$= \left\{ \frac{2}{3(3.00 \text{ mol})\left[8.31 \text{ J/(mol} \cdot \text{K})\right]} \right\} \left[+2438 \text{ J} - (-962 \text{ J})\right] + 345 \text{ K} = 436 \text{ K}$$
Note that the heat is positive \((Q = +2438 \text{ J})\) since the system (the gas) gains heat, and the work is negative \((W = -962 \text{ J})\), since it is done on the system.

7. **SSM REASONING** The change \(\Delta U\) in the weight lifter’s internal energy is given by the first law of thermodynamics as \(\Delta U = Q - W\) (Equation 15.1), where \(Q\) is the heat and \(W\) is the work. The amount of heat is that required to evaporate the water (perspiration) and is \(mL_v\) (see Equation 12.5), where \(m\) is the mass of the water and \(L_v\) is the latent heat of vaporization of perspiration.

**SOLUTION**

a. The heat required to evaporate the water is energy that leaves the weight lifter’s body along with the evaporated water, so this heat \(Q\) in the first law of thermodynamics is negative. Therefore, we substitute \(Q = -mL_v\) into the first law and obtain

\[
\Delta U = Q - W = -mL_v - W
\]

\[
= -(0.150 \text{ kg})(2.42 \times 10^6 \text{ J/kg}) - (1.40 \times 10^5 \text{ J}) = -5.03 \times 10^5 \text{ J}
\]

b. Since 1 nutritional Calorie = 4186 J, the number of nutritional calories is

\[
\left(5.03 \times 10^5 \text{ J}\right)\left(\frac{1 \text{ Calorie}}{4186 \text{ J}}\right) = 1.20 \times 10^2 \text{ nutritional Calories}
\]

8. **REASONING** We will determine the heat by applying the first law of thermodynamics to the overall process. This law is \(\Delta U = Q - W\) (Equation 15.1). We will add the changes in the internal energy for the two steps in the process to obtain the overall change \(\Delta U\) and similarly add the work values to get the overall work \(W\).

**SOLUTION** Using the first law of thermodynamics from Equation 15.1, we have

\[
\Delta U = Q - W \quad \text{or} \quad Q = \Delta U + W
\] (1)

In both steps the internal energy increases, so overall we have \(\Delta U = 228 \text{ J} + 115 \text{ J} = +343 \text{ J}\). In both steps the work is negative according to our convention, since it is done on the system. Overall, then, we have \(W = -166 \text{ J} - 177 \text{ J} = -343 \text{ J}\). With these values for \(\Delta U\) and \(W\), Equation (1) reveals that

\[
Q = \Delta U + W = (+343 \text{ J}) + (-343 \text{ J}) = 0 \text{ J}
\]

Since the heat is zero, the overall process is **adiabatic**.
9. **REASONING** According to Equation 15.2, \( W = P \Delta V \), the average pressure \( \bar{P} \) of the expanding gas is equal to \( \bar{P} = W / \Delta V \), where the work \( W \) done by the gas on the bullet can be found from the work-energy theorem (Equation 6.3). Assuming that the barrel of the gun is cylindrical with radius \( r \), the volume of the barrel is equal to its length \( L \) multiplied by the area \( (\pi r^2) \) of its cross section. Thus, the change in volume of the expanding gas is \( \Delta V = L \pi r^2 \).

**SOLUTION** The work done by the gas on the bullet is given by Equation 6.3 as

\[
W = \frac{1}{2} m (v_{\text{final}}^2 - v_{\text{initial}}^2) = \frac{1}{2} (2.6 \times 10^{-3} \text{ kg}) [(370 \text{ m/s})^2 - 0] = 180 \text{ J}
\]

The average pressure of the expanding gas is, therefore,

\[
\bar{P} = \frac{W}{\Delta V} = \frac{180 \text{ J}}{(0.61 \text{ m}) \pi (2.8 \times 10^{-3} \text{ m})^2} = 1.2 \times 10^7 \text{ Pa}
\]

10. **REASONING** Equation 15.2 indicates that work \( W \) done at a constant pressure \( P \) is given by \( W = P \Delta V \). In this expression \( \Delta V \) is the change in volume; \( \Delta V = V_f - V_i \), where \( V_f \) is the final volume and \( V_i \) is the initial volume. Thus, the change in volume is

\[
\Delta V = \frac{W}{P}
\]

The pressure is known, and the work can be obtained from the first law of thermodynamics as \( W = Q - \Delta U \) (see Equation 15.1).

**SOLUTION** Substituting \( W = Q - \Delta U \) into Equation (1) gives

\[
\Delta V = \frac{W}{P} = \frac{Q - \Delta U}{P} = \frac{(+2780 \text{ J}) - (+3990 \text{ J})}{1.26 \times 10^5 \text{ Pa}} = -9.60 \times 10^{-3} \text{ m}^3
\]

Note that \( Q \) is positive (+2780 J) since the system gains heat; \( \Delta U \) is also positive (+3990 J) since the internal energy of the system increases. The change \( \Delta V \) in volume is negative, reflecting the fact that the final volume is less than the initial volume.

11. **REASONING** The work done in an isobaric process is given by Equation 15.2, \( W = P \Delta V \); therefore, the pressure is equal to \( P = W / \Delta V \). In order to use this expression, we must first determine a numerical value for the work done; this can be calculated using the first law of thermodynamics (Equation 15.1), \( \Delta U = Q - W \).

**SOLUTION** Solving Equation 15.1 for the work \( W \), we find

\[
W = Q - \Delta U = 1500 \text{ J} - (+4500 \text{ J}) = -3.0 \times 10^3 \text{ J}
\]
Therefore, the pressure is
\[
P = \frac{W}{\Delta V} = \frac{-3.0 \times 10^3 \text{ J}}{-0.010 \text{ m}^3} = 3.0 \times 10^5 \text{ Pa}
\]

The change in volume \(\Delta V\), which is the final volume minus the initial volume, is negative because the final volume is 0.010 m\(^3\) less than the initial volume.

12. **REASONING** The work done in the process is equal to the “area” under the curved line between \(A\) and \(B\) in the drawing. From the graph, we find that there are about 78 “squares” under the curve. Each square has an “area” of

\[
\left(2.0 \times 10^4 \text{ Pa}\right)(2.0 \times 10^{-3} \text{ m}^3) = 4.0 \times 10^1 \text{ J}
\]

**SOLUTION**

a. The work done in the process has a magnitude of

\[
W = (78)(4.0 \times 10^1 \text{ J}) = 3100 \text{ J}
\]

b. The final volume is smaller than the initial volume, so the gas is compressed. Therefore, work is done on the gas so the work is **negative**.

13. **SSM** **REASONING AND SOLUTION**

a. Starting at point \(A\), the work done during the first (vertical) straight-line segment is

\[
W_1 = P_1\Delta V_1 = P_1(0 \text{ m}^3) = 0 \text{ J}
\]

For the second (horizontal) straight-line segment, the work is

\[
W_2 = P_2\Delta V_2 = 10(1.0 \times 10^4 \text{ Pa})(2.0 \times 10^{-3} \text{ m}^3) = 1200 \text{ J}
\]

For the third (vertical) straight-line segment the work is

\[
W_3 = P_3\Delta V_3 = P_3(0 \text{ m}^3) = 0 \text{ J}
\]

For the fourth (horizontal) straight-line segment the work is

\[
W_4 = P_4\Delta V_4 = 15(1.0 \times 10^4 \text{ Pa})(2.0 \times 10^{-3} \text{ m}^3) = 1800 \text{ J}
\]

The total work done is

\[
W = W_1 + W_2 + W_3 + W_4 = +3.0 \times 10^3 \text{ J}
\]

b. Since the total work is positive, work is done **by the system**.

14. **REASONING** We will determine the heat by applying the first law of thermodynamics to the overall process. This law is \(\Delta U = Q - W\) (Equation 15.1). To determine the heat \(Q\), values for the change \(\Delta U\) in the internal energy and the work \(W\) will be needed. We will use
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2

U = \frac{3}{2} nRT \quad (Equation 14.7) for the internal energy of an ideal gas and W = P\Delta V (Equation 15.2) for the work done at constant pressure. In Equation 14.7, we do not know the Kelvin temperature T. However, we can use the ideal gas law PV = nRT (Equation 14.1) to deal with this lack of information.

**SOLUTION**

a. Using the first law of thermodynamics from Equation 15.1, we have

\[ \Delta U = Q - W \quad \text{or} \quad Q = \Delta U + W \quad (1) \]

Using Equation 14.7 for the internal energy and Equation 14.1 for the ideal gas law, we have

\[ U = \frac{3}{2} nRT = \frac{3}{2} PV \quad \text{or} \quad \Delta U = \frac{3}{2} P\Delta V \quad (2) \]

where we have taken advantage of the fact that the pressure P is constant. Using Equation (2) and \( W = P\Delta V \) (Equation 15.2) to substitute into Equation (1), we obtain

\[ Q = \Delta U + W = \frac{3}{2} P\Delta V + P\Delta V = \frac{5}{2} P\Delta V \]

\[ = \frac{5}{2} \left( 2.6 \times 10^5 \text{ Pa} \right) \left( +6.2 \times 10^{-3} \text{ m}^3 \right) = 4.0 \times 10^3 \text{ J} \]

b. The heat \( Q \) is positive. Therefore, according to our convention, heat flows into the gas.

15. **REASONING AND SOLUTION**  The work done by the expanding gas is

\[ W = Q - \Delta U = 2050 \text{ J} - 1730 \text{ J} = 320 \text{ J} \]

The work, according to Equation 6.1, is also the magnitude \( F \) of the force exerted on the piston times the magnitude \( s \) of its displacement. But the force is equal to the weight \( mg \) of the block and piston, so that the work is \( W = Fs = mgs \). Thus, we have

\[ s = \frac{W}{mg} = \frac{320 \text{ J}}{(135 \text{ kg})(9.80 \text{ m/s}^2)} = 0.24 \text{ m} \]

16. **REASONING AND SOLUTION**  The rod’s volume increases by an amount \( \Delta V = \beta V_0 \Delta T \), according to Equation 12.3. The work done by the expanding aluminum is, from Equation 15.2,

\[ W = P\Delta V = P\beta V_0 \Delta T = (1.01 \times 10^5 \text{ Pa})(69 \times 10^{-6}/\text{C}^\circ)(1.4 \times 10^{-3} \text{ m}^3)(3.0 \times 10^2 \text{ C}^\circ) = 2.9 \text{ J} \]

17. **SSM  REASONING**  The pressure \( P \) of the gas remains constant while its volume increases by an amount \( \Delta V \). Therefore, the work \( W \) done by the expanding gas is given by \( W = P\Delta V \) (Equation 15.2). \( \Delta V \) is known, so if we can obtain a value for \( W \), we can use this expression to calculate the pressure. To determine \( W \), we turn to the first law of
thermodynamics $\Delta U = Q - W$ (Equation 15.1), where $Q$ is the heat and $\Delta U$ is the change in the internal energy. $\Delta U$ is given, so to use the first law to determine $W$ we need information about $Q$. According to Equation 12.4, the heat needed to raise the temperature of a mass $m$ of material by an amount $\Delta T$ is $Q = cm\Delta T$ where $c$ is the material’s specific heat capacity.

**SOLUTION** According to Equation 15.2, the pressure $P$ of the expanding gas can be determined from the work $W$ and the change $\Delta V$ in volume of the gas according to

$$P = \frac{W}{\Delta V}$$

Using the first law of thermodynamics, we can write the work as $W = Q - \Delta U$ (Equation 15.1). With this substitution, the expression for the pressure becomes

$$P = \frac{W}{\Delta V} = \frac{Q - \Delta U}{\Delta V} \quad (1)$$

Using Equation 12.4, we can write the heat as $Q = cm\Delta T$, which can then be substituted into Equation (1). Thus,

$$P = \frac{Q - \Delta U}{\Delta V} = \frac{cm\Delta T - \Delta U}{\Delta V} = \frac{[1080 \text{ J/}(\text{kg} \cdot \text{C}^\circ)](24.0 \times 10^{-3} \text{ kg})(53.0 \text{ C}^\circ) - 939 \text{ J}}{1.40 \times 10^{-3} \text{ m}^3} = 3.1 \times 10^5 \text{ Pa}$$

18. **REASONING AND SOLUTION** According to the first law of thermodynamics, the change in internal energy is $\Delta U = Q - W$. The work can be obtained from the area under the graph. There are sixty squares of area under the graph, so the positive work of expansion is

$$W = 60(1.0 \times 10^4 \text{ Pa})(2.0 \times 10^{-3} \text{ m}^3) = 1200 \text{ J}$$

Since $Q = 2700 \text{ J}$, the change in internal energy is

$$\Delta U = Q - W = 2700 \text{ J} - 1200 \text{ J} = 1500 \text{ J}$$

19. **REASONING AND SOLUTION** Since the pan is open, the process takes place at constant (atmospheric) pressure $P_0$. The work involved in an isobaric process is given by Equation 15.2: $W = P_0 \Delta V$. The change in volume of the liquid as it is heated is given according to Equation 12.3 as $\Delta V = \beta V_0 \Delta T$, where $\beta$ is the coefficient of volume expansion. Table 12.1 gives $\beta = 207 \times 10^{-6} \text{ (C}^\circ\text{)}^{-1}$ for water. The heat absorbed by the water is given by
Equation 12.4 as \( Q = cm\Delta T \), where \( c = 4186 \text{ J/(kg·°C)} \) is the specific heat capacity of liquid water according to Table 12.2. Therefore,

\[
\frac{W}{Q} = \frac{P_0 \beta V_{0}\Delta T}{cm\Delta T} = \frac{P_0 \beta}{c \rho} = \frac{P_0 \beta}{c \rho}
\]

where \( \rho = 1.00 \times 10^3 \text{ kg/m}^3 \) is the density of the water (see Table 11.1). Thus, we find

\[
W = \frac{P_0 \beta}{c \rho} \left[ 4186 \text{ J/(kg·°C)} \right] \left[ 1.00 \times 10^3 \text{ kg/m}^3 \right] = 4.99 \times 10^{-6}
\]

20. **REASONING** The work \( W \) done when an ideal gas expands isothermally at a Kelvin temperature \( T \) is \( W = nRT \ln \left( \frac{V_f}{V_i} \right) \) (Equation 15.3), where \( V_f \) and \( V_i \) are the final and initial volumes of the gas, respectively, \( R = 8.31 \text{ J/(mol·K)} \) is the universal gas constant, and \( n \) is the number of moles of the gas. Solving this expression for \( \frac{V_f}{V_i} \) gives the desired ratio.

**SOLUTION** Rearranging Equation 15.3, we have

\[
W = nRT \ln \left( \frac{V_f}{V_i} \right) \quad \text{or} \quad \ln \left( \frac{V_f}{V_i} \right) = \frac{W}{nRT} \quad (1)
\]

As Section 14.1 discusses, the number of moles \( n \) is given by the mass \( m \) divided by the mass per mole:

\[
n = \frac{m}{\text{Mass per mole}} = \frac{6.0 \text{ g}}{4.0 \text{ g/mol}} = 1.5 \text{ mol}
\]

Using Equation (1), we obtain

\[
\ln \left( \frac{V_f}{V_i} \right) = \frac{W}{nRT} = \frac{9600 \text{ J}}{1.5 \text{ mol} \cdot 8.31 \text{ J/(mol·K)} \cdot 370 \text{ K}} = 2.08
\]

Therefore, \( \frac{V_f}{V_i} = e^{2.08} = 8.0 \).

21. **SSM REASONING** Since the gas is expanding adiabatically, the work done is given by Equation 15.4 as \( W = \frac{\frac{2}{3}nRT_i}{T_f} \). Once the work is known, we can use the first law of thermodynamics to find the change in the internal energy of the gas.

**SOLUTION**

a. The work done by the expanding gas is
\[ W = \frac{3}{2} nR \left( T_f - T_i \right) = \frac{3}{2} (5.0 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (370 \text{ K} - 290 \text{ K}) = +5.0 \times 10^3 \text{ J} \]

b. Since the process is adiabatic, \( Q = 0 \text{ J} \), and the change in the internal energy is
\[ \Delta U = Q - W = 0 - 5.0 \times 10^3 \text{ J} = -5.0 \times 10^3 \text{ J} \]

22. **REASONING** When \( n \) moles of an ideal gas change quasistatically to a final volume \( V_f \) from an initial volume \( V_i \) at a constant temperature \( T \), the work \( W \) done is (see Equation 15.3)
\[ W = nRT \ln \left( \frac{V_f}{V_i} \right) \quad \text{or} \quad T = \frac{W}{nR \ln \left( \frac{V_f}{V_i} \right)} \quad (1) \]

where \( R \) is the universal gas constant. To determine \( T \) from Equation (1), we need a value for the work, which we do not have. However, we do have a value for the heat \( Q \). To take advantage of this value, we note that Section 14.3 discusses the fact that the internal energy of an ideal gas is directly proportional to its Kelvin temperature. Since the temperature is constant (the neon expands isothermally), the internal energy remains constant. According to the first law of thermodynamics (Equation 15.1), the change \( \Delta U \) in the internal energy is given by \( \Delta U = Q - W \). Since the internal energy \( U \) is constant, \( \Delta U = 0 \), so that \( W = Q \).

**SOLUTION** Substituting \( W = Q \) into the expression for \( T \) in Equation (1), we find that the temperature of the gas during the isothermal expansion is
\[ T = \frac{Q}{nR \ln \left( \frac{V_f}{V_i} \right)} = \frac{4.75 \times 10^3 \text{ J}}{(3.00 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] \ln \left( \frac{0.250 \text{ m}^3}{0.100 \text{ m}^3} \right)} = 208 \text{ K} \]

23. **REASONING** We can use the first law of thermodynamics, \( \Delta U = Q - W \) (Equation 15.1) to find the work \( W \). The heat is \( Q = -4700 \text{ J} \), where the minus sign denotes that the system (the gas) loses heat. The internal energy \( U \) of a monatomic ideal gas is given by \( U = \frac{3}{2} nRT \) (Equation 14.7), where \( n \) is the number of moles, \( R \) is the universal gas constant, and \( T \) is the Kelvin temperature. If the temperature remains constant during the process, the internal energy does not change, so \( \Delta U = 0 \text{ J} \).

**SOLUTION** The work done during the isothermal process is
\[ W = Q - \Delta U = -4700 \text{ J} + 0 \text{ J} = -4700 \text{ J} \]

The negative sign indicates that work is done on the system.
24. **REASONING** An adiabatic process is one for which no heat enters or leaves the system, so \( Q = 0 \) J. The work is given as \( W = +610 \) J, where the plus sign denotes that the gas does work, according to our convention. Knowing the heat and the work, we can use the first law of thermodynamics to find the change \( \Delta U \) in internal energy as \( \Delta U = Q - W \) (Equation 15.1). Knowing the change in the internal energy, we can find the change in the temperature by recalling that the internal energy of a monatomic ideal gas is \( U = \frac{3}{2}nRT \), according to Equation 14.7. As a result, it follows that \( \Delta U = \frac{3}{2}nR\Delta T \).

**SOLUTION** Using the first law from Equation 15.5 and the change in internal energy from Equation 14.7, we have

\[
\Delta U = Q - W \quad \text{or} \quad \frac{3}{2}nR\Delta T = Q - W
\]

Therefore, we find

\[
\Delta T = \frac{2(Q - W)}{3nR} = \frac{2(0 \text{ J}) - (610 \text{ J})}{3(0.50 \text{ mol})(8.31 \text{ J/(mol·K)})} = \boxed{-98 \text{ K}}
\]

The change in temperature is a decrease.

---

25. **SSM REASONING** When the expansion is isothermal, the work done can be calculated from Equation (15.3): \( W = nRT \ln\left(\frac{V_f}{V_i}\right) \). When the expansion is adiabatic, the work done can be calculated from Equation 15.4: \( W = \frac{3}{2}nR(T_i - T_f) \).

Since the gas does the same amount of work whether it expands adiabatically or isothermally, we can equate the right hand sides of these two equations. We also note that since the initial temperature is the same for both cases, the temperature \( T \) in the isothermal expansion is the same as the initial temperature \( T_i \) for the adiabatic expansion. We then have

\[
nRT_i \ln\left(\frac{V_f}{V_i}\right) = \frac{3}{2}nR(T_i - T_f) \quad \text{or} \quad \ln\left(\frac{V_f}{V_i}\right) = \frac{\frac{3}{2}(T_i - T_f)}{T_i}
\]

**SOLUTION** Solving for the ratio of the volumes gives

\[
\frac{V_f}{V_i} = e^{\frac{3}{2}(T_i - T_f)/T_i} = e^{\frac{3}{2}(405 \text{ K} - 245 \text{ K})/(405 \text{ K})} = \boxed{1.81}
\]

---

26. **REASONING** According to the first law of thermodynamics \( \Delta U = Q - W \) (Equation 15.1), where \( \Delta U \) is the change in the internal energy, \( Q \) is the heat, and \( W \) is the work. This expression may be solved for the heat. \( \Delta U \) can be evaluated by remembering that the internal energy of a monatomic ideal gas is \( U = \frac{3}{2}nRT \) (Equation 14.7), where \( n \) is the number of moles, \( R = 8.31 \text{ J/(mol·K)} \) is the universal gas constant, and \( T \) is the Kelvin temperature. Since heat is being added isothermally, the temperature remains constant and
so does the internal energy of the gas. Therefore, \( \Delta U = 0 \) J. To evaluate \( W \) we use

\[ W = nRT \ln \left( \frac{V_f}{V_i} \right) \]  

(Equation 15.3), where \( V_f \) and \( V_i \) are the final and initial volumes of the gas, respectively.

**SOLUTION**  According to the first law of thermodynamics, as given in Equation 15.1, the heat added to the gas is

\[ Q = \Delta U + W \]

Using the fact that \( \Delta U = 0 \) J for an ideal gas undergoing an isothermal process and the fact that \( W = nRT \ln \left( \frac{V_f}{V_i} \right) \) (Equation 15.3), we can rewrite the expression for the heat as follows:

\[ Q = \Delta U + W = nRT \ln \left( \frac{V_f}{V_i} \right) \]

Since the volume of the gas doubles, we know that \( V_f = 2V_i \). Thus, it follows that

\[ Q = nRT \ln \left( \frac{V_f}{V_i} \right) = (2.5 \, \text{mol}) \left[ 8.31 \, \text{J/mol} \cdot \text{K} \right] (430 \, \text{K}) \ln \left( \frac{2V_i}{V_f} \right) = 6200 \, \text{J} \]

27. **REASONING** During an adiabatic process, no heat flows into or out of the gas (\( Q = 0 \) J). For an ideal gas, the final pressure and volume \((P_f \text{ and } V_f)\) are related to the initial pressure and volume \((P_i \text{ and } V_i)\) by \( P_i V_i^\gamma = P_f V_f^\gamma \) (Equation 15.5), where \( \gamma \) is the ratio of the specific heat capacities at constant pressure and constant volume (\( \gamma = \frac{7}{5} \) in this problem). The initial and final pressures are not given. However, the initial and final temperatures are known, so we can use the ideal gas law, \( PV = nRT \) (Equation 14.1) to relate the temperatures to the pressures. We will then be able to find \( V_i/V_f \) in terms of the initial and final temperatures.

**SOLUTION** Substituting the ideal gas law, \( PV = nRT \), into \( P_i V_i^\gamma = P_f V_f^\gamma \) gives

\[ \left( \frac{nRT_i}{V_i} \right) V_i^\gamma = \left( \frac{nRT_f}{V_f} \right) V_f^\gamma \quad \text{or} \quad T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \]

Solving this expression for the ratio of the initial volume to the final volume yields

\[ \frac{V_i}{V_f} = \left( \frac{T_f}{T_i} \right)^{\frac{1}{\gamma-1}} \]

The initial and final Kelvin temperatures are \( T_i = (21 \, ^\circ\text{C} + 273) = 294 \, \text{K} \) and \( T_f = (688 \, ^\circ\text{C} + 273) = 961 \, \text{K} \). The ratio of the volumes is
\[ \frac{V_i}{V_f} = \left( \frac{T_f}{T_i} \right)^{\frac{1}{\gamma - 1}} = \left( \frac{961 \text{ K}}{294 \text{ K}} \right)^{\frac{1}{\frac{7}{2} - 1}} = 19.3 \]
28. **REASONING**

a. The work done by the gas is equal to the area under the pressure-versus-volume curve. We will measure this area by using the graph given with the problem.

![Diagram](image)

b. Since the gas is an ideal gas, it obeys the ideal gas law, \( PV = nRT \) (Equation 14.1). This implies that \( \frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} \). All the variables except for \( T_B \) in this relation are known. Therefore, we can use this expression to find the temperature at point B.

c. The heat \( Q \) that has been added to or removed from the gas can be obtained from the first law of thermodynamics, \( Q = \Delta U + W \) (Equation 15.1), where \( \Delta U \) is the change in the internal energy of the gas and \( W \) is the work done by the gas. The work \( W \) is known from part (a) of the problem. The change \( \Delta U \) in the internal energy of the gas can be obtained from Equation 14.7, \( \Delta U = U_B - U_A = \frac{3}{2} nR(T_B - T_A) \), where \( n \) is the number of moles, \( R \) is the universal gas constant, and \( T_B \) and \( T_A \) are the final and initial Kelvin temperatures. We do not know \( n \), but we can use the ideal gas law \((PV = nRT)\) to replace \( nRT_B \) by \( P_B V_B \) and to replace \( nRT_A \) by \( P_A V_A \).

**SOLUTION**

a. From the drawing we see that the area under the curve is 5.00 “squares,” where each square has an area of \((2.00 \times 10^5 \text{ Pa})(2.00 \text{ m}^3) = 4.00 \times 10^5 \text{ J} \). Therefore, the work \( W \) done by the gas is

\[
W = (5.00 \text{ squares})(4.00 \times 10^5 \text{ J/square}) = 2.00 \times 10^6 \text{ J}
\]

b. In the Reasoning section, we have seen that \( \frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} \). Solving this relation for the temperature \( T_B \) at point B, using the fact that \( P_A = P_B \) (see the graph), and taking the values for \( V_B \) and \( V_A \) from the graph, we have that

\[
T_B = \left( \frac{P_B V_B}{P_A V_A} \right) T_A = \left( \frac{V_B}{V_A} \right) T_A = \left( \frac{10.0 \text{ m}^3}{2.00 \text{ m}^3} \right) (185 \text{ K}) = 925 \text{ K}
\]

c. From the Reasoning section we know that the heat \( Q \) that has been added to or removed from the gas is given by \( Q = \Delta U + W \). The change \( \Delta U \) in the internal energy of the gas is \( \Delta U = U_B - U_A = \frac{3}{2} nR(T_B - T_A) \). Thus, the heat can be expressed as
\[ Q = \Delta U + W = \frac{3}{2} nR(T_B - T_A) + W \]

We now use the ideal gas law \((PV = nRT)\) to replace \(nRT_B\) by \(P_BV_B\) and \(nRT_A\) by \(P_AV_A\). The result is

\[ Q = \frac{3}{2}(P_BV_B - P_AV_A) + W \]

Taking the values for \(P_B, V_B, P_A,\) and \(V_A\) from the graph and using the result from part a that \(W = 2.00 \times 10^6 \text{ J}\), we find that the heat is

\[ Q = \frac{3}{2}(P_BV_B - P_AV_A) + W \]

\[ = \frac{3}{2}[(2.00 \times 10^5 \text{ Pa})(10.0 \text{ m}^3) - (2.00 \times 10^5 \text{ Pa})(2.00 \text{ m}^3)] + 2.00 \times 10^6 \text{ J} = 4.40 \times 10^6 \text{ J} \]

29. **SSM REASONING AND SOLUTION**

**Step A \rightarrow B**

The internal energy of a monatomic ideal gas is \(U = (3/2)nRT\). Thus, the change is

\[ \Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2}(1.00 \text{ mol})[8.31 \text{ J/(mol\cdot K)}](800.0 \text{ K} - 400.0 \text{ K}) = 4990 \text{ J} \]

The work for this constant pressure step is \(W = P\Delta V\). But the ideal gas law applies, so

\[ W = P\Delta V = nR \Delta T = (1.00 \text{ mol})[8.31 \text{ J/(mol\cdot K)}](800.0 \text{ K} - 400.0 \text{ K}) = 3320 \text{ J} \]

The first law of thermodynamics indicates that the heat is

\[ Q = \Delta U + W = \frac{3}{2} nR \Delta T + nR \Delta T \]

\[ = \frac{5}{2}(1.00 \text{ mol})[8.31 \text{ J/(mol\cdot K)}](800.0 \text{ K} - 400.0 \text{ K}) = 8310 \text{ J} \]

**Step B \rightarrow C**

The internal energy of a monatomic ideal gas is \(U = (3/2)nRT\). Thus, the change is

\[ \Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2}(1.00 \text{ mol})[8.31 \text{ J/(mol\cdot K)}](400.0 \text{ K} - 800.0 \text{ K}) = -4990 \text{ J} \]

The volume is constant in this step, so the work done by the gas is \[ W = 0 \text{ J} \].

The first law of thermodynamics indicates that the heat is

\[ Q = \Delta U + W = \Delta U = -4990 \text{ J} \]
Step C → D
The internal energy of a monatomic ideal gas is \( U = (3/2)nRT \). Thus, the change is
\[
\Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (200.0 \text{ K} - 400.0 \text{ K}) = -2490 \text{ J}
\]
The work for this constant pressure step is \( W = P\Delta V \). But the ideal gas law applies, so
\[
W = P\Delta V = nR \Delta T = (1.00 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (200.0 \text{ K} - 400.0 \text{ K}) = -1660 \text{ J}
\]
The first law of thermodynamics indicates that the heat is
\[
Q = \Delta U + W = \frac{3}{2} nR \Delta T + nR \Delta T = \frac{5}{2} (1.00 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (200.0 \text{ K} - 400.0 \text{ K}) = -4150 \text{ J}
\]

Step D → A
The internal energy of a monatomic ideal gas is \( U = (3/2)nRT \). Thus, the change is
\[
\Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (400.0 \text{ K} - 200.0 \text{ K}) = 2490 \text{ J}
\]
The volume is constant in this step, so the work done by the gas is \( W = 0 \text{ J} \)
The first law of thermodynamics indicates that the heat is
\[
Q = \Delta U + W = \Delta U = 2490 \text{ J}
\]

30. **REASONING** The cylinder containing the gas is perfectly insulated, so no heat can enter or leave. Therefore, the compression of the gas is adiabatic, and the initial and final pressures \( (P_i, P_f) \) and volumes \( (V_i, V_f) \) of the gas are related by \( P_i V_i^\gamma = P_f V_f^\gamma \) (Equation 15.5), where \( \gamma = \frac{5}{3} \) because the gas is monatomic and ideal. The final Kelvin temperature \( T_f \) of the gas is twice the initial temperature \( T_i \), so we have that \( T_f = 2T_i \). We will use the ideal gas law \( PV = nRT \) (Equation 14.1) to determine the final pressure \( P_f \) of the gas from its volume and Kelvin temperature. In Equation 14.1, \( n \) is the number of moles of the gas and \( R = 8.31 \text{ J/(mol·K)} \) is the universal gas constant.

**SOLUTION** In order to make use of the ideal gas law \( PV = nRT \) (Equation 14.1), we rewrite \( P_i V_i^\gamma = P_f V_f^\gamma \) (Equation 15.5) so that the product \( PV \) is raised to the power \( \gamma \):
\[
P_i^{1-\gamma} P_i^{\gamma} V_i^\gamma = P_f^{1-\gamma} P_f^{\gamma} V_f^\gamma \quad \text{or} \quad P_i^{1-\gamma} (P_i V_i)^\gamma = P_f^{1-\gamma} (P_f V_f)^\gamma \quad (1)
\]
In Equation (1), we have used the fact that \( P(a+b) = P^a P^b \). Substituting \( P_i V_i = nRT_i \) and \( P_i V_i = nRT_i \) (Equation 14.1) into Equation (1) yields

\[
P_i^{1-\gamma} \left( \mu \kappa T_i \right)^{\gamma} = P_i^{1-\gamma} \left( \mu \kappa T_f \right)^{\gamma} \quad \text{or} \quad P_i^{1-\gamma} T_i^{\gamma} = P_i^{1-\gamma} T_f^{\gamma}
\]  

(2)

Solving Equation (2) for \( P_i^{1-\gamma} \) and then raising both sides to the power \( \frac{1}{1-\gamma} \), we obtain

\[
P_i^{1-\gamma} = P_i^{1-\gamma} \left( \frac{T_f}{T_i} \right)^{\gamma} \quad \text{or} \quad P_i = P_i \left( \frac{T_f}{T_i} \right)^{\gamma/(1-\gamma)}
\]  

(3)

Equation (3) makes use of the fact that \( (p^a)^{1/a} = p^{a/a} = P^1 = P \). Substituting \( T_f = 2T_i \) and \( \gamma = \frac{5}{3} \) into Equation (3), we find that

\[
P_i = (1.50 \times 10^5 \text{ Pa}) \left( \frac{\sqrt{\gamma}}{2 T_i} \right)^{\left(\frac{5}{3}\right)} \left[\frac{1}{1-(5/3)}\right] = (1.50 \times 10^5 \text{ Pa}) \left( \frac{1}{2} \right)^{(5/2)} = 8.49 \times 10^5 \text{ Pa}
\]

31. REASONING AND SOLUTION
a. Since the curved line between A and C is an isotherm, the initial and final temperatures are the same. Since the internal energy of an ideal monatomic gas is \( U = (3/2)nRT \), the initial and final energies are also the same, and the change in the internal energy is \( \Delta U = 0 \). The first law of thermodynamics, then, indicates that for the process \( A \rightarrow B \rightarrow C \), we have

\[
\Delta U = 0 = Q - W \quad \text{or} \quad Q = W
\]

The heat is equal to the work. Determining the work from the area beneath the straight line segments AB and BC, we find that

\[
Q = W = - \left( 4.00 \times 10^5 \text{ Pa} \right) \left( 0.400 \text{ m}^3 - 0.200 \text{ m}^3 \right) = -8.00 \times 10^4 \text{ J}
\]

b. The minus sign is included because the gas is compressed, so that work is done on the gas. Since the answer for \( Q \) is negative, we conclude that heat flows out of the gas.

32. REASONING AND SOLUTION
a. The final temperature of the adiabatic process is given by solving Equation 15.4 for \( T_f \)

\[
T_f = T_i - \frac{W}{\frac{3}{2} nR} = 393 \text{ K} - \frac{825 \text{ J}}{\frac{3}{2}(1.00 \text{ mol})[8.31 \text{ J/(mol·K)]}} = 327 \text{ K}
\]
b. According to Equation 15.5 for the adiabatic expansion of an ideal gas, $P_i V_i^\gamma = P_f V_f^\gamma$. Therefore,

$$V_f^\gamma = V_i^\gamma \left( \frac{P_i}{P_f} \right)$$

From the ideal gas law, $PV = nRT$; therefore, the ratio of the pressures is given by

$$\frac{P_i}{P_f} = \left( \frac{T_i}{T_f} \right) \left( \frac{V_f}{V_i} \right)$$

Combining the previous two equations gives

$$V_f^\gamma = V_i^\gamma \left( \frac{T_i}{T_f} \right) \left( \frac{V_f}{V_i} \right)$$

Solving for $V_f$ we obtain

$$\frac{V_f}{V_i} = \left( \frac{T_i}{T_f} \right) \left( \frac{V_f}{V_i} \right)$$

or

$$V_f = V_i \left( \frac{T_i}{T_f} \right) \frac{1}{\gamma - 1}$$

Therefore,

$$V_f = V_i \left( \frac{T_i}{T_f} \right) \frac{1}{\gamma - 1} = (0.100 \text{ m}^3) \left( \frac{393 \text{ K}}{327 \text{ K}} \right)^{1/(2/3)} = (0.100 \text{ m}^3) \left( \frac{393 \text{ K}}{327 \text{ K}} \right)^{3/2} = 0.132 \text{ m}^3$$

33. **SSM REASONING AND SOLUTION** Let the left be side 1 and the right be side 2. Since the partition moves to the right, side 1 does work on side 2, so that the work values involved satisfy the relation $W_1 = -W_2$. Using Equation 15.4 for each work value, we find that

$$\frac{3}{2} nR (T_{1i} - T_{1f}) = -\frac{3}{2} nR (T_{2i} - T_{2f})$$

or

$$T_{1f} + T_{2f} = T_{1i} + T_{2i} = 525 \text{ K} + 275 \text{ K} = 8.00 \times 10^2 \text{ K}$$

We now seek a second equation for the two unknowns $T_{1f}$ and $T_{2f}$. Equation 15.5 for an adiabatic process indicates that $P_{li} V_{li}^\gamma = P_{lf} V_{lf}^\gamma$ and $P_{2i} V_{2i}^\gamma = P_{2f} V_{2f}^\gamma$. Dividing these two equations and using the facts that $V_{li} = V_{2i}$ and $P_{li} = P_{2f}$, gives

$$\frac{P_{li} V_{li}^\gamma}{P_{2i} V_{2i}^\gamma} = \frac{P_{lf} V_{lf}^\gamma}{P_{2f} V_{2f}^\gamma}$$

or

$$\frac{P_{li}}{P_{2f}} = \left( \frac{V_{lf}}{V_{2i}} \right)^\gamma$$

$$\frac{P_{li}}{P_{2i}} = \left( \frac{V_{lf}}{V_{2f}} \right)^\gamma$$
Using the ideal gas law, we find that

\[
\frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^\gamma \quad \text{becomes} \quad \frac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\frac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^\gamma
\]

Since \( V_{1i} = V_{2i} \) and \( P_{1f} = P_{2f} \), the result above reduces to

\[
\frac{T_{1f}}{T_{2i}} = \left(\frac{T_{1f}}{T_{2f}}\right)^\gamma \quad \text{or} \quad \frac{T_{1f}}{T_{2f}} = \left(\frac{T_{li}}{T_{2i}}\right)^{1/\gamma} = \left(\frac{525 \text{ K}}{275 \text{ K}}\right)^{1/\gamma} = 1.474
\]

Using this expression for the ratio of the final temperatures in \( T_{1f} + T_{2f} = 8.00 \times 10^2 \text{ K} \), we find that

a. \( T_{1f} = 477 \text{ K} \) and b. \( T_{2f} = 323 \text{ K} \)

34. **REASONING** The heat \( Q \) that must be added to \( n \) moles of a monatomic ideal gas to raise its temperature by \( \Delta T \) Kelvin degrees under conditions of constant pressure is \( Q = C_p n \Delta T \) (Equation 15.6), where \( C_p = \frac{5}{2} R \) (Equation 15.7) is the molar specific heat capacity of a monatomic ideal gas and \( R = 8.31 \text{ J/(mol} \cdot \text{K)} \) is the universal gas constant.

**SOLUTION** Substituting Equation 15.7 into Equation 15.6 shows that the heat is

\[
Q = C_p n \Delta T = \left(\frac{5}{2} R\right)n \Delta T \tag{1}
\]

As Section 14.1 discusses, the number of moles \( n \) is given by the mass \( m \) divided by the mass per mole:

\[
n = \frac{m}{\text{Mass per mole}} = \frac{8.0 \text{ g}}{39.9 \text{ g/mol}} = 0.20 \text{ mol}
\]

With this value for \( n \), Equation (1) gives

\[
Q = C_p n \Delta T = \left(\frac{5}{2} R\right)n \Delta T = \left(\frac{5}{2} \left[8.31 \text{ J/(mol} \cdot \text{K)}\right]\right)(0.20 \text{ mol})(75 \text{ K}) = 310 \text{ J}
\]

35. **SSM REASONING AND SOLUTION** According to the first law of thermodynamics (Equation 15.1), \( \Delta U = U_f - U_i = Q - W \). Since the internal energy of this gas is doubled by the addition of heat, the initial and final internal energies are \( U \) and \( 2U \), respectively. Therefore,

\[
\Delta U = U_f - U_i = 2U - U = U
\]

Equation 15.1 for this situation then becomes \( U = Q - W \). Solving for \( Q \) gives
The initial internal energy of the gas can be calculated from Equation 14.7:

\[
U = \frac{3}{2} nRT = \frac{3}{2} (2.5 \text{ mol}) \left[ 8.31 \text{ J/(mol} \cdot \text{K}) \right] (350 \text{ K}) = 1.1 \times 10^4 \text{ J}
\]

a. If the process is carried out isochorically (i.e., at constant volume), then \( W = 0 \), and the heat required to double the internal energy is

\[
Q = U + W = U + 0 = 1.1 \times 10^4 \text{ J}
\]

b. If the process is carried out isobarically (i.e., at constant pressure), then \( W = P\Delta V \), and Equation (1) above becomes

\[
Q = U + W = U + P\Delta V
\]

From the ideal gas law, \( PV = nRT \), we have that \( P\Delta V = nR\Delta T \), and Equation (2) becomes

\[
Q = U + nR\Delta T
\]

The internal energy of an ideal gas is directly proportional to its Kelvin temperature. Since the internal energy of the gas is doubled, the final Kelvin temperature will be twice the initial Kelvin temperature, or \( \Delta T = 350 \text{ K} \). Substituting values into Equation (3) gives

\[
Q = 1.1 \times 10^4 \text{ J} + (2.5 \text{ mol})[8.31 \text{ J/(mol} \cdot \text{K})](350 \text{ K}) = 1.8 \times 10^4 \text{ J}
\]

36. **REASONING** Under constant-pressure conditions, the heat \( Q_p \) required to raise the temperature of an ideal gas is given by \( Q_p = C_p n \Delta T \) (Equation 15.6), where \( C_p \) is the molar specific heat capacity at constant pressure, \( n \) is the number of moles of the gas, and \( \Delta T \) is the change in the temperature. The molar specific heat capacity \( C_p \) at constant pressure is greater than the molar specific heat capacity at constant volume \( C_V \), as we see from \( C_p = C_V + R \) (Equation 15.10), where \( R = 8.31 \text{ J/(mol} \cdot \text{K}) \). Equation 15.6 also applies to the constant-volume process, so we have that \( Q_V = C_V n \Delta T \), where \( Q_V = 3500 \text{ J} \) is the heat required for the constant-volume process. We will use these relations, and the fact that the change in temperature and the number of moles of the gas are the same for both processes, to find the heat \( Q_p \) required for the constant-pressure process.

**SOLUTION** We begin by substituting \( C_p = C_V + R \) (Equation 15.10) into \( Q_p = C_p n \Delta T \) (Equation 15.6) to obtain

\[
Q_p = C_p n \Delta T = (C_V + R) n \Delta T
\]
Solving \( Q_V = C_V n \Delta T \) (Equation 15.6) for \( C_V \) yields \( C_V = \frac{Q_V}{n \Delta T} \). Substituting this result into Equation (1), we find that

\[
Q_p = (C_V + R)n \Delta T = \left( \frac{Q_V}{n \Delta T} + R \right)n \Delta T = Q_V + Rn \Delta T
\]

Therefore, the heat required for the constant-pressure process is

\[
Q_p = 3500 \text{ J} + (1.6 \text{ mol}) \left[ 8.31 \text{ J/(mol·K)} \right] (75 \text{ K}) = 4500 \text{ J}
\]

37. 

**REASONING** When the temperature of a gas changes as a result of heat \( Q \) being added, the change \( \Delta T \) in temperature is related to the amount of heat according to \( Q = Cn \Delta T \) (Equation 15.6), where \( C \) is the molar specific heat capacity, and \( n \) is the number of moles. The heat \( Q_V \) added under conditions of constant volume is \( Q_V = C_V n \Delta T_V \), where \( C_V \) is the specific heat capacity at constant volume and is given by \( C_V = \frac{3}{2} R \) (Equation 15.8) and \( R \) is the universal gas constant. The heat \( Q_p \) added under conditions of constant pressure is \( Q_p = C_p n \Delta T_p \), where \( C_p \) is the specific heat capacity at constant pressure and is given by \( C_p = \frac{5}{2} R \) (Equation 15.7). It is given that \( Q_V = Q_p \), and this fact will allow us to find the change in temperature of the gas whose pressure remains constant.

**SOLUTION** Setting \( Q_V = Q_p \), gives

\[
\frac{C_V n \Delta T_V}{Q_V} = \frac{C_p n \Delta T_p}{Q_p}
\]

Algebraically eliminating \( n \) and solving for \( \Delta T_p \), we obtain

\[
\Delta T_p = \left( \frac{C_V}{C_p} \right) \Delta T_V = \left( \frac{3}{2} R \right) \left( \frac{5}{2} R \right) (75 \text{ K}) = 45 \text{ K}
\]

38. 

**REASONING** The gas is in a rigid container, so we conclude that the heating takes place at constant volume. For a constant-volume process involving an ideal monatomic gas, the amount \( Q \) of heat transferred is given by \( Q = C_V n \Delta T \) (Equation 15.6), where \( C_V = \frac{3}{2} R \) (Equation 15.8) is the molar specific heat capacity at constant volume, \( R \) is the universal gas constant, \( n \) is the number of moles of gas, and \( \Delta T \) is the change in temperature.
**SOLUTION** Solving \( Q = C_V n \Delta T \) (Equation 15.6) for the number \( n \) of moles of the gas, we obtain

\[
n = \frac{Q}{C_V \Delta T}
\]  

(1)

Substituting \( C_V = \frac{3}{2} R \) (Equation 15.8) and \( \Delta T = T_f - T_i \) into Equation (1) yields

\[
n = \frac{Q}{C_V \Delta T} = \frac{Q}{\left(\frac{3}{2} R\right)(T_f - T_i)} = \frac{2Q}{3R(T_f - T_i)}
\]

\[
= \frac{2(8500 \text{ J})}{3[8.31 \text{ J/(mol} \cdot \text{K}](279 \text{ K} - 217 \text{ K})} = 11 \text{ mol}
\]

**39. REASONING AND SOLUTION**

a. The amount of heat needed to raise the temperature of the gas at constant volume is given by Equations 15.6 and 15.8, \( Q = n C_V \Delta T \). Solving for \( \Delta T \) yields

\[
\Delta T = \frac{Q}{nC_v} = \frac{5.24 \times 10^3 \text{ J}}{3.00 \text{ mol} \cdot \left(\frac{3}{2} R\right)} = 1.40 \times 10^2 \text{ K}
\]

b. The change in the internal energy of the gas is given by the first law of thermodynamics with \( W = 0 \), since the gas is heated at constant volume:

\[
\Delta U = Q - W = 5.24 \times 10^3 \text{ J} - 0 = 5.24 \times 10^3 \text{ J}
\]

c. The change in pressure can be obtained from the ideal gas law,

\[
\Delta P = \frac{nR \Delta T}{V} = \frac{(3.00 \text{ mol})R \left(1.40 \times 10^2 \text{ K}\right)}{1.50 \text{ m}^3} = 2.33 \times 10^3 \text{ Pa}
\]

**40. REASONING** According to Equations 15.6 and 15.7, the heat supplied to a monatomic ideal gas at constant pressure is \( Q = C_P n \Delta T \), with \( C_P = \frac{5}{2} R \). Thus, \( Q = \frac{5}{2} nR \Delta T \). The percentage of this heat used to increase the internal energy by an amount \( \Delta U \) is

\[
\text{Percentage} = \left(\frac{\Delta U}{Q}\right) \times 100\% = \left(\frac{\Delta U}{\frac{5}{2} nR \Delta T}\right) \times 100\%
\]

But according to the first law of thermodynamics, \( \Delta U = Q - W \). The work \( W \) is \( W = P \Delta V \), and for an ideal gas \( P \Delta V = nR \Delta T \). Therefore, the work \( W \) becomes \( W = P \Delta V = nR \Delta T \) and the change in the internal energy is \( \Delta U = Q - W = \frac{5}{2} nR \Delta T - nR \Delta T = \frac{3}{2} nR \Delta T \).
Combining this expression for $\Delta U$ with Equation (1) yields a numerical value for the percentage of heat that is used to increase its internal energy.

**SOLUTION**

a. The percentage is

$$\text{Percentage} = \left( \frac{\frac{3}{2}nR\Delta T}{\frac{3}{2}nR\Delta T} \right) \times 100\% = \left( \frac{3}{5} \right) \times 100\% = 60.0\%$$

b. The remainder of the heat, or 40.0%, is used for the work of expansion.

41. **REASONING AND SOLUTION**  
   The change in volume is $\Delta V = -sA$, where $s$ is the distance through which the piston drops and $A$ is the piston area. The minus sign is included because the volume decreases. Thus,

$$s = \frac{-\Delta V}{A}$$

The ideal gas law states that $\Delta V = nR\Delta T/P$. But $Q = C_p n \Delta T = \frac{5}{2} R n \Delta T$. Thus, $\Delta T = Q/(\frac{5}{2} R n)$. Using these expressions for $\Delta V$ and $\Delta T$, we find that

$$s = \frac{-nR \Delta T / P}{A} = \frac{-nR}{PA} \frac{Q}{\frac{5}{2} R n} = \frac{-Q}{\frac{5}{2} PA}$$

$$= \frac{\frac{5}{2}(1.01 \times 10^5 \text{ Pa})(3.14 \times 10^{-2} \text{ m}^2)}{(-2093 \text{ J})} = 0.264 \text{ m}$$

42. **REASONING**  
The heat added is given by Equation 15.6 as $Q = C_V n \Delta T$, where $C_V$ is the molar specific heat capacity at constant volume, $n$ is the number of moles, and $\Delta T$ is the change in temperature. But the heat is supplied by the heater at a rate of ten watts, or ten joules per second, so $Q = (10.0 \text{ W})t$, where $t$ is the on-time for the heater. In addition, we know that the ideal gas law applies: $PV = nRT$ (Equation 14.1). Since the volume is constant while the temperature changes by an amount $\Delta T$, the amount by which the pressure changes is $\Delta P$. This change in pressure is given by the ideal gas law in the form $(\Delta P)V = nR(\Delta T)$.

**SOLUTION**  
Using Equation 15.6 and the expression $Q = (10.0 \text{ W})t$ for the heat delivered by the heater, we have

$$Q = C_V n \Delta T \quad \text{or} \quad (10.0 \text{ W})t = C_V n \Delta T \quad \text{or} \quad t = \frac{C_V n \Delta T}{10.0 \text{ W}}$$

Using the ideal gas law in the form $(\Delta P)V = nR(\Delta T)$, we can express the change in temperature as $\Delta T = (\Delta P)V/nR$. With this substitution for $\Delta T$, the expression for the time becomes
According to Equation 15.8, $C_V = \frac{3}{2} R$ for a monatomic ideal gas, so we find

$$ t = \frac{C_V n (\Delta P)V}{(10.0 \text{ W}) n R} $$

The ideal gas law for this process gives $\Delta T_1 = \frac{2PV}{nR}$, so $Q_1 = 3PV$.

The second process is isobaric, so

$$ Q_2 = C_p n \Delta T_2 = \frac{(5R/2)n \Delta T_2}{2} $$

The ideal gas law for this process gives $\Delta T_2 = \frac{3PV}{nR}$, so $Q_2 = \frac{(15/2)PV}{nR}$. The total heat is $Q = Q_1 + Q_2 = \frac{21}{2}PV$.

But at conditions of standard temperature and pressure (see Section 14.2), $P = 1.01 \times 10^5 \text{ Pa}$ and $V = 22.4 \text{ liters} = 22.4 \times 10^{-3} \text{ m}^3$, so

$$ Q = \frac{21}{2}PV = \frac{21}{2} \left(1.01 \times 10^5 \text{ Pa}\right) \left(22.4 \times 10^{-3} \text{ m}^3\right) = 2.38 \times 10^4 \text{ J} $$

44. **REASONING** The efficiency of the engine is $e = \frac{|W|}{|Q_H|}$ (Equation 15.11), where $|W|$ is the magnitude of the work and $|Q_H|$ is the magnitude of the input heat. In addition, energy conservation requires that $|Q_H| = |W| + |Q_C|$ (Equation 15.12), where $|Q_C|$ is the rejected heat.

**SOLUTION**

a. Solving Equation 15.11 for $|Q_H|$, we can find the magnitude of the input heat:

$$ e = \frac{|W|}{|Q_H|} \text{ or } |Q_H| = \frac{|W|}{e} = \frac{5500 \text{ J}}{0.64} = 8600 \text{ J} $$

b. Solving Equation 15.12 for $|Q_C|$, we can find the magnitude of the rejected heat:
45. **REASONING**

a. The efficiency \( e \) of a heat engine is given by \( e = \frac{|W|}{|Q_H|} \) (Equation 15.11), where \(|Q_H|\) is the magnitude of the input heat needed to do an amount of work of magnitude \(|W|\). The “input energy” used by the athlete is equal to the magnitude \(|\Delta U|\) of the athlete’s internal energy change, so the efficiency of a “human heat engine” can be expressed as

\[
e = \frac{|W|}{|\Delta U|} \quad \text{or} \quad |\Delta U| = \frac{|W|}{e}
\]

b. We will use the first law of thermodynamics \( \Delta U = Q - W \) (Equation 15.1) to find the magnitude \(|Q|\) of the heat that the athlete gives off. We note that the internal energy change \( \Delta U \) is negative, since the athlete spends this energy in order to do work.

**SOLUTION**

a. Equation (1) gives the magnitude of the internal energy change:

\[
|\Delta U| = \frac{|W|}{e} = \frac{5.1 \times 10^4 \text{ J}}{0.11} = 4.6 \times 10^5 \text{ J}
\]

b. From part (a), the athlete’s internal energy change is \( \Delta U = -4.6 \times 10^5 \text{ J} \). The first law of dynamics \( \Delta U = Q - W \) (Equation 15.1), therefore, yields the heat \( Q \) given off by the athlete:

\[
Q = \Delta U + W = -4.6 \times 10^5 \text{ J} + 5.1 \times 10^4 \text{ J} = -4.1 \times 10^5 \text{ J}
\]

The magnitude of the heat given off is, thus, \( 4.1 \times 10^5 \text{ J} \).

46. **REASONING** According to the definition of efficiency given in Equation 15.11, an engine with an efficiency \( e \) does work of magnitude \(|W| = e|Q_H|\), where \(|Q_H|\) is the magnitude of the input heat. We will apply this expression to each engine and utilize the fact that in each case the same work is done. We expect to find that engine 2 requires less input heat to do the same amount of work, because it has the greater efficiency.

**SOLUTION** Applying Equation 15.11 to each engine gives

\[
|W| = e_1 |Q_{H1}| \quad \text{and} \quad |W| = e_2 |Q_{H2}|
\]

Engine 1

Engine 2

Since the work is the same for each engine, we have
It follows, then, that

\[ |Q_{H2}| = \left( \frac{e_1}{e_2} \right) |Q_{H1}| = \left( \frac{0.18}{0.26} \right) (5500 \text{ J}) = 3800 \text{ J} \]

which is less than the input heat for engine 1, as expected.

47. **REASONING** According to Equation 15.11, the efficiency of a heat engine is 
\[ e = \frac{|W|}{|Q_H|} \]
, where \(|W|\) is the magnitude of the work and \(|Q_H|\) is the magnitude of the input heat. Thus, the magnitude of the work is 
\[ |W| = e|Q_H| \]. We can apply this result before and after the tune-up to compute the extra work produced.

**SOLUTION** Using Equation 15.11, we find the work before and after the tune-up as follows:

\[ |W_{\text{Before}}| = e_{\text{Before}} |Q_H| \quad \text{and} \quad |W_{\text{After}}| = e_{\text{After}} |Q_H| \]

Subtracting the “before” equation from the “after” equation gives

\[ |W_{\text{After}}| - |W_{\text{Before}}| = e_{\text{After}} |Q_H| - e_{\text{Before}} |Q_H| = (e_{\text{After}} - e_{\text{Before}}) |Q_H| = 0.050(1300 \text{ J}) = 65 \text{ J} \]

48. **REASONING** The efficiency \( e \) of a heat engine is given by 
\[ e = \frac{|W|}{|Q_H|} \] (Equation 15.11),
where \(|W|\) is the magnitude of the work and \(|Q_H|\) is the magnitude of the input heat, which is the \( 4.1 \times 10^6 \text{ J} \) of energy generated by the climber’s metabolic processes.

The work \( W_{nc} \) done in climbing upward is related to the vertical height of the climb via the work-energy theorem (see Equation 6.8), which is

\[ W_{nc} = \frac{KE_f + PE_f}{\text{Final total mechanical energy}} - \frac{KE_0 + PE_0}{\text{Initial total mechanical energy}} \]

Here, \( W_{nc} \) is the net work done by nonconservative forces, in this case the work done by the climber in going upward. Since the climber starts at rest and finishes at rest, the final kinetic energy \( KE_f \) and the initial kinetic energy \( KE_0 \) are zero. As a result, we have 
\[ W_{nc} = |W| = PE_f - PE_0 \], where PE\(_f\) and PE\(_0\) are the final and initial gravitational potential energies, respectively. Equation 6.5 gives the gravitational potential energy as 
\[ PE = mgh \],
where \( m \) is the mass of the climber, \( g \) is the magnitude of the acceleration due to gravity, and \( h \) is the vertical height of the climb. Taking the height at her starting point to be zero, we then have 
\[ W_{nc} = |W| = mgh. \]
SOLUTION Using \( e = \frac{|W|}{|Q_H|} \) (Equation 15.11) for the efficiency of a heat engine and relating the magnitude \(|W|\) of the work to the height \( h \) of the climb via the work-energy theorem as \(|W| = mgh\), we find that

\[
e = \frac{|W|}{|Q_H|} = \frac{mgh}{|Q_H|} = \frac{(52 \text{ kg})(9.80 \text{ m/s}^2)(730 \text{ m})}{4.1 \times 10^6 \text{ J}} = 0.091
\]

49. SSM REASONING The efficiency \( e \) of an engine can be expressed as (see Equation 15.13) \( e = 1 - \frac{|Q_C|}{|Q_H|} \), where \(|Q_C|\) is the magnitude of the heat delivered to the cold reservoir and \(|Q_H|\) is the magnitude of the heat supplied to the engine from the hot reservoir. Solving this equation for \(|Q_C|\) gives \(|Q_C| = (1-e)|Q_H|\). We will use this expression twice, once for the improved engine and once for the original engine. Taking the ratio of these expressions will give us the answer that we seek.

SOLUTION Taking the ratio of the heat rejected to the cold reservoir by the improved engine to that for the original engine gives

\[
\frac{|Q_C, \text{ improved}|}{|Q_C, \text{ original}|} = \frac{(1-e, \text{ improved})|Q_H, \text{ improved}|}{(1-e, \text{ original})|Q_H, \text{ original}|}
\]

But the input heat to both engines is the same, so \(|Q_H, \text{ improved}| = |Q_H, \text{ original}|\). Thus, the ratio becomes

\[
\frac{|Q_C, \text{ improved}|}{|Q_C, \text{ original}|} = \frac{1-e, \text{ improved}}{1-e, \text{ original}} = \frac{1-0.42}{1-0.23} = 0.75
\]

50. REASONING The efficiency of either engine is given by Equation 15.13, \( e = 1 - \left(\frac{|Q_C|}{|Q_H|}\right)\). Since engine A receives three times more input heat, produces five times more work, and rejects two times more heat than engine B, it follows that \( |Q_{HA}| = 3|Q_{HB}| \), \( |W_A| = 5|W_B| \), and \( |Q_{CA}| = 2|Q_{CB}| \). As required by the principle of energy conservation for engine A (Equation 15.12),

\[
|Q_{HA}| = \frac{|Q_{CA}|}{3|Q_{in}|} + \frac{|W_A|}{5|W_s|}
\]

Thus,

\[
3|Q_{HB}| = 2|Q_{CB}| + 5|W_B| \quad (1)
\]
Since engine B also obeys the principle of energy conservation (Equation 15.12),

\[ \left| Q_{HB} \right| = \left| Q_{CB} \right| + \left| W_B \right| \tag{2} \]

Substituting \( \left| Q_{HB} \right| \) from Equation (2) into Equation (1) yields

\[ 3(\left| Q_{CB} \right| + \left| W_B \right|) = 2\left| Q_{CB} \right| + 5\left| W_B \right| \]

Solving for \( \left| W_B \right| \) gives

\[ \left| W_B \right| = \frac{1}{2}\left| Q_{CB} \right| \]

Therefore, Equation (2) predicts for engine B that

\[ \left| Q_{HB} \right| = \left| Q_{CB} \right| + \left| W_B \right| = \frac{3}{2}\left| Q_{CB} \right| \]

**SOLUTION**

a. Substituting \( \left| Q_{CA} \right| = 2\left| Q_{CB} \right| \) and \( \left| Q_{HA} \right| = 3\left| Q_{HB} \right| \) into Equation 15.13 for engine A, we have

\[ e_A = 1 - \frac{\left| Q_{CA} \right|}{\left| Q_{HA} \right|} = 1 - \frac{2\left| Q_{CB} \right|}{3\left| Q_{HB} \right|} = 1 - \frac{2\left| Q_{CB} \right|}{3\left( \frac{3}{2}\left| Q_{CB} \right| \right)} = 1 - \frac{4}{9} = \frac{5}{9} \]

b. Substituting \( \left| Q_{HB} \right| = \frac{3}{2}\left| Q_{CB} \right| \) into Equation 15.13 for engine B, we have

\[ e_B = 1 - \frac{\left| Q_{CB} \right|}{\left| Q_{HB} \right|} = 1 - \frac{\left| Q_{CB} \right|}{\frac{3}{2}\left| Q_{CB} \right|} = 1 - \frac{2}{3} = \frac{1}{3} \]

51. **REASONING** We will use the subscript “27” to denote the engine whose efficiency is 27.0% \((e_{27} = 0.270)\) and the subscript “32” to denote the engine whose efficiency is 32.0% \((e_{32} = 0.320)\). In general, the efficiency \( e_{\text{Carnot}} \) of a Carnot engine depends on the Kelvin temperatures, \( T_C \) and \( T_H \), of its cold and hot reservoirs through the relation (see Equation 15.15) \( e_{\text{Carnot}} = 1 - (T_C/T_H) \). Solving this equation for the temperature \( T_{C,32} \) of the engine whose efficiency is \( e_{32} \) gives \( T_{C,32} = (1 - e_{32})T_{H,32} \). We are given \( e_{32} \), but do not know the temperature \( T_{H,32} \). However, we are told that this temperature is the same as that of the hot reservoir of the engine whose efficiency is \( e_{27} \), so \( T_{H,32} = T_{H,27} \). The temperature \( T_{H,27} \) can be determined since we know the efficiency and cold reservoir temperature of this engine.

**SOLUTION** The temperature of the cold reservoir for engine whose efficiency is \( e_{32} \) is \( T_{C,32} = (1 - e_{32})T_{H,32} \). Since \( T_{H,32} = T_{H,27} \), we have that
The efficiency $e_{27}$ is given by Equation 15.15 as $e_{27} = 1 - \left( \frac{T_C, 27}{T_H, 27} \right)$. Solving this equation for the temperature $T_{H, 27}$ of the hot reservoir and substituting the result into Equation 1 yields

$$T_{C, 32} = (1 - e_{32})T_{H, 27}$$  \hspace{1cm} (1)

52. **REASONING** The maximum efficiency of the engine is the efficiency that a Carnot engine would have operating with the same hot and cold reservoirs. Thus, the maximum efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ (Equation 15.15), where $T_C$ and $T_H$ are the Kelvin temperatures of the cold and the hot reservoir, respectively.

**SOLUTION** Using Equation 15.15 for $e_{\text{Carnot}}$ and recognizing that the engine has an efficiency $e$ that is three-fifths the maximum or Carnot efficiency, we obtain

$$e = \frac{3}{5} e_{\text{Carnot}} = \frac{3}{5} \left( 1 - \frac{T_C}{T_H} \right) = \frac{3}{5} \left( 1 - \frac{620 \text{ K}}{950 \text{ K}} \right) = 0.21$$

53. **SSM REASONING** The efficiency $e$ of a Carnot engine is given by Equation 15.15, $e = 1 - (T_C / T_H)$, where, according to Equation 15.14, $|Q_C| / |Q_H| = T_C / T_H$. Since the efficiency is given along with $T_C$ and $|Q_C|$, Equation 15.15 can be used to calculate $T_H$. Once $T_H$ is known, the ratio $T_C / T_H$ is thus known, and Equation 15.14 can be used to calculate $|Q_H|$.

**SOLUTION**

a. Solving Equation 15.15 for $T_H$ gives

$$T_H = \frac{T_C}{1 - e} = \frac{378 \text{ K}}{1 - 0.700} = 1260 \text{ K}$$

b. Solving Equation 15.14 for $|Q_H|$ gives

$$|Q_H| = |Q_C| \left( \frac{T_H}{T_C} \right) = (5230 \text{ J}) \left( \frac{1260 \text{ K}}{378 \text{ K}} \right) = 1.74 \times 10^4 \text{ J}$$
54. **REASONING** We seek the ratio $T_{C,f}/T_{C,i}$ of the final Kelvin temperature of the cold reservoir to the initial Kelvin temperature of the cold reservoir. The Kelvin temperature $T_H$ of the hot reservoir is constant, and it is related to the temperature of the cold reservoir by the Carnot efficiency: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ (Equation 15.15). We will solve Equation 15.15 for the temperature $T_C$ of the cold reservoir, and use the resulting expression to determine the ratio $T_{C,f}/T_{C,i}$. Because the temperature of the cold reservoir increases, we expect the ratio to have a value greater than 1.

**SOLUTION** Solving $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ (Equation 15.15) for the cold-reservoir-temperature $T_C$ yields

$$\frac{T_C}{T_H} = 1 - e_{\text{Carnot}} \quad \text{or} \quad T_C = T_H \left(1 - e_{\text{Carnot}}\right) \quad (1)$$

Applying Equation (1) to both the initial temperature and the final temperature, we find that

$$T_{C,i} = T_H \left(1 - e_{\text{Carnot},i}\right) \quad \text{and} \quad T_{C,f} = T_H \left(1 - e_{\text{Carnot},f}\right) \quad (2)$$

Taking the ratio of Equations (2), we obtain

$$\frac{T_{C,f}}{T_{C,i}} = \frac{T_H \left(1 - e_{\text{Carnot},f}\right)}{T_H \left(1 - e_{\text{Carnot},i}\right)} = \frac{1 - e_{\text{Carnot},f}}{1 - e_{\text{Carnot},i}} = \frac{1 - 0.70}{1 - 0.75} = 1.2$$

55. **REASONING** The smallest possible temperature of the hot reservoir would occur when the engine is a Carnot engine, since it has the greatest efficiency of any engine operating between the same hot and cold reservoirs. The efficiency $e_{\text{Carnot}}$ of a Carnot engine is (see Equation 15.15) $e_{\text{Carnot}} = 1 - (T_C/T_H)$, where $T_C$ and $T_H$ are the Kelvin temperatures of its cold and hot reservoirs. Solving this equation for $T_H$ gives $T_H = T_C/(1 - e_{\text{Carnot}})$. We are given $T_C$, but do not know $e_{\text{Carnot}}$. However, the efficiency is defined as the magnitude $|W|$ of the work done by the engine divided by the magnitude $|Q_H|$ of the input heat from the hot reservoir, so $e_{\text{Carnot}} = |W|/|Q_H|$ (Equation 15.11). Furthermore, the conservation of energy requires that the magnitude $|Q_H|$ of the input heat equals the sum of the magnitude $|W|$ of the work done by the engine and the magnitude $|Q_C|$ of the heat it rejects to the cold reservoir, $|Q_H| = |W| + |Q_C|$. By combining these relations, we will be able to find the temperature of the hot reservoir of the Carnot engine.
**SOLUTION** From the Reasoning section, the temperature of the hot reservoir is 
\[ T_H = T_C/(1 - e_{\text{Carnot}}) \]. Writing the efficiency of the engine as 
\[ e_{\text{Carnot}} = |W|/|Q_H| \], the expression for the temperature becomes

\[ T_H = \frac{T_C}{1-e_{\text{Carnot}}} = \frac{T_C}{1-|W|/|Q_H|} \]

From the conservation of energy, we have that 
\[ |Q_H| = |W| + |Q_C| \]. Substituting this expression for \(|Q_H|\) into the one above for \(T_H\) gives

\[ T_H = \frac{T_C}{1-|W|/|Q_H|} = \frac{285 \text{ K}}{18500 \text{ J}} = \frac{6550 \text{ J}}{18500 \text{ J} + 6550 \text{ J}} = 1090 \text{ K} \]

56. **REASONING** The efficiency \(e_{\text{Carnot}}\) of a Carnot engine is 
\[ e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \]
(Equation 15.15), where \(T_C\) and \(T_H\) are, respectively, the Kelvin temperatures of the cold and hot reservoirs. After the changes are made to the temperatures, this same equation still applies, except that the variables must be labeled to denote the new values. We will use a “prime” for this purpose. From the original efficiency and the information given about the changes made to the temperatures, we will be able to obtain the new temperature ratio and, hence, the new efficiency.

**SOLUTION** After the reservoir temperatures are changed, the engine has an efficiency that, according to Equation 15.15, is

\[ e'_{\text{Carnot}} = 1 - \frac{T_C'}{T_H'} \]

where the “prime” denotes the new engine. Using unprimed symbols to denote the original engine, we know that \(T_C' = 2T_C\) and \(T_H' = 4T_H\). With these substitutions, the efficiency of the new engine becomes

\[ e'_{\text{Carnot}} = 1 - \frac{T_C}{4T_H} = 1 - \frac{2T_C}{4T_H} = 1 - \frac{1}{2} \left( \frac{T_C}{T_H} \right) \]  \hspace{1cm} (1)

To obtain the original ratio \(T_C/T_H\), we use Equation 15.15:

\[ e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \quad \text{or} \quad \frac{T_C}{T_H} = 1 - e_{\text{Carnot}} \]

Substituting this original temperature ratio into Equation (1) gives
\[ e'_{\text{Carnot}} = 1 - \frac{1}{2} \left( \frac{T_C}{T_H} \right) = 1 - \frac{1}{2} \left( 1 - e_{\text{Carnot}} \right) = \frac{1}{2} \left( 1 + e_{\text{Carnot}} \right) = \frac{1}{2} (1 + 0.40) = 0.70 \]

57. **REASONING AND SOLUTION** The efficiency of the engine is \( e = 1 - \left( \frac{T_C}{T_H} \right) \) so

(i) Increase \( T_H \) by 40 K: \( e = 1 - \left[ \frac{350 \text{ K}}{690 \text{ K}} \right] = 0.493 \)

(ii) Decrease \( T_C \) by 40 K: \( e = 1 - \left[ \frac{310 \text{ K}}{650 \text{ K}} \right] = 0.523 \)

The greater improvement is made by lowering the temperature of the cold reservoir.

58. **REASONING AND SOLUTION** The amount of work delivered by the engines can be determined from Equation 15.12, \( |Q_H| = |W| + |Q_C| \). Solving for \( |W| \) for each engine gives:

\[ |W_1| = |Q_{H1}| - |Q_{C1}| \quad \text{and} \quad |W_2| = |Q_{H2}| - |Q_{C2}| \]

The total work delivered by the two engines is

\[ |W| = |W_1| + |W_2| = (|Q_{H1}| - |Q_{C1}|) + (|Q_{H2}| - |Q_{C2}|) \]

But we know that \( |Q_{H2}| = |Q_{C1}| \), so that

\[ |W| = (|Q_{H1}| - |Q_{C1}|) + (|Q_{C1}| - |Q_{C2}|) = |Q_{H1}| - |Q_{C2}| \]

(1)

Since these are Carnot engines,

\[ \frac{|Q_{C1}|}{|Q_{H1}|} = \frac{T_{C1}}{T_{H1}} \quad \Rightarrow \quad |Q_{C1}| = |Q_{H1}| \frac{T_{C1}}{T_{H1}} = (4800 \text{ J}) \left( \frac{670 \text{ K}}{890 \text{ K}} \right) = 3.61 \times 10^3 \text{ J} \]

Similarly, noting that \( |Q_{H2}| = |Q_{C1}| \) and that \( T_{H2} = T_{C1} \), we have

\[ |Q_{C2}| = |Q_{H2}| \frac{T_{C2}}{T_{H2}} = |Q_{C1}| \frac{T_{C2}}{T_{C1}} = \left( 3.61 \times 10^3 \text{ J} \right) \left( \frac{420 \text{ K}}{670 \text{ K}} \right) = 2.26 \times 10^3 \text{ J} \]

Substituting into Equation (1) gives

\[ |W| = 4800 \text{ J} - 2.26 \times 10^3 \text{ J} = 2.5 \times 10^3 \text{ J} \]

59. **SSM REASONING AND SOLUTION** The temperature of the gasoline engine input is \( T_1 = 904 \text{ K} \), the exhaust temperature is \( T_2 = 412 \text{ K} \), and the air temperature is \( T_3 = 300 \text{ K} \). The efficiency of the engine/exhaust is
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The efficiency of the second engine is

\[ e_2 = 1 - \left( \frac{T_3}{T_2} \right) = 0.272 \]

The magnitude of the work done by each segment is

\[ W_1 = e_1 |Q_{H1}| \quad \text{and} \quad W_2 = e_2 |Q_{H2}| = e_2 |Q_{C1}| \]

since \[ |Q_{H2}| = |Q_{C1}| \]

Now examine \([W_1/W_1] + [W_2/W_1]\) to find the ratio of the total work produced by both engines to that produced by the first engine alone.

\[
\frac{|W_1| + |W_2|}{|W_1|} = e_1 |Q_{H1}| + e_2 |Q_{C1}| = 1 + \left( \frac{e_2}{e_1} \right) \left( \frac{|Q_{C1}|}{|Q_{H1}|} \right)
\]

But, \(e_1 = 1 - \left( |Q_{C1}|/|Q_{H1}| \right)\), so that \(|Q_{C1}|/|Q_{H1}| = 1 - e_1\). Therefore,

\[
\frac{|W_1| + |W_2|}{|W_1|} = 1 + \frac{e_2}{e_1} (1 - e_1) = 1 + \frac{e_2}{e_1} - e_2 = 1 + 0.500 - 0.272 = 1.23
\]

60. **REASONING** The maximum efficiency \(e\) at which the power plant can operate is given by Equation 15.15, \(e = 1 - \left( \frac{T_p}{T_h} \right)\). The power output is given; it can be used to find the magnitude \(|W|\) of the work output for a 24 hour period. With the efficiency and \(|W|\) known, Equation 15.11, \(e = |W|/|Q_{H1}|\), can be used to find \(|Q_{H1}|\), the magnitude of the input heat. The magnitude \(|Q_{C1}|\) of the exhaust heat can then be found from Equation 15.12, \(|Q_{H1}| = |W| + |Q_{C1}|\).

**SOLUTION**

a. The maximum efficiency is

\[
e = 1 - \frac{T_p}{T_h} = 1 - \frac{323 \text{ K}}{505 \text{ K}} = 0.360
\]

b. Since the power output of the power plant is \(P = 84,000\) kW, the required heat input \(|Q_{H1}|\) for a 24 hour period is

\[
|Q_{H1}| = \frac{|W|}{e} = \frac{P t}{e} = \frac{(8.4 \times 10^7 \text{ J/s})(24 \text{ h})}{0.360} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 2.02 \times 10^{13} \text{ J}
\]
Therefore, solving Equation 15.12 for \( |Q_C| \), we have

\[
|Q_C| = |Q_H| - |W| = 2.02 \times 10^{13} \text{ J} - 7.3 \times 10^{12} \text{ J} = 1.3 \times 10^{13} \text{ J}
\]

61. **SSM REASONING** The expansion from point a to point b and the compression from point c to point d occur isothermally, and we will apply the first law of thermodynamics to these parts of the cycle in order to obtain expressions for the input and rejected heats, magnitudes \( |Q_H| \) and \( |Q_C| \), respectively. In order to simplify the resulting expression for \( |Q_C|/|Q_H| \), we will then use the fact that the expansion from point b to point c and the compression from point d to point a are adiabatic.

**SOLUTION** According to the first law of thermodynamics, the change in internal energy \( \Delta U \) is given by \( \Delta U = Q - W \) (Equation 15.1), where \( Q \) is the heat and \( W \) is the work. Since the internal energy of an ideal gas is proportional to the Kelvin temperature and the temperature is constant for an isothermal process, it follows that \( \Delta U = 0 \text{ J} \) for such a case. The work of isothermal expansion or compression for an ideal gas is \( W = nRT \ln \left( \frac{V_f}{V_i} \right) \) (Equation 15.3), where \( n \) is the number of moles, \( R \) is the universal gas constant, \( T \) is the Kelvin temperature, \( V_f \) is the final volume of the gas, and \( V_i \) is the initial volume. We have, then, that

\[
\Delta U = Q - W \quad \text{or} \quad 0 = Q - nRT \ln \left( \frac{V_f}{V_i} \right) \quad \text{or} \quad Q = nRT \ln \left( \frac{V_f}{V_i} \right)
\]

Applying this result for \( Q \) to the isothermal expansion (temperature = \( T_H \)) from point a to point b and the isothermal compression (temperature = \( T_C \)) from point c to point d, we have

\[
Q_H = nRT_H \ln \left( \frac{V_b}{V_a} \right) \quad \text{and} \quad Q_C = nRT_C \ln \left( \frac{V_d}{V_c} \right)
\]

where \( V_f = V_b \) and \( V_i = V_a \) for the isotherm at \( T_H \) and \( V_f = V_d \) and \( V_i = V_c \) for the isotherm at \( T_C \). In this problem, we are interested in the magnitude of the heats. For \( Q_H \), this poses no difficulty, since \( V_b > V_a \), \( \ln \left( \frac{V_b}{V_a} \right) \) is positive, and we have

\[
|Q_H| = nRT_H \ln \left( \frac{V_b}{V_a} \right)
\]

(1)
However, for $Q_C$, we need to be careful, because $V_c > V_d$ and $\ln \left( \frac{V_d}{V_c} \right)$ is negative. Thus, we write for the magnitude of $Q_C$ that

$$|Q_C| = -nRT_C \ln \left( \frac{V_d}{V_c} \right) = nRT_C \ln \left( \frac{V_c}{V_d} \right)$$

(2)

According to Equations (1) and (2), the ratio of the magnitudes of the rejected and input heats is

$$\frac{|Q_C|}{|Q_H|} = \frac{nRT_C \ln \left( \frac{V_c}{V_d} \right)}{nRT_H \ln \left( \frac{V_b}{V_a} \right)} = \frac{T_C \ln \left( \frac{V_c}{V_d} \right)}{T_H \ln \left( \frac{V_b}{V_a} \right)}$$

(3)

We now consider the adiabatic parts of the Carnot cycle. For the adiabatic expansion or compression of an ideal gas the initial pressure and volume ($P_i$ and $V_i$) are related to the final pressure and volume ($P_f$ and $V_f$) according to

$$P_i V_i^\gamma = P_f V_f^\gamma$$

(15.5)

where $\gamma$ is the ratio of the specific heats at constant pressure and constant volume. It is also true that $P = nRT / V$ (Equation 14.1), according to the ideal gas law. Substituting this expression for the pressure into Equation 15.5 gives

$$\left( \frac{nRT_i}{V_i} \right) V_i^\gamma = \left( \frac{nRT_f}{V_f} \right) V_f^\gamma \quad \text{or} \quad T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Applying this result to the adiabatic expansion from point b to point c and to the adiabatic compression from point d to point a, we obtain

$$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \quad \text{and} \quad T_C V_d^{\gamma-1} = T_H V_a^{\gamma-1}$$

Dividing the first of these equations by the second shows that

$$\frac{T_H V_b^{\gamma-1}}{T_H V_a^{\gamma-1}} = \frac{T_C V_c^{\gamma-1}}{T_C V_d^{\gamma-1}} \quad \text{or} \quad \frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}} \quad \text{or} \quad \frac{V_b}{V_a} = \frac{V_c}{V_d}$$

With this result, Equation (3) becomes

$$\frac{|Q_C|}{|Q_H|} = \frac{T_C \ln \left( \frac{V_c}{V_d} \right)}{T_H \ln \left( \frac{V_b}{V_a} \right)} = \frac{T_C}{T_H}$$

62. **REASONING AND SOLUTION**  The efficiency $e$ of the power plant is three-fourths its Carnot efficiency so, according to Equation 15.15,
\[ e = 0.75 \left( 1 - \frac{T_C}{T_H} \right) = 0.75 \left( 1 - \frac{40 \text{ K} + 273 \text{ K}}{285 \text{ K} + 273 \text{ K}} \right) = 0.33 \]

The power output of the plant is \(1.2 \times 10^9\) watts. According to Equation 15.11, \(e = |W|/|Q_H| = (\text{Power} \cdot t)/|Q_H|\). Therefore, at 33% efficiency, the magnitude of the heat input per unit time is

\[ \frac{|Q_H|}{t} = \frac{\text{Power}}{e} = \frac{1.2 \times 10^9 \text{ W}}{0.33} = 3.6 \times 10^9 \text{ J/s} \]

From the principle of conservation of energy, the heat output per unit time must be

\[ \frac{|Q_C|}{t} = \frac{|Q_H|}{t} - \text{Power} = 2.4 \times 10^9 \text{ J/s} \]

The rejected heat is carried away by the flowing water and, according to Equation 12.4, \(|Q_C| = cm\Delta T\). Therefore,

\[ \frac{|Q_C|}{t} = \frac{cm\Delta T}{t} \quad \text{or} \quad \frac{|Q_C|}{t} = \frac{c}{m} \Delta T \]

Solving the last equation for \(\Delta T\), we have

\[ \Delta T = \frac{|Q_C|}{tc(m/t)} = \frac{|Q_C|/t}{c(m/t)} = \frac{2.4 \times 10^9 \text{ J/s}}{4186 \text{ J/(kg \cdot C\text{\degree})}} \left( \frac{1.0 \times 10^5 \text{ kg/s}}{\text{J}} \right) = 5.7 \text{ C\degree} \]

63. **REASONING** The coefficient of performance of an air conditioner is \(|Q_C|/|W|\), according to Equation 15.16, where \(|Q_C|\) is the magnitude of the heat removed from the house and \(|W|\) is the magnitude of the work required for the removal. In addition, we know that the first law of thermodynamics (energy conservation) applies, so that \(|W| = |Q_H| - |Q_C|\), according to Equation 15.12. In this equation \(|Q_H|\) is the magnitude of the heat discarded outside. While we have no direct information about \(|Q_C|\) and \(|Q_H|\), we do know that the air conditioner is a Carnot device. This means that Equation 15.14 applies: \(|Q_C|/|Q_H| = T_C/T_H\). Thus, the given temperatures will allow us to calculate the coefficient of performance.

**SOLUTION** Using Equation 15.16 for the definition of the coefficient of performance and Equation 15.12 for the fact that \(|W| = |Q_H| - |Q_C|\), we have

\[
\text{Coefficient of performance} = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{|Q_C|/|Q_H|}{1 - |Q_C|/|Q_H|}
\]
Equation 15.14 applies, so that $|Q_C|/|Q_H| = T_C / T_H$. With this substitution, we find

Coefficient of performance $= \frac{|Q_C|/|Q_H|}{1 - |Q_C|/|Q_H|} = \frac{T_C / T_H}{1 - T_C / T_H}$

$= \frac{T_C}{T_H - T_C} = \frac{297 \text{ K}}{(311 \text{ K}) - (297 \text{ K})} = 21$

64. **REASONING** The magnitude $|Q_C|$ of the heat removed from the inside compartment (cold reservoir) of the refrigerator is found from the first law of thermodynamics: $|Q_C| = |Q_H| - |W|$ (Equation 15.12), where $|Q_H|$ is the magnitude of the heat deposited in the kitchen (hot reservoir) and $|W|$ is the magnitude of the work done. For a Carnot refrigerator, the magnitudes of the heat removed and deposited are related by $|Q_C|/|Q_H| = T_C / T_H$ (Equation 15.14), where $T_C$ and $T_H$ are, respectively, the Kelvin temperatures of the inside compartment and the kitchen.

**SOLUTION** Solving $|Q_C|/|Q_H| = T_C / T_H$ (Equation 15.14) for $|Q_H|$ yields

$|Q_H| = |Q_C| \left(\frac{T_H}{T_C}\right)$

(1)

Substituting Equation (1) into $|Q_C| = |Q_H| - |W|$ (Equation 15.12), we obtain

$|Q_C| = |Q_C| \left(\frac{T_H}{T_C}\right) - |W| \quad$ or $\quad |Q_C| = \frac{|W|}{\frac{T_H}{T_C} - 1}$

Therefore, the magnitude of the heat that can be removed from the inside compartment is

$|Q_C| = \frac{|W|}{\frac{T_H}{T_C} - 1} = \frac{2500 \text{ J}}{\frac{299 \text{ K}}{277 \text{ K}} - 1} = 3.1 \times 10^4 \text{ J}$

65. **REASONING** The coefficient of performance COP is defined as $\text{COP} = |Q_C|/|W|$ (Equation 15.16), where $|Q_C|$ is the magnitude of the heat removed from the cold reservoir
and $|W|$ is the magnitude of the work done on the refrigerator. The work is related to the magnitude $|Q_H|$ of the heat deposited into the hot reservoir and $|Q_C|$ by the conservation of energy, $|W| = |Q_H| - |Q_C|$. Thus, the coefficient of performance can be written as (after some algebraic manipulations)

$$\text{COP} = \frac{1}{\frac{|Q_H|}{|Q_C|} - 1}$$

The maximum coefficient of performance occurs when the refrigerator is a Carnot refrigerator. For a Carnot refrigerator, the ratio $|Q_H|/|Q_C|$ is equal to the ratio $T_H/T_C$ of the Kelvin temperatures of the hot and cold reservoirs, $|Q_H|/|Q_C| = T_H/T_C$ (Equation 15.14).

**SOLUTION** Substituting $|Q_H|/|Q_C| = T_H/T_C$ into the expression above for the COP gives

$$\text{COP} = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{1}{\frac{296 \text{ K}}{275 \text{ K}} - 1} = 13$$

66. **REASONING** According to the first law of thermodynamics (see Equation 15.12), the magnitude $|W|$ of the work required is given by $|W| = |Q_H| - |Q_C|$, where $|Q_H|$ is the magnitude of the heat deposited in the hot reservoir and $|Q_C|$ is the magnitude of the heat removed from the cold reservoir. Since the air conditioners are Carnot devices, we know that the ratio $|Q_C|/|Q_H|$ is equal to the ratio $T_C/T_H$ of the Kelvin temperatures of the cold and hot reservoirs: $|Q_C|/|Q_H| = T_C/T_H$ (Equation 15.14).

**SOLUTION** Combining Equations 15.12 and 15.14, we have

$$|W| = |Q_H| - |Q_C| = \left(\frac{|Q_H|}{|Q_C|} - 1\right)|Q_C| = \left(\frac{T_H}{T_C} - 1\right)|Q_C|$$

Applying this result to each air conditioner gives

**Unit A**

$$|W| = \left(\frac{309.0 \text{ K}}{294.0 \text{ K}} - 1\right)(4330 \text{ J}) = 220 \text{ J}$$

**Unit B**

$$|W| = \left(\frac{309.0 \text{ K}}{301.0 \text{ K}} - 1\right)(4330 \text{ J}) = 120 \text{ J}$$

We can find the heat deposited outside directly from Equation 15.14 by solving it for $|Q_H|$. 

\[
\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \quad \text{or} \quad |Q_H| = \left(\frac{T_H}{T_C}\right)|Q_C|
\]

Applying this result to each air conditioner gives

**Unit A**

\[|Q_H| = \left(\frac{309.0 \text{ K}}{294.0 \text{ K}}\right)(4330 \text{ J}) = 4550 \text{ J}\]

**Unit B**

\[|Q_H| = \left(\frac{309.0 \text{ K}}{301.0 \text{ K}}\right)(4330 \text{ J}) = 4450 \text{ J}\]

67. **REASONING AND SOLUTION**

We know that

\[|Q_C| = |Q_H| - |W| = 14200 \text{ J} - 800 \text{ J} = 13400 \text{ J}\]

Therefore,

\[T_C = T_H \left(\frac{|Q_C|}{|Q_H|}\right) = (301 \text{ K})\left(\frac{13400 \text{ J}}{14200 \text{ J}}\right) = 284 \text{ K}\]

68. **REASONING**

The efficiency of the engine is \(e = |W|/|Q_H|\) (Equation 15.11), where \(|W|\) is the magnitude of the work and \(|Q_H|\) is the magnitude of the input heat. The coefficient of performance COP of the heat pump is \(\text{COP} = |Q_H|/|W|\), according to Equation 15.17.

**SOLUTION**

Using \(e = |W|/|Q_H|\) (Equation 15.11) to substitute into Equation 15.17 for the coefficient of performance, we obtain

\[\text{COP} = \frac{|Q_H|}{|W|} = \frac{1}{|W|/|Q_H|} = \frac{1}{e} = \frac{1}{0.55} = 1.8\]

69. **REASONING**

Since the refrigerator is a Carnot device, we know that

\[\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}\]

(Equation 15.14). We have values for \(T_H\) (the temperature of the kitchen) and \(|Q_C|\) (the magnitude of the heat removed from the food). Thus, we can use this expression to determine \(T_C\) (the temperature inside the refrigerator), provided that a value can be obtained for \(|Q_H|\) (the magnitude of the heat that the refrigerator deposits into the kitchen). Energy conservation dictates that \(|Q_H| = |W| + |Q_C|\) (Equation 15.12), where \(|W|\) is the magnitude of the work that the appliance uses and is known.

**SOLUTION**

From Equation 15.14, it follows that
70. REASONING  The mass \( m \) of ice that the ice maker can produce in one day depends upon the magnitude \( |Q_C| \) of the heat it can extract from water as the water cools from \( T_i = 15.0 \, ^\circ C \) to \( T_f = 0 \, ^\circ C \) and then freezes at 0 °C. To cool a mass \( m \) of water from \( T_i \) to \( T_f \) requires extracting an amount of heat \( Q_1 = cm \Delta T \) (Equation 12.4) from it, where \( c \) is the specific heat capacity of water (see Table 12.2 in the text), and \( \Delta T = 15 \, ^\circ C \) is the difference between the higher initial temperature \( T_i \) and the lower final temperature \( T_f \). For the freezing process, the heat loss is given by \( Q_2 = mL_f \) (Equation 12.5), where \( L_f \) is the latent heat of fusion of water (see Table 12.3 in the text). Therefore, the magnitude \( |Q_C| \) of the total heat to be extracted from the water each day is

\[
|Q_C| = \frac{cm \Delta T}{W + |Q_C|} + mL_f
\]  

(1)

The ice maker operates in the same way as a refrigerator or an air conditioner, so the ice maker’s coefficient of performance (COP) is given by \( \text{COP} = \frac{|Q_C|}{|W|} \) (Equation 15.16), where \( |W| \) is the magnitude of the work done by the ice maker in extracting the heat of magnitude \( |Q_C| \) from the water. Solving Equation 15.16 for \( |Q_C| \), we find that

\[
|Q_C| = |W|(\text{COP})
\]  

(2)

The wattage \( P \) of the ice maker is the rate at which it does work. This rate is given by \( P = \frac{|W|}{t} \) (Equation 6.10a). Therefore, the magnitude \( |W| \) of the total amount of work the ice maker performs in one day is

\[
|W| = Pt
\]  

(3)

SOLUTION  Solving Equation (1) for the desired mass \( m \) of ice, we find that

\[
m\left(c \Delta T + L_f\right) = |Q_C| \quad \text{or} \quad m = \frac{|Q_C|}{c \Delta T + L_f}
\]  

(4)

Substituting Equation (2) into Equation (4), then, yields
\[ m = \frac{|Q_C|}{c \Delta T + L_T} = \frac{|W|(\text{COP})}{c \Delta T + L_T} \]  

(5)

Lastly, substituting Equation (3) into Equation (5), we obtain

\[ m = \frac{|W|\text{COP}}{c \Delta T + L_T} = \frac{P_t(\text{COP})}{c \Delta T + L_T} \]

The elapsed time of \( t = 1 \) day is equivalent to \( t = 8.64 \times 10^4 \) s (see the inside of the front cover of the text). Therefore, the maximum amount of ice that the ice maker can produce in one day is

\[ m = \frac{P_t(\text{COP})}{c \Delta T + L_T} = \frac{(225 \text{ W})(8.64 \times 10^4 \text{ s})(3.60)}{4186 \text{ J/(kg} \cdot \text{C})} (15.0 \text{ C}^o + 33.5 \times 10^4 \text{ J/kg}) = 176 \text{ kg} \]

71. **REASONING** The net heat added to the room is \( |Q_H| - |Q_C| \), where \( |Q_H| \) is the magnitude of the heat put into the room by the unit and \( |Q_C| \) is the magnitude of the heat removed by the unit. To determine this net heat, we will use the fact that energy conservation applies, so that \( |Q_H| = |W| + |Q_C| \) (Equation 15.12). We will also use the fact that the coefficient of performance COP of the air conditioner is \( \text{COP} = |Q_C|/|W| \) (Equation 15.16). In Equations 15.12 and 15.16, \( |W| \) is the amount of work needed to operate the unit.

**SOLUTION** Rearranging Equation 15.12 for energy conservation, we have

\[ |Q_H| = |W| + |Q_C| \quad \text{or} \quad \text{Net heat} = |Q_H| - |Q_C| = |W| \]

(1)

Thus, the net heat is just the work \( |W| \) needed to operate the unit. We can obtain this work directly from the coefficient of performance as specified by Equation 15.16:

\[ \text{COP} = \frac{|Q_C|}{|W|} \quad \text{or} \quad |W| = \frac{|Q_C|}{\text{COP}} \]

(2)

Substituting \( |W| \) from Equation (2) into Equation (1), we find that

\[ \text{Net heat} = |Q_H| - |Q_C| = |W| = \frac{|Q_C|}{\text{COP}} = \frac{7.6 \times 10^4 \text{ J}}{2.0} = 3.8 \times 10^4 \text{ J} \]

72. **REASONING** Power is energy per unit time. Therefore, the heat \( Q_{\text{space heater}} \) that the space heater would put into the kitchen is the heater’s power output \( P \) times the time \( t \) during
which the heater is on: \( Q_{\text{space heater}} = P t \) (Equation 6.10b). The magnitude of the heat that the refrigerator puts into the kitchen is \( |Q_H| \). To determine this heat, we will use the fact that energy conservation applies, so that \( |Q_H| = |W| + |Q_C| \) (Equation 15.12). We will also use the fact that the coefficient of performance COP of the refrigerator is \( \text{COP} = |Q_C|/|W| \) (Equation 15.16). In Equations 15.12 and 15.16, \( |W| \) is the amount of work needed to operate the refrigerator and \( |Q_C| \) is the magnitude of the heat that must be removed from the water to make the ice. Heat must be removed from the water for two reasons. First, the water must be cooled by 20.0 \(^\circ\text{C}\) (from 20.0 \(^\circ\text{C}\) to 0.0 \(^\circ\text{C}\)). The amount of heat removed to accomplish the cooling is \( Q_{\text{cooling}} = cm\Delta T \) (Equation 12.4), where \( c = 4186 \text{J/(kg} \cdot \text{C}) \) is the specific heat capacity of water as given in Table 12.2, \( m \) is the mass of the water, and \( \Delta T \) is the amount of the temperature drop. After the water reaches 0.0 \(^\circ\text{C}\), additional heat must be removed to make the water freeze. The amount of heat removed to accomplish the freezing is \( Q_{\text{freezing}} = mL_f \) (Equation 12.5), where \( L_f = 33.5 \times 10^4 \text{J/kg} \) is the latent heat of fusion of water as given in Table 12.3.

**SOLUTION** According to Equation 6.10b, the run-time \( t \) of the heater is \( t = Q_{\text{space heater}} / P \). We know that \( Q_{\text{space heater}} = |Q_H| \), so that the run time is

\[
t = \frac{Q_{\text{space heater}}}{P} = \frac{|Q_H|}{P} \quad (1)
\]

Using energy conservation in the form of Equation 15.12, we know that \( |Q_H| = |W| + |Q_C| \), so that Equation (1) becomes

\[
t = \frac{|Q_H|}{P} = \frac{|W| + |Q_C|}{P} \quad (2)
\]

Since the coefficient of performance COP of the refrigerator is \( \text{COP} = |Q_C|/|W| \) (Equation 15.16), we can substitute \( |W| = |Q_C|/\text{(COP)} \) into Equation (2) and obtain

\[
t = \frac{|W| + |Q_C|}{P} = \left( \frac{|Q_C|}{P} \text{(COP)} \right) + \frac{|Q_C|}{P} = \left( \frac{1}{\text{COP}} + 1 \right) \frac{|Q_C|}{P} \quad (3)
\]

Using \( Q_{\text{cooling}} = cm \Delta T \) (Equation 12.4) and \( Q_{\text{freezing}} = mL_f \) (Equation 12.5), we can write the magnitude \( |Q_C| \) of the heat that must be removed from the water to make the ice in the following way: \( |Q_C| = cm \Delta T + mL_f \). Substituting this result into Equation (3), we find that
\[ t = \left( \frac{1}{\text{COP}} + 1 \right) \left| \frac{Q_C}{P} \right| = \left( \frac{1}{3.00} + 1 \right) \left( \frac{4186 \, \text{J/(kg C\textsuperscript{o})}}{3.00 \times 10^3 \, \text{W}} \right) \left( 1.50 \, \text{kg} \right) (20.0 \, \text{C\textsuperscript{o}}) + (1.50 \, \text{kg}) (33.5 \times 10^4 \, \text{J/kg}) \right] = 279 \, \text{s} \]

**73. SSM REASONING** Let the coefficient of performance be represented by the symbol COP. Then according to Equation 15.16, \( \text{COP} = \frac{|Q_C|}{|W|} \). From the statement of energy conservation for a Carnot refrigerator (Equation 15.12), \(|W| = |Q_H| - |Q_C|\). Combining Equations 15.16 and 15.12 leads to

\[
\text{COP} = \frac{|Q_C|}{|Q_H| - |Q_C|} = \left( \frac{|Q_H|}{|Q_C|} \right) - 1
\]

Replacing the ratio of the heats with the ratio of the Kelvin temperatures, according to Equation 15.14, leads to

\[
\text{COP} = \frac{1}{T_H/T_C - 1} \quad \text{(1)}
\]

The heat \( |Q_C| \) that must be removed from the refrigerator when the water is cooled can be calculated using Equation 12.4, \( |Q_C| = cm \Delta T \); therefore,

\[
|W| = \frac{|Q_C|}{\text{COP}} = \frac{cm \Delta T}{\text{COP}} \quad \text{(2)}
\]

**SOLUTION**

a. Substituting values into Equation (1) gives

\[
\text{COP} = \frac{1}{T_H/T_C - 1} = \frac{1}{\left( \frac{20.0 + 273.15}{6.0 + 273.15} \right) \, \text{K} - 1} = 2.0 \times 10^1
\]

b. Substituting values into Equation (2) gives

\[
|W| = \frac{cm \Delta T}{\text{COP}} = \frac{4186 \, \text{J/(kg C\textsuperscript{o})}}{2.0 \times 10^1} \left( 5.00 \, \text{kg} \right) (20.0 \, \text{C\textsuperscript{o}} - 6.0 \, \text{C\textsuperscript{o}}) = 1.5 \times 10^4 \, \text{J}
\]

**74. REASONING** The efficiency of the Carnot engine is, according to Equation 15.15,

\[
e = 1 - \frac{T_C}{T_H} = 1 - \frac{842 \, \text{K}}{1684 \, \text{K}} = \frac{1}{2}
\]
The magnitude of the work delivered by the engine is, according to Equation 15.11,
\[ |W| = e |Q_H| = \frac{1}{2} |Q_H| \]

The heat pump removes an amount of heat \( |Q_H| \) from the cold reservoir. Thus, the amount of heat \( |Q'| \) delivered to the hot reservoir of the heat pump is
\[ |Q'| = |Q_H| + |W| = |Q_H| + \frac{1}{2} |Q_H| = \frac{3}{2} |Q_H| \]

Therefore, \( |Q'|/|Q_H| = 3/2 \). According to Equation 15.14, \( |Q'|/|Q_H| = T'/T_C \), so \( T'/T_C = 3/2 \).

**SOLUTION** Solving for \( T' \) gives
\[ T' = \frac{3}{2} T_C = \frac{3}{2} (842 \text{ K}) = 1.26 \times 10^3 \text{ K} \]

**75. REASONING** The total entropy change \( \Delta S_{\text{universe}} \) of the universe is the sum of the entropy changes of the hot and cold reservoirs. For each reservoir, the entropy change is given by
\[ \Delta S = \left( \frac{Q}{T} \right)_R \] (Equation 15.18). As indicated by the label “R,” this equation applies only to reversible processes. For the irreversible engines, therefore, we apply this equation to an imaginary process that removes the given heat from the hot reservoir reversibly and rejects the given heat to the cold reservoir reversibly. According to the second law of thermodynamics stated in terms of entropy (see Section 15.11), the reversible engine is the one for which \( \Delta S_{\text{universe}} = 0 \text{ J/K} \), and the irreversible engine that could exist is the one for which \( \Delta S_{\text{universe}} > 0 \text{ J/K} \). The irreversible engine that could not exist is the one for which \( \Delta S_{\text{universe}} < 0 \text{ J/K} \).

**SOLUTION** Using Equation 15.18, we write the total entropy change of the universe as the sum of the entropy changes of the hot (H) and cold (C) reservoirs.
\[ \Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C} \]

In this expression, we have used \(-|Q_H|\) for the heat from the hot reservoir because that reservoir loses heat. We have used \(+|Q_C|\) for the heat rejected to the cold reservoir because that reservoir gains heat. Applying this expression to the three engines gives the following results:
Chapter 15  Problems

Engine I
\[ \Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C} = -\frac{1650 \text{ J}}{550 \text{ K}} + \frac{1120 \text{ J}}{330 \text{ K}} = +0.4 \text{ J/K} \]

Engine II
\[ \Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C} = -\frac{1650 \text{ J}}{550 \text{ K}} + \frac{990 \text{ J}}{330 \text{ K}} = 0 \text{ J/K} \]

Engine III
\[ \Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C} = -\frac{1650 \text{ J}}{550 \text{ K}} + \frac{660 \text{ J}}{330 \text{ K}} = -1.0 \text{ J/K} \]

Since \( \Delta S_{\text{universe}} = 0 \text{ J/K} \) for Engine II, it is reversible. Since \( \Delta S_{\text{universe}} > 0 \text{ J/K} \) for Engine I, it is irreversible and could exist. Since \( \Delta S_{\text{universe}} < 0 \text{ J/K} \) for Engine III, it is irreversible and could not exist.

76. **REASONING** Equation 15.19 gives the unavailable work as \( W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} \), where \( T_0 \) is the Kelvin temperature of the coldest reservoir into which heat can be rejected and \( \Delta S_{\text{universe}} \) is the total entropy change of the universe. We can use this expression to find \( Q \), because \( \Delta S_{\text{universe}} \) involves \( Q \). In this case, the “universe” means the reservoir where the heat originates and the reservoir into which the heat spontaneously flows. The reservoir out of which the heat flows loses entropy, while the reservoir into which the heat flows gains entropy. \( \Delta S_{\text{universe}} \) is the sum of the two changes.

**SOLUTION** To determine each of the two contributions to \( \Delta S_{\text{universe}} \), we use \( \Delta S = \left( \frac{Q}{T} \right)_R \) (Equation 15.18). In this expression the subscript “R” reminds us that we must imagine a reversible process by which the heat exits the reservoir at 394 K and arrives in the reservoir at 298 K. Thus, we have

\[
\Delta S_{\text{universe}} = -\frac{Q}{394 \text{ K}} + \frac{Q}{298 \text{ K}}
\]

Negative entropy change Positive entropy change
due to loss of heat from due to gain of heat by
original reservoir target reservoir

With this expression for \( \Delta S_{\text{universe}} \), Equation 15.19 for the unavailable work becomes

\[
W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} = T_0 \left( -\frac{Q}{394 \text{ K}} + \frac{Q}{298 \text{ K}} \right)
\]

Solving this equation for \( Q \) reveals that

\[
Q = \frac{W_{\text{unavailable}}}{T_0 \left( -\frac{1}{394 \text{ K}} + \frac{1}{298 \text{ K}} \right)} = \frac{2800 \text{ J}}{(248 \text{ K}) \left( -\frac{1}{394 \text{ K}} + \frac{1}{298 \text{ K}} \right)} = 1.4 \times 10^4 \text{ J}
\]
77. **REASONING AND SOLUTION** The change in entropy \( \Delta S \) of a system for a process in which heat \( Q \) enters or leaves the system reversibly at a constant temperature \( T \) is given by Equation 15.18, \( \Delta S = (Q/T)_R \). For a phase change, \( Q = mL \), where \( L \) is the latent heat (see Section 12.8).

a. If we imagine a reversible process in which 3.00 kg of ice melts into water at 273 K, the change in entropy of the water molecules is

\[
\Delta S = \left( \frac{Q}{T} \right)_R = \left( \frac{mL}{T} \right)_R = \frac{(3.00 \text{ kg})(3.35 \times 10^5 \text{ J/kg})}{273 \text{ K}} = 3.68 \times 10^3 \text{ J/K}
\]

b. Similarly, if we imagine a reversible process in which 3.00 kg of water changes into steam at 373 K, the change in entropy of the water molecules is

\[
\Delta S = \left( \frac{Q}{T} \right)_R = \left( \frac{mL_v}{T} \right)_R = \frac{(3.00 \text{ kg})(2.26 \times 10^6 \text{ J/kg})}{373 \text{ K}} = 1.82 \times 10^4 \text{ J/K}
\]

c. Since the change in entropy is greater for the vaporization process than for the fusion process, the vaporization process creates more disorder in the collection of water molecules.

78. **REASONING** According to the discussion on Section 15.11, the change \( \Delta S_{universe} \) in the entropy of the universe is the sum of the change in entropy \( \Delta S_C \) of the cold reservoir and the change in entropy \( \Delta S_H \) of the hot reservoir, or \( \Delta S_{universe} = \Delta S_C + \Delta S_H \). The change in entropy of each reservoir is given by Equation 15.18 as \( \Delta S = (Q/T)_R \), where \( Q \) is the heat removed from or delivered to the reservoir and \( T \) is the Kelvin temperature of the reservoir. In applying this equation we imagine a process in which the heat is lost by the house and gained by the outside in a reversible fashion.

**SOLUTION** Since heat is lost from the hot reservoir (inside the house), the change in entropy is negative: \( \Delta S_H = -Q_H/T_H \). Since heat is gained by the cold reservoir (the outdoors), the change in entropy is positive: \( \Delta S_C = +Q_C/T_C \). Here we are using the symbols \( Q_H \) and \( Q_C \) to denote the magnitudes of the heats. The change in the entropy of the universe is

\[
\Delta S_{universe} = \Delta S_C + \Delta S_H = \frac{Q_C}{T_C} - \frac{Q_H}{T_H} = \frac{24500 \text{ J}}{258 \text{ K}} - \frac{24500 \text{ J}}{294 \text{ K}} = 11.6 \text{ J/K}
\]

In this calculation we have used the fact that \( T_C = 273 - 15 ^\circ C = 258 \text{ K} \) and \( T_H = 273 + 21 ^\circ C = 294 \text{ K} \).
79. **REASONING**  The change $\Delta S_{\text{universe}}$ in entropy of the universe for this process is the sum of the entropy changes for (1) the warm water ($\Delta S_{\text{water}}$) as it cools down from its initial temperature of $85.0 \, ^\circ\text{C}$ to its final temperature $T_f$, (2) the ice ($\Delta S_{\text{ice}}$) as it melts at $0 \, ^\circ\text{C}$, and (3) the ice water ($\Delta S_{\text{ice water}}$) as it warms up from $0 \, ^\circ\text{C}$ to the final temperature $T_f$:

$$\Delta S_{\text{universe}} = \Delta S_{\text{water}} + \Delta S_{\text{ice}} + \Delta S_{\text{ice water}}$$

To find the final temperature $T_f$, we will follow the procedure outlined in Sections 12.7 and 12.8, where we set the heat lost by the warm water as it cools down equal to the heat gained by the melting ice and the resulting ice water as it warms up. The heat $Q$ that must be supplied or removed to change the temperature of a substance of mass $m$ by an amount $\Delta T$ is $Q = cm\Delta T$ (Equation 12.4), where $c$ is the specific heat capacity. The heat that must be supplied to melt a mass $m$ of a substance is $Q = mL_f$ (Equation 12.5), where $L_f$ is the latent heat of fusion.

**SOLUTION**

a. We begin by finding the final temperature $T_f$ of the water. Setting the heat lost equal to the heat gained gives

$$cm_{\text{water}}(85.0 \, ^\circ\text{C} - T_f) = m_{\text{ice}}L_f + m_{\text{ice}}c(T_f - 0.0 \, ^\circ\text{C})$$

Solving this relation for the final temperature $T_f$ yields

$$T_f = \frac{cm_{\text{water}}(85.0 \, ^\circ\text{C}) - m_{\text{ice}}L_f}{c(m_{\text{ice}} + m_{\text{water}})} = \frac{[4186 \text{ J/}(\text{kg} \cdot ^\circ\text{C})](6.00 \, \text{kg})(85.0 \, ^\circ\text{C}) - (3.00 \, \text{kg})(33.5 \times 10^4 \text{ J/kg})}{[4186 \text{ J/}(\text{kg} \cdot ^\circ\text{C})](3.00 \, \text{kg} + 6.00 \, \text{kg})} = 30.0 \, ^\circ\text{C}$$

We have taken the specific heat capacity of $4186 \text{ J/}(\text{kg} \cdot ^\circ\text{C})$ for water from Table 12.2 and the latent heat of $33.5 \times 10^4 \text{ J/kg}$ from Table 12.3. This temperature is equivalent to $T_f = (273 + 30.0 \, ^\circ\text{C}) = 303 \, \text{K}$.

The change $\Delta S_{\text{universe}}$ in the entropy of the universe is the sum of three contributions:

**[Contribution 1]**

$$\Delta S_{\text{water}} = m_{\text{water}}c \ln \left( \frac{T_f}{T_i} \right) = (6.00 \, \text{kg})[4186 \text{ J/}(\text{kg} \cdot ^\circ\text{C})] \ln \left( \frac{303 \, \text{K}}{358 \, \text{K}} \right) = -4190 \text{ J/K}$$

where $T_i = 273 + 85.0 \, ^\circ\text{C} = 358 \, \text{K}$. 
[Contribution 2]

\[ \Delta S_{\text{ice}} = \frac{Q}{T} = \frac{mL_f}{T} = \left(3.00 \text{ kg}\right)\left(33.5 \times 10^4 \text{ J/kg}\right) \frac{273 \text{ K}}{273 \text{ K}} = +3680 \text{ J/K} \]

[Contribution 3]

\[ \Delta S_{\text{ice water}} = m_{\text{ice}} c \ln \left(\frac{T_f}{T_i}\right) = (3.00 \text{ kg})\left[4186 \text{ J/(kg \cdot C)}\right] \ln \left(\frac{303 \text{ K}}{273 \text{ K}}\right) = +1310 \text{ J/K} \]

The change in the entropy of the universe is

\[ \Delta S_{\text{universe}} = \Delta S_{\text{water}} + \Delta S_{\text{ice}} + \Delta S_{\text{ice water}} = +8.0 \times 10^2 \text{ J/K} \]

b. The entropy of the universe increases, because the mixing process is irreversible.

80. REASONING We think of the entire universe as divided into two parts: the sun and the rest of the universe. As the sun spontaneously radiates heat into space, the change in its entropy each second is \( \Delta S_{\text{sun}} \), and the change in the entropy of the rest of the universe is \( \Delta S_{\text{rest}} \) each second. The total entropy change of the universe in one second is the sum of the two entropy changes

\[ \Delta S_{\text{universe}} = \Delta S_{\text{sun}} + \Delta S_{\text{rest}} \quad (1) \]

The spontaneous thermal radiation of the sun is an irreversible process, but if we imagine a reversible process by which the sun loses an amount \( Q \) of heat, and a reversible process by which the rest of the universe absorbs this heat, then we can make use of \( \Delta S = \left(\frac{Q}{T}\right)_R \) (Equation 15.18) to calculate the entropy changes of the sun and of the rest of the universe:

\[ \Delta S_{\text{sun}} = -\frac{|Q|}{T_{\text{sun}}} \quad \text{and} \quad \Delta S_{\text{rest}} = +\frac{|Q|}{T_{\text{rest}}} \quad (2) \]

In Equations (2), \( T_{\text{sun}} \) is the sun’s surface temperature, and \( T_{\text{rest}} \) is the average temperature of the rest of the universe. The explicit algebraic signs (− and +) in Equations (2) indicate, respectively, that the sun loses heat and the rest of the universe gains heat.

The heat \( Q \) radiated by the sun to the rest of the universe in a time \( t = 1.0 \text{ s} \) is given by Equation 13.2:

\[ Q = e\sigma T_{\text{sun}}^4 At \quad (13.2) \]
In Equation 13.2, \( e \) is the emissivity of the sun (assumed to be 1, because the sun is taken to be a perfect blackbody), \( \sigma = 5.67 \times 10^{-8} \text{ J/(s·m}^2·\text{K}^4) \) is the Stefan-Boltzmann constant, and \( A \) is the sun’s surface area. Because the sun is a sphere with a radius \( r \), its surface area is

\[
A = 4\pi r^2 = 4\pi \left(6.96 \times 10^8 \text{ m} \right)^2 = 6.09 \times 10^{18} \text{ m}^2
\]

(3)

**SOLUTION** Substituting Equations (2) into Equation (1), we obtain

\[
\Delta S_{\text{universe}} = \Delta S_{\text{sun}} + \Delta S_{\text{rest}} = \frac{|Q|}{T_{\text{sun}}} + |Q| \left( \frac{1}{T_{\text{rest}}} - \frac{1}{T_{\text{sun}}} \right) = |Q| \left( \frac{1}{T_{\text{rest}}} - \frac{1}{T_{\text{sun}}} \right)
\]

(4)

Substituting Equation 13.2 into Equation (4) yields

\[
\Delta S_{\text{universe}} = |Q| \left( \frac{1}{T_{\text{rest}}} - \frac{1}{T_{\text{sun}}} \right) = e\sigma T^4 \Delta t \left( \frac{1}{T_{\text{rest}}} - \frac{1}{T_{\text{sun}}} \right)
\]

Therefore, the net entropy change of the entire universe due to one second of thermal radiation from the sun is

\[
\Delta S_{\text{universe}} =
\]

\[
= (1) \left[ 5.67 \times 10^{-8} \text{ J/(s·m}^2·\text{K}^4) \right] (5800 \text{ K})^4 \left(6.09 \times 10^{18} \text{ m}^2 \right)(1.0 \text{ s}) \left( \frac{1}{2.73 \text{ K}} - \frac{1}{5800 \text{ K}} \right)
\]

\[
= 1.4 \times 10^{26} \text{ J/K}
\]

81. **REASONING**

a. According to the discussion in Section 15.11, the change \( \Delta S_{\text{universe}} \) in the entropy of the universe is the sum of the change in entropy \( \Delta S_H \) of the hot reservoir and the change in entropy \( \Delta S_C \) of the cold reservoir, so \( \Delta S_{\text{universe}} = \Delta S_H + \Delta S_C \). The change in entropy of each reservoir is given by Equation 15.18 as \( \Delta S = (Q/T)_R \). The engine is irreversible, so we must imagine a process in which the heat \( Q \) is added to or removed from the reservoirs reversibly. \( T \) is the Kelvin temperature of a reservoir. Since heat is lost from the hot reservoir, the change in entropy is negative: \( \Delta S_H = -|Q_H|/T_H \). Since heat is gained by the cold reservoir, the change in entropy is positive: \( \Delta S_C = +|Q_C|/T_C \). The change in entropy of the universe is

\[
\Delta S_{\text{universe}} = \Delta S_H + \Delta S_C = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C}
\]

b. The magnitude \( |W| \) of the work done by any engine depends on its efficiency \( e \) and input heat \( |Q_H| \) via \( |W| = e|Q_H| \) (Equation 15.11). For a reversible engine, the efficiency is related
to the Kelvin temperatures of its hot and cold reservoirs by \( e = 1 - (T_C/T_H) \), Equation 15.15. Combining these two relations will allow us to determine \(|W|\).

c. The difference in the work produced by the two engines is labeled \( W_{\text{unavailable}} \) in Section 15.11, where \( W_{\text{unavailable}} = W_{\text{reversible}} - W_{\text{irreversible}} \). The difference in the work is related to the change in the entropy of the universe by \( W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} \) (Equation 15.19), where \( T_0 \) is the Kelvin temperature of the coldest heat reservoir. In this case \( T_0 = T_C \).

**SOLUTION**

a. From part a of the **REASONING**, the change in entropy of the universe is

\[
\Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_C|}{T_C}
\]

The magnitude \(|Q_C|\) of the heat rejected to the cold reservoir is related to the magnitude \(|Q_H|\) of the heat supplied to the engine from the hot reservoir and the magnitude \(|W|\) of the work done by the engine via \(|Q_C| = |Q_H| - |W|\) (Equation 15.12). Thus, \( \Delta S_{\text{universe}} \) becomes

\[
\Delta S_{\text{universe}} = -\frac{|Q_H|}{T_H} + \frac{|Q_H| - |W|}{T_C} = -\frac{1285 \text{ J}}{852 \text{ K}} + \frac{1285 \text{ J} - 264 \text{ J}}{314 \text{ K}} = +1.74 \text{ J/K}
\]

b. The magnitude \(|W|\) of the work done by an engine depends on its efficiency \( e \) and input heat \(|Q_H|\) via \(|W| = e|Q_H|\) (Equation 15.11). For a reversible engine, the efficiency is related to the temperatures of its hot and cold reservoirs by \( e = 1 - (T_C/T_H) \), Equation 15.15. The work done by the reversible engine is

\[
|W_{\text{reversible}}| = e|Q_H| = \left(1 - \frac{T_C}{T_H}\right)|Q_H| = \left(1 - \frac{314 \text{ K}}{852 \text{ K}}\right)(1285 \text{ J}) = 811 \text{ J}
\]

c. According to the discussion in part c of the **REASONING**, the difference between the work done by the reversible and irreversible engines is

\[
W_{\text{reversible}} - W_{\text{irreversible}} = T_C \Delta S_{\text{universe}} = (314 \text{ K})(1.74 \text{ J/K}) = 546 \text{ J}
\]

82. **REASONING** During an adiabatic process, no heat flows into or out of the gas \((Q = 0 \text{ J})\). For an ideas gas, the final pressure and volume \((P_f \text{ and } V_f)\) are related to the initial pressure and volume \((P_i \text{ and } V_i)\) by \( P_f V_f^\gamma = P_i V_i^\gamma \) (Equation 15.5), where \( \gamma \) is the ratio of the specific....
heat capacities at constant pressure and constant volume \((\gamma = \frac{5}{3}\text{ in this problem})\). We will use this relation to find \(V_f/V_i\).

**SOLUTION** Solving \(PV_1^{\gamma} = PV_f^{\gamma}\) for \(V_f/V_i\) and noting that the pressure doubles \((P_f/P_i = 2.0)\) during the compression, we have

\[
\frac{V_f}{V_i} = \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} = \left(\frac{1}{2.0}\right)^{\frac{5}{3}} = 0.66
\]

---

**83. SSM Reasoning** According to the first law of thermodynamics (Equation 15.1), 
\[\Delta U = Q - W.\]
For a monatomic ideal gas (Equation 14.7), \(U = \frac{3}{2}nRT\). Therefore, for the process in question, the change in the internal energy is \(\Delta U = \frac{3}{2}nR\Delta T\). Combining the last expression for \(\Delta U\) with Equation 15.1 yields

\[
\frac{3}{2}nR\Delta T = Q - W
\]

This expression can be solved for \(\Delta T\).

**SOLUTION**

a. The heat is \(Q = +1200 J\), since it is absorbed by the system. The work is \(W = +2500 J\), since it is done by the system. Solving the above expression for \(\Delta T\) and substituting the values for the data given in the problem statement, we have

\[
\Delta T = \frac{Q-W}{\frac{3}{2}nR} = \frac{1200 J - 2500 J}{\frac{3}{2}(0.50 \text{ mol})(8.31 J/(\text{mol} \cdot \text{K}))} = -2.1 \times 10^2 \text{ K}
\]

b. Since \(\Delta T = T_{\text{final}} - T_{\text{initial}}\) is negative, \(T_{\text{initial}}\) must be greater than \(T_{\text{final}}\); this change represents a decrease in temperature.

Alternatively, one could deduce that the temperature decreases from the following physical argument. Since the system loses more energy in doing work than it gains in the form of heat, the internal energy of the system decreases. Since the internal energy of an ideal gas depends only on the temperature, a decrease in the internal energy must correspond to a decrease in the temperature.

---

**84. Reasoning** According to Equation 15.11, the efficiency \(e\) of a heat engine is 
\[e = |W|/|Q_H|,\]
where \(|W|\) is the magnitude of the work done by the engine and \(|Q_H|\) is the magnitude of the input heat that the engine uses. \(|Q_H|\) is given, but \(|W|\) is unknown. However, energy conservation requires that \(|Q_H| = |W| + |Q_C|\) (Equation 15.12), where \(|Q_C|\) is the magnitude of the heat rejected by the engine and is given. From this equation, therefore, a value for \(|W|\) can be obtained.
**SOLUTION** According to Equation 15.11, the efficiency is

\[
e = \frac{|W|}{|Q_H|}
\]

Since \(|Q_H| = |W| + |Q_C|\) (Equation 15.12), we can solve for \(|W|\) to show that \(|W| = |Q_H| - |Q_C|\).

Substituting this result into the efficiency expression gives

\[
e = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} = \frac{5.6 \times 10^4 \text{ J} - 1.8 \times 10^4 \text{ J}}{5.6 \times 10^4 \text{ J}} = 0.68
\]

85. **REASONING** When a gas expands under isobaric conditions, its pressure remains constant. The work \(W\) done by the expanding gas is \(W = P (V_f - V_i)\), Equation 15.2, where \(P\) is the pressure and \(V_f\) and \(V_i\) are the final and initial volumes. Since all the variables in this relation are known, we can solve for the final volume.

**SOLUTION** Solving \(W = P (V_f - V_i)\) for the final volume gives

\[
V_f = \frac{W}{P} + V_i = \frac{480 \text{ J}}{1.6 \times 10^5 \text{ Pa}} + 1.5 \times 10^{-3} \text{ m}^3 = 4.5 \times 10^{-3} \text{ m}^3
\]

86. **REASONING** The magnitude \(|W|\) of the work done by a heat engine is related to its efficiency \(e\) by \(e = \frac{|W|}{|Q_H|}\) (Equation 15.11), where \(|Q_H|\) is the magnitude of heat extracted from the hot reservoir. Solving Equation 15.11 for \(|W|\), we obtain

\[
|W| = e|Q_H| \quad (1)
\]

The conservation of energy requires that the magnitude \(|Q_H|\) of the extracted heat must be equal to the magnitude \(|W|\) of the work done plus the magnitude \(|Q_C|\) of the heat rejected by the engine:

\[
|Q_H| = |W| + |Q_C| \quad (15.12)
\]

Because we know the efficiency \(e\) and \(|Q_C|\), we can use Equations (1) and (15.12) to determine the magnitude \(|W|\) of the work the engine does in one second.

**SOLUTION** Substituting Equation (15.12) into Equation (1) yields

\[
|W| = e|Q_H| = e(|W| + |Q_C|) \quad (2)
\]

Solving Equation (2) for the magnitude \(|W|\) of the work done by the engine, we obtain
\[ |W| = e|W| + e|Q_C| \quad \text{or} \quad |W|(1 - e) = e|Q_C| \quad \text{or} \quad |W| = \frac{e|Q_C|}{1 - e} \quad \text{(3)} \]

The engine rejects \(|Q_C| = 9900 \text{ J}\) of heat every second, at an efficiency of \(e = 0.22\), so the magnitude of the work it does in one second is, by Equation (3),

\[ |W| = \frac{e|Q_C|}{1 - e} = \frac{0.22(9900 \text{ J})}{1 - 0.22} = 2800 \text{ J} \]

87. **REASONING AND SOLUTION**

a. Since the energy that becomes unavailable for doing work is zero for the process, we have from Equation 15.19, \(W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} = 0\). Therefore, \(\Delta S_{\text{universe}} = 0\) and according to the discussion in Section 15.11, the process is reversible.

b. Since the process is reversible, we have (see Section 15.11)

\[ \Delta S_{\text{universe}} = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} = 0 \]

Therefore,

\[ \Delta S_{\text{surroundings}} = -\Delta S_{\text{system}} = -125 \text{ J/K} \]

88. **REASONING AND SOLUTION** Suppose this device were a Carnot engine instead of a heat pump. We know that its efficiency \(e\) would be

\[ e = 1 - \left( \frac{T_C}{T_H} \right) = 1 - \left[ \frac{265 \text{ K}}{298 \text{ K}} \right] = 0.111 \]

The efficiency, however, is also given by

\[ e = \frac{|W|}{|Q_H|} \]

Since the heat pump’s coefficient of performance COP is \(\text{COP} = \frac{|Q_H|}{|W|}\), we have that

\[ \text{COP} = \frac{|Q_H|}{|W|} = \frac{1}{e} = \frac{1}{0.111} = 9.03 \]

89. **REASONING AND SOLUTION** Equation 15.14 holds for a Carnot air conditioner as well as a Carnot engine. Therefore, solving Equation 15.14 for \(|Q_C|\), we have

\[ |Q_C| = |Q_H| \left( \frac{T_C}{T_H} \right) = \left(6.12 \times 10^5 \text{ J} \right) \left( \frac{299 \text{ K}}{312 \text{ K}} \right) = 5.86 \times 10^5 \text{ J} \]
90. **REASONING** According to Equation 15.11, the efficiency \( e \) of a heat engine is given by 
\[
 e = \frac{|W|}{|Q_H|},
\]
where \(|W|\) is the magnitude of the work and \(|Q_H|\) is the magnitude of the input heat. Thus, the magnitude of the work is \(|W| = e|Q_H|\).

The efficiency \( e_{\text{Carnot}} \) of a Carnot engine is given by Equation 15.15 as 
\[
e_{\text{Carnot}} = 1 - \frac{T_C}{T_H},
\]
where \(T_C\) and \(T_H\) are, respectively, the Kelvin temperatures of the cold and hot reservoirs. This expression can be used to determine \(T_C\).

**SOLUTION** Using Equation 15.11, we find the work delivered by each engine as follows:

**Engine A**
\[
|W| = e|Q_H| = (0.60)(1200 \text{ J}) = 720 \text{ J}
\]

**Engine B**
\[
|W| = e|Q_H| = (0.80)(1200 \text{ J}) = 960 \text{ J}
\]

Equation 15.15, which gives the efficiency \( e_{\text{Carnot}} \) of a Carnot engine, can be solved for the temperature \(T_C\) of the cold reservoir:
\[
e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \quad \text{or} \quad T_C = \left(1 - e_{\text{Carnot}}\right)T_H
\]

Applying this result to each engine gives

**Engine A**
\[
T_C = (1 - 0.60)(650 \text{ K}) = 260 \text{ K}
\]

**Engine B**
\[
T_C = (1 - 0.80)(650 \text{ K}) = 130 \text{ K}
\]

91. **SSM REASONING** For segment AB, there is no work, since the volume is constant. For segment BC the process is isobaric and Equation 15.2 applies. For segment CA, the work can be obtained as the area under the line CA in the graph.

**SOLUTION**

a. For segment \(AB\), the process is isochoric, that is, the volume is constant. For a process in which the volume is constant, no work is done, so \(W = 0 \text{ J}\).

b. For segment BC, the process is isobaric, that is, the pressure is constant. Here, the volume is increasing, so the gas is expanding against the outside environment. As a result, the gas does work, which is positive according to our convention. Using Equation 15.2 and the data in the drawing, we obtain
\[
W = P(V_f - V_i)
\]
\[
= (7.0 \times 10^5 \text{ Pa}) \left(5.0 \times 10^{-3} \text{ m}^3\right) - (2.0 \times 10^{-3} \text{ m}^3) = +2.1 \times 10^3 \text{ J}
\]
c. For segment CA, the volume of the gas is decreasing, so the gas is being compressed and work is being done on it. Therefore, the work is negative, according to our convention. The magnitude of the work is the area under the segment CA. We estimate that this area is $15$ of the squares in the graphical grid. The area of each square is

$$(1.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3) = 1.0 \times 10^2 \text{ J}$$

The work, then, is

$$W = -15(1.0 \times 10^2 \text{ J}) = -1.5 \times 10^3 \text{ J}$$

92. **REASONING AND SOLUTION**

a. The work is the area under the path $ACB$. There are $48$ "squares" under the path, so that

$$W = -48 \left(2.0 \times 10^4 \text{ Pa}\right) \left(2.0 \times 10^{-3} \text{ m}^3\right) = -1900 \text{ J}$$

The minus sign is included because the gas is compressed, so that work is done on it. Since there is no temperature change between $A$ and $B$ (the line $AB$ is an isotherm) and the gas is ideal, $\Delta U = 0$, so

$$Q = \Delta U + W = W = -1900 \text{ J}$$

b. The negative answer for $W$ means that heat flows out of the gas.

93. **REASONING** The change in the internal energy of the gas can be found using the first law of thermodynamics, since the heat added to the gas is known and the work can be calculated by using Equation 15.2, $W = P \Delta V$. The molar specific heat capacity at constant pressure can be evaluated by using Equation 15.6 and the ideal gas law.

**SOLUTION**

a. The change in the internal energy is

$$\Delta U = Q - W = Q - P\Delta V$$

$$= 31.4 \text{ J} - \left(1.40 \times 10^4 \text{ Pa}\right) \left(8.00 \times 10^{-4} \text{ m}^3 - 3.00 \times 10^{-4} \text{ m}^3\right) = 24.4 \text{ J}$$

b. According to Equation 15.6, the molar specific heat capacity at constant pressure is $C_p = Q/(n \Delta T)$. The term $n \Delta T$ can be expressed in terms of the pressure and change in volume by using the ideal gas law:

$$P \Delta V = n R \Delta T \quad \text{or} \quad n \Delta T = P \Delta V/R$$

Substituting this relation for $n \Delta T$ into $C_p = Q/(n \Delta T)$, we obtain
\[ C_P = \frac{Q}{P \Delta V} = \frac{31.4 \text{ J}}{\left(1.40 \times 10^4 \text{ Pa}\right)\left(5.00 \times 10^{-4} \text{ m}^3\right)} = \frac{37.3 \text{ J/(mol} \cdot \text{K)}}{8.31 \text{ J/(mol} \cdot \text{K)}} \]

94. **REASONING** According to the conservation of energy, the work \( W \) done by the electrical energy is \( |W| = |Q_H| - |Q_C| \), where \( |Q_H| \) is the magnitude of the heat delivered to the outside (the hot reservoir) and \( |Q_C| \) is the magnitude of the heat removed from the house (the cold reservoir). Dividing both sides of this relation by the time \( t \), we have

\[
\frac{|W|}{t} = \frac{|Q_H|}{t} - \frac{|Q_C|}{t}
\]

The term \( |W|/t \) is the magnitude of the work per second that must be done by the electrical energy, and the terms \( |Q_H|/t \) and \( |Q_C|/t \) are, respectively, the magnitude of the heat per second delivered to the outside and removed from the house. Since the air conditioner is a Carnot air conditioner, we know that \( |Q_H|/|Q_C| = T_H/T_C \) is equal to the ratio \( T_H/T_C \) of the Kelvin temperatures of the hot and cold reservoirs, \( |Q_H|/|Q_C| = T_H/T_C \) (Equation 15.14). This expression, along with the one above for \( |W|/t \), will allow us to determine the magnitude of the work per second done by the electrical energy.

**SOLUTION** Solving the expression \( |Q_H|/|Q_C| = T_H/T_C \) for \( |Q_H| \), substituting the result into the relation \( \frac{|W|}{t} = \frac{|Q_H|}{t} - \frac{|Q_C|}{t} \), and recognizing that \( |Q_C|/t = 10 \, 500 \, \text{J/s} \), give

\[
\frac{|W|}{t} = \frac{|Q_H|}{t} - \frac{|Q_C|}{t} = \frac{|Q_C|}{t} \frac{T_H}{T_C} - \frac{|Q_C|}{t} = \left( \frac{|Q_C|}{t} \right) \left( \frac{T_H}{T_C} - 1 \right) = (10 \, 500 \, \text{J/s}) \left( \frac{306.15 \, \text{K}}{292.15 \, \text{K}} - 1 \right) = 5.0 \times 10^2 \, \text{J/s}
\]

In this result we have used the fact that \( T_H = 273.15 + 33.0 \, ^\circ\text{C} = 306.15 \, \text{K} \) and the fact that \( T_C = 273.15 + 19.0 \, ^\circ\text{C} = 292.15 \, \text{K} \).

95. **REASONING AND SOLUTION** The total heat generated by the students is

\[ Q = (200)(130 \, \text{W})(3000 \, \text{s}) = 7.8 \times 10^7 \, \text{J} \]

For the isochoric process,

\[ Q = C_V n \Delta T = (5R/2) n \Delta T \]
The number of moles of air in the room is found from the ideal gas law to be

\[ n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(1200 \text{ m}^3)}{[8.31 \text{ J/(mol} \cdot \text{K})](294 \text{ K})} = 5.0 \times 10^4 \text{ mol} \]

Now

\[ \Delta T = \frac{Q}{\frac{5}{2} R n} = \frac{7.8 \times 10^7 \text{ J}}{\frac{5}{2}[8.31 \text{ J/(mol} \cdot \text{K})](5.0 \times 10^4 \text{ mol})} = 75 \text{ K} \]

96. **REASONING** When an amount \( Q \) of heat flows spontaneously from the hot reservoir at temperature \( T_H = 373 \text{ K} \) to the cold reservoir at temperature \( T_C = 273 \text{ K} \), the amount of energy rendered unavailable for work is

\[ W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} \quad (15.19) \]

In Equation 15.19, \( T_0 \) is the Kelvin temperature of the lowest-temperature reservoir \( (T_0 = 173 \text{ K}) \) that is available, and \( \Delta S_{\text{universe}} \) is the change in entropy of the universe as a result of the heat transfer. This entropy change is the sum of the entropy change \( \Delta S_H \) of the hot reservoir and the entropy change \( \Delta S_C \) of the cold reservoir:

\[ \Delta S_{\text{universe}} = \Delta S_H + \Delta S_C \quad (1) \]

To find the entropy changes of the hot and cold reservoirs, we employ \( \Delta S = \left( \frac{Q}{T} \right)_R \) (Equation 15.18), imagining reversible processes where an amount of heat \( Q \) is transferred from the hot reservoir to the cold reservoir. The entropy changes given by Equation 15.18 will be the same as for the irreversible process that occurs as heat \( Q \) flows from the hot reservoir, through the copper rod, and into the cold reservoir, because entropy is a function of state. Equation 15.18, then, yields

\[ \Delta S_H = -\frac{|Q|}{T_H} \quad \text{and} \quad \Delta S_C = +\frac{|Q|}{T_C} \quad (2) \]

The hot reservoir loses heat, and the cold reservoir gains heat, as heat flows from the hot reservoir to the cold reservoir. The explicit algebraic signs (− and +) in Equation (2) reflect these facts.

Because the heat flow from the hot reservoir to the cold reservoir occurs as a process of thermal conduction, the heat \( Q \) that flows through the rod in a time \( t \) is found from

\[ Q = \frac{(kA \Delta T) t}{L} \quad (13.1) \]
Here, $k = 390 \text{ J/(s} \cdot \text{m} \cdot \text{C})$ is the thermal conductivity of copper (see Table 13.1 in the text), $A$ and $L$ are the cross-sectional area and length of the rod, respectively, and $\Delta T$ is the difference in temperature between the ends of the rod: $\Delta T = T_H - T_C$.

**SOLUTION** Substituting Equation (1) into Equation 15.19 yields

$$W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} = T_0 (\Delta S_H + \Delta S_C)$$

(3)

Substituting Equations (2) into Equation (3) and simplifying, we find that

$$W_{\text{unavailable}} = T_0 (\Delta S_H + \Delta S_C) = T_0 \left( \frac{-|Q|}{T_H} + \frac{|Q|}{T_C} \right) = |Q| \left( -\frac{T_0}{T_H} + \frac{T_0}{T_C} \right)$$

(4)

Lastly, substituting Equation 13.1 into Equation (4), we obtain a final expression for the amount of energy that becomes unavailable for doing work in this process:

$$W_{\text{unavailable}} = |Q| \left( -\frac{T_0}{T_H} + \frac{T_0}{T_C} \right) = \frac{(kA \Delta T)t}{L} \left( -\frac{T_0}{T_H} + \frac{T_0}{T_C} \right)$$

(5)

Expressed in seconds, the elapsed time is $t = \left(\frac{2.0 \text{ min}}{1 \text{ min}}\right) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 120 \text{ s}$. In addition, the temperature difference $\Delta T = 373 \text{ K} - 273 \text{ K} = 1.00 \times 10^2 \text{ K}$ is equivalent to $1.00 \times 10^2 \text{ C}$, because one kelvin is equivalent to one Celsius degree. Equation (5), then, yields the desired result:

$$W_{\text{unavailable}} = \left[\frac{390 \text{ J/(s} \cdot \text{m} \cdot \text{C})}{0.35 \text{ m}}\right] \left(9.4 \times 10^{-4} \text{ m}^2 \times 1.00 \times 10^2 \text{ C} \times 120 \text{ s} \times \frac{173 \text{ K}}{373 \text{ K}} + \frac{173 \text{ K}}{273 \text{ K}}\right)$$

$$= 2100 \text{ J}$$

97. **REASONING** The power rating $\bar{P}$ of the heater is equal to the heat $Q$ supplied to the gas divided by the time $t$ the heater is on, $\bar{P} = Q/t$ (Equation 6.10b). Therefore, $t = Q/\bar{P}$. The heat required to change the temperature of a gas under conditions of constant pressure is given by $Q = C_p n \Delta T$ (Equation 15.6), where $C_p$ is the molar specific heat capacity at constant pressure, $n$ is the number of moles, and $\Delta T = T_f - T_i$ is the change in temperature. For a monatomic ideal gas, the specific heat capacity at constant pressure is $C_p = \frac{5}{2} R$, Equation (15.7), where $R$ is the universal gas constant. We do not know $n$, $T_f$ and $T_i$, but we can use the ideal gas law, $PV = nRT$, (Equation 14.1) to replace $nRT_i$ by $P_iV_i$ and to replace $nRT_f$ by $P_fV_f$, quantities that we do know.
**SOLUTION**  Substituting  \( Q = C_p n \Delta T = C_p n (T_f - T_i) \) into  \( t = Q / \bar{P} \) and using the fact that  \( C_p = \frac{5}{2} R \) give

\[
t = \frac{Q}{\bar{P}} = \frac{C_p n (T_f - T_i)}{\bar{P}} = \frac{\frac{5}{2} R n (T_f - T_i)}{\bar{P}}
\]

Replacing  \( R n T \) by  \( P_i V_f \) and  \( R n T \) by  \( P_i V_i \) and remembering that  \( P_i = P_f \), we find

\[
t = \frac{\frac{5}{2} P_i (V_f - V_i)}{\bar{P}}
\]

Since the volume of the gas increases by 25.0\%,  \( V_f = 1.250 V_i \). The time that the heater is on is

\[
t = \frac{\frac{5}{2} P_i (V_f - V_i)}{\bar{P}} = \frac{\frac{5}{2} P_i (1.250 V_i - V_i)}{\bar{P}} = \frac{\frac{5}{2} P_i (0.250) V_i}{\bar{P}}
\]

\[
= \frac{\frac{5}{2} (7.60 \times 10^5 \text{ Pa})(0.250)(1.40 \times 10^{-3} \text{ m}^3)}{15.0 \text{ W}} = 44.3 \text{ s}
\]
REASONING The first law of thermodynamics states that due to heat $Q$ and work $W$, the internal energy of the system changes by an amount $\Delta U$ according to $\Delta U = Q - W$ (Equation 15.1). This law will enable us to find the various quantities for the three processes.

**Process $A \rightarrow B$** There is no work done for the process $A \rightarrow B$. The reason is that the volume is constant (see the drawing), which means that the change $\Delta V$ in the volume is zero. Thus, the area under the plot of pressure versus volume is zero, and $W_{A \rightarrow B} = 0$ J. Since $Q$ is known, the first law can then be used to find $\Delta U$.

**Process $B \rightarrow C$** Since the change $\Delta U_{B \rightarrow C}$ in the internal energy of the gas and the work $W_{B \rightarrow C}$ are known for the process $B \rightarrow C$, the heat $Q_{B \rightarrow C}$ can be determined directly by using the first law of thermodynamics: $Q_{B \rightarrow C} = \Delta U_{B \rightarrow C} + W_{B \rightarrow C}$ (Equation 15.1).

**Process $C \rightarrow A$** For the process $C \rightarrow A$ it is possible to find the change in the internal energy of the gas once the changes in the internal energy for the processes $A \rightarrow B$ and $B \rightarrow C$ are known. The total change $\Delta U_{\text{total}}$ in the internal energy for the three processes is $\Delta U_{\text{total}} = \Delta U_{A \rightarrow B} + \Delta U_{B \rightarrow C} + U_{C \rightarrow A}$, and we can use this equation to find $\Delta U_{C \rightarrow A}$, with the aid of values for $\Delta U_{A \rightarrow B}$ and $\Delta U_{B \rightarrow C}$. This is possible because $\Delta U_{\text{total}}$ is the change in the internal energy for the total process $A \rightarrow B \rightarrow C \rightarrow A$. This process begins and ends at the same place on the pressure-versus-volume plot. Therefore, the value of the internal energy $U$ is the same at the start and the end, with the result that $\Delta U_{\text{total}} = 0$ J.

**SOLUTION**

a. Since the change in volume is zero ($\Delta V = 0$ m$^3$), the area under the plot of pressure versus volume is zero, with the result that the work is $W_{A \rightarrow B} = 0$ J.

b. The change in the internal energy of the gas for the process $A \rightarrow B$ is found from the first law of thermodynamics:

$$\Delta U_{A \rightarrow B} = Q_{A \rightarrow B} - W_{A \rightarrow B} = +561 \text{ J} - 0 \text{ J} = +561 \text{ J}$$

c. According to the first law of thermodynamics, the change in the internal energy of the gas for the process $B \rightarrow C$ is $\Delta U_{B \rightarrow C} = Q_{B \rightarrow C} - W_{B \rightarrow C}$. Thus, the heat for this process is

$$Q_{B \rightarrow C} = \Delta U_{B \rightarrow C} + W_{B \rightarrow C} = +4303 \text{ J} + 3740 \text{ J} = +8043 \text{ J}$$
Since this heat is positive, it is added to the gas.

d. The change $\Delta U_{\text{total}}$ in the total internal energy for the three processes is $\Delta U_{\text{total}} = \Delta U_{A \rightarrow B} + \Delta U_{B \rightarrow C} + \Delta U_{C \rightarrow A}$. Solving this equation for $\Delta U_{C \rightarrow A}$ gives $\Delta U_{C \rightarrow A} = \Delta U_{\text{total}} - \Delta U_{A \rightarrow B} - \Delta U_{B \rightarrow C}$. As discussed in the REASONING (Process C$\rightarrow$A), $\Delta U_{\text{total}} = 0 \text{ J}$, since the third process ends at point A, which is the start of the first process. Therefore,

$$\Delta U_{C \rightarrow A} = 0 \text{ J} - 561 \text{ J} - 4303 \text{ J} = -4864 \text{ J}$$

e. According to the first law of thermodynamics, the change in the internal energy of the gas for the process C$\rightarrow$A is $\Delta U_{C \rightarrow A} = Q_{C \rightarrow A} - W_{C \rightarrow A}$. The heat for this process is

$$Q_{C \rightarrow A} = \Delta U_{C \rightarrow A} + W_{C \rightarrow A} = -4864 \text{ J} + (-2867 \text{ J}) = -7731 \text{ J}$$

Since this heat is negative, it is removed from the gas.

99. **REASONING AND SOLUTION**  We wish to find an expression for the overall efficiency $e$ in terms of the efficiencies $e_1$ and $e_2$. From the problem statement, the overall efficiency of the two-engine device is

$$e = \frac{|W_1| + |W_2|}{|Q_H|}$$  \hspace{1cm} (1)

where $|Q_H|$ is the input heat to engine 1. The efficiency of a heat engine is defined by Equation 15.11, $e = |W|/|Q_H|$, so we can write

$$|W_1| = e_1 |Q_H|$$  \hspace{1cm} (2)

and

$$|W_2| = e_2 |Q_{H2}|$$

Since the heat rejected by engine 1 is used as input heat for the second engine, $|Q_{H2}| = |Q_{C1}|$, and the expression above for $|W_2|$ can be written as

$$|W_2| = e_2 |Q_{C1}|$$  \hspace{1cm} (3)

According to Equation 15.12, we have $|Q_{C1}| = |Q_H| - |W_1|$, so that Equation (3) becomes
\[ |W_2| = e_2 \left( |Q_{H1}| - |W_1| \right) \]  

(4)

Substituting Equations (2) and (4) into Equation (1) gives

\[
e = \frac{e_1 |Q_{H1}| + e_2 \left( |Q_{H1}| - |W_1| \right)}{|Q_{H1}|} = e_1 |Q_{H1}| + e_2 \left( |Q_{H1}| - e_1 |Q_{H1}| \right)
\]

Algebraically canceling the \(|Q_{H1}|\)'s in the right hand side of the last expression gives the desired result:

\[ e = e_1 + e_2 - e_1 e_2 \]

100. **REASONING** We seek the difference \(W_{\text{diff}}\) in work between the work \(W_{\text{ad}}\) done by the gas during the adiabatic process and the work \(W_{\text{alt}}\) done by the gas during the alternative process:

\[ W_{\text{diff}} = W_{\text{ad}} - W_{\text{alt}} \]  

(1)

Considering the alternative process first, we bear in mind that, according to \(W = P(V_f - V_i)\) (Equation 15.2), a gas only does work if its volume increases. As the pressure of the gas decreases to \(P_f\) from \(P_i\) during the first step, its volume remains constant, and, therefore, the gas does no work. Consequently, the total work \(W_{\text{alt}}\) done by the gas during the entire alternative process occurs during the second step, when the gas expands to a volume \(V_f\) from a volume \(V_i\) at a constant pressure \(P_f\):

\[ W_{\text{alt}} = P_f \left( V_f - V_i \right) \]  

(2)

Only the initial volume \(V_i\) is given, so the final volume \(V_f\) in Equation (2) must be determined. Because the gas is ideal and monatomic, and can undergo an adiabatic process that takes it from the initial volume \(V_i\) to the final volume \(V_f\), we will use \(P_{V_i}^{\gamma} = P_{V_f}^{\gamma}\) (Equation 15.5), with \(\gamma = \frac{5}{3}\), to determine the final volume \(V_f\). Solving Equation 15.5 for the final volume yields

\[ V_f^{\gamma} = \frac{P_{V_f}^{\gamma}}{P_f} \], or

\[ V_f = \left( \frac{P_{V_i}^{\gamma}}{P_f} \right)^{\frac{1}{\gamma}} = V_i \left( \frac{P_i}{P_f} \right)^{\frac{1}{\gamma}} = \left( 6.34 \times 10^{-3} \text{ m}^3 \right) \left( \frac{2.20 \times 10^5 \text{ Pa}}{8.15 \times 10^4 \text{ Pa}} \right)^{\frac{1}{3}} = 1.15 \times 10^{-2} \text{ m}^3 \]
In the adiabatic process, the gas does an amount of work \( W_{ad} \) given by

\[
W_{ad} = \frac{3}{2} nR (T_i - T_f)
\]  

(15.4)

In Equation 15.4, \( n \) is the number of moles of the gas, \( R \) is the universal gas constant, and \( T_i \) and \( T_f \) are the initial and final Kelvin temperatures of the gas. In order to determine the initial and final temperatures, we will employ the ideal gas law: \( PV = nRT \) (Equation 14.1). Solving Equation 14.1 for the initial and final Kelvin temperatures yields

\[
T_i = \frac{PV_i}{nR} \quad \text{and} \quad T_f = \frac{PV_f}{nR}
\]  

(3)

**SOLUTION** Substituting Equations (3) into Equation 15.4 gives

\[
W_{ad} = \frac{3}{2} nR (T_i - T_f) = \frac{3}{2} \mu R \left( \frac{PV_i}{\mu R} - \frac{PV_f}{\mu R} \right) = \frac{3}{2} (PV_i - PV_f)
\]  

(4)

Substituting Equations (4) and (2) into Equation (1), we obtain

\[
W_{diff} = W_{ad} - W_{alt} = \frac{3}{2} (PV_i - PV_f) - P_i (V_f - V_i)
\]

\[
= \frac{3}{2} PV_i - \frac{3}{2} PV_f - P_i V_f + P_i V_i = \frac{3}{2} PV_i - \frac{3}{2} PV_f + P_i V_i
\]

\[
= \frac{3}{2} PV_i + P_f \left( V_i - \frac{5}{2} V_f \right)
\]

Substituting in the given values of \( P_i, P_f, V_i, \) and the calculated value of \( V_f \), yields

\[
W_{diff} = \frac{3}{2} PV_i + P_f \left( V_i - \frac{5}{2} V_f \right)
\]

\[
= \frac{3}{2} \left( 2.20 \times 10^5 \text{ Pa} \right) \left( 6.34 \times 10^{-3} \text{ m}^3 \right)
\]

\[
+ \left( 8.15 \times 10^4 \text{ Pa} \right) \left[ 6.34 \times 10^{-3} \text{ m}^3 - \frac{5}{2} \left( 1.15 \times 10^{-2} \text{ m}^3 \right) \right] = 270 \text{ J}
\]
1. (c) It is indeed the direction in which the disturbance occurs that distinguishes the two types of waves.

2. (a) As one domino topples against the next one in line, it is moving partly perpendicular and partly parallel to the direction along which the disturbance propagates.

3. (d) The amplitude specifies the maximum excursion of the spot from the spot’s undisturbed position, and the spot moves through this distance four times during each cycle. For instance, in one cycle starting from its undisturbed position, the spot moves upward a distance \( A \), downward a distance \( A \) (returning to its undisturbed position), downward again a distance \( A \), and finally upward again a distance \( A \) (returning to its undisturbed position).

4. (d) According to Equation 16.1, the speed \( v \) of the wave is related to the wavelength \( \lambda \) and the frequency \( f \) by \( v = f \lambda \). The speed depends only on the properties of the string (tension, mass, and length) and remains constant as the frequency is doubled. According to Equation 16.1, then, the wavelength must be cut in half.

5. 0.80 m

6. (b) The amplitude specifies the maximum excursion of a particle from its undisturbed position and has nothing to do with the wave speed. The amplitude does affect the speed of the simple harmonic motion, however. Equation 10.8 indicates that \( v_{\text{max}} \) is proportional to the amplitude.

7. (c) As discussed in Section 16.3, the speed of the wave is greater when the tension in the rope is greater, which it is in the upper portion of the rope. The rope has a mass \( m \) and, hence, a weight. The upper portion of the rope has a greater tension than does the lower portion, because the upper portion supports the weight of a greater length of rope hanging below.

8. (e) According to Equation 16.2, the speed is proportional to the square root of the tension. Since the tension in string 1 is three times that in string 2, the speed in string 1 is \( \sqrt{3} = 1.73 \) times the speed in string 2.

9. 0.0039 kg/m

10. 161 m/s

11. (e) Condensations are regions where the air pressure is increased above the normal pressure, and rarefactions are regions where the air pressure is decreased below the normal pressure. When the amplitude decreases to zero, the pressure of the air is no longer being increased
above or being decreased below the normal air pressure. Therefore, there are no longer any condensations or rarefactions.

12. (b) Sound travels faster in liquids than in gases. The greater speed in water ensures that the echo will return more quickly in water than in air.

13. (d) The frequency of the sound is determined by the vibrating diaphragm of the horn. The sound wave travels through the air and contacts the surface of the water, where it causes the water molecules to vibrate at the same frequency as the molecules in the air. However, the speed of sound in air is smaller than in air. According to Equation 16.1, the wavelength is proportional to the speed, when the frequency is constant. As a result, the wavelength is smaller in the air than in the water.

14. (a) According to Equation 16.5, the speed of sound in an ideal gas is directly proportional to the square root of the Kelvin temperature. This means that the speed at the higher temperature of \(30.0 + 273.15 = 303\) K is greater than the speed at the lower temperature of \(15.0 + 273.15 = 288\) K by a factor of \(\frac{\sqrt{303\ K}}{\sqrt{288\ K}}\). Thus, the desired speed is \((275\ m/s)\frac{\sqrt{303\ K}}{\sqrt{288\ K}}\).

15. 9.0 W

16. 4.77 dB

17. 0.45

18. (c) The Doppler effect causes the observed frequency to be shifted toward higher values when the source moves toward a stationary observer and when the observer moves toward a stationary source. The shift to higher observed frequencies is even greater when the source and the observer move toward each other, as they do here. In Equation 16.15 the plus sign applies in the numerator and the minus sign in the denominator.

19. (d) In order for the Doppler effect to be large, the speed \(v_s\) of the source and/or the speed \(v_o\) of the observer must be appreciable fractions of the speed \(v\) of sound. The Doppler effect depends on \(\frac{v_s}{v}\) or \(\frac{v_o}{v}\) or on both of these ratios (See Equations 16.11 – 16.15.) For given values of \(v_s\) and \(v_o\), these ratios decrease, and the Doppler effect decreases as the speed of sound increases. The speed of sound in air (assumed to be an ideal gas) increases with temperature, according to Equation 16.5. Therefore, the Doppler effect decreases with increasing temperature, no matter if the source moves, the observer moves, or both move.

20. 992 Hz
1. **SSM REASONING** Since light behaves as a wave, its speed \( v \), frequency \( f \), and wavelength \( \lambda \) are related to each other according to \( v = f \lambda \) (Equation 16.1). We can solve this equation for the frequency in terms of the speed and the wavelength.

**SOLUTION** Solving Equation 16.1 for the frequency, we find that

\[
 f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{5.45 \times 10^{-7} \text{ m}} = 5.50 \times 10^{14} \text{ Hz}
\]

2. **REASONING AND SOLUTION**
   a. Since 15 boxcars pass by in 12.0 s, the boxcars pass by with a frequency of

\[
 f = \frac{15}{120 \text{ s}} = 1.25 \text{ Hz}
\]

   b. Since the length of a boxcar corresponds to the wavelength \( \lambda \) of a wave, we have, from Equation 16.1, that

\[
 v = f \lambda = (1.25 \text{ Hz})(14.0 \text{ m}) = 17.5 \text{ m/s}
\]

3. **REASONING**
   a. The period is the time required for one complete cycle of the wave to pass. The period is also the time for two successive crests to pass the person.

   b. The frequency is the reciprocal of the period, according to Equation 10.5.

   c. The wavelength is the horizontal length of one cycle of the wave, or the horizontal distance between two successive crests.

   d. The speed of the wave is equal to its frequency times its wavelength (see Equation 16.1).

   e. The amplitude \( A \) of a wave is the maximum excursion of a water particle from the particle’s undisturbed position.

**SOLUTION**
   a. After the initial crest passes, 5 additional crests pass in a time of 50.0 s. The period \( T \) of the wave is

\[
 T = \frac{50.0 \text{ s}}{5} = 10.0 \text{ s}
\]
b. Since the frequency $f$ and period $T$ are related by $f = 1/T$ (Equation 10.5), we have

$$f = \frac{1}{T} = \frac{1}{10.0 \text{ s}} = 0.100 \text{ Hz}$$

c. The horizontal distance between two successive crests is given as 32 m. This is also the wavelength $\lambda$ of the wave, so

$$\lambda = 32 \text{ m}$$

d. According to Equation 16.1, the speed $v$ of the wave is

$$v = f \lambda = (0.100 \text{ Hz})(32 \text{ m}) = 3.2 \text{ m/s}$$

e. There is no information given, either directly or indirectly, about the amplitude of the wave. Therefore,

it is not possible to determine the amplitude.

4. **REASONING** The speed of a Tsunami is equal to the distance $x$ it travels divided by the time $t$ it takes for the wave to travel that distance. The frequency $f$ of the wave is equal to its speed divided by the wavelength $\lambda$, $f = \frac{v}{\lambda}$ (Equation 16.1). The period $T$ of the wave is related to its frequency by Equation 10.5, $T = \frac{1}{f}$.

**SOLUTION**

a. The speed of the wave is (in m/s)

$$v = \frac{x}{t} = \frac{3700 \times 10^3 \text{ m}}{5.3 \text{ s}} = \frac{1 \text{ km}}{3600 \text{ s}} = 190 \text{ m/s}$$

b. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{190 \text{ m/s}}{750 \times 10^3 \text{ m}} = 2.5 \times 10^{-4} \text{ Hz}$$

(16.1)

c. The period of any wave is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{2.5 \times 10^{-4} \text{ Hz}} = 4.0 \times 10^3 \text{ s}$$

(10.5)

5. **SSM** **REASONING** When the end of the Slinky is moved up and down continuously, a transverse wave is produced. The distance between two adjacent crests on the wave, is, by definition, one wavelength. The wavelength $\lambda$ is related to the speed and frequency of a periodic wave by $\lambda = \frac{v}{f}$ (Equation 16.1). In order to use Equation 16.1, we must first determine the frequency of the wave. The wave on the Slinky will have the same frequency as the simple harmonic motion of the hand. According to the data given in the problem statement, the frequency is $f = (2.00 \text{ cycles})/(1 \text{ s}) = 2.00 \text{ Hz}$.
SOLUTION  Substituting the values for $\lambda$ and $f$, we find that the distance between crests is

$$\lambda = \frac{v}{f} = \frac{0.50 \text{ m/s}}{2.00 \text{ Hz}} = 0.25 \text{ m}$$

6. REASONING  The speed $v$ of the wave is $v = f \lambda$ (Equation 16.1), where $f$ is the frequency and $\lambda$ is the wavelength of the wave. The frequency is $f = \frac{1}{T}$ (Equation 10.5), where $T$ is the period. The period is the time between successive crests. The wavelength is the distance between two successive crests.

SOLUTION  Substituting Equation 10.5 for the frequency into Equation 16.1 for the speed, we obtain

$$v = f \lambda = \frac{\lambda}{T}$$

The wavelength is given as $\lambda = 4.0 \text{ m}$. Since four crests pass by in 7.0 s and the period is the time between successive crests, the period is $T = (7.0 \text{ s})/3$. Therefore, Equation (1) reveals that the speed of the wave is

$$v = \frac{\lambda}{T} = \frac{4.0 \text{ m}}{(7.0 \text{ s})/3} = 1.7 \text{ m/s}$$

7. REASONING  The speed $v$ of a wave is equal to its frequency $f$ times its wavelength $\lambda$ (Equation 16.1). The wavelength is the horizontal length of one cycle of the wave. From the left graph in the text it can be seen that this distance is 0.040 m. The frequency is the reciprocal of the period, according to Equation 10.5, and the period is the time required for one complete cycle of the wave to pass. From the right graph in the text, it can be seen that the period is 0.20 s, so the frequency is $1/(0.20 \text{ s})$.

SOLUTION  Since the wavelength is $\lambda = 0.040 \text{ m}$ and the period is $T = 0.20 \text{ s}$, the speed of the wave is

$$v = \lambda f = \lambda \left( \frac{1}{T} \right) = (0.040 \text{ m}) \left( \frac{1}{0.20 \text{ s}} \right) = 0.20 \text{ m/s}$$

8. REASONING AND SOLUTION  First find the speed of the record at a distance of 0.100 m from the center:

$$v = r\omega = (0.100 \text{ m})(3.49 \text{ rad/s}) = 0.349 \text{ m/s} \quad (8.9)$$

The wavelength is, then,

$$\lambda = \frac{v}{f} = \frac{(0.349 \text{ m/s})(5.00 \times 10^3 \text{ Hz})}{6.98 \times 10^{-5} \text{ m}} = 6.98 \times 10^{-5} \text{ m} \quad (16.1)$$
9. **REASONING** During each cycle of the wave, a particle of the string moves through a total distance that equals $4A$, where $A$ is the amplitude of the wave. The number of wave cycles per second is the frequency $f$ of the wave. Therefore, the distance moved per second by a string particle is $4Af$. The time to move through a total distance $L$, then, is $t = L/(4Af)$. According to Equation 16.1, however, we have that $f = v/\lambda$, so that

$$t = \frac{L}{4Af} = \frac{L}{4A(v/\lambda)}$$

**SOLUTION** Using the result obtained above, we find that

$$t = \frac{L\lambda}{4Av} = \frac{(1.0 \times 10^3 \text{ m})(0.18 \text{ m})}{4(2.0 \times 10^{-3} \text{ m})(450 \text{ m/s})} = 5.0 \times 10^1 \text{ s}$$

10. **REASONING** The waves are traveling at a velocity $v_{WS}$ relative to the shore, and the jetskier is traveling in the same direction as the waves at a velocity $v_{JS}$ relative to the shore. Therefore, as explained in Section 3.4, the velocity of the jetskier relative to the shore is given by

$$v_{JS} = v_{JW} + v_{WS} \quad (1)$$

where $v_{JW}$ is the velocity of the jetskier relative to the waves. From the jetskier’s point of view, the waves have a speed that is the magnitude of $v_{JW}$ (or $v_{JW}$), a wavelength $\lambda$ equal to the distance between crests, and a frequency $f$ equal to the frequency of the bumps he experiences. These three quantities are related by Equation 16.1:

$$v_{JW} = f \lambda \quad (2)$$

**SOLUTION** Solving Equation (1) for $v_{JW}$ yields $v_{JW} = v_{JS} - v_{WS}$. We assume that both the jetskier and the waves move in the positive direction. Since the jetskier is traveling faster than the waves, $v_{JS}$ (the magnitude of $v_{JS}$) is greater than $v_{WS}$ (the magnitude of $v_{WS}$). Therefore, our vector result $v_{JW} = v_{JS} - v_{WS}$ becomes $v_{JW} = v_{JS} - v_{WS}$, which is an expression relating the speeds. Substituting this result into Equation (2), we obtain

$$v_{JW} = v_{JS} - v_{WS} = f \lambda \quad (3)$$

Solving Equation (3) for the wave speed $v_{WS}$, we obtain

$$v_{WS} = v_{JS} - f \lambda = 8.4 \text{ m/s} - (1.2 \text{ Hz})(5.8 \text{ m}) = 1.4 \text{ m/s}$$

11. **REASONING AND SOLUTION** When traveling with the waves, the skier "sees" the waves to be traveling with a velocity of $v_w - v_s$ and with a period of $T_1 = 0.600$ s. Suppressing the units for convenience, we find that the distance between crests is (in meters)
\[ \lambda = (v_s - v_w)T_1 = (12.0 - v_w)(0.600) = 7.20 - 0.600 v_w \]  
(1)

Similarly, for the skier traveling opposite the waves

\[ \lambda = (v_s + v_w)T_2 = 6.00 + 0.500 v_w \]  
(2)

a. Subtracting Equation (2) from Equation (1) and solving for \( v_w \) gives \( v_w = 1.09 \text{ m/s} \).

b. Substituting the value for \( v_w \) into Equation (1) gives \( \lambda = 6.55 \text{ m} \).

12. **REASONING** The length \( L \) of the string is one factor that affects the speed of a wave traveling on it, in so far as the speed \( v \) depends on the mass per unit length \( m/L \) according to

\[ v = \sqrt{\frac{F}{m/L}} \]  
(Equation 16.2). The other factor affecting the speed is the tension \( F \). The speed is not directly given here. However, the frequency \( f \) and the wavelength \( \lambda \) are given, and the speed is related to them according to \( v = f\lambda \) (Equation 16.1). Substituting Equation 16.1 into Equation 16.2 will give an equation that can be solved for the length \( L \).

**SOLUTION** Substituting Equation 16.1 into Equation 16.2 gives

\[ v = f\lambda = \sqrt{\frac{F}{m/L}} \]

Solving for the length \( L \), we find that

\[ L = \frac{f^2\lambda^2m}{F} = \frac{(260 \text{ Hz})^2(0.60 \text{ m})^2(5.0 \times 10^{-3} \text{ kg})}{180 \text{ N}} = 0.68 \text{ m} \]

13. **SSM REASONING** According to Equation 16.2, the linear density of the string is given by \( (m/L) = F/v^2 \), where the speed \( v \) of waves on the middle C string is given by Equation 16.1, \( v = f\lambda = \left( \frac{1}{T} \right) \lambda \), where \( T \) is the period.

**SOLUTION** Combining Equations 16.2 and 16.1 and using the given data, we obtain

\[ \frac{m}{L} = \frac{F}{v^2} = \frac{FT^2}{\lambda^2} = \frac{(944 \text{ N})(3.82 \times 10^{-3} \text{ s})^2}{(1.26 \text{ m})^2} = 8.68 \times 10^{-3} \text{ kg/m} \]

14. **REASONING** The speed \( v \) of a transverse wave on a wire is given by \( v = \sqrt{F/(m/L)} \) (Equation 16.2), where \( F \) is the tension and \( m/L \) is the mass per unit length (or linear density) of the wire. We are given that \( F \) and \( m \) are the same for the two wires, and that one is twice as long as the other. This information, along with knowledge of the wave speed on the shorter wire, will allow us to determine the speed of the wave on the longer wire.
**SOLUTION** The speeds on the longer and shorter wires are:

**[Longer wire]** \[ v_{\text{longer}} = \sqrt{\frac{F}{m/L_{\text{longer}}}} \]

**[Shorter wire]** \[ v_{\text{shorter}} = \sqrt{\frac{F}{m/L_{\text{shorter}}}} \]

Dividing the expression for \( v_{\text{longer}} \) by that for \( v_{\text{shorter}} \) gives

\[
\frac{v_{\text{longer}}}{v_{\text{shorter}}} = \sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}}
\]

Since \( v_{\text{shorter}} = 240 \text{ m/s} \) and \( L_{\text{longer}} = 2L_{\text{shorter}} \), the speed of the wave on the longer wire is

\[
v_{\text{longer}} = v_{\text{shorter}} \sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}} = (240 \text{ m/s}) \sqrt{\frac{2L_{\text{shorter}}}{L_{\text{shorter}}}} = (240 \text{ m/s}) \sqrt{2} = 340 \text{ m/s}
\]

15. **REASONING** The speed \( v \) of the transverse pulse on the wire is determined by the tension \( F \) in the wire and the mass per unit length \( m/L \) of the wire, according to \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2). The ball has a mass \( M \). Since the wire supports the weight \( Mg \) of the ball and since the weight of the wire is negligible, it is only the ball’s weight that determines the tension in the wire, \( F = Mg \). Therefore, we can use Equation 16.2 with this value of the tension and solve it for the acceleration \( g \) due to gravity. The speed of the transverse pulse is not given, but we know that the pulse travels the length \( L \) of the wire in a time \( t \) and that the speed is \( v = L/t \).

**SOLUTION** Substituting the tension \( F = Mg \) and the speed \( v = L/t \) into Equation 16.2 for the speed of the pulse on the string gives

\[
v = \sqrt{\frac{F}{m/L}} \quad \text{or} \quad \frac{L}{t} = \sqrt{\frac{Mg}{m/L}}
\]

Solving for the acceleration \( g \) due to gravity, we obtain

\[
g = \left( \frac{L}{t} \right)^2 \left( \frac{m}{L} \right) \left( \frac{0.95 \text{ m}}{0.016 \text{ s}} \right)^2 \left( 1.2 \times 10^{-4} \text{ kg/m} \right) = \frac{7.7 \text{ m/s}^2}{0.055 \text{ kg}} = 140 \text{ m/s}^2
\]

16. **REASONING** Each pulse travels a distance that is given by \( vt \), where \( v \) is the wave speed and \( t \) is the travel time up to the point when they pass each other. The sum of the distances
traveled by each pulse must equal the 50.0-m length of the wire, since each pulse starts out from opposite ends of the wires.

**SOLUTION** Using $v_A$ and $v_B$ to denote the speeds on either wire, we have

$$v_A t + v_B t = 50.0 \text{ m}$$

Solving for the time $t$ and using Equation 16.2

$$v = \sqrt{\frac{F}{m/L}}$$

we find

$$t = \frac{50.0 \text{ m}}{v_A + v_B} = \frac{50.0 \text{ m}}{\sqrt{\frac{6.00 \times 10^2 \text{ N}}{0.020 \text{ kg/m}}} + \sqrt{\frac{3.00 \times 10^2 \text{ N}}{0.020 \text{ kg/m}}}} = 0.17 \text{ s}$$

17. **REASONING** The speed $v$ of a transverse wave on a string is given by

$$v = \sqrt{\frac{F}{m/L}}$$

(Equation 16.2), where $F$ is the tension and $m/L$ is the mass per unit length (or linear density) of the string. The strings are identical, so they have the same mass per unit length. However, the tensions are different. In part (a) of the text drawing, the string supports the entire weight of the 26-N block, so the tension in the string is 26 N. In part (b), the block is supported by the part of the string on the left side of the middle pulley and the part of the string on the right side. Each part supports one-half of the block’s weight, or 13 N. Thus, the tension in the string is 13 N.

**SOLUTION**

a. The speed of the transverse wave in part (a) of the text drawing is

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{26 \text{ N}}{0.065 \text{ kg/m}}} = 2.0 \times 10^1 \text{ m/s}$$

b. The speed of the transverse wave in part (b) of the drawing is

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{13 \text{ N}}{0.065 \text{ kg/m}}} = 1.4 \times 10^1 \text{ m/s}$$

18. **REASONING** The speed $v$ of the wave on the cable is

$$v = \sqrt{\frac{F}{m/L}}$$

(Equation 16.2), where $F$ is the tension in the cable and $m/L$ is the mass per unit length of the cable. The mass per unit length is called the linear density of the cable. The tension is given. The linear density is not given. However, it can be obtained from the value of 7860 kg/m$^3$ given for the density $\rho$ of steel and the given cross-sectional area of the cable.
**SOLUTION** The speed of the wave on the cable is

\[ v = \sqrt{\frac{F}{m/L}} \]  \hspace{1cm} (16.2)

To obtain the linear density \( m/L \), we proceed as follows. The density is \( \rho = \frac{m}{V} \) (Equation 11.1), where \( m \) is the mass of a volume \( V \) of material. But the volume of the steel in the cable is its length \( L \) times its cross-sectional area \( A \), or \( V = LA \). Substituting this result into Equation 11.1 gives

\[ \rho = \frac{m}{V} = \frac{m}{LA} = \frac{m/L}{A} \quad \text{or} \quad m/L = \rho A \]

Substituting this result for the linear density into Equation 16.2, we obtain

\[ v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{F}{\rho A}} = \sqrt{\frac{1.00 \times 10^4 \text{ N}}{(7860 \text{ kg/m}^3)(2.83 \times 10^{-3} \text{ m}^2)}} = 21.2 \text{ m/s} \]

---

19. **REASONING**

a. The wavelength is the horizontal distance between two successive crests. The horizontal distance between successive crests of wave B is two times greater than that of wave A. Therefore, B has the greater wavelength.

b. The frequency \( f \) of a wave is related to its speed \( v \) and wavelength \( \lambda \) by Equation 16.1, \( f = v/\lambda \). Since the speed is the same for both waves, the wave with the smaller wavelength has the larger frequency. Therefore wave A, having the smaller wavelength, has the larger frequency.

c. The maximum speed of a particle moving in simple harmonic motion is given by Equation 10.8 as \( v_{\text{max}} = A \omega \), where \( A \) is the amplitude of the wave and \( \omega \) is the angular frequency, \( \omega = 2\pi f \). Wave A has both a larger amplitude and frequency, so the maximum particle speed is greater for A.

**SOLUTION**

a. From the drawing, we determine the wavelength of each wave to be

\[ \lambda_A = \mathbf{2.0 \text{ m}} \quad \text{and} \quad \lambda_B = \mathbf{4.0 \text{ m}} \]

b. The frequency of each wave is given by Equation 16.1 as:

\[ f_A = \frac{v}{\lambda_A} = \frac{12 \text{ m/s}}{2.0 \text{ m}} = \mathbf{6.0 \text{ Hz}} \quad \text{and} \quad f_B = \frac{v}{\lambda_B} = \frac{12 \text{ m/s}}{4.0 \text{ m}} = \mathbf{3.0 \text{ Hz}} \]

c. The maximum speed for a particle moving in simple harmonic motion is given by Equation 10.8 as \( v_{\text{max}} = A \omega \). The amplitude of each wave can be obtained from the drawing: \( A_A = 0.50 \text{ m} \) and \( A_B = 0.25 \text{ m} \).
Wave A

\[ v_{\text{max}} = A_\text{A} \omega_\text{A} = A_\text{A} 2\pi f_\text{A} = (0.50 \text{ m}) 2\pi (6.0 \text{ Hz}) = 19 \text{ m/s} \]

Wave B

\[ v_{\text{max}} = A_\text{B} \omega_\text{B} = A_\text{B} 2\pi f_\text{B} = (0.25 \text{ m}) 2\pi (3.0 \text{ Hz}) = 4.7 \text{ m/s} \]

20. **REASONING** A particle of the string is moving in simple harmonic motion. The maximum speed of the particle is given by Equation 10.8 as \( v_{\text{max}} = A \omega \), where \( A \) is the amplitude of the wave and \( \omega \) is the angular frequency. The angular frequency is related to the frequency \( f \) by Equation 10.6, \( \omega = 2\pi f \), so the maximum speed can be written as \( v_{\text{max}} = 2\pi f A \). The speed \( v \) of a wave on a string is related to the frequency \( f \) and wavelength \( \lambda \) by Equation 16.1, \( v = f\lambda \). The ratio of the maximum particle speed to the speed of the wave is

\[ \frac{v_{\text{max}}}{v} = \frac{2\pi f A}{f \lambda} = \frac{2\pi A}{\lambda} \]

The equation can be used to find the wavelength of the wave.

**SOLUTION** Solving the equation above for the wavelength, we have

\[ \lambda = \frac{2\pi A}{v_{\text{max}}} = \frac{2\pi (4.5 \text{ cm})}{3.1} = 9.1 \text{ cm} \]

21. **REASONING** Using the procedures developed in Chapter 4 for using Newton’s second law to analyze the motion of bodies and neglecting the weight of the wire relative to the tension in the wire lead to the following equations of motion for the two blocks:

\[ \sum F_x = F - m_1 g \sin 30.0^\circ = 0 \quad \text{(1)} \]

\[ \sum F_y = F - m_2 g = 0 \quad \text{(2)} \]

where \( F \) is the tension in the wire. In Equation (1) we have taken the direction of the +x axis for block 1 to be parallel to and up the incline. In Equation (2) we have taken the direction of the +y axis to be upward for block 2. This set of equations consists of two equations in three unknowns, \( m_1, m_2, \) and \( F \). Thus, a third equation is needed in order to solve for any of the unknowns. A useful third equation can be obtained by solving Equation 16.2 for \( F \):

\[ F = (m/L)v^2 \quad \text{(3)} \]

Combining Equation (3) with Equations (1) and (2) leads to

\[ (m/L)v^2 - m_1 g \sin 30.0^\circ = 0 \quad \text{(4)} \]

\[ (m/L)v^2 - m_2 g = 0 \quad \text{(5)} \]

Equations (4) and (5) can be solved directly for the masses \( m_1 \) and \( m_2 \).
**SOLUTION** Substituting values into Equation (4), we obtain

\[
m_1 = \frac{(m/L)v^2}{g \sin 30^\circ} = \frac{(0.0250 \text{ kg/m})(75.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \sin 30^\circ} = \boxed{28.7 \text{ kg}}
\]

Similarly, substituting values into Equation (5), we obtain

\[
m_2 = \frac{(m/L)v^2}{g} = \frac{(0.0250 \text{ kg/m})(75.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} = \boxed{14.3 \text{ kg}}
\]

22. **REASONING AND SOLUTION** According to Equation 16.2, the tension in the wire initially is

\[
F_0 = (m/L)v^2 = (9.8 \times 10^{-3} \text{ kg/m})(46 \text{ m/s})^2 = 21 \text{ N}
\]

As the temperature is lowered, the wire will attempt to shrink by an amount \(\Delta L = \alpha L \Delta T\) (Equation 12.2), where \(\alpha\) is the coefficient of thermal expansion. Since the wire cannot shrink, a stress will develop (see Equation 10.17 and Section 10.8), according to

\[
\text{Stress} = Y\Delta L/L = Y\alpha \Delta T
\]

where \(Y\) is Young’s modulus. This stress corresponds (see Section 10.8) to an additional tension \(F^\prime\):

\[
F^\prime = (\text{Stress})A = YA\alpha \Delta T = (1.1 \times 10^{11} \text{ Pa})(1.1 \times 10^{-6} \text{ m}^2)(17 \times 10^{-6}/\text{C}\circ)(14 \text{ C}\circ) = 29 \text{ N}
\]

The total tension in the wire at the lower temperature is now \(F = F_0 + F^\prime\), so that the new speed of the waves on the wire is

\[
v = \sqrt{\frac{F_0 + F^\prime}{m/L}} = \sqrt{\frac{21 \text{ N} + 29 \text{ N}}{9.8 \times 10^{-3} \text{ kg/m}}} = \boxed{71 \text{ m/s}}
\]

23. **REASONING AND SOLUTION** If the string has length \(L\), the time required for a wave on the string to travel from the center of the circle to the ball is

\[
t = \frac{L}{v_{\text{wave}}}
\]

The speed of the wave is given by text Equation 16.2

\[
v_{\text{wave}} = \sqrt{\frac{F}{m_{\text{string}}/L}}
\]

The tension \(F\) in the string provides the centripetal force on the ball, so that

\[
F = m_{\text{ball}}\omega^2 r = m_{\text{ball}}\omega^2 L
\]
Eliminating the tension $F$ from Equations (2) and (3) above yields

\[ v_{\text{wave}} = \sqrt{\frac{m_{\text{ball}} \omega^2}{m_{\text{string}} / L}} = \sqrt{\frac{m_{\text{ball}} \omega^2 L^2}{m_{\text{string}}}} = L \sqrt{\frac{m_{\text{ball}} \omega^2}{m_{\text{string}}}} \]

Substituting this expression for $v_{\text{wave}}$ into Equation (1) gives

\[ t = \frac{L}{\sqrt{\frac{m_{\text{ball}} \omega^2}{m_{\text{string}}}}} = \sqrt{\frac{m_{\text{string}}}{m_{\text{ball}} \omega^2}} = \sqrt{\frac{0.0230 \text{ kg}}{(15.0 \text{ kg})(12.0 \text{ rad/s})^2}} = 3.26 \times 10^{-3} \text{ s} \]

---

24. **REASONING** The speed $v$ of the wave is related to its wavelength $\lambda$ and frequency $f$ according to $v = f \lambda$ (Equation 16.1), so we will need to determine the wavelength and frequency. The mathematical description of this wave has the form $y = A \sin \left(2 \pi ft + \frac{2 \pi x}{\lambda}\right)$ (Equation 16.4), which applies to waves moving in the $-x$ direction. Identifying like terms in the given mathematical description of the wave and Equation 16.4 will permit us to determine the wavelength $\lambda$ and frequency $f$.

**SOLUTION** The dimensionless term $2 \pi f t$ in Equation 16.4 corresponds to the term $8.2 \pi t$ in the given wave equation. The time $t$ is measured in seconds (s), so in order for the quantity $8.2 \pi t$ to be dimensionless, the units of the numerical factor 8.2 must be $s^{-1} = \text{Hz}$, and we have that

\[ 2 \pi f / t = (8.2 \text{ Hz}) \neq / t \quad \text{or} \quad 2f = 8.2 \text{ Hz} \quad \text{or} \quad f = \frac{8.2 \text{ Hz}}{2} = 4.1 \text{ Hz} \]

Similarly, the dimensionless term $2 \pi x / \lambda$ in Equation 16.4 corresponds to the term $0.54 \pi x$ in the mathematical description of this wave. Because $x$ is measured in meters (m), the term $0.54 \pi x$ is dimensionless if the numerical factor 0.54 has units of $m^{-1}$. Thus,

\[ \frac{2 \pi / \lambda}{\lambda} = (0.54 \text{ m}^{-1}) \neq / \lambda \quad \text{or} \quad \frac{2}{\lambda} = 0.54 \text{ m}^{-1} \quad \text{or} \quad \lambda = \frac{2}{0.54 \text{ m}^{-1}} = 3.7 \text{ m} \]

Applying $v = f \lambda$ (Equation 16.1), we obtain the speed of the wave:

\[ v = \lambda f = (3.7 \text{ m})(4.1 \text{ Hz}) = 15 \text{ m/s} \]

---

25. **REASONING** The mathematical form for the displacement of a wave traveling in the $-x$ direction is given by Equation 16.4: $y = A \sin \left(2 \pi ft + \frac{2 \pi x}{\lambda}\right)$.

**SOLUTION** Using Equation 16.1 and the fact that $f = 1/T$, we obtain the following numerical values for $f$ and $\lambda$:
\[
f = \frac{1}{T} = \frac{1}{0.77 \text{ s}} = 1.3 \text{ Hz} \quad \text{and} \quad \lambda = \frac{v}{f} = \frac{12 \text{ m/s}}{1.3 \text{ Hz}} = 9.2 \text{ m}
\]

Using the given value for the amplitude \( A \) and substituting the above values for \( f \) and \( \lambda \) into Equation 16.4 gives

\[
y = A \sin \left( 2\pi ft + \frac{2\pi x}{\lambda} \right) = (0.37 \text{ m}) \sin \left[ 2\pi (1.3 \text{ Hz})t + \left( \frac{2\pi}{9.2 \text{ m}} \right)x \right]
\]

\[
y = (0.37 \text{ m}) \sin \left[ (8.2 \text{ rad/s})t + \left( 0.68 \text{ m}^{-1} \right)x \right]
\]

26. **REASONING** Since the wave is traveling in the +x direction, the appropriate mathematical expression is \( y = A \sin \left( 2\pi ft - \frac{2\pi x}{\lambda} \right) \) (Equation 16.3). From the graph that accompanies the problem statement, we can obtain the amplitude \( A \) and the period \( T \) of the wave. The frequency \( f \) is \( f = 1/T \) (Equation 10.5). The wavelength \( \lambda \) can be obtained from \( v = f \lambda \) (Equation 16.1), where \( v \) is the speed of the wave, which is given.

**SOLUTION** Referring to the graph that accompanies the problem statement, we find that the amplitude is \( A = 0.010 \text{ m} \). The period is the time for one complete cycle of the wave, and the graph indicates that the period is \( T = 0.30 \text{ s} - 0.10 \text{ s} = 0.20 \text{ s} \). Equation 10.5 shows that the frequency is

\[
f = \frac{1}{T} = \frac{1}{0.20 \text{ s}} = 5.0 \text{ Hz}
\]

Solving Equation 16.1 for the wavelength, we obtain

\[
v = f \lambda \quad \text{or} \quad \lambda = \frac{v}{f} = \frac{0.15 \text{ m/s}}{5.0 \text{ Hz}} = 0.030 \text{ m}
\]

Using these values for \( A, f, \) and \( \lambda \) in Equation 16.3, we find that

\[
y = A \sin \left( 2\pi ft - \frac{2\pi x}{\lambda} \right) = (0.010 \text{ m}) \sin \left[ 2\pi (5.0 \text{ Hz})t - \frac{2\pi x}{(0.030 \text{ m})} \right]
\]

\[
y = (0.010 \text{ m}) \sin (31t - 210x)
\]

In this answer the number 31 has units of radians/second, and the number 210 has units of radians/meter.

27. **SSM REASONING** Since the wave is traveling in the +x direction, its form is given by Equation 16.3 as

\[
y = A \sin \left( 2\pi ft - \frac{2\pi x}{\lambda} \right)
\]
We are given that the amplitude is \( A = 0.35 \) m. However, we need to evaluate \( 2\pi f \) and \( \frac{2\pi}{\lambda} \).

Although the wavelength \( \lambda \) is not stated directly, it can be obtained from the values for the speed \( v \) and the frequency \( f \), since we know that \( v = f\lambda \) (Equation 16.1).

**SOLUTION** Since the frequency is \( f = 14 \) Hz, we have

\[
2\pi f = 2\pi(14 \text{ Hz}) = 88 \text{ rad/s}
\]

It follows from Equation 16.1 that

\[
\frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi(14 \text{ Hz})}{5.2 \text{ m/s}} = 17 \text{ m}^{-1}
\]

Using these values for \( 2\pi f \) and \( \frac{2\pi}{\lambda} \) in Equation 16.3, we have

\[
y = A\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right)
\]

\[
y = (0.35 \text{ m})\sin\left[(88 \text{ rad/s})t - (17 \text{ m}^{-1})x\right]
\]

---

28. **REASONING** The tension \( F \) in a string can be found from \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2), where \( v \) is the speed of a wave traveling along the string and \( m/L \) is the string’s linear density. The speed \( v \) of the wave depends upon the frequency \( f \) and wavelength \( \lambda \) of the wave, according to \( v = f\lambda \) (Equation 16.1). We will determine the frequency and wavelength of the wave by comparing the given equation for the displacement \( y \) of a particle from its equilibrium position to the general equation \( y = A\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right) \) (Equation 16.3). Once we have identified the frequency and wavelength, we will be able to determine the wave speed \( v \) via Equation 16.1. We will then use Equation 16.2 to obtain the tension \( F \) in the string.

**SOLUTION** Comparing \( y = (0.021 \text{ m})\sin(25t - 2.0x) \) with the general equation \( y = A\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right) \) (Equation 16.3), we see that \( 2\pi ft = 25t \), or \( 2\pi f = 25 \), and that \( \frac{2\pi x}{\lambda} = 2.0x \), or \( \frac{2\pi}{\lambda} = 2.0 \). Therefore, we have that

\[
f = \frac{25}{2\pi} \text{ Hz} \quad \text{and} \quad \lambda = \frac{2\pi}{2.0} \text{ m} = \pi \text{ m} \quad (1)
\]

Squaring both sides of \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2) and solving for \( F \), we obtain
\[ v^2 = \frac{F}{m/L} \quad \text{or} \quad F = v^2 (m/L) \]  

(2)

Substituting \( v = f \lambda \) (Equation 16.1) into Equation (2) yields

\[ F = v^2 (m/L) = (f \lambda)^2 (m/L) \]  

(3)

Substituting Equations (1) and the linear density into Equation (3), we find that

\[ F = (f \lambda)^2 (m/L) = \left[ \frac{25}{(2 \pi \text{ Hz})(\pi \text{ m})} \right]^2 (1.6 \times 10^{-2} \text{ kg/m}) = 2.5 \text{ N} \]

29. **SSM REASONING**  The speed of a wave on the string is given by Equation 16.2 as

\[ v = \sqrt{\frac{F}{m/L}}, \]  

where \( F \) is the tension in the string and \( m/L \) is the mass per unit length (or linear density) of the string. The wavelength \( \lambda \) is the speed of the wave divided by its frequency \( f \) (Equation 16.1).

**SOLUTION**

a. The speed of the wave on the string is

\[ v = \sqrt{\frac{15 \text{ N}}{0.85 \text{ kg/m}}} = 4.2 \text{ m/s} \]

b. The wavelength is

\[ \lambda = \frac{v}{f} = \frac{4.2 \text{ m/s}}{12 \text{ Hz}} = 0.35 \text{ m} \]

c. The amplitude of the wave is \( A = 3.6 \text{ cm} = 3.6 \times 10^{-2} \text{ m} \). Since the wave is moving along the \(-x\) direction, the mathematical expression for the wave is given by Equation 16.4 as

\[ y = A \sin \left( 2\pi f t + \frac{2\pi x}{\lambda} \right) \]

Substituting in the numbers for \( A, f, \) and \( \lambda \), we have

\[ y = A \sin \left( 2\pi \frac{12 \text{ Hz} t + 2\pi \frac{3.6 \times 10^{-2} \text{ m}}{0.35 \text{ m}}}{0.35 \text{ m}} \right) \]

\[ = (3.6 \times 10^{-2} \text{ m}) \sin \left[ (75 \text{ rad/s}) t + (18 \text{ m}^{-1}) x \right] \]
30. **Reasoning** Using either Equation 16.3 or 16.4 with \( x = 0 \) m, we obtain

\[
y = A \sin(2\pi f t)
\]

Applying this equation with \( f = 175 \) Hz, we can find the times at which \( y = 0.10 \) m.

**Solution** Using the given values for \( y \) and \( A \), we obtain

\[
0.10 \text{ m} = (0.20 \text{ m}) \sin(2\pi f t) \quad \text{or} \quad 2\pi f t = \sin^{-1}\left(\frac{0.10 \text{ m}}{0.20 \text{ m}}\right) = \sin^{-1}(0.50)
\]

The smallest two angles for which the sine function is 0.50 are 30° and 150°. However, the values for \( 2\pi f t \) must be expressed in radians. Thus, we find that the smallest two angles for which the sine function is 0.50 are expressed in radians as follows:

\[
(30^\circ)\left(\frac{2\pi}{360^\circ}\right) \quad \text{and} \quad (150^\circ)\left(\frac{2\pi}{360^\circ}\right)
\]

The times corresponding to these angles can be obtained as follows:

\[
2\pi f t_1 = (30^\circ)\left(\frac{2\pi}{360^\circ}\right) \quad \text{and} \quad 2\pi f t_2 = (150^\circ)\left(\frac{2\pi}{360^\circ}\right)
\]

\[
t_1 = \frac{30^\circ}{f(360^\circ)} \quad \text{and} \quad t_2 = \frac{150^\circ}{f(360^\circ)}
\]

Subtracting and using \( f = 175 \) Hz gives

\[
t_2 - t_1 = \frac{150^\circ}{f(360^\circ)} - \frac{30^\circ}{f(360^\circ)} = \frac{(150^\circ - 30^\circ)}{(175 \text{ Hz})(360^\circ)} = 1.9 \times 10^{-3} \text{ s}
\]

31. **SSM Reasoning** The speed \( v \), frequency \( f \), and wavelength \( \lambda \) of the sound are related according to \( v = f\lambda \) (Equation 16.1). This expression can be solved for the wavelength in terms of the speed and the frequency. The speed of sound in seawater is 1522 m/s, as given in Table 16.1. While the frequency is not given directly, the period \( T \) is known and is related to the frequency according to \( f = 1/T \) (Equation 10.5).

**Solution** Substituting the frequency from Equation 10.5 into Equation 16.1 gives

\[
v = f\lambda = \left(\frac{1}{T}\right)\lambda
\]

Solving this result for the wavelength yields

\[
\lambda = vT = (1522 \text{ m/s})\left(71 \times 10^{-3} \text{ s}\right) = 110 \text{ m}
\]

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32. **REASONING** The period \( T \) and the frequency \( f \) of the sound wave are related according to \( f = 1/T \) (Equation 10.5). Therefore, if we can obtain the frequency, we can determine the frequency. Since the wavelength \( \lambda \) and the speed of sound \( v \) in seawater are given, we can obtain the frequency \( f \) from \( v = f \lambda \) (Equation 16.1).

**SOLUTION** According to \( f = 1/T \) (Equation 10.5), the period of the sound wave is

\[
T = \frac{1}{f}
\]

(1)

Solving \( v = f \lambda \) (Equation 16.1) for the frequency gives

\[
f = \frac{v}{\lambda}
\]

(2)

Substituting Equation (2) into Equation (1), we find that

\[
T = \frac{1}{f} = \frac{1}{v/\lambda} = \frac{\lambda}{v} = \frac{0.015 \text{ m}}{1522 \text{ m/s}} = 9.9 \times 10^{-6} \text{ s}
\]

33. **REASONING AND SOLUTION** The speed of sound in an ideal gas is given by text Equation 16.5

\[
v = \sqrt{\frac{\gamma k T}{m}}
\]

where \( m \) is the mass of a single gas particle (atom or molecule). Solving for \( T \) gives

\[
T = \frac{m v^2}{\gamma k}
\]

(1)

The mass of a single helium atom is

\[
\frac{4.003 \text{ g/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) = 6.650 \times 10^{-27} \text{ kg}
\]

The speed of sound in oxygen at 0 °C is 316 m/s (see Table 16.1). Since helium is a monatomic gas, \( \gamma = 1.67 \). Then, substituting into Equation (1) gives

\[
T = \frac{\left( 6.65 \times 10^{-27} \text{ kg} \right) (316 \text{ m/s})^2}{1.67 (1.38 \times 10^{-23} \text{ J/K})} = 28.8 \text{ K}
\]

34. **REASONING** A rail can be approximated as a long slender bar, so the speed of sound in the rail is given by Equation 16.7. With this equation and the given data for Young’s modulus and the density of steel, we can determine the speed of sound in the rail. Then, we will be able to compare this speed to the speed of sound in air at 20 °C, which is 343 m/s.
SOLUTION The speed of sound in the rail is

\[
v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.0 \times 10^{11} \text{ N/m}^2}{7860 \text{ kg/m}^3}} = 5.0 \times 10^3 \text{ m/s}
\]

This speed is greater than the speed of sound in air at 20 °C by a factor of

\[
\frac{v_{\text{rail}}}{v_{\text{air}}} = \frac{5.0 \times 10^3 \text{ m/s}}{343 \text{ m/s}} = 15
\]

35. SSM REASONING AND SOLUTION The speed of sound in an ideal gas is given by Equation 16.5, \( v = \sqrt{\gamma kT/m} \). The ratio of the speed of sound \( v_2 \) in the container (after the temperature change) to the speed \( v_1 \) (before the temperature change) is

\[
\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}
\]

Thus, the new speed is

\[
v_2 = v_1 \sqrt{\frac{T_2}{T_1}} = (1220 \text{ m/s}) \sqrt{\frac{405 \text{ K}}{201 \text{ K}}} = 1730 \text{ m/s}
\]

36. REASONING The speed of sound in an ideal gas depends on its temperature \( T \) through Equation 16.5 as \( v = \sqrt{\frac{\gamma kT}{m}} \), where \( \gamma = 1.40 \) for air, \( k \) is Boltzmann’s constant, and \( m \) is the mass of a molecule in the air. Thus, the speed of sound depends on the air temperature, with a lower temperature giving rise to a smaller speed of sound. The speed of sound is greater in the afternoon at the warmer temperature and is the speed that the car must exceed.

SOLUTION The speed of sound in air is \( v = \sqrt{\frac{\gamma kT}{m}} \), where the temperature \( T \) must be expressed on the Kelvin scale \((T = T_c + 273, \text{ Equation 12.1})\). Taking the ratio of the speed of sound at 43 °C to that at 0 °C, we have

\[
\frac{v_{43 ^\circ C}}{v_{0 ^\circ C}} = \sqrt{\frac{\gamma k (43 + 273)}{m}} = \frac{316 \text{ K}}{273 \text{ K}}
\]

The speed of sound at 43 °C is

\[
v_{43 ^\circ C} = v_{0 ^\circ C} \sqrt{\frac{316 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s}) \sqrt{\frac{316 \text{ K}}{273 \text{ K}}} = 356 \text{ m/s}
\]

Therefore, the speed of your car must exceed \( v_{\text{car}} = 356 \text{ m/s} \).
37. **REASONING AND SOLUTION** The wheel must rotate at a frequency of 
\[ f = \frac{2200 \text{ Hz}}{20} = 110 \text{ Hz} \]. The angular speed \( \omega \) of the wheel is
\[ \omega = 2\pi f = 2\pi (110 \text{ Hz}) = 690 \text{ rad/s} \]

38. **REASONING** Traveling at a speed \( v \) in a material for a time \( t \), a sound wave travels a distance \( x \) that is given by \( x = vt \). In traveling from its point of origin to the bottom of the lake and back again to the point of origin, the sound in this problem travels through both air and water. The expression \( x = vt \) applies to the travel through both materials, assuming that the appropriate speed of sound is used. The distance traveled through air is \( 2d_{\text{air}} \), where \( d_{\text{air}} = 10.0 \text{ m} \). The distance traveled through water is \( 2d_{\text{water}} \), where \( d_{\text{water}} \) is the depth of the lake. Moreover, the travel time in air plus the travel time in water equals the given total travel time of 0.110 s.

**SOLUTION** The distance traveled through water is \( 2d_{\text{water}} = v_{\text{water}}t_{\text{water}} \), where \( v_{\text{water}} \) is the speed of sound in fresh water and \( t_{\text{water}} \) is the travel time in the water. Solving this expression for the depth of the lake gives
\[ d_{\text{water}} = \frac{v_{\text{water}}t_{\text{water}}}{2} \]

The total travel time in both materials is \( t = t_{\text{air}} + t_{\text{water}} \). Solving this expression for \( t_{\text{water}} \) and substituting the result into Equation (1) gives
\[ d_{\text{water}} = \frac{v_{\text{water}}t_{\text{water}}}{2} = \frac{v_{\text{water}}(t - t_{\text{air}})}{2} \]

The travel time \( t_{\text{air}} \) in Equation (2) can be obtained by remembering that the distance traveled through air is \( 2d_{\text{air}} = v_{\text{air}}t_{\text{air}} \), where \( v_{\text{air}} \) is the speed of sound in air. Solving this expression for \( t_{\text{air}} \) gives \( t_{\text{air}} = 2d_{\text{air}} / v_{\text{air}} \). Substituting this result into Equation (2) reveals that
\[ d_{\text{water}} = \frac{v_{\text{water}}(t - t_{\text{air}})}{2} = \frac{v_{\text{water}}}{2} \left( t - \frac{2d_{\text{air}}}{v_{\text{air}}} \right) \]
\[ = \left( \frac{1482 \text{ m/s}}{2} \right) \left[ (0.110 \text{ s}) - \frac{2(10.0 \text{ m})}{(343 \text{ m/s})} \right] = 38 \text{ m} \]

Note that we have taken the speeds of sound in air and in fresh water at 20 °C from Table 16.1.
39. **REASONING** Under the assumptions given in the problem, both the bullet and the sound of the rifle discharge travel at constant speeds. Therefore, we can use \( v = \frac{d}{t} \) (Equation 2.1, where \( v \) is speed, \( d \) is distance, and \( t \) is elapsed time) to determine the distance that the bullet travels, as well as the time it takes the rifle report to reach the observer. The speed of the bullet is given, and we know that the speed of sound in air is 343 m/s, because the air temperature is 20 °C (See Table 16.1).

**SOLUTION** Applying Equation 2.1, we can express the distance \( d_b \) traveled by the bullet as

\[
d_b = v_b t
\]

where \( v_b = 840 \) m/s and \( t \) is the time needed for the report of the rifle to reach the observer. The observer is a distance \( d = 25 \) m from the marksman, so we can use Equation 2.1 with the speed of sound \( v = 343 \) m/s to determine \( t \):

\[
t = \frac{d}{v}
\]

Substituting Equation (2) into Equation (1), we find the distance traveled by the bullet before the observer hears the report:

\[
d_b = v_b t = v_b \left( \frac{d}{v} \right) = \frac{v_b d}{v} = \frac{(840 \text{ m/s})(25 \text{ m})}{343 \text{ m/s}} = 61 \text{ m}
\]

40. **REASONING** Generally, the speed of sound in a liquid like water is greater than in a gas. And, in fact, according to Table 16.1, the speed of sound in water is greater than the speed of sound in air. Since the speed of sound in water is greater than in air, an underwater ultrasonic pulse returns to the ruler in a shorter time than a pulse in air. The ruler has been designed for use in air, not in water, so this quicker return time fools the ruler into believing that the object is much closer than it actually is. Therefore, the reading on the ruler is less than the actual distance.

**SOLUTION** Let \( x \) be the actual distance from the ruler to the object. The time it takes for the ultrasonic pulse to reach the object and return to the ruler, a distance of \( 2x \), is equal to the distance divided by the speed of sound in water \( v_{\text{water}} \): \( t = \frac{2x}{v_{\text{water}}} \). The speed of sound in water is given by Equation 16.6 as \( v_{\text{water}} = \sqrt{\frac{B_{\text{ad}}}{\rho}} \), where \( B_{\text{ad}} \) is the adiabatic bulk modulus and \( \rho \) is the density of water. Thus, the time it takes for the pulse to return is

\[
t = \frac{2x}{v_{\text{water}}} = \frac{2x}{\sqrt{\frac{B_{\text{ad}}}{\rho}}} = \frac{2(25.0 \text{ m})}{\sqrt{\frac{2.37 \times 10^9 \text{ Pa}}{1025 \text{ kg/m}^3}}} = 3.29 \times 10^{-2} \text{ s}
\]

The ruler measures this value for the time and computes the distance to the object by using the speed of sound in air, 343 m/s. The distance \( x_{\text{ruler}} \) displayed by the ruler is equal to the speed of sound in air multiplied by the time \( \frac{1}{2} t \) it takes for the pulse to go from the ruler to the object:
\[ x_{\text{ruler}} = v_{\text{air}} \left( \frac{1}{2} t \right) = (343 \text{ m/s}) \left( \frac{1}{2} \right) \left( 3.29 \times 10^{-2} \text{ s} \right) = 5.64 \text{ m} \]

Thus, the ruler displays a distance of \( x_{\text{ruler}} = 5.64 \text{ m} \). As expected, the reading on the ruler’s display is less than the actual distance of 25.0 m.

41. **REASONING AND SOLUTION**
   
   a. In order to determine the order of arrival of the three waves, we need to know the speeds of each wave. The speeds for air, water and the metal are
   
   \[ v_{a} = 343 \text{ m/s}, \quad v_{w} = 1482 \text{ m/s}, \quad v_{m} = 5040 \text{ m/s} \]
   
   The order of arrival is **metal wave first, water wave second, air wave third**.

   b. Calculate the length of time each wave takes to travel 125 m.
   
   \[ t_{m} = \frac{125 \text{ m}}{5040 \text{ m/s}} = 0.025 \text{ s} \]
   \[ t_{w} = \frac{125 \text{ m}}{1482 \text{ m/s}} = 0.084 \text{ s} \]
   \[ t_{a} = \frac{125 \text{ m}}{343 \text{ m/s}} = 0.364 \text{ s} \]
   
   Therefore, the delay times are
   
   \[ \Delta t_{12} = t_{w} - t_{m} = 0.084 \text{ s} - 0.025 \text{ s} = 0.059 \text{ s} \]
   \[ \Delta t_{13} = t_{a} - t_{m} = 0.364 \text{ s} - 0.025 \text{ s} = 0.339 \text{ s} \]

42. **REASONING** Since the sound wave travels twice as far in neon as in krypton in the same time, the speed of sound in neon must be twice that in krypton:

   \[ v_{\text{neon}} = 2v_{\text{krypton}} \tag{1} \]

   Furthermore, the speed of sound in an ideal gas is given by
   \[ v = \sqrt{\frac{\gamma kT}{m}} \]
   according to Equation 16.5. In this expression, \( \gamma \) is the ratio of the specific heat capacities at constant pressure and constant volume and is the same for either gas (see Section 15.6), \( k \) is Boltzmann’s constant, \( T \) is the Kelvin temperature, and \( m \) is the mass of an atom. This expression for the speed can be used for both gases in Equation (1) and the result solved for the temperature of the neon.

   **SOLUTION** Using Equation 16.5 in Equation (1), we have

   \[ \sqrt{\frac{\gamma k T_{\text{neon}}}{m_{\text{neon}}}} = 2 \sqrt{\frac{\gamma k T_{\text{krypton}}}{m_{\text{krypton}}}} \]

   Squaring this result and solving for the temperature of the neon give
\[
\frac{\gamma k T_{\text{neon}}}{m_{\text{neon}}} = 4 \left(\frac{\gamma k T_{\text{krypton}}}{m_{\text{krypton}}}\right) \quad \text{or} \quad T_{\text{neon}} = 4 \left(\frac{m_{\text{neon}}}{m_{\text{krypton}}}\right) T_{\text{krypton}}
\]

We note here that the mass \( m \) of an atom is proportional to its atomic mass in atomic mass units (u). As a result, the temperature of the neon is

\[
T_{\text{neon}} = 4 \left(\frac{m_{\text{neon}}}{m_{\text{krypton}}}\right) T_{\text{krypton}} = 4 \left(\frac{20.2 \text{ u}}{83.8 \text{ u}}\right) (293 \text{ K}) = 283 \text{ K}
\]

43. **SSM REASONING** The sound will spread out uniformly in all directions. For the purposes of counting the echoes, we will consider only the sound that travels in a straight line parallel to the ground and reflects from the vertical walls of the cliff.

Let the distance between the hunter and the closer cliff be \( x_1 \) and the distance from the hunter to the further cliff be \( x_2 \).

The first echo arrives at the location of the hunter after traveling a total distance \( 2x_1 \) in a time \( t_1 \), so that, if \( v_s \) is the speed of sound, \( t_1 = \frac{2x_1}{v_s} \). Similarly, the second echo arrives after reflection from the far wall and in an amount of time \( t_2 \) after the firing of the gun. The quantity \( t_2 \) is related to the distance \( x_2 \) and the speed of sound \( v_s \) according to \( t_2 = \frac{2x_2}{v_s} \). The time difference between the first and second echo is, therefore

\[
\Delta t = t_2 - t_1 = \frac{2}{v_s} (x_2 - x_1)
\]

The third echo arrives in a time \( t_3 \) after the second echo. It arises from the sound of the second echo that is reflected from the closer cliff wall. Thus, \( t_3 = \frac{2x_1}{v_s} \), or, solving for \( x_1 \), we have

\[
x_1 = \frac{v_s t_3}{2}
\]

Combining Equations (1) and (2), we obtain

\[
\Delta t = t_2 - t_1 = \frac{2}{v_s} \left( x_2 - \frac{v_s t_3}{2} \right)
\]

Solving for \( x_2 \), we have

\[
x_2 = \frac{v_s}{2} (\Delta t + t_3)
\]

The distance between the cliffs can be found from \( d = x_1 + x_2 \), where \( x_1 \) and \( x_2 \) can be determined from Equations (2) and (3), respectively.
SOLUTION According to Equation (2), the distance $x_1$ is

$$x_1 = \frac{(343 \text{ m/s})(1.1 \text{ s})}{2} = 190 \text{ m}$$

According to Equation (3), the distance $x_2$ is

$$x_2 = \frac{(343 \text{ m/s})}{2} (1.6 \text{ s} + 1.1 \text{ s}) = 460 \text{ m}$$

Therefore, the distance between the cliffs is

$$d = x_1 + x_2 = 190 \text{ m} + 460 \text{ m} = 650 \text{ m}$$

44. REASONING AND SOLUTION The speed of sound in an ideal gas is given by text Equation 16.5:

$$v = \sqrt{\frac{\gamma kT}{m}}$$

where $\gamma = c_p/c_v$, $k$ is Boltzmann's constant, $T$ is the Kelvin temperature of the gas, and $m$ is the mass of a single gas molecule. If $m_{\text{TOTAL}}$ is the mass of the gas sample and $N$ is the total number of molecules in the sample, then the above equation can be written as

$$v = \sqrt{\frac{\gamma kT}{m_{\text{TOTAL}}/N}} = \sqrt{\frac{\gamma kT m_{\text{TOTAL}}}{N}}$$

(1)

For an ideal gas, $PV = NkT$, so that Equation (1) becomes,

$$v = \sqrt{\frac{\gamma PV}{m_{\text{TOTAL}}}} = \sqrt{\frac{(1.67)(3.5 \times 10^5 \text{ Pa})(2.5 \text{ m}^3)}{2.3 \text{ kg}}} = 8.0 \times 10^2 \text{ m/s}$$

45. REASONING Equation 16.7 relates the Young's modulus $Y$, the mass density $\rho$, and the speed of sound $v$ in a long slender solid bar. According to Equation 16.7, the Young's modulus is given by $Y = \rho v^2$. The data given in the problem can be used to compute values for both $\rho$ and $v$.

SOLUTION Using the values of the data given in the problem statement, we find that the speed of sound in the bar is

$$v = \frac{L}{t} = \frac{0.83 \text{ m}}{1.9 \times 10^{-4} \text{ s}} = 4.4 \times 10^3 \text{ m/s}$$

where $L$ is the length of the rod and $t$ is the time required for the wave to travel the length of the rod. The mass density of the bar is, from Equation 11.1, $\rho = m/V = m/(LA)$, where $m$ and $A$ are, respectively, the mass and the cross-sectional area of the rod. The density of the rod is, therefore,
\[ \rho = \frac{m}{LA} = \frac{2.1 \text{ kg}}{(0.83 \text{ m})(1.3 \times 10^{-4} \text{ m}^2)} = 1.9 \times 10^4 \text{ kg/m}^3 \]

Using these values, we find that Young’s modulus for the material of the rod is

\[ Y = \rho v^2 = \left(1.9 \times 10^4 \text{ kg/m}^3\right)\left(4.4 \times 10^3 \text{ m/s}\right)^2 = 3.7 \times 10^{11} \text{ N/m}^2 \]

Comparing this value to those given in Table 10.1, we conclude that the bar is most likely made of tungsten.

46. **REASONING** Two facts allow us to solve this problem. First, the sound reaches the microphones at different times, because the distances between the microphones and the source of sound are different. Since sound travels at a speed \( v = 343 \text{ m/s} \) and the sound arrives at microphone 2 later by an interval of \( \Delta t = 1.46 \times 10^{-3} \text{ s} \), it follows that

\[ L_2 - L_1 = v \Delta t \]  \hspace{1cm} (1)

Second, the microphones and the source of sound are located at the corners of a right triangle. Therefore, the Pythagorean theorem applies:

\[ L_2^2 = D^2 + L_1^2 \]  \hspace{1cm} (2)

Since \( v \), \( \Delta t \), and \( D \) are known, these two equations may be solved simultaneously for the distances \( L_1 \) and \( L_2 \).

**SOLUTION** Solving Equation (1) for \( L_2 \) gives

\[ L_2 = L_1 + v \Delta t \]

Substituting this result into Equation (2) gives

\[ (L_1 + v \Delta t)^2 = D^2 + L_1^2 \]

\[ L_1^2 + 2L_1v \Delta t + (v \Delta t)^2 = D^2 + L_1^2 \]

\[ 2L_1v \Delta t + (v \Delta t)^2 = D^2 \]

Solving for \( L_1 \), we find that

\[ L_1 = \frac{D^2 - (v \Delta t)^2}{2v \Delta t} = \frac{(1.50 \text{ m})^2 - (343 \text{ m/s})^2 \left(1.46 \times 10^{-3} \text{ s}\right)^2}{2(343 \text{ m/s})(1.46 \times 10^{-3} \text{ s})} = 2.00 \text{ m} \]

Solving Equation (2) for \( L_2 \) and substituting the result for \( L_1 \) reveal that

\[ L_2 = \sqrt{D^2 + L_1^2} = \sqrt{(1.50 \text{ m})^2 + (2.00 \text{ m})^2} = 2.50 \text{ m} \]
47. **REASONING** Let \( v_p \) represent the speed of the primary wave and \( v_s \) the speed of the secondary wave. The travel times for the primary and secondary waves are \( t_p \) and \( t_s \), respectively. If \( x \) is the distance from the earthquake to the seismograph, then \( t_p = \frac{x}{v_p} \) and \( t_s = \frac{x}{v_s} \). The difference in the arrival times is

\[
 t_s - t_p = \frac{x}{v_s} - \frac{x}{v_p} = x \left( \frac{1}{v_s} - \frac{1}{v_p} \right)
\]

We can use this equation to find the distance from the earthquake to the seismograph.

**SOLUTION** Solving the equation above for \( x \) gives

\[
x = \frac{t_s - t_p}{\frac{1}{v_s} - \frac{1}{v_p}} = \frac{78 \text{ s}}{\frac{1}{4.5 \times 10^3 \text{ m/s}} - \frac{1}{8.0 \times 10^3 \text{ m/s}}} = 8.0 \times 10^5 \text{ m}
\]

48. **REASONING** The speed \( v_{\text{truck}} \) of the truck is equal to the distance \( x \) it travels divided by the time \( t_{\text{truck}} \) it takes to travel that distance:

\[
v_{\text{truck}} = \frac{x}{t_{\text{truck}}}
\]

Since sound also moves at a constant speed, the distance \( x \) it travels in reaching the truck is the product of the speed \( v \) of sound and the time \( t_{\text{sound}} \) for it to travel that distance, or \( x = v \cdot t_{\text{sound}} \). The time for the sound to reach the truck is one-half the round-trip time \( t_{\text{RT}} \), so \( t_{\text{sound}} = \frac{1}{2} t_{\text{RT}} \). Thus, the distance traveled by the sound can be written as

\[
x = v \left( \frac{1}{2} t_{\text{RT}} \right)
\]

Substituting this expression for \( x \) into Equation (1) gives

\[
v_{\text{truck}} = \frac{x}{t_{\text{truck}}} = \frac{v \left( \frac{1}{2} t_{\text{RT}} \right)}{t_{\text{truck}}}
\]

Since the air is assumed to be an ideal gas, the speed of sound is related to the Kelvin temperature \( T \) and the average mass \( m \) of an air molecule by Equation 16.5:

\[
v = \sqrt{\frac{\gamma k T}{m}}
\]

where \( \gamma \) is the ratio of the specific heat capacity of air at constant pressure to that at constant volume, and \( k \) is Boltzmann’s constant. Substituting this expression for \( v \) into Equation (2) gives

\[
v_{\text{truck}} = \sqrt{\frac{\gamma k T}{m} \left( \frac{1}{2} t_{\text{RT}} \right)}
\]
**SOLUTION** The temperature of the air must be its Kelvin temperature, which is related to the Celsius temperature $T_c$ by $T = T_c + 273.15$ (Equation 12.1): $T = 56 \degree C + 273.15 = 329 K$. The average mass $m$ (in kg) of an air molecule is the same as that determined in Example 4, namely, $m = 4.80 \times 10^{-26}$ kg. Thus, the speed of the truck is

$$v_{\text{truck}} = \frac{\sqrt{\gamma kT}}{m} \left( \frac{1}{2} t_{\text{RT}} \right) = \sqrt{\frac{\frac{7}{5}(1.38 \times 10^{-23} \text{ J/K})(329 \text{ K})}{4.80 \times 10^{-26} \text{ kg}}} \left[ \frac{1}{2} (0.120 \text{ s}) \right]$$

$$= \frac{5.46 \text{ m/s}}{4.00 \text{ s}}$$

**49. REASONING AND SOLUTION** The speed of sound in an ideal gas is

$$v = \sqrt{\frac{\gamma kT}{m}}$$

so that the mass of a gas molecule is

$$m = \frac{\gamma k T}{v^2} = \frac{(1.67)(1.38 \times 10^{-23} \text{ J/K})(3.00 \times 10^2 \text{ K})}{(363 \text{ m/s})^2} = 5.25 \times 10^{-26} \text{ kg} = 5.25 \times 10^{-23} \text{ g}$$

We now need to determine what fraction of the gas is argon and what fraction is neon. First find the mass of each molecule.

$$m_{\text{ar}} = \frac{39.9 \text{ g/mol}}{6.022 \times 10^{23} \text{/mol}} = 6.63 \times 10^{-23} \text{ g}$$

$$m_{\text{ne}} = \frac{20.2 \text{ g/mol}}{6.022 \times 10^{23} \text{/mol}} = 3.35 \times 10^{-23} \text{ g}$$

Let $q$ be the fraction of gas that is in the form of argon and $p$ be the fraction that is neon. We know that $q + p = 1$. Also, $qm_{\text{ar}} + pm_{\text{ne}} = m$. Substituting $p = 1 - q$ in this equation gives

$$qm_{\text{ar}} + (1 - q)m_{\text{ne}} = m$$

and we can now solve for the fraction

$$q = \frac{m - m_{\text{ne}}}{m_{\text{ar}} - m_{\text{ne}}}$$

Suppressing the units and algebraically canceling the factor of $10^{-23}$, we find

$$q = \frac{5.25 - 3.35}{6.63 - 3.35} = 0.57 = 57\% \text{ argon}$$

and

$$p = 1 - q = 1 - 0.57 = 0.43 = 43\% \text{ neon}$$
50. **REASONING** Traveling at a speed $v$ for the round trip between the theodolite and the target the sound wave travels a distance $2x = vt$, where $x$ is the one-way distance to the target and $t$ is the round trip travel time. In solving this problem, we will apply this expression to both the correct and the incorrect distance measurement. The speed of sound in air (assumed to behave as an ideal gas) is $v = \sqrt[\gamma/kT/m}$ (Equation 16.5), where $\gamma$ is the ratio of specific heats, $k$ is Boltzmann’s constant, $m$ is the mass of a molecule of the air, and $T$ is the Kelvin temperature.

**SOLUTION** The percentage error in the incorrect value measured by the theodolite is

$$\% \text{ Error } = \left( \frac{x_{\text{incorrect}} - x_{\text{correct}}}{x_{\text{correct}}} \right) \times 100\% = \left( \frac{x_{\text{incorrect}}}{x_{\text{correct}}} - 1 \right) \times 100\% \quad \text{(1)}$$

Using the expression $2x = vt$, we write the one-way correct and incorrect distances measured by the theodolite as follows:

$$x_{\text{correct}} = \frac{1}{2} v_{291} t_{291} \quad \text{Correct distance measured at 291 K}$$

$$x_{\text{incorrect}} = \frac{1}{2} v_{298} t_{298} \quad \text{Incorrect distance measured at 298 K}$$

Using these expressions, we see that the ratio of the incorrect to the correct distance needed in Equation (1) is

$$\frac{x_{\text{incorrect}}}{x_{\text{correct}}} = \frac{t_{298}}{t_{291}} \quad \text{(2)}$$

If the theodolite were also calibrated correctly for the measurement at 298 K, then both measurements would yield the same one-way correct distance, and we would have

$$x_{\text{correct}} = \frac{1}{2} v_{291} t_{291} \quad \text{Correct distance measured at 291 K}$$

$$x_{\text{correct}} = \frac{1}{2} v_{298} t_{298} \quad \text{Correct distance measured at 298 K}$$

Since the two correct values of the one-distance must be the same, it follows that

$$\frac{1}{2} v_{291} t_{291} = \frac{1}{2} v_{298} t_{298} \quad \text{or} \quad \frac{t_{298}}{t_{291}} = \frac{v_{291}}{v_{298}} \quad \text{(3)}$$

Assuming that air behaves as an ideal gas, we know that the speed of sound is $v = \sqrt[\gamma/kT/m]$ (Equation 16.5), so that the result in Equation (3) becomes

$$\frac{t_{298}}{t_{291}} = \frac{v_{291}}{v_{298}} = \sqrt[\gamma/kT_{291}/m] = \sqrt[\gamma/kT_{298}/m] = \frac{T_{291}}{T_{298}} \quad \text{(4)}$$

Substituting Equation (4) into Equation (2) gives

$$\frac{x_{\text{incorrect}}}{x_{\text{correct}}} = \frac{t_{298}}{t_{291}} = \frac{T_{291}}{T_{298}} \quad \text{(5)}$$
Finally, substituting Equation (5) into Equation (1), we obtain

\[
\% \text{ Error} = \left( \frac{x_{\text{incorrect}}}{x_{\text{correct}}} - 1 \right) \times 100\% = \left( \frac{\sqrt{\frac{T}{291}} - 1}{\sqrt{\frac{T}{298}} - 1} \right) \times 100\% = \left( \frac{291 K}{298 K} - 1 \right) \times 100\% = -1.2\%
\]

The negative answer indicates that the incorrect value of the distance is smaller than the correct value.

51. **SSM REASONING** We must determine the time \( t \) for the warning to travel the vertical distance \( h = 10.0 \text{ m} \) from the prankster to the ears of the man when he is just under the window. The desired distance above the man's ears is the distance that the balloon would travel in this time and can be found with the aid of the equations of kinematics.

**SOLUTION** Since sound travels with constant speed \( v_s \), the distance \( h \) and the time \( t \) are related by \( h = v_s t \). Therefore, the time \( t \) required for the warning to reach the ground is

\[
t = \frac{h}{v_s} = \frac{10.0 \text{ m}}{343 \text{ m/s}} = 0.0292 \text{ s}
\]

We now proceed to find the distance that the balloon travels in this time. To this end, we must find the balloon's final speed \( v_y \), after falling from rest for 10.0 m. Since the balloon is dropped from rest, we use Equation 3.6b \((v_y^2 = v_{0y}^2 + 2a_y y)\) with \( v_{0y} = 0 \text{ m/s} \):

\[
v_y = \sqrt{v_{0y}^2 + 2a_y y} = \sqrt{(0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 14.0 \text{ m/s}
\]

Using this result, we can find the balloon's speed 0.0292 seconds before it hits the man by solving Equation 3.3b \((v_y = v_{0y} + a_y t)\) for \( v_{0y} \):

\[
v_{0y} = v_y - a_y t = [14.0 \text{ m/s} - (9.80 \text{ m/s}^2)(0.0292 \text{ s})] = 13.7 \text{ m/s}
\]

Finally, we can find the desired distance \( y \) above the man's head from Equation 3.5b:

\[
y = v_{0y} t + \frac{1}{2} a_y t^2 = (13.7 \text{ m/s})(0.0292 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(0.0292 \text{ s})^2 = 0.404 \text{ m}
\]

52. **REASONING AND SOLUTION** We have \( I = P/A \). Therefore,

\[
P = IA = (3.2 \times 10^{-6} \text{ W/m}^2)(2.1 \times 10^{-3} \text{ m}^2) = 6.7 \times 10^{-9} \text{ W}
\]
53. **REASONING AND SOLUTION** Since the sound radiates uniformly in all directions, at a distance $r$ from the source, the energy of the sound wave is distributed over the area of a sphere of radius $r$. Therefore, according to $I = \frac{P}{4\pi r^2}$ (Equation 16.9) with $r = 3.8 \text{ m}$, the power radiated from the source is

$$P = 4\pi I r^2 = 4\pi (3.6 \times 10^{-2} \text{ W/m}^2)(3.8 \text{ m})^2 = 6.5 \text{ W}$$

54. **REASONING**

a. The source emits sound uniformly in all directions, so the sound intensity $I$ at any distance $r$ is given by Equation 16.9 as $I = P / (4\pi r^2)$, where $P$ is the sound power emitted by the source. Since patches 1 and 2 are at the same distance from the source of sound, the sound intensity at each location is the same, so $I_1 = I_2$. Patch 3 is farther from the sound source, so the intensity $I_3$ is smaller for points on that patch. Therefore, patches 1 and 2 have equal intensities, each of which is greater than the intensity at patch 3.

b. According to Equation 16.8, the sound intensity $I$ is defined as the sound power $P$ that passes perpendicularly through a surface divided by the area $A$ of that surface, $I = P/A$. The area of the surface is, then, $A = P/I$. Since the same sound power passes through patches 1 and 2, and the intensity at each one is the same, their areas must also be the same, $A_1 = A_2$. The same sound power passes through patch 3, but the intensity at that surface is smaller than that at patches 1 and 2. Thus, the area $A_3$ of patch 3 is larger than that of surface 1 or 2. In summary, $A_3$ is the largest area, followed by $A_1$ and $A_2$, which are equal.

**SOLUTION**

a. The sound intensity at the inner spherical surface is given by Equation 16.9 as

$$I_A = \frac{P}{4\pi r_A^2} = \frac{2.3 \text{ W}}{4\pi (0.60 \text{ m})^2} = 0.51 \text{ W/m}^2$$

This intensity is the same at all points on the inner surface, since all points are equidistant from the sound source. Therefore, the sound intensity at patches 1 and 2 are equal; $I_1 = I_2 = 0.51 \text{ W/m}^2$.

The sound intensity at the outer spherical surface is

$$I_B = \frac{P}{4\pi r_B^2} = \frac{2.3 \text{ W}}{4\pi (0.80 \text{ m})^2} = 0.29 \text{ W/m}^2$$

This intensity is the same at all points on outer surface. Therefore, the sound intensity at patch 3 is $I_3 = 0.29 \text{ W/m}^2$. 
b. The area of a patch is given by Equation 16.8 as the sound power passing perpendicularly through that area divided by the sound intensity, \( A = P/I \). The areas of the three patches are:

\[
\begin{align*}
\text{Patch 1} & : A_1 = \frac{P}{I_1} = \frac{1.8 \times 10^{-3} \text{ W}}{0.51 \text{ W/m}^2} = 3.5 \times 10^{-3} \text{ m}^2 \\
\text{Patch 2} & : A_2 = \frac{P}{I_2} = \frac{1.8 \times 10^{-3} \text{ W}}{0.51 \text{ W/m}^2} = 3.5 \times 10^{-3} \text{ m}^2 \\
\text{Patch 3} & : A_3 = \frac{P}{I_3} = \frac{1.8 \times 10^{-3} \text{ W}}{0.29 \text{ W/m}^2} = 6.2 \times 10^{-3} \text{ m}^2
\end{align*}
\]

These answers are consistent with our discussion in the REASONING.

55. **REASONING AND SOLUTION** The intensity of the "direct" sound is given by text Equation 16.9,

\[
I_{\text{DIRECT}} = \frac{P}{4\pi r^2}
\]

The total intensity at the point in question is

\[
I_{\text{TOTAL}} = I_{\text{DIRECT}} + I_{\text{REFLECTED}}
\]

\[
= \left[ \frac{1.1 \times 10^{-3} \text{ W}}{4\pi (3.0 \text{ m})^2} \right] + 4.4 \times 10^{-6} \text{ W/m}^2 = 1.4 \times 10^{-5} \text{ W/m}^2
\]

56. **REASONING** Since the sound is emitted uniformly in all directions, the intensity \( I \) at a distance \( r \) from the source is \( I = \frac{P}{4\pi r^2} \) (Equation 16.9), where \( P \) is the sound power emitted by the source. We will apply this expression to both distances.

**SOLUTION** Applying Equation 16.9 to both distances, we have

\[
I_{22} = \frac{P}{4\pi r_{22}^2} \quad \text{and} \quad I_{78} = \frac{P}{4\pi r_{78}^2}
\]

\( I_{22} \) Intensity at a position 22 m from the source

Dividing the expression on the right by the expression on the left gives
\[
I_{78} = I_{22} \left( \frac{r_{22}}{r_{78}} \right)^2 = \left( 3.0 \times 10^{-4} \text{ W/m}^2 \right) \left( \frac{22 \text{ m}}{78 \text{ m}} \right)^2 = 2.4 \times 10^{-5} \text{ W/m}^2
\]

57. **REASONING AND SOLUTION** According to Equation 16.8, the power radiated by the speaker is \( P = IA = I \pi r^2 \), where \( r \) is the radius of the circular opening. Thus, the radiated power is

\[
P = (17.5 \text{ W/m}^2)(\pi)(0.0950 \text{ m})^2 = 0.496 \text{ W}
\]

As a percentage of the electrical power, this is

\[
\frac{0.496 \text{ W}}{25.0 \text{ W}} \times 100 \% = 1.98 \%
\]

58. **REASONING** The sound intensity \( I \) a distance \( r \) from a source broadcasting sound uniformly in all directions is given by \( I = \frac{P}{4\pi r^2} \) (Equation 16.9), where \( P \) is the sound power of the source. The total sound intensity \( I_{\text{tot}} \) that the man hears at either position is the sum of the intensities \( I_1 \) and \( I_2 \) due to the two speakers.

**SOLUTION**

a. When the man is halfway between the speakers, his distance \( r \) from either speaker is half the 30.0-m distance between the speakers: \( r = 15.0 \text{ m} \). The total sound intensity at that position is, therefore, twice the intensity \( I = \frac{P}{4\pi r^2} \) (Equation 16.9) of either speaker alone:

\[
I_{\text{tot}} = 2 \left( \frac{P}{4\pi r^2} \right) = \frac{P}{2\pi r^2} = \frac{0.500 \text{ W}}{2\pi (15.0 \text{ m})^2} = 3.54 \times 10^{-4} \text{ W/m}^2
\]

b. In part (a), the man is \( r = 15.0 \text{ m} \) from either speaker. After he has walked 4.0 m towards speaker 1, his distance from that speaker is \( r_1 = 15.0 \text{ m} - 4.0 \text{ m} = 11.0 \text{ m} \), and his distance from speaker 2 is \( r_2 = 15.0 \text{ m} + 4.0 \text{ m} = 19.0 \text{ m} \). The total sound intensity at his final position is, therefore,
\[
I_{\text{tot}} = I_1 + I_2 = \frac{P}{4\pi r_1^2} + \frac{P}{4\pi r_2^2} = \frac{P}{4\pi} \left( \frac{1}{r_1^2} + \frac{1}{r_2^2} \right) = 0.500 \text{ W} \left[ \frac{1}{(11.0 \text{ m})^2} + \frac{1}{(19.0 \text{ m})^2} \right] \\
= 4.39 \times 10^{-4} \text{ W/m}^2
\]

59. **SSM REASONING** Intensity \( I \) is power \( P \) divided by the area \( A \), or \( I = \frac{P}{A} \), according to Equation 16.8. The area is given directly, but the power is not. Therefore, we need to recast this expression in terms of the data given in the problem. Power is the change in energy per unit time, according to Equation 6.10b. In this case the energy is the heat \( Q \) that causes the temperature of the lasagna to increase. Thus, the power is \( P = \frac{Q}{t} \), where \( t \) denotes the time.

As a result, Equation 16.8 for the intensity becomes

\[
I = \frac{P}{A} = \frac{Q}{tA} \quad (1)
\]

According to Equation 12.4, the heat that must be supplied to increase the temperature of a substance of mass \( m \) by an amount \( \Delta T \) is \( Q = cm\Delta T \), where \( c \) is the specific heat capacity. Substituting this expression into Equation (1) gives

\[
I = \frac{Q}{tA} = \frac{cm\Delta T}{tA} \quad (2)
\]

**SOLUTION** Equation (2) reveals that the intensity of the microwaves is

\[
I = \frac{cm\Delta T}{tA} = \left[ \frac{3200 \text{ J/}(\text{kg} \cdot \text{C}^\circ)}{1200 \text{ J/}(\text{kg} \cdot \text{C}^\circ)} \right] \left( 0.35 \text{ kg} \right) \left( 72 \text{ C}^\circ \right) \left( 480 \text{ s} \right) \left( 2.2 \times 10^{-2} \text{ m}^2 \right) = 7.6 \times 10^3 \text{ W/m}^2
\]

60. **REASONING** Because source 1 (the source at the origin) is more powerful than source 2, both locations of equal sound intensity will be closer to source 2 than to source 1. One point, \( A \), is between the sources, and the other, \( B \), is on the opposite side of source 2 (see the drawing). Let \( x \) be the distance from the source 1 to point \( A \), and \( d = 123 \text{ m} \) be the distance between the sources. The intensities \( I_1 \) and \( I_2 \) of the sound from the sources can be found from

\[
I = \frac{P}{4\pi r^2} \quad \text{(Equation 16.9)}, \text{ where } P \text{ is the power output of a source and } r \text{ is the distance between the source and point } A. \text{ The sound intensities are equal at point } A, \text{ so we have that}
\]
Additionally, we know that the power $P_1$ emitted by source 1 is four times greater than the power $P_2$ emitted by source 2:

$$P_1 = 4P_2 \quad (2)$$

**SOLUTION** In Equation (1), $r_1 = x$, and $r_2 = d - x$ when $A$ is taken as the point of equal sound intensity. Alternatively, we could say that $x$ is the position of point $B$ relative to the origin (source 1), so that the distance $r_2$ between source 2 and point $B$ would be $r_2 = x - d$. It does not make a difference which we choose, since $r_2$ is squared in Equation (1), and

$$r_2^2 = (d - x)^2 = (x - d)^2.$$  

In fact, because Equation (1) is quadratic, either choice will yield two values of $x$, one for position $A$ and one for position $B$. Substituting Equation (2), $r_1 = x$, and $r_2 = d - x$ into Equation (1) yields

$$4 \frac{P_2'}{\pi x^2} = \frac{P_2'}{\pi (d - x)^2} \quad \text{or} \quad 4 \frac{1}{x^2} = \frac{1}{(d - x)^2} \quad (2)$$

Cross-multiplying Equation (2) and taking the square root of both sides, we obtain

$$x^2 = 4(d - x)^2 \quad \text{or} \quad x = \pm 2(d - x) \quad (3)$$

Solving Equation (3) separately for the positive root and the negative root gives

**Positive root**

$$x = +2(d - x) = 2d - 2x$$

$$3x = 2d$$

$$x = 2d/3 = 2(123 \text{ m})/3 = 82.0 \text{ m}$$

**Negative root**

$$x = -2(d - x) = -2d + 2x$$

$$-x = -2d$$

$$x = 2d = 2(123 \text{ m}) = 246 \text{ m}$$

61. **REASONING** According to Equation 16.8, the intensity $I$ of a sound wave is equal to the sound power $P$ divided by the area $A$ through which the power passes; $I = P/A$. The area is that of a circle, so $A = \pi r^2$, where $r$ is the radius. The power, on the other hand, is the energy $E$ per unit time, $P = E/t$, according to Equation 6.10b. Thus, we have

$$I = \frac{P}{A} = \frac{E}{\pi r^2}$$

All the variables in this equation are known except for the time.

**SOLUTION** Solving the expression above for the time yields

$$t = \frac{E}{I \pi r^2} = \frac{4800 \text{ J}}{(5.9 \times 10^3 \text{ W/m}^2) \pi \left(1.8 \times 10^{-2} \text{ m}\right)^2} = 8.0 \times 10^2 \text{ s}$$
62. **REASONING** Since the sound is emitted from the rocket uniformly in all directions, the energy carried by the sound wave spreads out uniformly over concentric spheres of increasing radii \( r_1, r_2, r_3, \ldots \) as the wave propagates. Let \( r_1 \) represent the radius when the measured intensity at the ground is \( I \) (position 1 for the rocket) and \( r_2 \) represent the radius when the measured intensity at the ground is \( \frac{1}{3} I \) (position 2 of the rocket). The time required for the energy to spread out over the sphere of radius \( r_1 \) is \( t_1 = \frac{r_1}{v_s} \), where \( v_s \) is the speed of sound. Similarly, the time required for the energy to spread out over a sphere of radius \( r_2 \) is \( t_2 = \frac{r_2}{v_s} \). As the sound wave emitted at position 1 spreads out uniformly, the rocket continues to accelerate upward to position 2 with acceleration \( a_y \) for a time \( t_{12} \). Therefore, the time that elapses between the two intensity measurements is

\[
t = t_2 - t_1 + t_{12}
\]

From Equation 3.3b, the time \( t_{12} \) is

\[
t_{12} = \frac{(v_2 - v_1)}{a_y}
\]

where \( v_1 \) and \( v_2 \) are the speeds of the rocket at positions 1 and 2, respectively. Combining Equations (1) and (2), we obtain

\[
t = t_2 - t_1 + \frac{(v_2 - v_1)}{a_y}
\]

The respective values of \( v_1 \) and \( v_2 \) can be found from Equation 3.6b ( \( v_y^2 = v_{0y}^2 + 2a_y y \) ) with \( v_{0y} = 0 \) m/s, and \( y = r \). Once values for \( v_1 \) and \( v_2 \) are known, Equation (3) can be solved directly.

**SOLUTION** From the data given in the problem statement, \( r_1 = 562 \) m. We can find \( r_2 \) using the following reasoning: from the definition of intensity, \( I_1 = \frac{P}{4\pi r_1^2} \) and \( I_2 = \frac{P}{4\pi r_2^2} \). Since \( I_1 = 3I_2 \), we have

\[
\frac{P}{4\pi r_1^2} = 3\frac{P}{4\pi r_2^2} \quad \text{or} \quad r_2 = r_1 \sqrt{3} = 973 \text{ m}
\]

The times \( t_1 \) and \( t_2 \) are

\[
t_1 = \frac{r_1}{v_s} = \frac{562 \text{ m}}{343 \text{ m/s}} = 1.64 \text{ s}
\]

and

\[
t_2 = \frac{r_2}{v_s} = \frac{973 \text{ m}}{343 \text{ m/s}} = 2.84 \text{ s}
\]
Then, taking up as the positive direction, we have from Equation 3.6b,

\[ v_1 = \sqrt{2a_y r_1} = \sqrt{2(58.0 \text{ m/s}^2)(562 \text{ m})} = 255 \text{ m/s} \]

and

\[ v_2 = \sqrt{2a_y r_2} = \sqrt{2(58.0 \text{ m/s}^2)(973 \text{ m})} = 336 \text{ m/s} \]

Substituting these values into Equation (3), we have

\[ t = 2.84 \text{ s} - 1.64 \text{ s} + \frac{(336 \text{ m/s} - 255 \text{ m/s})}{58.0 \text{ m/s}^2} = [2.6 \text{ s}] \]

---

63. **REASONING**

a. The intensity \( I_1 \) of the sound from the motor when she is a distance \( d \) away is given by

\[ I_1 = \frac{P}{4\pi d^2} \]  
(Equation 16.9), where \( P \) is the total sound power emitted by the motor. As she moves away from the motor, its total sound power \( P \) does not change. We will use this fact, together with Equation 16.9, to determine the sound intensity \( I_2 \) when she is a distance \( 2d \) from the motor.

b. Once the sound intensity \( I_2 \) at the woman’s final position has been determined, the sound intensity level \( \beta_2 \) at that position will be found from

\[ \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \]  
(Equation 16.10),

where \( I_0 \) is the reference sound intensity, in this case the threshold of hearing.

**SOLUTION**

a. Applying Equation 16.9 to the woman’s second position, a distance \( 2d \) from the motor, we find that

\[ I_2 = \frac{P}{4\pi (2d)^2} = \frac{P}{16\pi d^2} \]  
(1)

Solving \( I_1 = \frac{P}{4\pi d^2} \) (Equation 16.9) for the total sound power \( P \) of the motor, we find that

\[ P = 4\pi d^2 I_1 \]  
(2)

Substituting Equation (2) into Equation (1) yields

\[ I_2 = \frac{P}{16\pi d^2} = \frac{4\pi d^2 I_1}{16\pi d^2} = \frac{I_1}{4} = \frac{3.2 \times 10^{-3} \text{ W/m}^2}{4} = 8.0 \times 10^{-4} \text{ W/m}^2 \]

b. From Equation 16.10, the sound intensity level twice as far from the motor is
\[ \beta_2 = (10 \text{ dB}) \log \left( \frac{I_2}{I_0} \right) = (10 \text{ dB}) \log \left( \frac{8.0 \times 10^{-4} \text{ W/m}^2}{1.0 \times 10^{-12} \text{ W/m}^2} \right) = 89 \text{ dB} \]

where we have used \( I_0 = 1.0 \times 10^{-12} \text{ W/m}^2 \) for the threshold of hearing.

64. **REASONING** The sound intensity level \( \beta \) in decibels (dB) is related to the sound intensity \( I \) according to \( \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \) (Equation 16.10), where \( I_0 \) is the intensity of the reference level. We will apply this expression to each of the given intensity levels.

**SOLUTION** The sound intensity level is changed from \( \beta_1 = 23 \text{ dB} \) to \( \beta_2 = 61 \text{ dB} \). Therefore, the amount of the change is

\[ \beta_2 - \beta_1 = (10 \text{ dB}) \log \left( \frac{I_2}{I_0} \right) - (10 \text{ dB}) \log \left( \frac{I_1}{I_0} \right) \]

It is a property of logarithms (see Equation D-12 in Appendix D) that \( \log A - \log B = \log \left( \frac{A}{B} \right) \). Therefore, our expression for the change in sound intensity level becomes

\[ \beta_2 - \beta_1 = (10 \text{ dB}) \log \left( \frac{I_2}{I_0} \right) - (10 \text{ dB}) \log \left( \frac{I_1}{I_0} \right) = (10 \text{ dB}) \log \left( \frac{I_2}{I_0} \frac{I_0}{I_1} \right) = (10 \text{ dB}) \log \left( \frac{I_2}{I_1} \right) \]

\( (61 \text{ dB}) - (23 \text{ dB}) = 38 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I_2}{I_1} \right) \) or \( \log \left( \frac{I_2}{I_1} \right) = 3.8 \)

Thus, according to Equation D-8 in Appendix D, we find that the desired ratio is

\[ \frac{I_2}{I_1} = 10^{3.8} = 6300 \]

65. **SSM REASONING** This is a situation in which the intensities \( I_{\text{man}} \) and \( I_{\text{woman}} \) (in watts per square meter) detected by the man and the woman are compared using the intensity level \( \beta \), expressed in decibels. This comparison is based on Equation 16.10, which we rewrite as follows:

\[ \beta = (10 \text{ dB}) \log \left( \frac{I_{\text{man}}}{I_{\text{woman}}} \right) \]

**SOLUTION** Using Equation 16.10, we have
\[ \beta = 7.8 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I_{\text{man}}}{I_{\text{woman}}} \right) \quad \text{or} \quad \log \left( \frac{I_{\text{man}}}{I_{\text{woman}}} \right) = \frac{7.8 \text{ dB}}{10 \text{ dB}} = 0.78 \]

Solving for the intensity ratio gives

\[ \frac{I_{\text{man}}}{I_{\text{woman}}} = 10^{0.78} = 6.0 \]

66. **REASONING** A threshold of hearing of –8.00 dB means that person 1 can hear a sound whose intensity is less than \( I_0 = 1.00 \times 10^{-12} \text{ W/m}^2 \), which is the intensity of the reference level. Person 2, with a threshold of hearing of +12.0 dB, can only hear sounds that have intensities greater than \( I_0 = 1.00 \times 10^{-12} \text{ W/m}^2 \). Thus, person 1 has the better hearing, because he can hear sounds with intensities less than \( 1.00 \times 10^{-12} \text{ W/m}^2 \). We expect, then that the ratio \( I_1/I_2 \) is less than one.

**SOLUTION** The relation between the sound intensity level \( \beta \) and the sound intensity \( I \) is given by Equation 16.10:

\[ \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \quad \text{or} \quad I = I_0 \frac{\beta}{10 \text{ dB}} \]

The threshold-of-hearing intensities for the two people are

\[ I_1 = I_0 \frac{\beta_1}{10 \text{ dB}} \quad \text{and} \quad I_2 = I_0 \frac{\beta_2}{10 \text{ dB}} \]

Taking the ratio \( I_1/I_2 \) gives

\[ \frac{I_1}{I_2} = \frac{I_0 \frac{\beta_1}{10 \text{ dB}}}{I_0 \frac{\beta_2}{10 \text{ dB}}} = \frac{10^{-8.00 \text{ dB}}}{10^{+12.0 \text{ dB}}} = \boxed{1.00 \times 10^{-2}} \]

This answer is less than one, as expected.

67. **REASONING** According to Equation 16.10, the sound intensity level \( \beta \) in decibels (dB) is related to the sound intensity \( I \) according to \( \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \), where the quantity \( I_0 \) is the reference intensity. Since the sound is emitted uniformly in all directions, the intensity, or power per unit area, is given by \( I = P/(4\pi r^2) \). Thus, the sound intensity at position 1 can be written as \( I_1 = P/(4\pi r_1^2) \), while the sound intensity at position 2 can be written as \( I_2 = P/(4\pi r_2^2) \). We can obtain the sound intensity levels from Equation 16.10 for these two positions by using these expressions for the intensities.
**SOLUTION** Using Equation 16.10 and the expressions for the intensities at the two positions, we can write the difference in the sound intensity levels \( \beta_{21} \) between the two positions as follows:

\[
\beta_{21} = \beta_2 - \beta_1 = (10 \text{ dB}) \log \left( \frac{I_2}{I_0} \right) - (10 \text{ dB}) \log \left( \frac{I_1}{I_0} \right)
\]

\[
= (10 \text{ dB}) \log \left( \frac{I_2 / I_0}{I_1 / I_0} \right) = (10 \text{ dB}) \log \left( \frac{I_2}{I_1} \right)
\]

\[
\beta_{21} = (10 \text{ dB}) \log \left[ \frac{P / (4\pi r_2^2)}{P / (4\pi r_1^2)} \right] = (10 \text{ dB}) \log \left( \frac{r_1^2}{r_2^2} \right) = (10 \text{ dB}) \log \left( \frac{r_1}{r_2} \right)^2
\]

\[
= (20 \text{ dB}) \log \left( \frac{r_1}{r_2} \right) = (20 \text{ dB}) \log \left( \frac{r_1}{2r_1} \right) = (20 \text{ dB}) \log \left( \frac{1}{2} \right) = -6.0 \text{ dB}
\]

The negative sign indicates that the sound intensity level decreases.

---

68. **REASONING** Since power \( P \) is energy \( E \) per unit of time \( t \) (see Equation 6.10b), the energy passing through the window is \( E = Pt \). The sound power is \( P = IA \) according to Equation 16.8, where \( I \) is the sound intensity and \( A \) is the area through which the sound passes perpendicularly. Substituting this expression for the power into the expression for the energy shows that the desired energy can be obtained from \( E = IA t \). The sound intensity in watts/m\(^2\) is related to the sound intensity level in decibels (dB) according to

\[
\beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \quad \text{(Equation 16.10)}, \quad \text{where} \ I_0 = 1.00 \times 10^{-12} \text{ W/m}^2 \text{ is the intensity of the threshold of hearing.}
\]

**SOLUTION** As explained in the **REASONING**, the desired energy is

\[
E = IA t \quad \text{(1)}
\]

The intensity \( I \) can be obtained from Equation 16.10 for the sound intensity level in the following way, using Equation D-8 for logarithms in Appendix D:

\[
\beta = 95 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \quad \text{or} \quad \log \left( \frac{I}{I_0} \right) = \frac{95 \text{ dB}}{10 \text{ dB}} = 9.5 \quad \text{or} \quad \frac{I}{I_0} = 10^{9.5} \quad \text{(2)}
\]

Substituting \( I = I_0 \left( 10^{9.5} \right) \) from Equation (2) into Equation (1) reveals that

\[
E = IA t = I_0 \left( 10^{9.5} \right) At = \left( 1.00 \times 10^{-12} \right) \left( 10^{9.5} \right) \left( 1.1 \text{ m} \right) \left( 0.75 \text{ m} \right) \left( 3600 \text{ s} \right) = 9.4 \text{ J}
\]
69. **REASONING** Knowing that the threshold of hearing corresponds to an intensity of $I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$, we can solve Equation 16.10 directly for the desired intensity.

**SOLUTION** Using Equation 16.10, we find

\[ \beta = 115 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \quad \text{or} \quad \frac{I}{I_0} = 10^{(115 \text{ dB})/(10 \text{ dB})} = 10^{11.5} \]

Solving for $I$ gives

\[ I = I_0 10^{11.5} = (1.00 \times 10^{-12} \text{ W/m}^2) 10^{11.5} = 0.316 \text{ W/m}^2 \]

---

70. **REASONING** The energy incident on the eardrum is equal to the sound power $P$ (which is assumed to be constant) times the time interval $t$ (see Equation 6.10b):

\[ \text{Energy} = Pt \quad (1) \]

According to Equation 16.8, the sound power is equal to the intensity $I$ of the wave times the area $A$ of the eardrum through which the power passes, or $P = IA$. Substituting this relation into Equation (1) gives

\[ \text{Energy} = Pt = (IA)t \quad (2) \]

We can obtain an expression for the sound intensity by employing the definition of the sound intensity level $\beta$ (Equation 16.10)

\[ \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \]

where $I_0$ is the threshold of hearing. Solving this expression for $I$ gives

\[ I = I_0 10^{\frac{\beta}{10 \text{ dB}}} \]

Substituting this relation for $I$ into Equation (2), we have that

\[ \text{Energy} = (IA)t = \left( I_0 10^{\frac{\beta}{10 \text{ dB}}} \right) At \]

**SOLUTION** Noting that $I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$ and that $t = 9.0 \text{ h} = 3.24 \times 10^4 \text{ s}$, the energy incident on the eardrum is

\[
\text{Energy} = \left( I_0 10^{\frac{\beta}{10 \text{ dB}}} \right) At
= \left( (1.00 \times 10^{-12} \text{ W/m}^2) 10^{90.0 \text{ dB}} \right) (2.0 \times 10^{-4} \text{ m}^2) (3.24 \times 10^4 \text{ s}) = 6.5 \times 10^{-3} \text{ J}
\]
71. **REASONING** If $I_1$ is the sound intensity produced by a single person, then $NI_1$ is the sound intensity generated by $N$ people. The sound intensity level generated by $N$ people is given by Equation 16.10 as

$$\beta_N = (10 \text{ dB}) \log \left( \frac{NI_1}{I_0} \right)$$

where $I_0$ is the threshold of hearing. Solving this equation for $N$ yields

$$N = \left( \frac{I_0}{I_1} \right) \frac{\beta_N}{10^{10 \text{ dB}}} \quad (1)$$

We also know that the sound intensity level for one person is

$$\beta_1 = (10 \text{ dB}) \log \left( \frac{I_1}{I_0} \right) \quad \text{or} \quad I_1 = I_0 10^{\frac{\beta_1}{10 \text{ dB}}} \quad (2)$$

Equations (1) and (2) are all that we need in order to find the number of people at the football game.

**SOLUTION** Substituting the expression for $I_1$ from Equation (2) into Equation (1) gives the desired result.

$$N = \frac{I_0}{I_1} \frac{\beta_N}{10^{10 \text{ dB}}} = \frac{10^{10 \text{ dB}}}{10^{60.0 \text{ dB}}} = \frac{10^{109 \text{ dB}}}{10^{10 \text{ dB}}} = \boxed{79400}$$

72. **REASONING** We must first find the intensities that correspond to the given sound intensity levels (in decibels). The total intensity is the sum of the two intensities. Once the total intensity is known, Equation 16.10 can be used to find the total sound intensity level in decibels.

**SOLUTION** Since, according to Equation 16.10, $\beta = (10 \text{ dB}) \log (I/I_0)$, where $I_0$ is the reference intensity corresponding to the threshold of hearing ($I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$), it follows that $I = I_0 10^{\beta/(10 \text{ dB})}$. Therefore, if $\beta_1 = 75.0 \text{ dB}$ and $\beta_2 = 72.0 \text{ dB}$ at the point in question, the corresponding intensities are
\[ I_1 = I_0 \cdot 10^{\beta_1/(10\,\text{dB})} = (1.00 \times 10^{-12} \, \text{W/m}^2) \cdot 10^{(75.0 \,\text{dB})/(10\,\text{dB})} = 3.16 \times 10^{-5} \, \text{W/m}^2 \]

\[ I_2 = I_0 \cdot 10^{\beta_2/(10\,\text{dB})} = (1.00 \times 10^{-12} \, \text{W/m}^2) \cdot 10^{(72.0 \,\text{dB})/(10\,\text{dB})} = 1.58 \times 10^{-5} \, \text{W/m}^2 \]

Therefore, the total intensity \( I_{\text{total}} \) at the point in question is

\[ I_{\text{total}} = I_1 + I_2 = (3.16 \times 10^{-5} \, \text{W/m}^2) + (1.58 \times 10^{-5} \, \text{W/m}^2) = 4.74 \times 10^{-5} \, \text{W/m}^2 \]

and the corresponding intensity level \( \beta_{\text{total}} \) is

\[ \beta_{\text{total}} = 10 \,\text{dB} \log \left( \frac{I_{\text{total}}}{I_0} \right) = 10 \,\text{dB} \log \left( \frac{4.74 \times 10^{-5} \, \text{W/m}^2}{1.00 \times 10^{-12} \, \text{W/m}^2} \right) = 76.8 \,\text{dB} \]

73. **REASONING** The sound intensity level heard by the gardener increases by 10.0 dB because distance between him and the radio decreases. The intensity of the sound is greater when the radio is closer than when it is further away. Since the unit is emitting sound uniformly (neglecting any reflections), the intensity is inversely proportional to the square of the distance from the radio, according to Equation 16.9. Combining this equation with Equation 16.10, which relates the intensity level in decibels to the intensity \( I \), we can use the 10.0-dB change in the intensity level to find the final vertical position of the radio. Once that position is known, we can then use kinematics to determine the fall time.

**SOLUTION** Let the sound intensity levels at the initial and final positions of the radio be \( \beta_i \) and \( \beta_f \), respectively. Using Equation 16.10 for each, we have

\[ \beta_f - \beta_i = (10 \,\text{dB}) \log \left( \frac{I_f}{I_0} \right) - (10 \,\text{dB}) \log \left( \frac{I_i}{I_0} \right) = (10 \,\text{dB}) \log \left( \frac{I_f}{I_i} \right) \] \hspace{1cm} (1)

Since the radiation is uniform, Equation 16.9 can be used to substitute for the intensities \( I_i \) and \( I_f \), so that Equation (1) becomes

\[ \beta_f - \beta_i = (10 \,\text{dB}) \log \left( \frac{I_f}{I_i} \right) \left[ \frac{P/4\pi h_f^2}{P/4\pi h_i^2} \right] = (10 \,\text{dB}) \log \left( \frac{h_i}{h_f} \right)^2 = (20 \,\text{dB}) \log \left( \frac{h_i}{h_f} \right) \] \hspace{1cm} (2)

Since \( \beta_f - \beta_i = 10.0 \,\text{dB} \), Equation (2) gives

\[ \beta_f - \beta_i = 10.0 \,\text{dB} = (20 \,\text{dB}) \log \left( \frac{h_i}{h_f} \right) \quad \text{or} \quad \frac{h_i}{h_f} = 10^{(10.0 \,\text{dB})/(20 \,\text{dB})} = 10^{0.500} = 3.16 \]

We can now determine the final position of the radio as follows:
It follows, then, that the radio falls through a distance of 5.1 m – 1.61 m = 3.49 m. Taking upward as the positive direction and noting that the radio falls from rest ($v_0 = 0$ m/s), we can solve Equation 2.8 \( y = v_0 t + \frac{1}{2}at^2 \) from the equations of kinematics for the fall time $t$:

\[
t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-3.49 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.84 \text{ s}
\]

74. **REASONING AND SOLUTION** The intensity level at each point is given by

\[
I = \frac{P}{4\pi r^2}
\]

Therefore,

\[
\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2
\]

Since the two intensity levels differ by 2.00 dB, the intensity ratio is

\[
\frac{I_1}{I_2} = 10^{0.200} = 1.58
\]

Thus,

\[
\left(\frac{r_2}{r_1}\right)^2 = 1.58
\]

We also know that $r_2 - r_1 = 1.00 \text{ m}$. We can then solve the two equations simultaneously by substituting, i.e., $r_2 = r_1 \sqrt{1.58}$ gives

\[
r_1 \sqrt{1.58} - r_1 = 1.00 \text{ m}
\]

so that

\[
r_1 = (1.00 \text{ m})/[\sqrt{1.58} - 1] = 3.9 \text{ m} \quad \text{and} \quad r_2 = 1.00 \text{ m} + r_1 = 4.9 \text{ m}
\]

75. **SSM REASONING AND SOLUTION** The sound intensity level $\beta$ in decibels (dB) is related to the sound intensity $I$ according to Equation 16.10, \( \beta = 10 \log \left(\frac{I}{I_0}\right) \), where the quantity $I_0$ is the reference intensity. According to the problem statement, when the sound intensity level triples, the sound intensity also triples; therefore,
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Then,

$$3\beta = (10 \text{ dB}) \log \left( \frac{3I}{I_0} \right)$$

Thus, $2\beta = (10 \text{ dB}) \log 3$ and

$$\beta = (5 \text{ dB}) \log 3 = 2.39 \text{ dB}$$

76. **REASONING** The observer of the sound (the bird-watcher) is stationary, while the source (the bird) is moving toward the observer. Therefore, the Doppler-shifted observed frequency is given by Equation 16.11. This expression can be solved to give the ratio of the bird’s speed to the speed of sound, from which the desired percentage follows directly.

**SOLUTION** According to Equation 16.11, the observed frequency $f_o$ is related to the frequency $f_s$ of the source, and the ratio of the speed of the source $v_s$ to the speed of sound $v$ by

$$f_o = f_s \left( \frac{1}{1 - v_s / v} \right) \quad \text{or} \quad \frac{f_o}{f_s} = \frac{1}{1 - v_s / v} \quad \text{or} \quad \frac{f_s}{f_o} = 1 - \frac{v_s}{v}$$

Solving for $v_s / v$ gives

$$\frac{v_s}{v} = 1 - \frac{f_s}{f_o} = 1 - \frac{1250 \text{ Hz}}{1290 \text{ Hz}} = 0.031$$

This ratio corresponds to 3.1%.

77. **SSM REASONING** Since you detect a frequency that is smaller than that emitted by the car when the car is stationary, the car must be moving away from you. Therefore, according to Equation 16.12, the frequency $f_o$ heard by a stationary observer from a source moving away from the observer is given by

$$f_o = f_s \left( \frac{1}{1 + \frac{v_s}{v}} \right)$$

where $f_s$ is the frequency emitted from the source when it is stationary with respect to the observer, $v$ is the speed of sound, and $v_s$ is the speed of the moving source. This expression can be solved for $v_s$. 
SOLUTION  We proceed to solve for \( v_s \) and substitute the data given in the problem statement. Rearrangement gives

\[
\frac{v_s}{v} = \frac{f_s}{f_o} - 1
\]

Solving for \( v_s \) and noting that \( f_o / f_s = 0.86 \) yields

\[
v_s = v \left( \frac{f_s}{f_o} - 1 \right) = (343 \text{ m/s}) \left( \frac{1}{0.86} - 1 \right) = 56 \text{ m/s}
\]

78. REASONING  The dolphin is the source of the clicks, and emits them at a frequency \( f_s \). The marine biologist measures a lower, Doppler-shifted click frequency \( f_o \), because the dolphin is swimming directly away. The difference between the frequencies is the source frequency minus the observed frequency: \( f_s - f_o \). We will use \( f_o = f_s / \left( 1 + \frac{v_s}{v} \right) \) (Equation 16.12), where \( v_s \) is the speed of the dolphin and \( v \) is the speed of sound in seawater, to determine the difference between the frequencies.

SOLUTION  Solving \( f_o = f_s / \left( 1 + \frac{v_s}{v} \right) \) (Equation 16.12) for the unknown source frequency \( f_s \), we obtain

\[
f_s = f_o \left( 1 + \frac{v_s}{v} \right)
\]

Therefore, the difference between the source and observed frequencies is

\[
f_s - f_o = f_o \left( 1 + \frac{v_s}{v} \right) - f_o = f_o \left[ \left( 1 + \frac{v_s}{v} \right) - 1 \right]
\]

\[
= f_o \left( \frac{v_s}{v} \right) = (2500 \text{ Hz}) \left( \frac{8.0 \text{ m/s}}{1522 \text{ m/s}} \right) = 13 \text{ Hz}
\]

79. REASONING  This problem deals with the Doppler effect in a situation where the source of the sound is moving and the observer is stationary. Thus, the observed frequency is given by Equation 16.11 when the car is approaching the observer and Equation 16.12 when the car is moving away from the observer. These equations relate the frequency \( f_o \) heard by the observer to the frequency \( f_s \) emitted by the source, the speed \( v_s \) of the source, and the speed \( v \) of sound. They can be used directly to calculate the desired ratio of the observed frequencies. We note that no information is given about the frequency emitted by the source. We will see, however, that none is needed, since \( f_s \) will be eliminated algebraically from the solution.
**SOLUTION** Equations 16.11 and 16.12 are

\[
\begin{align*}
    f_o^{\text{Approach}} &= f_s \left( \frac{1}{1 - v_s / v} \right) \quad (16.11) \\
    f_o^{\text{Recede}} &= f_s \left( \frac{1}{1 + v_s / v} \right) \quad (16.12)
\end{align*}
\]

The ratio is

\[
\frac{f_o^{\text{Approach}}}{f_o^{\text{Recede}}} = \frac{f_s \left( \frac{1}{1 - v_s / v} \right)}{f_s \left( \frac{1}{1 + v_s / v} \right)} = \frac{1 + v_s / v}{1 - v_s / v} = \frac{1 + \frac{9.00 \text{ m/s}}{343 \text{ m/s}}}{1 - \frac{9.00 \text{ m/s}}{343 \text{ m/s}}} \approx 1.054
\]

As mentioned in the **REASONING**, the unknown source frequency \( f_s \) has been eliminated algebraically from this calculation.

---

80. **REASONING** The observed frequency changes because of the Doppler effect. As you drive toward the parked car (a stationary source of sound), the Doppler effect is that given by Equation 16.13. As you drive away from the parked car, Equation 16.14 applies.

**SOLUTION** Equations 16.13 and 16.14 give the observed frequency \( f_o \) in each case:

\[
\begin{align*}
    f_o, \text{ toward} &= f_s \left( 1 + v_o / v \right) \\
    \text{Driving toward parked car} \\
    f_o, \text{ away} &= f_s \left( 1 - v_o / v \right) \\
    \text{Driving away from parked car}
\end{align*}
\]

Subtracting the equation on the right from the one on the left gives the change in the observed frequency:

\[
f_o, \text{ toward} - f_o, \text{ away} = 2 f_s v_o / v
\]

Solving for the observer’s speed (which is your speed), we obtain

\[
    v_o = \frac{v \left( f_o, \text{ toward} - f_o, \text{ away} \right)}{2 f_s} = \frac{343 \text{ m/s}(95 \text{ Hz})}{2(960 \text{ Hz})} = 17 \text{ m/s}
\]

---

81. **REASONING** If the train were headed directly towards the car, the frequency \( f_o \) heard by the driver would be given by \( f_o = f_s \left( 1 - v_{\text{train}} / v \right) \) (Equation 16.11), where \( f_s \) is the frequency of the train’s horn, \( v \) is the speed of sound in air, and \( v_{\text{train}} \) is the speed of the train. However, the driver is 20.0 m south of the crossing, and at the instant when the horn is sounded, the train is 20.0 m west of the crossing. The train would have to be headed directly southeast (45° south of east) in order to be moving directly towards the car at this instant. Therefore, to calculate the Doppler-shifted frequency of the horn blast, we must...
consider only the component \( v_{SE} \) of the train’s velocity \( v_{\text{train}} \) that is directed southeast. We will use trigonometry to find the magnitude \( v_{SE} \) of this component of the train’s velocity, and then employ Equation 16.11 to determine the frequency \( f_o \) heard by the driver.

**SOLUTION** According to the drawing, the scalar component \( v_{SE} \) of the train’s velocity \( v_{\text{train}} \) that is directed towards the car is

\[
v_{SE} = v_{\text{train}} \cos 45.0^\circ
\]

Therefore, from Equation 16.11, we have that

\[
f_o = f_s \left( \frac{1}{1 - \frac{v_{SE}}{v}} \right) = f_s \left( \frac{1}{1 - \frac{v_{\text{train}} \cos 45.0^\circ}{v}} \right) = \left(289 \text{ Hz} \right) \left( \frac{1}{1 - \frac{(55.0 \text{ m/s}) \cos 45.0^\circ}{343 \text{ m/s}}} \right) = 326 \text{ Hz}
\]

---

82. **REASONING** The highest frequency that a healthy young person with normal hearing can hear is 20 000 Hz. Therefore, the car must be moving such that the Doppler effect causes the observed frequency to be at least 20 000 Hz or lower.

**SOLUTION**

a. If the car moves away from the person, the wavelength of the sound increases. The increase in wavelength gives rise to a decrease in frequency. We see, then, that **the car must be moving away from the person**.

b. Since the car is moving away from the stationary person, the frequency \( f_o \) heard by the observer (the person) is related to the frequency \( f_s \) emitted by the source (the loudspeaker in the car) by

\[
f_o = f_s \left( \frac{1}{1 + \frac{v_s}{v}} \right)
\]

where \( v_s \) is the speed of the source and \( v \) is the speed of sound in air. Solving Equation 16.12 for \( v_s \), and recognizing that the maximum frequency that can be heard by a healthy young person with normal hearing is \( f_o = 20 000 \text{ Hz} \), we find that the minimum speed of the car is

\[
v_s = v \left( \frac{f_s}{f_o} - 1 \right) = (343 \text{ m/s}) \left( \frac{20 510 \text{ Hz}}{20 000 \text{ Hz}} - 1 \right) = 8.7 \text{ m/s}
\]

---

83. **SSM REASONING** The Doppler shift that occurs here arises because both the source and the observer of the sound are moving. Therefore, the expression for the Doppler-shifted observed frequency \( f_o \) is given by Equation 16.15 as
\[ f_o = f_s \left( \frac{1 + v_o / v}{1 - v_s / v} \right) \]

where \( f_s \) is the frequency emitted by the source, \( v_o \) is the speed of the observer, \( v_s \) is the speed of the source, and \( v \) is the speed of sound. The observer is moving toward the source, so we use the plus sign in the numerator. The source is moving toward the observer, so we use the minus sign in the denominator. Thus, Equation 16.15 becomes

\[ f_o = f_s \left( \frac{1 + v_o / v}{1 - v_s / v} \right) \]

Recognizing that both trucks move at the same speed, we can substitute \( v_o = v_s = v_{\text{Truck}} \) and solve for \( v_{\text{Truck}} \).

**SOLUTION** Using Equation 16.15 as described in the REASONING and substituting \( v_o = v_s = v_{\text{Truck}} \), we have

\[ \frac{f_o}{f_s} - 1 = \frac{v_{\text{Truck}}}{v} + \left( \frac{f_o}{f_s} \right) \left( \frac{v_{\text{Truck}}}{v} \right) \quad \text{or} \quad \frac{v_{\text{Truck}}}{v} \left( 1 + \frac{f_o}{f_s} \right) = \frac{f_o}{f_s} - 1 \]

Rearranging, with a view toward solving for \( v_{\text{Truck}} / v \), gives

\[ \frac{v_{\text{Truck}}}{v} = \frac{\frac{f_o}{f_s} - 1}{1 + \frac{f_o}{f_s}} = \frac{1.14 - 1}{1 + 1.14} = 0.14 \quad \text{or} \quad v_{\text{Truck}} = \left( \frac{0.14}{2.14} \right) (343 \text{ m/s}) = 22 \text{ m/s} \]

84. **REASONING** Following its release from rest, the platform begins to move down the incline, picking up speed as it goes. Thus, the platform’s velocity points down the incline, and its magnitude increases with time. The reason for the increasing velocity is gravity. The acceleration due to gravity points vertically downward and has a component along the length of the incline. The changing velocity is related to the acceleration of the platform according to Equation 2.4, which gives the acceleration as the change in the velocity divided by the time interval during which the change occurs.

The frequency detected by the microphone at the instant the platform is released from rest is the same as the frequency broadcast by the speaker. However, as the platform begins to move away from the speaker, the microphone detects fewer wave cycles per second than the speaker broadcasts. In other words, the microphone detects a frequency that is smaller than that broadcast by the speaker. This is an example of the Doppler effect and occurs because
the platform is moving in the same direction as the sound is traveling. As the platform picks up speed, the microphone detects an ever decreasing frequency. The speaker is the source of the sound, and the microphone is the “observer.” Since the source is stationary and the observer is moving away from the source, the Doppler-shifted observed frequency is given by Equation 16.14.

**SOLUTION** The acceleration \( a \) is directed down the incline and is the change in the velocity divided by the time interval during which the change occurs. The change in the velocity is the velocity \( v_{o,t} \) at a later time \( t \) minus the velocity \( v_{o,t_0} \) at an earlier time \( t_0 \). Thus, according to Equation 2.4, the acceleration is

\[
a = \frac{v_{o,t} - v_{o,t_0}}{t - t_0}
\]

Equation 16.14 gives the frequency \( f_o \) detected by the microphone in terms of the frequency \( f_s \) emitted by the speaker, the speed \( v_o \), and the speed \( v \) of sound:

\[
f_o = f_s \left(1 - \frac{v_o}{v}\right)
\]

Solving for the speed of the mike gives

\[
v_o = v \left(1 - \frac{f_o}{f_s}\right)
\]

Using this result, we can determine the speed of the microphone-platform at the two given times:

\[\text{[t = 1.5 s]} \quad v_{o,1.5s} = (343 \text{ m/s}) \left(1 - \frac{9939 \text{ Hz}}{1.000 \times 10^4 \text{ Hz}}\right) = 2.1 \text{ m/s}\]

\[\text{[t = 3.5 s]} \quad v_{o,3.5s} = (343 \text{ m/s}) \left(1 - \frac{9857 \text{ Hz}}{1.000 \times 10^4 \text{ Hz}}\right) = 4.9 \text{ m/s}\]

Using these two values for the velocity, we can now obtain the acceleration using Equation 2.4

\[
a = \frac{v_{o,3.5s} - v_{o,1.5s}}{t - t_0} = \frac{4.9 \text{ m/s} - 2.1 \text{ m/s}}{3.5 \text{ s} - 1.5 \text{ s}} = 1.4 \text{ m/s}^2
\]

---

85. **REASONING** Since the car is accelerating, its velocity is changing. The acceleration \( a_s \) of the car (the “source”) is given by Equation 2.4 as

\[
a_s = \frac{v_{s, \text{final}} - v_{s, \text{initial}}}{t}
\]
where \( v_{s, \text{final}} \) and \( v_{s, \text{initial}} \) are the final and initial velocities of the source and \( t \) is the elapsed time. To find \( v_{s, \text{final}} \) and \( v_{s, \text{initial}} \) we note that the frequency \( f_{o} \) of the sound heard by the observer depends on the speed \( v_{s} \) of the source. When the source is moving away from a stationary observer, Equation 16.12 gives this relation as

\[
f_{o} = f_{s} \left( \frac{1}{1 + \frac{v_{s}}{v}} \right)
\]

where \( f_{s} \) is the frequency of the sound emitted by the source and \( v \) is the speed of sound.

Solving for \( v_{s} \) gives

\[
v_{s} = v \left( \frac{f_{s}}{f_{o}} - 1 \right)
\]

(2)

We assume that the car is accelerating along the +x axis, so its velocity is always positive. Since the speed of the car is the magnitude of the velocity, the speed has the same numerical value as the velocity. According to Equation (2), the final and initial velocities of the car are:

**[Final velocity]**

\[
v_{s, \text{final}} = v \left( \frac{f_{s}}{f_{o, \text{final}}} - 1 \right)
\]

(3)

**[Initial velocity]**

\[
v_{s, \text{initial}} = v \left( \frac{f_{s}}{f_{o, \text{initial}}} - 1 \right)
\]

(4)

Substituting Equations (3) and (4) into Equation (1) gives

\[
a_{s} = \frac{v_{s, \text{final}} - v_{s, \text{initial}}}{t} = \frac{v \left( \frac{f_{s}}{f_{o, \text{final}}} - 1 \right) - v \left( \frac{f_{s}}{f_{o, \text{initial}}} - 1 \right)}{t} = \left( \frac{v f_{s}}{t f_{o, \text{final}}} \right) \left( \frac{1}{f_{o, \text{final}}} - \frac{1}{f_{o, \text{initial}}} \right)
\]

**SOLUTION** The acceleration of the car is

\[
a_{s} = \left( \frac{v f_{s}}{t} \right) \left( \frac{1}{f_{o, \text{final}}} - \frac{1}{f_{o, \text{initial}}} \right) = \left( \frac{(343 \text{ m/s})(1.00 \times 10^{3} \text{ Hz})}{(14.0 \text{ s})} \right) \left( \frac{1}{912.0 \text{ Hz}} - \frac{1}{966.0 \text{ Hz}} \right) = 1.5 \text{ m/s}^{2}
\]

86. **REASONING**

a. Since the sound source and the observer are stationary, there is no Doppler effect. The wavelength remains the same and the frequency of the sound heard by the observer remains the same as that emitted by the sound source.

b. When the sound source moves toward a stationary observer, the wavelength decreases (see Figure 16.28b). This decrease arises because the condensations “bunch-up” as the
source moves toward the observer. The frequency heard by the observer increases, because, according to Equation 16.1, the frequency is inversely proportional to the wavelength; a smaller wavelength gives rise to a greater frequency.

c. The wavelength decreases for the same reason given in part (b). The increase in frequency is due to two effects; the decrease in wavelength, and the fact that the observer intercepts more wave cycles per second as she moves toward the sound source.

**SOLUTION**

a. The frequency of the sound is the same as that emitted by the siren; \( f_0 = f_s = 2450 \text{ Hz} \)

The wavelength is given by Equation 16.1 as

\[
\lambda = \frac{v}{f_s} = \frac{343 \text{ m/s}}{2450 \text{ Hz}} = 0.140 \text{ m}
\]

b. According to the discussion in Section 16.9 (see the subsection “Moving source”) the wavelength \( \lambda' \) of the sound is given by \( \lambda' = \lambda - \frac{v_s}{f_s} T \), where \( v_s \) is the speed of the source and \( T \) is the period of the sound. However, \( T = 1/f_s \) so that

\[
\lambda' = \lambda - \frac{v_s}{f_s} T = \lambda - \frac{v_s}{f_s} \frac{26.8 \text{ m/s}}{2450 \text{ Hz}} = 0.129 \text{ m}
\]

The frequency \( f_0 \) heard by the observer is equal to the speed of sound \( v \) divided by the shortened wavelength \( \lambda' \):

\[
f_0 = \frac{v}{\lambda'} = \frac{343 \text{ m/s}}{0.129 \text{ m}} = 2660 \text{ Hz}
\]

c. The wavelength is the same as that in part (b), so \( \lambda' = 0.129 \text{ m} \). The frequency heard by the observer can be obtained from Equation 16.15, where we use the fact that the observer is moving toward the sound source:

\[
f_0 = f_s \left(1 + \frac{v_s}{v} \right) = \left(2450 \text{ Hz}\right) \left(1 + \frac{14.0 \text{ m/s}}{343 \text{ m/s}} \right) = 2770 \text{ Hz}
\]

87. **SSM REASONING**

a. Since the two submarines are approaching each other head on, the frequency \( f_0 \) detected by the observer (sub B) is related to the frequency \( f_s \) emitted by the source (sub A) by
where \( v_o \) and \( v_s \) are the speed of the observer and source, respectively, and \( v \) is the speed of the underwater sound.

b. The sound reflected from submarine B has the same frequency that it detects, namely, \( f_o \).

Now sub B becomes the source of sound and sub A is the observer. We can still use Equation 16.15 to find the frequency detected by sub A.

**SOLUTION**

a. The frequency \( f_o \) detected by sub B is

\[
f_o = f_s \frac{1 + \frac{v_o}{v}}{1 - \frac{v_s}{v}} = (1550 \text{ Hz}) \frac{1 + \frac{8 \text{ m/s}}{1522 \text{ m/s}}}{1 - \frac{12 \text{ m/s}}{1522 \text{ m/s}}} = 1570 \text{ Hz}
\]

b. The sound reflected from submarine B has the same frequency that it detects, namely, 1570 Hz. Now sub B is the source of sound whose frequency is \( f_s = 1570 \text{ Hz} \). The speed of sub B is \( v_s = 8 \text{ m/s} \). The frequency detected by sub A (whose speed is \( v_o = 12 \text{ m/s} \)) is

\[
f_o = f_s \frac{1 + \frac{v_o}{v}}{1 - \frac{v_s}{v}} = (1570 \text{ Hz}) \frac{1 + \frac{12 \text{ m/s}}{1522 \text{ m/s}}}{1 - \frac{8 \text{ m/s}}{1522 \text{ m/s}}} = 1590 \text{ Hz}
\]

88. **REASONING AND SOLUTION** The maximum observed frequency is \( f_o^{\text{max}} \), and the minimum observed frequency is \( f_o^{\text{min}} \). We are given that \( f_o^{\text{max}} - f_o^{\text{min}} = 2.1 \text{ Hz} \), where

\[
f_o^{\text{max}} = f_s \left(1 + \frac{v_o}{v}\right)
\]

and

\[
f_o^{\text{min}} = f_s \left(1 - \frac{v_o}{v}\right)
\]

We have

\[
f_o^{\text{max}} - f_o^{\text{min}} = f_s \left(1 + \frac{v_o}{v}\right) - f_s \left(1 - \frac{v_o}{v}\right) = 2 f_s \left(\frac{v_o}{v}\right)
\]

We can now solve for the maximum speed \( v_o \) of the microphone:
Using $v_{\text{max}} = v_{\text{o}} = A\omega$, Equation 10.6, where $A$ is the amplitude of the simple harmonic motion and $\omega$ is the angular frequency, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0 \text{ s}} = 3.1 \text{ rad/s}$, we have

$$A = \frac{v_{\text{o}}}{\omega} = \frac{0.82 \text{ m/s}}{3.1 \text{ rad/s}} = 0.26 \text{ m}$$

89. **REASONING** The sound intensity level outside the room is

$$\beta_{\text{outside}} = (10 \text{ dB}) \log \left( \frac{I_{\text{outside}}}{I_0} \right) \quad (16.10)$$

where $I_0$ is the threshold of hearing. Solving for the intensity $I_{\text{outside}}$ gives

$$I_{\text{outside}} = I_0 10^{\frac{\beta_{\text{outside}}}{10 \text{ dB}}}$$

These two relations will allow us to find the sound intensity outside the room.

**SOLUTION** From the problem statement we know that $\beta_{\text{outside}} = \beta_{\text{inside}} + 44.0 \text{ dB}$. We can evaluate $\beta_{\text{inside}}$ by applying Equation 16.10 to the inside of the room:

$$\beta_{\text{inside}} = (10 \text{ dB}) \log \left( \frac{I_{\text{inside}}}{I_0} \right) = (10 \text{ dB}) \log \left( \frac{1.20 \times 10^{-10} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 20.8 \text{ dB}$$

Thus, the sound intensity level outside the room is $\beta_{\text{outside}} = 20.8 \text{ dB} + 44.0 \text{ dB} = 64.8 \text{ dB}$. The sound intensity outside the room is

$$I_{\text{outside}} = I_0 10^{\frac{\beta_{\text{outside}}}{10 \text{ dB}}} = \left(1.00 \times 10^{-12} \text{ W/m}^2\right) 10^{\frac{64.8 \text{ dB}}{10 \text{ dB}}} = 3.02 \times 10^{-6} \text{ W/m}^2$$

90. **REASONING** The speed $v$, the frequency $f$, and the wavelength $\lambda$ of the sound wave are related according to $v = f \lambda$ (Equation 16.1). We can apply this expression to the sound wave in the air and in the water, recognizing that the frequency is the same in both media.

**SOLUTION** Applying $v = f \lambda$ (Equation 16.1) to the sound wave in both the air and in the water, we have

$$v_{\text{air}} = f \lambda_{\text{air}} \quad \text{and} \quad v_{\text{water}} = f \lambda_{\text{water}}$$

Dividing the equation on the right by the equation on the left, we obtain
Solving this result for \( \lambda_{\text{water}} \), we find that
\[
\lambda_{\text{water}} = \lambda_{\text{air}} \left( \frac{v_{\text{water}}}{v_{\text{air}}} \right) = (2.74 \text{ m}) \left( \frac{1482 \text{ m/s}}{343 \text{ m/s}} \right) = 11.8 \text{ m}
\]
where we have taken the speeds of sound in fresh water and air at 20 °C from Table 16.1.

91. **REASONING AND SOLUTION** The speed of sound in a liquid is given by Equation 16.6, \( v = \sqrt{B_{\text{ad}}/\rho} \), where \( B_{\text{ad}} \) is the adiabatic bulk modulus and \( \rho \) is the density of the liquid. Solving for \( B_{\text{ad}} \), we obtain \( B_{\text{ad}} = v^2 \rho \). Values for the speed of sound in fresh water and in ethyl alcohol are given in Table 16.1. The ratio of the adiabatic bulk modulus of fresh water to that of ethyl alcohol at 20°C is, therefore,
\[
\frac{(B_{\text{ad}})_{\text{water}}}{(B_{\text{ad}})_{\text{ethyl alcohol}}} = \frac{v_{\text{water}}^2 \rho_{\text{water}}}{v_{\text{ethyl alcohol}}^2 \rho_{\text{ethyl alcohol}}} = \frac{(1482 \text{ m/s})^2 (998 \text{ kg/m}^3)}{(1162 \text{ m/s})^2 (789 \text{ kg/m}^3)} = 2.06
\]

92. **REASONING** Both the observer (you) and the source (the eagle) are moving toward each other. According to Equation 16.15, the frequency \( f_o \) heard by the observer is related to the frequency \( f_s \) of the sound emitted by the source by
\[
f_o = f_s \left( 1 + \frac{v_o}{v} \right) \left( 1 - \frac{v_s}{v} \right)
\]
where \( v_o \) and \( v_s \) are, respectively, the speeds of the observer and source.

**SOLUTION** Substituting in the given data, we find that the frequency heard by the observer is
\[
f_o = f_s \left( 1 + \frac{v_o}{v} \right) \left( 1 - \frac{v_s}{v} \right) = (3400 \text{ Hz}) \left( 1 + \frac{39 \text{ m/s}}{330 \text{ m/s}} \right) \left( 1 - \frac{18 \text{ m/s}}{330 \text{ m/s}} \right) = 4.0 \times 10^3 \text{ Hz}
\]

93. **REASONING** The tension \( F \) in the violin string can be found by solving Equation 16.2 for \( F \) to obtain \( F = mv^2/L \), where \( v \) is the speed of waves on the string and can be found from Equation 16.1 as \( v = f\lambda \).
Combining Equations 16.2 and 16.1 and using the given data, we obtain

\[ F = \frac{mv^2}{L} = (m/L) f^2 \lambda^2 = \left(7.8 \times 10^{-4} \text{ kg/m}\right) (440 \text{ Hz})^2 \left(65 \times 10^{-2} \text{ m}\right)^2 = 64 \text{ N} \]

94. **REASONING** We can think of the rumble strips as a wave carved into the surface of the road. The wave is stationary relative to the road, but its speed \( v \) relative to the car is equal to the car’s speed \( v \) relative to the road. The frequency \( f \) of the wave is equal to the frequency of the wheel vibrations caused by the wave. The distance between the centers of adjacent grooves, then, is the wavelength \( \lambda \) of the rumble-strip wave. The relationship among these three quantities is given by \( v = f\lambda \) (Equation 16.1), which we will use to find \( \lambda \).

**SOLUTION** Solving \( v = f\lambda \) (Equation 16.1) for the wavelength \( \lambda \), we obtain

\[ \lambda = \frac{v}{f} = \frac{23 \text{ m/s}}{82 \text{ Hz}} = 0.28 \text{ m} \]

95. **SSM REASONING AND SOLUTION** The intensity level \( \beta \) in decibels (dB) is related to the sound intensity \( I \) according to Equation 16.10:

\[ \beta = (10 \text{ dB}) \log \left(\frac{I}{I_0}\right) \]

where the quantity \( I_0 \) is the reference intensity. Therefore, we have

\[ \beta_2 - \beta_1 = (10 \text{ dB}) \log \left(\frac{I_2}{I_0}\right) - (10 \text{ dB}) \log \left(\frac{I_1}{I_0}\right) = (10 \text{ dB}) \log \left(\frac{I_2}{I_1}\frac{I_1}{I_0}\right) = (10 \text{ dB}) \log \left(\frac{I_2}{I_1}\right) \]

Solving for the ratio \( I_2 / I_1 \), we find

\[ 30.0 \text{ dB} = (10 \text{ dB}) \log \left(\frac{I_2}{I_1}\right) \quad \text{or} \quad \frac{I_2}{I_1} = 10^{3.0} = 1000 \]

Thus, we conclude that the sound intensity increases by a factor of 1000.

96. **REASONING AND SOLUTION** The energy carried by the sound into the ear is

\[ \text{Energy} = IAt = (3.2 \times 10^{-5} \text{ W/m}^2)(2.1 \times 10^{-3} \text{ m}^2)(3600 \text{ s}) = 2.4 \times 10^{-4} \text{ J} \]
97. **SSM REASONING** As the transverse wave propagates, the colored dot moves up and down in simple harmonic motion with a frequency of 5.0 Hz. The amplitude (1.3 cm) is the magnitude of the maximum displacement of the dot from its equilibrium position.

**SOLUTION** The period $T$ of the simple harmonic motion of the dot is $T = 1/f = 1/(5.0 \text{ Hz}) = 0.20 \text{ s}$. In one period the dot travels through one complete cycle of its motion, and covers a vertical distance of $4 \times (1.3 \text{ cm}) = 5.2 \text{ cm}$. Therefore, in 3.0 s the dot will have traveled a *total vertical distance* of

$$\left(\frac{3.0 \text{ s}}{0.20 \text{ s}}\right) (5.2 \text{ cm}) = 78 \text{ cm}$$

98. **REASONING** The wavelength $\lambda$ of a sound wave is equal to its speed $v$ divided by its frequency $f$ (Equation 16.1):

$$\lambda = \frac{v}{f} \quad (1)$$

The speed of sound in a gas is given by Equation 16.5 as $v = \sqrt{\gamma k T / m}$, where $T$ is the Kelvin temperature and $m$ is the mass of a single air molecule. The Kelvin temperature is related to the Celsius temperature $T_c$ by $T = T_c + 273.15$ (Equation 12.1), so the speed of sound can be expressed as

$$v = \sqrt{\frac{\gamma k (T_c + 273.15)}{m}} \quad (2)$$

We know that the speed of sound is 343 m/s at 20.0 °C, so that

$$343 \text{ m/s} = \sqrt{\frac{\gamma k (20.0 \degree \text{C} + 273.15)}{m}} \quad (3)$$

Dividing Equation (2) by (3) gives

$$\frac{v}{343 \text{ m/s}} = \sqrt{\frac{\gamma k (T_c + 273.15)}{\gamma k (20.0 \degree \text{C} + 273.15)}} = \sqrt{\frac{T_c + 273.15}{20.0 \degree \text{C} + 273.15}} \quad (4)$$

**SOLUTION** Solving Equation (4) for $v$ and substituting the result into Equation (1) gives

$$\lambda = \frac{v}{f} = \frac{(343 \text{ m/s}) \sqrt{T_c + 273.15}}{20.0 \degree \text{C} + 273.15} = \frac{(343 \text{ m/s}) \sqrt{35 \degree \text{C} + 273.15}}{20.0 \degree \text{C} + 273.15} = 3.9 \times 10^{-3} \text{ m}$$
99. **SSM REASONING** You hear a frequency $f_o$ that is 1.0% lower than the frequency $f_s$ emitted by the source. This means that the frequency you observe is 99.0% of the emitted frequency, so that $f_o = 0.990 f_s$. You are an observer who is moving away from a stationary source of sound. Therefore, the Doppler-shifted frequency that you observe is specified by Equation 16.14, which can be solved for the bicycle speed $v_o$.

**SOLUTION** Equation 16.14, in which $v$ denotes the speed of sound, states that

$$f_o = f_s \left(1 - \frac{v_o}{v}\right)$$

Solving for $v_o$ and using the fact that $f_o = 0.990 f_s$ reveal that

$$v_o = v \left(1 - \frac{f_o}{f_s}\right) = (343 \text{ m/s}) \left(1 - \frac{0.990 f_s}{f_s}\right) = 3.4 \text{ m/s}$$

100. **REASONING** If we treat the sample of argon atoms like an ideal monatomic gas ($\gamma = 1.67$) at 298 K, Equation 14.6 \left(\frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT\right) can be solved for the root-mean-square speed $v_{\text{rms}}$ of the argon atoms. The speed of sound in argon can be found from Equation 16.5: $v = \sqrt{\gamma kT/m}$.

**SOLUTION** We first find the mass of an argon atom. Since the molecular mass of argon is 39.9 u, argon has a mass per mole of $39.9 \times 10^{-3}$ kg/mol. Thus, the mass of a single argon atom is

$$m = \frac{39.9 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 6.63 \times 10^{-26} \text{ kg}$$

a. Solving Equation 14.6 for $v_{\text{rms}}$ and substituting the data given in the problem statement, we find

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(298 \text{ K})}{6.63 \times 10^{-26} \text{ kg}}} = 431 \text{ m/s}$$

b. The speed of sound in argon is, according to Equation 16.5,

$$v = \sqrt{\frac{\gamma kT}{m}} = \sqrt{\frac{(1.67)(1.38 \times 10^{-23} \text{ J/K})(298 \text{ K})}{6.63 \times 10^{-26} \text{ kg}}} = 322 \text{ m/s}$$
101. **SSM REASONING** The intensity level $\beta$ is related to the sound intensity $I$ according to Equation 16.10:

$$\beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right)$$

where $I_0$ is the reference level sound intensity. Solving for $I$ gives

$$I = I_0 10^{\frac{\beta}{10 \text{ dB}}}$$

**SOLUTION** Taking the ratio of the sound intensity $I_{\text{rock}}$ at the rock concert to the intensity $I_{\text{jazz}}$ at the jazz fest gives

$$\frac{I_{\text{rock}}}{I_{\text{jazz}}} = \frac{I_0 10^{\frac{\beta_{\text{rock}}}{10 \text{ dB}}}}{I_0 10^{\frac{\beta_{\text{jazz}}}{10 \text{ dB}}}} = \frac{10^{115 \text{ dB}}}{10^{95 \text{ dB}}} = 10^{20 \text{ dB}} = 1.0 \times 10^2$$

102. **REASONING AND SOLUTION** The intensity level in dB is $\beta = (10 \text{ dB})\log \left( \frac{I}{I_0} \right)$, Equation 16.10, where $\beta = 14$ dB. Therefore,

$$\frac{I}{I_0} = 10^{\frac{\beta}{10 \text{ dB}}} = 10^{1.4} = 25$$

103. **REASONING** Suppose we knew the sound intensity ratio $\frac{I_A}{I_B}$. In addition, suppose we knew the sound intensity ratio $\frac{I_C}{I_A}$. Then, all that would be necessary to obtain the desired ratio $\frac{I_C}{I_B}$ would be to multiply the two known ratios:

$$\left( \frac{I_A}{I_B} \right) \left( \frac{I_C}{I_A} \right) = \frac{I_C}{I_B}$$

This is exactly the procedure we will follow, except that first we will obtain the intensity ratios from the given intensity levels (expressed in decibels). The intensity level $\beta$ in decibels is given by Equation 16.10.
SOLUTION Applying Equation 16.10 to persons A and B gives

\[ \beta_{A/B} = 1.5 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I_A}{I_B} \right) \]

\[ \log \left( \frac{I_A}{I_B} \right) = \frac{1.5 \text{ dB}}{10 \text{ dB}} = 0.15 \quad \text{or} \quad \frac{I_A}{I_B} = 10^{0.15} \]

In a similar fashion for persons C and A we obtain

\[ \beta_{C/A} = 2.7 \text{ dB} = (10 \text{ dB}) \log \left( \frac{I_C}{I_A} \right) \]

\[ \log \left( \frac{I_C}{I_A} \right) = \frac{2.7 \text{ dB}}{10 \text{ dB}} = 0.27 \quad \text{or} \quad \frac{I_C}{I_A} = 10^{0.27} \]

Multiplying the two intensity ratios reveals that

\[ \frac{I_C}{I_B} = \left( \frac{I_A}{I_B} \right) \left( \frac{I_C}{I_A} \right) = \left( 10^{0.15} \right) \left( 10^{0.27} \right) = 10^{0.42} = 2.6 \]

104. REASONING AND SOLUTION The maximum acceleration of the dot occurs at the extreme positions and is

\[ a_{\text{max}} = (2\pi f)^2 A = \left[ 2\pi(4.0 \text{ Hz}) \right]^2 \left( 5.4 \times 10^{-3} \text{ m} \right) = 3.4 \text{ m/s}^2 \]  

(10.10)

105. REASONING According to Equation 16.2, the speed of the wave at a distance \( y \) above the bottom of the rope depends on the tension in the rope at that spot, and the tension is greater near the top than near the bottom. This is because the rope has weight. Consider the section of the rope between the bottom end and a point at a distance \( y \) above the bottom end. The part of the rope above this point must support the weight of this section, which is the mass of the section times the magnitude \( g \) of the acceleration due to gravity. Since the rope is uniform, the mass of the section is simply the total mass \( m \) of the rope times the fraction \( y/L \), which is the length of the section divided by the total length of the rope. Thus, the weight of the section is \( m \left( \frac{y}{L} \right) g \). It is this weight that determines the tension and, hence, the speed of the wave.

SOLUTION

a. According to Equation 16.2, the speed \( v \) of the wave is

\[ v = \sqrt{\frac{F}{m/L}} \]
where $F$ is the tension, $m$ is the total mass of the rope, and $L$ is the length of the rope. At a point $y$ meters above the bottom end, the rope is supporting the weight of the section beneath that point, which is $m \left( \frac{y}{L} \right) g$, as discussed in the REASONING. The rope supports the weight by virtue of the tension in the rope. Since the rope does not accelerate upward or downward, the tension must be equal to $m \left( \frac{y}{L} \right) g$, according to Newton’s second law of motion. Substituting this tension for $F$ in Equation 16.2 reveals that the speed at a point $y$ meters above the bottom end is

$$v = \sqrt{\frac{m \left( \frac{y}{L} \right) g}{m/L}} = \sqrt{yg}$$

b. Using the expression just derived, we find the following speeds

- $[y = 0.50 \text{ m}]$ \hspace{1cm} $v = \sqrt{yg} = \sqrt{(0.50 \text{ m})(9.80 \text{ m/s}^2)} = 2.2 \text{ m/s}$

- $[y = 2.0 \text{ m}]$ \hspace{1cm} $v = \sqrt{yg} = \sqrt{(2.0 \text{ m})(9.80 \text{ m/s}^2)} = 4.4 \text{ m/s}$

106. REASONING The speed $v$ of the waves on the strand of silk is given by $v = \sqrt{\frac{F}{m/L}}$ (Equation 16.2), where $F$ is the tension in the strand and $m/L$ is the ratio of the mass $m$ of the strand to its length $L$. The tension $F$ is directly proportional to the spider’s mass $M$, because the tension force exerted on the spider by the silk is equal in magnitude to the spider’s weight $Mg$, where $g$ is the magnitude of the acceleration due to gravity:

$$F = Mg \quad (1)$$

Neither the mass $m$ nor the length $L$ of the silk strand are given, but we know the density $\rho$ of the silk, which is the ratio of the mass $m$ of the strand to its volume $V$, according to $\rho = \frac{m}{V}$ (Equation 11.1). The strand is a cylinder, so its volume $V$ is the product of its length $L$ and its cross-sectional area $A = \pi r^2$:

$$V = AL = \pi r^2 L \quad (2)$$

Equation 11.1 and Equation (2) will permit us to determine the mass per unit length $m/L$ of the silk strand.

SOLUTION Substituting Equation (1) into $v = \sqrt{\frac{F}{m/L}}$ (Equation 16.2), we obtain
Squaring both sides of Equation (3) and solving for the mass $M$ of the spider yields

$$v^2 = \frac{Mg}{m/L} \quad \text{or} \quad M = \frac{v^2 (m/L)}{g}$$

(4)

We must now determine the value of the ratio $m/L$ in terms of the density $\rho$ and radius $r$ of the silk strand. Substituting Equation (2) for the volume $V$ of the silk strand into $\rho = \frac{m}{V}$ (Equation 11.1), we obtain

$$\rho = \frac{m}{V} = \frac{m}{\pi r^2 L} = \frac{1}{\pi r^2} \left( \frac{m}{L} \right)$$

(5)

Therefore, the ratio $m/L$ is given by

$$m/L = \rho \pi r^2$$

(6)

Substituting Equation (6) into Equation (4), then, yields the mass $M$ of the spider:

$$M = \frac{v^2 (m/L)}{g} = \frac{v^2 \rho \pi r^2}{g} = \frac{(280 \text{ m/s})^2 \left(1300 \text{ kg/m}^3 \right) \pi \left(4.0 \times 10^{-6} \text{ m} \right)^2}{9.80 \text{ m/s}^2} = 5.2 \times 10^{-4} \text{ kg}$$

107. **Reasoning** Newton's second law can be used to analyze the motion of the blocks using the methods developed in Chapter 4. We can thus determine an expression that relates the magnitude $P$ of the pulling force to the magnitude $F$ of the tension in the wire. Equation 16.2 [$v = \sqrt{F/(m/L)}$] can then be used to find the tension in the wire.

**Solution** The following drawings show a schematic of the situation described in the problem and the free-body diagrams for each block, where $m_1 = 42.0$ kg and $m_2 = 19.0$ kg. The pulling force is $P$, and the tension in the wire gives rise to the forces $F$ and $-F$, which act on $m_1$ and $m_2$, respectively.

Newton's second law for block 1 is, taking forces that point to the right as positive, $F = m_1a$, or $a = F/m_1$. For block 2, we obtain $P - F = m_2a$. Using the expression for $a$ obtained from the equation for block 1, we have


\[ P - F = F \left( \frac{m_2}{m_1} \right) \quad \text{or} \quad P = F \left( \frac{m_2}{m_1} + 1 \right) \]

According to Equation 16.2, \( F = v^2 (m/L) \), where \( m/L \) is the mass per unit length of the wire. Combining this expression for \( F \) with the expression for \( P \), we have

\[ P = v^2 (m/L) \left( \frac{m_2}{m_1} + 1 \right) = (352 \text{ m/s})^2 (8.50 \times 10^{-4} \text{ kg/m}) \left( \frac{19.0 \text{ kg}}{42.0 \text{ kg}} + 1 \right) = 153 \text{ N} \]

108. REASONING The earplugs reduce the sound intensity by a factor of 350, so the sound intensity \( I_{\text{without}} \) that the crew member would experience without the earplugs is 350 times larger than the sound intensity \( I_{\text{with}} \) experienced with the earplugs in place:

\[ I_{\text{without}} = 350 I_{\text{with}} \tag{1} \]

The sound intensity levels \( \beta_{\text{without}} \) and \( \beta_{\text{with}} \) corresponding to these intensities are found from \( \beta = (10 \text{ dB}) \log \left( \frac{I}{I_0} \right) \) (Equation 16.10), where \( I_0 \) is the reference level sound intensity.

SOLUTION According to Equation 16.10, the sound intensity level \( \beta_{\text{without}} \) is

\[ \beta_{\text{without}} = (10 \text{ dB}) \log \left( \frac{I_{\text{without}}}{I_0} \right) \tag{2} \]

Substituting Equation (1) into Equation (2) yields

\[ \beta_{\text{without}} = (10 \text{ dB}) \log \left( \frac{350 I_{\text{with}}}{I_0} \right) \tag{3} \]

It is convenient to express the quantity \( \frac{350 I_{\text{with}}}{I_0} \) in Equation (3) as a product:

\[ \frac{350 I_{\text{with}}}{I_0} = 350 \left( \frac{I_{\text{with}}}{I_0} \right) \]. This permits us to put Equation (3) into a more useful form:

\[ \beta_{\text{without}} = (10 \text{ dB}) \log \left[ 350 \left( \frac{I_{\text{with}}}{I_0} \right) \right] = (10 \text{ dB}) \left[ \log 350 + \log \left( \frac{I_{\text{with}}}{I_0} \right) \right] \]

\[ = (10 \text{ dB}) \log 350 + (10 \text{ dB}) \log \left( \frac{I_{\text{with}}}{I_0} \right) \tag{4} \]
The term \((10 \, \text{dB}) \log \left( \frac{I_{\text{with}}}{I_0} \right)\) in Equation (4) is the sound intensity level \(\beta_{\text{with}}\), as we see from Equation 16.10. Therefore, the sound intensity level experienced without earplugs is

\[
\beta_{\text{without}} = (10 \, \text{dB}) \log 350 + \beta_{\text{with}} = (10 \, \text{dB}) \log 350 + 88 \, \text{dB} = 113 \, \text{dB}
\]

### 109. REASONING AND SOLUTION

The sound emitted by the plane at A reaches the person after a time \(t\). This time, \(t\), required for the sound wave at A to reach the person is the same as the time required for the plane to fly from A to B.

The figure at the right shows the relevant geometry. During the time \(t\), the plane travels the distance \(x\), while the sound wave travels the distance \(d\). The sound wave travels with constant speed. The plane has a speed of \(v_0\) at A and a speed \(v\) at B and travels with constant acceleration. Thus,

\[
d = v_{\text{sound}} t
\]

and

\[
x = \frac{1}{2} (v + v_0) t
\]

From the drawing, we have

\[
\sin \theta = \frac{x}{d} = \frac{1}{2} (v + v_0) t \cdot \frac{v + v_0}{v_{\text{sound}} t} = \frac{v + v_0}{2v_{\text{sound}}}
\]

Solving for \(v\) gives

\[
v = 2v_{\text{sound}} \sin \theta - v_0 = 2(343 \, \text{m/s}) \sin 36.0^\circ - 164 \, \text{m/s} = 239 \, \text{m/s}
\]

### 110. REASONING

Both the sound reflected from the ceiling and from the floor of the cavern travel the same distance through the ground to reach the microphones. But the sound reflected from the floor travels from the ceiling to the floor and back again, a distance \(2h\) farther than the sound reflected from the ceiling, where \(h\) is the height of the cavern. Therefore, the time \(\Delta t = 0.0437 \, \text{s} - 0.0245 \, \text{s} = 0.0192 \, \text{s}\) that elapses between the arrival of the first and second reflected sounds is the time it takes the sound that penetrates the cavern to travel the distance \(2h\) in the cavern. If the speed of sound in the air of the cavern is \(v\), then Equation 2.1 gives

\[
v = \frac{2h}{\Delta t} \quad (2.1)
\]
We are assuming that air behaves like an ideal gas, so the speed $v$ of sound is given by $v = \sqrt{\frac{\gamma kT}{m}}$ (Equation 16.5), where $\gamma = c_p/c_v$ is the ratio of the specific heat capacities of air at constant pressure and constant volume, $k$ is Boltzmann's constant, $m$ is the mass of a single molecule of air, and $T$ is the Kelvin temperature of the air. Since we do not have the values of $\gamma$ or $m$, we will use the fact that, at $T_0 = 273.15 \text{ K} + 20.0 \, ^\circ\text{C}$ the speed of sound in air is $v_0 = 343 \text{ m/s}$ (see Table 16.1) to determine, with Equation 16.5, the speed $v$ of sound when the temperature is $T = 273.15 \text{ K} + 9.0 \, ^\circ\text{C}$.

**SOLUTION** Solving Equation 2.1 for the height $h$ of the cavern yields

$$h = \frac{v\Delta t}{2}$$  \hspace{1cm} (1)

Equation 16.5 gives both the speed $v$ of sound in air at $T = 273.15 \text{ K} + 9.0 \, ^\circ\text{C}$ and the speed $v_0$ of sound in air at $T_0 = 273.15 \text{ K} + 20.0 \, ^\circ\text{C}$:

$$v = \sqrt{\frac{\gamma kT}{m}} \quad \text{and} \quad v_0 = \sqrt{\frac{\gamma kT_0}{m}}$$  \hspace{1cm} (2)

Taking the ratio of Equations (2), we obtain

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \frac{T}{T_0} \quad \text{or} \quad v = v_0 \frac{T}{T_0}$$  \hspace{1cm} (3)

Substituting Equation (3) into Equation (1) yields

$$h = \frac{v\Delta t}{2} = \left( \frac{v_0 \sqrt{T}}{T_0} \right) \frac{\Delta t}{2} = \frac{v_0 \Delta t}{2} \frac{T}{T_0}$$

$$= \left( \frac{343 \text{ m/s}}{2} \frac{0.0192 \text{ s}}{2} \right) \frac{273.15 \text{ K} + 9 \, ^\circ\text{C}}{273.15 \text{ K} + 20.0 \, ^\circ\text{C}} = 3.23 \text{ m}$$
ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (d) If we add pulses 1 and 4 as per the principle of linear superposition, the resultant is a straight horizontal line that extends across the entire graph.

2. (a) These two pulses combine to produce a peak that is 4 units high and a valley that is 2 units deep. No other combination gives greater values.

3. (c) The smallest difference in path lengths for destructive interference to occur is one-half a wavelength \( \frac{1}{2} \lambda \). As the frequency goes up, the wavelength goes down, so the separation between the cellists decreases.

4. Smallest separation = 1.56 m

5. (b) According to Equation 17.2, the diffraction angle \( \theta \) is related to the wavelength and diameter by \( \sin \theta = 1.22 \left( \frac{\lambda}{D} \right) \) and is determined by the ratio \( \lambda/D \). Here the ratio is \( 2\lambda_0 / D_0 \) and is the largest of any of the choices, so it yields the largest diffraction angle.

6. \( \theta = 30.0 \) degrees

7. (e) Since the wavelength is directly proportional to the speed of the sound wave (see Section 16.2), the wavelength is greatest in the helium-filled room. The greater the wavelength, the greater the diffraction angle \( \theta \) (see Section 17.3). Thus, the greatest diffraction occurs in the helium-filled room.

8. (d) The trombones produce 6 beats every 2 seconds, so the beat frequency is 3 Hz. The second trombone can be producing a sound whose frequency is either 438 Hz – 3 Hz = 435 Hz or 438 Hz + 3 Hz = 441 Hz.

9. Beat frequency = 3.0 Hz

10. (d) According to the discussion in Section 17.5, one loop of a transverse standing wave corresponds to one-half a wavelength. The two loops in the top picture mean that the wavelength of 1.2 m is also the distance \( L \) between the walls, so \( L = 1.2 \) m. The bottom picture contains three loops in a distance of 1.2 m, so its wavelength is \( \frac{2}{3}(1.2 \text{ m}) = 0.8 \text{ m} \).
11. (b) The frequency of a standing wave is directly proportional to the speed of the traveling waves that form it (see Equation 17.3). The speed of the waves, on the other hand, depends on the mass \( m \) of the string through the relation \( v = \sqrt{\frac{F}{mL}} \), so the smaller the mass, the greater is the speed and, hence, the greater the frequency of the standing wave.

12. (c) For a string with a fixed length, tension, and linear density, the frequency increases when the harmonic number \( n \) increases from 4 to 5 (see Equation 17.3). According to \( \lambda = \frac{v}{f} \) (Equation 16.1), the wavelength decreases when the frequency increases.

13. Fundamental frequency = \( 2.50 \times 10^2 \) Hz

14. (c) One loop of a longitudinal standing wave corresponds to one-half a wavelength. Since this standing wave has two loops, its wavelength is equal to the length of the tube, or 0.80 m.

15. (b) There are two loops in this longitudinal standing wave. This means that the 2nd harmonic is being generated. According to Equation 17.4, the \( n \)th harmonic frequency is

\[
f_n = n \left( \frac{v}{2L} \right), \text{ where } \frac{v}{2L} \text{ is the fundamental frequency.}
\]

Since \( f_2 = 440 \) Hz and \( n = 2 \), we have

\[
\frac{v}{2L} = \frac{440 \text{ Hz}}{2} = 220 \text{ Hz}.
\]

16. (c) The standing wave pattern in the drawing corresponds to \( n = 3 \) (the 3rd harmonic) for a tube open at only one end. Using Equation 17.5, the length of the tube is

\[
L = \frac{n v}{4 f_n} = \frac{(3) (343 \text{ m/s})}{4 (660 \text{ Hz})} = 0.39 \text{ m}.
\]

17. Frequency of 3rd harmonic = \( 9.90 \times 10^2 \) Hz
CHAPTER 17 | THE PRINCIPLE OF LINEAR SUPERPOSITION AND INTERFERENCE PHENOMENA

PROBLEMS

1. **REASONING** For destructive interference to occur, the difference in travel distances for the sound waves must be an integer number of half wavelengths. For larger and larger distances between speaker B and the observer at C, the difference in travel distances becomes smaller and smaller. Thus, the largest possible distance between speaker B and the observer at C occurs when the difference in travel distances is just one half wavelength.

**SOLUTION** Since the triangle ABC in Figure 17.7 is a right triangle, we can apply the Pythagorean theorem to obtain the distance \( d_{AC} \) as \( \sqrt{(5.00 \text{ m})^2 + d_{BC}^2} \). Therefore, the difference in travel distances is

\[
\sqrt{(5.00 \text{ m})^2 + d_{BC}^2} - d_{BC} = \frac{\lambda}{2} = \frac{v}{2f}
\]

where we have used Equation 16.1 to express the wavelength \( \lambda \) as \( \lambda = \frac{v}{f} \). Solving for the distance \( d_{BC} \) gives

\[
\sqrt{(5.00 \text{ m})^2 + d_{BC}^2} = d_{BC} + \frac{v}{2f} \quad \text{or} \quad (5.00 \text{ m})^2 + d_{BC}^2 = \left(d_{BC} + \frac{v}{2f}\right)^2
\]

\[
(5.00 \text{ m})^2 + d_{BC}^2 = d_{BC}^2 + \frac{d_{BC}v}{f} + \frac{v^2}{4f^2} \quad \text{or} \quad (5.00 \text{ m})^2 = \frac{d_{BC}v}{f} + \frac{v^2}{4f^2}
\]

\[
d_{BC} = \frac{(5.00 \text{ m})^2 - \frac{v^2}{4f^2}}{\frac{v}{f}} = \frac{(5.00 \text{ m})^2 - (343 \text{ m/s})^2}{4(125 \text{ Hz})^2} = 8.42 \text{ m}
\]

2. **REASONING** When the difference in path lengths traveled by the two sound waves is a half-integer number \( \left(\frac{1}{2}, \frac{1}{2}, 2\frac{1}{2}, \ldots\right) \) of wavelengths, the waves are out of phase and destructive interference occurs at the listener. The smallest separation \( d \) between the speakers is when the difference in path lengths is \( \frac{1}{2} \) of a wavelength, so \( d = \frac{1}{2} \lambda \). The
wavelength is, according to Equation 16.1, is equal to the speed $v$ of sound divided by the frequency $f$; \( \lambda = \frac{v}{f} \).

**SOLUTION** Substituting $\lambda = \frac{v}{f}$ into \( d = \frac{1}{2} \lambda \) gives
\[
d = \frac{1}{2} \lambda = \frac{1}{2} \left( \frac{v}{f} \right) = \frac{1}{2} \left( \frac{343 \text{ m/s}}{245 \text{ Hz}} \right) = 0.700 \text{ m}
\]

3. **REASONING** According to the principle of linear superposition, when two or more waves are present simultaneously at the same place, the resultant wave is the sum of the individual waves. We will use the fact that both pulses move at a speed of 1 cm/s to locate the pulses at the times $t = 1 \text{ s}, 2 \text{ s}, 3 \text{ s},$ and $4 \text{ s}$ and, by applying this principle to the places where the pulses overlap, determine the shape of the string.

**SOLUTION** The shape of the string at each time is shown in the following drawings:

4. **REASONING** The speakers are vibrating in phase. Therefore, in order for constructive interference to occur at point C, the difference in path lengths for the sound from speakers A and B must be zero or an integer number of wavelengths. Since the sound from speaker A
travels a greater distance in reaching point C than the sound from speaker B does, the difference in path lengths cannot be zero. The smallest value of \( d \) must, then, be associated with a path difference of one wavelength. The wavelength \( \lambda \) of a sound that has a frequency \( f \) and travels at a speed \( v \) is \( \lambda = \frac{v}{f} \) (Equation 16.1).

**SOLUTION** Using the Pythagorean theorem to express the distance between speaker A and point C, we set the difference in path lengths equal to one wavelength \( \lambda \):

\[
\sqrt{d^2 + d^2} - d = \lambda \quad \text{or} \quad \left( \sqrt{2} - 1 \right) d = \lambda
\]

Substituting \( \lambda = \frac{v}{f} \) (Equation 16.1) into this result gives

\[
\left( \sqrt{2} - 1 \right) d = \frac{v}{f} \quad \text{or} \quad d = \frac{v}{(\sqrt{2} - 1) f} = \frac{343 \text{ m/s}}{(\sqrt{2} - 1)(250 \text{ Hz})} = 3.3 \text{ m}
\]

5. **SSM REASONING** According to the principle of linear superposition, the resultant displacement due to two waves is the sum of the displacements due to each wave. In order to find the net displacement at the stated time and positions, then, we will calculate the individual displacements \( y_1 \) and \( y_2 \) and then find their sum. We note that the phase angles are measured in radians rather than degrees, so calculators must be set to the radian mode in order to yield valid results.

**SOLUTION**

a. At \( t = 4.00 \text{ s} \) and \( x = 2.16 \text{ m} \), the net displacement \( y \) of the string is

\[
y = y_1 + y_2 = (24.0 \text{ mm}) \sin \left( (9.00 \pi \text{ rad/s})(4.00 \text{ s}) - (1.25 \pi \text{ rad/m})(2.16 \text{ m}) \right) \\
+ (35.0 \text{ mm}) \sin \left( (2.88 \pi \text{ rad/s})(4.00 \text{ s}) + (0.400 \pi \text{ rad/m})(2.16 \text{ m}) \right) \\
= \boxed{+13.3 \text{ mm}}
\]

b. The time is still \( t = 4.00 \text{ s} \), but the position is now \( x = 2.56 \text{ m} \). Therefore, the net displacement \( y \) is

\[
y = y_1 + y_2 = (24.0 \text{ mm}) \sin \left( (9.00 \pi \text{ rad/s})(4.00 \text{ s}) - (1.25 \pi \text{ rad/m})(2.56 \text{ m}) \right) \\
+ (35.0 \text{ mm}) \sin \left( (2.88 \pi \text{ rad/s})(4.00 \text{ s}) + (0.400 \pi \text{ rad/m})(2.56 \text{ m}) \right) \\
= \boxed{+48.8 \text{ mm}}
\]
6. **REASONING** In Drawing 1 the two speakers are equidistant from the observer O. Since each wave travels the same distance in reaching the observer, the difference in travel-distances is zero, and constructive interference will occur for any frequency. Different frequencies will correspond to different wavelengths, but the path difference will always be zero. Condensations will always meet condensations and rarefactions will always meet rarefactions at the observation point. Since any frequency is acceptable in Drawing 1, our solution will focus on Drawing 2. In Drawing 2, destructive interference occurs only when the difference in travel distances for the two waves is an odd integer number \( n \) of half-wavelengths. Only certain frequencies, therefore, will be consistent with this requirement.

**SOLUTION** The frequency \( f \) and wavelength \( \lambda \) are related by \( \lambda = \frac{v}{f} \) (Equation 16.1), where \( v \) is the speed of the sound. Using this equation together with the requirement for destructive interference in drawing 2, we have

\[
\frac{\sqrt{L^2 + L^2} - L}{\text{Difference in travel distances}} = \frac{n \lambda}{2} = n \frac{v}{2f}
\]

where \( n = 1, 3, 5, \ldots \)

Here we have used the Pythagorean theorem to determine the length of the diagonal of the square. Solving for the frequency \( f \) gives

\[
f = \frac{nv}{2(\sqrt{2} - 1)L}
\]

The problem asks for the minimum frequency, so we choose \( n = 1 \) and obtain

\[
f = \frac{343 \text{ m/s}}{2(\sqrt{2} - 1)(0.75 \text{ m})} = 550 \text{ Hz}
\]

---

7. **SSM REASONING** The geometry of the positions of the loudspeakers and the listener is shown in the following drawing.

![Diagram](image)

The listener at C will hear either a loud sound or no sound, depending upon whether the interference occurring at C is constructive or destructive. If the listener hears no sound, destructive interference occurs, so
\[ d_2 - d_1 = \frac{n\lambda}{2} \quad n = 1, 3, 5, \ldots \] (1)

**SOLUTION**  Since \( v = \lambda f \), according to Equation 16.1, the wavelength of the tone is

\[
\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{68.6 \text{ Hz}} = 5.00 \text{ m}
\]

Speaker B will be closest to Speaker A when \( n = 1 \) in Equation (1) above, so

\[
d_2 = \frac{n\lambda}{2} + d_1 = \frac{5.00 \text{ m}}{2} + 1.00 \text{ m} = 3.50 \text{ m}
\]

From the figure above we have that,

\[
x_1 = (1.00 \text{ m}) \cos 60.0^\circ = 0.500 \text{ m}
\]

\[
y = (1.00 \text{ m}) \sin 60.0^\circ = 0.866 \text{ m}
\]

Then

\[
x_2^2 + y^2 = d_2^2 = (3.50 \text{ m})^2 \quad \text{or} \quad x_2 = \sqrt{(3.50 \text{ m})^2 - (0.866 \text{ m})^2} = 3.39 \text{ m}
\]

Therefore, the closest that speaker A can be to speaker B so that the listener hears no sound is \( x_1 + x_2 = 0.500 \text{ m} + 3.39 \text{ m} = 3.89 \text{ m} \).

8. **REASONING**  The two speakers are vibrating exactly out of phase.  This means that the conditions for constructive and destructive interference are opposite of those that apply when the speakers vibrate in phase, as they do in Example 1 in the text.  Thus, for two wave sources vibrating exactly out of phase, a difference in path lengths that is zero or an integer number \((1, 2, 3, \ldots)\) of wavelengths leads to destructive interference; a difference in path lengths that is a half-integer number \(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \ldots\) of wavelengths leads to constructive interference.  First, we will determine the wavelength being produced by the speakers.  Then, we will determine the difference in path lengths between the speakers and the observer and compare the differences to the wavelength in order to decide which type of interference occurs.

**SOLUTION**  According to Equation 16.1, the wavelength \( \lambda \) is related to the speed \( v \) and frequency \( f \) of the sound as follows:

\[
\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{429 \text{ Hz}} = 0.800 \text{ m}
\]
Since ABC in Figure 17.7 is a right triangle, the Pythagorean theorem applies and the difference $\Delta d$ in the path lengths is given by

$$\Delta d = d_{AC} - d_{BC} = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC}$$

We will now apply this expression for parts (a) and (b).

a. When $d_{BC} = 1.15 \text{ m}$, we have

$$\Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50 \text{ m})^2 + (1.15 \text{ m})^2} - 1.15 \text{ m} = 1.60 \text{ m}$$

Since $1.60 \text{ m} = 2(0.800 \text{ m}) = 2\lambda$, it follows that the interference is destructive (the speakers vibrate out of phase).

b. When $d_{BC} = 2.00 \text{ m}$, we have

$$\Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50 \text{ m})^2 + (2.00 \text{ m})^2} - 2.00 \text{ m} = 1.20 \text{ m}$$

Since $1.20 \text{ m} = 1.5(0.800 \text{ m}) = \left(1\frac{1}{2}\right)\lambda$, it follows that the interference is constructive (the speakers vibrate out of phase).

9. **REASONING** The fact that a loud sound is heard implies constructive interference, which occurs when the difference in path lengths is an integer number (1, 2, 3, ...) of wavelengths. This difference is $50.5 \text{ m} - 26.0 \text{ m} = 24.5 \text{ m}$. Therefore, constructive interference occurs when $24.5 \text{ m} = n\lambda$, where $n = 1, 2, 3, \ldots$. The wavelength is equal to the speed $v$ of sound divided by the frequency $f$; $\lambda = \frac{v}{f}$ (Equation 16.1). Substituting this relation for $\lambda$ into $24.5 \text{ m} = n\lambda$, and solving for the frequency gives

$$f = \frac{nv}{24.5 \text{ m}}$$

This relation will allow us to find the two lowest frequencies that the listener perceives as being loud due to constructive interference.

**SOLUTION** The lowest frequency occurs when $n = 1$:

$$f = \frac{nv}{24.5 \text{ m}} = \frac{(1)(343 \text{ m/s})}{24.5 \text{ m}} = 14 \text{ Hz}$$

This frequency lies below 20 Hz, so it cannot be heard by the listener. For $n = 2$ and $n = 3$, the frequencies are $28$ and $42 \text{ Hz}$, which are the two lowest frequencies that the listener perceives as being loud.
10. **Reasoning** When the listener is standing midway between the speakers, both sound waves travel the same distance from the speakers to the listener. Since the speakers are vibrating out of phase, when the diaphragm of one speaker is moving outward (creating a condensation), the diaphragm of the other speaker is moving inward (creating a rarefaction). Whenever a condensation from one speaker reaches the listener, it is met by a rarefaction from the other, and vice versa. Therefore, the two sound waves produce destructive interference, and the listener hears no sound.

When the listener begins to move sideways, the distance between the listener and each speaker is no longer the same. Consequently, the sound waves no longer produce destructive interference, and the sound intensity begins to increase. When the difference in path lengths $\ell_1 - \ell_2$ traveled by the two sounds is one-half a wavelength, or $\ell_1 - \ell_2 = \frac{1}{2} \lambda$, constructive interference occurs, and a loud sound will be heard.

**Solution** The two speakers are vibrating out of phase. Therefore, when the difference in path lengths $\ell_1 - \ell_2$ traveled by the two sounds is one-half a wavelength, or $\ell_1 - \ell_2 = \frac{1}{2} \lambda$, constructive interference occurs. Note that this condition is different than that for two speakers vibrating in phase. The frequency $f$ of the sound is equal to the speed $v$ of sound divided by the wavelength $\lambda$; $f = v/\lambda$. (Equation 16.1). Thus, we have that

$$\ell_1 - \ell_2 = \frac{1}{2} \lambda = \frac{v}{2f} \quad \text{or} \quad f = \frac{v}{2(\ell_1 - \ell_2)}$$

The distances $\ell_1$ and $\ell_2$ can be determined by applying the Pythagorean theorem to the right triangles in the drawing:

$$\ell_1 = \sqrt{(4.00 \text{ m})^2 + (1.50 \text{ m} + 0.92 \text{ m})^2} = 4.68 \text{ m}$$

$$\ell_2 = \sqrt{(4.00 \text{ m})^2 + (1.50 \text{ m} - 0.92 \text{ m})^2} = 4.04 \text{ m}$$

The frequency of the sound is

$$f = \frac{v}{2(\ell_1 - \ell_2)} = \frac{343 \text{ m/s}}{2(4.68 \text{ m} - 4.04 \text{ m})} = 270 \text{ Hz}$$

11. **Reasoning and Solution** Since $v = \lambda f$, the wavelength of the tone is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{73.0 \text{ Hz}} = 4.70 \text{ m}$$
The following drawing shows the line between the two speakers and the distances in question.

![Diagram](image)

Constructive interference will occur when the difference in the distances traveled by the two sound waves in reaching point P is an integer number of wavelengths. That is, when

\[(L - x) - x = n\lambda\]

where \(n\) is an integer (or zero). Solving for \(x\) gives

\[x = \frac{L - n\lambda}{2}\]  \hspace{1cm} (1)

When \(n = 0\), \(x = L/2 = (7.80 \text{ m})/2 = 3.90 \text{ m}\). This corresponds to the point halfway between the two speakers. Clearly in this case, each wave has traveled the same distance and therefore, they will arrive in phase.

When \(n = 1\),

\[x = \frac{(7.80 \text{ m}) - (4.70 \text{ m})}{2} = 1.55 \text{ m}\]

Thus, there is a point of constructive interference 1.55 m from speaker A. The points of constructive interference will occur symmetrically about the center point at \(L/2\), so there is also a point of constructive interference 1.55 m from speaker B, that is at the point 7.80 m – 1.55 m = 6.25 m from speaker A.

When \(n > 1\), the values of \(x\) obtained from Equation (1) will be negative. These values correspond to positions of constructive interference that lie to the left of A or to the right of C. They do not lie on the line between the speakers.

12. **REASONING**  The diffraction angle for the first minimum for a circular opening is given by Equation 17.2: \(\sin \theta = 1.22\lambda/D\), where \(D\) is the diameter of the opening.

**SOLUTION**  

a. Using Equation 16.1, we must first find the wavelength of the 2.0-kHz tone:

\[\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{2.0 \times 10^3 \text{ Hz}} = 0.17 \text{ m}\]

The diffraction angle for a 2.0-kHz tone is, therefore,
\[ \theta = \sin^{-1}\left(1.22 \times \frac{0.17 \text{ m}}{0.30 \text{ m}}\right) = 44^\circ \]

b. The wavelength of a 6.0-kHz tone is
\[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{6.0 \times 10^3 \text{ Hz}} = 0.057 \text{ m} \]

Therefore, if we wish to generate a 6.0-kHz tone whose diffraction angle is as wide as that for the 2.0-kHz tone in part (a), we will need a speaker of diameter \( D \), where
\[ D = \frac{1.22 \lambda}{\sin \theta} = \frac{(1.22)(0.057 \text{ m})}{\sin 44^\circ} = 0.10 \text{ m} \]

13. **SSM REASONING** Equation 17.1 specifies the diffraction angle \( \theta \) according to \( \sin \theta = \lambda / D \), where \( \lambda \) is the wavelength of the sound and \( D \) is the width of the opening. The wavelength depends on the speed and frequency of the sound. Since the frequency is the same in winter and summer, only the speed changes with the temperature. We can account for the effect of the temperature on the speed by assuming that the air behaves as an ideal gas, for which the speed of sound is proportional to the square root of the Kelvin temperature.

**SOLUTION** Equation 17.1 indicates that
\[ \sin \theta = \frac{\lambda}{D} \]

Into this equation, we substitute \( \lambda = v / f \) (Equation 16.1), where \( v \) is the speed of sound and \( f \) is the frequency:
\[ \sin \theta = \frac{\lambda}{D} = \frac{v / f}{D} \]

Assuming that air behaves as an ideal gas, we can use \( v = \sqrt{\gamma kT / m} \) (Equation 16.5), where \( \gamma \) is the ratio of the specific heat capacities at constant pressure and constant volume, \( k \) is Boltzmann’s constant, \( T \) is the Kelvin temperature, and \( m \) is the average mass of the molecules and atoms of which the air is composed:
\[ \sin \theta = \frac{v}{f D} = \frac{1}{f D} \sqrt{\frac{\gamma kT}{m}} \]

Applying this result for each temperature gives
\[ \sin \theta_{\text{summer}} = \frac{1}{f D} \sqrt{\frac{\gamma kT_{\text{summer}}}{m}} \quad \text{and} \quad \sin \theta_{\text{winter}} = \frac{1}{f D} \sqrt{\frac{\gamma kT_{\text{winter}}}{m}} \]

Dividing the summer-equation by the winter-equation, we find
\[
\frac{\sin \theta_{\text{summer}}}{\sin \theta_{\text{winter}}} = \frac{1}{fD} \sqrt{\frac{\gamma k T_{\text{summer}}}{m}} = \frac{T_{\text{summer}}}{T_{\text{winter}}}
\]

Thus, it follows that
\[
\sin \theta_{\text{summer}} = \sin \theta_{\text{winter}} \sqrt{\frac{T_{\text{summer}}}{T_{\text{winter}}}} = \sin 15.0^\circ \sqrt{\frac{311 \text{ K}}{273 \text{ K}}} = 0.276 \quad \text{or} \quad \theta_{\text{summer}} = \sin^{-1}(0.276) = 16.0^\circ
\]

14. **REASONING** The diffraction angle \( \theta \) depends upon the wavelength \( \lambda \) of the sound wave and the width \( D \) of the doorway according to \( \sin \theta = \frac{\lambda}{D} \) (Equation 17.1). We will determine the wavelength from the speed \( v \) and frequency \( f \) of the sound wave via \( v = f \lambda \) (Equation 16.1). We note that, in part (a), the frequency of the sound wave is given in kilohertz (kHz), so we will use the equivalence \( 1 \text{ kHz} = 10^3 \text{ Hz} \).

**SOLUTION**

a. Solving \( \sin \theta = \frac{\lambda}{D} \) (Equation 17.1) for \( \theta \), we obtain
\[
\theta = \sin^{-1}\left(\frac{\lambda}{D}\right)
\] (1)

Solving \( v = f \lambda \) (Equation 16.1) for \( \lambda \) yields \( \lambda = \frac{v}{f} \). Substituting this result into Equation (1), we find that
\[
\theta = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{v}{Df}\right)
\] (2)

Therefore, when the frequency of the sound wave is 5.0 kHz = 5.0\times10^3 \text{ Hz}, the diffraction angle is
\[
\theta = \sin^{-1}\left(\frac{v}{Df}\right) = \sin^{-1}\left[\frac{343 \text{ m/s}}{(0.77 \text{ m})(5.0\times10^3 \text{ Hz})}\right] = 5.1^\circ
\]

b. When the frequency of the sound wave is 5.0\times10^2 \text{ Hz}, the diffraction angle is
\[
\theta = \sin^{-1}\left(\frac{v}{Df}\right) = \sin^{-1}\left[\frac{343 \text{ m/s}}{(0.77 \text{ m})(5.0\times10^2 \text{ Hz})}\right] = 63^\circ
\]

15. **REASONING** For a rectangular opening ("single slit") such as a doorway, the diffraction angle \( \theta \) at which the first minimum in the sound intensity occurs is given by \( \sin \theta = \frac{\lambda}{D} \) (Equation 17.1), where \( \lambda \) is the wavelength of the sound and \( D \) is the width of the opening.
This relation can be used to find the angle provided we realize that the wavelength $\lambda$ is related to the speed $v$ of sound and the frequency $f$ by $\lambda = \frac{v}{f}$ (Equation 16.1).

**SOLUTION**

a. Substituting $\lambda = \frac{v}{f}$ into Equation 17.1 and using $D = 0.700$ m (only one door is open) gives

$$\sin \theta = \frac{\lambda}{D} = \frac{\frac{v}{f}}{D} = \frac{343 \text{ m/s}}{607 \text{ Hz} \times 0.700 \text{ m}} = 0.807 \quad \theta = \arcsin (0.807) = 53.8^\circ$$

b. When both doors are open, $D = 2 \times 0.700$ m and the diffraction angle is

$$\sin \theta = \frac{\lambda}{D} = \frac{\frac{v}{f}}{D} = \frac{343 \text{ m/s}}{607 \text{ Hz} \times 2 \times 0.700 \text{ m}} = 0.404 \quad \theta = \arcsin (0.404) = 23.8^\circ$$

16. **REASONING** The diffraction angle $\theta$ is determined by the ratio of the wavelength $\lambda$ of the sound to the diameter $D$ of the speaker, according to $\sin \theta = 1.22 \frac{\lambda}{D}$ (Equation 17.2). The wavelength is related to the frequency $f$ and the speed $v$ of the wave by $\lambda = \frac{v}{f}$ (Equation 16.1).

**SOLUTION** Substituting Equation 16.1 into Equation 17.2, we have

$$\sin \theta = 1.22 \frac{\frac{v}{f}}{D} = 1.22 \frac{v}{D f}$$

Since the speed of sound is a constant, this result indicates that the diffraction angle $\theta$ will be the same for each of the three speakers, provided that the diameter $D$ times the frequency $f$ has the same value. Thus, we pair the diameter and the frequency as follows:

<table>
<thead>
<tr>
<th>Diameter $D$</th>
<th>Frequency $f$</th>
<th>Product $Df$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050 m</td>
<td>$12.0 \times 10^3$ Hz</td>
<td>$6.0 \times 10^2$ m/s</td>
</tr>
<tr>
<td>0.10 m</td>
<td>$6.0 \times 10^3$ Hz</td>
<td>$6.0 \times 10^2$ m/s</td>
</tr>
<tr>
<td>0.15 m</td>
<td>$4.0 \times 10^3$ Hz</td>
<td>$6.0 \times 10^2$ m/s</td>
</tr>
</tbody>
</table>

The common value of the diffraction angle, then, is

$$\theta = \arcsin \left( 1.22 \frac{\frac{v}{D f}}{D f} \right) = \arcsin \left[ \frac{1.22 \times 343 \text{ m/s}}{6.0 \times 10^2 \text{ m/s}} \right] = 44^\circ$$

17. **REASONING AND SOLUTION** At 0 °C the speed of sound in air is given as 331 m/s in Table 16.1 in the text. This corresponds to a wavelength of

$$\lambda_1 = \frac{v}{f} = \frac{331 \text{ m/s}}{3.00 \times 10^3 \text{ Hz}} = 0.1103 \text{ m}$$

The diffraction angle is given by Equation 17.2 as
\[ \theta_1 = \sin^{-1}\left(\frac{1.22 \lambda}{D}\right) = \sin^{-1}\left[\frac{1.22(0.1103 \text{ m})}{0.175 \text{ m}}\right] = 50.3^\circ \]

For an ideal gas, the speed of sound is proportional to the square root of the Kelvin temperature, according to Equation 16.5. Therefore, the speed of sound at 29 °C is

\[ v = \left(331 \text{ m/s}\right)\sqrt{\frac{302 \text{ K}}{273 \text{ K}}} = 348 \text{ m/s} \]

The wavelength at this temperature is \( \lambda_2 = \frac{(348 \text{ m/s})/(3.00 \times 10^3 \text{ Hz})}{0.116 \text{ m}} \). This gives a diffraction angle of \( \theta_2 = 54.0^\circ \). The change in the diffraction angle is thus

\[ \Delta \theta = 54.0^\circ - 50.3^\circ = 3.7^\circ \]

18. **REASONING** The person does not hear a sound because she is sitting at a position where the first minimum in the single slit diffraction pattern occurs. This position is specified by the angle \( \theta \) according to \( \sin \theta = \frac{\lambda}{D} \) (Equation 17.1), where \( \lambda \) is the wavelength of the sound and \( D \) is the width of the slit (or diffraction horn). The wavelength \( \lambda \) of a sound that has a frequency \( f \) and travels at a speed \( v \) is \( \lambda = \frac{v}{f} \) (Equation 16.1). We will apply these equations to determine the angle \( \alpha \) for the frequency of 8100 Hz. Knowing \( \alpha \), we will apply the equations a second time for the angle \( \alpha/2 \) and thereby determine the unknown frequency.

**SOLUTION** The first diffraction minimum for an angle \( \theta \) is specified as follows by Equation 17.1:

\[ \sin \theta = \frac{\lambda}{D} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{\lambda}{D}\right) \]

Substituting \( \lambda = \frac{v}{f} \) (Equation 16.1) into this result we obtain

\[ \sin \theta = \frac{v}{fD} \quad (1) \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{v}{fD}\right) \quad (2) \]

Using Equation (2) to determine the angle \( \alpha \) for the frequency \( f = 8100 \text{ Hz} \), we obtain

\[ \alpha = \sin^{-1}\left(\frac{v}{fD}\right) = \sin^{-1}\left[\frac{343 \text{ m/s}}{(8100 \text{ Hz})(0.060 \text{ m})}\right] = 44.9^\circ \]

where we have carried an extra significant figure for this intermediate result. When the person moves to an angle \( \alpha/2 \), the unknown frequency at which the first diffraction minimum now occurs is different and can be found from Equation (1) with \( \theta = \alpha/2 \):

\[ \sin\left(\frac{\alpha}{2}\right) = \frac{v}{fD} \quad \text{or} \quad f = \frac{v}{D \sin(\alpha/2)} = \frac{343 \text{ m/s}}{(0.060 \text{ m})\sin(44.9^\circ/2)} = 1.5 \times 10^4 \text{Hz} \]
19. **Reasoning** The beat frequency of two sound waves is the difference between the two sound frequencies. From the graphs, we see that the period of the wave in the upper text figure is 0.020 s, so its frequency is $f_1 = 1/T_1 = 1/(0.020 \text{ s}) = 5.0 \times 10^1 \text{ Hz}$. The frequency of the wave in the lower figure is $f_2 = 1/(0.024 \text{ s}) = 4.2 \times 10^1 \text{ Hz}$.

**Solution** The beat frequency of the two sound waves is

$$f_{\text{beat}} = f_1 - f_2 = 5.0 \times 10^1 \text{ Hz} - 4.2 \times 10^1 \text{ Hz} = 8 \text{ Hz}$$

20. **Reasoning** The time between successive beats is the period of the beat frequency. The period that corresponds to any frequency is the reciprocal of that frequency. The beat frequency itself is the magnitude of the difference between the two sound frequencies produced by the pianos. Each of the piano frequencies can be determined as the speed of sound divided by the wavelength.

**Solution** According to Equation 10.5, the period $T$ (the time between successive beats) that corresponds to the beat frequency $f_{\text{beat}}$ is $T = 1/f_{\text{beat}}$. The beat frequency is the magnitude of the difference between the two sound frequencies $f_A$ and $f_B$, so that $f_{\text{beat}} = |f_A - f_B|$. With this substitution, the expression for the period becomes

$$T = \frac{1}{f_{\text{beat}}} = \frac{1}{|f_A - f_B|}$$

(1)

The frequencies $f_A$ and $f_B$ are given by Equation 16.1 as

$$f_A = \frac{v}{\lambda_A} \quad \text{and} \quad f_B = \frac{v}{\lambda_B}$$

Substituting these expressions into Equation (1) gives

$$T = \frac{1}{|f_A - f_B|} = \frac{1}{v} \left( \frac{\lambda_A}{\lambda_B} \right) = \frac{1}{343 \text{ m/s}} \left( \frac{343 \text{ m/s}}{0.769 \text{ m}} \right) = 0.25 \text{ s}$$

21. **Reasoning** The beat frequency is the difference between two sound frequencies. Therefore, the original frequency of the guitar string (before it was tightened) was either 3 Hz lower than that of the tuning fork (440.0 Hz − 3 Hz = 337 Hz) or 3 Hz higher (440.0 Hz + 3 Hz = 443 Hz):

\[
\begin{align*}
443 \text{ Hz} & \quad \{ \text{3-Hz beat frequency} \\ 
440.0 \text{ Hz} & \quad \{ \text{3-Hz beat frequency} \\ 
437 \text{ Hz} & \quad \{ \text{3-Hz beat frequency}
\end{align*}
\]
To determine which of these frequencies is the correct one (437 or 443 Hz), we will use the information that the beat frequency decreases when the guitar string is tightened.

**SOLUTION** When the guitar string is tightened, its frequency of vibration (either 437 or 443 Hz) increases. As the drawing below shows, when the 437-Hz frequency increases, it becomes closer to 440.0 Hz, so the beat frequency decreases. When the 443-Hz frequency increases, it becomes farther from 440.0 Hz, so the beat frequency increases. Since the problem states that the beat frequency decreases, the original frequency of the guitar string was **437 Hz**.

<table>
<thead>
<tr>
<th>443 Hz</th>
<th>440.0 Hz</th>
<th>Beat frequency increases</th>
</tr>
</thead>
<tbody>
<tr>
<td>437 Hz</td>
<td>440.0 Hz</td>
<td>Beat frequency decreases</td>
</tr>
</tbody>
</table>

**Tuning fork**  **Original string**  **Tightened string**

22. **REASONING** Two ultrasonic sound waves combine and form a beat frequency that is in the range of human hearing. The beat frequency is the difference between the two ultrasonic frequencies, that is, the larger minus the smaller of the two frequencies. One of the ultrasonic frequencies is 70 kHz. We can determine the smallest and the largest value for the other ultrasonic frequency by considering the implications of a beat frequency of 20 kHz on the one hand and 20 Hz on the other hand, since these two values define the limits of hearing for a healthy young person.

**SOLUTION** Let us first assume that the beat frequency $f_{\text{beat}}$ is 20 kHz. Noting that the symbols “$>$” and “$<$” mean, respectively, “greater than” and “less than”, we have the two following possibilities for the unknown ultrasonic frequency $f$:

- $f > 70 \text{ kHz}$
  
  $f_{\text{beat}} = 20 \text{ kHz} = f - 70 \text{ kHz} \quad \text{or} \quad f = (20 \text{ kHz}) + (70 \text{ kHz}) = 90 \text{ kHz}$

- $f < 70 \text{ kHz}$
  
  $f_{\text{beat}} = 20 \text{ kHz} = (70 \text{ kHz}) - f \quad \text{or} \quad f = (70 \text{ kHz}) - (20 \text{ kHz}) = 50 \text{ kHz}$

Next, we assume that the beat frequency $f_{\text{beat}}$ is 20 Hz. Ignoring significant figures, we now have the two following possibilities for the unknown ultrasonic frequency $f$:

- $f > 70 \text{ kHz}$
  
  $f_{\text{beat}} = 20 \times 10^{-3} \text{ kHz} = f - 70 \text{ kHz}$
  
  or $f = (20 \times 10^{-3} \text{ kHz}) + (70 \text{ kHz}) = 70.020 \text{ kHz}$

- $f < 70 \text{ kHz}$
  
  $f_{\text{beat}} = 20 \times 10^{-3} \text{ kHz} = (70 \text{ kHz}) - f$ (This is incorrect, should be $f_{\text{beat}} = 20 \text{ kHz} - f$)
  
  or $f = (70 \text{ kHz}) - (20 \times 10^{-3} \text{ kHz}) = 69.980 \text{ kHz}$
a. From these four values, we can see that the smallest possible frequency for the other ultrasonic wave is \(50\ \text{kHz}\).

b. From these four values, we can see that the largest possible frequency for the other ultrasonic wave is \(90\ \text{kHz}\).

23. \textbf{REASONING} When two frequencies are sounded simultaneously, the beat frequency produced is the difference between the two. Thus, knowing the beat frequency between the tuning fork and one flute tone tells us only the difference between the known frequency and the tuning-fork frequency. It does not tell us whether the tuning-fork frequency is greater or smaller than the known frequency. However, two different beat frequencies and two flute frequencies are given. Consideration of both beat frequencies will enable us to find the tuning-fork frequency.

\textbf{SOLUTION} The fact that a 1-Hz beat frequency is heard when the tuning fork is sounded along with the 262-Hz tone implies that the tuning-fork frequency is either 263 Hz or 261 Hz. We can eliminate one of these values by considering the fact that a 3-Hz beat frequency is heard when the tuning fork is sounded along with the 266-Hz tone. This implies that the tuning-fork frequency is either 269 Hz or 263 Hz. Thus, the tuning-fork frequency must be \(263\ \text{Hz}\).

24. \textbf{REASONING} The beat frequency heard by the bystander is the difference between the frequency of the sound that the bystander hears from the moving car and that from the (stationary) parked car. The bystander hears a frequency \(f_o\) from the moving horn that is greater than the emitted frequency \(f_s\). This is because of the Doppler effect (Section 16.9). The bystander is the observer of the sound wave emitted by the horn. Since the horn is moving toward the observer, more condensations and rarefactions of the wave arrive at the observer’s ear per second than would otherwise be the case. More cycles per second means that the observed frequency is greater than the emitted frequency.

The bystander also hears a frequency from the horn of the stationary car that is equal to the frequency \(f_s\) produced by the horn. Since this horn is stationary, there is no Doppler effect.

\textbf{SOLUTION} According to Equation 16.11, the frequency \(f_o\) that the bystander hears from the moving horn is

\[f_o = f_s \left( \frac{1}{1 - \frac{v_s}{v}} \right)\]

where \(f_s\) is the frequency of the sound emitted by the horn, \(v_s\) is the speed of the moving horn, and \(v\) is the speed of sound. The beat frequency heard by the bystander is \(f_o - f_s\), so we find that
25. **REASONING** When the wave created by the tuning fork is superposed on the sound wave traveling through the seawater, the beat frequency $f_{\text{beat}}$ heard by the underwater swimmer is equal to the difference between the two frequencies:

$$f_{\text{beat}} = f_{\text{wave}} - f_{\text{fork}}$$

We know the frequency $f_{\text{fork}}$ of the tuning fork, and we will determine the frequency $f_{\text{wave}}$ of the sound wave from its wavelength $\lambda$ and speed $v$ via $v = f_{\text{wave}}\lambda$ (Equation 16.1). The speed of a sound wave in a liquid is given by $v = \sqrt{\frac{B_{\text{ad}}}{\rho}}$ (Equation 16.6), where $B_{\text{ad}}$ is the adiabatic bulk modulus of the liquid, and $\rho$ is its density.

**SOLUTION** Solving $v = f_{\text{wave}}\lambda$ (Equation 16.1) for $f_{\text{wave}}$ yields $f_{\text{wave}} = \frac{v}{\lambda}$. Substituting this result into Equation (1), we obtain

$$f_{\text{beat}} = f_{\text{wave}} - f_{\text{fork}} = \frac{v}{\lambda} - f_{\text{fork}}$$

Substituting $v = \sqrt{\frac{B_{\text{ad}}}{\rho}}$ (Equation 16.6) into Equation (2), we find that

$$f_{\text{beat}} = \frac{v}{\lambda} - f_{\text{fork}} = \sqrt{\frac{2.31 \times 10^9 \text{ Pa}}{1025 \text{ kg/m}^3}} - \frac{440.0 \text{ Hz}}{3.35 \text{ m}} = 8 \text{ Hz}$$

26. **REASONING AND SOLUTION** The speed of the speakers is

$$v_s = 2\pi r/t = 2\pi(9.01 \text{ m})/(20.0 \text{ s}) = 2.83 \text{ m/s}$$

The sound that an observer hears coming from the right speaker is Doppler shifted to a new frequency given by Equation 16.11 as

$$f_{\text{OR}} = \frac{f_s}{1-v_s/v} = \frac{100.0 \text{ Hz}}{1-[(2.83 \text{ m/s})/(343.00 \text{ m/s})]} = 100.83 \text{ Hz}$$

The sound that an observer hears coming from the left speaker is shifted to a new frequency given by Equation 16.12 as

$$f_{\text{OL}} = \frac{f_s}{1+v_s/v}$$
The beat frequency heard by the observer is then

\[ 100.83 \text{ Hz} - 99.18 \text{ Hz} = 1.7 \text{ Hz} \]

27. **REASONING** The time it takes for a wave to travel the length \( L \) of the string is \( t = \frac{L}{v} \), where \( v \) is the speed of the wave. The speed can be obtained since the fundamental frequency is known, and Equation 17.3 (with \( n = 1 \)) gives the fundamental frequency as \( f_1 = \frac{v}{(2L)} \). The length is not needed, since it can be eliminated algebraically between this expression and the expression for the time.

**SOLUTION** Solving Equation 17.3 for the speed gives \( v = 2Lf_1 \). With this result for the speed, the time for a wave to travel the length of the string is

\[ t = \frac{L}{v} = \frac{L}{2Lf_1} = \frac{1}{2f_1} = \frac{1}{2(256 \text{ Hz})} = 1.95 \times 10^{-3} \text{ s} \]

28. **REASONING** For a vibrating string that is fixed at both ends, there are an integer number of half-wavelengths between the ends. Each half-wavelength gives rise to one loop in the standing wave. From this information, we can determine the wavelength. Once the wavelength \( \lambda \) is known, the speed \( v \) of the waves follows from \( v = f\lambda \) (Equation 16.1), since the frequency \( f \) is given in the statement of the problem.

**SOLUTION**

a. In the distance of 2.50 m there are five loops, so there are five half-wavelengths. Thus, the wavelength of the wave can be obtained by noting that

\[ 5 \left( \frac{\lambda}{2} \right) = 2.50 \text{ m} \quad \text{or} \quad \lambda = 1.00 \text{ m} \]

b. The speed of the wave can be obtained directly from Equation 16.1:

\[ v = f\lambda = (85.0 \text{ Hz})(1.00 \text{ m}) = 85.0 \text{ m/s} \]

c. The fundamental frequency arises when there is only one loop in the standing wave. The wavelength in that case is \( (1)(\lambda/2) = 2.50 \text{ m} \), or \( \lambda = 5.00 \text{ m} \). The fundamental frequency of the string can be obtained now from Equation 16.1:

\[ f = \frac{v}{\lambda} = \frac{85.0 \text{ m/s}}{5.00 \text{ m}} = 17.0 \text{ Hz} \]
29. **REASONING** The fundamental frequency \( f_1 \) is given by Equation 17.3 with \( n = 1 \): 
\[ f_1 = \frac{v}{2L} \]
Since values for \( f_1 \) and \( L \) are given in the problem statement, we can use this expression to find the speed of the waves on the cello string. Once the speed is known, the tension \( F \) in the cello string can be found by using Equation 16.2, 
\[ v = \sqrt{\frac{F}{m/L}} \]

**SOLUTION** Combining Equations 17.3 and 16.2 yields

\[ 2Lf_1 = \sqrt{\frac{F}{m/L}} \]

Solving for \( F \), we find that the tension in the cello string is

\[ F = 4L^2 f_1^2 (m/L) = 4(0.800 \text{ m})^2 (65.4 \text{ Hz})^2 (1.56 \times 10^{-2} \text{ kg/m}) = 171 \text{ N} \]

30. **REASONING** The harmonic frequencies are integer multiples of the fundamental frequency. Therefore, for wire A (on which there is a second-harmonic standing wave), the fundamental frequency is one half of 660 Hz, or 330 Hz. Similarly, for wire B (on which there is a third-harmonic standing wave), the fundamental frequency is one third of 660 Hz, or 220 Hz. The fundamental frequency \( f_1 \) is related to the length \( L \) of the wire and the speed \( v \) at which individual waves travel back and forth on the wire by \( f_1 = v/(2L) \) (Equation 17.3, with \( n = 1 \)). This relation will allow us to determine the speed of the wave on each wire.

**SOLUTION** Using Equation 17.3 with \( n = 1 \), we find

\[ f_1 = \frac{v}{2L} \quad \text{or} \quad v = 2Lf_1 \]

**Wire A** \[ v = 2(1.2 \text{ m})(330 \text{ Hz}) = 790 \text{ m/s} \]

**Wire B** \[ v = 2(1.2 \text{ m})(220 \text{ Hz}) = 530 \text{ m/s} \]

31. **REASONING** According to Equation 17.3, the fundamental \((n = 1)\) frequency of a string fixed at both ends is related to the wave speed \( v \) by \( f_1 = v/2L \), where \( L \) is the length of the string. Thus, the speed of the wave is \( v = 2Lf_1 \). Combining this with Equation 16.2, 
\[ v = \sqrt{\frac{F}{m/L}} \]
we find, after some rearranging, that

\[ \frac{F}{L^2} = 4f_1^2 (m/L) \]
Since the strings have the same tension and the same lengths between their fixed ends, we have
\[ f_{1E}^2 (m/L)_E = f_{1G}^2 (m/L)_G \]
where the symbols “E” and “G” represent the E and G strings on the violin. This equation can be solved for the linear density of the G string.

**SOLUTION** The linear density of the G string is
\[ (m/L)_G = \frac{f_{1E}^2}{f_{1G}^2} (m/L)_E = \left( \frac{f_{1E}}{f_{1G}} \right)^2 (m/L)_E \]
\[ = \left( \frac{659.3 \text{ Hz}}{196.0 \text{ Hz}} \right)^2 \left( 3.47 \times 10^{-4} \text{ kg/m} \right) = 3.93 \times 10^{-3} \text{ kg/m} \]

32. **REASONING** The series of natural frequencies for a wire fixed at both ends is given by \( f_n = n v/(2L) \) (Equation 17.3), where the harmonic number \( n \) takes on the integer values 1, 2, 3, etc. This equation can be solved for \( n \). The natural frequency \( f_n \) is the lowest frequency that the human ear can detect, and the length \( L \) of the wire is given. The speed \( v \) at which waves travel on the wire can be obtained from the given values for the tension and the wire’s linear density.

**SOLUTION** According to Equation 17.3, the harmonic number \( n \) is
\[ n = \frac{f_n 2L}{v} \]
The speed \( v \) is \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2), where \( F \) is the tension and \( m/L \) is the linear density. Substituting this expression into Equation (1) gives
\[ n = \frac{f_n 2L}{v} = \frac{f_n 2L}{\sqrt{\frac{F}{m/L}}} = \frac{m/L}{F} = (20.0 \text{ Hz}) 2(7.60 \text{ m}) \sqrt{0.0140 \text{ kg/m} \over 323 \text{ N}} = [2] \]

33. **REASONING** A standing wave is composed of two oppositely traveling waves. The speed \( v \) of these waves is given by \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2), where \( F \) is the tension in the string and \( m/L \) is its linear density (mass per unit length). Both \( F \) and \( m/L \) are given in the statement of the problem. The wavelength \( \lambda \) of the waves can be obtained by visually inspecting the standing wave pattern. The frequency of the waves is related to the speed of the waves and their wavelength by \( f = v/\lambda \) (Equation 16.1).
**SOLUTION**

a. The speed of the waves is

\[ v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{280 \text{ N}}{8.5 \times 10^{-3} \text{ kg/m}}} = 180 \text{ m/s} \]

b. Two loops of any standing wave comprise one wavelength. Since the string is 1.8 m long and consists of three loops (see the drawing), the wavelength is

\[ \lambda = \frac{2}{3} (1.8 \text{ m}) = 1.2 \text{ m} \]

c. The frequency of the waves is

\[ f = \frac{v}{\lambda} = \frac{180 \text{ m/s}}{1.2 \text{ m}} = 150 \text{ Hz} \]

34. **REASONING** Equation 17.3 (with \( n = 1 \)) gives the fundamental frequency as \( f_1 = \frac{v}{(2L)} \), where \( L \) is the wire’s length and \( v \) is the wave speed on the wire. The speed is given by Equation 16.2 as \( v = \sqrt{\frac{F}{m/L}} \), where \( F \) is the tension and \( m \) is the mass of the wire.

**SOLUTION** Using Equations 17.3 and 16.2, we obtain

\[ f_1 = \frac{v}{2L} = \frac{1}{2} \sqrt{\frac{F}{mL}} = \frac{1}{2} \sqrt{\frac{160 \text{ N}}{6.0 \times 10^{-3} \text{ kg} \cdot 0.41 \text{ m}}} = 130 \text{ Hz} \]

35. **REASONING** The fundamental frequency \( f_1 \) of the wire is given by \( f_1 = \frac{v}{(2L)} \) (Equation 17.3, with \( n = 1 \)), where \( v \) is the speed at which the waves travel on the wire and \( L \) is the length of the wire. The speed is related to the tension \( F \) in the wire according to \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2), where \( m/L \) is the mass per unit length of the wire.

The tension in the wire in Part 2 of the text drawing is less than the tension in Part 1. The reason is related to Archimedes’ principle (see Equation 11.6). This principle indicates that when an object is immersed in a fluid, the fluid exerts an upward buoyant force on the object. In Part 2 the upward buoyant force from the mercury supports part of the block’s weight, thus reducing the amount of the weight that the wire must support.

**SOLUTION** Substituting Equation 16.2 into Equation 17.3, we can obtain the fundamental frequency of the wire:
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\[ f_1 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F}{m/L}} \]  \hspace{1cm} (1)

In Part 1 of the text drawing, the tension \( F \) balances the weight of the block, keeping it from falling. The weight of the block is its mass times the acceleration due to gravity (see Equation 4.5). The mass, according to Equation 11.1 is the density \( \rho_{\text{copper}} \) times the volume \( V \) of the block. Thus, the tension in Part 1 is

**Part 1 tension** \[ F = (\text{mass}) g = \rho_{\text{copper}} V g \]

In Part 2 of the text drawing, the tension is reduced from this amount by the amount of the upward buoyant force. According to Archimedes’ principle, the buoyant force is the weight of the liquid mercury displaced by the block. Since half of the block’s volume is immersed, the volume of mercury displaced is \( \frac{1}{2} V \). The weight of this mercury is the mass times the acceleration due to gravity. Once again, according to Equation 11.1, the mass is the density \( \rho_{\text{mercury}} \) times the volume, which is \( \frac{1}{2} V \). Thus, the tension in Part 2 is

**Part 2 tension** \[ F = \rho_{\text{copper}} V g - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) g \]

With these two values for the tension we can apply Equation (1) to both parts of the drawing and obtain

**Part 1** \[ f_1 = \frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} V g}{m/L}} \]

**Part 2** \[ f_1 = \frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} V g - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) g}{m/L}} \]

Dividing the fundamental frequency of Part 2 by that of Part 1 gives

\[ \frac{f_{1, \text{Part 2}}}{f_{1, \text{Part 1}}} = \frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} V g - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) g}{m/L}} = \sqrt{\frac{\rho_{\text{copper}} - \frac{1}{2} \rho_{\text{mercury}}}{\rho_{\text{copper}}}} \]

\[ = \sqrt{\frac{8890 \text{ kg/m}^3 - \frac{1}{2}(13600 \text{ kg/m}^3)}{8890 \text{ kg/m}^3}} = 0.485 \]

36. **REASONING** The frequencies \( f_n \) of the standing waves allowed on a string fixed at both ends are given by Equation 17.3 as \( f_n = n \left( \frac{v}{2L} \right) \), where \( n \) is an integer that specifies the harmonic number, \( v \) is the speed of the traveling waves that make up the standing waves,
and \( L \) is the length of the string. The speed \( v \) is related to the tension \( F \) in the string and the linear density \( m/L \) via \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2). Therefore, the frequencies of the standing waves can be written as

\[
f_n = n \left( \frac{v}{2L} \right) = n \left( \frac{\sqrt{F/m/L}}{2L} \right) = n \frac{\sqrt{F}}{2L} \sqrt{m/L}
\]

The tension \( F \) in each string is provided by the weight \( W \) (either \( W_A \) or \( W_B \)) that hangs from the right end, so \( F = W \). Thus, the expression for \( f_n \) becomes \( f_n = n \frac{\sqrt{W}}{2L} \sqrt{m/L} \). We will use this relation to find the weight \( W_B \).

**SOLUTION**  String A has one loop so \( n = 1 \), and the frequency \( f_1^A \) of this standing wave is

\[
f_1^A = \frac{1}{2L} \sqrt{\frac{W_A}{m/L}}.
\]

String B has two loops so \( n = 2 \), and the frequency \( f_2^B \) of this standing wave is

\[
f_2^B = \frac{2}{2L} \sqrt{\frac{W_B}{m/L}}.
\]

We are given that the two frequencies are equal, so

\[
\frac{1}{2L} \sqrt{\frac{W_A}{m/L}} = \frac{2}{2L} \sqrt{\frac{W_B}{m/L}}
\]

Solving this expression for \( W_B \) gives

\[
W_B = \frac{1}{4} W_A = \frac{1}{4} (44 \text{ N}) = 11 \text{ N}
\]

37. **SSM REASONING** We can find the extra length that the D-tuner adds to the E-string by calculating the length of the D-string and then subtracting from it the length of the E string. For standing waves on a string that is fixed at both ends, Equation 17.3 gives the frequencies as \( f_n = n(v/2L) \). The ratio of the fundamental frequency of the D-string to that of the E-string is
\[
\frac{f_D}{f_E} = \frac{v/(2L_D)}{v/(2L_E)} = \frac{L_E}{L_D}
\]

This expression can be solved for the length \(L_D\) of the D-string in terms of quantities given in the problem statement.

**SOLUTION**  The length of the D-string is

\[
L_D = L_E \left( \frac{f_E}{f_D} \right) = (0.628 \text{ m}) \left( \frac{41.2 \text{ Hz}}{36.7 \text{ Hz}} \right) = 0.705 \text{ m}
\]

The length of the E-string is extended by the D-tuner by an amount

\[
L_D - L_E = 0.705 \text{ m} - 0.628 \text{ m} = 0.077 \text{ m}
\]

38. **REASONING** The beat frequency is equal to the higher frequency of the shorter string minus the lower frequency of the longer string. The reason the longer string has the lower frequency can be seen from the drawing, where it is evident that both strings are vibrating at their fundamental frequencies. The fundamental frequency of vibration \((n = 1)\) for a string fixed at each end is given by \(f_1 = v/(2L)\) (Equation 17.3). Since the speed \(v\) is the same for both strings (see the following paragraph), but the length \(L\) is greater for the longer string, the longer string vibrates at the lower frequency.

The waves on the longer string have the same speed as those on the shorter string. The speed \(v\) of a transverse wave on a string is given by \(v = \sqrt{F/(m/L)}\) (Equation 16.2), where \(F\) is the tension in the string and \(m/L\) is the mass per unit length (or linear density). Since \(F\) and \(m/L\) are the same for both strings, the speed of the waves is the same.

**SOLUTION** The beat frequency is the frequency of the shorter string minus the frequency of the longer string: \(f_{\text{shorter}} - f_{\text{longer}}\). We are given that \(f_{\text{shorter}} = 225 \text{ Hz}\).

According to Equation 17.3 with \(n = 1\), we have \(f_{\text{longer}} = v/(2L_{\text{longer}})\), where \(L_{\text{longer}}\) is the length of the longer string. According to the drawing, we have \(L_{\text{longer}} = L + 0.0057 \text{ m}\). Thus,

\[
f_{\text{longer}} = \frac{v}{2L_{\text{longer}}} = \frac{v}{2(L + 0.0057 \text{ m})}
\]

Since the speed \(v\) of the waves on the longer string is the same as those on the shorter string, \(v = 41.8 \text{ m/s}\). The length \(L\) of the shorter string can be obtained directly from Equation 17.3:
Substituting this number back into the expression for $f_{\text{longer}}$ yields

$$f_{\text{longer}} = \frac{v}{2(L + 0.0057 \text{ m})} = \frac{41.8 \text{ m/s}}{2(0.0929 \text{ m} + 0.0057 \text{ m})} = 212 \text{ Hz}$$

The beat frequency is $f_{\text{shorter}} - f_{\text{longer}} = 225 \text{ Hz} - 212 \text{ Hz} = 13 \text{ Hz}$.

**39. SSM REASONING**  The beat frequency produced when the piano and the other instrument sound the note (three octaves higher than middle C) is $f_{\text{beat}} = f - f_0$, where $f$ is the frequency of the piano and $f_0$ is the frequency of the other instrument ($f_0 = 2093 \text{ Hz}$). We can find $f$ by considering the temperature effects and the mechanical effects that occur when the temperature drops from 25.0 °C to 20.0 °C.

**SOLUTION**  The fundamental frequency $f_0$ of the wire at 25.0 °C is related to the tension $F_0$ in the wire by

$$f_0 = \frac{v}{2L_0} = \frac{\sqrt{F_0/(m/L)}}{2L_0}$$

where Equations 17.3 and 16.2 have been combined.

The amount $\Delta L$ by which the piano wire attempts to contract is (see Equation 12.2)

$$\Delta L = \alpha L_0 \Delta T$$

where $\alpha$ is the coefficient of linear expansion of the wire, $L_0$ is its length at 25.0 °C, and $\Delta T$ is the amount by which the temperature drops. Since the wire is prevented from contracting, there must be a stretching force exerted at each end of the wire. According to Equation 10.17, the magnitude of this force is

$$\Delta F = Y \left( \frac{\Delta L}{L_0} \right) A$$

where $Y$ is the Young's modulus of the wire, and $A$ is its cross-sectional area. Combining this relation with Equation 12.2, we have

$$\Delta F = Y \left( \frac{\alpha L_0 \Delta T}{L_0} \right) A = \alpha(\Delta T)YA$$

Thus, the frequency $f$ at the lower temperature is

$$f = \frac{v}{2L_0} = \frac{\sqrt{(F_0 + \Delta F)/(m/L)}}{2L_0} = \frac{\sqrt{F_0 + \alpha(\Delta T)YA}/(m/L)}}{2L_0}$$

(2)
Using Equations (1) and (2), we find that the frequency \( f \) is

\[
f = f_0 \sqrt{\frac{F_0 + \alpha(\Delta T)Y A}{(m/L)}} = f_0 \sqrt{\frac{F_0 + \alpha(\Delta T)Y A}{F_0}}
\]

\[
f = (2093 \text{ Hz}) \sqrt{\frac{818.0 \text{ N} + (12 \times 10^{-6} / \text{C}^2)(5.0 \text{ C}^2)(2.0 \times 10^{11} \text{ N/m}^2)(7.85 \times 10^{-7} \text{ m}^2)}{818.0 \text{ N}}} = 2105 \text{ Hz}
\]

Therefore, the beat frequency is \( 2105 \text{ Hz} - 2093 \text{ Hz} = 12 \text{ Hz} \).

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40. **REASONING AND SOLUTION** We are given that \( f_j = \sqrt[12]{2} f_{j-1} \).

a. The length of the unfretted string is \( L_0 = v/(2f_0) \) and the length of the string when it is pushed against fret 1 is \( L_1 = v/(2f_1) \). The distance between the frets is

\[
L_0 - L_1 = \frac{v}{2f_0} - \frac{v}{2f_1} = \left( \frac{v}{2f_0} \right) \left( 1 - \frac{f_0}{f_1} \right) = \left( \frac{v}{2f_0} \right) \left( 1 - \frac{1}{\sqrt[12]{2}} \right)
\]

\[
= L_0 \left( 1 - \frac{1}{\sqrt[12]{2}} \right) = (0.628 \text{ m})(0.0561) = 0.0352 \text{ m}
\]

b. The frequencies corresponding to the sixth and seventh frets are \( f_6 = \left( \sqrt[12]{2} \right)^6 f_0 \) and \( f_7 = \left( \sqrt[12]{2} \right)^7 f_0 \). The distance between fret 6 and fret 7 is

\[
L_6 - L_7 = \frac{v}{2f_6} - \frac{v}{2f_7}
\]

\[
= \frac{v}{2 \left( \sqrt[12]{2} \right)^6 f_0} - \frac{v}{2 \left( \sqrt[12]{2} \right)^7 f_0} = \left( \frac{v}{2f_0} \right) \left[ \frac{1}{\left( \sqrt[12]{2} \right)^6} - \frac{1}{\left( \sqrt[12]{2} \right)^7} \right]
\]

\[
= L_0 \left[ \frac{1}{\left( \sqrt[12]{2} \right)^6} - \frac{1}{\left( \sqrt[12]{2} \right)^7} \right] = (0.628 \text{ m})(0.0397) = 0.0249 \text{ m}
\]

---

41. **SSM REASONING** Equation 17.5 (with \( n = 1 \)) gives the fundamental frequency as \( f_1 = v/(4L) \), where \( L \) is the length of the auditory canal and \( v \) is the speed of sound.

**SOLUTION** Using Equation 17.5, we obtain
42. **REASONING** The fundamental \((n = 1)\) frequency for a tube open at both ends is \(f_1 = v/(2L)\) (Equation 17.4). The fundamental frequency of a tube open at only one end is \(f_1 = v/(4L)\) (Equation 17.5). Since we know the fundamental frequency of the tube open at only one end, we can use these relations to determine the other fundamental frequency.

**SOLUTION**

a. The ratio of the fundamental frequency of a tube open at both ends to that of a tube open at only one end is

\[
\frac{(f_1)_{\text{open at both ends}}}{(f_1)_{\text{open at only one end}}} = \frac{\frac{v}{2L}}{\frac{v}{4L}} = 2
\]

Thus,

\[
(f_1)_{\text{open at both ends}} = 2(f_1)_{\text{open at only one end}} = 2(130.8 \text{ Hz}) = 261.6 \text{ Hz}
\]

b. The length \(L\) of the tube can be found from either Equation 17.4 or 17.5. Solving \(f_1 = v/(2L)\) (Equation 17.4) for \(L\) reveals that

\[
L = \frac{v}{2f_1} = \frac{343 \text{ m/s}}{2(261.6 \text{ Hz})} = 0.656 \text{ m}
\]

43. **REASONING** The frequency of a pipe open at both ends is given by Equation 17.4 as

\[
f_n = n \left( \frac{v}{2L} \right)
\]

where \(n\) is an integer specifying the harmonic number, \(v\) is the speed of sound, and \(L\) is the length of the pipe.

**SOLUTION** Solving the equation above for \(L\), and recognizing that \(n = 3\) for the third harmonic, we have

\[
L = n \left( \frac{v}{2f_n} \right) = 3 \left[ \frac{343 \text{ m/s}}{2(262 \text{ Hz})} \right] = 1.96 \text{ m}
\]

44. **REASONING AND SOLUTION** We know that \(L = v/(2f)\). For 20.0 Hz

\[
L = (343 \text{ m/s})/[2(20.0 \text{ Hz})] = 8.6 \text{ m}
\]

For 20.0 kHz

\[
L = (343 \text{ m/s})/[2(20.0 \times 10^3 \text{ Hz})] = 8.6 \times 10^{-3} \text{ m}
\]
45. **REASONING** The fundamental frequency \( f_1^A \) of air column A, which is open at both ends, is given by Equation 17.4 with \( n = 1 \): 
\[
f_1^A = \left(1\right) \left(\frac{v}{2L_A}\right)
\]
where \( v \) is the speed of sound in air and \( L_A \) is the length of the air column. Similarly, the fundamental frequency \( f_1^B \) of air column B, which is open at only one end, can be expressed using Equation 17.5 with \( n = 1 \): 
\[
f_1^B = \left(1\right) \left(\frac{v}{4L_B}\right)
\]
These two relations will allow us to determine the length of air column B.

**SOLUTION** Since the fundamental frequencies of the two air columns are the same 
\( f_1^A = f_1^B \), so that 
\[
\left(1\right) \left(\frac{v}{2L_A}\right) = \left(1\right) \left(\frac{v}{4L_B}\right)
\]
or 
\[
L_B = \frac{1}{2} L_A = \frac{1}{2} (0.70 \text{ m}) = 0.35 \text{ m}
\]

46. **REASONING** For a tube open at both ends, the sequence of natural frequencies \( f_n \) is given by \( f_n = n \left(\frac{v}{2L}\right) \) (Equation 17.4), where \( n = 1, 2, 3, 4, ... \). The speed of sound is \( v \), and the length of the tube is \( L \). For the fundamental frequency \( n = 1 \). This equation can be solved for the lengths of the piccolo and the flute, and the desired ratio obtained from the results.

**SOLUTION** Solving the expression for the fundamental frequency \( f_1 \) (Equation 17.4 with \( n = 1 \)) for the length \( L \), we obtain 
\[
f_1 = \frac{v}{2L} \quad \text{or} \quad L = \frac{v}{2f_1}
\]
Dividing \( L_{\text{piccolo}} \) by \( L_{\text{flute}} \) gives 
\[
\frac{L_{\text{piccolo}}}{L_{\text{flute}}} = \frac{\frac{v}{2f_1}}{\frac{v}{2(f_1)_{\text{flute}}}} = \frac{(f_1)_{\text{flute}}}{(f_1)_{\text{piccolo}}} = \frac{261.6 \text{ Hz}}{587.3 \text{ Hz}} = 0.445
\]

47. **SSM REASONING** The natural frequencies of a tube open at only one end are given by Equation 17.5 as 
\( f_n = n \left(\frac{v}{4L}\right) \), where \( n \) is any odd integer \( (n = 1, 3, 5, ...) \), \( v \) is the speed of sound, and \( L \) is the length of the tube. We can use this relation to find the value for \( n \) for the 450-Hz sound and to determine the length of the pipe.
**SOLUTION**

a. The frequency $f_n$ of the 450-Hz sound is given by $450 \text{ Hz} = n \left( \frac{v}{4L} \right)$. Likewise, the frequency of the next higher harmonic is $750 \text{ Hz} = (n + 2) \left( \frac{v}{4L} \right)$, because $n$ is an odd integer and this means that the value of $n$ for the next higher harmonic must be $n + 2$. Taking the ratio of these two relations gives

$$\frac{750 \text{ Hz}}{450 \text{ Hz}} = \frac{\left( n + 2 \right) \left( \frac{v}{4L} \right)}{n \left( \frac{v}{4L} \right)} = \frac{n + 2}{n}$$

Solving this equation for $n$ gives $n = 3$.

b. Solving the equation $450 \text{ Hz} = n \left( \frac{v}{4L} \right)$ for $L$ and using $n = 3$, we find that the length of the tube is

$$L = n \left( \frac{v}{4f_n} \right) = 3 \left[ \frac{343 \text{ m/s}}{4(450 \text{ Hz})} \right] = 0.57 \text{ m}$$

---

48. **REASONING** According to Equation 16.1, the frequency $f$ of the vibrations in the rod is related to the wavelength $\lambda$ and speed $v$ of the sound waves by

$$f = \frac{v}{\lambda} \quad (16.1)$$

The distance between adjacent antinodes of a longitudinal standing wave is equal to half a wavelength $\left( \frac{1}{2} \lambda \right)$. This rod has antinodes at the ends only, so the length $L$ of the rod is equal to half a wavelength. Put another way, the wavelength is twice the length of the rod:

$$\lambda = 2L \quad (1)$$

The speed $v$ of sound in the rod depends upon the values of Young’s modulus $Y$ and the density $\rho$ of aluminum via Equation 16.7:

$$v = \sqrt{\frac{Y}{\rho}} \quad (16.7)$$

**SOLUTION** Substituting Equation (1) into Equation 16.1 yields

$$f = \frac{v}{\lambda} = \frac{v}{2L} \quad (2)$$

Substituting Equation 16.7 into Equation (2), we obtain

$$f = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{Y}{\rho}} = \frac{1}{2(1.2 \text{ m})} \sqrt{\frac{6.9\times10^{10} \text{ N/m}^2}{2700 \text{ kg/m}^3}} = 2100 \text{ Hz}$$
49. **REASONING** The well is open at the top and closed at the bottom, so it can be approximated as a column of air that is open at only one end. According to Equation 17.5, the natural frequencies for such an air column are

\[ f_n = n \left( \frac{v}{4L} \right) \quad \text{where} \quad n = 1, 3, 5, \ldots \]

The depth \( L \) of the well can be calculated from the speed of sound, \( v = 343 \text{ m/s} \), and a knowledge of the natural frequencies \( f_n \).

**SOLUTION** We know that two of the natural frequencies are 42 and 70.0 Hz. The ratio of these two frequencies is

\[ \frac{70.0 \text{ Hz}}{42 \text{ Hz}} = \frac{5}{3} \]

Therefore, the value of \( n \) for each frequency is \( n = 3 \) for the 42-Hz sound, and \( n = 5 \) for the 70.0-Hz sound. Using \( n = 3 \), for example, the depth of the well is

\[ L = \frac{nv}{4f_3} = \frac{3(343 \text{ m/s})}{4(42 \text{ Hz})} = 6.1 \text{ m} \]

50. **REASONING** The pressure \( P_2 \) at a depth \( h \) in a static fluid such as the mercury column is given by \( P_2 = P_{\text{atm}} + \rho gh \) (Equation 11.4), where \( P_{\text{atm}} = 1.01 \times 10^5 \text{ Pa} \) is the air pressure at the surface of the fluid, \( \rho \) is the density of the fluid, and \( g \) is the magnitude of the acceleration due to gravity. Because the air-filled portion of the tube is open at one end, it can have standing waves with natural frequencies given by \( f_n = n \left( \frac{v}{4L} \right) \) (Equation 17.5), where \( n \) can take on only odd integral values \((n = 1, 3, 5, \ldots)\), and \( v \) is the speed of sound in air. The mercury decreases the effective length of the air-filled portion of the tube from its initial length \( L_0 = 0.75 \text{ m} \) to its final length \( L \). The third harmonic \( f_{3,0} \) of the original tube is found by choosing \( n = 3 \) in Equation 17.5, and the fundamental frequency \( f_1 \) of the shortened tube is found by choosing \( n = 1 \). We note that the height \( h \) of the mercury column is equal to the difference between the original and final lengths of the air in the tube: \( h = L_0 - L \).

**SOLUTION** Substituting \( h = L_0 - L \) into \( P_2 = P_{\text{atm}} + \rho gh \) (Equation 11.4), we obtain

\[ P_2 = P_{\text{atm}} + \rho g (L_0 - L) \quad (1) \]

The third harmonic frequency of the tube at its initial length \( L_0 \) and the fundamental frequency \( f_1 \) of the tube at its final length \( L \) are equal, so from \( f_n = n \left( \frac{v}{4L} \right) \) (Equation 17.5), we find that
\[ f_{3,0} = 3 \left( \frac{v}{AL_0} \right) = 1 \left( \frac{v}{AL} \right) = f_1 \quad \text{or} \quad \frac{3}{L_0} = \frac{1}{L} \quad \text{or} \quad L = \frac{L_0}{3} \]  

(2)

Substituting Equation (2) into Equation (1), we obtain

\[ P_2 = P_{\text{atm}} + \rho g (L_0 - L) = P_{\text{atm}} + \rho g \left( L_0 - \frac{1}{3} L_0 \right) = P_{\text{atm}} + \frac{2}{3} \rho g L_0 \]

Therefore, the pressure at the bottom of the mercury column is

\[ P_2 = 1.01 \times 10^5 \text{ Pa} + \frac{2}{3} (13600 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.75 \text{ m}) = 1.68 \times 10^5 \text{ Pa} \]

51. **REASONING** We will make use of the series of natural frequencies (including the first overtone frequency) given by \( f_n = \frac{n v}{2L} \) (Equation 17.4), for a tube open at both ends. In this expression, \( n \) takes on the integer values 1, 2, 3, etc. and has the value of \( n = 2 \) for the first overtone frequency that is given. We can solve Equation 17.4 for \( L \), but must deal with the fact that no value is given for the speed \( v \) of sound in gas B. To obtain the necessary value, we will use the fact that both gases are ideal gases and utilize the speed given for gas A and the masses of the two types of molecules.

**SOLUTION** According to Equation 17.4 as applied to gas B, we have \( f_n = \frac{n v_B}{2L} \), which can be solved for \( L \) to show that

\[ L = \frac{nv_B}{2f_n} \]  

(1)

Since both gases are ideal gases, the speed is given by \( v = \sqrt{\frac{\gamma kT}{m}} \) (Equation 16.5), where \( \gamma \) is the ratio of the specific heat capacities at constant pressure and constant volume, \( k \) is Boltzmann’s constant, \( T \) is the Kelvin temperature, and \( m \) is the mass of a molecule of the gas. Noting that \( \gamma \) and \( T \) are the same for each gas, we can apply this expression to both gases:

\[ v_A = \sqrt{\frac{\gamma kT}{m_A}} \quad \text{and} \quad v_B = \sqrt{\frac{\gamma kT}{m_B}} \]

Dividing the expression for gas B by that for gas A gives

\[ \frac{v_B}{v_A} = \sqrt{\frac{m_A}{m_B}} = \sqrt{\frac{m_A}{m_B}} \quad \text{or} \quad v_B = v_A \sqrt{\frac{m_A}{m_B}} \]

Substituting this result for \( v_B \) into Equation (1), we find that

\[ L = \frac{nv_B}{2f_n} = \frac{nv_A}{2f_n} \sqrt{\frac{m_A}{m_B}} \]  

\[ = \frac{2(259 \text{ m/s})}{2(386 \text{ Hz})} \sqrt{\frac{7.31 \times 10^{-26} \text{ kg}}{1.06 \times 10^{-25} \text{ kg}}} = 0.557 \text{ m} \]
52. **REASONING AND SOLUTION** The original tube has a fundamental given by \( f = \frac{v}{4L} \), so that its length is \( L = \frac{v}{4f} \). The cut tube that has one end closed has a length of \( L_c = \frac{v}{4f_c} \), while the cut tube that has both ends open has a length \( L_o = \frac{v}{2f_o} \).

We know that \( L = L_c + L_o \). Substituting the expressions for the lengths and solving for \( f \) gives

\[
\frac{f_o f_c}{2f_c + f_o} = \frac{(425 \text{ Hz})(675 \text{ Hz})}{2(675 \text{ Hz}) + 425 \text{ Hz}} = 162 \text{ Hz}
\]

53. **REASONING** According to Equation 17.3, the length \( L \) of the string is related to its third harmonic (\( n = 3 \)) frequency \( f_3 \) and the speed \( v \) of the waves on the string by

\[
f_3 = 3\left(\frac{v}{2L}\right) \quad \text{or} \quad L = \frac{3v}{2f_3}
\]

The speed \( v \) of the waves is found from Equation 16.2:

\[
v = \sqrt{\frac{F}{m/L}} \quad (16.2)
\]

Here, \( F \) is the tension in the string and the ratio \( m/L \) is its linear density.

**SOLUTION** Substituting Equation 16.2 into Equation (1), then, gives the length of the string as:

\[
L = \frac{3v}{2f_3} = \frac{3}{2f_3} \sqrt{\frac{F}{m/L}}
\]

Although \( L \) appears on both sides of Equation (2), no further algebra is required. This is because \( L \) appears in the ratio \( m/L \) on the right side. This ratio is the linear density of the string, which has a known value of \( 5.6 \times 10^{-3} \) kg/m. Therefore, the length of the string is

\[
L = \frac{3}{2f_3} \sqrt{\frac{3.3 \text{ N}}{2(130 \text{ Hz}) \sqrt{5.6 \times 10^{-3} \text{ kg/m}}}} = 0.28 \text{ m}
\]

54. **REASONING** The speed \( v \) of sound is related to its frequency \( f \) and wavelength \( \lambda \) by \( v = f \lambda \) (Equation 16.1). At the end of the tube where the tuning fork is, there is an antinode, because the gas molecules there are free to vibrate. At the plunger, there is a node, because the gas molecules there are not free to vibrate. Since there is an antinode at one end of the tube and a node at the other, the smallest value of \( L \) occurs when the length of the tube is one quarter of a wavelength.

**SOLUTION** Since the smallest value for \( L \) is a quarter of a wavelength, we have \( L = \frac{1}{4} \lambda \) or \( \lambda = 4L \). According to Equation 16.1, the speed of sound is

\[
v = f \lambda = f (4L) = (485 \text{ Hz})(4 \times 0.264 \text{ m}) = 512 \text{ m/s}
\]
55. **REASONING** When constructive interference occurs again at point C, the path length difference is two wavelengths, or \( \Delta s = 2\lambda = 3.20 \text{ m} \). Therefore, we can write the expression for the path length difference as

\[
\Delta s = s_{AC} - s_{BC} = \sqrt{s_{AB}^2 + s_{BC}^2} - s_{BC} = 3.20 \text{ m}
\]

This expression can be solved for \( s_{AB} \).

**SOLUTION** Solving for \( s_{AB} \), we find that

\[
s_{AB} = \sqrt{(3.20 \text{ m} + 2.40 \text{ m})^2 - (2.40 \text{ m})^2} = 5.06 \text{ m}
\]

56. **REASONING** Let \( L_A \) be length of the first pipe, and \( L_B \) be the final length of the second pipe. The length \( \Delta L \) removed from the second pipe, then, is

\[
\Delta L = L_A - L_B
\]

Both pipes are open at one end only, so their lengths \( L \) are related to their fundamental frequencies \( f_1 \) and the speed \( v \) of sound in air by \( f_1 = \frac{v}{4L} \) (Equation 17.5 with \( n = 1 \)), or

\[
L = \frac{v}{4f_1}
\]

The beat frequency \( f_{\text{beat}} \) that occurs when both pipes are vibrating at their fundamental frequencies is the difference between the higher frequency \( f_{1,B} \) of the second pipe and the lower frequency \( f_{1,A} \) of the first pipe:

\[
f_{\text{beat}} = f_{1,B} - f_{1,A}
\]

**SOLUTION** Substituting Equation (2) into Equation (1) yields

\[
\Delta L = L_A - L_B = \frac{v}{4f_{1,A}} - \frac{v}{4f_{1,B}} = \frac{v}{4} \left( \frac{1}{f_{1,A}} - \frac{1}{f_{1,B}} \right)
\]

Solving Equation (3) for the unknown frequency of the shortened pipe, we find that

\[
f_{1,B} = f_{1,A} + f_{\text{beat}}
\]

Substituting Equation (5) into Equation (4), we obtain

\[
\Delta L = \frac{v}{4} \left( \frac{1}{f_{1,A}} - \frac{1}{f_{1,B}} \right) = \frac{v}{4} \left( \frac{1}{f_{1,A}} - \frac{1}{f_{1,A} + f_{\text{beat}}} \right) = \left( \frac{343 \text{ m/s}}{4} \right) \left( \frac{1}{256 \text{ Hz}} - \frac{1}{256 \text{ Hz} + 12 \text{ Hz}} \right) = 0.015 \text{ m}
\]

Therefore, the second pipe has been shortened by cutting off

\[
\left( 0.015 \text{ m} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) = 1.5 \text{ cm}
\]
57. **REASONING** For a tube open at only one end, the series of natural frequencies is given by
\[ f_n = \frac{n\nu}{4L} \] (Equation 17.5), where \( n \) has the values 1, 3, 5, etc., \( \nu \) is the speed of sound, and \( L \) is the tube length. We will apply this expression to both the air-filled and the helium-filled tube in order to determine the desired ratio.

**SOLUTION** According to Equation 17.5, we have
\[ f_{n,\text{air}} = \frac{n\nu_{\text{air}}}{4L} \quad \text{and} \quad f_{n,\text{helium}} = \frac{n\nu_{\text{helium}}}{4L} \]

Dividing the expression for helium by the expression for air, we find that
\[ \frac{f_{n,\text{helium}}}{f_{n,\text{air}}} = \frac{n\nu_{\text{helium}}}{4L} = \frac{\nu_{\text{helium}}}{\nu_{\text{air}}} = \frac{1.00 \times 10^3 \text{ m/s}}{343 \text{ m/s}} = 2.92 \]

58. **REASONING** According to the principle of linear superposition, when two or more waves are present simultaneously at the same place, the resultant wave is the sum of the individual waves. We will use the fact that the pulses move at a speed of 1 cm/s to locate the pulses at the times \( t = 1 \text{ s}, 2 \text{ s}, 3 \text{ s}, \text{ and } 4 \text{ s} \) and, by applying this principle to the places where the pulses overlap, determine the shape of the string.

**SOLUTION** The shape of the string at each time is shown in the following drawings:
59. **REASONING** For standing waves on a string that is clamped at both ends, Equations 17.3 and 16.2 indicate that the standing wave frequencies are

\[ f_n = n \left( \frac{v}{2L} \right) \quad \text{where} \quad v = \sqrt{\frac{F}{m/L}} \]

Combining these two expressions, we have, with \( n = 1 \) for the fundamental frequency,

\[ f_1 = \frac{1}{2L} \sqrt{\frac{F}{m/L}} \]

This expression can be used to find the ratio of the two fundamental frequencies.

**SOLUTION** The ratio of the two fundamental frequencies is

\[ \frac{f_{\text{old}}}{f_{\text{new}}} = \frac{1}{2L} \sqrt{\frac{F_{\text{old}}}{m/L}} = \sqrt{\frac{F_{\text{old}}}{F_{\text{new}}}} \]

Since \( F_{\text{new}} = 4F_{\text{old}} \), we have

\[ f_{\text{new}} = f_{\text{old}} \sqrt{\frac{F_{\text{new}}}{F_{\text{old}}}} = f_{\text{old}} \sqrt{4} = f_{\text{old}} \sqrt{4} = (55.0 \text{ Hz}) (2) = 1.10 \times 10^2 \text{ Hz} \]

60. **REASONING** Each string has a node at each end, so the frequency of vibration is given by Equation 17.3 as \( f_n = n \nu/(2L) \), where \( n = 1, 2, 3, \ldots \) The speed \( \nu \) of the wave can be determined from Equation 16.2 as \( \nu = \sqrt{F/(m/L)} \). We will use these two relations to find the lowest frequency that permits standing waves in both strings with a node at the junction.

**SOLUTION** Since the frequency of the left string is equal to the frequency of the right string, we can write

\[ \frac{n_{\text{left}} \sqrt{\frac{F}{(m/L)_{\text{left}}}}}{2L_{\text{left}}} = \frac{n_{\text{right}} \sqrt{\frac{F}{(m/L)_{\text{right}}}}}{2L_{\text{right}}} \]

Substituting in the data given in the problem yields

\[ \frac{n_{\text{left}} \sqrt{190.0 \text{ N}}}{2(3.75 \text{ m})} = \frac{n_{\text{right}} \sqrt{190.0 \text{ N}}}{2(1.25 \text{ m})} \]
This expression gives \( n_{\text{left}} = 6n_{\text{right}} \). Letting \( n_{\text{left}} = 6 \) and \( n_{\text{right}} = 1 \), the frequency of the left string (which is also equal to the frequency of the right string) is

\[
 f_6 = \frac{(6) \sqrt{\frac{190.0 \text{ N}}{6.00 \times 10^{-2} \text{ kg/m}}}}{2(3.75 \text{ m})} = 45.0 \text{ Hz}
\]

61. **REASONING**  When the difference \( \ell_1 - \ell_2 \) in path lengths traveled by the two sound waves is a half-integer number \( \left( \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots \right) \) of wavelengths, destructive interference occurs at the listener. When the difference in path lengths is zero or an integer number \( (1, 2, 3, \ldots) \) of wavelengths, constructive interference occurs. Therefore, we will divide the distance \( \ell_1 - \ell_2 \) by the wavelength of the sound to determine if constructive or destructive interference occurs. The wavelength is, according to Equation 16.1, the speed \( v \) of sound divided by the frequency \( f \); \( \lambda = \frac{v}{f} \).

**SOLUTION**

a. The distances \( \ell_1 \) and \( \ell_2 \) can be determined by applying the Pythagorean theorem to the two right triangles in the drawing:

\[
\ell_1 = \sqrt{(2.200 \text{ m})^2 + (1.813 \text{ m})^2} = 2.851 \text{ m}
\]

\[
\ell_2 = \sqrt{(2.200 \text{ m})^2 + (1.187 \text{ m})^2} = 2.500 \text{ m}
\]

Therefore, \( \ell_1 - \ell_2 = 0.351 \text{ m} \). The wavelength of the sound is \( \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1466 \text{ Hz}} = 0.234 \text{ m} \). Dividing the distance \( \ell_1 - \ell_2 \) by the wavelength \( \lambda \) gives the number of wavelengths in this distance:

\[
\text{Number of wavelengths} = \frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351 \text{ m}}{0.233 \text{ m}} = 1.5
\]

Since the number of wavelengths is a half-integer number \( (1\frac{1}{2}) \), destructive interference occurs at the listener.

b. The wavelength of the sound is now \( \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{977 \text{ Hz}} = 0.351 \text{ m} \). Dividing the distance \( \ell_1 - \ell_2 \) by the wavelength \( \lambda \) gives the number of wavelengths in that distance:
Number of wavelengths = $\frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351 \text{ m}}{0.351 \text{ m}} = 1$

Since the number of wavelengths is an integer number (1), constructive interference occurs at the listener.

62. **REASONING** The blows of the carpenters’ hammers fall at frequencies $f_1$ and $f_2$, which are related to the period $T$ between blows by $T = 1/f$ (Equation 10.5). The beat frequency $f_{\text{beat}} = |f_1 - f_2|$ of their hammer blows is the frequency at which the blows fall at the same instant. The time between these simultaneous blows is the period $T_{\text{beat}}$ of the beats.

**SOLUTION**
a. When the second carpenter hammers more rapidly than the first, the frequency of the second carpenter’s hammer blows is larger than that of the first carpenter’s blows: $f_2 > f_1$. Therefore, the beat frequency is given by

$$f_{\text{beat}} = |f_1 - f_2| = f_2 - f_1 \quad (1)$$

Solving Equation (1) for $f_2$, we obtain

$$f_2 = f_1 + f_{\text{beat}} \quad (2)$$

The relation $T = 1/f$ (Equation 10.5) tells us that the period $T_2$ of the second carpenter’s hammer blows is the reciprocal of the frequency $f_2$. Therefore,

$$T_2 = \frac{1}{f_2} = \frac{1}{f_1 + f_{\text{beat}}} \quad (3)$$

Again invoking Equation 10.5, we can rewrite Equation (3) in terms of periods, rather than frequencies:

$$T_2 = \frac{1}{T_{\text{beat}}} + \frac{1}{T_1} = \frac{1}{\frac{1}{0.75 \text{ s}} + \frac{1}{4.6 \text{ s}}} = \frac{1}{0.64 \text{ s}}$$

b. If the second carpenter hammers less rapidly than the first carpenter, then $f_2 < f_1$, and the beat frequency is $f_{\text{beat}} = |f_1 - f_2| = f_1 - f_2$. Therefore, the frequency of the second carpenter’s hammering is $f_2 = f_1 - f_{\text{beat}}$, and we have that

$$T_2 = \frac{1}{f_2} = \frac{1}{f_1 - f_{\text{beat}}} = \frac{1}{T_1 - \frac{T_{\text{beat}}}{T_1}} = \frac{1}{\frac{1}{0.75 \text{ s}} - \frac{1}{4.6 \text{ s}}} = \frac{1}{0.90 \text{ s}}$$
63. **REASONING** The natural frequencies of the cord are, according to Equation 17.3, 
\[ f_n = \frac{nv}{2L}, \] 
where \( n = 1, 2, 3, \ldots \) The speed \( v \) of the waves on the cord is, according to Equation 16.2, 
\[ v = \sqrt{\frac{F}{m/L}}, \] 
where \( F \) is the tension in the cord. Combining these two expressions, we have
\[ f_n = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{F}{m/L}} \quad \text{or} \quad \left( \frac{f_n 2L}{n} \right)^2 = \frac{F}{m/L}. \]

Applying Newton's second law of motion, \( \Sigma F = ma \), to the forces that act on the block and are parallel to the incline gives
\[ F - Mg \sin \theta = Ma = 0 \quad \text{or} \quad F = Mg \sin \theta \]
where \( Mg \sin \theta \) is the component of the block's weight that is parallel to the incline. Substituting this value for the tension into the equation above gives
\[ \left( \frac{f_n 2L}{n} \right)^2 = \frac{Mg \sin \theta}{m/L}. \]

This expression can be solved for the angle \( \theta \) and evaluated at the various harmonics. The answer can be chosen from the resulting choices.

**SOLUTION** Solving this result for \( \sin \theta \) shows that
\[ \sin \theta = \frac{(m/L) \left( \frac{f_n 2L}{n} \right)^2}{Mg} = \frac{1.20 \times 10^{-2} \text{ kg/m}}{(15.0 \text{ kg})(9.80 \text{ m/s}^2)} \left[ \frac{(165 \text{ Hz})2(0.600 \text{ m})}{n} \right]^2 = \frac{3.20}{n^2}. \]

Thus, we have
\[ \theta = \sin^{-1} \left( \frac{3.20}{n^2} \right). \]

Evaluating this for the harmonics corresponding to the range of \( n \) from \( n = 2 \) to \( n = 4 \), we have
\[ \theta = \sin^{-1} \left( \frac{3.20}{2^2} \right) = 53.1^\circ \text{ for } n = 2 \]
\[ \theta = \sin^{-1} \left( \frac{3.20}{3^2} \right) = 20.8^\circ \text{ for } n = 3 \]
\[ \theta = \sin^{-1} \left( \frac{3.20}{4^2} \right) = 11.5^\circ \text{ for } n = 4 \]

The angles between 15.0° and 90.0° are \( \theta = 20.8^\circ \) and \( \theta = 53.1^\circ \).
PREFACE for Volume II

This volume contains the answers to the Focus on Concepts questions and the complete solutions to the problems for chapters 18 through 32. These chapters include electricity and magnetism, ac circuits, light and optics, and modern physics. The solutions for chapters 1 through 17 [mechanics (including fluids), thermal physics and wave motion] are contained in Volume 1.

Each chapter is organized so that the answers to the Focus on Concepts questions appear first, followed by the solutions to the problems.

An electronic version of this manual is available on the Instructor’s Companion Website. The files are available in three formats: Microsoft Word 97-2003, Microsoft Word 2007, and PDF files of each individual solution.

The icon SSM at the beginning of some of the problems indicates that the solution is also available in the Student Solutions Manual.

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| CHAPTER 18 | Answers to Focus on Concepts Questions | Problems | 895 | 898 |
| CHAPTER 19 | Answers to Focus on Concepts Questions | Problems | 954 | 956 |
| CHAPTER 20 | Answers to Focus on Concepts Questions | Problems | 1000 | 1003 |
| CHAPTER 21 | Answers to Focus on Concepts Questions | Problems | 1076 | 1079 |
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| CHAPTER 30 | Answers to Focus on Concepts Questions | Problems | 1466 | 1468 |
| CHAPTER 31 | Answers to Focus on Concepts Questions | Problems | 1503 | 1505 |
| CHAPTER 32 | Answers to Focus on Concepts Questions | Problems | 1540 | 1542 |
1. \(1.9 \times 10^{13}\)

2. (b) Suppose that A is positive and B is negative. Since C and A also attract each other, C must be negative. Thus, B and C repel each other, because they have like charges (both negative). Suppose, however, that A is negative and B is positive. Since C and A also attract each other, C must be positive. Again we conclude that B and C repel each other, because they have like charges (both positive).

3. (a) The ball is electrically neutral (net charge equals zero). However, it is made from a conducting material, so it contains electrons that are free to move. The rod attracts some of these (negative) electrons to the side of the ball nearest the rod, leaving the opposite side of the ball positively charged. Since the negative side of the ball is closer to the positive rod than the positive side, a net attractive force arises.

4. (d) The fact that the positive rod repels one object indicates that that object carries a net positive charge. The fact that the rod repels the other object indicates that that object carries a net negative charge. Since both objects are identical and made from conducting material, they share the combined net charges equally after they are touched together. Since the rod repels each object after they are touched, each object must then carry a net positive charge. But the net electric charge of any isolated system is conserved, so the total net charge initially must also have been positive. This means that the initial positive charge had the greater magnitude.

5. (c) This distribution is not possible because of the law of conservation of electric charge. The total charge on the three objects here is \(\frac{2}{3}q\), whereas only \(q\) was present initially.

6. (c) This is an example of charging by induction. The negatively charged rod repels free electrons in the metal. These electrons move through the point of contact and into the sphere farthest away from the rod, giving it an induced charge of \(-q\). The sphere nearest the rod acquires an induced charge of \(+q\). As long as the rod is kept in place while the spheres are separated, these induced charges cannot recombine and remain on the spheres.

7. (b) Coulomb’s law states that the magnitude of the force is given by \(F = k \frac{|q_1| |q_2|}{r^2}\). Doubling the magnitude of each charge as in A would increase the numerator by a factor of four, but this is offset by the change in separation, which increases the denominator by a factor of \(2^2 = 4\). Doubling the magnitude of only one charge as in D would increase the numerator by
a factor of two, but this is offset by the change in separation, which increases the
denominator by a factor of \((\sqrt{2})^2 = 2\).

8. (e) Coulomb’s law states that the magnitude of the force is given by \(F = k \frac{|q_1||q_2|}{r^2}\). The force
is directed along the line between the charges and is an attraction for unlike charges and a
repulsion for like charges. Charge B is attracted by charge A with a force of magnitude
\(k \frac{|q_1||q_2|}{d^2}\) and repelled by charge C with a force of the same magnitude. Since both forces
point to the left, the net force acting on B has a magnitude of \(2k \frac{|q_1||q_2|}{d^2}\). Charge A is attracted
by charge B with a force of \(k \frac{|q_1||q_2|}{d^2}\) and also by charge C with a force of \(k \frac{|q_1||q_2|}{(2d)^2}\). Since both
forces point to the right, the net force acting on A has a magnitude of \((1.25)k \frac{|q_1||q_2|}{d^2}\). Charge
C is pushed to the right by B with a force of \(k \frac{|q_1||q_2|}{d^2}\) and pulled to the left by A with a force
of \(k \frac{|q_1||q_2|}{(2d)^2}\). Since these two forces have different directions, the net force acting on C has a
magnitude of \((0.75)k \frac{|q_1||q_2|}{d^2}\).

9. (b) According to Coulomb’s law, the magnitude of the force that any one of the point charges
exerts on another point charge is given by \(F = k \frac{|q_1||q_2|}{d^2}\), where \(d\) is the length of each side of
the triangle. The charge at B experiences a repulsive force from the charge at A and an
attractive force from the charge at C. Both forces have vertical components, but one points
in the \(+y\) direction and the other in the \(-y\) direction. These vertical components have equal
magnitudes and cancel, leaving a resultant that is parallel to the \(x\) axis.

10. 8.5 \(\mu\)C

11. (e) According to Equation 18.2, the force exerted on a charge by an electric field is
proportional to the magnitude of the charge. Since the negative charge has twice the
magnitude of the positive charge, the negative charge experiences twice the force.
Furthermore, the direction of the force on the positive charge is in the same direction as the
field, so that we can conclude that the field points due west. The force on the negative
charge points opposite to the field and, therefore, points due east.
12. (c) The electric field created by a point charge has a magnitude \[ E = \frac{k|q|}{r^2} \] and is inversely proportional to the square of the distance \( r \). If \( r \) doubles, the charge magnitude must increase by a factor of \( 2^2 = 4 \) to keep the field the same.

13. (b) To the left of the positive charge the two contributions to the total field have opposite directions. There is a spot in this region at which the field from the smaller, but closer, positive charge exactly offsets the field from the greater, but more distant, negative charge.

14. (e) Consider the charges on opposite corners. In all of the arrangements these are like charges. This means that the two field contributions created at the center of the square point in opposite directions and, therefore, cancel. Thus, only the charge opposite the empty corner determines the magnitude of the net field at the center of the square. Since the point charges all have the same magnitude, the net field there has the same magnitude in each arrangement.

15. \( 1.8 \times 10^{-6} \) C/m²

16. (c) The tangent to the field line gives the direction of the electric field at a point. At A the tangent points due south, at B southeast, and at C due east.

17. (a) The electric field has a greater magnitude where the field lines are closer together. They are closest together at B and farthest apart at A. Therefore, the field has the greatest magnitude at B and the smallest magnitude at A.

18. (d) In a conductor electric charges can readily move in response to an electric field. In A, B, and C the electric charges experience an electric field and, hence, a force from neighboring charges and will move outward, away from each other. They will rearrange themselves so that the electric field within the metal is zero at equilibrium. This means that they will reside on the outermost surface. Thus, only D could represent charges in equilibrium.

19. 1.3 N·m²/C

20. 0.45 N·m²/C
1. **SSM REASONING** The charge of a single proton is $+e$, and the charge of a single electron is $-e$, where $e = 1.60 \times 10^{-19}$ C. The net charge of the ionized atom is the sum of the charges of its constituent protons and electrons.

**SOLUTION** The ionized atom has 26 protons and 7 electrons, so its net electric charge $q$ is

$$q = 26(+e) + 7(-e) = +19e = +19(1.60 \times 10^{-19} \text{ C}) = +3.04 \times 10^{-18} \text{ C}$$

2. **REASONING** Since the object has a charge of $-2.0 \mu\text{C}$ to begin with, it becomes neutral when that charge is removed. To create a charge of $+3.0 \mu\text{C}$ on the object, an additional charge of $-3.0 \mu\text{C}$ must be removed. Thus, the total charge that must be removed from the object is $-5.0 \mu\text{C}$. The number of electrons that correspond to this charge can be obtained by dividing the charge by the charge on a single electron, which is $-e = -160 \times 10^{-19}$ C.

**SOLUTION** The number $N$ of electrons corresponding to $-5.0 \mu\text{C}$ is

$$N = \frac{-5.0 \times 10^{-6} \text{ C}}{-1.60 \times 10^{-19} \text{ C}} = \frac{-5.0 \times 10^{-6}}{-1.60 \times 10^{-19}} = 3.1 \times 10^{13}$$

3. **REASONING**
   a. Since the objects are metallic and identical, the charges on each combine and produce a net charge that is shared equally by each object. Thus, each object ends up with one-fourth of the net charge.

   b. The number of electrons (or protons) that make up the final charge on each object is equal to the final charge divided by the charge of an electron (or proton).

**SOLUTION**

   a. The net charge is the algebraic sum of the individual charges. The charge $q$ on each object after contact and separation is one-fourth the net charge, or

$$q = \frac{1}{4}(1.6 \mu\text{C} + 6.2 \mu\text{C} - 4.8 \mu\text{C} - 9.4 \mu\text{C}) = -1.6 \mu\text{C}$$
b. Since the charge on each object is negative, the charge is comprised of electrons. The number of electrons on each object is the charge \( q \) divided by the charge \( -e \) of a single electron:

\[
\text{Number of electrons} = \frac{q}{-e} = \frac{-1.6 \times 10^{-6} \text{ C}}{-1.60 \times 10^{-19} \text{ C}} = 1.0 \times 10^{13}
\]

4. **REASONING** The conservation of electric charge states that, during any process, the net electric charge of an isolated system remains constant (is conserved). Therefore, the net charge \((q_1 + q_2)\) on the two spheres before they touch is the same as the net charge after they touch. When the two identical metal spheres touch, the net charge will spread out equally over both of them. When the spheres are separated, the charge on each is the same.

**SOLUTION**

a. Since the final charge on each sphere is +5.0 \( \mu \text{C} \), the final net charge on both spheres is \(2(+5.0 \ \mu \text{C}) = +10.0 \ \mu \text{C} \). The initial net charge must also be +10.0 \( \mu \text{C} \). The only spheres whose net charge is +10.0 \( \mu \text{C} \) are

\[B \ (q_B = -2.0 \ \mu \text{C}) \text{ and } D \ (q_D = +12.0 \ \mu \text{C})\]

b. Since the final charge on each sphere is +3.0 \( \mu \text{C} \), the final net charge on the three spheres is \(3(+3.0 \ \mu \text{C}) = +9.0 \ \mu \text{C} \). The initial net charge must also be +9.0 \( \mu \text{C} \). The only spheres whose net charge is +9.0 \( \mu \text{C} \) are

\[A \ (q_A = -8.0 \ \mu \text{C}), \ C \ (q_C = +5.0 \ \mu \text{C}) \text{ and } D \ (q_D = +12.0 \ \mu \text{C})\]

c. Since the final charge on a given sphere in part (b) is +3.0 \( \mu \text{C} \), we would have to add \(-3.0 \ \mu \text{C} \) to make it electrically neutral. Since the charge on an electron is \(-1.6 \times 10^{-19} \text{ C} \), the number of electrons that would have to be added is

\[
\text{Number of electrons} = \frac{-3.0 \times 10^{-6} \text{ C}}{-1.6 \times 10^{-19} \text{ C}} = 1.9 \times 10^{13}
\]

5. **SSM REASONING** Identical conducting spheres equalize their charge upon touching. When spheres A and B touch, an amount of charge \(+q\), flows from A and instantaneously neutralizes the \(-q\) charge on B leaving B momentarily neutral. Then, the remaining amount of charge, equal to \(+4q\), is equally split between A and B, leaving A and B each with equal amounts of charge \(+2q\). Sphere C is initially neutral, so when A and C touch, the \(+2q\) on A splits equally to give \(+q\) on A and \(+q\) on C. When B and C touch, the \(+2q\) on B and the \(+q\) on C combine to give a total charge of \(+3q\), which is then equally divided between the spheres B and C; thus, B and C are each left with an amount of charge \(+1.5q\).
**SOLUTION**  Taking note of the initial values given in the problem statement, and summarizing the final results determined in the *REASONING* above, we conclude the following:

a. Sphere C ends up with an amount of charge equal to $+1.5q$.

b. The charges on the three spheres before they were touched, are, according to the problem statement, $+5q$ on sphere A, $-q$ on sphere B, and zero charge on sphere C. Thus, the total charge on the spheres is $+5q - q + 0 = +4q$.

c. The charges on the spheres after they are touched are $+q$ on sphere A, $+1.5q$ on sphere B, and $+1.5q$ on sphere C. Thus, the total charge on the spheres is $+q + 1.5q + 1.5q = +4q$.

---

6. **REASONING**  When $N$ electrons, each carrying a charge $-e = -1.6 \times 10^{-19}$ C, are transferred from the plate to the rod, the system consisting of the plate and the rod is isolated. Therefore, the total charge $q_{1i} + q_{2i}$ of the system is unchanged by the process, where $q_{1i}$ is the initial charge of the plate and $q_{2i}$ is the initial charge of the rod. At the end, the rod and the plate each have the same final charge $q_{1f} = q_{2f}$. Therefore, each must have a charge equal to half the total charge of the system: $q_{1f} = q_{2f} = \frac{1}{2}(q_{1i} + q_{2i})$.

**SOLUTION**  The final charge $q_{2f}$ on the rod is equal to its initial charge $q_{2i}$ plus the charge transferred to it, which is equal to the product of the number $N$ of electrons transferred and the charge $-e$ of each electron. Therefore,

$$q_{2f} = q_{2i} + N(-e) = q_{2i} - Ne$$  \(1\)

Since the final charge on the rod is equal to half the total initial charge of the system, we can substitute $q_{2f} = \frac{1}{2}(q_{1i} + q_{2i})$ into Equation (1) and solve for $N$:

$$\frac{1}{2}(q_{1i} + q_{2i}) = q_{2i} - Ne \quad \text{or} \quad Ne = q_{2i} - \frac{1}{2}(q_{1i} + q_{2i}) = \frac{1}{2}(q_{2i} - q_{1i}) \quad \text{or} \quad N = \frac{(q_{2i} - q_{1i})}{2e}$$

Therefore, the number of electrons that must be transferred to the rod is

$$N = \frac{(q_{2i} - q_{1i})}{2e} = \frac{+2.0 \times 10^{-6} \text{ C} - (-3.0 \times 10^{-6} \text{ C})}{2(1.6 \times 10^{-19} \text{ C})} \approx 1.6 \times 10^{13}$$

---

7. **REASONING**

a. The number $N$ of electrons is 10 times the number of water molecules in 1 liter of water. The number of water molecules is equal to the number $n$ of moles of water molecules times Avogadro’s number $N_A$: $N = 10nN_A$.

b. The net charge of all the electrons is equal to the number of electrons times the change on one electron.
**SOLUTION**

a. The number $N$ of water molecules is equal to $10 \, n \, N_A$, where $n$ is the number of moles of water molecules and $N_A$ is Avogadro’s number. The number of moles is equal to the mass $m$ of 1 liter of water divided by the mass per mole of water. The mass of water is equal to its density $\rho$ times the volume, as expressed by Equation 11.1. Thus, the number of electrons is

$$N = 10 \, n \, N_A = 10 \left( \frac{m}{18.0 \text{ g/mol}} \right) N_A = 10 \left( \frac{\rho \, V}{18.0 \text{ g/mol}} \right) N_A$$

$$= 10 \left[ \left( \frac{1000 \text{ kg/m}^3}{1 \text{ kg}} \right) \left( 1.00 \times 10^{-3} \text{ m}^3 \right) \left( \frac{1000 \text{ g}}{1 \text{ kg}} \right) \right] \left( 6.022 \times 10^{23} \text{ mol}^{-1} \right)$$

$$= 3.35 \times 10^{26} \text{ electrons}$$

b. The net charge $Q$ of all the electrons is equal to the number of electrons times the change on one electron: $Q = \left( 3.35 \times 10^{26} \right) \left( -1.60 \times 10^{-19} \text{ C} \right) = -5.36 \times 10^7 \text{ C}.$

---

8. **REASONING** The magnitude of the electrostatic force that acts on particle 1 is given by Coulomb’s law as $F = k |q_1| |q_2| / r^2$. This equation can be used to find the magnitude $|q_2|$ of the charge.

**SOLUTION** Solving Coulomb’s law for the magnitude $|q_2|$ of the charge gives

$$|q_2| = \frac{F \, r^2}{k |q_1|} = \frac{(3.4 \text{ N})(0.26 \text{ m})^2}{\left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( 3.5 \times 10^{-6} \text{ C} \right)} = 7.3 \times 10^{-6} \text{ C} \tag{18.1}$$

Since $q_1$ is positive and experiences an attractive force, the charge $q_2$ must be negative.

---

9. **SSM REASONING** The number $N$ of excess electrons on one of the objects is equal to the charge $q$ on it divided by the charge of an electron ($-e$), or $N = q/(-e)$. Since the charge on the object is negative, we can write $q = -|q|$, where $|q|$ is the magnitude of the charge. The magnitude of the charge can be found from Coulomb’s law (Equation 18.1), which states that the magnitude $F$ of the electrostatic force exerted on each object is given by $F = k|q||q|/r^2$, where $r$ is the distance between them.
SOLUTION The number \( N \) of excess electrons on one of the objects is

\[
N = \frac{q}{-e} = \frac{-|q|}{e} = |q| \tag{1}
\]

To find the magnitude of the charge, we solve Coulomb’s law, \( F = k\frac{|q||q|}{r^2} \), for \(|q|\):

\[
|q| = \frac{F r^2}{k}
\]

Substituting this result into Equation (1) gives

\[
N = \frac{|q|}{e} = \frac{\sqrt{\frac{F r^2}{k}}}{e} = \frac{\sqrt{(4.55\times10^{-21} \text{ N})(1.80\times10^{-3} \text{ m})^2}}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 8
\]

10. REASONING
   a. The magnitude of the electrostatic force that acts on each sphere is given by Coulomb’s law as \( F = k\frac{|q_1||q_2|}{r^2} \), where \(|q_1|\) and \(|q_2|\) are the magnitudes of the charges, and \( r \) is the distance between the centers of the spheres.

b. When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact. Since the spheres are identical, the charge on each after being separated is one-half the net charge. Coulomb’s law can be applied again to determine the magnitude of the electrostatic force that each sphere experiences.

SOLUTION
   a. The magnitude of the force that each sphere experiences is given by Coulomb’s law as:

\[
F = \frac{k|q_1||q_2|}{r^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(20.0\times10^{-6} \text{ C})(50.0\times10^{-6} \text{ C})}{(2.50\times10^{-2} \text{ m})^2} = 1.44\times10^4 \text{ N}
\]

Because the charges have opposite signs, the force is attractive.

b. The net charge on the spheres is \(-20.0 \mu\text{C} + 50.0 \mu\text{C} = +30.0 \mu\text{C}\). When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact, or \(+30.0 \mu\text{C}\). Since the spheres are identical, the charge on each after being separated is one-half the net charge, so \( q_1 = q_2 = +15.0 \mu\text{C} \). The electrostatic force that acts on each sphere is now

\[
F = \frac{k|q_1||q_2|}{r^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(15.0\times10^{-6} \text{ C})(15.0\times10^{-6} \text{ C})}{(2.50\times10^{-2} \text{ m})^2} = 3.24\times10^3 \text{ N}
\]

Since the charges now have the same signs, the force is repulsive.
11. **REASONING** Initially, the two spheres are neutral. Since negative charge is removed from the sphere which loses electrons, it then carries a net positive charge. Furthermore, the neutral sphere to which the electrons are added is then negatively charged. Once the charge is transferred, there exists an electrostatic force on each of the two spheres, the magnitude of which is given by Coulomb's law (Equation 18.1), \( F = k \frac{|q_1||q_2|}{r^2} \).

**SOLUTION**

a. Since each electron carries a charge of \( -1.60 \times 10^{-19} \text{ C} \), the amount of negative charge removed from the first sphere is

\[
3.0 \times 10^{13} \text{ electrons} \left( \frac{1.60 \times 10^{-19} \text{ C}}{1 \text{ electron}} \right) = 4.8 \times 10^{-6} \text{ C}
\]

Thus, the first sphere carries a charge +4.8 \( \times \) \( 10^{-6} \) C, while the second sphere carries a charge -4.8 \( \times \) \( 10^{-6} \) C. The magnitude of the electrostatic force that acts on each sphere is, therefore,

\[
F = k \frac{|q_1||q_2|}{r^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times (4.8 \times 10^{-6} \text{ C})^2}{(0.50 \text{ m})^2} = 0.83 \text{ N}
\]

b. Since the spheres carry charges of opposite sign, the force is attractive.

12. **REASONING** The magnitude of the force of attraction between the charges is given by Coulomb’s law: \( F = k \frac{|q_1||q_2|}{r^2} \) (Equation 18.1), where \( |q_1| \) and \( |q_2| \) are the magnitudes of the charges and \( r \) is the separation of the charges. We will apply this equation twice, once when the separation is \( r_A \) and the magnitude of the force is \( F_A = 1.5 \text{ N} \) and then again when the separation is \( r_B = r_A / 9 \) and the magnitude of the force is \( F_B \).

**SOLUTION** Applying Coulomb’s law when the separations are \( r_A \) and \( r_B \), we obtain

\[
F_A = k \frac{|q_1||q_2|}{r_A^2} \quad \text{and} \quad F_B = k \frac{|q_1||q_2|}{r_B^2}
\]

In order to eliminate the unknown charge magnitudes and the constant \( k \), we divide the equation on the right by the equation on the left:

\[
\frac{F_B}{F_A} = k \frac{|q_1||q_2|/r_B^2}{|q_1||q_2|/r_A^2} = \left( \frac{r_A}{r_B} \right)^2 \quad \text{or} \quad F_B = F_A \left( \frac{r_A}{r_B} \right)^2
\]

Using the fact that \( r_B = r_A / 9 \), we find that the unknown force-magnitude is
13. **REASONING** Let $\mathbf{F}_2$ and $\mathbf{F}_1$ represent the forces exerted on the charge $q$ at the origin by the point charges $q_1$ and $q_2$, respectively. According to Equation 18.1, the magnitudes of these forces are given by

$$F_1 = k \frac{|q_1||q|}{r_1^2} \quad \text{and} \quad F_2 = k \frac{|q_2||q|}{r_2^2} \quad (1)$$

where $r_1$ is the distance between $q_1$ and $q$, $r_2$ is the distance between $q_2$ and $q$, and $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. The directions of the forces are determined by the signs of each charge-pair. The sign of $q_1$ is opposite that of $q$, so $\mathbf{F}_1$ is an attractive force, pointing in the positive $y$ direction. The signs of $q_2$ and $q$ are both positive, so $\mathbf{F}_2$ is a repulsive force, pointing in the negative $y$ direction (see the drawing). Because the net force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ acting on $q$ points in the positive $y$ direction, the force $\mathbf{F}_1$ must have a greater magnitude than the force $\mathbf{F}_2$. Therefore, the magnitude $F$ of the net electric force acting on $q$ is equal to the magnitude of the attractive force $F_1$ minus that of the repulsive force $F_2$:

$$F = F_1 - F_2 \quad (2)$$

**SOLUTION** Substituting Equations (1) into Equation (2) yields

$$F = F_1 - F_2 = k \frac{|q_1||q|}{r_1^2} - k \frac{|q_2||q|}{r_2^2} \quad (3)$$

Solving Equation (3) for $|q_2|$, we obtain

$$k \frac{|q_2||q|}{r_2^2} = k \frac{|q_1||q|}{r_1^2} - F \quad \text{or} \quad |q_2| = r_2^2 \left( \frac{|q_1||q|}{r_1^2} - \frac{F}{k|q|} \right)$$

Substituting the given values, we find that

$$|q_2| = (0.34 \text{ m})^2 \left[ \frac{-25 \times 10^{-6} \text{ C}}{(0.22 \text{ m})^2} - \frac{27 \text{ N}}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)} + 8.4 \times 10^{-6} \text{ C} \right] = 1.8 \times 10^{-5} \text{ C}$$
14. **REASONING** The electrical force that each charge exerts on charge 2 is shown in the following drawings. $\mathbf{F}_{21}$ is the force exerted on 2 by 1, and $\mathbf{F}_{23}$ is the force exerted on 2 by 3. Each force has the same magnitude, because the charges have the same magnitude and the distances are equal.

![Diagrams showing forces on charge 2](image)

The net electric force $\mathbf{F}$ that acts on charge 2 is shown in the following diagrams.

![Diagrams showing net forces on charge 2](image)

It can be seen from the diagrams that the largest electric force occurs in (a), followed by (c), and then by (b).

**SOLUTION** The magnitude $F_{21}$ of the force exerted on 2 by 1 is the same as the magnitude $F_{23}$ of the force exerted on 2 by 3, since the magnitudes of the charges are the same and the distances are the same. Coulomb’s law gives the magnitudes as

$$F_{21} = F_{23} = \frac{k|q||q|}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.6 \times 10^{-6} \text{ C})(8.6 \times 10^{-6} \text{ C})}{(3.8 \times 10^{-3} \text{ m})^2} = 4.6 \times 10^4 \text{ N}$$

In part (a) of the drawing showing the net electric force acting on charge 2, both $\mathbf{F}_{21}$ and $\mathbf{F}_{23}$ point to the left, so the net force has a magnitude of

$$F = 2F_{12} = 2(4.6 \times 10^4 \text{ N}) = 9.2 \times 10^4 \text{ N}$$

In part (b) of the drawing showing the net electric force acting on charge 2, $\mathbf{F}_{21}$ and $\mathbf{F}_{23}$ point in opposite directions, so the net force has a magnitude of $0 \text{ N}$.
In part (c) showing the net electric force acting on charge 2, the magnitude of the net force can be obtained from the Pythagorean theorem:

\[
F = \sqrt{F_{21}^2 + F_{23}^2} = \sqrt{(4.6 \times 10^4 \text{ N})^2 + (4.6 \times 10^4 \text{ N})^2} = 6.5 \times 10^4 \text{ N}
\]

15. **REASONING AND SOLUTION**

a. Since the gravitational force between the spheres is one of attraction and the electrostatic force must balance it, the electric force must be one of repulsion. Therefore, the charges must have the same algebraic signs, both positive or both negative.

b. There are two forces that act on each sphere; they are the gravitational attraction \(F_G\) of one sphere for the other, and the repulsive electric force \(F_E\) of one sphere on the other. From the problem statement, we know that these two forces balance each other, so that \(F_G = F_E\). The magnitude of \(F_G\) is given by Newton's law of gravitation (Equation 4.3: \(F_G = Gm_1m_2/r^2\)), while the magnitude of \(F_E\) is given by Coulomb's law (Equation 18.1: \(F_E = k|q_1||q_2|/r^2\)). Therefore, we have

\[
\frac{Gm_1m_2}{r^2} = \frac{k|q_1||q_2|}{r^2} \quad \text{or} \quad Gm^2 = k|q|^2
\]

since the spheres have the same mass \(m\) and carry charges of the same magnitude \(|q|\). Solving for \(|q|\), we find

\[
|q| = m\sqrt{\frac{G}{k}} = (2.0 \times 10^{-6} \text{ kg})\sqrt{\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 1.7 \times 10^{-16} \text{ C}
\]

16. **REASONING** The drawing at the right shows the set-up. The force on the \(+q\) charge at the origin due to the other \(+q\) charge is given by Coulomb's law (Equation 18.1), as is the force due to the \(+2q\) charge. These two forces point to the left, since each is repulsive. The sum of the two is twice the force on the \(+q\) charge at the origin due to the other \(+q\) charge alone.

**SOLUTION** Applying Coulomb's law, we have
Chapter 18 Problems

17. **SSM REASONING** Each particle will experience an electrostatic force due to the presence of the other charge. According to Coulomb's law (Equation 18.1), the magnitude of the force felt by each particle can be calculated from $F = k |q_1| |q_2|/r^2$, where $|q_1|$ and $|q_2|$ are the respective charges on particles 1 and 2 and $r$ is the distance between them. According to Newton's second law, the magnitude of the force experienced by each particle is given by $F = ma$, where $a$ is the acceleration of the particle and we have assumed that the electrostatic force is the only force acting.

**SOLUTION**

a. Since the two particles have identical positive charges, $|q_1| = |q_2| = |q|$, and we have, using the data for particle 1,

$$\frac{k|q|^2}{r^2} = m_1a_1$$

Solving for $|q|$, we find that

$$|q| = \sqrt{\frac{m_1a_1r^2}{k}} = \sqrt{\frac{(6.00 \times 10^{-6} \text{ kg})(4.60 \times 10^3 \text{ m/s}^2)(2.60 \times 10^{-2} \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 4.56 \times 10^{-8} \text{ C}$$

b. Since each particle experiences a force of the same magnitude (From Newton's third law), we can write $F_1 = F_2$, or $m_1a_1 = m_2a_2$. Solving this expression for the mass $m_2$ of particle 2, we have

$$m_2 = \frac{m_1a_1}{a_2} = \frac{(6.00 \times 10^{-6} \text{ kg})(4.60 \times 10^3 \text{ m/s}^2)}{8.50 \times 10^3 \text{ m/s}^2} = 3.25 \times 10^{-6} \text{ kg}$$
18. **REASONING AND SOLUTION** Calculate the magnitude of each force acting on the center charge. Using Coulomb’s law, we can write

\[ F_{43} = \frac{k |q_4||q_3|}{r_{43}^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(4.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})}{(0.100 \text{ m})^2} \]

\[ = 10.8 \text{ N (toward the south)} \]

\[ F_{53} = \frac{k |q_5||q_3|}{r_{53}^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-6} \text{ C})(3.00 \times 10^{-6} \text{ C})}{(0.100 \text{ m})^2} \]

\[ = 13.5 \text{ N (toward the east)} \]

Adding \( F_{43} \) and \( F_{53} \) as vectors, we have

\[ F = \sqrt{F_{43}^2 + F_{53}^2} = \sqrt{(10.8 \text{ N})^2 + (13.5 \text{ N})^2} = 17.3 \text{ N} \]

\[ \theta = \tan^{-1} \left( \frac{F_{43}}{F_{53}} \right) = \tan^{-1} \left( \frac{10.8 \text{ N}}{13.5 \text{ N}} \right) = 38.7^\circ \text{ S of E} \]

19. **REASONING** According to Newton’s second law, the centripetal acceleration experienced by the orbiting electron is equal to the centripetal force divided by the electron’s mass. Recall from Section 5.3 that the centripetal force \( F_c \) is the name given to the net force required to keep an object on a circular path of radius \( r \). For an electron orbiting about two protons, the centripetal force is provided almost exclusively by the electrostatic force of attraction between the electron and the protons. This force points toward the center of the circle and its magnitude is given by Coulomb’s law.

**SOLUTION** The magnitude \( a_c \) of the centripetal acceleration is equal to the magnitude \( F_c \) of the centripetal force divided by the electron’s mass: \( a_c = F_c / m \) (Equation 5.3). The centripetal force is provided almost entirely by the electrostatic force, so \( F_c = F \), where \( F \) is the magnitude of the electrostatic force of attraction between the electron and the two protons, Thus, \( a_c = F / m \). The magnitude of the electrostatic force is given by Coulomb’s law, \( F = k |q_1||q_2| / r^2 \) (Equation 18.1), where \( |q_1| = |e| \) and \( |q_2| = |+2e| \) are the magnitudes of the charges, \( r \) is the radius of the orbit, and \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \). Substituting this expression for \( F \) into \( a_c = F / m \), and using \( m = 9.11 \times 10^{-31} \text{ kg} \) for the mass of the electron, we find that
20. **REASONING** The unknown charges can be determined using Coulomb’s law to express the electrostatic force that each unknown charge exerts on the 4.00 μC charge. In applying this law, we will use the fact that the net force points downward in the drawing. This tells us that the unknown charges are both negative and have the same magnitude, as can be understood with the help of the free-body diagram for the 4.00 μC charge that is shown at the right. The diagram shows the attractive force F from each negative charge directed along the lines between the charges. Only when each force has the same magnitude (which is the case when both unknown charges have the same magnitude) will the resultant force point vertically downward. This occurs because the horizontal components of the forces cancel, one pointing to the right and the other to the left (see the diagram). The vertical components reinforce to give the observed downward net force.

**SOLUTION** Since we know from the **REASONING** that the unknown charges have the same magnitude, we can write Coulomb’s law as follows:

\[
F = k \frac{(4.00 \times 10^{-6} \text{ C})|q_A|}{r^2} = k \frac{(4.00 \times 10^{-6} \text{ C})|q_B|}{r^2}
\]

The magnitude of the net force acting on the 4.00 μC charge, then, is the sum of the magnitudes of the two vertical components \(F \cos 30.0^\circ\) shown in the free-body diagram:

\[
\Sigma F = k \frac{(4.00 \times 10^{-6} \text{ C})|q_A|}{r^2} \cos 30.0^\circ + k \frac{(4.00 \times 10^{-6} \text{ C})|q_B|}{r^2} \cos 30.0^\circ
\]

\[
= 2k \frac{(4.00 \times 10^{-6} \text{ C})|q_A|}{r^2} \cos 30.0^\circ
\]

Solving for the magnitude of the charge gives

\[
a_c = \frac{F}{m} = \frac{k |e|+2e}{r^2} = \frac{k |e|+2e}{m r^2}
\]

\[
= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)|-1.60 \times 10^{-19} \text{ C}||+2 \times 1.60 \times 10^{-19} \text{ C}|}{(9.11 \times 10^{-31} \text{ kg})(2.65 \times 10^{-11} \text{ m})^2} = 7.19 \times 10^{23} \text{ m/s}^2
\]
\[ |q_A| = \frac{(\Sigma F)r^2}{2k(4.00 \times 10^{-6} \text{C}) \cos 30.0^\circ} \]

\[ = \frac{(405 \text{N})(0.0200 \text{ m})^2}{2\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)(4.00 \times 10^{-6} \text{C}) \cos 30.0^\circ} = 2.60 \times 10^{-6} \text{C} \]

Thus, we have \( q_A = q_B = -2.60 \times 10^{-6} \text{C} \).

21. **REASONING**

a. There are two electrostatic forces that act on \( q_1 \); that due to \( q_2 \) and that due to \( q_3 \). The magnitudes of these forces can be found by using Coulomb’s law. The magnitude and direction of the net force that acts on \( q_1 \) can be determined by using the method of vector components.

b. According to Newton’s second law, Equation 4.2b, the acceleration of \( q_1 \) is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force.

**SOLUTION**

a. The magnitude \( F_{12} \) of the force exerted on \( q_1 \) by \( q_2 \) is given by Coulomb’s law, Equation 18.1, where the distance is specified in the drawing:

\[ F_{12} = k \frac{|q_1||q_2|}{r_{12}^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(8.00 \times 10^{-6} \text{C})(5.00 \times 10^{-6} \text{C})}{(1.30 \text{ m})^2} = 0.213 \text{ N} \]

Since the magnitudes of the charges and the distances are the same, the magnitude of \( F_{13} \) is the same as the magnitude of \( F_{12} \), or \( F_{13} = 0.213 \text{ N} \). From the drawing it can be seen that the \( x \)-components of the two forces cancel, so we need only to calculate the \( y \) components of the forces.

<table>
<thead>
<tr>
<th>Force</th>
<th>( y ) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{12} )</td>
<td>(+F_{12} \sin 23.0^\circ = +0.213 \text{ N} \sin 23.0^\circ = +0.0832 \text{ N})</td>
</tr>
<tr>
<td>( F_{13} )</td>
<td>(+F_{13} \sin 23.0^\circ = +0.213 \text{ N} \sin 23.0^\circ = +0.0832 \text{ N})</td>
</tr>
<tr>
<td><strong>( F )</strong></td>
<td>( F_y = +0.166 \text{ N} )</td>
</tr>
</tbody>
</table>

Thus, the net force is \( \boxed{\textbf{F} = +0.166 \text{ N (directed along the +y axis)}} \).
b. According to Newton’s second law, Equation 4.2b, the acceleration of \( q_1 \) is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force:

\[
a = \frac{F}{m} = \frac{+0.166 \text{ N}}{1.50 \times 10^{-3} \text{ kg}} = +111 \text{ m/s}^2
\]

where the plus sign indicates that the acceleration is along the \(+y\) axis.

---

22. **REASONING** When the airplane and the other end of the guideline carry point charges \(+q\) and \( -q \), the airplane is subject to an attractive electric force of magnitude

\[
F = k \frac{|q_1||q_2|}{r^2} = k \frac{|q||-q|}{r^2} = k \frac{q^2}{r^2} \quad \text{(Equation 18.1)}
\]

where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \) and \( r \) is the length of the guideline. This electric force provides part of the centripetal force \( F_{c2} = \frac{mv_2^2}{r} \) (Equation 5.3) necessary to keep the airplane (mass = \( m \)) flying along its circular path at the higher speed \( v_2 \) (which is associated with the greater kinetic energy \( KE_2 \)). The remainder of the centripetal force is provided by the maximum tension \( T_{max} \) in the guideline, so we have that

\[
F + T_{max} = F_{c2} = \frac{mv_2^2}{r} \quad \text{(1)}
\]

When the airplane is neutral and flying at the slower speed \( v_1 \) (which is associated with the smaller kinetic energy \( KE_1 \)), there is no electrical force, so the centripetal force \( F_{c1} = \frac{mv_1^2}{r} \) acting on the airplane is due solely to the maximum tension \( T_{max} \) in the guideline:

\[
T_{max} = F_{c1} = \frac{mv_1^2}{r} \quad \text{(2)}
\]

We note that the kinetic energy of the airplane is given by \( KE = \frac{1}{2}mv^2 \) (Equation 6.2), so that the quantities in the numerators of Equations (1) and (2) are proportional to the kinetic energies \( KE_2 \) and \( KE_1 \) of the airplane:

\[
mv_2^2 = 2(KE_2) \quad \text{and} \quad mv_1^2 = 2(KE_1) \quad \text{(3)}
\]

**SOLUTION** Solving \( F = k \frac{q^2}{r^2} \) (Equation 18.1) for \( q^2 \) and taking the square root of both sides, we obtain

\[
q^2 = \frac{Fr^2}{k} \quad \text{or} \quad q = \sqrt{\frac{Fr^2}{k}} \quad \text{(4)}
\]

Substituting Equations (3) into Equations (1) and (2) yields
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\[ F + T_{\text{max}} = \frac{2(KE_2)}{r} \quad \text{and} \quad T_{\text{max}} = \frac{2(KE_1)}{r} \]  
(5)

Substituting the second of Equations (5) into the first of Equations (5), we obtain

\[ F + \frac{2(KE_1)}{r} = \frac{2(KE_2)}{r} \quad \text{or} \quad F = \frac{2(KE_2)}{r} - \frac{2(KE_1)}{r} = \frac{2(KE_2 - KE_1)}{r} \]  
(6)

Substituting Equation (6) into Equation (4), we find that

\[ q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{2(KE_2 - KE_1)}{r^2}} = \sqrt{\frac{2(KE_2 - KE_1)}{kr}} \]

Therefore, the magnitude \( q \) of the charge on the airplane is

\[ q = \sqrt{\frac{2(51.8 \ J - 50.0 \ J)(3.0 \ m)}{8.99 \times 10^9 \ N \cdot m^2/C^2}} = 3.5 \times 10^{-5} \ C \]

23. **REASONING** The kinetic energy of the orbiting electron is \( KE = \frac{1}{2} mv^2 \) (Equation 6.2), where \( m \) and \( v \) are its mass and speed, respectively. We can obtain the speed by noting that the electron experiences a centripetal force whose magnitude \( F_c \) is given by \( F_c = mv^2 / r \) (Equation 5.3), where \( r \) is the radius of the orbit. The centripetal force is provided almost exclusively by the electrostatic force of attraction \( F \) between the electron and the protons, so \( F_c = F \). The electrostatic force points toward the center of the circle and its magnitude is given by Coulomb’s law as \( F = k|q_1||q_2|/r^2 \) (Equation 18.1), where \( |q_1| \) and \( |q_2| \) are the magnitudes of the charges.

**SOLUTION** The kinetic energy of the electron is

\[ KE = \frac{1}{2} mv^2 \]  
(6.2)

Solving the centripetal-force expression, \( F_c = mv^2 / r \) (Equation 5.3), for the speed \( v \), and substituting the result into Equation 6.2 gives

\[ KE = \frac{1}{2} mv^2 = \frac{1}{2} m' \left( \frac{rF_c}{m'} \right) = \frac{1}{2} rF_c \]  
(1)

The centripetal force is provided almost entirely by the electrostatic force, so \( F_c = F \), where \( F \) is the magnitude of the electrostatic force of attraction between the electron and the three protons. This force is given by Coulomb’s law, \( F = k|q_1||q_2|/r^2 \) (Equation 18.1). Substituting Coulomb’s law into Equation 1 yields

\[ KE = \frac{1}{2} rF_c = \frac{1}{2} rF = \frac{1}{2} \left( \frac{k|q_1||q_2|}{r^2} \right) = \frac{k|q_1||q_2|}{2r} \]
Setting $|q_1| = -e$ and $|q_2| = +3e$, we have

\[
\text{KE} = \frac{k|-e|+3e}{2r} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)|-1.60 \times 10^{-19} \text{ C}| + 3 \times 1.60 \times 10^{-19} \text{ C}|}{2(1.76 \times 10^{-11} \text{ m})} = 1.96 \times 10^{-17} \text{ J}
\]

24. **REASONING** Since the spring is stretched because of the charges, the charges must be exerting a repulsive force on each other. Only like charges repel each other. The magnitude of the repulsive force between the charges is given by Coulomb’s law: 

\[
F = k \frac{|q_1||q_2|}{r^2}
\]

(Equation 18.1), where $|q_1|$ and $|q_2|$ are the magnitudes of the charges and $r$ is the separation of the charges. This repulsive force provides the applied force needed to stretch the spring, which is $F_{x \text{Applied}} = k_{\text{spring}} x$ (Equation 10.1), where $k_{\text{spring}}$ is the spring constant and $x$ is the displacement of the spring from its unstrained length.

**SOLUTION**

a. Since only like charges repel each other, the charges must be

| either both positive or both negative |

b. Since the charges have equal magnitudes, we know that $|q_1| = |q_2| = |q|$, so that Coulomb’s law can be used to determine $|q|:

\[
F = k \frac{|q_1||q_2|}{r^2} = k \frac{|q|^2}{r^2} \quad \text{or} \quad |q| = \sqrt{\frac{Fr^2}{k}}
\]

In this result for $|q|$ we know that the electrostatic force $F$ provides the applied force needed to stretch the spring, so that $F = F_{x \text{Applied}} = k_{\text{spring}} x$ (Equation 10.1). With this substitution for $F$, our expression for $|q|$ becomes

\[
|q| = \sqrt{\frac{Fr^2}{k}} = \sqrt{k_{\text{spring}}x r^2} = \sqrt{\frac{(220 \text{ N/m})(0.020 \text{ m})[(0.32 \text{ m}) + (0.020 \text{ m})]^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 7.5 \times 10^{-6} \text{ C}
\]
REASONING Consider the drawing at the right. It is given that the charges \( q_A, q_1, \) and \( q_2 \) are each positive. Therefore, the charges \( q_1 \) and \( q_2 \) each exert a repulsive force on the charge \( q_A \). As the drawing shows, these forces have magnitudes \( F_{A1} \) (vertically downward) and \( F_{A2} \) (horizontally to the left). The unknown charge placed at the empty corner of the rectangle is \( q_U \), and it exerts a force on \( q_A \) that has a magnitude \( F_{AU} \). In order that the net force acting on \( q_A \) point in the vertical direction, the horizontal component of \( F_{AU} \) must cancel out the horizontal force \( F_{A2} \). Therefore, \( F_{AU} \) must point as shown in the drawing, which means that it is an attractive force and \( q_U \) must be negative, since \( q_A \) is positive.

SOLUTION The basis for our solution is the fact that the horizontal component of \( F_{AU} \) must cancel out the horizontal force \( F_{A2} \). The magnitudes of these forces can be expressed using Coulomb’s law \( F = k|q||q'|/r^2 \), where \( r \) is the distance between the charges \( q \) and \( q' \). Thus, we have

\[
F_{AU} = \frac{k|q_A||q_U|}{(4d)^2 + d^2} \quad \text{and} \quad F_{A2} = \frac{k|q_A||q_2|}{(4d)^2}
\]

where we have used the fact that the distance between the charges \( q_A \) and \( q_U \) is the diagonal of the rectangle, which is \( \sqrt{(4d)^2 + d^2} \) according to the Pythagorean theorem, and the fact that the distance between the charges \( q_A \) and \( q_2 \) is \( 4d \). The horizontal component of \( F_{AU} \) is \( F_{AU} \cos \theta \), which must be equal to \( F_{A2} \), so that we have

\[
\frac{k|q_A||q_U|}{(4d)^2 + d^2} \cos \theta = \frac{k|q_A||q_2|}{(4d)^2} \quad \text{or} \quad \frac{|q_U|}{17} \cos \theta = \frac{|q_2|}{16}
\]

The drawing in the REASONING, reveals that \( \cos \theta = (4d)/\sqrt{(4d)^2 + d^2} = 4/\sqrt{17} \). Therefore, we find that

\[
\frac{|q_U|}{17} \left(\frac{4}{\sqrt{17}}\right) = \frac{|q_2|}{16} \quad \text{or} \quad |q_U| = \frac{17\sqrt{17}}{64} |q_2| = \frac{17\sqrt{17}}{64} (3.0 \times 10^{-6} \text{ C}) = 3.3 \times 10^{-6} \text{ C}
\]

As discussed in the REASONING, the algebraic sign of the charge \( q_U \) is negative.
26. **REASONING AND SOLUTION** In order for the net force on any charge to be directed inward toward the center of the square, the charges must be placed with alternate + and – signs on each successive corner. The magnitude of the force on any charge due to an adjacent charge located at a distance $r$ is

$$F = k\frac{|q|^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.0 \times 10^{-6} \text{ C})^2}{(0.30 \text{ m})^2} = 0.40 \text{ N}$$

The forces due to two adjacent charges are perpendicular to one another and produce a resultant force that has a magnitude of

$$F_{\text{adjacent}} = \sqrt{2F^2} = \sqrt{2(0.40 \text{ N})^2} = 0.57 \text{ N}$$

The magnitude of the force due to the diagonal charge that is located at a distance of $r\sqrt{2}$ is

$$F_{\text{diagonal}} = k\frac{|q|^2}{(r\sqrt{2})^2} = k\frac{|q|^2}{2r^2} = \frac{0.40 \text{ N}}{2} = 0.20 \text{ N}$$

since the diagonal distance is $r\sqrt{2}$. The force $F_{\text{diagonal}}$ is directed opposite to $F_{\text{adjacent}}$ (since the diagonal charges are of the same sign). Therefore, the net force acting on any of the charges is directed inward and has a magnitude

$$F_{\text{net}} = F_{\text{adjacent}} - F_{\text{diagonal}} = 0.57 \text{ N} - 0.20 \text{ N} = 0.37 \text{ N}$$

27. **SSM REASONING** The charged insulator experiences an electric force due to the presence of the charged sphere shown in the drawing in the text. The forces acting on the insulator are the downward force of gravity (i.e., its weight, $W = mg$), the electrostatic force $F = k\frac{|q_1||q_2|}{r^2}$ (see Coulomb’s law, Equation 18.1) pulling to the right, and the tension $T$ in the thread pulling up and to the left at an angle $\theta$ with respect to the vertical, as shown in the drawing in the problem statement. We can analyze the forces to determine the desired quantities $\theta$ and $T$.

**SOLUTION.**

a. We can see from the diagram given with the problem statement that

$$T_x = F \quad \text{which gives} \quad T \sin \theta = k\frac{|q_1||q_2|}{r^2}$$

and

$$T_y = W \quad \text{which gives} \quad T \cos \theta = mg$$

Dividing the first equation by the second yields

$$\frac{T \sin \theta}{T \cos \theta} = \tan \theta = \frac{k\frac{|q_1||q_2|}{r^2}}{mg}$$
Solving for $\theta$, we find that

$$\theta = \tan^{-1}\left( \frac{k|q_1||q_2|}{mgr^2} \right)$$

$$= \tan^{-1}\left[ \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.600 \times 10^{-6} \text{ C})(0.900 \times 10^{-6} \text{ C})}{(8.00 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m})^2} \right] = 15.4^\circ$$

b. Since $T \cos \theta = mg$, the tension can be obtained as follows:

$$T = \frac{mg}{\cos \theta} = \frac{(8.00 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)}{\cos 15.4^\circ} = 0.813 \text{ N}$$

28. REASONING AND SOLUTION Since the objects attract each other initially, one object has a negative charge, and the other object has a positive charge. Assume that the negative charge has a magnitude of $|q_1|$ and that the positive charge has a magnitude of $|q_2|$. Assume also that $|q_2|$ is greater than $|q_1|$. The magnitude $F$ of the initial attractive force between the objects is

$$F = \frac{k|q_1||q_2|}{r^2} \quad (1)$$

After the objects are brought into contact and returned to their initial positions, the charge on each object is the same and has a magnitude of $(|q_2| - |q_1|)/2$. The magnitude $F$ of the force between the objects is now

$$F = \frac{k[(|q_2| - |q_1|)/2]^2}{r^2} \quad (2)$$

It is given that $F$ is the same in Equations (1) and (2). Therefore, we can equate Equations (1) and (2) and rearrange the result to find that

$$|q_1|^2 - 6|q_2||q_1| + |q_2|^2 = 0 \quad (3)$$

The solutions to this quadratic equation are

$$|q_1| = 5.828|q_2| \quad \text{and} \quad |q_1| = 0.1716|q_2| \quad (4)$$

Since we have assumed that $|q_2|$ is greater than $|q_1|$, we must choose the solution $|q_1| = 0.1716|q_2|$. Substituting this result into Equation (1) and using the given values of $F = 1.20 \text{ N}$ and $r = 0.200 \text{ m}$, we find that
\[ |q_1| = 0.957 \, \mu C \quad \text{and} \quad |q_2| = 5.58 \, \mu C \]  

(5)

Note that we need not have assumed that \( |q_2| \) is greater than \( |q_1| \). We could have assumed that \( |q_2| \) is less than \( |q_1| \). Had we done so, we would have found that

\[ |q_1| = 5.58 \, \mu C \quad \text{and} \quad |q_2| = 0.957 \, \mu C \]  

(6)

Considering Equations (5) and (6) and remembering that \( q_1 \) is the negative charge, we conclude that the two possible solutions to this problem are

\[ q_1 = -0.957 \, \mu C \quad \text{and} \quad q_2 = +5.58 \, \mu C \quad \text{or} \quad q_1 = -5.58 \, \mu C \quad \text{and} \quad q_2 = +0.957 \, \mu C \]

29. **REASONING** The electric field created by a point charge is inversely proportional to the square of the distance from the charge, according to Equation 18.3. Therefore, we expect the distance \( r_2 \) to be greater than the distance \( r_1 \), since the field is smaller at \( r_2 \) than it is at \( r_1 \). The ratio \( r_2/r_1 \), then, should be greater than one.

**SOLUTION** Applying Equation 18.3 to each position relative to the charge, we have

\[ E_1 = \frac{k |q|}{r_1^2} \quad \text{and} \quad E_2 = \frac{k |q|}{r_2^2} \]

Dividing the expression for \( E_1 \) by the expression for \( E_2 \) gives

\[ \frac{E_1}{E_2} = \frac{k |q|/r_1^2}{k |q|/r_2^2} = \frac{r_2^2}{r_1^2} \]

Solving for the ratio \( r_2/r_1 \) gives

\[ \frac{r_2}{r_1} = \sqrt{\frac{E_1}{E_2}} = \sqrt{\frac{248 \, \text{N/C}}{132 \, \text{N/C}}} = 1.37 \]

As expected, this ratio is greater than one.

30. **REASONING**

a. The magnitude of the electric field is obtained by dividing the magnitude of the force (obtained from the meter) by the magnitude of the charge. Since the charge is positive, the direction of the electric field is the same as the direction of the force.

b. As in part (a), the magnitude of the electric field is obtained by dividing the magnitude of the force by the magnitude of the charge. Since the charge is negative, however, the direction of the force (as indicated by the meter) is opposite to the direction of the electric field. Thus, the direction of the electric field is opposite to that of the force.
**Solution**
a. According to Equation 18.2, the magnitude of the electric field is

\[ E = \frac{F}{q} = \frac{40.0 \, \mu \text{N}}{20.0 \, \mu \text{C}} = 2.0 \, \text{N/C} \]

As mentioned in the *Reasoning*, the direction of the electric field is the same as the direction of the force, or **due east**.

b. The magnitude of the electric field is

\[ E = \frac{F}{q} = \frac{20.0 \, \mu \text{N}}{10.0 \, \mu \text{C}} = 2.0 \, \text{N/C} \]

Since the charge is negative, the direction of the electric field is opposite to the direction of the force, or **due east**. Thus, the electric fields in parts (a) and (b) are the same.

---

### 31. Solution

Knowing the electric field at a spot allows us to calculate the force that acts on a charge placed at that spot, without knowing the nature of the object producing the field. This is possible because the electric field is defined as \( E = \frac{F}{q_0} \), according to Equation 18.2. This equation can be solved directly for the force \( F \), if the field \( E \) and charge \( q_0 \) are known.

**Solution** Using Equation 18.2, we find that the force has a magnitude of

\[ F = E|q_0| = (260,000 \, \text{N/C})(7.0 \times 10^{-6} \, \text{C}) = 1.8 \, \text{N} \]

If the charge were positive, the direction of the force would be due west, the same as the direction of the field. But the charge is negative, so the force points in the opposite direction or due east. Thus, the force on the charge is **1.8 N due east**.

---

### 32. Reasoning and Solution

The electric field lines must originate on the positive charges and terminate on the negative charge. They cannot cross one another. Furthermore, the number of field lines beginning or terminating on any charge must be proportional to the magnitude of the charge. Thus, for every field line that leaves the charge +q, two field lines must leave the charge +2q. These three lines must terminate on the −3q charge. If the sketch is to have six field lines, two of them must originate on +q, and four of them must originate on the charge +2q.
33. **REASONING** Each charge creates an electric field at the center of the square, and the four fields must be added as vectors to obtain the net field. Since the charges all have the same magnitude and since each corner is equidistant from the center of the square, the magnitudes of the four individual fields are identical. Each is given by Equation 18.3 as \( E = \frac{k|q|}{r^2} \). The directions of the various contributions are not the same, however. The field created by a positive charge points away from the charge, while the field created by a negative charge points toward the charge.

**SOLUTION** The drawing at the right shows each of the field contributions at the center of the square (see black dot). Each is directed along a diagonal of the square. Note that \( \mathbf{E}_D \) and \( \mathbf{E}_B \) point in opposite directions and, therefore, cancel, since they have the same magnitude. In contrast \( \mathbf{E}_A \) and \( \mathbf{E}_C \) point in the same direction toward corner A and, therefore, combine to give a net field that is twice the magnitude of \( \mathbf{E}_A \) or \( \mathbf{E}_C \). In other words, the net field at the center of the square is given by the following vector equation:

\[
\sum \mathbf{E} = \mathbf{E}_A + \mathbf{E}_B + \mathbf{E}_C + \mathbf{E}_D = \mathbf{E}_A + \mathbf{E}_B + \mathbf{E}_C - \mathbf{E}_B = \mathbf{E}_A + \mathbf{E}_C = 2\mathbf{E}_A
\]

Using Equation 18.3, we find that the magnitude of the net field is

\[
\sum E = 2E_A = 2 \frac{k|q|}{r^2}
\]

In this result \( r \) is the distance from a corner to the center of the square, which is one half of the diagonal distance \( d \). Using \( L \) for the length of a side of the square and taking advantage of the Pythagorean theorem, we have \( r = \frac{1}{2} d = \frac{1}{2} \sqrt{L^2 + L^2} \). With this substitution for \( r \), the magnitude of the net field becomes

\[
\sum E = 2 \frac{k|q|}{\left(\frac{1}{2} \sqrt{L^2 + L^2}\right)^2} = \frac{4k|q|}{L^2} = \frac{4 \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \left(2.4 \times 10^{-12} \text{ C}\right)}{(0.040 \text{ m})^2} = 54 \text{ N/C}
\]

34. **REASONING**

**Part (a)** of the drawing given in the text. The electric field produced by a charge points away from a positive charge and toward a negative charge. Therefore, the electric field \( \mathbf{E}_{+2} \) produced by the +2.0 \( \mu \text{C} \) charge points away from it, and the electric fields \( \mathbf{E}_{-3} \) and \( \mathbf{E}_{-5} \) produced by the −3.0 \( \mu \text{C} \) and −5.0 \( \mu \text{C} \) charges point toward them (see the left-hand side of the following drawing). The magnitude of the electric field produced by a point charge is
given by Equation 18.3 as \( E = k|q|/r^2 \). Since the distance from each charge to the origin is the same, the magnitude of the electric field is proportional only to the magnitude \( |q| \) of the charge. Thus, the \( x \) component \( E_x \) of the net electric field is proportional to 5.0 \( \mu \text{C} \) (2.0 \( \mu \text{C} \) + 3.0 \( \mu \text{C} \)). Since only one of the charges produces an electric field in the \( y \) direction, the \( y \) component \( E_y \) of the net electric field is proportional to the magnitude of this charge, or 5.0 \( \mu \text{C} \). Thus, the \( x \) and \( y \) components are equal, as indicated at the right-hand side of the following drawing, where the net electric field \( E \) is also shown.

**Part (b) of the drawing given in the text.** Using the same arguments as earlier, we find that the electric fields produced by the four charges are shown at the left-hand side of the following drawing. These fields also produce the same net electric field \( E \) as before, as indicated at the right-hand side of the following drawing.

**SOLUTION**

**Part (a) of the drawing given in the text.** The net electric field in the \( x \) direction is

\[
E_x = \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times 2.0 \times 10^{-6} \text{ C}}{(0.061 \text{ m})^2} \right) + \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times 3.0 \times 10^{-6} \text{ C}}{(0.061 \text{ m})^2} \right)
\]

\[
= 1.2 \times 10^7 \text{ N/C}
\]

The net electric field in the \( y \) direction is
The magnitude of the net electric field is

\[
E = \sqrt{E_x^2 + E_y^2} = \sqrt{(1.2 \times 10^7 \text{ N/C})^2 + (1.2 \times 10^7 \text{ N/C})^2} = 1.7 \times 10^7 \text{ N/C}
\]

**Part (b) of the drawing given in the text.** The magnitude of the net electric field is the same as determined for part (a); \( E = 1.7 \times 10^7 \text{ N/C} \)
c. At $y = +0.15$ m, the electric fields $\mathbf{E}_{\text{point}}$ and $\mathbf{E}_{\text{ext}}$ are perpendicular (see the drawing). This makes them, in effect, the $x$-component ($\mathbf{E}_{\text{ext}}$) and $y$-component ($\mathbf{E}_{\text{point}}$) of the net electric field $\mathbf{E}$. The magnitude $E$ of the net electric field, then, is given by the Pythagorean theorem (Equation 1.7):

$$E = \sqrt{E_{\text{ext}}^2 + E_{\text{point}}^2} = \sqrt{(4500 \text{ N/C})^2 + (3200 \text{ N/C})^2} = 5500 \text{ N/C}$$

36. **REASONING**

a. The magnitude $E$ of the electric field is given by $E = \sigma / \varepsilon_0$ (Equation 18.4), where $\sigma$ is the charge density (or charge per unit area) and $\varepsilon_0$ is the permittivity of free space.

b. The magnitude $F$ of the electric force that would be exerted on a $\mathbf{K}^+$ ion placed inside the membrane is the product of the magnitude $|q_0|$ of the charge and the magnitude $E$ of the electric field (see Equation 18.2), or $F = |q_0|E$.

**SOLUTION**

a. The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{7.1 \times 10^{-6} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 8.0 \times 10^5 \text{ N/C}$$

b. The magnitude $F$ of the force exerted on a $\mathbf{K}^+$ ion ($q_0 = +e$) is

$$F = |q_0|E = |e|E = |1.60 \times 10^{-19} \text{ C}|(8.0 \times 10^5 \text{ N/C}) = 1.3 \times 10^{-13} \text{ N}$$

37. **SSM REASONING** The drawing at the right shows the set-up. Here, the electric field $\mathbf{E}$ points along the $+y$ axis and applies a force of $+\mathbf{F}$ to the $+q$ charge and a force of $-\mathbf{F}$ to the $-q$ charge, where $q = 8.0 \mu\text{C}$ denotes the magnitude of each charge. Each force has the same magnitude of $F = E |q|$, according to Equation 18.2.

The torque is measured as discussed in Section 9.1. According to Equation 9.1, the torque produced by each force has a magnitude given by the magnitude of the force times the lever arm, which is the perpendicular distance between the point of application of the force and the axis of rotation. In the drawing the $z$ axis is the axis of rotation and is midway between the ends of the rod. Thus, the lever arm for each force is half the length $L$ of the rod or $L/2$, and the magnitude of the torque produced by each force is $(E |q|)(L/2)$. 
SOLUTION The +F and the –F force each cause the rod to rotate in the same sense about the z axis. Therefore, the torques from these forces reinforce one another. Using the expression \( (E |q|)(L/2) \) for the magnitude of each torque, we find that the magnitude of the net torque is

\[
\text{Magnitude of net torque} = E|q| \left( \frac{L}{2} \right) + E|q| \left( \frac{L}{2} \right) = E|q|L
\]

\[
= (5.0 \times 10^3 \text{ N/C})(8.0 \times 10^{-6} \text{ C})(4.0 \text{ m}) = 0.16 \text{ N\cdot m}
\]

38. REASONING Before the 3.0-\( \mu \text{C} \) point charge \( q \) is introduced into the region, the region contains a uniform electric field \( E \) of magnitude \( 1.6 \times 10^4 \text{ N/C} \). After the 3.0-\( \mu \text{C} \) charge is introduced into the region, the net electric field changes. In addition to the uniform electric field \( E \), the region will also contain the electric field \( E_q \) due to the point charge \( q \). The field at any point in the region is the vector sum of \( E \) and \( E_q \). The field \( E_q \) is radial as discussed in the text, and its magnitude at any distance \( r \) from the charge \( q \) is given by \( E_q = k|q|/r^2 \) (Equation 18.3). There will be one point \( P \) in the region where the net electric field \( E_{\text{net}} \) is zero. This point is located where the field \( E \) has the same magnitude and points in the direction opposite to the field \( E_q \). We will use this reasoning to find the distance \( r_0 \) from the charge \( q \) to the point \( P \).

SOLUTION Let us assume that the field \( E \) points to the right and that the charge \( q \) is negative (the problem is done the same way if \( q \) is positive, although then the relative positions of \( P \) and \( q \) will be reversed). Since \( q \) is negative, its electric field is radially inward (i.e., toward \( q \)); therefore, in order for the field \( E_q \) to point in the opposite direction to \( E \), the charge \( q \) will have to be to the left of the point \( P \) where \( E_{\text{net}} \) is zero, as shown in the drawing at the right. Using \( E_q = k|q|/r_0^2 \) (Equation 18.3) and solving for the distance \( r_0 \), we find

\[
r_0 = \sqrt{k|q|/E_q}
\]

Since the magnitude \( E_q \) must be equal to the magnitude of \( E \) at the point \( P \), we have

\[
r_0 = \sqrt{k|q|/E} = \sqrt{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})/1.6 \times 10^4 \text{ N/C}} = 1.3 \text{ m}
\]

39. [SSM] REASONING Two forces act on the charged ball (charge \( q \)); they are the downward force of gravity \( mg \) and the electric force \( F \) due to the presence of the charge \( q \) in the electric field \( E \). In order for the ball to float, these two forces must be equal in magnitude and opposite in direction, so that the net force on the ball is zero (Newton's
second law). Therefore, \( \mathbf{F} \) must point upward, which we will take as the positive direction. According to Equation 18.2, \( \mathbf{F} = q \mathbf{E} \). Since the charge \( q \) is negative, the electric field \( \mathbf{E} \) must point downward, as the product \( q \mathbf{E} \) in the expression \( \mathbf{F} = q \mathbf{E} \) must be positive, since the force \( \mathbf{F} \) points upward. The magnitudes of the two forces must be equal, so that \( mg = \lvert q \rvert E \). This expression can be solved for \( E \).

**SOLUTION**  The magnitude of the electric field \( \mathbf{E} \) is

\[
E = \frac{mg}{\lvert q \rvert} = \frac{(0.012 \text{ kg})(9.80 \text{ m/s}^2)}{18 \times 10^{-6} \text{ C}} = 6.5 \times 10^3 \text{ N/C}
\]

As discussed in the reasoning, this electric field points downward.

40. **REASONING** The proton and the electron have the same charge magnitude \( e \), so the electric force that each experiences has the same magnitude. The directions are different, however. The proton, being positive, experiences a force in the same direction as the electric field (due east). The electron, being negative, experiences a force in the opposite direction (due west).

Newton’s second law indicates that the direction of the acceleration is the same as the direction of the net force, which, in this case, is the electric force. The proton’s acceleration is in the same direction (due east) as the electric field. The electron’s acceleration is in the opposite direction (due west) as the electric field.

Newton’s second law indicates that the magnitude of the acceleration is equal to the magnitude of the electric force divided by the mass. Although the proton and electron experience the same force magnitude, they have different masses. Thus, they have accelerations of different magnitudes.

**SOLUTION** According to Newton’s second law, Equation 4.2, the acceleration \( a \) of an object is equal to the net force divided by the object’s mass \( m \). Here there is only one force, the electric force \( F \), so it is the net force. According to Equation 18.2, the magnitude of the electric force is equal to the product of the magnitude of the charge and the magnitude of the electric field, or \( F = \lvert q \rvert E \). Thus, the magnitude of the acceleration can be written as

\[
a = \frac{F}{m} = \frac{\lvert q \rvert E}{m}
\]

The magnitude of the acceleration of the proton is

\[
a = \frac{\lvert q \rvert E}{m} = \frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(8.0 \times 10^4 \text{ N/C}\right)}{1.67 \times 10^{-27} \text{ kg}} = 7.7 \times 10^{12} \text{ m/s}^2
\]

The magnitude of the acceleration of the electron is

\[
a = \frac{\lvert q \rvert E}{m} = \frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(8.0 \times 10^4 \text{ N/C}\right)}{9.11 \times 10^{-31} \text{ kg}} = 1.4 \times 10^{16} \text{ m/s}^2
\]
41. **REASONING AND SOLUTION** Figure 1 at the right shows the configuration given in text Figure 18.21a. The electric field at the center of the rectangle is the resultant of the electric fields at the center due to each of the four charges. As discussed in Conceptual Example 11, the magnitudes of the electric field at the center due to each of the four charges are equal. However, the fields produced by the charges in corners 1 and 3 are in opposite directions. Since they have the same magnitudes, they combine to give zero resultant. The fields produced by the charges in corners 2 and 4 point in the same direction (toward corner 2). Thus, \( E_C = E_{C2} + E_{C4} \), where \( E_C \) is the magnitude of the electric field at the center of the rectangle, and \( E_{C2} \) and \( E_{C4} \) are the magnitudes of the electric field at the center due to the charges in corners 2 and 4 respectively. Since both \( E_{C2} \) and \( E_{C4} \) have the same magnitude, we have \( E_C = 2E_{C2} \).

The distance \( r \), from any of the charges to the center of the rectangle, can be found using the Pythagorean theorem (see Figure 2 at the right):

\[
d = \sqrt{(3.00 \text{ cm})^2 + (5.00 \text{ cm})^2} = 5.83 \text{ cm}
\]

Therefore, \( r = \frac{d}{2} = 2.92 \text{ cm} = 2.92 \times 10^{-2} \text{ m} \)

The electric field at the center has a magnitude of

\[
E_C = 2E_{C2} = \frac{2k|q_2|}{r^2} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.60 \times 10^{-12} \text{ C})}{(2.92 \times 10^{-2} \text{ m})^2} = 1.81 \times 10^2 \text{ N/C}
\]

Figure 3 at the right shows the configuration given in text Figure 18.21b. All four charges contribute a non-zero component to the electric field at the center of the rectangle. As discussed in Conceptual Example 11, the contribution from the charges in corners 2 and 4 point toward corner 2 and the contribution from the charges in corners 1 and 3 point toward corner 1.

Notice also, the magnitudes of \( E_{24} \) and \( E_{13} \) are equal, and, from the first part of this problem, we know that

\[
E_{24} = E_{13} = 1.81 \times 10^2 \text{ N/C}
\]
The electric field at the center of the rectangle is the vector sum of $E_{24}$ and $E_{13}$. The $x$ components of $E_{24}$ and $E_{13}$ are equal in magnitude and opposite in direction; hence

$$(E_{13})_x - (E_{24})_x = 0$$

Therefore,

$$E_C = (E_{13})_y + (E_{24})_y = 2(E_{13})_y = 2(E_{13}) \sin \theta$$

From Figure 2, we have that

$$\sin \theta = \frac{5.00 \text{ cm}}{d} = \frac{5.00 \text{ cm}}{5.83 \text{ cm}} = 0.858$$

and

$$E_C = 2(E_{13}) \sin \theta = 2\left(1.81 \times 10^2 \text{ N/C}\right)(0.858) = 3.11 \times 10^2 \text{ N/C}$$

---

42. **REASONING AND SOLUTION** The magnitude of the force on $q_1$ due to $q_2$ is given by Coulomb's law:

$$F_{12} = \frac{k|q_1||q_2|}{r_{12}^2} \quad (1)$$

The magnitude of the force on $q_1$ due to the electric field of the capacitor is given by

$$F_{1C} = |q_1|E_C = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right) \quad (2)$$

Equating the right hand sides of Equations (1) and (2) above gives

$$\frac{k|q_1||q_2|}{r_{12}^2} = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right)$$

Solving for $r_{12}$ gives

$$r_{12} = \sqrt{\frac{\varepsilon_0 k|q_2|}{\sigma}}$$

$$= \sqrt{\frac{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)}{0.858}(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})} = 5.53 \times 10^{-2} \text{ m}$$

---

43. **REASONING** The electric field is given by Equation 18.2 as the force $F$ that acts on a test charge $q_0$, divided by $q_0$. Although the force is not known, the acceleration and mass of the charged object are given. Therefore, we can use Newton’s second law to determine the force as the mass times the acceleration and then determine the magnitude of the field directly
from Equation 18.2. The force has the same direction as the acceleration. The direction of
the field, however, is in the direction opposite to that of the acceleration and force. This is
because the object carries a negative charge, while the field has the same direction as the
force acting on a positive test charge.

**SOLUTION** According to Equation 18.2, the magnitude of the electric field is

\[ E = \frac{F}{|q_0|} \]

According to Newton’s second law, the net force acting on an object of mass \( m \) and
acceleration \( a \) is \( \Sigma F = ma \). Here, the net force is the electrostatic force \( F \), since that force
alone acts on the object. Thus, the magnitude of the electric field is

\[ E = \frac{F}{|q_0|} = \frac{ma}{|q_0|} = \frac{(3.0 \times 10^{-3} \text{ kg})(2.5 \times 10^3 \text{ m/s}^2)}{34 \times 10^{-6} \text{ C}} = 2.2 \times 10^5 \text{ N/C} \]

The direction of this field is opposite to the direction of the acceleration. Thus, the field
points along the \(-x\) axis.

44. **REASONING** The external electric field \( E \) exerts a force \( F_E = qE \)
(Equation 18.2) on the sphere, where \( q = +6.6 \mu\text{C} \) is the net charge
of the sphere. The external electric field \( E \) is directed upward, so the force \( F_E \) it exerts on the positively charged sphere is also
directed upward (see the free-body diagram). Balancing this
upward force are two downward forces: the weight \( mg \) of the
sphere (where \( m \) is the mass of the sphere and \( g \) is the acceleration
due to gravity) and the force \( F_s \) exerted on the sphere by the spring
(see the free-body diagram). We know that the spring exerts a
downward force on the sphere because the equilibrium length
\( L = 0.059 \text{ m} \) of the spring is shorter than its unstrained length
\( L_0 = 0.074 \text{ m} \). The magnitude \( F_s \) of the spring force is given by
\( F_s = kx \) (Equation 10.2, without the minus sign), where \( k \) is the spring constant of the spring and
\( x = L_0 - L = 0.074 \text{ m} - 0.059 \text{ m} = 0.015 \text{ m} \) is the distance by which the spring has been
compressed.

**SOLUTION** Solving \( F_E = qE \) (Equation 18.2) for the magnitude \( E \) of the external electric
field, we find that

\[ E = \frac{F_E}{q} \]  

(1)

The sphere is in equilibrium, so the upward force \((F_E)\) must exactly balance the two
downward forces \((mg \text{ and } F_s)\). Therefore, the magnitudes of the three forces are related by
\[ F_E = mg + F_s \]  
\[ E = \frac{mg + F_s}{q} \]

Lastly, substituting \( F_s = kx \) (Equation 10.2, without the minus sign) into Equation (3), we obtain the desired electric field magnitude:

\[ E = \frac{mg + kx}{q} = \frac{(5.1 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) + (2.4 \text{ N/m})(0.015 \text{ m})}{6.6 \times 10^{-6} \text{ C}} = 1.3 \times 10^4 \text{ N/C} \]

45. SSM REASONING The two charges lying on the x axis produce no net electric field at the coordinate origin. This is because they have identical charges, are located the same distance from the origin, and produce electric fields that point in opposite directions. The electric field produced by \( q_3 \) at the origin points away from the charge, or along the \(-y\) direction. The electric field produced by \( q_4 \) at the origin points toward the charge, or along the \(+y\) direction. The net electric field is, then, \( E = -E_3 + E_4 \), where \( E_3 \) and \( E_4 \) can be determined by using Equation 18.3.

SOLUTION The net electric field at the origin is

\[ E = -E_3 + E_4 = \frac{-k|q_3|}{r_3^2} + \frac{k|q_4|}{r_4^2} \]

\[ = \frac{-\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(3.0 \times 10^{-6} \text{ C})}{\left(5.0 \times 10^{-2} \text{ m}\right)^2} + \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(8.0 \times 10^{-6} \text{ C})}{\left(7.0 \times 10^{-2} \text{ m}\right)^2} \]

\[ = 3.9 \times 10^6 \text{ N/C} \]

The plus sign indicates that the net electric field points along the \(+y\) direction.

46. REASONING The electric field is a vector. Therefore, the total field \( \mathbf{E} \) is the vector sum of its two parts, or \( \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \). We will carry out this vector addition by using the method of components (see Section 1.8).
SOLUTION  The drawing at the right shows the two vectors \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \), together with their \( x \) and \( y \) components. In the following table, we calculate the components of each vector. We also show the \( x \) component \( E_x \) of the total field as the sum of the individual \( x \) components of \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \) and the \( y \) component \( E_y \) of the total field as the sum of the individual \( y \) components of \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \). Note that the calculations in the table carry additional significant figures. Rounding off to the correct number of significant figures will be done when we calculate the final answers.

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x ) component</th>
<th>( y ) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{E}_1 )</td>
<td>( E_{1x} = E_1 \cos \theta_1 = (1200 \text{ N/C}) \cos 35^\circ ) ( = 983 \text{ N/C} )</td>
<td>( E_{1y} = E_1 \sin \theta_1 = (1200 \text{ N/C}) \sin 35^\circ ) ( = 688 \text{ N/C} )</td>
</tr>
<tr>
<td>( \mathbf{E}_2 )</td>
<td>( E_{2x} = E_2 \cos \theta_2 = (1700 \text{ N/C}) \cos 55^\circ ) ( = 975 \text{ N/C} )</td>
<td>( E_{2y} = E_2 \sin \theta_2 = (1700 \text{ N/C}) \sin 55^\circ ) ( = 1393 \text{ N/C} )</td>
</tr>
<tr>
<td>( \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 )</td>
<td>( E_x = E_{1x} + E_{2x} = 983 \text{ N/C} + 975 \text{ N/C} ) ( = 1958 \text{ N/C} )</td>
<td>( E_y = E_{1y} + E_{2y} = 688 \text{ N/C} + 1393 \text{ N/C} ) ( = 2081 \text{ N/C} )</td>
</tr>
</tbody>
</table>

Since the components \( E_x \) and \( E_y \) of the total field are perpendicular, we can use the Pythagorean theorem to calculate the magnitude \( E \) of the total field and trigonometry to calculate the directional angle \( \theta \):

\[
E = \sqrt{E_x^2 + E_y^2} = \sqrt{(1958 \text{ N/C})^2 + (2081 \text{ N/C})^2} = 2900 \text{ N/C}
\]

\[
\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{2081 \text{ N/C}}{1958 \text{ N/C}}\right) = 47^\circ
\]

47. REASONING Since we know the initial velocity and time, we can determine the particle’s displacement from an equation of kinematics, provided its acceleration can be determined. The acceleration is given by Newton’s second law as the net force acting on the particle divided by its mass. The net force is the electrostatic force, since the particle is moving in an electric field. The electrostatic force depends on the particle’s charge and the electric field, both of which are known.

SOLUTION To obtain the displacement \( x \) of the particle we employ Equation 3.5a from the equations of kinematics: \( x = v_{0x} t + \frac{1}{2} a_x t^2 \). We use this equation because two of the variables, the initial velocity \( v_{0x} \) and the time \( t \), are known. The initial velocity is zero, since
the particle is released from rest. The acceleration \( a_x \) can be found from Newton’s second law, as given by Equation 4.2a, as the net force \( \Sigma F_x \) acting on the particle divided by its mass \( m \): 

\[ a_x = \frac{\Sigma F_x}{m} \]

Only the electrostatic force \( F_x \) acts on the proton, so it is the net force. Setting \( \Sigma F_x = F_x \) in Newton’s second law gives 

\[ a_x = \frac{F_x}{m} \]

Substituting this result into Equation 3.5a, we have that

\[
x = v_{0x}t + \frac{1}{2} a_x t^2 = v_{0x}t + \frac{1}{2} \left( \frac{F_x}{m} \right) t^2
\]

(1)

Since the particle is moving in a uniform electric field \( E_x \), it experiences an electrostatic force \( F_x \) given by 

\[ F_x = q_0 E_x \]

(Equation 18.2), where \( q_0 \) is the charge. Substituting this expression for \( F_x \) into Equation (1) gives

\[
x = v_{0x}t + \frac{1}{2} \left( \frac{q_0 E_x}{m} \right) t^2
\]

\[
= (0 \text{ m/s}) (1.6 \times 10^{-2} \text{ s}) + \frac{1}{2} \left[ \frac{(+12 \times 10^{-6} \text{ C})(+480 \text{ N/C})}{3.8 \times 10^{-5} \text{ kg}} \right] (1.6 \times 10^{-2} \text{ s})^2 = +1.9 \times 10^{-2} \text{ m}
\]

48. REASONING

a. The drawing at the right shows the electric fields at point P due to the two charges in the case that the second charge is positive. The presence of the second charge causes the magnitude of the net field at P to be twice as great as it is when only the first charge is present. Since both fields have the same direction, the magnitude of \( E_2 \) must, then, be the same as the magnitude of \( E_1 \). But the second charge is further away from point P than is the first charge, and more distant charges create weaker fields. To offset the weakness that comes from the greater distance, the second charge must have a greater magnitude than that of the first charge.

b. The drawing at the right shows the electric fields at point P due to the two charges in the case that the second charge is negative. The presence of the second charge causes the magnitude of the net field at P to be twice as great as it is when only the first charge is present. Since the fields now have opposite directions, the magnitude of \( E_2 \) must be greater than the magnitude of \( E_1 \). This is necessary so that \( E_2 \) can offset \( E_1 \) and still lead to a net field with twice the magnitude as \( E_1 \). To create this greater field \( E_2 \), the second charge must now have a greater magnitude than it did in question (a).
**SOLUTION**

a. The magnitudes of the field contributions of each charge are given according to Equation 18.3 as \( E = \frac{k|q|}{r^2} \). With \( q_2 \) present, the magnitude of the net field at P is twice what it is when only \( q_1 \) is present. Using Equation 18.3, we can express this fact as follows:

\[
\frac{k|q_1|}{d^2} + \frac{k|q_2|}{(2d)^2} = 2 \frac{k|q_1|}{d^2} \quad \text{or} \quad \frac{k|q_2|}{(2d)^2} = \frac{k|q_1|}{d^2}
\]

Solving for \(|q_2|\) gives

\[
|q_2| = 4|q_1| = 4(0.50 \mu C) = 2.0 \mu C
\]

Thus, the second charge is \( q_2 = +2.0 \mu C \).

b. Now that the second charge is negative, we have

\[
\frac{k|q_2|}{(2d)^2} - \frac{k|q_1|}{d^2} = 2 \frac{k|q_1|}{d^2} \quad \text{or} \quad \frac{k|q_2|}{(2d)^2} = 3 \frac{k|q_1|}{d^2}
\]

Solving for \(|q_2|\) gives

\[
|q_2| = 12|q_1| = 12(0.50 \mu C) = 6.0 \mu C
\]

Thus, the second charge is \( q_2 = -6.0 \mu C \).

---

49. **REASONING AND SOLUTION** From kinematics, \( v_y^2 = v_{0y}^2 + 2a_y y \). Since the electron starts from rest, \( v_{0y} = 0 \) m/s. The acceleration of the electron is given by

\[
a_y = \frac{F}{m} = \frac{eE}{m}
\]

where \( e \) and \( m \) are the electron's charge magnitude and mass, respectively, and \( E \) is the magnitude of the electric field. The magnitude of the electric field between the plates of a parallel plate capacitor is \( E = \sigma e_0 \), where \( \sigma \) is the magnitude of the charge per unit area on each plate. Thus, \( a_y = e\sigma (me_0) \). Combining this expression for \( a \) with the kinematics equation we have

\[
v_y^2 = 2 \left( \frac{e\sigma}{me_0} \right) y
\]

Solving for \( v_y \) gives
50. **REASONING** The following drawing shows the two particles in the electric field $E_x$. They are separated by a distance $d$. If the particles are to stay at the same distance from each other after being released, they must have the same acceleration, so $a_{x,1} = a_{x,2}$. According to Newton’s second law (Equation 4.2a), the acceleration $a_x$ of each particle is equal to the net force $\Sigma F_x$ acting on it divided by its mass $m$, or $a_x = \Sigma F_x / m$.

![Diagram of two particles in an electric field](image)

$q_1 = -7.0 \mu C$

$m_1 = 1.4 \times 10^{-5} \text{ kg}$

$q_2 = +18 \mu C$

$m_2 = 2.6 \times 10^{-5} \text{ kg}$

**SOLUTION** The net force acting on each particle and its resulting acceleration are:

$q_1$: The charge $q_1$ experiences a force $q_1E_x$ due to the electric field (see Equation 18.2). The charge also experiences an attractive force in the $+x$ direction due to the presence of $q_2$. This force is given by Coulomb’s law as $+k|q_1||q_2|/d^2$ (see Equation 18.1). The net force acting on $q_1$ is

$$\Sigma F_{x,1} = q_1E_x + k\frac{|q_1||q_2|}{d^2}$$

The acceleration of $q_1$ is

$$a_{x,1} = \frac{\Sigma F_{x,1}}{m_1} = \frac{q_1E_x + k\frac{|q_1||q_2|}{d^2}}{m_1}$$

$q_2$: The charge $q_2$ experiences a force $q_2E_x$ due to the electric field. It also experiences an attractive force in the $-x$ direction due to the presence of $q_1$. This force is given by Coulomb’s law as $-k|q_1||q_2|/d^2$. The net force acting on $q_2$ is

$$\Sigma F_{x,2} = q_2E_x - k\frac{|q_1||q_2|}{d^2}$$

The acceleration of $q_2$ is
\[ a_{x,2} = \frac{\sum F_{x,2}}{m_2} = \frac{q_2 E_x - k \frac{|q_1||q_2|}{d^2}}{m_2} \]

Setting \( a_{x,1} = a_{x,2} \) gives

\[ \frac{q_1 E_x + k \frac{|q_1||q_2|}{d^2}}{m_1} = \frac{q_2 E_x - k \frac{|q_1||q_2|}{d^2}}{m_2} \]

Solving this expression for \( d \), we find that

\[ d = \frac{k |q_1||q_2|}{E_x} \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \]

\[ = \sqrt{\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^2}{\left( -7.0 \times 10^{-6} \text{ C} \right) + 18 \times 10^{-6} \text{ C} \left( \frac{1}{1.4 \times 10^{-5} \text{ kg}} + \frac{1}{2.6 \times 10^{-5} \text{ kg}} \right)}} \]

\[ = \frac{(+2500 \text{ N/C})}{(2.6 \times 10^{-5} \text{ kg} - 1.4 \times 10^{-5} \text{ kg})} = 6.5 \text{ m} \]

51. **SSM REASONING AND SOLUTION** The net electric field at point \( P \) in Figure 1 is the vector sum of the fields \( E_+ \) and \( E_- \), which are due, respectively, to the charges \( +q \) and \( -q \). These fields are shown in Figure 2.

According to Equation 18.3, the magnitudes of the fields \( E_+ \) and \( E_- \) are the same, since the triangle is an isosceles triangle with equal sides of length \( \ell \). Therefore, \( E_+ = E_- = k \frac{|q|}{\ell^2} \).
The vertical components of these two fields cancel, while the horizontal components reinforce, leading to a total field at point $P$ that is horizontal and has a magnitude of

$$E_p = E_+ \cos \alpha + E_- \cos \alpha = 2k|q| \cos \alpha$$

At point $M$ in Figure 1, both $E_+$ and $E_-$ are horizontal and point to the right. Again using Equation 18.3, we find

$$E_M = E_+ + E_- = \frac{k|q|}{d^2} + \frac{k|q|}{d^2} = \frac{2k|q|}{d^2}$$

Since $E_M/E_P = 9.0$, we have

$$\frac{E_M}{E_P} = \frac{2k|q|/d^2}{2k|q|((\cos \alpha)/\ell^2)} = \frac{1}{(\cos \alpha) d^2/\ell^2} = 9.0$$

But from Figure 1, we can see that $d/\ell = \cos \alpha$. Thus, it follows that

$$\frac{1}{\cos^2 \alpha} = 9.0 \quad \text{or} \quad \cos \alpha = \sqrt[2]{9.0} = 0.48$$

The value for $\alpha$ is, then, $\alpha = \cos^{-1}(0.48) = [61^\circ]$. 

52. **REASONING** In the drawing that accompanies the problem statement, we assume that the electron is initially moving in the $+x$ direction and then begins moving upward in the $+y$ direction as it moves through the capacitor. The upward part of the motion occurs because the electric field $E$ of the capacitor, which points downward from the positive plate toward the negative plate, exerts a force $F$ on the electron. The electric field is given by $E = F/q_0$ (Equation 18.2), where $q_0 = -e$ is the charge on the electron. Newton’s second law applies to the force (assumed to be the only force present), so that $F = ma_y$ (Equation 4.1), where $m$ is the mass of the electron and $a_y$ is the electron’s acceleration in the $y$ direction. The kinematics equations apply to the motion in both the $x$ and $y$ directions, and with their aid we can determine the acceleration $a_y$. Knowing the acceleration, we will be able to determine the force and, hence, the electric field.

**SOLUTION** According to $E = F/q_0$ (Equation 18.2) and $F = ma_y$ (Equation 4.1), we have

$$E = \frac{F}{q_0} = \frac{ma_y}{q_0} \quad (1)$$

From kinematics we know that the electron’s displacement in the upward direction at the time $t$ that it exits the capacitor is $y = 0.150 \times 10^{-2}$ m and is given by $y = \frac{1}{2} a_y t^2$ (Equation 3.5b with $v_{0y} = 0 \text{ m/s}$ since the electron is initially not moving in the
y direction). We can solve this equation to obtain \( a_y = 2y/t^2 \) and substitute this result into Equation (1):

\[
E = \frac{ma_y}{q_0} = \frac{m2y}{q_0 t^2}
\]

We also know that the electron’s displacement in the horizontal direction at the time \( t \) that it exits the capacitor is \( x = 2.00 \times 10^{-2} \text{ m} \) and is given by \( x = v_{0x} t \), since the horizontal speed of the electron is \( v_{0x} = 7.00 \times 10^6 \text{ m/s} \) and remains constant during the motion. We can solve this equation to obtain \( t = x/v_{0x} \) and substitute this result into Equation (2):

\[
E = \frac{m2y}{q_0 t^2} = \frac{m2y}{q_0 (x/v_{0x})^2} = \frac{m2yv_{0x}^2}{q_0 x^2}
\]

With \( q_0 = -e = -1.60 \times 10^{-19} \text{ C} \) as the electron charge and \( m = 9.11 \times 10^{-31} \text{ kg} \) as the electron mass, Equation (3) gives

\[
E = \frac{m2yv_{0x}^2}{q_0 x^2} = \frac{\left(9.11 \times 10^{-31} \text{ kg}\right)2\left(0.150 \times 10^{-2} \text{ m}\right)\left(7.00 \times 10^6 \text{ m/s}\right)^2}{\left(-1.60 \times 10^{-19} \text{ C}\right)\left(2.00 \times 10^{-2} \text{ m}\right)^2} = -2090 \text{ N/C}
\]

This result is negative, because the electric field points downward in the drawing that accompanies the problem, which is the direction that we assumed to be the negative \( y \) direction. The magnitude of the electric field, then, is \( [2090 \text{ N/C}] \).

53. **REASONING AND SOLUTION** Since the thread makes an angle of 30.0° with the vertical, it can be seen that the electric force on the ball, \( F_e \), and the gravitational force, \( mg \), are related by

\[
F_e = mg \tan 30.0^\circ
\]

The force \( F_e \) is due to the charged ball being in the electric field of the parallel plate capacitor. That is,

\[
F_e = E |q_{\text{ball}}|
\]

where \( |q_{\text{ball}}| \) is the magnitude of the ball’s charge and \( E \) is the magnitude of the field due to the plates. According to Equation 18.4 \( E \) is

\[
E = \frac{q}{\varepsilon_0 A}
\]

where \( q \) is the magnitude of the charge on each plate and \( A \) is the area of each plate. Substituting Equation 18.4 into Equation (1) gives
\[ F_e = mg \tan 30.0^\circ = \frac{q|q_{\text{ball}}|}{\varepsilon_0 A} \]

Solving for \( q \) yields
\[ q = \frac{\varepsilon_0 A m g \tan 30.0^\circ}{|q_{\text{ball}}|} \]
\[ = \frac{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)}{0.0150 \text{ m}^2}(6.50 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan 30.0^\circ \]
\[ = 3.25 \times 10^{-8} \text{ C} \]

54. **REASONING AND SOLUTION** Gauss' Law is given by text Equation 18.7: \( \Phi_E = \frac{Q}{\varepsilon_0} \), where \( Q \) is the net charge enclosed by the Gaussian surface.

a. \( \Phi_E = \frac{3.5 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 4.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \)

b. \( \Phi_E = \frac{-2.3 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = -2.6 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \)

c. \( \Phi_E = \frac{(3.5 \times 10^{-6} \text{ C}) + (-2.3 \times 10^{-6} \text{ C})}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)} = 1.4 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \)

55. **SSM REASONING** As discussed in Section 18.9, the magnitude of the electric flux \( \Phi_E \) through a surface is equal to the magnitude of the component of the electric field that is normal to the surface multiplied by the area of the surface, \( \Phi_E = E_\perp A \), where \( E_\perp \) is the magnitude of the component of \( E \) that is normal to the surface of area \( A \). We can use this expression and the figure in the text to determine the desired quantities.

**SOLUTION**

a. The magnitude of the flux through surface 1 is \( \left( \Phi_E \right)_1 = (E \cos 35^\circ)A_1 = (250 \text{ N/C})(\cos 35^\circ)(1.7 \text{ m}^2) = 350 \text{ N} \cdot \text{m}^2/\text{C} \)

b. Similarly, the magnitude of the flux through surface 2 is \( \left( \Phi_E \right)_2 = (E \cos 55^\circ)A_2 = (250 \text{ N/C})(\cos 55^\circ)(3.2 \text{ m}^2) = 460 \text{ N} \cdot \text{m}^2/\text{C} \)
56. **REASONING** In each case, Gauss’ law can be used to determine the electric flux \( \Phi_E \). For a Gaussian surface that encloses a net charge \( Q \), this law is \( \Phi_E = \frac{Q}{\varepsilon_0} \) (Equation 18.7), where \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{ N} \cdot \text{m}^2) \).

**SOLUTION**

a. Since the net charge surrounded by the surface is \( Q = +2.0 \times 10^{-6} \text{ C} \), Gauss’ law shows that

\[
\Phi_E = \frac{Q}{\varepsilon_0} = \frac{2.0 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/(\text{ N} \cdot \text{m}^2)} = 2.3 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}
\]

b. Since the net charge surrounded by the surface is the same as in part a and since Gauss’ law holds for a closed surface of any shape, the flux is the same as in part a;

\[\Phi_E = 2.3 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \] .

c. Since the net charge surrounded by the surface is the same as in part a and since Gauss’ law holds for a closed surface of any shape, the flux is the same as in part a;

\[\Phi_E = 2.3 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \] .

57. **REASONING** The magnitude of the electric flux \( \Phi_E \) through the circular surface is determined by the angle \( \phi \) (less than 90º) between the electric field and the normal to the surface, as well as the magnitude \( E \) of the electric field and the area \( A \) of the surface:

\[
\Phi_E = (E \cos \phi) A \quad (18.6)
\]

We will use Equation 18.6 to determine the angle \( \phi \).

**SOLUTION** Solving Equation 18.6 for the angle \( \phi \), we obtain

\[
\cos \phi = \frac{\Phi_E}{EA} \quad \text{or} \quad \phi = \cos^{-1}\left(\frac{\Phi_E}{EA}\right) \quad (1)
\]

The surface is circular, with a radius \( r \), so its area is \( A = \pi r^2 \). Making this substitution in Equation (1) yields

\[
\phi = \cos^{-1}\left(\frac{\Phi_E}{EA}\right) = \cos^{-1}\left(\frac{\Phi_E}{E\pi r^2}\right) = \cos^{-1}\left[\frac{78 \text{ N} \cdot \text{m}^2/\text{C}}{(1.44 \times 10^4 \text{ N/C})\pi (0.057 \text{ m})^2}\right] = 58º
\]
58. **REASONING** The charge \( Q \) inside the rectangular box is related to the electric flux \( \Phi_E \) that passes through the surfaces of the box by Gauss’ law, \( Q = \varepsilon_0 \Phi_E \) (Equation 18.7), where \( \varepsilon_0 \) is the permittivity of free space. The electric flux is the algebraic sum of the flux through each of the six surfaces.

**SOLUTION** The charge inside the box is

\[
Q = \varepsilon_0 \Phi_E = \varepsilon_0 \left( \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 \right)
\]

\[
= \left[ 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \right] \left( +1500 \frac{N \cdot m^2}{C} + 2200 \frac{N \cdot m^2}{C} + 4600 \frac{N \cdot m^2}{C} \right.
\]

\[
- 1800 \frac{N \cdot m^2}{C} - 3500 \frac{N \cdot m^2}{C} - 5400 \frac{N \cdot m^2}{C} \left)
\right)
\]

\[
= -2.1 \times 10^{-8} \text{ C}
\]

59. **REASONING** We use a Gaussian surface that is a sphere centered within the solid sphere. The radius \( r \) of this surface is smaller than the radius \( R \) of the solid sphere. Equation 18.7 gives Gauss’ law as follows:

\[
\sum (E \cos \phi) \Delta A = \frac{Q}{\varepsilon_0}
\]

**SOLUTION** The positive charge is spread uniformly throughout the solid sphere and, therefore, is spherically symmetric. Consequently, the electric field is directed radially outward, and for each element of area \( \Delta A \) is perpendicular to the surface. This means that the angle \( \phi \) between the normal to the surface and the field is \( 0^\circ \), as the drawing shows. Furthermore, the electric field has the same magnitude everywhere on the Gaussian surface. Because of these considerations, we can write the electric flux as follows:

\[
\sum (E \cos \phi) \Delta A = \sum (E \cos 0^\circ) \Delta A = E \left( \sum \Delta A \right)
\]

The term \( \sum \Delta A \) is the entire area of the spherical Gaussian surface or \( 4\pi r^2 \). With this substitution, the electric flux becomes

\[
\sum (E \cos \phi) \Delta A = E \left( \sum \Delta A \right) = E \left( 4\pi r^2 \right)
\]

\[ (1) \]
Now consider the net charge $Q$ within the Gaussian surface. This charge is the charge density times the volume $\frac{4}{3}\pi r^3$ encompassed by that surface:

$$Q = \frac{q}{\frac{4}{3}\pi R^3} \times \left(\frac{4}{3}\pi r^3\right) = \frac{qr^3}{R^3} \tag{2}$$

Substituting Equations (1) and (2) into Equation 18.7 gives

$$E\left(4\pi r^2\right) = \frac{qr^3}{\epsilon_0 R^3}$$

Rearranging this result shows that

$$E = \frac{qr^3}{\left(4\pi r^2\right)\epsilon_0} = \frac{qr}{4\pi\epsilon_0 R^3}$$

60. **REASONING** Gauss’ Law, $\sum (E \cos \phi) \Delta A = \frac{Q}{\epsilon_0}$ (Equation 18.7), relates the electric field magnitude $E$ on a Gaussian surface to the net charge $Q$ enclosed by that surface; $\Phi_E = \sum (E \cos \phi) \Delta A$ (Equation 18.6) is the electric flux through the Gaussian surface (divided into many tiny sections of area $\Delta A$) and $\epsilon_0$ is the permittivity of free space. We are to determine the magnitude $E$ of the electric field due to electric charges that are spread uniformly over the surfaces of two concentric spherical shells. The electric field due to these charges possesses spherical symmetry, so we will choose Gaussian surfaces in the shape of spheres concentric with the shells. The radius $r$ of each Gaussian surface will be equal to the distance from the common center of the shells and will be the distance at which we are to evaluate the electric field.

Because the electric field has spherical symmetry, the magnitude $E$ of the electric field is constant at all points on any such spherical Gaussian surface. Furthermore, the electric field is directed either radially outward (if the net charge within the Gaussian surface is positive) or radially inward (if the net charge within the Gaussian surface is negative). This means that the angle $\phi$ between the electric field and the normal to any such spherical Gaussian surface is either $0.0^\circ$ or $180^\circ$. The quantity $E \cos \phi$, therefore, is constant, and may be factored out of the summation in Equation 18.6:

$$\Phi_E = \sum (E \cos \phi) \Delta A = (E \cos \phi) \sum \Delta A \tag{1}$$

The sum $\sum \Delta A$ of all the tiny sections of area $\Delta A$ that compose a spherical Gaussian surface is the total surface area $\sum \Delta A = A = 4\pi r^2$ of a sphere of radius $r$. Thus, Equation (1) becomes

$$\Phi_E = (E \cos \phi) \sum \Delta A = 4\pi r^2 E \cos \phi \tag{2}$$
SOLUTION

a. The outer shell has a radius \( r_2 = 0.15 \) m, and we are to determine the electric field at a distance \( r = 0.20 \) m from the common center of the shells. Therefore, we will choose a spherical Gaussian surface (radius \( r = 0.20 \) m) that encloses both shells and shares their common center. According to Gauss’ law and Equation (2), we have that the net electric flux \( \Phi_E \) through this sphere is

\[
\sum (E \cos \phi) \Delta A = 4\pi r^2 E \cos \phi = \frac{Q}{\varepsilon_0}
\]  

(3)

Solving Equation (3) for \( E \) yields

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2 \cos \phi}
\]  

(4)

Because the chosen Gaussian surface encloses both shells, the net charge \( Q \) enclosed by the surface is \( Q = q_1 + q_2 \). The positive charge \( q_2 \) on the outer shell has a larger magnitude than the negative charge \( q_1 \) on the inner shell, so that \( Q \) is a positive net charge. Therefore, the electric field is directed \( \text{radially outward} \), and the angle between the electric field and the normal to the surface of the spherical Gaussian surface is \( \phi = 0.0^\circ \). Therefore, Equation (4) gives the electric field magnitude as

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2 \cos \phi} = \frac{q_1 + q_2}{4\pi \varepsilon_0 r^2 \cos \phi} = \frac{-1.6 \times 10^{-6} \text{ C} + 5.1 \times 10^{-6} \text{ C}}{4\pi \left[8.85 \times 10^{-12} \text{ C}^2/\left(\text{N} \cdot \text{m}^2\right)\right]}(0.20 \text{ m})^2 \cos 0.0^\circ
\]

\[= 7.9 \times 10^5 \text{ N/C}\]

b. We again choose a spherical Gaussian surface concentric with the shells, this time of radius \( r = 0.10 \) m. The radius of this sphere is greater than the radius \( (r_1 = 0.050 \) m) of the inner shell but less than the radius \( (r_2 = 0.15 \) m) of the outer shell. Therefore, this Gaussian surface is located \( \text{between} \) the two shells and encloses only the charge on the inner shell: \( Q = q_1 \). This is a negative charge, so that the electric field is directed \( \text{radially inward} \), and the angle between the electric field and the normal to the surface of the Gaussian sphere is \( \phi = 180^\circ \). From Equation (4), then, we have that

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2 \cos \phi} = \frac{q_1}{4\pi \varepsilon_0 r^2 \cos \phi} = \frac{-1.6 \times 10^{-6} \text{ C}}{4\pi \left[8.85 \times 10^{-12} \text{ C}^2/\left(\text{N} \cdot \text{m}^2\right)\right]}(0.10 \text{ m})^2 \cos 180^\circ
\]

\[= 1.4 \times 10^6 \text{ N/C}\]

c. Choosing a spherical Gaussian surface with a radius of \( r = 0.025 \) m, we see that it is entirely inside the inner shell \( (r_1 = 0.050 \text{ m}) \). Therefore, the enclosed charge is zero: \( Q = 0 \text{ C} \). Equation (4) shows that the electric field at this distance from the common center is zero:

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2 \cos \phi} = \frac{0 \text{ C}}{4\pi \varepsilon_0 r^2 \cos \phi} = 0 \text{ N/C}
\]
61. **REASONING** The electric flux through each face of the cube is given by \( \Phi_E = (E \cos \phi)A \) (see Section 18.9) where \( E \) is the magnitude of the electric field at the face, \( A \) is the area of the face, and \( \phi \) is the angle between the electric field and the outward normal of that face. We can use this expression to calculate the electric flux \( \Phi_E \) through each of the six faces of the cube.

**SOLUTION**

a. On the bottom face of the cube, the outward normal points parallel to the \(-y\) axis, in the opposite direction to the electric field, and \( \phi = 180^\circ \). Therefore,

\[
(\Phi_E)_{\text{bottom}} = (1500 \text{ N/C})(\cos 180^\circ)(0.20 \text{ m})^2 = -6.0 \times 10^1 \text{ N m}^2/\text{C}
\]

On the top face of the cube, the outward normal points parallel to the \(+y\) axis, and \( \phi = 0.0^\circ \). The electric flux is, therefore,

\[
(\Phi_E)_{\text{top}} = (1500 \text{ N/C})(\cos 0.0)(0.20 \text{ m})^2 = +6.0 \times 10^1 \text{ N m}^2/\text{C}
\]

On each of the other four faces, the outward normals are perpendicular to the direction of the electric field, so \( \phi = 90^\circ \). So for each of the four side faces,

\[
(\Phi_E)_{\text{sides}} = (1500 \text{ N/C})(\cos 90^\circ)(0.20 \text{ m})^2 = 0 \text{ N m}^2/\text{C}
\]

b. The total flux through the cube is

\[
(\Phi_E)_{\text{total}} = (\Phi_E)_{\text{top}} + (\Phi_E)_{\text{bottom}} + (\Phi_E)_{\text{sides}} + (\Phi_E)_{\text{sides}} + (\Phi_E)_{\text{sides}} + (\Phi_E)_{\text{sides}}
\]

Therefore,

\[
(\Phi_E)_{\text{total}} = (+6.0 \times 10^1 \text{ N m}^2/\text{C}) + (-6.0 \times 10^1 \text{ N m}^2/\text{C}) + 0 + 0 + 0 + 0 = 0 \text{ N m}^2/\text{C}
\]

62. **REASONING** Because the charge is distributed uniformly along the straight wire, the electric field is directed radially outward, as the following end view of the wire illustrates.
And because of symmetry, the magnitude of the electric field is the same at all points equidistant from the wire. In this situation we will use a Gaussian surface that is a cylinder concentric with the wire. The drawing shows that this cylinder is composed of three parts, the two flat ends (1 and 3) and the curved wall (2). We will evaluate the electric flux for this three-part surface and then set it equal to $\frac{Q}{\varepsilon_0}$ (Gauss’ law) to find the magnitude of the electric field.

**SOLUTION** Surfaces 1 and 3 – the flat ends of the cylinder – are parallel to the electric field, so $\cos \phi = \cos 90^\circ = 0$. Thus, there is no flux through these two surfaces: $\Phi_1 = \Phi_3 = 0 \text{ N} \cdot \text{m}^2/\text{C}$.

Surface 2 – the curved wall – is everywhere perpendicular to the electric field $E$, so $\cos \phi = \cos 0^\circ = 1$. Furthermore, the magnitude $E$ of the electric field is the same for all points on this surface, so it can be factored outside the summation in Equation 18.6:

$$\Phi_2 = \Sigma \left( E \cos 0^\circ \right) \Delta A = E \Sigma A$$

The area $\Sigma A$ of this surface is just the circumference $2\pi r$ of the cylinder times its length $L$: $\Sigma A = (2\pi r)L$. The electric flux through the entire cylinder is, then,

$$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 = 0 + E(2\pi rL) + 0 = E(2\pi rL)$$

Following Gauss’ law, we set $\Phi_E$ equal to $\frac{Q}{\varepsilon_0}$, where $Q$ is the net charge inside the Gaussian cylinder: $E(2\pi rL) = \frac{Q}{\varepsilon_0}$. The ratio $\frac{Q}{L}$ is the charge per unit length of the wire and is known as the linear charge density $\lambda$: $\lambda = \frac{Q}{L}$. Solving for $E$, we find that

$$E = \frac{Q/L}{2\pi \varepsilon_0 r} = \frac{\lambda}{2\pi \varepsilon_0 r}$$
63. **REASONING** The electric field lines must originate on the positive charges and terminate on the negative charges. They cannot cross one another. Furthermore, the number of field lines beginning or ending on any charge must be proportional to the magnitude of the charge.

**SOLUTION** If 10 electric field lines leave the +5q charge, then six lines must originate from the +3q charge, and eight lines must end on each −4q charge. The drawing shows the electric field lines that meet these criteria.

64. **REASONING** The gravitational force is an attractive force. To neutralize this force, the electrical force must be a repulsive force. Therefore, the charges must both be positive or both negative. Newton’s law of gravitation, Equation 4.3, states that the gravitational force depends inversely on the square of the distance between the earth and the moon. Coulomb’s law, Equation 18.1 states that the electrical force also depends inversely on the square of the distance. When these two forces are added together to give a zero net force, the distance can be algebraically eliminated. Thus, we do not need to know the distance between the two bodies.

**SOLUTION** Since the repulsive electrical force neutralizes the attractive gravitational force, the magnitudes of the two forces are equal:

\[
\frac{k |q| |q|}{r^2} = \frac{GM_e M_m}{r^2}
\]

Electrical force, Equation 18.1
Gravitational force, Equation 4.3

Solving this equation for the magnitude \(|q|\) of the charge on either body, we find

\[
|q| = \sqrt{\frac{GM_e M_m}{k}} = \sqrt{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(7.35 \times 10^{22} \text{ kg}\right)\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)} = 5.71 \times 10^{13} \text{ C}
\]

65. **REASONING AND SOLUTION** The +2q of charge initially on the sphere lies entirely on the outer surface. When the +q charge is placed inside of the sphere, then a −q charge will still be induced on the interior of the sphere. An additional +q will appear on the outer surface, giving a net charge of +3q.
66. **REASONING** Since the charged droplet (charge = $q$) is suspended motionless in the electric field $E$, the net force on the droplet must be zero. There are two forces that act on the droplet, the force of gravity $W = mg$, and the electric force $F = qE$ due to the electric field. Since the net force on the droplet is zero, we conclude that $mg = |q|E$. We can use this reasoning to determine the sign and the magnitude of the charge on the droplet.

**SOLUTION**

a. Since the net force on the droplet is zero, and the weight of magnitude $W$ points downward, the electric force of magnitude $F = |q|E$ must point upward. Since the electric field points upward, the excess charge on the droplet must be **positive** in order for the force $F$ to point upward.

b. Using the expression $mg = |q|E$, we find that the magnitude of the excess charge on the droplet is

$$|q| = \frac{mg}{E} = \frac{(3.50 \times 10^{-9} \text{ kg})(9.80 \text{ m/s}^2)}{8480 \text{ N/C}} = 4.04 \times 10^{-12} \text{ C}$$

The charge on a proton is $1.60 \times 10^{-19} \text{ C}$, so the excess number of protons is

$$\left(4.04 \times 10^{-12} \text{ C}\right)\left(\frac{1 \text{ proton}}{1.60 \times 10^{-19} \text{ C}}\right) = 2.53 \times 10^7 \text{ protons}$$

67. **SSM REASONING**

a. The drawing shows the two point charges $q_1$ and $q_2$. Point A is located at $x = 0$ cm, and point B is at $x = +6.0$ cm.

Since $q_1$ is positive, the electric field points away from it. At point A, the electric field $E_1$ points to the left, in the $-x$ direction. Since $q_2$ is negative, the electric field points toward it. At point A, the electric field $E_2$ points to the right, in the $+x$ direction. The net electric field is $E = -E_1 + E_2$. We can use Equation 18.3, $E = k|q|/r^2$, to find the magnitude of the electric field due to each point charge.
b. The drawing shows the electric fields produced by the charges $q_1$ and $q_2$ at point B, which is located at $x = +6.0$ cm.

Since $q_1$ is positive, the electric field points away from it. At point B, the electric field points to the right, in the $+x$ direction. Since $q_2$ is negative, the electric field points toward it. At point B, the electric field points to the right, in the $+x$ direction. The net electric field is $E = +E_1 + E_2$.

**SOLUTION**

a. The net electric field at the origin (point A) is $E = -E_1 + E_2$:

$$E = -E_1 + E_2 = -\frac{k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= -\left(\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{3.0 \times 10^{-2} \text{ m}}\right)\left(8.5 \times 10^{-6} \text{ C}\right) + \left(\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{9.0 \times 10^{-2} \text{ m}}\right)\left(21 \times 10^{-6} \text{ C}\right)$$

$$= -6.2 \times 10^7 \text{ N/C}$$

The minus sign tells us that the net electric field points along the $-x$ axis.

b. The net electric field at $x = +6.0$ cm (point B) is $E = E_1 + E_2$:

$$E = E_1 + E_2 = \frac{k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= \left(\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{3.0 \times 10^{-2} \text{ m}}\right)\left(8.5 \times 10^{-6} \text{ C}\right) + \left(\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{3.0 \times 10^{-2} \text{ m}}\right)\left(21 \times 10^{-6} \text{ C}\right)$$

$$= +2.9 \times 10^8 \text{ N/C}$$

The plus sign tells us that the net electric field points along the $+x$ axis.
68. **REASONING** The magnitude \( F \) of the forces that point charges \( q_1 \) and \( q_2 \) exert on each other varies with the distance \( r \) separating them according to \( F = k \frac{|q_1||q_2|}{r^2} \) (Equation 18.1), where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \). We note that both charges are given in units of microcoulombs (\( \mu \text{C} \)), rather than the base SI units of coulombs (C). We will replace the prefix \( \mu \) with \( 10^{-6} \) when calculating the distance \( r \) from Equation 18.1.

**SOLUTION** Solving \( F = k \frac{|q_1||q_2|}{r^2} \) (Equation 18.1) for the distance \( r \), we obtain

\[
r^2 = k \frac{|q_1||q_2|}{F} \quad \text{or} \quad r = \sqrt{k \frac{|q_1||q_2|}{F}}
\]

Therefore, when the force magnitude \( F \) is 0.66 N, the distance between the charges must be

\[
r = \sqrt{k \frac{|q_1||q_2|}{F}} = \sqrt{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(\frac{8.4 \times 10^{-6} \text{ C}}{0.66 \text{ N}})(5.6 \times 10^{-6} \text{ C})} = 0.80 \text{ m}
\]

69. **SSM** **REASONING** The electrons transferred increase the magnitudes of the positive and negative charges from 2.00 \( \mu \text{C} \) to a greater value. We can calculate the number \( N \) of electrons by dividing the change in the magnitude of the charges by the magnitude \( e \) of the charge on an electron. The greater charge that exists after the transfer can be obtained from Coulomb’s law and the value given for the magnitude of the electrostatic force.

**SOLUTION** The number \( N \) of electrons transferred is

\[
N = \frac{|q_{\text{after}}| - |q_{\text{before}}|}{e}
\]

where \( |q_{\text{after}}| \) and \( |q_{\text{before}}| \) are the magnitudes of the charges after and before the transfer of electrons occurs. To obtain \( |q_{\text{after}}| \), we apply Coulomb’s law with a value of 68.0 N for the electrostatic force:

\[
F = k \frac{|q_{\text{after}}|^2}{r^2} \quad \text{or} \quad |q_{\text{after}}| = \sqrt{k \frac{Fr^2}{r^2}}
\]

Using this result in the expression for \( N \), we find that

\[
N = \frac{\sqrt{k \frac{Fr^2}{r^2}} - |q_{\text{before}}|}{e} = \frac{(68.0 \text{ N})(0.0300 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 - 2.00 \times 10^{-6} \text{ C}} = 3.8 \times 10^{12}
\]
70. **REASONING** The fact that the net electric field points upward along the vertical axis holds the key to this problem. The drawing at the right shows the fields from each charge, together with the horizontal components of each. The reason that the net field points upward is that these horizontal components point in opposite directions and cancel. Since they cancel, they must have equal magnitudes, a fact that will quickly lead us to a solution.

**SOLUTION** Setting the magnitudes of the horizontal components of the fields equal gives

\[
E_2 \sin 60.0^\circ = E_1 \sin 30.0^\circ
\]

The magnitude of the electric field created by a point charge is given by Equation 18.3. Using this expression for \(E_1\) and \(E_2\) and noting that each point charge is the same distance \(r\) from the center of the circle, we obtain

\[
\frac{k|q_2|}{r^2} \sin 60.0^\circ = \frac{k|q_1|}{r^2} \sin 30.0^\circ \quad \text{or} \quad |q_2| \sin 60.0^\circ = |q_1| \sin 30.0^\circ
\]

Solving for the ratio of the charge magnitudes gives

\[
\frac{|q_2|}{|q_1|} = \frac{\sin 30.0^\circ}{\sin 60.0^\circ} = 0.577
\]

71. **SSM** **REASONING** The drawing shows the arrangement of the three charges. Let \(\vec{E}_q\) represent the electric field at the empty corner due to the \(-q\) charge. Furthermore, let \(\vec{E}_1\) and \(\vec{E}_2\) be the electric fields at the empty corner due to charges \(+q_1\) and \(+q_2\), respectively.

According to the Pythagorean theorem, the distance from the charge \(-q\) to the empty corner along the diagonal is given by \(\sqrt{(2d)^2 + d^2} = \sqrt{5d^2} = d\sqrt{5}\). The magnitude of each electric field is given by Equation 18.3, \(E = k|q|/r^2\). Thus, the magnitudes of each of the electric fields at the empty corner are given as follows:
\[
E_q = \frac{k|q|}{r^2} = \frac{k|q|}{(d\sqrt{3})^2} = \frac{k|q|}{5d^2}
\]

\[
E_1 = \frac{k|q_1|}{(2d)^2} = \frac{k|q_1|}{4d^2} \quad \text{and} \quad E_2 = \frac{k|q_2|}{d^2}
\]

The angle \(\theta\) that the diagonal makes with the horizontal is \(\theta = \tan^{-1}(d/2d) = 26.57^\circ\). Since the net electric field \(E_{\text{net}}\) at the empty corner is zero, the horizontal component of the net field must be zero, and we have

\[
E_1 - E_q \cos 26.57^\circ = 0 \quad \text{or} \quad \frac{k|q_1|}{4d^2} - \frac{k|q|\cos 26.57^\circ}{5d^2} = 0
\]

Similarly, the vertical component of the net field must be zero, and we have

\[
E_2 - E_q \sin 26.57^\circ = 0 \quad \text{or} \quad \frac{k|q_2|}{d^2} - \frac{k|q|\sin 26.57^\circ}{5d^2} = 0
\]

These last two expressions can be solved for the charge magnitudes \(|q_1|\) and \(|q_2|\).

**SOLUTION** Solving the last two expressions for \(|q_1|\) and \(|q_2|\), we find that

\[
|q_1| = \frac{4}{5} q \cos 26.57^\circ = 0.716 q
\]

\[
|q_2| = \frac{1}{5} q \sin 26.57^\circ = 0.0895 q
\]

---

**72. REASONING**

The drawing at the right shows the forces that act on the charges at each corner. For example, \(\mathbf{F}_{AB}\) is the force exerted on the charge at corner A by the charge at corner B. The directions of the forces are consistent with the fact that like charges repel and unlike charges attract. Coulomb’s law indicates that all of the forces shown have the same magnitude, namely, \(F = k|q|^2/L^2\), where \(|q|\) is the magnitude of each of the charges and \(L\) is the length of each side of the equilateral triangle. The magnitude is the same for each force, because \(|q|\) and \(L\) are the same for each pair of charges.
The net force acting at each corner is the sum of the two force vectors shown in the drawing, and the net force is greatest at corner A. This is because the angle between the two vectors at A is 60°. With the angle less than 90°, the two vectors partially reinforce one another. In comparison, the angles between the vectors at corners B and C are both 120°, which means that the vectors at those corners partially offset one another.

The net forces acting at corners B and C have the same magnitude, since the magnitudes of the individual vectors are the same and the angles between the vectors at both B and C are the same (120°). Thus, vector addition by either the tail-to-head method (see Section 1.6) or the component method (see Section 1.8) will give resultant vectors that have different directions but the same magnitude. The magnitude of the net force is the smallest at these two corners.

**SOLUTION** As pointed out in the **REASONING**, the magnitude of any individual force vector is \( F = k \left| q \right|^2 / L^2 \).

With this in mind, we apply the component method for vector addition to the forces at corner A, which are shown in the drawing at the right, together with the appropriate components. The \( x \) component \( \Sigma F_x \) and the \( y \) component \( \Sigma F_y \) of the net force are

\[
\left( \Sigma F_x \right)_A = F_{AB} \cos 60.0° + F_{AC} = F \left( \cos 60.0° + 1 \right)
\]

\[
\left( \Sigma F_y \right)_A = F_{AB} \sin 60.0° = F \sin 60.0°
\]

where we have used the fact that \( F_{AB} = F_{AC} = F \). The Pythagorean theorem indicates that the magnitude of the net force at corner A is

\[
\left( \Sigma F \right)_A = \sqrt{\left( \Sigma F_x \right)_A^2 + \left( \Sigma F_y \right)_A^2} = \sqrt{F^2 \left( \cos 60.0° + 1 \right)^2 + (F \sin 60.0°)^2}
\]

\[
= F \sqrt{\left( \cos 60.0° + 1 \right)^2 + (\sin 60.0°)^2} = k \frac{\left| q \right|^2}{L^2} \sqrt{\left( \cos 60.0° + 1 \right)^2 + (\sin 60.0°)^2}
\]

\[
= \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \frac{\left( 5.0 \times 10^{-6} \text{ C} \right)^2}{(0.030 \text{ m})^2} \sqrt{\left( \cos 60.0° + 1 \right)^2 + (\sin 60.0°)^2}
\]

\[
= 430 \text{ N}
\]
We now apply the component method for vector addition to the forces at corner B. These forces, together with the appropriate components are shown in the drawing at the right. We note immediately that the two vertical components cancel, since they have opposite directions. The two horizontal components, in contrast, reinforce since they have the same direction. Thus, we have the following components for the net force at corner B:

\[
(\Sigma F_x)_B = -F_{BC} \cos 60.0^\circ - F_{BA} \cos 60.0^\circ = -2F \cos 60.0^\circ
\]

\[
(\Sigma F_y)_B = 0
\]

where we have used the fact that \( F_{BC} = F_{BA} = F \). The Pythagorean theorem indicates that the magnitude of the net force at corner B is

\[
(\Sigma F)_B = \sqrt{(\Sigma F_x)_B^2 + (\Sigma F_y)_B^2} = \sqrt{(-2F \cos 60.0^\circ)^2 + (0)^2} = 2F \cos 60.0^\circ
\]

\[
= 2 \frac{|q|^2}{L^2} \cos 60.0^\circ = 2 \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \left(\frac{5.0 \times 10^{-6} \text{ C}}{(0.030 \text{ m})^2}\right) \cos 60.0^\circ
\]

\[
= 250 \text{ N}
\]

As discussed in the **REASONING**, the magnitude of the net force acting on the charge at corner C is the same as that acting on the charge at corner B, so \( (\Sigma F)_C = 250 \text{ N} \).

73. **REASONING** The magnitude \( E \) of the electric field is the magnitude \( F \) of the electric force exerted on a small test charge divided by the magnitude of the charge: \( E = \frac{F}{|q|} \). According to Newton’s second law, Equation 4.2, the net force acting on an object is equal to its mass \( m \) times its acceleration \( a \). Since there is only one force acting on the object, it is the net force. Thus, the magnitude of the electric field can be written as

\[
E = \frac{F}{|q|} = \frac{ma}{|q|}
\]
The acceleration is related to the initial and final velocities, \( v_0 \) and \( v \), and the time \( t \) through Equation 2.4, as \( a = \frac{v - v_0}{t} \). Substituting this expression for \( a \) into the one above for \( E \) gives

\[
E = ma = m\left(\frac{v - v_0}{t}\right) = \frac{m(v - v_0)}{|q|t}
\]

**SOLUTION** The magnitude \( E \) of the electric field is

\[
E = \frac{m(v - v_0)}{|q|t} = \frac{(9.0 \times 10^{-5} \text{ kg})(2.0 \times 10^3 \text{ m/s} - 0 \text{ m/s})}{(7.5 \times 10^{-6} \text{ C})(0.96 \text{ s})} = 2.5 \times 10^4 \text{ N/C}
\]

74. **REASONING** We will use Coulomb’s law to calculate the force that any one charge exerts on another charge. Note that in such calculations there are three separations to consider. Some of the charges are a distance \( d \) apart, some a distance \( 2d \), and some a distance \( 3d \). The greater the distance, the smaller the force. The net force acting on any one charge is the vector sum of three forces. In the following drawing we represent each of those forces by an arrow. These arrows are not drawn to scale and are meant only to “symbolize” the three different force magnitudes that result from the three different distances used in Coulomb’s law. In the drawing the directions are determined by the facts that like charges repel and unlike charges attract. By examining the drawing we will be able to identify the greatest and the smallest net force.

![Drawing of charges A, B, C, and D with arrows showing forces](image)

The greatest net force occurs for charge C, because all three force contributions point in the same direction and two of the three have the greatest magnitude, while the third has the next greatest magnitude. The smallest net force occurs for charge B, because two of the three force contributions cancel.

**SOLUTION** Using Coulomb’s law for each contribution to the net force, we calculate the ratio of the greatest to the smallest net force as follows:

\[
\frac{(\Sigma F)_C}{(\Sigma F)_B} = \frac{k \frac{|q|^2}{d^2} + k \frac{|q|^2}{d^2} + k \frac{|q|^2}{(2d)^2}}{k \frac{|q|^2}{d^2} - k \frac{|q|^2}{d^2} + k \frac{|q|^2}{(2d)^2}} = \frac{1 + 1 + \frac{1}{4}}{1} = 9.0
\]
75. **REASONING** The magnitude of the electric field between the plates of a parallel plate capacitor is given by Equation 18.4 as $E = \frac{\sigma}{\varepsilon_0}$, where $\sigma$ is the charge density for each plate and $\varepsilon_0$ is the permittivity of free space. It is the charge density that contains information about the radii of the circular plates, for charge density is the charge per unit area. The area of a circle is $\pi r^2$. The second capacitor has a greater electric field, so its plates must have the greater charge density. Since the charge on the plates is the same in each case, the plate area and, hence, the plate radius, must be smaller for the second capacitor. As a result, we expect that the ratio $r_2/r_1$ is less than one.

**SOLUTION** Using $|q|$ to denote the magnitude of the charge on the capacitor plates and $A = \pi r^2$ for the area of a circle, we can use Equation 18.4 to express the magnitude of the field between the plates of a parallel plate capacitor as follows:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{|q|}{\varepsilon_0 \pi r^2}$$

Applying this result to each capacitor gives

$$E_1 = \frac{|q|}{\varepsilon_0 \pi r_1^2} \quad \text{and} \quad E_2 = \frac{|q|}{\varepsilon_0 \pi r_2^2}$$

First capacitor Second capacitor

Dividing the expression for $E_1$ by the expression for $E_2$ gives

$$\frac{E_1}{E_2} = \frac{|q| \left(\frac{\varepsilon_0 \pi r_1^2}{r_2^2}\right)}{|q| \left(\frac{\varepsilon_0 \pi r_2^2}{r_1^2}\right)} = \frac{r_2^2}{r_1^2}$$

Solving for the ratio $r_2/r_1$ gives

$$\frac{r_2}{r_1} = \sqrt{\frac{E_1}{E_2}} = \sqrt{\frac{2.2 \times 10^5 \text{ N/C}}{3.8 \times 10^5 \text{ N/C}}} = 0.76$$

As expected, this ratio is less than one.

76. **REASONING AND SOLUTION**

a. To find the charge on each ball we first need to determine the electric force acting on each ball. This can be done by noting that each thread makes an angle of 18° with respect to the vertical.

$$F_e = mg \tan 18^\circ = (8.0 \times 10^{-4} \text{ kg})(9.80 \text{ m/s}^2) \tan 18^\circ = 2.547 \times 10^{-3} \text{ N}$$
We also know that

\[ F_e = \frac{k|q_1||q_2|}{r^2} = \frac{k|q|^2}{r^2} \]

where \( r = 2(0.25 \text{ m}) \sin 18^\circ = 0.1545 \text{ m} \). Now

\[ |q| = r \sqrt{\frac{F_e}{k}} = (0.1545 \text{ m}) \sqrt{\frac{2.547 \times 10^{-3} \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = 8.2 \times 10^{-8} \text{ C} \]

b. The tension is due to the combination of the weight of the ball and the electric force, the two being perpendicular to one another. The tension is therefore,

\[ T = \sqrt{(mg)^2 + F_e^2} = \sqrt{\left[(8.0 \times 10^{-4} \text{ kg})(9.80 \text{ m/s}^2)\right]^2 + (2.547 \times 10^{-3} \text{ N})^2} = 8.2 \times 10^{-3} \text{ N} \]
1. (d) The force on the positive charge is in the same direction as the electric field, and the displacement of the charge is opposite to the direction of the force. Therefore, the work is negative (see Section 6.1). The electric potential energy at point \( B \) differs from that at \( A \) by \( \Delta \text{EPE} = \Delta W_{AB} \) (Equation 19.1). Since \( \Delta W_{AB} \) is negative, \( \text{EPE}_B \) is greater than \( \text{EPE}_A \).

2. (d) The electric potential energy \( \text{EPE} \) is related to the electric potential \( V \) by \( \text{EPE} = q_0 V \) (Equation 19.3). So, even though the electric potentials at two locations are the same, the electric potential energies are different since the charges placed at these locations are different.

3. \( V_B - V_A = -5.0 \times 10^2 \text{ V} \)

4. (a) The change in the proton’s electric potential energy \( (\Delta \text{EPE}_B - \Delta \text{EPE}_A) \) in going from \( A \) to \( B \) is related to the change in the potential \( (V_B - V_A) \) by Equation 19.4 as \( \Delta \text{EPE}_B - \Delta \text{EPE}_A = (+e)(V_B - V_A) \), where \(+e\) is the charge on the proton. On the other hand, the change in the electron’s electric potential energy \( (\Delta \text{EPE}_A - \Delta \text{EPE}_B) \) in going from \( B \) to \( A \) is related to the change in the potential \( (V_A - V_B) \) by \( \Delta \text{EPE}_A - \Delta \text{EPE}_B = (-e)(V_A - V_B) \), where \(-e\) is the charge on the electron. Comparing the right-hand sides of these two equations shows that the change in the proton’s electric potential energy is the same as the change in the electron’s electric potential energy.

5. (c) According to Equation 19.6, the potential produced by the charge \( q \) is \( V = kq/r \). The smaller the value of \( r \), the greater is the potential. The potential, however, does not depend on the charge \((q_0 \text{ or } 2q_0)\) placed at points \( P_1 \) or \( P_2 \). See Section 19.3.

6. (e) The total electric potential at the origin is the algebraic sum of the potentials due to all the charges. Since each potential is of the form \( V = kq/r \) (Equation 19.6) and \( r \) is same for each charge, the total electric potential is proportional to the sum of the charges. The sum of the charges in \( A \) \((+4q)\) equals the sum in \( C \) \((+4q)\), which is greater than the sum in \( B \) \((+2q)\).

7. (b) The electric potential energy of the two charges in the top drawing is \( \text{EPE} = q_0 V \), where \( V = kq/r \) is the electric potential produced by the charge at the origin (see Equation 19.6). Thus, \( \text{EPE} = kqq_0/r \). In a similar fashion, the electric potential energy for the charges in the bottom drawing is \( kq(2q_0)/(2r) = kqq_0/r \), which is the same as that in the top drawing.

8. \( V_B - V_A = 6.7 \times 10^3 \text{ V} \)
9. (a) The electric potential energy EPE of two charges $q_1$ and $q_2$ is $EPE = kq_1q_2/r$, where $r$ is the distance between them (see Section 19.3). Since the distance $r$ is the same for all four pairs of charges, the electric potential energy is proportional to the product $q_1q_2$ of the charges. The products of the charges in A and C are the same ($+12q^2$), and the products of the charges in B and D are the same ($-12q^2$).

10. (c) The electric field $E$ is related to the electric potential difference $\Delta V$ by $E = -\Delta V/\Delta s$ (Equation 19.7), where $\Delta s$ is the displacement of one point in the region relative to another point. If the potential is the same everywhere, then $\Delta V = 0$ V, so $E$ is zero everywhere.

11. (c) The magnitude of the electric field between the plates is $|E| = -\Delta V/\Delta s$ (Equation 19.7).

Since $\Delta s$ is the same for all three capacitors, $|E|$ is proportional to the potential difference $\Delta V = V_{\text{right}} - V_{\text{left}}$ between the right and left plates. $\Delta V$ for capacitor C (300 V) is greater than that for capacitor B (250 V), which is greater than that for capacitor A (200 V).

12. (e) The magnitude of the electric field between the plates is $|E| = -\Delta V/\Delta s$ (Equation 19.7).

Since $\Delta s$ is the same for all four segments, $|E|$ is proportional to the potential difference $\Delta V$ in each segment. $\Delta V$ is 2 units for segment D, 1 unit for segment B, and 0 units for segments A and C.

13. $E = -7.5$ V/m

14. $E = -2.0 \times 10^3$ V/m

15. (a) The capacitance of a parallel plate capacitor is $C = \kappa \varepsilon_0 A/d$ (Equation 19.10). Since the area $A$ increases by a factor of 4 and the spacing $d$ between the plates increases by a factor of 2, the capacitance increases by a factor of $4/2 = 2$.

16. (b) The amount of charge on each plate is $q = CV$ (Equation 19.8), where $C = \kappa \varepsilon_0 A/d$ (Equation 19.10). If the plate separation $d$ increases, $C$ decreases. If $C$ decreases while $V$ is held constant, the amount of charge decreases.

17. (c) The energy stored in a capacitor is directly proportional to its capacitance (see Equation 19.11). The capacitance, on the other hand, is directly proportional to the dielectric constant and the area of each plate and is inversely proportional to the plate separation (Equation 19.10). Therefore, inserting a dielectric, increasing the area of each plate, and decreasing the plate separation increases the energy stored.

18. Energy = $6.0 \times 10^{-4}$ J
1. **REASONING** When the electron moves from the ground to the cloud, the change in its electric potential energy is $\Delta(EPE) = EPE_{\text{cloud}} - EPE_{\text{ground}}$. (Remember that the change in any quantity is its final value minus its initial value.) The change in the electric potential energy is related to the change $\Delta V$ in the potential by $\Delta(EPE) = q_0 \Delta V$ (Equation 19.4), where $q_0$ is the charge on the electron. This relation will allow us to find the change in the electron’s potential energy.

**SOLUTION** The difference in the electric potentials between the cloud and the ground is $\Delta V = V_{\text{cloud}} - V_{\text{ground}} = 1.3 \times 10^8 \text{ V}$, and the charge on an electron is $q_0 = -1.60 \times 10^{-19} \text{ C}$. Thus, the change in the electron’s electric potential energy when the electron moves from the ground to the cloud is

$$\Delta(EPE) = q_0 \Delta V = (-1.60 \times 10^{-19} \text{ C})(1.3 \times 10^8 \text{ V}) = -2.1 \times 10^{-11} \text{ J}$$

2. **REASONING** A positive charge accelerates from a region of higher potential toward a region of lower potential. In contrast, a negative charge accelerates from a region of lower potential toward a region of higher potential. The electric potential $V$ is the electric potential energy $EPE$ divided by the charge $q_0$ on the particle: $V = \frac{EPE}{q_0}$ (Equation 19.3). Since the only force that acts on the particle is the conservative electric force, the total mechanical energy of the particle is conserved during the motion. Therefore, the initial total energy equals the final total energy. This fact will allow us to determine the difference between the initial and final electric potential energies. Knowing the difference in the electric potential energies, we will determine the potential difference $V_B - V_A$ by using Equation 19.3.

**SOLUTION**

a. **Point B** is at the higher potential, because the particle has a negative charge and a negative charge accelerates from a lower potential to a higher potential.

b. Using Equation 19.3 to relate the electric potential at each point to the electric potential energy at each point, we have

$$V_B - V_A = \frac{EPE_B - EPE_A}{q_0}$$

(1)
To determine $\text{EPE}_B - \text{EPE}_A$ we use the fact that the total mechanical energy of the particle is conserved during the motion:

$$\frac{1}{2}mv_A^2 + \text{EPE}_A = \frac{1}{2}mv_B^2 + \text{EPE}_B$$

or

$$\text{EPE}_B - \text{EPE}_A = \frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2$$

Substituting this result into Equation (1) and noting that $v_A = 0 \text{ m/s}$ (the particle starts from rest) gives

$$V_B - V_A = \frac{\text{EPE}_B - \text{EPE}_A}{q_0} = \frac{\frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2}{q_0}$$

$$= \frac{-mv_B^2}{2q_0} = \frac{-\left(2.5 \times 10^{-6} \text{ kg}\right)(42 \text{ m/s})^2}{2\left(-1.5 \times 10^{-6} \text{ C}\right)} = 1500 \text{ V}$$

This answer is positive, as it must be, since we know from part a that point B is at the higher potential.

3. **SSM REASONING AND SOLUTION** Combining Equations 19.1 and 19.3, we have

$$W_{AB} = \text{EPE}_A - \text{EPE}_B = q_0(V_A - V_B) = (+1.6 \times 10^{-19} \text{ C})(0.070 \text{ V}) = 1.1 \times 10^{-20} \text{ J}$$

4. **REASONING** Equation 19.1 indicates that the work done by the electric force as the particle moves from point A to point B is $W_{AB} = \text{EPE}_A - \text{EPE}_B$. For motion through a distance $s$ along the line of action of a constant force of magnitude $F$, the work is given by Equation 6.1 as either $+Fs$ (if the force and the displacement have the same direction) or $-Fs$ (if the force and the displacement have opposite directions). Here, $\text{EPE}_A - \text{EPE}_B$ is given to be positive, so we can conclude that the work is $W_{AB} = +Fs$ and that the force points in the direction of the motion from point A to point B. The electric field is given by Equation 18.2 as $E = F/q_0$, where $q_0$ is the charge.

**SOLUTION** a. Using Equation 19.1 and the fact that $W_{AB} = +Fs$, we find

$$W_{AB} = +Fs = \text{EPE}_A - \text{EPE}_B$$

$$F = \frac{\text{EPE}_A - \text{EPE}_B}{s} = \frac{9.0 \times 10^{-4} \text{ J}}{0.20 \text{ m}} = 4.5 \times 10^{-3} \text{ N}$$

As discussed in the reasoning, the direction of the force is **from A toward B**.

b. From Equation 18.2, we find that the electric field has a magnitude of
\[ E = \frac{F}{q_0} = \frac{4.5 \times 10^{-3} \text{ N}}{1.5 \times 10^{-6} \text{ C}} = 3.0 \times 10^3 \text{ N/C} \]

The direction is the same as that of the force on the positive charge, namely from \( A \) toward \( B \).

5. **REASONING** The energy to accelerate the car comes from the energy stored in the battery pack. Work is done by the electric force as the charge moves from point \( A \) (the positive terminal), through the electric motor, to point \( B \) (the negative terminal). The work \( W_{AB} \) done by the electric force is given by Equation 19.4 as the product of the charge and the potential difference \( V_A - V_B \), or \( W_{AB} = q_0(V_A - V_B) \). The power supplied by the battery pack is the work divided by the time, as expressed by Equation 6.10a.

**SOLUTION** According to Equation 6.10a, the power \( P \) supplied by the battery pack is

\[ P = \frac{W_{AB}}{t} = \frac{q_0(V_A - V_B)}{t} = \frac{(1200 \text{ C})(290 \text{ V})}{7.0 \text{ s}} = 5.0 \times 10^4 \text{ W} \]

Since 745.7 W = 1 hp (see the page facing the inside of the front cover of the text), the power rating, in horsepower, is

\[ (5.0 \times 10^4 \text{ W}) \left( \frac{1 \text{ hp}}{745.7 \text{ W}} \right) = 67 \text{ hp} \]

6. **REASONING** Let the electrons accelerate to point \( B \) from a condition of rest at point \( A \). The speed \( v_B \) of an electron at \( B \) is found from its kinetic energy \( \text{KE}_B = \frac{1}{2} m v_B^2 \) (Equation 6.2) at \( B \). Because only the conservative electric and gravitational forces act on the electron, its total mechanical energy (the sum of its kinetic energy and potential energies) is conserved. Furthermore, if we assume that the electron only moves horizontally, then its gravitational potential energy \( mgh \) is constant during its acceleration, and we conclude that the sum of its kinetic energy \( \text{KE}_B \) and electric potential energy \( \text{EPE}_B \) at \( B \) is equal to the sum of its kinetic energy \( \text{KE}_A \) and electric potential energy \( \text{EPE}_A \) at \( A \):

\[ \frac{1}{2} m v_B^2 + \text{EPE}_B = \frac{1}{2} m v_A^2 + \text{EPE}_A \]  \hspace{1cm} (1)

The difference between the electric potential energy of the electron at \( B \) and at \( A \), in turn, depends upon the charge \( q = -e \) of the electron and the potential difference \( V_B - V_A \) that it accelerates through:

\[ \text{EPE}_B - \text{EPE}_A = q(V_B - V_A) = -e(V_B - V_A) \]  \hspace{1cm} (19.4)

We note that, as the electron has a negative charge, it freely accelerates from a *lower* potential \( V_A \) toward a *higher* potential \( V_B \). Therefore the potential difference between \( B \) and \( A \) must be \( V_B - V_A = 25 000 \text{ V} \).
**SOLUTION**  The electron is initially at rest, so we have that $v_A = 0 \text{ m/s}$. Substituting this value into Equation (1) and solving for $v_B^2$, we obtain

\[
\frac{1}{2} mv_B^2 = E_{PE_A} - E_{PE_B} \quad \text{or} \quad v_B^2 = \frac{2(E_{PE_A} - E_{PE_B})}{m}
\]

(2)

Taking the square root of both sides of Equation (2), we find that

\[
v_B = \sqrt{\frac{2(E_{PE_A} - E_{PE_B})}{m}} = \sqrt{-2\frac{(E_{PE_B} - E_{PE_A})}{m}}
\]

(3)

Note that, in order to match the expression on the left side of Equation 19.4, we have exchanged the electric potential energy terms in the last step of Equation (3): $-(E_{PE_B} - E_{PE_A}) = (E_{PE_A} - E_{PE_B})$. Substituting Equation 19.4 into Equation (3), yields

\[
v_f = \sqrt{-2\frac{(E_{PE_B} - E_{PE_A})}{m}} = \sqrt{-2\frac{(-e)(V_B - V_A)}{m}} = \sqrt{\frac{2e(V_B - V_A)}{m}}
\]

Therefore, the speed of an electron just before it strikes the television screen is

\[
v_B = \sqrt{\frac{2e(V_B - V_A)}{m}} = \sqrt{\frac{2(1.6\times10^{-19} \text{ C})(+25000 \text{ V})}{9.11\times10^{-31} \text{ kg}}} = 9.4\times10^7 \text{ m/s}
\]

7. **SSM REASONING**  The translational speed of the particle is related to the particle’s translational kinetic energy, which forms one part of the total mechanical energy that the particle has. The total mechanical energy is conserved, because only the gravitational force and an electrostatic force, both of which are conservative forces, act on the particle (see Section 6.5). Thus, we will determine the speed at point $A$ by utilizing the principle of conservation of mechanical energy.

**SOLUTION**  The particle’s total mechanical energy $E$ is

\[
E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + mgh + \frac{1}{2} kx^2 + E_{PE}
\]

Translational Rotational Gravitational Elastic Electric
kinetic kinetic potential potential potential energy energy

Since the particle does not rotate the angular speed $\omega$ is always zero, and since there is no elastic force we may omit the terms $\frac{1}{2} I\omega^2$ and $\frac{1}{2} kx^2$ from this expression. With this in mind, we express the fact that $E_B = E_A$ (energy is conserved) as follows:
\[ \frac{1}{2}mv_B^2 + mgh_B + EPE_B = \frac{1}{2}mv_A^2 + mgh_A + EPE_A \]

This equation can be simplified further, since the particle travels horizontally, so that \( h_B = h_A \), with the result that

\[ \frac{1}{2}mv_B^2 + EPE_B = \frac{1}{2}mv_A^2 + EPE_A \]

Solving for \( v_A \) gives

\[ v_A = \sqrt{v_B^2 + \frac{2(EPE_B - EPE_A)}{m}} \]

According to Equation 19.4, the difference in electric potential energies \( EPE_B - EPE_A \) is related to the electric potential difference \( V_B - V_A \):

\[ EPE_B - EPE_A = q_0(V_B - V_A) \]

Substituting this expression into the expression for \( v_A \) gives

\[ v_A = \sqrt{v_B^2 + \frac{2q_0(V_B - V_A)}{m}} = \sqrt{(0 \text{ m/s})^2 + \frac{2(-2.0 \times 10^{-5} \text{ C})(-36 \text{ V})}{4.0 \times 10^{-6} \text{ kg}}} = 19 \text{ m/s} \]

8. **REASONING** The electric potential difference \( \Delta V \) experienced by the electron has the same magnitude as the electric potential difference experienced by the proton. Moreover, the charge \( q_0 \) on either particle has the same magnitude. According to \( \Delta EPE = q_0 \Delta V \) (Equation 19.4), the losses in \( EPE \) for the electron and the proton are the same. Conservation of energy, then, dictates that the electron gains the same amount of kinetic energy as does the proton.

Both particles start from rest, so the gain in kinetic energy is equal to the final kinetic energies in each case, which are the same for each particle. However, kinetic energy is \( \frac{1}{2}mv^2 \) (Equation 6.2), and the mass of the electron is much less than the mass of the proton (see the inside of the front cover). Since the kinetic energies are the same, the speed \( v_e \) of the electron will be greater than the speed \( v_p \) of the proton.

**SOLUTION** Let us assume that the proton accelerates from point A to point B. According to the energy-conservation principle (including only kinetic and electric potential energies), we have

\[ KE_{p, B} + EPE_{p, B} = KE_{p, A} + EPE_{p, A} \quad \text{or} \quad KE_{p, B} = EPE_{p, A} - EPE_{p, B} \]

Here we have used the fact that the initial kinetic energy is zero since the proton starts from rest. According to Equation 19.3, the electric potential energy of a charge \( q_0 \) is \( EPE = q_0V \),
where $V$ is the potential experienced by the charge. We can, therefore, write the final kinetic energy of the proton as follows:

$$\text{KE}_{p, B} = \text{EPE}_{p, A} - \text{EPE}_{p, B} = q_p (V_A - V_B) \quad (1)$$

The electron has a negative charge, so it accelerates in the direction opposite to that of the proton, or from point B to point A. Energy conservation applied to the electron gives

$$\text{KE}_{e, A} + \text{EPE}_{e, A} = \text{KE}_{e, B} + \text{EPE}_{e, B} \quad \text{or} \quad \text{KE}_{e, A} = \text{EPE}_{e, B} - \text{EPE}_{e, A}$$

Here we have used the fact that the initial kinetic energy is zero since the electron starts from rest. Again using Equation 19.3, we can write the final kinetic energy of the electron as follows:

$$\text{KE}_{e, A} = \text{EPE}_{e, B} - \text{EPE}_{e, A} = q_e (V_B - V_A)$$

But an electron has a negative charge that is equal in magnitude to the charge on a proton, so $q_e = -q_p$. With this substitution, we can write the kinetic energy of the electron as

$$\text{KE}_{e, A} = \text{EPE}_{e, B} - \text{EPE}_{e, A} = -q_p (V_B - V_A) = q_p (V_A - V_B) \quad (2)$$

Comparing Equations (1) and (2), we can see that

$$\text{KE}_{p, B} = \text{KE}_{e, A} \quad \text{or} \quad \frac{1}{2} m_p v_p^2 = \frac{1}{2} m_e v_e^2$$

Solving for the ratio $v_e/v_p$ and referring to the inside of the front cover for the masses of the electron and the proton, we obtain

$$\frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}}} = 42.8$$

9. **SSM REASONING** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the charge remains constant. Applying the principle of conservation of energy between locations A and B, we obtain

$$\frac{1}{2} m v_A^2 + \text{EPE}_A = \frac{1}{2} m v_B^2 + \text{EPE}_B$$

Since the charged particle starts from rest, $v_A = 0$. The difference in potential energies is related to the difference in potentials by Equation 19.4, $\text{EPE}_B - \text{EPE}_A = q(V_B - V_A)$. Thus, we have

$$q(V_A - V_B) = \frac{1}{2} m v_B^2 \quad (1)$$

Similarly, applying the conservation of energy between locations C and B gives

$$q(V_C - V_B) = \frac{1}{2} m (2v_B)^2 \quad (2)$$
Dividing Equation (1) by Equation (2) yields
\[ \frac{V_A - V_B}{V_C - V_B} = \frac{1}{4} \]
This expression can be solved for \( V_B \).

**SOLUTION** Solving for \( V_B \), we find that
\[ V_B = \frac{4V_A - V_C}{3} = \frac{4(452 \text{ V}) - 791 \text{ V}}{3} = 339 \text{ V} \]

10. **REASONING** The particle slows down as it moves from the lower potential at \( A \) to the higher potential at \( B \), indicating that the charge \( q \) of the particle is positive. The work \( W_{AB} \) done on the particle by the electric force is proportional to \( q \), according to \( W_{AB} = -q(V_B - V_A) \) (Equation 19.4), where \( V_B - V_A \) is the electric potential difference between positions \( B \) and \( A \). According to the work-energy theorem, the work done on the particle by the electric force is also equal to the difference between the final and initial kinetic energies of the particle: \( W_{AB} = KE_B - KE_A \) (Equation 6.3). The kinetic energies are given in electron-volts (eV), so we will use the equivalence 1 eV = \( 1.60 \times 10^{-19} \text{ J} \) to convert from electron-volts to joules.

**SOLUTION** Solving \( W_{AB} = -q(V_B - V_A) \) (Equation 19.4) for \( q \), we obtain
\[ q = -\frac{W_{AB}}{V_B - V_A} \] (1)
Substituting \( W_{AB} = KE_B - KE_A \) (Equation 6.3) into Equation (1) yields
\[ q = -\frac{KE_B - KE_A}{V_B - V_A} \] (2)
Since 1 eV = \( 1.60 \times 10^{-19} \text{ J} \), Equation (2) gives the particle’s charge as
\[ q = -\frac{KE_B - KE_A}{V_B - V_A} = -\frac{(7060 \text{ eV} - 9520 \text{ eV})}{[27.0 \text{ V} - (-55.0 \text{ V})]} \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = +4.80 \times 10^{-18} \text{ C} \]

11. **REASONING**
a. The work \( W_{AB} \) done by the electric force in moving a charge \( q \) from \( A \) to \( B \) is related to the potential difference \( V_A - V_B \) between the two points by Equation 19.4 as \( W_{AB} = q(V_A - V_B) \). Letting \( A = \text{ground} \) and \( B = \text{cloud} \), the work done can be written as
\[ W_{\text{ground-cloud}} = q(V_{\text{ground}} - V_{\text{cloud}}) = -q(V_{\text{cloud}} - V_{\text{ground}}) \] (1)
b. According to the work-energy theorem (Equation 6.3), the work \( W \) done on an object of mass \( m \) is equal to its final kinetic energy \( \frac{1}{2}mv_f^2 \) minus its initial kinetic energy \( \frac{1}{2}mv_0^2 \).

Setting \( W = W_{\text{ground-cloud}} \) and noting that \( v_0 = 0 \) m/s, since the automobile starts from rest, the work-energy theorem takes the form \( W_{\text{ground-cloud}} = \frac{1}{2}mv_f^2 \). Solving this equation for \( v_f \) and substituting Equation (1) for \( W_{\text{ground-cloud}} \), the final speed of the car is

\[
v_f = \sqrt{\frac{2W_{\text{ground-cloud}}}{m}} = \sqrt{\frac{-2q(V_{\text{cloud}} - V_{\text{ground}})}{m}} \tag{2}
\]

c. If the work \( W_{\text{ground-cloud}} \) were converted completely into heat \( Q \), this heat could be used to raise the temperature of water. The relation between \( Q \) and the change \( \Delta T \) in the temperature of the water is \( Q = cm\Delta T \) (Equation 12.4), where \( c \) is the specific heat capacity of water and \( m \) is its mass. Solving this expression for \( m \) and substituting Equation (1) for \( W_{\text{ground-cloud}} \), we have

\[
m = \frac{Q}{c\Delta T} = \frac{W_{\text{ground-cloud}}}{c\Delta T} = \frac{-q(V_{\text{cloud}} - V_{\text{ground}})}{c\Delta T} \tag{3}
\]

**SOLUTION**

a. The work done on the charge as it moves from the ground to the cloud is \( W_{\text{ground-cloud}} = -q(V_{\text{cloud}} - V_{\text{ground}}) \). Setting \( q = -25 \) C (the charge is negative) and \( V_{\text{cloud}} - V_{\text{ground}} = 1.2 \times 10^9 \) V, we find that the work is

\[
W_{\text{ground-cloud}} = -q(V_{\text{cloud}} - V_{\text{ground}}) = -(-25 \text{ C})(1.2 \times 10^9 \text{ V}) = 3.0 \times 10^{10} \text{ J}
\]

b. From Equation (2) we have for the speed of the automobile that

\[
v_f = \sqrt{\frac{-2q(V_{\text{cloud}} - V_{\text{ground}})}{m}} = \sqrt{\frac{-2(-25 \text{ C})(1.2 \times 10^9 \text{ V})}{1100 \text{ kg}}} = 7.4 \times 10^3 \text{ m/s}
\]

c. The mass of water that can be heated is given by \( m = \frac{-q(V_{\text{cloud}} - V_{\text{ground}})}{c\Delta T} \) [see Equation (3)]. Setting \( \Delta T = 100 \text{ C}^\circ \) and using \( c = 4186 \text{ J/(kg·C}^\circ) \) from Table 12.2, we find that the mass of water is

\[
m = \frac{-q(V_{\text{cloud}} - V_{\text{ground}})}{c\Delta T} = \frac{-(25 \text{ C})(1.2 \times 10^9 \text{ V})}{[4186 \text{ J/(kg·C}^\circ)](100 \text{ C}^\circ)} = 7.2 \times 10^4 \text{ kg}
\]
12. **REASONING** The gravitational and electric forces are conservative forces, so the total energy of the particle remains constant as it moves from point \(A\) to point \(B\). Recall from Equation 6.5 that the gravitational potential energy (GPE) of a particle of mass \(m\) is \(\text{GPE} = mgh\), where \(h\) is the height of the particle above the earth’s surface. The conservation of energy is written as

\[
\frac{1}{2}m v_A^2 + mgh_A + \text{EPE}_A = \frac{1}{2}m v_B^2 + mgh_B + \text{EPE}_B
\]  

(1)

We will use this equation several times to determine the initial speed \(v_A\) of the negatively charged particle.

**SOLUTION**

When the negatively charged particle is thrown upward, it attains a maximum height of \(h\). For this particle we have:

\[
\begin{align*}
v_A & = ? \\
EPE_A & = (-q) V_A \\
h_A & = 0 \text{ (ground level)}
\end{align*}
\]

\[
\begin{align*}
v_B & = 0 \text{ (at maximum height)} \\
EPE_B & = (-q)V_B \\
h_B & = h \text{ (the maximum height)}
\end{align*}
\]

Solving the conservation of energy equation, Equation (1), for \(v_A\) and substituting in the data above gives

\[
v_A = \sqrt{\frac{2}{m} \left[ mgh + (-q)(V_B - V_A) \right]}
\]

(2)

Equation (2) cannot be solved as it stands because the height \(h\) and the potential difference \((V_B - V_A)\) are not known. We now make use of the fact that a positively charged particle, when thrown straight upward with an initial speed of 30.0 m/s, also reaches the maximum height \(h\). For this particle we have:

\[
\begin{align*}
v_A & = 30.0 \text{ m/s} \\
EPE_A & = (+q) V_A \\
h_A & = 0 \text{ (ground level)}
\end{align*}
\]

\[
\begin{align*}
v_B & = 0 \text{ (at maximum height)} \\
EPE_B & = (+q)V_B \\
h_B & = h
\end{align*}
\]

Solving the conservation of energy equation, Equation (1), for the potential difference \((V_B - V_A)\) and substituting in the data above gives

\[
V_B - V_A = \frac{1}{+q} \left[ \frac{1}{2} m (30.0 \text{ m/s})^2 - mgh \right]
\]

(3)

Substituting Equation (3) into Equation (2) gives, after some algebraic simplifications,

\[
v_A = \sqrt{4gh - (30.0 \text{ m/s})^2}
\]

(4)

Equation (4) cannot be solved because the height \(h\) is still unknown. We now make use of the fact that the uncharged particle, when thrown straight upward with an initial speed of 25.0 m/s, also reaches the maximum height \(h\). For this particle we have:
\[ v_A = 25.0 \text{ m/s} \quad v_B = 0 \text{ (at maximum height)} \]
\[ \text{EPE}_A = qV_A = 0 \text{ (since } q = 0) \quad \text{EPE}_B = qV_B = 0 \text{ (since } q = 0) \]
\[ h_A = 0 \text{ (ground level)} \quad h_A = h \]

Solving Equation (1) with this data for the maximum height \( h \) yields
\[
\frac{(25.0 \text{ m/s})^2}{2g} = \frac{(25.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 31.9 \text{ m}
\]

Substituting \( h = 31.9 \text{ m} \) into Equation (4) gives \( v_A = 18.7 \text{ m/s} \).

13. **REASONING** The electric potential at a distance \( r \) from a point charge \( q \) is given by \( V = kq/r \) (Equation 19.6). The total electric potential due to the two charges is the algebraic sum of the individual potentials.

**SOLUTION** Using Equation 19.6, we find that the total electric potential due to the two charges is
\[
V = \frac{kq_1}{r} + \frac{kq_2}{r} = \frac{k}{r}(q_1 + q_2)
\]
The distance \( r \) is one-half the distance between the charges, so \( r = \frac{1}{2}(1.20 \text{ m}) \) for both charges. Thus, the total electric potential midway between the charges is
\[
V = \frac{k}{2}(q_1 + q_2) = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{\frac{1}{2}(1.20 \text{ m})}(+3.40 \times 10^{-6} \text{ C} - 6.10 \times 10^{-6} \text{ C}) = -4.05 \times 10^4 \text{ V}
\]

14. **REASONING** The potential \( V \) at a distance \( r \) from a proton is \( V = k(+e)/r \) (see Equation 19.6), where \(+e\) is the charge of the proton. When an electron \((q = -e)\) is placed at a distance \( r \) from the proton, the electric potential energy is \( \text{EPE} = -eV \), as per Equation 19.3.

**SOLUTION** The difference in the electric potential energies when the electron and proton are separated by \( r_{\text{final}} = 5.29 \times 10^{-11} \text{ m} \) and when they are very far apart \((r_{\text{initial}} = \infty)\) is
\[
\text{EPE}_{\text{final}} - \text{EPE}_{\text{initial}} = \frac{(-e)ke}{r_{\text{final}}} - \frac{(-e)ke}{r_{\text{initial}}}
\]
\[
= - (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2 \times \left(\frac{1}{5.29 \times 10^{-11} \text{ m}} - \frac{1}{\infty}\right) = -4.35 \times 10^{-18} \text{ J}
\]
15. **REASONING** The potential of each charge $q$ at a distance $r$ away is given by Equation 19.6 as $V = kq/r$. By applying this expression to each charge, we will be able to find the desired ratio, because the distances are given for each charge.

**SOLUTION** According to Equation 19.6, the potentials of each charge are

$$V_A = \frac{kq_A}{r_A} \quad \text{and} \quad V_B = \frac{kq_B}{r_B}$$

Since we know that $V_A = V_B$, it follows that

$$\frac{kq_A}{r_A} = \frac{kq_B}{r_B} \quad \text{or} \quad \frac{q_B}{q_A} = \frac{r_B}{r_A} = \frac{0.43 \text{ m}}{0.18 \text{ m}} = 2.4$$

16. **REASONING** The electric potential energy $EPE$ of the third charge $q_3$ is given by $EPE = q_3 V_{\text{Total}}$ (see Equation 19.3), where $V_{\text{Total}}$ is the total potential at the point where the third charge is placed. The total potential at either of the empty corners is the sum of the individual potentials created by each of the charges. The individual potential created by a point charge $q$ is $V = kq/r$ (Equation 19.6), where $r$ is the distance between the charge and an empty corner.

**SOLUTION** We begin by noting that each side of the square has a length $L$, so that the diagonal has a length of $\sqrt{2}L$, according to the Pythagorean theorem. Using Equation 19.6, we can express the total potential at corners $A$ and $B$ as follows:

$$V_{\text{Total, A}} = \frac{kq_2}{L} + \frac{kq_1}{\sqrt{2}L} \quad \text{and} \quad V_{\text{Total, B}} = \frac{kq_1}{L} + \frac{kq_2}{\sqrt{2}L}$$

Using Equation 19.3, we find that the electric potential energy of the third charge at each corner is:

$$EPE_A = q_3 V_{\text{Total, A}} = q_3 \left( \frac{kq_2}{L} + \frac{kq_1}{\sqrt{2}L} \right) = \frac{q_3k}{L} \left( \frac{q_2}{L} + \frac{q_1}{\sqrt{2}} \right)$$

$$= \frac{(-6.0 \times 10^{-9} \text{ C})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.25 \text{ m})}{\left( +4.0 \times 10^{-9} \text{ C} + \frac{1.5 \times 10^{-9} \text{ C}}{\sqrt{2}} \right)} = -1.1 \times 10^{-6} \text{ J}$$

$$EPE_B = q_3 V_{\text{Total, B}} = q_3 \left( \frac{kq_1}{L} + \frac{kq_2}{\sqrt{2}L} \right) = \frac{q_3k}{L} \left( \frac{q_1}{L} + \frac{q_2}{\sqrt{2}} \right)$$

$$= \frac{(-6.0 \times 10^{-9} \text{ C})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.25 \text{ m})}{\left( +4.0 \times 10^{-9} \text{ C} + \frac{1.5 \times 10^{-9} \text{ C}}{\sqrt{2}} \right)} = -0.93 \times 10^{-6} \text{ J}$$
17. **REASONING** The electric potential at a distance $r$ from a point charge $q$ is given by Equation 19.6 as $V = \frac{kq}{r}$. The total electric potential at location $P$ due to the four point charges is the algebraic sum of the individual potentials.

**SOLUTION** The total electric potential at $P$ is (see the drawing)

$$V = \frac{k(-q)}{d} + \frac{k(+q)}{2d} + \frac{k(+q)}{d} + \frac{k(-q)}{d} = \frac{-kq}{2d}$$

Substituting in the numbers gives

$$V = \frac{-kq}{2d} = \frac{-\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(2.0 \times 10^{-6} \text{ C}\right)}{2(0.96 \text{ m})} = -9.4 \times 10^3 \text{ V}$$

18. **REASONING** The work $W_{AB}$ done by the electric force when a charge $q_0$ moves from location $A$ to location $B$ is $W_{AB} = -q_0(V_B - V_A)$ (Equation 19.4), where $V_A$ and $V_B$ are the electric potentials at the two locations. A point charge $q$ creates an electric potential $V$ at a spot that is a distance $r$ away from the charge, according to $V = \frac{kq}{r}$ (Equation 19.6), where $k$ is the constant in Coulomb’s law.

**SOLUTION** Let $q = +125 \mu\text{C}$ be the charge at the center of the square and $q_0 = +7.0 \mu\text{C}$ be the charge at corner $A$ of the square. Let $B$ represent any one of the three empty corners of the square. Using Equations 19.4 and 19.6, we can write the work done by the electric force as charge $q_0$ is moved from location $A$ to location $B$ in the following way:

$$W_{AB} = -q_0(V_B - V_A) = -q_0\left(\frac{kq}{r_B} - \frac{kq}{r_A}\right)$$

(1)

Note that the distances of each corner of the square from the center of the square are the same, so that $r_A = r_B$ in Equation (1). Therefore, we conclude that $W_{AB} = 0 \text{ J}$.

19. **REASONING** The electric potential at a distance $r$ from a point charge $q$ is given by Equation 19.6 as $V = \frac{kq}{r}$. The total electric potential at location $P$ due to the six point charges is the algebraic sum of the individual potentials.
**SOLUTION** Starting at the upper left corner of the rectangle, we proceed clockwise and add up the six contributions to the total electric potential at $P$ (see the drawing):

$$V = \frac{k(+7.0q)}{d^2 + \left(\frac{d}{2}\right)^2} + \frac{k(+3.0q)}{d^2 + \left(\frac{d}{2}\right)^2} + \frac{k(+5.0q)}{d^2 + \left(\frac{d}{2}\right)^2} + \frac{k(+7.0q)}{d^2 + \left(\frac{d}{2}\right)^2} + \frac{k(-3.0q)}{d^2 + \left(\frac{d}{2}\right)^2} + \frac{k(-5.0q)}{d^2 + \left(\frac{d}{2}\right)^2}$$

$$= \frac{k(+14.0q)}{d^2 + \left(\frac{d}{2}\right)^2}$$

Substituting $q = 9.0 \times 10^{-6}$ C and $d = 0.13$ m gives

$$V = \frac{k(+14.0q)}{d^2 + \left(\frac{d}{2}\right)^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{(0.13 \text{ m})^2 + \left(\frac{0.13 \text{ m}}{2}\right)^2} = +7.8 \times 10^6 \text{ V}$$

20. **REASONING** The potential at a distance $r$ from a point charge $q$ is given by Equation 19.6 as $V = kq/r$. Therefore, the potential difference between the locations $B$ and $A$ can be written as

$$V_B - V_A = \frac{kq}{r_B} - \frac{kq}{r_A}$$

We can use this relation to find the charge $q$.

**SOLUTION** Solving the equation above for $q$ yields

$$q = \frac{V_B - V_A}{k \left(\frac{1}{r_B} - \frac{1}{r_A}\right)} = \frac{45.0 \text{ V}}{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{1}{4.00 \text{ m}} - \frac{1}{3.00 \text{ m}}\right)} = -6.0 \times 10^{-8} \text{ C}$$

21. **REASONING** The potential $V$ created by a point charge $q$ at a spot that is located at a distance $r$ is given by Equation 19.6 as $V = kq/r$, where $q$ can be either a positive or negative quantity, depending on the nature of the charge. We will apply this expression to obtain the potential created at the empty corner by each of the three charges fixed to the square. The total potential at the empty corner is the sum of these three contributions. Setting this sum equal to zero will allow us to obtain the unknown charge.
**SOLUTION**  The drawing at the right shows the three charges fixed to the corners of the square. The length of each side of the square is denoted by $L$. Note that the distance between the unknown charge $q$ and the empty corner is $L$. Note also that the distance between one of the 1.8-$\mu$C charges and the empty corner is $r = L$, but that the distance between the other 1.8-$\mu$C charge and the empty corner is $r = \sqrt{L^2 + L^2} = \sqrt{2}L$, according to the Pythagorean theorem. Using Equation 19.6 to express the potential created by the unknown charge $q$ and by each of the 1.8-$\mu$C charges, we find that the total potential at the empty corner is

$$V_{\text{total}} = \frac{kq}{L} + \frac{k\left(1.8 \times 10^{-6} \text{ C}\right)}{L} + \frac{k\left(1.8 \times 10^{-6} \text{ C}\right)}{\sqrt{2}L} = 0$$

In this result the constant $k$ and the length $L$ can be eliminated algebraically, leading to the following result for $q$:

$$q + 1.8 \times 10^{-6} \text{ C} + \frac{1.8 \times 10^{-6} \text{ C}}{\sqrt{2}} = 0 \quad \text{or} \quad q = \left(-1.8 \times 10^{-6} \text{ C}\right)\left(1 + \frac{1}{\sqrt{2}}\right) = -3.1 \times 10^{-6} \text{ C}$$

---

22. **REASONING**  The potential from the positive charge is positive, while the potential from the negative charge is negative at the spot in question. The magnitude of the positive potential must be equal to the magnitude of the negative potential. Only then will the algebraic sum of the two potentials give zero for the total potential.

The drawing shows the arrangement of the charges. The two charges and the spot in question form a right triangle. Therefore, the distance between the positive charge and the zero-potential spot is the hypotenuse of the right triangle and, according to the Pythagorean theorem, is greater than $L$.

There are two spots on the dashed line where the total potential is zero. One spot is shown in the drawing. The other spot is on the dashed line at a distance $L$ below the negative charge.

**SOLUTION**  We begin by noting that the distance between the positive charge $+2q$ and the zero-potential spot is given by the Pythagorean theorem as $\sqrt{L^2 + (2.00 \text{ m})^2}$. With this in mind and using $V = kq/r$ (Equation 19.6), we write the total potential at the spot in question as follows:

$$V_{\text{Total}} = \frac{k(-q)}{L} + \frac{k(2q)}{\sqrt{L^2 + (2.00 \text{ m})^2}} = 0 \quad \text{or} \quad \frac{1}{L} = \frac{+2}{\sqrt{L^2 + (2.00 \text{ m})^2}}$$
Squaring both sides of this result gives
\[
\frac{1}{L^2} = \frac{4}{L^2 + (2.00 \text{ m})^2}
\]
or
\[
L^2 + (2.00 \text{ m})^2 = 4L^2
\]
Solving for \( L \), we obtain
\[
3L^2 = (2.00 \text{ m})^2 \quad \text{or} \quad L = \frac{2.00 \text{ m}}{\sqrt{3}} = 1.15 \text{ m}
\]

23. **SSM REASONING** Initially, the three charges are infinitely far apart. We will proceed as in Example 8 by adding charges to the triangle, one at a time, and determining the electric potential energy at each step. According to Equation 19.3, the electric potential energy \( \text{EPE} \) is the product of the charge \( q \) and the electric potential \( V \) at the spot where the charge is placed, \( \text{EPE} = qV \). The total electric potential energy of the group is the sum of the energies of each step in assembling the group.

**SOLUTION** Let the corners of the triangle be numbered clockwise as 1, 2 and 3, starting with the top corner. When the first charge \( q_1 = 8.00 \mu \text{C} \) is placed at a corner 1, the charge has no electric potential energy, \( \text{EPE}_1 = 0 \). This is because the electric potential \( V_1 \) produced by the other two charges at corner 1 is zero, since they are infinitely far away.

Once the 8.00-\( \mu \text{C} \) charge is in place, the electric potential \( V_2 \) that it creates at corner 2 is
\[
V_2 = \frac{kq_1}{r_{21}}
\]
where \( r_{21} = 5.00 \text{ m} \) is the distance between corners 1 and 2, and \( q_1 = 8.00 \mu \text{C} \). When the 20.0-\( \mu \text{C} \) charge is placed at corner 2, its electric potential energy \( \text{EPE}_2 \) is
\[
\text{EPE}_2 = q_2V_2 = q_2 \left( \frac{kq_1}{r_{21}} \right)
\]
\[
= \left( 20.0 \times 10^{-6} \text{ C} \right) \left[ \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \cdot (8.00 \times 10^{-6} \text{ C})}{5.00 \text{ m}} \right] = 0.288 \text{ J}
\]
The electric potential \( V_3 \) at the remaining empty corner is the sum of the potentials due to the two charges that are already in place on corners 1 and 2:
\[
V_3 = \frac{kq_1}{r_{31}} + \frac{kq_2}{r_{32}}
\]
where \( q_1 = 8.00 \mu \text{C}, r_{31} = 3.00 \text{ m}, q_2 = 20.0 \mu \text{C}, \) and \( r_{32} = 4.00 \text{ m} \). When the third charge \( (q_3 = -15.0 \mu \text{C}) \) is placed at corner 3, its electric potential energy \( \text{EPE}_3 \) is
The electric potential energy of the entire array is given by
\[
EPE = EPE_1 + EPE_2 + EPE_3 = 0 + 0.288 \text{ J} + (-1.034 \text{ J}) = -0.746 \text{ J}
\]

24. **REASONING** The speed \( v_B \) of the test charge at position \( B \), the center of the square, is related to its kinetic energy by \( KE_B = \frac{1}{2} m v_B^2 \) (Equation 6.2). No forces other than the conservative electric force act on the test charge, so the total mechanical energy of the test charge is conserved as it accelerates from its initial position \( A \) at one corner of the square to its final position \( B \). There is no rotational motion and no elastic potential energy, and the test charge is at rest at position \( A \), so the conservation of energy takes the following form:
\[
\frac{1}{2} m v_B^2 + EPE_B = \frac{1}{2} m (0 \text{ m/s})^2 + EPE_A \quad \text{or} \quad \frac{1}{2} m v_B^2 + EPE_B = EPE_A \quad (1)
\]

In Equation (1), \( m \) is the mass of the test charge. At both position \( A \) and position \( B \), the electric potential energy \( EPE \) of the test charge is determined by \( EPE = qV \) (Equation 19.3), where \( V \) is the total electric potential due to both of the charges \( q \). We will use \( V = \frac{kq}{r} \) (Equation 19.6), where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \) to obtain the total electric potential at positions \( A \) and \( B \).

**SOLUTION** At positions \( A \) and \( B \), the total electric potential is the sum of the electric potentials due to the charges \( q \). Since the charges are identical, and both are equidistant from positions \( A \) and \( B \), the total potentials \( V_A \) and \( V_B \) are twice the potentials due to either charge alone. Therefore, from \( V = \frac{kq}{r} \) (Equation 19.6), we have that
\[
V_A = 2 \left( \frac{kq}{r_A} \right) = \frac{2kq}{r_A} \quad \text{and} \quad V_B = 2 \left( \frac{kq}{r_B} \right) = \frac{2kq}{r_B} \quad (2)
\]
In Equation (2), \( r_A = 0.480 \) m is the length of one side of the square, and \( r_B \) is half the length of the diagonal \( r_A \sqrt{2} \) of the square, so that \( r_B = \frac{1}{2}(r_A \sqrt{2}) = \frac{1}{2} \sqrt{2}(0.480 \text{ m}) = 0.339 \text{ m} \). Substituting Equations (2) into \( EPE = q_0 V \) (Equation 19.3) yields

\[
EPE_A = q_0 V_A = q_0 \left( \frac{2kq}{r_A} \right) = \frac{2kq_0 q}{r_A} \quad \text{and} \quad EPE_B = q_0 V_B = q_0 \left( \frac{2kq}{r_B} \right) = \frac{2kq_0 q}{r_B}
\]  

(3)

Substituting Equations (3) into Equation (1), we obtain

\[
\frac{1}{2} m v_B^2 + \frac{2kq_0 q}{r_B} = \frac{2kq_0 q}{r_A}
\]  

(4)

Solving Equation (4) for \( v_B^2 \), we find that

\[
\frac{1}{2} m v_B^2 = \frac{2kq_0 q}{r_A} - \frac{2kq_0 q}{r_B} \quad \text{or} \quad \frac{1}{2} m v_B^2 = 2kq_0 q \left( \frac{1}{r_A} - \frac{1}{r_B} \right) \quad \text{or} \quad v_B^2 = \frac{4kq q_0}{m} \left( \frac{1}{r_A} - \frac{1}{r_B} \right)
\]

Therefore, the speed of the test charge at the center of the square is

\[
v_B = \sqrt{\frac{4kq q_0}{m} \left( \frac{1}{r_A} - \frac{1}{r_B} \right)}
\]

\[
= \sqrt{\frac{4 \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( 7.20 \times 10^{-6} \text{ C} \right) \left( -2.40 \times 10^{-8} \text{ C} \right) \left( \frac{1}{0.480 \text{ m}} - \frac{1}{0.339 \text{ m}} \right)}{6.60 \times 10^{-8} \text{ kg}}}
\]

= 290 \text{ m/s}

25. **REASONING** The only force acting on each proton is the conservative electric force. Therefore, the total energy (kinetic energies plus electric potential energy) is conserved at all points along the motion. For two points, \( A \) and \( B \), the conservation of energy is expressed as follows:

\[
\frac{1}{2} m v_A^2 + \frac{1}{2} m v_A^2 + EPE_A = \frac{1}{2} m v_B^2 + \frac{1}{2} m v_B^2 + EPE_B
\]

The electric potential energy of two protons (charge on each proton = \( +e \)) that are separated by a distance \( r \) can be found by combining the relations \( EPE = eV \) (Equation 19.3) and \( V = ke/r \) (Equation 19.6) to give \( EPE = ke^2/r \). By using this expression for \( EPE \) in the conservation of energy relation, we will be able to determine the distance of closest approach.
**SOLUTION** When the protons are very far apart \((r_A = \infty)\), so that \(\text{EPE}_A = 0 \text{ J}\). At the distance \(r_B\) of closest approach, the speed of each proton is momentarily zero \((v_B = 0 \text{ m/s})\). With these substitutions, the conservation of energy equation reduces to

\[
\frac{1}{2} mv_A^2 + \frac{1}{2} mv_B^2 = \frac{ke^2}{r_B}
\]

Solving for \(r_B\), the distance of closest approach, gives

\[
r_B = \frac{ke^2}{mv_A^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^6 \text{ m/s})^2} = 1.53 \times 10^{-14} \text{ m}
\]

We have taken the mass of a proton from the inside of the front cover of the text.

26. **REASONING** It will not matter in what order the group is assembled. For convenience, we will assemble the group from one end of the line to the other. The potential energy of each charge added to the group will be determined and the four values added together to get the total. At each step, the electric potential energy of an added charge \(q_0\) is given by Equation 19.3 as \(\text{EPE} = q_0V_{\text{Total}}\), where \(V_{\text{Total}}\) is the potential at the point where the added charge is placed. The potential \(V_{\text{Total}}\) will be determined by adding together the contributions from the charges previously put in position, each according to Equation 19.6 \((V = kq/r)\).

**SOLUTION** The first charge added to the group has no electric potential energy, since the spot where it goes has a total potential of \(V_{\text{Total}} = 0 \text{ V}\), there being no charges in the vicinity to create it:

\[
\text{EPE}_1 = q_0V_{\text{Total}} = (2.0 \times 10^{-6} \text{ C})(0 \text{ V}) = 0 \text{ J}
\]

The second charge experiences a total potential that is created by the first charge:

\[
V_{\text{Total}} = \frac{kv}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(+2.0 \times 10^{-6} \text{ C})}{0.40 \text{ m}} = 4.5 \times 10^4 \text{ V}
\]

\[
\text{EPE}_2 = q_0V_{\text{Total}} = (2.0 \times 10^{-6} \text{ C})(4.5 \times 10^4 \text{ V}) = 0.090 \text{ J}
\]

The third charge experiences a total potential that is created by the first and the second charges:
The fourth charge experiences a total potential that is created by the first, second, and third charges:

\[
V_{\text{Total}} = \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{0.80 \text{ m}} \right) \left( +2.0 \times 10^{-6} \text{ C} \right)
\]

\[
+ \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{0.40 \text{ m}} \right) \left( +2.0 \times 10^{-6} \text{ C} \right) = 6.7 \times 10^4 \text{ V}
\]

\[
EPE_3 = q_0 V_{\text{Total}} = \left( 2.0 \times 10^{-6} \text{ C} \right) \left( 6.7 \times 10^4 \text{ V} \right) = 0.13 \text{ J}
\]

The total electric potential energy of the group is

\[
EPE_{\text{Total}} = EPE_1 + EPE_2 + EPE_3 + EPE_4
\]

\[
= 0 \text{ J} + 0.090 \text{ J} + 0.13 \text{ J} + 0.16 \text{ J} = 0.38 \text{ J}
\]

27. **SSM REASONING** The only force acting on the moving charge is the conservative electric force. Therefore, the sum of the kinetic energy KE and the electric potential energy EPE is the same at points A and B:

\[
\frac{1}{2} m v_A^2 + EPE_A = \frac{1}{2} m v_B^2 + EPE_B
\]

Since the particle comes to rest at B, \( v_B = 0 \). Combining Equations 19.3 and 19.6, we have

\[
EPE_A = q V_A = q \left( \frac{kq_1}{d} \right)
\]

and

\[
EPE_B = q V_B = q \left( \frac{kq_1}{r} \right)
\]
where \(d\) is the initial distance between the fixed charge and the moving charged particle, and \(r\) is the distance between the charged particles after the moving charge has stopped. Therefore, the expression for the conservation of energy becomes

\[
\frac{1}{2}mv_A^2 + \frac{kq q_1}{d} = \frac{kq q_1}{r}
\]

This expression can be solved for \(r\). Once \(r\) is known, the distance that the charged particle moves can be determined.

**SOLUTION** Solving the expression above for \(r\) gives

\[
r = \frac{kq q_1}{\frac{1}{2}mv_A^2 + \frac{kq q_1}{d}} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(-8.00 \times 10^{-6} \text{ C})(-3.00 \times 10^{-6} \text{ C})}{\frac{1}{2}(7.20 \times 10^{-3} \text{ kg})(65.0 \text{ m/s})^2 + \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(-8.00 \times 10^{-6} \text{ C})(-3.00 \times 10^{-6} \text{ C})}{0.0450 \text{ m}}
\]

\[
= 0.0108 \text{ m}
\]

Therefore, the charge moves a distance of \(0.0450 \text{ m} - 0.0108 \text{ m} = 0.0342 \text{ m}\).

28. **REASONING** The figure at the right shows two identical charges (●) fixed to diagonally opposite corners of a square. The initial potential \((V_A)_{initial}\) at corner A (or corner B) is caused by the presence of the two identical charges. After the third charge is placed at the center of the square, the final potential \((V_A)_{final}\) at corner A (or corner B) is caused by the presence of the two identical charges and the third charge. From the problem statement, we know that the addition of the third charge causes the potential at A (and B) to change sign without changing magnitude. This means that

\[(V_A)_{final} = -(V_A)_{initial}\]  

(1)

To evaluate \((V_A)_{initial}\) and \((V_A)_{final}\) we will apply \(V = kq / r\) (Equation 19.6), which gives the potential \(V\) created by a point charge \(q\) at a distance \(r\) away from the charge.

**SOLUTION** As the drawing indicates, each of the two identical charges \(q\) is the same distance \(r\) from corner A (or B) and, therefore, contributes the same amount to the initial potential at corner A (or B), according to Equation 19.6. Thus, we have

\[
(V_A)_{initial} = \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}
\]

(2)
When the third charge $q_3$ is placed at the center of the square, the potential at corner A (or B) becomes

$$(V_A)_{\text{final}} = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3}{(d/2)}$$

where $d$ is the length of the diagonal of the square (see the drawing). To find $d$ we use the Pythagorean theorem (see the drawing):

$$d = \sqrt{r^2 + r^2} = \sqrt{2}r \quad \text{or} \quad \frac{d}{2} = \frac{r}{\sqrt{2}}$$

With this substitution for $d/2$ the expression for $(V_A)_{\text{final}}$ becomes

$$(V_A)_{\text{final}} = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r} \quad (3)$$

Substituting Equations (2) and (3) into Equation (1) shows that

$$\frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r} = -\frac{2kq}{r}$$

Solving for $q_3$ gives

$$q_3 = -(2\sqrt{2})q = -(2\sqrt{2})(1.7 \times 10^{-6} \text{ C}) = -4.8 \times 10^{-6} \text{ C}$$

29. **REASONING** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the initial separation of the particles.

**SOLUTION** For two points, $A$ and $B$, along the motion, the conservation of energy is

$$\frac{1}{2}m_1v_{1,A}^2 + \frac{1}{2}m_2v_{2,A}^2 + \frac{kq_1q_2}{r_A} = \frac{1}{2}m_1v_{1,B}^2 + \frac{1}{2}m_2v_{2,B}^2 + \frac{kq_1q_2}{r_B}$$

Solving this equation for $1/r_A$ and setting $v_{1,A} = v_{2,A} = 0$ m/s, since the particles are initially at rest, we obtain

$$\frac{1}{r_A} = \frac{1}{r_B} + \frac{1}{kq_1q_2}\left(\frac{1}{2}m_1v_{1,B}^2 + \frac{1}{2}m_2v_{2,B}^2\right) \quad (1)$$
This equation cannot be solved for the initial separation \( r_A \), because the final speed \( v_{2,B} \) of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

\[
\frac{m_1v_{1,A} + m_2v_{2,A}}{m_1v_{1,B} + m_2v_{2,B}} = \frac{m_1v_{1,B} + m_2v_{2,B}}{m_1v_{1,B} + m_2v_{2,B}}
\]

Initial linear momentum Final linear momentum

Setting \( v_{1,A} = v_{2,A} = 0 \) and solving for \( v_{2,B} \) gives

\[
v_{2,B} = -\frac{m_1}{m_2}v_{1,B} = -\frac{3.00 \times 10^{-3} \text{ kg}}{6.00 \times 10^{-3} \text{ kg}}(125 \text{ m/s}) = -62.5 \text{ m/s}
\]

Substituting this value for \( v_{2,B} \) into Equation (1) yields

\[
\frac{1}{r_A} = \frac{1}{0.100 \text{ m}} + \frac{1}{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(8.00 \times 10^{-6} \text{ C})^2}
\times \left[\frac{1}{2}(3.00 \times 10^{-3} \text{ kg})(125 \text{ m/s})^2 + \frac{1}{2}(6.00 \times 10^{-3} \text{ kg})(-62.5 \text{ m/s})^2\right]
\]

\[
r_A = 1.41 \times 10^{-2} \text{ m}
\]

30. **REASONING AND SOLUTION**

a. Let \( d \) be the distance between the charges. The potential at the point \( x_1 = 4.00 \text{ cm} \) to the left of the negative charge is

\[
V = 0 = \frac{kq_1}{d - x_1} - \frac{kq_2}{x_1}
\]

which gives

\[
\frac{q_1}{q_2} = \frac{d}{x_1} - 1 \quad (1)
\]

Similarly, at the point \( x_2 = 7.00 \text{ cm} \) to the right of the negative charge we have

\[
V = 0 = \frac{kq_1}{x_2 + d} - \frac{kq_2}{x_2}
\]

which gives

\[
\frac{q_1}{q_2} = \frac{d}{x_2} + 1 \quad (2)
\]

Equating Equations (1) and (2) and solving for \( d \) gives \( d = 0.187 \text{ m} \).

b. Using the above value for \( d \) in Equation (1) yields \( \frac{q_1}{q_2} = 3.67 \).
31. **REASONING** The electric potential \( V \) at a distance \( r \) from a point charge \( q \) is \( V = kq/r \) (Equation 19.6). The potential is the same at all points on a spherical surface whose distance from the charge is \( r = kq/V \). We will use this relation to find the distance between the two equipotential surfaces.

**SOLUTION** The radial distance \( r_{75} \) from the charge to the 75.0-V equipotential surface is \( r_{75} = kq/V_{75} \), and the distance to the 190-V equipotential surface is \( r_{190} = kq/V_{190} \). The distance between these two surfaces is

\[
r_{75} - r_{190} = \frac{kq}{V_{75}} - \frac{kq}{V_{190}} = kq \left( \frac{1}{V_{75}} - \frac{1}{V_{190}} \right)
\]

\[
= \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( +1.50 \times 10^{-8} \text{ C} \right) \left( \frac{1}{75.0 \text{ V}} - \frac{1}{190 \text{ V}} \right) = 1.1 \text{ m}
\]

32. **REASONING** The equipotential surfaces that surround a point charge \( q \) are spherical. Each has a potential \( V \) that is given by \( V = \frac{kq}{r} \) (Equation 19.6), where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \) and \( r \) is the radius of the sphere. Equation 19.6 can be solved for \( q \), provided that we can obtain a value for the radius \( r \). The radius can be found from the given surface area \( A \), since the surface area of a sphere is \( A = 4\pi r^2 \).

**SOLUTION** Solving \( V = \frac{kq}{r} \) (Equation 19.6) for \( q \) gives

\[ q = \frac{Vr}{k} \]

Since the surface area of a sphere is \( A = 4\pi r^2 \), it follows that \( r = \sqrt{\frac{A}{4\pi}} \). Substituting this result into the expression for \( q \) gives

\[
q = \frac{Vr}{k} = \frac{V}{k} \sqrt{\frac{A}{4\pi}} = \frac{490 \text{ V}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \sqrt{\frac{1.1 \text{ m}^2}{4\pi}} = 1.6 \times 10^{-8} \text{ C}
\]

33. **SSM REASONING AND SOLUTION** From Equation 19.7a we know that \( E = -\frac{\Delta V}{\Delta s} \), where \( \Delta V \) is the potential difference between the two surfaces of the membrane, and \( \Delta s \) is the distance between them. If \( A \) is a point on the positive surface and \( B \) is a point on the negative surface, then \( \Delta V = V_A - V_B = 0.070 \text{ V} \). The electric field between the surfaces is

\[
E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = \frac{V_A - V_B}{\Delta s} = \frac{0.070 \text{ V}}{8.0 \times 10^{-9} \text{ m}} = 8.8 \times 10^6 \text{ V/m}
\]
34. **REASONING** Since the electric force does negative work, the electric force must point opposite to the displacement of the test charge. The point charge is positive, so it exerts an outward-directed force on the positive test charge. Since the force and the displacement of the test charge have opposite directions, the displacement must be directed inward toward the point charge. Therefore, the radius \( r_B \) is less than the radius \( r_A \).

**SOLUTION** According to Equation 19.4, we have

\[
V_B - V_A = \frac{-W_{AB}}{q_0}
\]

where \( V_B \) and \( V_A \) are the potentials of the equipotential surfaces, and \( W_{AB} \) is the work done in moving the charge \( q_0 \) from surface \( A \) to surface \( B \). The potential of a point charge \( q \) is given by \( V = kq/r \) (Equation 19.6). With this substitution for \( V_B \) and \( V_A \), we have

\[
\frac{kq}{r_B} - \frac{kq}{r_A} = \frac{-W_{AB}}{q_0}
\]

or

\[
\frac{kq}{r_B} = \frac{kq}{r_A} - \frac{W_{AB}}{q_0}
\]

or

\[
\frac{1}{r_B} = \frac{1}{r_A} - \frac{W_{AB}}{kqq_0}
\]

Solving for \( r_B \) gives:

\[
\frac{1}{r_B} = \frac{1}{r_A} - \frac{W_{AB}}{kqq_0}
\]

\[
\frac{1}{r_B} = \frac{1}{1.8 \text{ m}} - \frac{8.1 \times 10^{-9} \text{ J}}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(+0.5 \times 10^{-8} \text{ C})(4.5 \times 10^{-11} \text{ C})} = 0.83 \text{ m}^{-1}
\]

\[
r_B = \frac{1}{0.83 \text{ m}^{-1}} = 1.2 \text{ m}
\]

35. **SSM** **REASONING** The magnitude \( E \) of the electric field is given by Equation 19.7a (without the minus sign) as \( E = \frac{\Delta V}{\Delta s} \), where \( \Delta V \) is the potential difference between the two metal conductors of the spark plug, and \( \Delta s \) is the distance between the two conductors. We can use this relation to find \( \Delta V \).

**SOLUTION** The potential difference between the conductors is

\[
\Delta V = E \Delta s = (4.7 \times 10^7 \text{ V/m})(0.75 \times 10^{-3} \text{ m}) = 3.5 \times 10^4 \text{ V}
\]
36. **REASONING** The drawing shows a set of equipotential surfaces, and position \( D \) is halfway between the +450-V equipotential and the +550-V equipotential. We can use \( E = \frac{\Delta V}{\Delta s} \) (Equation 19.7a, without the minus sign) to determine the magnitude \( E \) of the electric field, where \( \Delta V \) is the potential difference (a positive number) between the two equipotential surfaces and \( \Delta s \) is the distance between them.

**SOLUTION** The distance between the equipotentials is equal to the width of two squares on the grid, each of which is 2.0 cm wide. Therefore, \( \Delta s = 2(2.0 \text{ cm}) = 4.0 \text{ cm} \). In SI base units, this is

\[
\Delta s = \left(4.0 \text{ cm}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) = 4.0 \times 10^{-2} \text{ m}
\]

Now, according to \( E = \frac{\Delta V}{\Delta s} \) (Equation 19.7a, without the minus sign), the magnitude of the electric field at position \( D \) is

\[
E = \frac{\Delta V}{\Delta s} = \frac{550.0 \text{ V} - 450.0 \text{ V}}{4.0 \times 10^{-2} \text{ m}} = 2500 \text{ V/m}
\]

Because the electric field points from high potential to low potential, at position \( D \) the electric field must point from the +550.0-V equipotential to the +450.0-V equipotential. Therefore, the direction of the electric field at \( D \) is towards the bottom of the drawing.

37. **SSM REASONING** The drawing shows the electric field \( \mathbf{E} \) and the three points, \( A \), \( B \), and \( C \), in the vicinity of point \( P \), which we take as the origin. We choose the upward direction as being positive. Thus, \( E = -4.0 \times 10^3 \text{ V/m} \), since the electric field points straight down. The electric potential at points \( A \) and \( B \) can be determined from Equation 19.7a as \( \Delta V = -E \Delta s \), since \( E \) and \( \Delta s \) are known. Since the path from \( P \) to \( C \) is perpendicular to the electric field, no work is done in moving a charge along such a path. Thus, the potential difference between these two points is zero.

**SOLUTION**

a. The potential difference between points \( P \) and \( A \) is \( V_A - V_P = -E \Delta s \). The potential at \( A \) is

\[
V_A = V_P - E \Delta s = 155 \text{ V} - \left(-4.0 \times 10^3 \text{ V/m}\right)\left(6.0 \times 10^{-3} \text{ m}\right) = 179 \text{ V}
\]

b. The potential difference between points \( P \) and \( B \) is \( V_B - V_P = -E \Delta s \). The potential at \( B \) is

\[
V_B = V_P - E \Delta s = 155 \text{ V} - \left(-4.0 \times 10^3 \text{ V/m}\right)\left(3.0 \times 10^{-3} \text{ m}\right) = 143 \text{ V}
\]
c. Since the path from \( P \) to \( C \) is perpendicular to the electric field and no work is done in moving a charge along such a path, it follows that \( \Delta V = 0 \) V. Therefore, \( V_C = V_P = 155 \) V.

38. **REASONING** The electric force \( \mathbf{F} \) is a conservative force, so the total energy (kinetic energy plus electric potential energy) remains constant as the electron moves across the capacitor. Thus, as the electron accelerates and its kinetic energy increases, its electric potential energy decreases. According to Equation 19.4, the change in the electron’s electric potential energy is equal to the charge on the electron \((-e)\) times the potential difference between the plates, or

\[
\text{EPE}_{\text{positive}} - \text{EPE}_{\text{negative}} = (-e)(V_{\text{positive}} - V_{\text{negative}}) \tag{19.4}
\]

The electric field \( \mathbf{E} \) is related to the potential difference between the plates and the displacement \( \Delta s \) by \( E = -\frac{(V_{\text{positive}} - V_{\text{negative}})}{\Delta s} \) (Equation 19.7a). Note that \( \Delta s \) and \((V_{\text{positive}} - V_{\text{negative}})\) are positive numbers, so the electric field is a negative number, denoting that it points to the left in the drawing:

**SOLUTION** The total energy of the electron is conserved, so its total energy at the positive plate is equal to its total energy at the negative plate:

\[
\frac{\text{KE}_{\text{positive}} + \text{EPE}_{\text{positive}}}{\text{Total energy at positive plate}} = \frac{\text{KE}_{\text{negative}} + \text{EPE}_{\text{negative}}}{\text{Total energy at negative plate}}
\]

Since the electron starts from rest at the negative plate, \( \text{KE}_{\text{negative}} = 0 \) J. Thus, the kinetic energy of the electron at the positive plate is \( \text{KE}_{\text{positive}} = -(\text{EPE}_{\text{positive}} - \text{EPE}_{\text{negative}}) \). We know from Equation 19.4 in the **REASONING** section that \( \text{EPE}_{\text{positive}} - \text{EPE}_{\text{negative}} = (-e)(V_{\text{positive}} - V_{\text{negative}}) \), so the kinetic energy can be written as...
ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL

\[ KE_{\text{positive}} = -(\text{EPE}_{\text{positive}} - \text{EPE}_{\text{negative}}) = e (V_{\text{positive}} - V_{\text{negative}}) \]

Since the potential difference is related to the electric field \( E \) and the displacement \( \Delta s \) by \( V_{\text{positive}} - V_{\text{negative}} = -E\Delta s \) (Equation 19.7a), we have that

\[ KE_{\text{positive}} = e \left( V_{\text{positive}} - V_{\text{negative}} \right) = e(-E\Delta s) \]

\[ = \left( 1.60 \times 10^{-19} \text{ C} \right) \left[ -(2.1 \times 10^6 \text{ V/m}) \left( +0.012 \text{ m} \right) \right] = 4.0 \times 10^{-15} \text{ J} \]

In arriving at this result, we have used the fact that the electric field is negative, since it points to the left in the drawing.

39. **REASONING** The electric field \( E \) that exists between two points in space is, according to Equation 19.7a, proportional to the electric potential difference \( \Delta V \) between the points divided by the distance \( \Delta x \) between them: \( E = -\Delta V / \Delta x \).

**SOLUTION**

a. The electric field in the region from \( A \) to \( B \) is

\[ E = -\frac{\Delta V}{\Delta x} = -\frac{5.0 \text{ V} - 5.0 \text{ V}}{0.20 \text{ m} - 0 \text{ m}} = 0 \text{ V/m} \]

b. The electric field in the region from \( B \) to \( C \) is

\[ E = -\frac{\Delta V}{\Delta x} = -\frac{3.0 \text{ V} - 5.0 \text{ V}}{0.40 \text{ m} - 0.20 \text{ m}} = 1.0 \times 10^1 \text{ V/m} \]

c. The electric field in the region from \( C \) to \( D \) is

\[ E = -\frac{\Delta V}{\Delta x} = -\frac{1.0 \text{ V} - 3.0 \text{ V}}{0.80 \text{ m} - 0.40 \text{ m}} = 5.0 \text{ V/m} \]

40. **REASONING** The starting point is at a distance of 1.60 m away from the charge. As we move radially outward from this point, we reach an ending point where the magnitude of the charge’s electric field has decreased to one-half of its initial value. Simultaneously the magnitude of the electric potential that the charge creates also decreases. We will determine the amount by which the electric potential decreases between the starting point and the ending point. To find the number of equipotential surfaces that are crossed as we move from the starting to the ending point, we will divide the amount of the decrease in the potential by the \( 1.00 \times 10^3 \text{ V} \) difference between successive surfaces.

**SOLUTION** Let \( V_{\text{start}} \) represent the electric potential at the starting point and \( V_{\text{end}} \) the electric potential at the ending point. Then, the amount by which the potential decreases
between the starting and ending points is \( V_{\text{start}} - V_{\text{end}} \). The number \( N \) of equipotential surfaces crossed as we move from the starting to the ending point is

\[
N = \frac{V_{\text{start}} - V_{\text{end}}}{1.00 \times 10^3 \text{ V}} = \frac{V_{\text{start}} - V_{\text{end}}}{1.00 \times 10^3 \text{ V}}
\]

\[(1)\]

A point charge \( q \) creates an electric potential \( V \) at a spot that is a distance \( r \) away from the charge, according to \( V = kq / r \) (Equation 19.6), where \( k \) is the constant in Coulomb’s law. Using this equation for \( V_{\text{start}} \) and \( V_{\text{end}} \) in Equation (1), we obtain

\[
N = \frac{kq}{1.00 \times 10^3 \text{ V}} \left( \frac{1}{r_{\text{start}}} - \frac{1}{r_{\text{end}}} \right)
\]

\[(2)\]

We know that \( r_{\text{start}} = 1.60 \text{ m} \). To determine \( r_{\text{end}} \) we use the fact that the starting and ending electric fields are related according to \( E_{\text{end}} = E_{\text{start}} / 2 \). These electric fields are specified by \( E = k |q| / r^2 \) (Equation 18.3). Thus, we have

\[
E_{\text{end}} = \frac{1}{2} E_{\text{start}} \quad \text{or} \quad \frac{k|q|}{r_{\text{end}}^2} = \left( \frac{1}{2} \right) \frac{k|q|}{r_{\text{start}}^2} \quad \text{or} \quad r_{\text{end}} = \left( \sqrt{2} \right) r_{\text{start}} = \left( \sqrt{2} \right) (1.60 \text{ m}) = 2.26 \text{ m}
\]

Using this result for \( r_{\text{end}} \) in Equation (2) reveals that

\[
N = \frac{kq}{1.00 \times 10^3 \text{ V}} \left( \frac{1}{r_{\text{start}}} - \frac{1}{r_{\text{end}}} \right) = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left( 2.00 \times 10^{-6} \text{ C} \right) \left( \frac{1}{1.60 \text{ m}} - \frac{1}{2.26 \text{ m}} \right)}{1.00 \times 10^3 \text{ V}} = 3.28
\]

Therefore, the number of equipotential surfaces crossed is \( 3 \).

41. **REASONING**
   a. The potential difference \( V_B - V_A \) between points \( A \) and \( B \) is related to the work \( W_{AB} \) done by the electric force when a charge \( q_0 \) moves from \( A \) to \( B \); \( V_B - V_A = -W_{AB} / q_0 \) (Equation 19.4). Since the displacement from \( A \) to \( B \) is perpendicular to the electric force, no work is done, so \( W_{AB} = 0 \) J (see Section 6.1).

   b. The potential difference \( \Delta V = V_C - V_B \) between \( B \) and \( C \) is related to the electric field \( E \) and the displacement \( \Delta s \) by \( \Delta V = -E \Delta s \) (Equation 19.7a). Since \( E \) and \( \Delta s \) are known, we can use this expression to find \( \Delta V \).
c. Since the electric force is a conservative force, it does not matter which path we take between points $A$ and $C$ to find the potential difference $V_A - V_C$. Therefore, we can retrace our steps in parts a and b to find this potential difference.

\[ V_B - V_A = \frac{-W_{AB}}{q_0} = \frac{0 \text{ J}}{q_0} = 0 \text{ V} \]

b. We have that $V_C - V_B = -E \Delta s$ (Equation 19.7a). Since the electric field points in the negative $y$ direction, $E = -3600 \text{ N/C}$. The displacement from $B$ to $C$ is positive, so $\Delta s = +0.080 \text{ m}$. Thus, the potential difference is

\[ V_C - V_B = -E \Delta s = -(-3600 \text{ N/C})(+0.080 \text{ m}) = +290 \text{ V} \]

c. To determine the potential difference $V_A - V_C$, we may take any path from $C$ to $A$ that is convenient. We will choose the paths $C \rightarrow B$ and $B \rightarrow A$, since we know the potential differences for each of these segments. Thus, the potential difference $V_A - V_C$ can be written as $V_A - V_C = (V_A - V_B) + (V_B - V_C)$. Since $V_B - V_C = -(V_C - V_B) = -290 \text{ V}$, and $V_A - V_B = -(V_B - V_A) = 0 \text{ V}$, we have that

\[ V_A - V_C = -(V_B - V_A) - (V_C - V_B) = 0 \text{ V} - 290 \text{ V} = -290 \text{ V} \]

42. **Reasoning** The magnitude $q$ of the charge on each plate of a capacitor is directly proportional to the magnitude $V$ of the electric potential difference (i.e., the voltage) between the plates: $q = CV$ (Equation 19.8), where $C$ is the capacitance of the capacitor.

**Solution** Solving Equation 19.8 for the capacitance $C$, we find that

\[ C = \frac{q}{V} = \frac{4.3 \times 10^{-6} \text{ C}}{1.5 \text{ V}} = 2.9 \times 10^{-6} \text{ F} \]
43. **REASONING** According to Equation 19.11b, the energy stored in a capacitor with capacitance $C$ and potential $V$ across its plates is $\text{Energy} = \frac{1}{2}CV^2$.

**SOLUTION** Therefore, solving Equation 19.11b for $V$, we have

$$V = \sqrt{\frac{2(\text{Energy})}{C}} = \sqrt{\frac{2(73 \text{ J})}{120 \times 10^{-6} \text{ F}}} = 1.1 \times 10^3 \text{ V}$$

44. **REASONING** The energy stored by a capacitor is $\text{Energy} = \frac{1}{2}CV^2$ (Equation 19.11b), where $C$ is the capacitance of the capacitor and $V$ is the magnitude $V$ of the electric potential difference (i.e., the voltage) between the plates. Applying this expression to each capacitor will allow us to determine the unknown voltage, since $C$ is the same for each capacitor and the two energies and the voltage applied to one of the capacitors are given.

**SOLUTION** Applying Equation 19.11 to each capacitor gives

$$\left(\text{Energy}\right)_A = \frac{1}{2}CV_A^2 \quad \text{and} \quad \left(\text{Energy}\right)_B = \frac{1}{2}CV_B^2$$

Dividing the $A$-equation by the $B$-equation, we find

$$\frac{\left(\text{Energy}\right)_A}{\left(\text{Energy}\right)_B} = \frac{\frac{1}{2}CV_A^2}{\frac{1}{2}CV_B^2} \quad \text{or} \quad \frac{\left(\text{Energy}\right)_A}{\left(\text{Energy}\right)_B} = \frac{V_A^2}{V_B^2}$$

$$V_A = V_B \sqrt{\frac{\left(\text{Energy}\right)_A}{\left(\text{Energy}\right)_B}} = (12 \text{ V}) \sqrt{\frac{3.1 \times 10^{-3} \text{ J}}{3.4 \times 10^{-4} \text{ J}}} = 36 \text{ V}$$

45. **REASONING**

a. The energy used to produce the flash is stored in the capacitor as electrical energy. The energy stored depends on the capacitance $C$ of the capacitor and the potential difference $V$ between its plates; $\text{Energy} = \frac{1}{2}CV^2$ (Equation 19.11b).

b. The power of the flash is the energy consumed divided by the duration of the flash (see Equation 6.10b).

**SOLUTION**

a. The energy used to produce the flash is

$$\text{Energy} = \frac{1}{2}CV = \frac{1}{2}(850 \times 10^{-6} \text{ F})(280 \text{ V})^2 = 33 \text{ J}$$

b. The power developed by the flash is

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{33 \text{ J}}{3.9 \times 10^{-3} \text{ s}} = 8500 \text{ W}$$
46. **REASONING** The magnitude \( q_B \) of the charge on each plate of capacitor B is directly proportional to the magnitude \( V \) of the electric potential difference (i.e., the voltage) between the plates: \( q_B = C_B V \) (Equation 19.8), where \( C_B \) is the capacitance of the capacitor. The capacitance is given, but the voltage is not given. To find the voltage we will utilize the information for capacitor A. The energy stored by capacitor A is \( (\text{Energy})_A = \frac{1}{2} q_A V \) (Equation 19.11a), where \( q_A \) is the magnitude of the charge on each plate and \( V \) is the magnitude of the electric potential difference (i.e., the voltage) between the plates. Equation 19.11a can be solved for the unspecified voltage.

**SOLUTION** The desired charge magnitude is \( q_B = C_B V \) (Equation 19.8). Solving Equation 19.11a for \( V \), we obtain

\[
(E\text{nergy})_A = \frac{1}{2} q_A V \quad \text{or} \quad V = \frac{2(E\text{nergy})_A}{q_A}
\]

Substituting this result for \( V \) into Equation 19.8 gives

\[
q_B = C_B V = C_B \left[ \frac{2(E\text{nergy})_A}{q_A} \right] = \left( 6.7 \times 10^{-6} \right) \left( \frac{2 \left( 5.0 \times 10^{-5} \right)}{11 \times 10^{-6}} \right) = 6.1 \times 10^{-5} \text{ C}
\]

47. **SSM REASONING AND SOLUTION** Equation 19.10 gives the capacitance for a parallel plate capacitor filled with a dielectric of constant \( \kappa \): \( C = \kappa \varepsilon_0 A / d \). Solving for \( \kappa \), we have

\[
\kappa = \frac{Cd}{\varepsilon_0 A} = \frac{(7.0 \times 10^{-6} F)(1.0 \times 10^{-5} \text{ m})}{(8.85 \times 10^{-12} \text{ F/m})(1.5 \text{ m}^2)} = 5.3
\]

48. **REASONING** The energy stored in a capacitor is given by \( \text{Energy} = \frac{1}{2} CV^2 \) (Equation 19.11b), where \( C \) is the capacitance of the capacitor and \( V \) is the potential difference across its plates. The only difference between the two capacitors is the dielectric material (dielectric constant \( \kappa = 4.50 \)) inside the filled capacitor. Therefore, the filled capacitor’s capacitance \( C_2 \) is greater than the capacitance \( C_1 \) of the empty capacitor by a factor of \( \kappa \):

\[
C_2 = \kappa C_1 \quad (1)
\]

Because both capacitors store the same amount of energy, from \( \text{Energy} = \frac{1}{2} CV^2 \) (Equation 19.11b), we have that

\[
\frac{1}{2} C_2 V_2^2 = \frac{1}{2} C_1 V_1^2 \quad (2)
\]
where $V_2$ is the potential difference across the plates of the filled capacitor, and $V_1 = 12.0$ V is the potential difference across the plates of the empty capacitor.

**SOLUTION** Solving Equation (2) for $V_2$, we obtain

$$V_2^2 = \frac{CV_1^2}{C_2} \quad \text{or} \quad V_2 = \sqrt{\frac{CV_1^2}{C_2}}$$

(3)

Substituting Equation (1) into Equation (3) yields

$$V_2 = \sqrt{\frac{CV_1^2}{\kappa C_1}} = \frac{V_1}{\sqrt{\kappa}} = \frac{12.0 \text{ V}}{\sqrt{4.50}} = 5.66 \text{ V}$$

49. **REASONING** The charge that resides on the outer surface of the cell membrane is $q = CV$, according to Equation 19.8. Before we can use this expression, however, we must first determine the capacitance of the membrane. If we assume that the cell membrane behaves like a parallel plate capacitor filled with a dielectric, Equation 19.10 ($C = \kappa \varepsilon_0 A/d$) applies as well.

**SOLUTION** The capacitance of the cell membrane is

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{(5.0)(8.85 \times 10^{-12} \text{ F/m})(5.0 \times 10^{-9} \text{ m}^2)}{1.0 \times 10^{-8} \text{ m}} = 2.2 \times 10^{-11} \text{ F}$$

a. The charge on the outer surface of the membrane is, therefore,

$$q = CV = (2.2 \times 10^{-11} \text{ F})(60.0 \times 10^{-3} \text{ V}) = 1.3 \times 10^{-12} \text{ C}$$

b. If the charge in part (a) is due to positive ions with charge $+e$ ($e = 1.6 \times 10^{-19} \text{ C}$), the number of ions present on the outer surface of the membrane is

$$\text{Number of } K^+ \text{ ions} = \frac{1.3 \times 10^{-12} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 8.1 \times 10^6$$

50. **REASONING** The energy $Q$ needed to melt ice is given by Equation 12.5 as $Q = mL_f$, where $m$ is the mass and $L_f$ is the latent heat of fusion for water. The energy needed to boil away water is given by $Q = mL_v$, where $L_v$ is the latent heat of vaporization for water. The latent heat of vaporization is greater than the latent heat of fusion (see Table 12.3). Therefore, it requires more energy to boil away one kilogram of water than to melt one kilogram of ice.
According to Equation 19.11b, the energy stored in a capacitor is $\text{Energy} = \frac{1}{2} CV^2$, where $C$ is the capacitance and $V$ is the voltage across the plates. Since the voltage is the same for both capacitors, the capacitor storing the greater energy has the greater capacitance. Capacitor B contains more energy, since it can boil the water. Therefore, capacitor B must have the greater capacitance.

**SOLUTION** Using the relations $\text{Energy} = \frac{1}{2} CV^2$ (Equation 19.11b) and $Q = mL$ (Equation 12.5), we find

$$\text{Energy}_A = mL_A = \frac{1}{2} C_A V_A^2 \quad \text{and} \quad \text{Energy}_B = mL_B = \frac{1}{2} C_B V_B^2$$

Dividing these two results gives

$$\frac{mL_B}{mL_A} = \frac{\frac{1}{2} C_B V_B^2}{\frac{1}{2} C_A V_A^2} \quad \text{or} \quad \frac{L_B}{L_A} = \frac{C_B}{C_A} \quad \text{or} \quad C_B = C_A \frac{L_B}{L_A}$$

Taking the values for the latent heats from Table 12.3, we find

$$C_B = C_A \frac{L_B}{L_A} = \left(9.3 \times 10^{-6} \text{ F}\right) \left(\frac{22.6 \times 10^5 \text{ J/kg}}{33.5 \times 10^4 \text{ J/kg}}\right) = 6.3 \times 10^{-5} \text{ F}$$

51. **REASONING** According to Equation 19.11b, the energy stored in a capacitor with a capacitance $C$ and potential $V$ across its plates is $\text{Energy} = \frac{1}{2} CV^2$. Once we determine how much energy is required to operate a 75-W light bulb for one minute, we can then use the expression for the energy to solve for $V$.

**SOLUTION** The energy stored in the capacitor, which is equal to the energy required to operate a 75-W bulb for one minute ($= 60 \text{ s}$), is

$$\text{Energy} = Pt = (75 \text{ W})(60 \text{ s}) = 4500 \text{ J}$$

Therefore, solving Equation 19.11b for $V$, we have

$$V = \sqrt{\frac{2(\text{Energy})}{C}} = \sqrt{\frac{2(4500 \text{ J})}{3.3 \text{ F}}} = 52 \text{ V}$$

52. **REASONING**

a. The conducting shells are equipotential surfaces, so the average magnitude $E$ of the electric field between them is given by $E = \frac{\Delta V}{\Delta s}$ (Equation 19.7a, minus sign omitted), where $\Delta V$ is the magnitude of the potential difference between the shells and $\Delta s$ is the distance between them. This distance is equal to the radius $r_{outer}$ of the outer shell minus the radius $r_{inner}$ of the inner shell.
b. Because this is not a parallel-plate capacitor, we cannot use $C = \frac{\kappa \varepsilon_0 A}{d}$ (Equation 19.10) to determine the capacitance $C$. Instead, we will make use of $q = CV$ (Equation 19.8), where $q$ is the magnitude of the charge on one of the cylindrical shells and $V$ is the magnitude of the potential difference between them. The value of $V$ is equal to the value $\Delta V$ found in part (a).

**SOLUTION**

a. Solving $E = \frac{\Delta V}{\Delta s}$ (Equation 19.7a, minus sign omitted) for $\Delta V$, we find

$$\Delta V = E\Delta s = E(r_{\text{outer}} - r_{\text{inner}}) = \left(4.2 \times 10^4 \text{ V/m}\right) \left(2.50 \times 10^{-3} \text{ m} - 2.35 \times 10^{-3} \text{ m}\right) = 6.3 \text{ V}$$

b. Solving $q = CV$ (Equation 19.8) for $C$, and noting that $V$, the potential difference between the shells is identical to the potential difference $\Delta V = 6.3 \text{ V}$ found in (a), we obtain

$$C = \frac{q}{V} = \frac{1.7 \times 10^{-10}}{6.3 \text{ V}} = 2.7 \times 10^{-11} \text{ F}$$

53. **REASONING AND SOLUTION** The charge on the empty capacitor is $q_0 = C_0 V_0$. With the dielectric in place, the charge remains the same. However, the new capacitance is $C = \kappa C_0$ and the new voltage is $V$. Thus,

$$q_0 = CV = \kappa C_0 V = C_0 V_0$$

Solving for the new voltage yields

$$V = V_0 / \kappa = (12.0 \text{ V}) / 2.8 = 4.3 \text{ V}$$

The potential difference is $12.0 - 4.3 = 7.7 \text{ V}$. The change in potential is a **decrease**.

54. **REASONING** The charge $q$ stored on the plates of a capacitor connected to a battery of voltage $V$ is $q = CV$ (Equation 19.8). The capacitance $C$ is $C = \frac{\kappa \varepsilon_0 A}{d}$ (Equation 19.10), where $\kappa$ is the dielectric constant of the material between the plates, $\varepsilon_0$ is the permittivity of free space, $A$ is the area of each plate, and $d$ is the distance between the plates. Once the capacitor is charged and disconnected from the battery, there is no way for the charge on the plates to change. Therefore, as the distance between the plates is doubled, the charge $q$ must remain constant. However, Equation 19.10 indicates that the capacitance is inversely proportional to the distance $d$, so the capacitance decreases as the distance increases. In Equation 19.8, as $C$ decreases, the voltage $V$ must increase in order that $q$ remains constant. The voltage increases as a result of the work done in moving the plates farther apart. In solving this problem, we will apply Equations 19.8 and 19.10 to the capacitor twice, once with the smaller and once with the larger value of the distance between the plates.
ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL

**SOLUTION** Using $q = CV$ (Equation 19.8) and $C = \frac{\kappa \varepsilon_0 A}{d}$ (Equation 19.10), we can express the charge on the capacitor as follows:

$$q = CV = \left(\frac{\kappa \varepsilon_0 A}{d}\right)V = \frac{\varepsilon_0 AV}{d}$$

where we have made use of the fact that $\kappa = 1$, since the capacitor is empty. Applying this result to the capacitor with smaller and larger values of the distance $d$, we have

$$q = \frac{\varepsilon_0 AV_{\text{smaller}}}{d_{\text{smaller}}} \quad \text{and} \quad q = \frac{\varepsilon_0 AV_{\text{larger}}}{d_{\text{larger}}}$$

Since $q$ is the same in each of these expressions, it follows that

$$\frac{\varepsilon_0 AV_{\text{smaller}}}{d_{\text{smaller}}} = \frac{\varepsilon_0 AV_{\text{larger}}}{d_{\text{larger}}} \quad \text{or} \quad \frac{V_{\text{smaller}}}{d_{\text{smaller}}} = \frac{V_{\text{larger}}}{d_{\text{larger}}}$$

Thus, we find that the voltage increases to a value of

$$V_{\text{larger}} = V_{\text{smaller}} \left(\frac{d_{\text{larger}}}{d_{\text{smaller}}}\right) = (9.0 \text{ V}) \left(\frac{2d_{\text{smaller}}}{d_{\text{larger}}}\right) = 18 \text{ V}$$

55. [SSM REASONING] If we assume that the motion of the proton and the electron is horizontal in the $+x$ direction, the motion of the proton is determined by Equation 2.8, $x = v_0 t + \frac{1}{2} a_p t^2$, where $x$ is the distance traveled by the proton, $v_0$ is its initial speed, and $a_p$ is its acceleration. If the distance between the capacitor places is $d$, then this relation becomes $\frac{1}{2} d = v_0 t + \frac{1}{2} a_p t^2$, or

$$d = 2v_0 t + a_p t^2 \quad (1)$$

We can solve Equation (1) for the initial speed $v_0$ of the proton, but, first, we must determine the time $t$ and the acceleration $a_p$ of the proton. Since the proton strikes the negative plate at the same instant the electron strikes the positive plate, we can use the motion of the electron to determine the time $t$.

For the electron, $\frac{1}{2} d = \frac{1}{2} a_e t^2$, where we have taken into account the fact that the electron is released from rest. Solving this expression for $t$ we have $t = \sqrt{\frac{d}{a_e}}$. Substituting this expression into Equation (1), we have

$$d = 2v_0 \sqrt{\frac{d}{a_e}} + \left(\frac{a_p}{a_e}\right) d \quad (2)$$
The accelerations can be found by noting that the magnitudes of the forces on the electron and proton are equal, since these particles have the same magnitude of charge. The force on the electron is \( F = eE = eV/d \), and the acceleration of the electron is, therefore,

\[
a_e = \frac{F}{m_e} = \frac{eV}{m_ed} \tag{3}
\]

Newton's second law requires that \( m_e a_e = m_p a_p \), so that

\[
a_p = \frac{m_e}{m_p} a_e \tag{4}
\]

Combining Equations (2), (3) and (4) leads to the following expression for \( v_0 \), the initial speed of the proton:

\[
v_0 = \frac{1}{2} \left( 1 - \frac{m_e}{m_p} \right) \sqrt{\frac{eV}{m_e}}
\]

**SOLUTION** Substituting values into the expression above, we find

\[
v_0 = \frac{1}{2} \left( 1 - \frac{9.11 \times 10^{-31} \text{kg}}{1.67 \times 10^{-27} \text{kg}} \right) \sqrt{\frac{(1.60 \times 10^{-19} \text{C})(175 \text{ V})}{9.11 \times 10^{-31} \text{kg}}} = 2.77 \times 10^6 \text{ m/s}
\]

56. **REASONING** The electric potential energy stored in the capacitor is given by Energy \( = \frac{1}{2} CV^2 \) (Equation 19.11b), where \( C \) is the capacitance of the capacitor. We choose Equation 19.11b to express the energy because it contains the potential difference \( V \) across the plates of the capacitor. This quantity is related to the magnitude \( E \) of the electric field between the capacitor’s plates via \( E = \frac{V}{d} \) (Equation 19.7b), where \( d \) is the distance between the plates. We will use Equations 19.11b and 19.7b to obtain an expression for the energy stored in the capacitor in terms of the magnitude \( E \) of the electric field between its plates and its capacitance \( C \). The capacitance itself is given by \( C = \frac{\kappa \varepsilon_0 A}{d} \) (Equation 19.10), where \( \kappa \) is the dielectric constant of the material between the plates, \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \) is the permittivity of free space, and \( A \) is the surface area of one plate. The dielectric constant for air is, to two significant figures, \( \kappa_{\text{air}} = 1.0 \), and that for neoprene rubber is \( \kappa_{\text{neo}} = 6.7 \) (see Table 19.1). We note that the plate area \( A \) and the plate separation \( d \) are not given, but are identical for both capacitors.

**SOLUTION** Solving \( E = \frac{V}{d} \) (Equation 19.7b) for \( V \) yields \( V = Ed \). Substituting this into Energy \( = \frac{1}{2} CV^2 \) (Equation 19.11b), we obtain
Energy = $\frac{1}{2} CV^2 = \frac{1}{2} C(Ed)^2 = \frac{1}{2} CE^2 d^2$

(1)

Substituting $C = \frac{\kappa\varepsilon_0 A}{d}$ (Equation 19.10) into Equation (1), then, yields

Energy = $\frac{1}{2} CE^2 d^2 = \frac{1}{2} \left( \frac{\kappa\varepsilon_0 A}{d} \right) E^2 d^2 = \frac{1}{2} \kappa\varepsilon_0 A E^2 d$

(2)

From Equation (2), we obtain expressions for the energy stored in each type of capacitor:

Energy\_{\text{air}} = \frac{1}{2} \kappa_{\text{air}} \varepsilon_0 A E^2_{\text{air}} d \quad \text{and} \quad \text{Energy\_{\text{neo}}} = \frac{1}{2} \kappa_{\text{neo}} \varepsilon_0 A E^2_{\text{neo}} d

(3)

Taking the ratio of Equations (3) eliminates the unknown plate area $A$ and plate separation $d$, leaving

\[
\frac{\text{Energy\_{\text{neo}}}}{\text{Energy\_{\text{air}}}} = \frac{\frac{1}{2} \kappa_{\text{neo}} \varepsilon_0 E^2_{\text{neo}} d}{\frac{1}{2} \kappa_{\text{air}} \varepsilon_0 E^2_{\text{air}} d} = \frac{\kappa_{\text{neo}} E^2_{\text{neo}}}{\kappa_{\text{air}} E^2_{\text{air}}}
\]

(4)

Solving Equation (4) for the energy in the neoprene-rubber-filled capacitor, we find that

Energy\_{\text{neo}} = \left[ \frac{\kappa_{\text{neo}} E^2_{\text{neo}}}{\kappa_{\text{air}} E^2_{\text{air}}} \right] \text{Energy\_{\text{air}}} = \left( \frac{6.7 \times 10^7}{1.0 \times 3.0 \times 10^6} \right) \left( 0.075 \text{ J} \right) = 8.0 \text{ J}

57. **SSM REASONING AND SOLUTION** The capacitance is given by

\[
C = \frac{k\varepsilon_0 A}{d} = \frac{5 \left( 8.85 \times 10^{-12} \text{ F/m} \right) \left( 5 \times 10^{-6} \text{ m}^2 \right)}{1 \times 10^{-8} \text{ m}} = 2 \times 10^{-8} \text{ F}
\]

58. **REASONING** The maximum operation time is the energy used by the shaver divided by the rate of usage, which is given. The energy that the shaver uses is the energy carried by the charge that passes between the terminals of the battery. This energy is the charge times the battery voltage. The charge can be determined from the number of particles specified and the charge on each particle.

**SOLUTION** The energy used by the shaver is that obtained by the electric charge as it passes from the positive terminal of the battery (terminal $A$), where the electric potential energy $EPE_A$ is higher, to the negative terminal (terminal $B$), where the electric potential energy $EPE_B$ is lower. The energy acquired by the charge is $EPE_A - EPE_B$. The rate at which the shaver uses energy is the power $P$, which is given as 4.0 W. According to Equation 6.10b, the power is the energy $EPE_A - EPE_B$ divided by the time $t$, so that $P = \frac{EPE_A - EPE_B}{t}$. Solving for the time gives
According to Equation 19.4, this total energy is \( E_{PE A} - E_{PE B} = q_0 (V_A - V_B) \), where \( q_0 \) is the charge and \( V_A - V_B \) is the electric potential difference between the battery terminals. Substituting this expression into Equation (1) gives

\[
t = \frac{E_{PE A} - E_{PE B}}{P} = \frac{q_0 (V_A - V_B)}{P}
\]  

(2)

The charge \( q_0 \) is the number \( n \) of charged particles times the charge \((e=1.6\times10^{-19} \text{ C})\) on each particle; in other words, \( q_0 = ne \). Substituting this value for \( q_0 \) into Equation (2), we find that

\[
t = \frac{q_0 (V_A - V_B)}{P} = \frac{ne (V_A - V_B)}{P} = \frac{3.0\times10^{22} \left(1.6\times10^{-19} \text{ C}\right)(1.5 \text{ V})}{4.0 \text{ W}} = 1800 \text{ s}
\]

59. **REASONING**  The electric potential difference between the two points is \( V_B - V_A = \frac{ke}{r_B} - \frac{ke}{r_A} \) (Equation 19.5). We can use this expression directly to calculate the electric potential difference.

**SOLUTION**  According to Equation 19.5, the electric potential difference is

\[
V_B - V_A = \frac{ke}{r_B} - \frac{ke}{r_A} = kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)
\]

\[
= \left(8.99\times10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \left(2.1\times10^{-9} \text{ C}\right) \left(\frac{1}{0.50 \text{ m}} - \frac{1}{0.25 \text{ m}}\right) = 38 \text{ V}
\]

60. **REASONING**  The net work \( W_{AB} \) done by the electric force on the point charge \( q_0 \) as it moves from \( A \) to \( B \) is proportional to the potential difference \( V_B - V_A \) between those positions, according to \( V_B - V_A = \frac{-W_{AB}}{q_0} \) (Equation 19.4). Positions \( A \) and \( B \) are on different equipotential surfaces, so we will read the potentials \( V_B \) and \( V_A \) from the drawing. We will employ the same procedure to solve part (b).

**SOLUTION**

a. Solving \( V_B - V_A = \frac{-W_{AB}}{q_0} \) (Equation 19.4) for \( W_{AB} \), we obtain
\[ W_{AB} = -q_0(V_B - V_A) \]  

(1)

From the drawing, we see that \( V_A = +350.0 \text{ V} \) and \( V_B = +550.0 \text{ V} \). Therefore, from Equation (1),

\[ W_{AB} = -\left( +2.8 \times 10^{-7} \text{ C} \right) \left( 550.0 \text{ V} - 350.0 \text{ V} \right) = -5.6 \times 10^{-5} \text{ J} \]

b. Positions \( A \) and \( C \) are both on the +350.0-V equipotential surface. Adapting Equation (1), then, we obtain

\[ W_{AC} = -q_0(V_C - V_A) = -\left( +2.8 \times 10^{-7} \text{ C} \right) \left( 350.0 \text{ V} - 350.0 \text{ V} \right) = 0 \text{ J} \]

61. **SSM REASONING** The work \( W_{AB} \) done by an electric force in moving a charge \( q_0 \) from point \( A \) to point \( B \) and the electric potential difference \( V_B - V_A \) are related according to

\[ V_B - V_A = \frac{-W_{AB}}{q_0} \quad \text{(Equation 19.4)} \]

This expression can be solved directly for \( q_0 \).

**SOLUTION** Solving Equation 19.4 for the charge \( q_0 \) gives

\[ q_0 = \frac{-W_{AB}}{V_B - V_A} = \frac{W_{AB}}{V_A - V_B} = \frac{2.70 \times 10^{-3} \text{ J}}{50.0 \text{ V}} = 5.40 \times 10^{-5} \text{ C} \]

62. **REASONING** Equation 19.10 gives the capacitance as \( C = \kappa \varepsilon_0 A/d \), where \( \kappa \) is the dielectric constant, and \( A \) and \( d \) are, respectively, the plate area and separation. Other things being equal, the capacitor with the larger plate area has the greater capacitance. The diameter of the circle equals the length of a side of the square, so the circle fits within the square. The square, therefore, has the larger area, and the capacitor with the square plates would have the greater capacitance.

To make the capacitors have equal capacitances, the dielectric constant must compensate for the larger area of the square plates. Therefore, since capacitance is proportional to the dielectric constant, the capacitor with square plates must contain a dielectric material with a smaller dielectric constant. Thus, the capacitor with circular plates contains the material with the greater dielectric constant.

**SOLUTION** The area of the circular plates is \( A_{\text{circle}} = \pi \left( \frac{1}{2} L \right)^2 \), while the area of the square plates is \( A_{\text{square}} = L^2 \). Using these areas and applying Equation 19.10 to each capacitor gives

\[ C = \kappa_{\text{circle}} \varepsilon_0 \frac{\pi \left( \frac{1}{2} L \right)^2}{d} \quad \text{and} \quad C = \kappa_{\text{square}} \varepsilon_0 \frac{L^2}{d} \]
Since the values for \( C \) are the same, we have

\[
\frac{\kappa_{\text{circle}} \varepsilon_0 \left( \frac{1}{2} L \right)^2}{d} = \frac{\kappa_{\text{square}} \varepsilon_0 L^2}{d} \quad \text{or} \quad \frac{\kappa_{\text{circle}}}{\kappa_{\text{square}}} = \frac{4}{\pi}
\]

Thus,

\[
\kappa_{\text{circle}} = \frac{4 \kappa_{\text{square}}}{\pi} = \frac{4(3.00)}{\pi} = 3.82
\]

63. **REASONING** The radius \( R \) of the circle is the distance between each charge and the center of the circle, where we know the total electric potential \( V \). The total electric potential is the sum of the contributions \( V_1, V_2, \) and \( V_3 \) made by the three charges. We will use \( V = \frac{kq}{r} \) (Equation 19.6), where \( k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \) and \( r = R \) to determine each of the three contributions to the total electric potential. Summing the contributions and equating the result to \( V = -2100 \text{ V} \), we will obtain the radius \( R \) of the circle.

**SOLUTION** From \( V = \frac{kq}{r} \) (Equation 19.6), the total potential \( V \) at the center of the circle is

\[
V = V_1 + V_2 + V_3 = \frac{kq_1}{R} + \frac{kq_2}{R} + \frac{kq_3}{R} = \frac{k(q_1 + q_2 + q_3)}{R}
\]

Solving Equation (1) for \( R \), we find that

\[
R = \frac{k(q_1 + q_2 + q_3)}{V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-5.8 \times 10^{-9} \text{ C} - 9.0 \times 10^{-9} \text{ C} + 7.3 \times 10^{-9} \text{ C})}{-2100 \text{ V}} = 0.032 \text{ m}
\]

64. **REASONING** The outside force is the only nonconservative force acting on the particle. Therefore, the work \( W_{nc} \) done by the outside force changes the total mechanical energy \( E = KE + EPE \) of the particle according to \( W_{nc} = \Delta KE + \Delta EPE \) (Equation 6.7b), where \( \Delta KE = KE_B - KE_A \) is the difference between the kinetic energy of the particle at equipotential surface \( B \) and that at equipotential surface \( A \), and \( \Delta EPE = EPE_B - EPE_A \) is the corresponding difference between the electric potential energies of the particle. The kinetic energy of the particle is found from its mass \( m \) and speed \( v \) via \( KE = \frac{1}{2} mv^2 \) (Equation 6.2):

\[
KE_A = \frac{1}{2} m v_A^2 \quad \text{and} \quad KE_B = \frac{1}{2} m v_B^2
\]

The electric potential energy of the particle is given by \( EPE = qV \) (Equation 19.3), where \( q \) is the charge of the particle, and \( V \) is the potential at an equipotential surface. Therefore, we have that
EPE_A = qV_A and EPE_B = qV_B  \hspace{1cm} (2)

**SOLUTION** Substituting \( \Delta KE = KE_B - KE_A \) and \( \Delta EPE = EPE_B - EPE_A \) into the relation \( W_{nc} = \Delta KE + \Delta EPE \) (Equation 6.7b), we obtain

\[
W_{nc} = \Delta KE + \Delta EPE = (KE_B - KE_A) + (EPE_B - EPE_A) \hspace{1cm} (3)
\]

Substituting Equations (1) and (2) into Equation (3) yields

\[
W_{nc} = \left( \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 \right) + (qV_B - qV_A) = \frac{1}{2}m(v_B^2 - v_A^2) + q(V_B - V_A)
\]

Therefore, the work done by the outside force in moving the particle from A to B is

\[
W_{nc} = \frac{1}{2} \left( 5.00 \times 10^{-2} \text{ kg} \right) \left[ (3.00 \text{ m/s})^2 - (2.00 \text{ m/s})^2 \right] + (4.00 \times 10^{-5} \text{ C})(7850 \text{ V} - 5650 \text{ V})
\]

\[
= 0.213 \text{ J}
\]

**REASONING** The charge \( q_0 \) on the empty capacitor is related to its capacitance \( C_0 \) and the potential difference \( V \) across the plates by \( q_0 = C_0V \) (Equation 19.8). The charge \( q \) on the capacitor filled with a dielectric is related to its capacitance \( C \) and the potential difference \( V \) across the plates by \( q = CV \). We know that the presence of the dielectric increases the capacitance such that \( C = \kappa C_0 \), where \( \kappa \) is the dielectric constant (see Equation 19.10 and the discussion that follows). Since the magnitude of the surface charge on the dielectric is equal to the difference in the charge on the plates with and without the dielectric, we have

\[
q - q_0 = CV - C_0V = (\kappa C_0)V - C_0V = C_0V(\kappa - 1)
\]

**SOLUTION** Since \( C_0 = 3.2 \times 10^{-6} \text{ F}, \ V = 12 \text{ V}, \) and \( \kappa = 4.5 \), the magnitude of the surface charge is

\[
q - q_0 = C_0V(\kappa - 1) = (3.2 \times 10^{-6} \text{ F})(12 \text{ V})(4.5 - 1) = 1.3 \times 10^{-4} \text{ C}
\]

**REASONING** The drawing at the right shows the charges \( q_1 \) (positive) and \( q_2 \) (negative). Also shown on the line through the charges are the two spots A and B where the total electric potential is zero and the spot C where the net electric field is zero. The fact that C is to the right of \( q_2 \) means that the magnitude of \( q_2 \) is smaller than the magnitude of \( q_1 \). This follows from the fact that the electric field of the nearby charge (smaller magnitude) balances to zero the field of the more distant charge (larger magnitude).
It is because the magnitude of \( q_2 \) is smaller than the magnitude of \( q_1 \) that spots A and B are shown closer to \( q_2 \) than to \( q_1 \).

**SOLUTION** The potential \( V \) created by a point charge \( q \) at a distance \( r \) away from the charge is \( V = kq / r \) (Equation 19.6). Using this equation for the potential contributed by \( q_1 \) and \( q_2 \) at the spots A and B, we have (suppressing units for clarity)

\[
\frac{kq_1}{3.00 - d_A} + \frac{kq_2}{d_A} = 0 \quad \text{(1)} \quad \text{and} \quad \frac{kq_1}{3.00 + d_B} + \frac{kq_2}{d_B} = 0 \quad \text{(2)}
\]

These two equations cannot be solved for the distances \( d_A \) and \( d_B \), since they contain the charges \( q_2 \) and \( q_1 \). To deal with the charges, we turn now to the fact that at the spot C the net electric field is zero. The electric field of magnitude \( E \) created by a point charge \( q \) at a distance \( r \) away from the charge is \( E = kq / r^2 \) (Equation 18.3), where \( |q| \) is the magnitude of the charge. At C the field from \( q_1 \) points to the right and the field from \( q_2 \) points to the left. Since the net field is zero, the magnitudes of the two contributions must be equal. Thus, using Equation 18.3 for each contribution and again suppressing units for clarity, we have

\[
E_1 = E_2 \quad \text{or} \quad \frac{k|q_1|}{(3.00 + 1.00)^2} = \frac{k|q_2|}{(1.00)^2} \quad \text{or} \quad |q_1| = 16.0|q_2|
\]

Since \( q_1 \) is positive and \( q_2 \) is negative, this result can be written as \( q_1 = -16.0q_2 \). Substituting this result into Equations (1) and (2), we obtain

\[
\frac{k(-16.0q_2)}{3.00 - d_A} + \frac{kq_2}{d_A} = 0 \quad \text{(3)} \quad \text{and} \quad \frac{k(-16.0q_2)}{3.00 + d_B} + \frac{kq_2}{d_B} = 0 \quad \text{(4)}
\]

It is now possible to eliminate \( q_2 \) from both Equations (3) and (4), which gives

\[
\frac{-16.0}{3.00 - d_A} + \frac{1}{d_A} = 0 \quad \text{or} \quad d_A = 0.176 \text{ m to the left of the negative charge}
\]

\[
\frac{-16.0}{3.00 + d_B} + \frac{1}{d_B} = 0 \quad \text{or} \quad d_B = 0.200 \text{ m to the right of the negative charge}
\]

67. **SSM REASONING** According to Equation 19.10, the capacitance of a parallel plate capacitor filled with a dielectric is \( C = \kappa \varepsilon_0 A / d \), where \( \kappa \) is the dielectric constant, \( A \) is the area of one plate, and \( d \) is the distance between the plates.

From the definition of capacitance (Equation 19.8), \( q = CV \). Thus, using Equation 19.10, we see that the charge \( q \) on a parallel plate capacitor that contains a dielectric is given by
\[ q = (\kappa \varepsilon_0 A/d)V. \] Since each dielectric occupies one-half of the volume between the plates, the area of each plate in contact with each material is \( A/2 \). Thus,

\[ q_1 = \frac{\kappa_1 \varepsilon_0 (A/2)}{d}V = \frac{\kappa_1 \varepsilon_0 A}{2d}V \quad \text{and} \quad q_2 = \frac{\kappa_2 \varepsilon_0 (A/2)}{d}V = \frac{\kappa_2 \varepsilon_0 A}{2d}V \]

According to the problem statement, the total charge stored by the capacitor is

\[ q_1 + q_2 = CV \quad \text{(1)} \]

where \( q_1 \) and \( q_2 \) are the charges on the plates in contact with dielectrics 1 and 2, respectively.

Using the expressions for \( q_1 \) and \( q_2 \) above, Equation (1) becomes

\[ CV = \frac{\kappa_1 \varepsilon_0 A}{2d}V + \frac{\kappa_2 \varepsilon_0 A}{2d}V = \frac{\kappa_1 \varepsilon_0 A + \kappa_2 \varepsilon_0 A}{2d}V = \frac{(\kappa_1 + \kappa_2) \varepsilon_0 A}{2d}V \]

This expression can be solved for \( C \).

**SOLUTION** Solving for \( C \), we obtain

\[ C = \frac{\varepsilon_0 A(\kappa_1 + \kappa_2)}{2d} \]

68. **REASONING** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the final speed of each particle.

**SOLUTION** For two points, \( A \) and \( B \), along the motion, the conservation of energy is

\[
\frac{1}{2}mv_{1,A}^2 + \frac{1}{2}mv_{2,A}^2 + \frac{kq_1q_2}{r_A} = \frac{1}{2}mv_{1,B}^2 + \frac{1}{2}mv_{2,B}^2 + \frac{kq_1q_2}{r_B} \]

Setting \( v_{1,A} = v_{2,A} = 0 \) since the particles are initially at rest, and letting \( r_B = \frac{1}{3}r_A \), the conservation of energy equation becomes

\[
\frac{1}{2}mv_{1,B}^2 + \frac{1}{2}mv_{2,B}^2 = -\frac{2kq_1q_2}{r_A} \quad \text{(1)}
\]

This equation cannot be solved for \( v_{1,B} \) because the final speed \( v_{2,B} \) of the second particle is not known. To find this speed, we will use the conservation of linear momentum:
\[ m v_{1A} + m v_{2A} = m v_{1B} + m v_{2B} \]

Initial linear momentum  Final linear momentum

Setting \( v_{1A} = v_{2A} = 0 \) and solving for \( v_{2B} \) gives \( v_{2B} = -v_{1B} \). Substituting this result into Equation (1) and solving for \( v_{1B} \) yields

\[
v_{1B} = \sqrt{\frac{-2kq_1q_2}{mr_A}}\\
= \sqrt{\frac{-2 \left( 9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( +5.0 \times 10^{-6} \text{ C} \right) \left( -5.0 \times 10^{-6} \text{ C} \right)} {\left( 6.0 \times 10^{-3} \text{ kg} \right) \left( 0.80 \text{ m} \right)}} = 9.7 \text{ m/s}
\]

This is also the speed of \( v_{2B} \).
CHAPTER 20 | ELECTRIC CIRCUITS

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. 1.5 A

2. (c) Ohm’s law states that the voltage \( V \) is directly proportional to the current \( I \), according to \( V = IR \), where \( R \) is the resistance. Thus, a plot of voltage versus current is a straight line that passes through the origin.

3. (e) Since both wires are made from the same material, the resistivity \( \rho \) is the same for each. The resistance \( R \) is given by \( R = \rho \frac{L}{A} \) (Equation 20.3), where \( L \) is the length and \( A \) is the cross-sectional area of the wire. With twice the length and one-half the radius (one-fourth the cross-sectional area), the second wire has \( \frac{L}{A} = \frac{2}{1/4} = 8 \) times the resistance as the first wire.

4. 250 C°

5. (a) Power \( P \) is the current \( I \) times the voltage \( V \) or \( P = IV \) (Equation 20.6a). However, since Ohm’s law applies to a resistance \( R \), the power is also \( P = I^2 R \) (Equation 20.6b) and \( P = \frac{V^2}{R} \) (Equation 20.6c). Therefore, all three of the changes specified leave the power unchanged.

6. 27 W

7. 0.29 A

8. (d) According to Ohm’s law, the voltage across the resistance \( R_1 \) is \( V_1 = IR_1 \). The two resistors are connected in series, and their equivalent resistance is, therefore, \( R_1 + R_2 \). According to Ohm’s law, the current in the circuit is \( I = \frac{V}{R_1 + R_2} \). Substituting this expression into the expression for \( V_1 \) gives \( V_1 = \left( \frac{V}{R_1 + R_2} \right) R_1 \).
9. (b) The series connection has an equivalent resistance of \( R_s = R + R = 2R \). The parallel connection has an equivalent resistance that can be determined from \( \frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \).

Therefore, it follows that \( R_p = \frac{1}{2} R \). The ratio of these values is \( \frac{R_s}{R_p} = \frac{2R}{\frac{1}{2} R} = 4 \).

10. (e) Since the two resistors are connected in parallel across the battery terminals, the same voltage is applied to each. Thus, according to Ohm’s law, the current in each resistor is inversely proportional to the resistance, so that \( \frac{I_1}{I_2} = \frac{V/R_1}{V/R_2} = \frac{R_2}{R_1} \).

11. 0.019 A

12. (c) In arrangement B the two resistors in series have a combined resistance of \( 2R \). Each series combination is in parallel, so that the reciprocal of the equivalent resistance is \( \frac{1}{R_{eq, B}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R} \), or \( R_{eq, B} = R \). Following the technique outlined in Section 20.8, we also find that \( R_{eq, C} = \frac{3}{4} R \) and \( R_{eq, A} = \frac{2}{5} R \).

13. (d) The internal resistance \( r \) of the battery and the resistance \( R \) are in series, so that the current from the battery can be calculated via Ohm’s law as \( I = \frac{V}{R + r} \). The voltage between the terminals is, then, \( \frac{1}{2} V = IR = \frac{VR}{R + r} \). This result can be solved to show that \( R = r \).

14. (b) Kirchhoff’s junction rule states that the sum of the magnitudes of the currents directed into a junction equals the sum of the magnitudes of the currents directed out of the junction. Here, this rule implies that \( I_1 + I_3 = I_2 \).

15. (d) Once the current in the resistor is drawn, the markings of the plus and minus sign are predetermined. This is because conventional current always flows from a higher toward a lower potential. Thus, since the current \( I_4 \) is directed from right to left, the right side of \( R_4 \) must be marked plus and the left side minus.

16. (b) Kirchhoff’s loop rule states that, around any closed loop, the sum of the potential drops equals the sum of the potential rises.

17. (a) The ammeter must be connected so that the current that flows through the resistor \( R_2 \) also flows through the ammeter. The voltmeter must be connected across the resistor \( R_2 \).
18. (e) The two capacitors in series have an equivalent capacitance \( C_S \) that can be determined from
\[
\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C},
\]
so that \( C_S = \frac{1}{2} C \). This capacitance is in parallel with a capacitance \( C \), so that the total equivalent capacitance is
\[
C_{eq} = \frac{1}{2} C + C = \frac{3}{2} C.
\]

19. (c) The time constant is given by the product of the resistance and the capacitance. Therefore, when the resistance is reduced to one-third of its initial value, the capacitance must be tripled, in order that the time constant remains unchanged.

20. 1.8 s
CHAPTER 20 | ELECTRIC CIRCUITS

PROBLEMS

1. **REASONING** The current $I$ is defined in Equation 20.1 as the amount of charge $\Delta q$ per unit of time $\Delta t$ that flows in a wire. Therefore, the amount of charge is the product of the current and the time interval. The number of electrons is equal to the charge that flows divided by the magnitude of the charge on an electron.

**SOLUTION**

a. The amount of charge that flows is

$$\Delta q = I \Delta t = (18 \text{ A})(2.0 \times 10^{-3} \text{ s}) = 3.6 \times 10^{-2} \text{ C}$$

b. The number of electrons $N$ is equal to the amount of charge divided by $e$, the magnitude of the charge on an electron.

$$N = \frac{\Delta q}{e} = \frac{3.6 \times 10^{-2} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 2.3 \times 10^{17}$$

2. **REASONING** We are given the average current $I$ and its duration $\Delta t$. We will employ $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1) to determine the amount $\Delta q$ of charge delivered to the ground by the lightning flash.

**SOLUTION** Solving $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1) for $\Delta q$, we obtain

$$\Delta q = I\Delta t = (1.26 \times 10^3 \text{ A})(0.138 \text{ s}) = 174 \text{ C}$$

3. **SSM REASONING AND SOLUTION** First determine the total charge delivered to the battery using Equation 20.1:

$$\Delta q = I\Delta t = (6.0 \text{ A})(5.0 \text{ h})\left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 1.1 \times 10^5 \text{ C}$$

To find the energy delivered to the battery, multiply this charge by the energy per unit charge (i.e., the voltage) to get

$$\text{Energy} = (\Delta q)V = (1.1 \times 10^5 \text{ C})(12 \text{ V}) = 1.3 \times 10^6 \text{ J}$$
4. **REASONING** Knowing the resistance $R$ (14 $\Omega$) of the heating element and the voltage $V$ being applied to it (120 V), we can use $V = IR$ (Ohm’s law, Equation 20.2) to determine the current $I$.

**SOLUTION** Using Ohm’s law, we have

$$V = IR \quad \text{or} \quad I = \frac{V}{R} = \frac{120 \text{ V}}{14 \Omega} = 8.6 \text{ A}$$

5. **REASONING**
   a. According to Ohm’s law, the current is equal to the voltage between the cell walls divided by the resistance.

   b. The number of Na$^+$ ions that flow through the cell wall is the total charge that flows divided by the charge of each ion. The total charge is equal to the current multiplied by the time.

   **SOLUTION**
   a. The current is
   
   $$I = \frac{V}{R} = \frac{75 \times 10^{-3} \text{ V}}{5.0 \times 10^9 \Omega} = 1.5 \times 10^{-11} \text{ A}$$
   
   b. The number of Na$^+$ ions is the total charge $\Delta q$ that flows divided by the charge $+e$ on each ion, or $\frac{\Delta q}{e}$. The charge is the product of the current $I$ and the time $\Delta t$, according to Equation 20.1, so that
   
   $$\text{Number of Na}^+ \text{ ions} = \frac{\Delta q}{e} = \frac{I \Delta t}{e} = \frac{(1.5 \times 10^{-11} \text{ A})(0.50 \text{ s})}{1.60 \times 10^{-19} \text{ C}} = 4.7 \times 10^7$$

6. **REASONING AND SOLUTION**
   a. The total charge that can be delivered is
   
   $$\Delta q = (220 \text{ A} \cdot \text{h}) \left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 7.9 \times 10^5 \text{ C}$$

   b. The maximum current is
   
   $$I = \frac{220 \text{ A} \cdot \text{h}}{(38 \text{ min}) \left(\frac{1 \text{ h}}{60 \text{ min}}\right)} = 350 \text{ A}$$

7. **REASONING** According to Ohm’s law, the resistance is the voltage of the battery divided by the current that the battery delivers. The current is the charge divided by the time during which it flows, as stated in Equation 20.1. We know the time, but are not given the charge directly. However, we can determine the charge from the energy delivered to the resistor, because this energy comes from the battery, and the potential difference between the battery
terminals is the difference in electric potential energy per unit charge, according to Equation 19.4. Thus, a 9.0-V battery supplies 9.0 J of energy to each coulomb of charge passing through it. To calculate the charge, then, we need only divide the energy from the battery by the 9.0 V potential difference.

**SOLUTION** Ohm’s law indicates that the resistance $R$ is the voltage $V$ of the battery divided by the current $I$, or $R = V/I$. According to Equation 20.1, the current $I$ is the amount of charge $\Delta q$ divided by the time $\Delta t$, or $I = \Delta q/\Delta t$. Using these two equations, we have

$$R = \frac{V}{I} = \frac{V}{\Delta q/\Delta t} = \frac{V \Delta t}{\Delta q}$$

According to Equation 19.4, the potential difference $\Delta V$ is the difference $\Delta (EPE)$ in the electric potential energy divided by the charge $\Delta q$, or $\Delta V = \frac{\Delta (EPE)}{\Delta q}$. However, it is customary to denote the potential difference across a battery by $V$, rather than $\Delta V$, so $V = \frac{\Delta (EPE)}{\Delta q}$. Solving this expression for the charge gives $\Delta q = \frac{\Delta (EPE)}{V}$. Using this result in the expression for the resistance, we find that

$$R = \frac{V \Delta t}{\Delta q} = \frac{V \Delta t}{\Delta (EPE)/V} = \frac{V^2 \Delta t}{\Delta (EPE)} = \frac{(9.0 \text{ V})^2 (6 \times 3600 \text{ s})}{1.1 \times 10^5 \text{ J}} = 16 \Omega$$

8. **REASONING** Voltage is a measure of energy per unit charge (joules per coulomb). Therefore, when an amount $\Delta q$ of charge passes through the toaster and there is a potential difference across the toaster equal to the voltage $V$ of the outlet, the energy that the charge delivers to the toaster is given by

$$\text{Energy} = V \Delta q$$

(1)

The charge $\Delta q$ that flows through the toaster in a time $\Delta t$ depends upon the magnitude $I$ of the electric current according to $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1). Therefore, we have that

$$\Delta q = I \Delta t$$

(2)

We will employ Ohm’s law, $I = \frac{V}{R}$ (Equation 20.2), to calculate the current in the toaster in terms of the voltage $V$ of the outlet and the resistance $R$ of the toaster.

**SOLUTION** Substituting Equation (2) into Equation (1) yields

$$\text{Energy} = V \Delta q = VI \Delta t$$

(3)
Substituting \( I = \frac{V}{R} \) (Equation 20.2) into Equation (3), we find that

\[
\text{Energy} = VI\Delta t = \left(\frac{V}{R}\right)\Delta t = \frac{V^2}{R} \frac{\Delta t}{\Delta t} = \frac{(120 \text{ V})^2}{14 \Omega} (60.0 \text{ s}) = 6.2 \times 10^4 \text{ J}
\]

9. **SSM REASONING** The number \( N \) of protons that strike the target is equal to the amount of electric charge \( \Delta q \) striking the target divided by the charge \( e \) of a proton, \( N = \frac{\Delta q}{e} \). From Equation 20.1, the amount of charge is equal to the product of the current \( I \) and the time \( \Delta t \). We can combine these two relations to find the number of protons that strike the target in 15 seconds.

The heat \( Q \) that must be supplied to change the temperature of the aluminum sample of mass \( m \) by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of aluminum. The heat is provided by the kinetic energy of the protons and is equal to the number of protons that strike the target times the kinetic energy per proton. Using this reasoning, we can find the change in temperature of the block for the 15 second-time interval.

**SOLUTION**

a. The number \( N \) of protons that strike the target is

\[
N = \frac{\Delta q}{e} = \frac{I \Delta t}{e} = \frac{(0.50 \times 10^{-6} \text{ A})(15 \text{ s})}{1.6 \times 10^{-19} \text{ C}} = 4.7 \times 10^{13}
\]

b. The amount of heat \( Q \) provided by the kinetic energy of the protons is

\[
Q = (4.7 \times 10^{13} \text{ protons})(4.9 \times 10^{-12} \text{ J/proton}) = 230 \text{ J}
\]

Since \( Q = cm\Delta T \) and since Table 12.2 gives the specific heat of aluminum as

\( c = 9.00 \times 10^2 \text{ J/(kg·C°)} \), the change in temperature of the block is

\[
\Delta T = \frac{Q}{cm} = \frac{230 \text{ J}}{9.00 \times 10^2 \frac{\text{J}}{\text{kg·C°}}(15 \times 10^{-3} \text{ kg})} = 17 \text{ C°}
\]

10. **REASONING**

a. The resistance \( R \) of a piece of material is related to its length \( L \) and cross-sectional area \( A \) by Equation 20.3, \( R = \rho L/A \), where \( \rho \) is the resistivity of the material. In order to rank the resistances, we need to evaluate \( L \) and \( A \) for each configuration in terms of \( L_0 \), the unit of length.
Chapter 20 Problems

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Resistance

<table>
<thead>
<tr>
<th></th>
<th>Resistance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>( R = \rho \frac{4L_0}{L_0 \times 2L_0} = \rho \left( \frac{2}{L_0} \right) )</td>
<td>1</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>( R = \rho \frac{L_0}{2L_0 \times 4L_0} = \rho \left( \frac{1}{8L_0} \right) )</td>
<td>3</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>( R = \rho \frac{2L_0}{L_0 \times 4L_0} = \rho \left( \frac{1}{2L_0} \right) )</td>
<td>2</td>
</tr>
</tbody>
</table>

Therefore, we expect that **a** has the largest resistance, followed by **c**, and then by **b**.

b. Equation 20.2 states that the current \( I \) is equal to the voltage \( V \) divided by the resistance, \( I = \frac{V}{R} \). Since the current is inversely proportional to the resistance, the largest current arises when the resistance is smallest, and vice versa. Thus, we expect that **b** has the largest current, followed by **c**, and then by **a**.

**SOLUTION**

a. The resistances can be found by using the results from the **REASONING**:

\( a \)  \( R = \rho \left( \frac{2}{L_0} \right) \left( 1.50 \times 10^{-2} \, \Omega \cdot m \right) \left( \frac{2}{5.00 \times 10^{-2} \, m} \right) = 0.600 \, \Omega \)

\( b \)  \( R = \rho \left( \frac{1}{8L_0} \right) \left( 1.50 \times 10^{-2} \, \Omega \cdot m \right) \left( \frac{1}{8 \times 5.00 \times 10^{-2} \, m} \right) = 0.0375 \, \Omega \)

\( c \)  \( R = \rho \left( \frac{1}{2L_0} \right) \left( 1.50 \times 10^{-2} \, \Omega \cdot m \right) \left( \frac{1}{2 \times 5.00 \times 10^{-2} \, m} \right) = 0.150 \, \Omega \)

b. The current in each case is given by Equation 20.2, where the value of the resistance is obtained from part (a):

\( a \)  \( I = \frac{V}{R} = \frac{3.00 \, V}{0.600 \, \Omega} = 5.00 \, A \)

\( b \)  \( I = \frac{V}{R} = \frac{3.00 \, V}{0.0375 \, \Omega} = 80.0 \, A \)

\( c \)  \( I = \frac{V}{R} = \frac{3.00 \, V}{0.150 \, \Omega} = 20.0 \, A \)

______________________________________________________________________________

11. **REASONING** The resistance \( R \) of a wire that has a length \( L \) and a cross-sectional area \( A \) is given by Equation 20.3 as \( R = \rho \frac{L}{A} \). Both wires have the same length and cross-sectional area. Only the resistivity \( \rho \) of the wire differs, and Table 20.1 gives the following values:
\[ \rho_{\text{Aluminum}} = 2.82 \times 10^{-8} \, \Omega \cdot \text{m} \text{ and } \rho_{\text{Copper}} = 1.72 \times 10^{-8} \, \Omega \cdot \text{m}. \] Applying Equation 20.3 to both wires and dividing the two equations will allow us to eliminate the unknown length and cross-sectional area algebraically and solve for the resistance of the copper wire.

**SOLUTION** Applying Equation 20.3 to both wires gives

\[ R_{\text{Copper}} = \frac{\rho_{\text{Copper}} L}{A} \text{ and } R_{\text{Aluminum}} = \frac{\rho_{\text{Aluminum}} L}{A} \]

Dividing these two equations, eliminating \( L \) and \( A \) algebraically, and solving the result for \( R_{\text{Copper}} \) give

\[ R_{\text{Copper}} = R_{\text{Aluminum}} \left( \frac{\rho_{\text{Copper}}}{\rho_{\text{Aluminum}}} \right) = (0.20 \Omega) \left( \frac{1.72 \times 10^{-8} \, \Omega \cdot \text{m}}{2.82 \times 10^{-8} \, \Omega \cdot \text{m}} \right) = 0.12 \Omega \]

**12. REASONING** The materials in Table 20.1 are listed according to their resistivities, so we need to find the resistivity of this material. The resistivity depends on the resistance \( R \), the length \( L \), and the cross-sectional \( A \) of the wire (see Equation 20.3). We know the length of the wire, and the cross-sectional area can be found from the radius \( r \) since \( A = \pi r^2 \). The resistance depends on the voltage \( V \) and current \( I \) through the relation \( R = \frac{V}{I} \), (Equation 20.2).

**SOLUTION** The resistivity is

\[ \rho = \frac{RA}{L} = \frac{R(\pi r^2)}{L} \quad (20.3) \]

We also know that \( R = \frac{V}{I} \), so the resistivity becomes

\[ \rho = \frac{\left( \frac{V}{I} \right) \pi r^2}{L} = \frac{V \pi r^2}{IL} = \frac{(0.0320 \, \text{V}) \pi (1.03 \times 10^{-3} \, \text{m})^2}{(1.35 \, \text{A})(2.80 \, \text{m})} = 2.82 \times 10^{-8} \, \Omega \cdot \text{m} \]

An inspection of Table 20.1 shows that the material that has this resistivity is aluminum.

**13. REASONING AND SOLUTION** Solving Equation 20.5 for \( \alpha \) yields

\[ \alpha = \frac{R}{R_0} \left( \frac{T_0 - 1}{T - T_0} \right) = \frac{43.7 \, \Omega}{38.0 \, \Omega} \left( \frac{55 \, ^\circ\text{C} - 25 \, ^\circ\text{C}}{55 \, ^\circ\text{C} - 25 \, ^\circ\text{C}} \right) = 0.0050 \left( ^\circ\text{C} \right)^{-1} \]
14. **REASONING** The resistance $R$ of the spooled wire decreases as its length $L$ decreases, according to $R = \rho \frac{L}{A}$ (Equation 20.3). The resistivity $\rho$ and cross-sectional area $A$ of the wire do not change. Because the same battery is used, the potential difference $V$ across the wire is the same in both cases. Therefore, $R = \frac{V}{I}$ (Equation 20.2) explains why the current $I$ increases as the resistance $R$ of the wire decreases.

**SOLUTION** Solving $R = \rho \frac{L}{A}$ (Equation 20.3) for $L$, we obtain $L = \frac{RA}{\rho}$. Thus, the initial length $L_0$ and final length $L_f$ of the wire are given by

$$L_0 = \frac{R_0 A}{\rho} \quad \text{and} \quad L_f = \frac{R_f A}{\rho} \quad \text{(1)}$$

where $R_0$ is the initial resistance, and $R_f$ the final resistance, of the wire. Taking the ratio of Equations (1) eliminates the unknown quantities $A$ and $\rho$, allowing us to solve for $L_f$ in terms of $L_0$ and the initial and final resistances:

$$\frac{L_f}{L_0} = \frac{\frac{R_f A}{\rho}}{\frac{R_0 A}{\rho}} = \frac{R_f}{R_0} \quad \text{or} \quad L_f = \left(\frac{R_f}{R_0}\right) L_0 \quad \text{(2)}$$

From $R = \frac{V}{I}$ (Equation 20.2), the initial resistance $R_0$ and final resistance $R_f$ of the spooled wire are

$$R_0 = \frac{V}{I_0} \quad \text{and} \quad R_f = \frac{V}{I_f} \quad \text{(3)}$$

Substituting Equations (3) into Equation (2) yields

$$L_f = \left(\frac{\frac{V}{I_f}}{\frac{V}{I_0}}\right) L_0 = \left(\frac{I_0}{I_f}\right) L_0 = \left(\frac{2.4 \text{ A}}{3.1 \text{ A}}\right) (75 \text{ m}) = 58 \text{ m}$$

15. **SSM REASONING** The resistance of a metal wire of length $L$, cross-sectional area $A$ and resistivity $\rho$ is given by Equation 20.3: $R = \rho L / A$. Solving for $A$, we have $A = \rho L / R$. We can use this expression to find the ratio of the cross-sectional area of the aluminum wire to that of the copper wire.
SOLUTION  Forming the ratio of the areas and using resistivity values from Table 20.1, we have

\[
\frac{A_{\text{aluminum}}}{A_{\text{copper}}} = \frac{\rho_{\text{aluminum}} L/R}{\rho_{\text{copper}} L/R} = \frac{2.82 \times 10^{-8} \, \Omega \cdot \text{m}}{1.72 \times 10^{-8} \, \Omega \cdot \text{m}} = 1.64
\]

16. REASONING AND SOLUTION  Using Equation 20.3 and the resistivity of aluminum from Table 20.1, we find

\[
R = \frac{\rho L}{A} = \frac{(2.82 \times 10^{-8} \, \Omega \cdot \text{m})(10.0 \times 10^3 \, \text{m})}{4.9 \times 10^{-4} \, \text{m}^2} = 0.58 \, \Omega
\]

17. REASONING  Assuming that the resistance is \( R \) at a temperature \( T \) and \( R_0 \) at a temperature \( T_0 \), we can write the percentage change \( p \) in resistance as

\[
p = \frac{R - R_0}{R_0} \times 100
\]

Equation 20.5, on the other hand, gives the resistance as a function of temperature as follows:

\[
R = R_0 \left[ 1 + \alpha (T - T_0) \right]
\]

where \( \alpha \) is the temperature coefficient of resistivity. Substituting this expression into the expression for the percentage change in resistance gives

\[
p = \frac{R - R_0}{R_0} \times 100 = \frac{R_0 + R_0 \alpha (T - T_0) - R_0}{R_0} \times 100 = \alpha (T - T_0) \times 100 = \alpha (T - T_0) 100 \quad (1)
\]

The change in temperature is unknown, but it is the same for both wires. Therefore, we will apply Equation (1) to each wire and divide the two expressions to eliminate the unknown change in temperature. From the result we will be able to calculate the percentage change in the resistance of the tungsten wire.

SOLUTION  Applying Equation (1) to both wires gives

\[
p_{\text{Tungsten}} = \alpha_{\text{Tungsten}} (T - T_0) 100 \quad \text{and} \quad p_{\text{Gold}} = \alpha_{\text{Gold}} (T - T_0) 100
\]

Dividing these two expressions, eliminating \((T - T_0)\) algebraically, and solving for \( p_{\text{Tungsten}} \) give
\[ \frac{p_{\text{Tungsten}}}{p_{\text{Gold}}} = \frac{\alpha_{\text{Tungsten}}(T - T_0)100}{\alpha_{\text{Gold}}(T - T_0)100} = \frac{\alpha_{\text{Tungsten}}}{\alpha_{\text{Gold}}} \]

\[ p_{\text{Tungsten}} = p_{\text{Gold}} \left( \frac{\alpha_{\text{Tungsten}}}{\alpha_{\text{Gold}}} \right) = (7.0\%) \frac{0.0045 \text{ (C°)}^{-1}}{0.0034 \text{ (C°)}^{-1}} = 9.3\% \]

18. **REASONING** The resistance \( R \) of the wire depends on its length \( L \), so we can use Equation 20.3 to express the length in terms of the resistance:

\[ L = \frac{RA}{\rho} \]

where \( A \) is the cross-sectional area of the wire and \( \rho \) is the resistivity of tungsten. The cross-sectional area can be expressed in terms of the radius \( r \) of the wire since \( A = \pi r^2 \). The resistance depends on the voltage \( V \) and current \( I \) through the relation \( R = V / I \) (Equation 20.2). Thus, the length of the wire can be expressed as

\[ L = \frac{RA}{\rho} = \frac{V}{I} \left( \pi r^2 \right) \]

The resistivity \( \rho \) at the temperature \( T \) depends on the resistivity \( \rho_0 \) at the temperature \( T_0 \) through the relation \( \rho = \rho_0 \left[ 1 + \alpha(T - T_0) \right] \) (Equation 20.4), where \( \alpha \) is the temperature coefficient of resistivity. Substituting this expression for \( \rho \) into the expression for the length of the wire, we have

\[ L = \frac{\left( \frac{V}{I} \right) \left( \pi r^2 \right)}{\rho_0 \left[ 1 + \alpha(T - T_0) \right]} \]

**SOLUTION** The length of the wire is

\[ L = \frac{\left( \frac{V}{I} \right) \left( \pi r^2 \right)}{\rho_0 \left[ 1 + \alpha(T - T_0) \right]} \]

\[ = \frac{\left( \frac{120 \text{ V}}{1.5 \text{ A}} \right) \pi \left( 0.075 \times 10^{-3} \text{ m} \right)^2}{\left( 5.6 \times 10^{-8} \text{ Ω m} \right) \left[ 1 + \left( 4.5 \times 10^{-3} \text{ (C°)}^{-1} \right) \left( 1320 \text{ °C} - 20.0 \text{ °C} \right) \right]} = 3.7 \text{ m} \]

The resistivity at 20.0 °C, \( \rho_0 = 5.6 \times 10^{-8} \text{ Ω m} \), was obtained from Table 20.1.
19. **SSM REASONING** We will ignore any changes in length due to thermal expansion. Although the resistance of each section changes with temperature, the total resistance of the composite does not change with temperature. Therefore,

\[
\left( R_{\text{tungsten}} \right)_0 + \left( R_{\text{carbon}} \right)_0 = \left( R_{\text{tungsten}} \right)_0 + \left( R_{\text{carbon}} \right)_0
\]

At room temperature

At temperature \( T \)

From Equation 20.5, we know that the temperature dependence of the resistance for a wire of resistance \( R_0 \) at temperature \( T_0 \) is given by \( R = R_0 [1 + \alpha (T - T_0)] \), where \( \alpha \) is the temperature coefficient of resistivity. Thus,

\[
\left( R_{\text{tungsten}} \right)_0 + \left( R_{\text{carbon}} \right)_0 = \left( R_{\text{tungsten}} \right)_0 (1 + \alpha_{\text{tungsten}} \Delta T) + \left( R_{\text{carbon}} \right)_0 (1 + \alpha_{\text{carbon}} \Delta T)
\]

Since \( \Delta T \) is the same for each wire, this simplifies to

\[
\left( R_{\text{tungsten}} \right)_0 \alpha_{\text{tungsten}} = -\left( R_{\text{carbon}} \right)_0 \alpha_{\text{carbon}}
\]

(1)

This expression can be used to find the ratio of the resistances. Once this ratio is known, we can find the ratio of the lengths of the sections with the aid of Equation 20.3 \( (L = RA/\rho) \).

**SOLUTION** From Equation (1), the ratio of the resistances of the two sections of the wire is

\[
\frac{\left( R_{\text{tungsten}} \right)_0}{\left( R_{\text{carbon}} \right)_0} = -\frac{\alpha_{\text{carbon}}}{\alpha_{\text{tungsten}}} = -\frac{-0.0005 [(\text{C}^0)\text{]}^{-1}}{0.0045 [(\text{C}^0)\text{]}^{-1}} = \frac{1}{9}
\]

Thus, using Equation 20.3, we find the ratio of the tungsten and carbon lengths to be

\[
\frac{L_{\text{tungsten}}}{L_{\text{carbon}}} = \frac{\left( R_0 A/\rho \right)_{\text{tungsten}}}{\left( R_0 A/\rho \right)_{\text{carbon}}} = \frac{\left( R_{\text{tungsten}} \right)_0 \rho_{\text{carbon}}}{\left( R_{\text{carbon}} \right)_0 \rho_{\text{tungsten}}} = \left( \frac{1}{9} \right) \left( \frac{3.5 \times 10^{-5} \text{\Omega} \cdot \text{m}}{5.6 \times 10^{-8} \text{\Omega} \cdot \text{m}} \right) = 70
\]

where we have used resistivity values from Table 20.1 and the fact that the two sections have the same cross-sectional areas.

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20. **REASONING AND SOLUTION** The voltage \( V_{\text{Cu}} \) between the ends of the copper rod is given by Ohm’s law as \( V_{\text{Cu}} = IR_{\text{Cu}} \), where \( R_{\text{Cu}} \) is the resistance of the copper rod. The current \( I \) in the circuit is equal to the voltage \( V \) of the battery that is connected across the free ends of the copper-iron rod divided by the equivalent resistance of the rod. The copper and iron rods are joined end-to-end, so the same current passes through each. Thus, they are connected in series, so the equivalent resistance \( R_S \) is \( R_S = R_{\text{Cu}} + R_{\text{Fe}} \). Thus, the current is
\[ I = \frac{V}{R_S} = \frac{V}{R_{Cu} + R_{Fe}} \]

The voltage across the copper rod is
\[ V_{Cu} = IR_{Cu} = \frac{V}{R_{Cu} + R_{Fe}} R_{Cu} \]

The resistance of the copper and iron rods is given by
\[ R_{Cu} = \frac{\rho_{Cu} L}{A} \quad \text{and} \quad R_{Fe} = \frac{\rho_{Fe} L}{A}, \]
where the length \( L \) and cross-sectional area \( A \) are the same for both rods and \( \rho_{Cu} \) and \( \rho_{Fe} \) denote the resistivities. Substituting these expressions for the resistances into the equation above and using resistivities from Table 20.1 yield
\[ V_{Cu} = \left( \frac{V}{\rho_{Cu} + \rho_{Fe}} \right) \rho_{Cu} \]
\[ V_{Cu} = \left( \frac{12 \text{ V}}{1.72 \times 10^{-8} \Omega \cdot \text{m} + 9.7 \times 10^{-8} \Omega \cdot \text{m}} \right) (1.72 \times 10^{-8} \Omega \cdot \text{m}) = 1.8 \text{ V} \]

21. **REASONING AND SOLUTION** The resistance of the thermistor decreases by 15% relative to its normal value of 37.0 °C. That is,
\[ \frac{\Delta R}{R_0} = \frac{R - R_0}{R_0} = -0.15 \]

According to Equation 20.5, we have
\[ R = R_0 [1 + \alpha (T - T_0)] \quad \text{or} \quad (R - R_0) = \alpha R_0 (T - T_0) \quad \text{or} \quad \frac{R - R_0}{R_0} = \alpha (T - T_0) = -0.15 \]

Rearranging this result gives
\[ T = T_0 + \frac{-0.15}{\alpha} = 37.0 ^\circ \text{C} + \frac{-0.15}{-0.060 \text{ (C)}^{-1}} = 39.5 ^\circ \text{C} \]

22. **REASONING** Knowing the power \( P \) (140 W) consumed by the blanket and the voltage \( V \) (120 V) being applied to it, we can use \( P = \frac{V^2}{R} \) (Equation 20.6c) to determine the resistance \( R \).

**SOLUTION** According to Equation 20.6c, we have
\[ P = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{140 \text{ W}} = 1.0 \times 10^2 \Omega \]
23. **REASONING AND SOLUTION**  According to Equation 20.6c, the power delivered to the iron is

\[
P = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{24 \text{ } \Omega} = 6.0 \times 10^2 \text{ W}
\]

24. **REASONING AND SOLUTION**  The power delivered is \( P = VI \), so that we have

a. \( P_{bd} = VI_{bd} = (120 \text{ V})(11 \text{ A}) = 1300 \text{ W} \)

b. \( P_{vc} = VI_{vc} = (120 \text{ V})(4.0 \text{ A}) = 480 \text{ W} \)

c. The energy is \( E = Pt \), so that we have

\[
\frac{E_{bd}}{E_{vc}} = \frac{P_{bd}t_{bd}}{P_{vc}t_{vc}} = \frac{(1300 \text{ W})(15 \text{ min})}{(480 \text{ W})(30.0 \text{ min})} = 1.4
\]

25. **REASONING**  The total cost of keeping all the TVs turned on is equal to the number of TVs times the cost to keep each one on. The cost for one TV is equal to the energy it consumes times the cost per unit of energy ($0.12 per kW·h). The energy that a single set uses is, according to Equation 6.10b, the power it consumes times the time of use.

**SOLUTION**  The total cost is

\[
\text{Total cost} = (110 \text{ million sets}) \left( \text{Cost per set} \right)
\]

\[
= (110 \text{ million sets}) \left[ \text{Energy (in kW·h) used per set} \right] \left( \frac{\$0.12}{1 \text{ kW} \cdot \text{h}} \right)
\]

The energy (in kW·h) used per set is the product of the power and the time, where the power is expressed in kilowatts and the time is in hours:

\[
\text{Energy used per set} = Pt = (75 \text{ W}) \left( \frac{1 \text{ kW}}{1000 \text{ W}} \right) (6.0 \text{ h}) \quad (6.10b)
\]

The total cost of operating the TV sets is

\[
\text{Total cost} = (110 \text{ million sets}) \left[ (75 \text{ W}) \left( \frac{1 \text{ kW}}{1000 \text{ W}} \right) (6.0 \text{ h}) \right] \left( \frac{\$0.12}{1 \text{ kW} \cdot \text{h}} \right) = 5.9 \times 10^6
\]
26. **REASONING** To find the current, we can use the fact that the power \( P \) is the product of the current \( I \) and the voltage \( V \), since the power and voltage are known.

**SOLUTION** Solving \( P = IV \) (Equation 20.6a) for the current, we have

\[
I = \frac{P}{V} = \frac{0.095 \text{ W}}{3.7 \text{ V}} = 0.026 \text{ A}
\]

---

27. **SSM REASONING** According to Equation 6.10b, the energy used is Energy = \( Pt \), where \( P \) is the power and \( t \) is the time. According to Equation 20.6a, the power is \( P = IV \), where \( I \) is the current and \( V \) is the voltage. Thus, Energy = \( IVt \), and we apply this result first to the dryer and then to the computer.

**SOLUTION** The energy used by the dryer is

\[
\text{Energy} = Pt = IVt = (16 \text{ A})(240 \text{ V})(45 \text{ min}) \left( \frac{60 \text{ s}}{1.00 \text{ min}} \right) = 1.04 \times 10^7 \text{ J}
\]

Converting minutes to seconds

For the computer, we have

\[
\text{Energy} = 1.04 \times 10^7 \text{ J} = IVt = (2.7 \text{ A})(120 \text{ V})t
\]

Solving for \( t \) we find

\[
t = \frac{1.04 \times 10^7 \text{ J}}{(2.7 \text{ A})(120 \text{ V})} = 3.21 \times 10^4 \text{ s} = \left( 3.21 \times 10^4 \text{ s} \right) \left( \frac{1.00 \text{ h}}{3600 \text{ s}} \right) = 8.9 \text{ h}
\]

---

28. **REASONING** A certain amount of time \( t \) is needed for the heater to deliver the heat \( Q \) required to raise the temperature of the water, and this time depends on the power produced by the heater. The power \( P \) is the energy (heat in this case) per unit time, so the time is the heat divided by the power or \( t = Q/P \). The heat required to raise the temperature of a mass \( m \) of water by an amount \( \Delta T \) is given by Equation 12.4 as \( Q = cm\Delta T \), where \( c \) is the specific heat capacity of water \([4186 \text{ J/(kg·Cº)}\), see Table 12.2]. The power dissipated in a resistance \( R \) is given by Equation 20.6c as \( P = V^2 / R \), where \( V \) is the voltage across the resistor. Using these expressions for \( Q \) and \( P \) will allow us to determine the time \( t \).

**SOLUTION** Substituting Equations 12.4 and 20.6c into the expression for the time and recognizing that the normal boiling point of water is 100.0 ºC, we find that

\[
t = \frac{Q}{P} = \frac{cm\Delta T}{V^2 / R} = \frac{Rcm\Delta T}{V^2}
\]

\[
= \left( 15 \Omega \right) \left[ 4186 \text{ J/(kg·Cº)} \right] (0.50 \text{ kg})(100.0 \text{ ºC} - 13 \text{ ºC}) \left( \frac{120 \text{ V}}{2} \right)^2 = 190 \text{ s}
\]
29. **REASONING** The total volume of ice melted is the product of the thickness \( h \) and the area \( A \) of the layer that melts, which permits us to determine the thickness in terms of the volume and the area:

\[
hA = \text{Volume} \quad \text{or} \quad h = \frac{\text{Volume}}{A} \quad (1)
\]

Given the density \( \rho = 917 \text{ kg/m}^3 \), we can find the volume of the ice from \( \rho = \frac{m}{\text{Volume}} \) (Equation 11.1), where \( m \) is the mass of the ice.

\[
\text{Volume} = \frac{m}{\rho} \quad (2)
\]

The mass \( m \) of the ice determines the heat \( Q \) required to melt it, according to Equation 12.5

\[
Q = mL \quad (12.5)
\]

In Equation 12.5, \( L = 33.5 \times 10^4 \text{ J/kg} \) is the latent heat of fusion for water (see Table 12.3). In order to find the maximum thickness of ice the defroster can melt, we will assume that all the heat generated by the defroster goes into melting the ice.

The power output \( P \) of the defroster is the rate at which it converts electrical energy to heat, so we have that

\[
P = \frac{Q}{t} \quad \text{or} \quad Q = Pt \quad (6.10b)
\]

where \( t \) is the elapsed time (3.0 minutes). The power output \( P \) is, in turn, found from the operating voltage \( V \) and current \( I \):

\[
P = IV \quad (20.6a)
\]

**SOLUTION** Substituting Equation (2) into Equation (1) yields

\[
h = \frac{\text{Volume}}{A} = \frac{(m/\rho)}{A} = \frac{m}{\rho A} \quad (3)
\]

Solving \( Q = mL \) (Equation 12.5) for \( m \), we find that \( m = \frac{Q}{L} \). Substituting this result into Equation (3), we obtain

\[
h = \frac{m}{\rho A} = \frac{(Q/L)}{\rho A} = \frac{Q}{L \rho A} \quad (4)
\]

Substituting Equation (6.10b) into Equation (4) yields

\[
h = \frac{Q}{L \rho A} = \frac{Pt}{L \rho A} \quad (5)
\]
We note that the elapsed time \( t \) is given in minutes, which must be converted to seconds. Substituting Equation 20.6a into Equation (5), we obtain the maximum thickness of the ice that the defroster can melt:

\[
h = \frac{P_l}{L \rho A} = \frac{IV_t}{L \rho A} = \frac{(23 \text{ A})(12 \text{ V})(3.0 \text{ min})}{(33.5 \times 10^4 \text{ J/kg})(917 \text{ kg/m}^3)(0.52 \text{ m}^2)} = 3.1 \times 10^{-4} \text{ m}
\]

30. **REASONING AND SOLUTION** We know that the resistance of the wire can be obtained from

\[
P = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{P}
\]

We also know that \( R = \rho L / A \). Solving for the length, noting that \( A = \pi r^2 \), and using \( \rho = 100 \times 10^{-8} \Omega \cdot \text{m} \) from Table 20.1, we find

\[
L = \frac{RA}{\rho} = \left( \frac{V^2}{P} \right) \left( \frac{\pi r^2}{\rho} \right) = \frac{V^2 \pi r^2}{\rho P} = \frac{(120 \text{ V})^2 \pi (6.5 \times 10^{-4} \text{ m})^2}{(100 \times 10^{-8} \Omega \cdot \text{m})(4.00 \times 10^2 \text{ W})} = 50 \text{ m}
\]

31. **SSM REASONING AND SOLUTION** As a function of temperature, the resistance of the wire is given by Equation 20.5: \( R = R_0 \left[ 1 + \alpha (T - T_0) \right] \), where \( \alpha \) is the temperature coefficient of resistivity. From Equation 20.6c, we have \( P = V^2 / R \). Combining these two equations, we have

\[
P = \frac{V^2}{R_0 \left[ 1 + \alpha (T - T_0) \right]} = \frac{P_0}{1 + \alpha (T - T_0)}
\]

where \( P_0 = V^2 / R_0 \), since the voltage is constant. But \( P = \frac{1}{2} P_0 \), so we find

\[
\frac{P_0}{2} = \frac{P_0}{1 + \alpha (T - T_0)} \quad \text{or} \quad 2 = 1 + \alpha (T - T_0)
\]

Solving for \( T \), we find

\[
T = \frac{1 + T_0}{\alpha} = \frac{1}{0.0045 \text{ (C}^{-1})} + 28^\circ = 250^\circ \text{C}
\]

32. **REASONING** Substituting \( V = \frac{1}{2} V_0 \) into Equation 20.7 gives a result that can be solved directly for the desired time.

**SOLUTION** From Equation 20.7 we have
\[ V = \frac{1}{2} V_0 = V_0 \sin 2\pi f t \quad \text{or} \quad \frac{1}{2} = \sin 2\pi f t \]

Using the inverse trigonometric sine function, we find

\[ 2\pi f t = \sin^{-1}\left(\frac{1}{2}\right) = 0.524 \]

In this result, the value of 0.524 is in radians and corresponds to an angle of 30.0°. Thus we find that the smallest value of \( t \) is

\[ t = \frac{0.524}{2\pi f} = \frac{0.524}{2\pi (60.0 \text{ Hz})} = 1.39 \times 10^{-3} \text{ s} \]

33. **REASONING**
   a. The average power \( \bar{P} \) delivered to the copy machine is equal to the square of the rms-current \( I_{\text{rms}} \) times the resistance \( R \), or \( \bar{P} = I_{\text{rms}}^2 R \) (Equation 20.15b). Both \( I_{\text{rms}} \) and \( R \) are known.

   b. According to the discussion in Section 20.5, the peak power \( P_{\text{peak}} \) is twice the average power, or \( P_{\text{peak}} = 2\bar{P} \).

**SOLUTION**
   a. The average power is

   \[ \bar{P} = I_{\text{rms}}^2 R = (6.50 \text{ A})^2 (18.6 \Omega) = 786 \text{ W} \] \hspace{1cm} (20.15b)

   b. The peak power is twice the average power, so

   \[ P_{\text{peak}} = 2\bar{P} = 2(786 \text{ W}) = 1572 \text{ W} \]

34. **REASONING** The peak voltage \( V_0 \) can be obtained from the rms voltage, since the two voltages are related according to \( V_{\text{rms}} = V_0 \sqrt{2} \) (Equation 20.13). Knowing the rms current \( I_{\text{rms}} \) (0.50 A) and the resistance \( R \) (47 Ω), we can use \( V_{\text{rms}} = I_{\text{rms}} R \) (Ohm’s law, Equation 20.14) to determine the rms voltage \( V_{\text{rms}} \).

**SOLUTION** According to Equation 20.13, we have

\[ V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \quad \text{or} \quad V_0 = \sqrt{2} V_{\text{rms}} \]

Using \( V_{\text{rms}} = I_{\text{rms}} R \) (Ohm’s law, Equation 20.14) to substitute into the expression for the peak voltage \( V_0 \), we obtain

\[ V_0 = \sqrt{2} V_{\text{rms}} = \sqrt{2} I_{\text{rms}} R = \sqrt{2} (0.50 \text{ A})(47 \Omega) = 33 \text{ V} \]
35. **REASONING**  Because we are ignoring the effects of temperature on the heater, the resistance $R$ of the heater is the same whether it is plugged into a 120-V outlet or a 230-V outlet. The average power output $\bar{P}$ of the heater is related to the rms outlet voltage $V_{\text{rms}}$ and the heater’s resistance $R$ by $\bar{P} = \frac{V_{\text{rms}}^2}{R}$ (Equation 20.15c).

**SOLUTION**  When plugged into an outlet in the US that has an rms voltage $V_{\text{rms}} = 120$ V, the average power output $\bar{P}$ of the heater is, from Equation 20.15c,

$$\bar{P} = \frac{V_{\text{rms}}^2}{R}$$

(20.15c)

When operated in Germany, the average power output $(\bar{P}_0 = 550 \text{ W})$ of the heater is given by $\bar{P}_0 = \frac{V_{\text{rms},0}^2}{R}$ (Equation 20.15c), where $V_{\text{rms},0} = 230$ V. Solving this relation for $R$, we obtain

$$R = \frac{V_{\text{rms},0}^2}{\bar{P}_0}$$

(1)

Substituting Equation (1) into Equation 20.15c yields

$$\bar{P} = \frac{V_{\text{rms}}^2}{R} = \frac{V_{\text{rms}}^2}{\left( \frac{V_{\text{rms},0}^2}{\bar{P}_0} \right)} = \frac{V_{\text{rms}}^2}{V_{\text{rms},0}^2} \bar{P}_0 = \left( \frac{120 \text{ V}}{230 \text{ V}} \right)^2 (550 \text{ W}) = 150 \text{ W}$$

36. **REASONING AND SOLUTION**  The power $P$ dissipated in the extension cord is $P = I^2 R$ (Equation 20.6b). The resistance $R$ is related to the length $L$ of the wire and its cross-sectional area $A$ by Equation 20.3, $R = \frac{\rho L}{A}$, where $\rho$ is the resistivity of copper. The cross-sectional area of the wire can be expressed as

$$A = \frac{\rho L}{R} = \frac{\rho I^2}{P}$$

where the ratio $P/L$ is the power per unit length of copper wire that the heater produces. The wire is cylindrical, so its cross-sectional area is $A = \pi r^2$. Thus, the smallest radius of wire that can be used is

$$r = I \sqrt{\frac{\rho}{\pi \left( \frac{P}{L} \right)}} = (18 \text{ A}) \sqrt{\frac{1.72 \times 10^{-8} \text{ } \Omega \cdot \text{m}}{\pi (1.0 \text{ W/m})}} = 1.3 \times 10^{-3} \text{ m}$$
Note that we have used 1.0 W/m as the power per unit length, rather than 2.0 W/m. This is because an extension cord is composed of two copper wires. If the maximum power per unit length that the extension cord itself can produce is 2.0 W/m, then each wire can produce only a maximum of 1.0 W/m.

37. **REASONING** The average power is given by Equation 20.15c as $P = \frac{V_{\text{rms}}^2}{R}$. In this expression the rms voltage $V_{\text{rms}}$ appears. However, we seek the peak voltage $V_0$. The relation between the two types of voltage is given by Equation 20.13 as $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, so we can obtain the peak voltage by using Equation 20.13 to substitute into Equation 20.15c.

**SOLUTION** Substituting $V_{\text{rms}}$ from Equation 20.13 into Equation 20.15c gives

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R} = \frac{V_0^2}{2R}$$

Solving for the peak voltage $V_0$ gives

$$V_0 = \sqrt{2RP} = \sqrt{2(4.0 \, \Omega)(55 \, W)} = 21 \, V$$

---

38. **REASONING** The higher the average power output $\bar{P}$ of the resistance heater, the faster it can supply the heat $Q$ needed to raise the water’s temperature. This follows from $\bar{P} = \frac{Q}{t}$ (Equation 6.10b), which implies that the recovery time is $t = \frac{Q}{\bar{P}}$. The average power output $\bar{P}$ of the resistance heater is given by $\bar{P} = \frac{V_{\text{rms}}^2}{R}$ (Equation 20.15c), where $R$ is the resistance of the heater and $V_{\text{rms}} = 120 \, V$ is the heater’s rms operating voltage. The heater must supply an amount of heat $Q = cm\Delta T$ (Equation 12.4) to cause an increase $\Delta T$ in the temperature of a mass $m$ of water, where $c = 4186 \, J/(kg \cdot ^\circ C)$ is the specific heat capacity of water (see Table 12.2). The mass $m$ of the water can be found from the density $\rho = 1.000 \times 10^3 \, kg/m^3$ (see Table 11.1) and the volume $V_{\text{water}}$ of the water held in the heater, according to $m = \rho V_{\text{water}}$ (Equation 11.1).

**SOLUTION** Substituting $\bar{P} = \frac{V_{\text{rms}}^2}{R}$ (Equation 20.15c) and $Q = cm\Delta T$ (Equation 12.4) into the expression $t = \frac{Q}{\bar{P}}$ for the recovery time, we obtain

$$t = \frac{Q}{\bar{P}} = \frac{cm\Delta T}{\left(\frac{V_{\text{rms}}^2}{R}\right)} = \frac{Rcm\Delta T}{V_{\text{rms}}^2}$$

(1)
Substituting $m = \rho V_{\text{water}}$ (Equation 11.1) into Equation (1) yields

$$t = \frac{R c m \Delta T}{V_{\text{rms}}^2} = \frac{R c \rho V_{\text{water}} \Delta T}{V_{\text{rms}}^2}$$

(Equation 2)

Before using Equation (2) to calculate the recovery time, we must convert the volume $V_{\text{water}}$ of the water in the unit from gallons to m$^3$, using the equivalence 1 gal = 3.79×10$^{-3}$ m$^3$:

$$V_{\text{water}} = (52 \text{ gal}) \left( \frac{3.79 \times 10^{-3} \text{ m}^3}{1.00 \text{ gal}} \right) = 0.20 \text{ m}^3$$

We note that the recovery time is to be expressed in hours, where 1.00 h = 3600 s. Therefore, from Equation (2), we find that

$$t = \frac{R c \rho V_{\text{water}} \Delta T}{V_{\text{rms}}^2} = \frac{(3.0 \Omega) \left[ 4186 \text{ J/(kg} \cdot \text{C}) \right] (1.000 \times 10^3 \text{ kg/m}^3)(0.20 \text{ m}^3)(53 \text{ °C} - 11 \text{ °C})}{(120 \text{ V})^2}$$

$$= 7300 \text{ s} \left( \frac{1.00 \text{ h}}{3600 \text{ s}} \right) = 2.0 \text{ h}$$

39. **SSM REASONING**

a. We can obtain the frequency of the alternating current by comparing this specific expression for the current with the more general one in Equation 20.8.

b. The resistance of the light bulb is, according to Equation 20.14, equal to the rms-voltage divided by the rms-current. The rms-voltage is given, and we can obtain the rms-current by dividing the peak current by $\sqrt{2}$, as expressed by Equation 20.12.

c. The average power is given by Equation 20.15a as the product of the rms-current and the rms-voltage.

**SOLUTION**

a. By comparing $I = (0.707 \text{ A}) \sin \left( (314 \text{ Hz}) t \right)$ with the general expression (see Equation 20.8) for the current in an ac circuit, $I = I_0 \sin 2\pi f t$, we see that

$$2\pi f t = (314 \text{ Hz}) t \quad \text{or} \quad f = \frac{314 \text{ Hz}}{2\pi} = [50.0 \text{ Hz}]$$

b. The resistance is equal to $V_{\text{rms}}/I_{\text{rms}}$, where the rms-current is related to the peak current $I_0$ by $I_{\text{rms}} = I_0 / \sqrt{2}$. Thus, the resistance of the light bulb is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\text{rms}}}{I_0 / \sqrt{2}} = \frac{\sqrt{2} (120.0 \text{ V})}{0.707 \text{ A}} = [2.40 \times 10^2 \Omega]$$

(20.14)
c. The average power is the product of the rms-current and rms-voltage:

\[
\bar{P} = I_{\text{rms}} V_{\text{rms}} = \left( \frac{I_0}{\sqrt{2}} \right) V_{\text{rms}} = \left( \frac{0.707 \, \text{A}}{\sqrt{2}} \right) (120.0 \, \text{V}) = 60.0 \, \text{W}
\]  

(20.15a)

40. **REASONING AND SOLUTION** The energy \( Q_1 \) that is released when the water cools from an initial temperature \( T \) to a final temperature of 0.0 °C is given by Equation 12.4 as

\[ Q_1 = cm(T - 0.0 \, \text{°C}) \]

The energy \( Q_2 \) released when the water turns into ice at 0.0 °C is \( Q_2 = mL_f \) where \( L_f \) is the latent heat of fusion for water. Since power \( P \) is energy divided by time, the power produced is

\[ P = \frac{Q_1 + Q_2}{t} = \frac{cm(T - 0.0 \, \text{°C}) + mL_f}{t} \]

The power produced by an electric heater is, according to Equation 20.6a, \( P = IV \). Substituting this expression for \( P \) into the equation above and solving for the current \( I \), we get

\[
I = \frac{cm(T - 0.0 \, \text{°C}) + mL_f}{tV}
\]

\[
I = \frac{(4186 \, \text{J/kg} \cdot \text{°C})(660 \, \text{kg})(10.0 \, \text{°C}) + (660 \, \text{kg})(33.5 \times 10^4 \, \text{J/kg})}{(9.0 \, \text{h}) \left( \frac{3600 \, \text{s}}{\text{h}} \right)(240 \, \text{V})} = 32 \, \text{A}
\]

41. **SSM REASONING** The equivalent series resistance \( R_s \) is the sum of the resistances of the three resistors. The potential difference \( V \) can be determined from Ohm's law according to \( V = IR_s \).

**SOLUTION**

a. The equivalent resistance is

\[
R_s = 25 \, \Omega + 45 \, \Omega + 75 \, \Omega = 145 \, \Omega
\]

b. The potential difference across the three resistors is

\[
V = IR_s = (0.51 \, \text{A})(145 \, \Omega) = 74 \, \text{V}
\]

42. **REASONING** According to Equation 20.2, the resistance \( R \) of the resistor is equal to the voltage \( V_R \) across it divided by the current \( I \), or \( R = V_R / I \). Since the resistor, the lamp, and the voltage source are in series, the voltage across the resistor is \( V_R = 120.0 \, \text{V} - V_L \), where \( V_L \) is the voltage across the lamp. Thus, the resistance is
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\[
R = \frac{120.0 \, \text{V} - V_L}{I}
\]

Since \( V_L \) is known, we need only determine the current in the circuit. Since we know the voltage \( V_L \) across the lamp and the power \( P \) dissipated by it, we can use Equation 20.6a to find the current: \( I = P/V_L \). The resistance can be written as

\[
R = \frac{120.0 \, \text{V} - V_L}{P/V_L}
\]

**SOLUTION** Substituting the known values for \( V_L \) and \( P \) into the equation above, the resistance is

\[
R = \frac{120.0 \, \text{V} - 25 \, \text{V}}{60.0 \, \text{W}} = 4.0 \times 10^4 \, \Omega
\]

43. **SSM REASONING** Using Ohm's law (Equation 20.2) we can write an expression for the voltage across the original circuit as \( V = I_0 R_0 \). When the additional resistor \( R \) is inserted in series, assuming that the battery remains the same, the voltage across the new combination is given by \( V = I(R + R_0) \). Since \( V \) is the same in both cases, we can write \( I_0 R_0 = I(R + R_0) \). This expression can be solved for \( R_0 \).

**SOLUTION** Solving for \( R_0 \), we have

\[
I_0 R_0 - IR_0 = IR \quad \text{or} \quad R_0 (I_0 - I) = IR
\]

Therefore, we find that

\[
R_0 = \frac{IR}{I_0 - I} = \frac{(12.0 \, \text{A})(8.00 \, \Omega)}{15.0 \, \text{A} - 12.0 \, \text{A}} = 32 \, \Omega
\]

44. **REASONING** The power \( P_n \) delivered to any one of the three resistors is equal to the product of the current squared \( (I^2) \) and the resistance \( R_n \), or \( P_n = I^2 R_n \), where \( n = 1, 2, \) or 3. In each case, the resistance is known, and Ohm’s law can be used to find the current. Ohm’s law states that the current in the circuit (which is also the current through each of the resistors) equals the voltage \( V \) of the battery divided by the equivalent resistance \( R_S \) of the three resistors: \( I = V/R_S \). Since the resistors are connected in series, we can obtain the equivalent resistance by adding the three resistances.
**SOLUTION** The power $P_n$ supplied to any one of the three resistors is

$$P_n = I^2 R_n \quad (n = 1, \ 2, \ or \ 3) \quad (20.6b)$$

The current $I$ depends on the voltage $V$ of the battery and the equivalent resistance $R_S$ of the three resistors through Ohm’s law:

$$I = \frac{V}{R_S} \quad (20.2)$$

Substituting Equation 20.2 into Equation 20.6b gives

$$P_n = I^2 R_n = \left(\frac{V}{R_S}\right)^2 R_n \quad (n = 1, \ 2, \ or \ 3) \quad (1)$$

Since the three resistors are wired in series, the equivalent resistance $R_S$ is the sum of the resistances: $R_S = R_1 + R_2 + R_3$ (Equation 20.16). Substituting this relation into Equation (1) yields

$$P_n = \left(\frac{V}{R_S}\right)^2 R_n = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_n \quad (n = 1, \ 2, \ or \ 3)$$

The power delivered to each resistor is:

$$P_1 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_1 = \left(\frac{24 \text{ V}}{2.0 \Omega + 4.0 \Omega + 6.0 \Omega}\right)^2 (2.0 \Omega) = 8.0 \text{ W}$$

$$P_2 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_2 = \left(\frac{24 \text{ V}}{2.0 \Omega + 4.0 \Omega + 6.0 \Omega}\right)^2 (4.0 \Omega) = 16 \text{ W}$$

$$P_3 = \left(\frac{V}{R_1 + R_2 + R_3}\right)^2 R_3 = \left(\frac{24 \text{ V}}{2.0 \Omega + 4.0 \Omega + 6.0 \Omega}\right)^2 (6.0 \Omega) = 24 \text{ W}$$

45. **REASONING** Since the two resistors are connected in series, they are equivalent to a single equivalent resistance that is the sum of the two resistances, according to Equation 20.16. Ohm’s law (Equation 20.2) can be applied with this equivalent resistance to give the battery voltage.

**SOLUTION** According to Ohm’s law, we find

$$V = IR_S = I \left(R_1 + R_2\right) = (0.12 \text{ A}) (47 \Omega + 28 \Omega) = 9.0 \text{ V}$$
46. **REASONING** The circuit containing the light bulb and resistor is shown in the drawing. The resistance $R_1$ of the light bulb is related to the power delivered to it by $R_1 = P_1 / I^2$ (Equation 20.6b), where $I$ is the current in the circuit. The power is known, and the current can be obtained from Ohm’s law as the voltage $V$ of the source divided by the equivalent resistance $R_S$ of the series circuit: $I = V / R_S$. Since the two resistors are wired in series, the equivalent resistance is the sum of the resistances, or $R_S = R_1 + R_2$.

![Diagram of the circuit containing a light bulb and two resistors. The light bulb has a power of 23.4 W. The voltage is 120.0 V.]

**SOLUTION** The resistance of the light bulb is

$$R_1 = \frac{P_1}{I^2}$$  \hspace{1cm} (20.6b)

Substituting $I = V / R_S$ (Equation 20.2) into Equation 20.6b gives

$$R_1 = \frac{P_1}{I^2} = \frac{P_1}{(V / R_S)^2} = \frac{P_1 R_S^2}{V^2}$$ \hspace{1cm} (1)

The equivalent resistance of the two resistors wired in series is $R_S = R_1 + R_2$ (Equation 20.16). Substituting this expression for $R_S$ into Equation (1) yields

$$R_1 = \frac{P_1 R_S^2}{V^2} = \frac{P_1 (R_1 + R_2)^2}{V^2}$$

Algebraically rearranging this equation, we find that

$$R_1^2 + \left(2R_2 - \frac{V^2}{P_1}\right)R_1 + R_2^2 = 0$$

This is a quadratic equation in the variable $R_1$. The solution can be found by using the quadratic formula (see Appendix C.4):
\[- \left( \frac{2R_2 - V^2}{P_1} \right) \pm \sqrt{ \left( \frac{2R_2 - V^2}{P_1} \right)^2 - 4R_2^2} \, \right] \over 2 \]

\[ R_1 = \frac{-2(144 \, \Omega) - (120.0 \, V)^2}{23.4 \, W} \pm \sqrt{\frac{2(144 \, \Omega) - (120.0 \, V)^2}{23.4 \, W}^2 - 4(144 \, \Omega)^2} \]

\[ = \frac{85.9 \, \Omega}{2} \text{ and } 242 \, \Omega \]

47. **SSM REASONING**

a. The greatest voltage for the battery is the voltage that generates the maximum current \( I \) that the circuit can tolerate. Once this maximum current is known, the voltage can be calculated according to Ohm's law, as the current times the equivalent circuit resistance for the three resistors in series. To determine the maximum current we note that the power \( P \) dissipated in each resistance \( R \) is \( P = I^2R \) according to Equation 20.6b. Since the power rating and resistance are known for each resistor, the maximum current that can be tolerated by a resistor is \( I = \sqrt{\frac{P}{R}} \). By examining this maximum current for each resistor, we will be able to identify the maximum current that the circuit can tolerate.

b. The battery delivers power to the circuit that is given by the battery voltage times the current, according to Equation 20.6a.

**SOLUTION**

a. Solving Equation 20.6b for the current, we find that the maximum current for each resistor is as follows:

\[ I = \frac{P}{R} = \sqrt{\frac{4.0 \, W}{2.0 \, \Omega}} = 1.4 \, A \]

\[ I = \frac{P}{R} = \sqrt{\frac{10.0 \, W}{12.0 \, \Omega}} = 0.913 \, A \]

\[ I = \frac{P}{R} = \sqrt{\frac{5.0 \, W}{3.0 \, \Omega}} = 1.3 \, A \]

The smallest of these three values is 0.913 A and is the maximum current that the circuit can tolerate. Since the resistors are connected in series, the equivalent resistance of the circuit is

\[ R_S = 2.0 \, \Omega + 12.0 \, \Omega + 3.0 \, \Omega = 17.0 \, \Omega \]

Using Ohm's law with this equivalent resistance and the maximum current of 0.913 A reveals that the maximum battery voltage is

\[ V = IR_S = (0.913 \, A)(17.0 \, \Omega) = 15.5 \, V \]

b. The power delivered by the battery in part (a) is given by Equation 20.6a as

\[ P = IV = (0.913 \, A)(15.5 \, V) = 14.2 \, W \]
48. **REASONING** The answer is not $340 \text{ W} + 240 \text{ W} = 580 \text{ W}$. The reason is that each heater contributes resistance to the circuit when they are connected in series across the battery. For a series connection, the resistances add together to give the equivalent total resistance, according to Equation 20.16. Thus, the total resistance is greater than the resistance of either heater. The greater resistance means that the current from the battery is less than when either heater is present by itself. Since the power for each heater is $P = I^2R$, according to Equation 20.6b, the smaller current means that the power delivered to an individual heater is less when both are connected than when that heater is connected alone. We approach this problem by remembering that the total power delivered to the series combination of the heaters is the power delivered to the equivalent series resistance.

**SOLUTION** Let the resistances of the two heaters be $R_1$ and $R_2$. Correspondingly, the powers delivered to the heaters when each is connected alone to the battery are $P_1$ and $P_2$. For the series connection, the equivalent total resistance is $R_1 + R_2$, according to Equation 20.16. Using Equation 20.6c, we can write the total power delivered to this equivalent resistance as

$$P = \frac{V^2}{R_1 + R_2}$$

But according to Equation 20.6c, as applied to the situations when each heater is connected by itself to the battery, we have

$$P_1 = \frac{V^2}{R_1} \quad \text{or} \quad R_1 = \frac{V^2}{P_1}$$

$$P_2 = \frac{V^2}{R_2} \quad \text{or} \quad R_2 = \frac{V^2}{P_2}$$

Substituting Equations (2) and (3) into Equation (1) gives

$$P = \frac{V^2}{P_1} + \frac{V^2}{P_2} = \frac{1}{P_1} + \frac{1}{P_2} = \frac{P_1P_2}{P_1 + P_2} = \frac{(340 \text{ W})(240 \text{ W})}{340 \text{ W} + 240 \text{ W}} = 140 \text{ W}$$

49. **REASONING** Ohm’s law provides the basis for our solution. We will use it to express the current from the battery when both resistors are connected and when only one resistor at a time is connected. When both resistors are connected, we will use Ohm’s law with the series equivalent resistance, which is $R_1 + R_2$, according to Equation 20.16. The problem statement gives values for amounts by which the current increases when one or the other resistor is removed. Thus, we will focus attention on the difference between the currents given by Ohm’s law.

**SOLUTION** When $R_2$ is removed, leaving only $R_1$ connected, the current increases by 0.20 A. In this case, using Ohm’s law to express the currents, we have
When $R_1$ is removed, leaving only $R_2$ connected, the current increases by 0.10 A. In this case, using Ohm’s law to express the currents, we have

\[ \frac{V}{R_2} - \frac{V}{R_1 + R_2} = \frac{VR_1}{R_2(R_1 + R_2)} = 0.10 \text{ A} \]  

(2)

Multiplying Equation (1) and Equation (2), we obtain

\[ \begin{bmatrix} \frac{VR_2}{R_1(R_1 + R_2)} \\ \frac{VR_1}{R_2(R_1 + R_2)} \end{bmatrix} = (0.20 \text{ A})(0.10 \text{ A}) \]

Simplifying this result algebraically shows that

\[ \frac{V^2}{(R_1 + R_2)^2} = (0.20 \text{ A})(0.10 \text{ A}) \quad \text{or} \quad \frac{V}{R_1 + R_2} = \sqrt{(0.20 \text{ A})(0.10 \text{ A})} = 0.14 \text{ A} \]  

(3)

a. Using the result for $V/(R_1 + R_2)$ from Equation (3) to substitute into Equation (1) gives

\[ \frac{V}{R_1} - 0.14 \text{ A} = 0.20 \text{ A} \quad \text{or} \quad R_1 = \frac{V}{0.20 \text{ A} + 0.14 \text{ A}} = \frac{12 \text{ V}}{0.20 \text{ A} + 0.14 \text{ A}} = 35 \Omega \]

b. Using the result for $V/(R_1 + R_2)$ from Equation (3) to substitute into Equation (2) gives

\[ \frac{V}{R_2} - 0.14 \text{ A} = 0.10 \text{ A} \quad \text{or} \quad R_2 = \frac{V}{0.10 \text{ A} + 0.14 \text{ A}} = \frac{12 \text{ V}}{0.10 \text{ A} + 0.14 \text{ A}} = 5.0 \times 10^1 \Omega \]

50. **REASONING** The total power $P$ is given by $P = V^2 / R_p$ (Equation 20.6c), where $V$ is the outlet voltage (120 V) and $R_p$ is the equivalent parallel resistance of the coffee-maker and the toaster. The equivalent parallel resistance can be determined with the aid of the following expression: $\frac{1}{R_p} = \frac{1}{R_{\text{coffee-maker}}} + \frac{1}{R_{\text{toaster}}}$ (Equation 20.17).

**SOLUTION** Using Equation 20.6c for the total power and Equation 20.17 to deal with the equivalent parallel resistance of the two appliances, we have
### 51. REASONING AND SOLUTION

The power \( P \) dissipated in a resistance \( R \) is given by Equation 20.6c as \( P = \frac{V^2}{R} \). The resistance \( R_{50} \) of the 50.0-W filament is

\[
R_{50} = \frac{V^2}{P} = \frac{(120.0 \text{ V})^2}{50.0 \text{ W}} = 288 \Omega
\]

The resistance \( R_{100} \) of the 100.0-W filament is

\[
R_{100} = \frac{V^2}{P} = \frac{(120.0 \text{ V})^2}{100.0 \text{ W}} = 144 \Omega
\]

### 52. REASONING

#### a.
The three resistors are in series, so the same current goes through each resistor: \( I_1 = I_2 = I_3 \). The voltage across each resistor is given by Equation 20.2 as \( V = IR \). Because the current through each resistor is the same, the voltage across each is proportional to the resistance. Since \( R_1 > R_2 > R_3 \), we expect the ranking of the voltages to be \( V_1 > V_2 > V_3 \).

#### b.
The three resistors are in parallel, so the same voltage exists across each: \( V_1 = V_2 = V_3 \). The current through each resistor is given by Equation 20.2 as \( I = \frac{V}{R} \). Because the voltage across each resistor is the same, the current through each is inversely proportional to the resistance. Since \( R_1 > R_2 > R_3 \), we expect the ranking of the currents to be \( I_3 > I_2 > I_1 \).

#### SOLUTION

#### a.
The current through the three resistors is given by \( I = \frac{V}{R_s} \), where \( R_s \) is the equivalent resistance of the series circuit. From Equation 20.16, the equivalent resistance is

\[
R_s = 50.0 \Omega + 25.0 \Omega + 10.0 \Omega = 85.0 \Omega
\]

The current through each resistor is

\[
I_1 = I_2 = I_3 = \frac{V}{R_s} = \frac{24.0 \text{ V}}{85.0 \Omega} = 0.282 \text{ A}
\]

The voltage across each resistor is

\[
V_1 = IR_1 = (0.282 \text{ A})(50.0 \Omega) = 14.1 \text{ V}
\]

\[
V_2 = IR_2 = (0.282 \text{ A})(25.0 \Omega) = 7.05 \text{ V}
\]

\[
V_3 = IR_3 = (0.282 \text{ A})(10.0 \Omega) = 2.82 \text{ V}
\]
b. The resistors are in parallel, so the voltage across each is the same as the voltage of the battery:

\[ V_1 = V_2 = V_3 = 24.0 \, \text{V} \]

The current through each resistor is equal to the voltage across each divided by the resistance:

\[ I_1 = \frac{V}{R_1} = \frac{24.0 \, \text{V}}{50.0 \, \Omega} = 0.480 \, \text{A} \]

\[ I_2 = \frac{V}{R_2} = \frac{24.0 \, \text{V}}{25.0 \, \Omega} = 0.960 \, \text{A} \]

\[ I_3 = \frac{V}{R_3} = \frac{24.0 \, \text{V}}{10.0 \, \Omega} = 2.40 \, \text{A} \]

53. **SSM REASONING** When the switch is open, no current goes to the resistor \( R_2 \). Current exists only in \( R_1 \), so it is the equivalent resistance. When the switch is closed, current is sent to both resistors. Since they are wired in parallel, we can use Equation 20.17 to find the equivalent resistance. Whether the switch is open or closed, the power \( P \) delivered to the circuit can be found from the relation \( P = \frac{V^2}{R} \) (Equation 20.6c), where \( V \) is the battery voltage and \( R \) is the equivalent resistance.

**SOLUTION**

a. When the switch is open, there is current only in resistor \( R_1 \). Thus, the equivalent resistance is \( R_1 = 65.0 \, \Omega \).

b. When the switch is closed, there is current in both resistors and, furthermore, they are wired in parallel. The equivalent resistance is

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{65.0 \, \Omega} + \frac{1}{96.0 \, \Omega} \quad \text{or} \quad R_p = 38.8 \, \Omega \quad (20.17) \]

c. When the switch is open, the power delivered to the circuit by the battery is given by \( P = \frac{V^2}{R_1} \), since the only resistance in the circuit is \( R_1 \). Thus, the power is

\[ P = \frac{V^2}{R_1} = \frac{(9.00 \, \text{V})^2}{65.0 \, \Omega} = 1.25 \, \text{W} \quad (20.6) \]

d. When the switch is closed, the power delivered to the circuit is \( P = \frac{V^2}{R_p} \), where \( R_p \) is the equivalent resistance of the two resistors wired in parallel:

\[ P = \frac{V^2}{R_p} = \frac{(9.00 \, \text{V})^2}{38.8 \, \Omega} = 2.09 \, \text{W} \quad (20.6) \]
54. **REASONING** The equivalent parallel resistance \( R_p \) can be determined with the aid of the following expression:

\[
\frac{1}{R_p} = \frac{1}{R_{16 \Omega \text{ speaker}}} + \frac{1}{R_{8 \Omega \text{ speaker}}} + \frac{1}{R_{4 \Omega \text{ speaker}}} \quad (\text{Equation 20.17}).
\]

**SOLUTION** Using Equation 20.17, we find that

\[
\frac{1}{R_p} = \frac{1}{16 \Omega} + \frac{1}{8 \Omega} + \frac{1}{4 \Omega} = 0.437 \, \Omega^{-1}
\]

\[
R_p = \frac{1}{0.437 \, \Omega^{-1}} = 2.3 \, \Omega
\]

Note that we have carried an extra significant figure in determining \( 1/R_p \) and rounded off to the correct number of significant figures in determining \( R_p \).

55. **SSM  REASONING** Since the resistors are connected in parallel, the voltage across each one is the same and can be calculated from Ohm's Law (Equation 20.2: \( V = IR \)). Once the voltage across each resistor is known, Ohm's law can again be used to find the current in the second resistor. The total power consumed by the parallel combination can be found calculating the power consumed by each resistor from Equation 20.6b: \( P = I^2 R \). Then, the total power consumed is the sum of the power consumed by each resistor.

**SOLUTION** Using data for the second resistor, the voltage across the resistors is equal to

\[
V = IR = (3.00 \, \text{A})(64.0 \, \Omega) = 192 \, \text{V}
\]

a. The current through the 42.0-\( \Omega \) resistor is

\[
I = \frac{V}{R} = \frac{192 \, \text{V}}{42.0 \, \Omega} = 4.57 \, \text{A}
\]

b. The power consumed by the 42.0-\( \Omega \) resistor is

\[
P = I^2 R = (4.57 \, \text{A})^2 (420 \, \Omega) = 877 \, \text{W}
\]

while the power consumed by the 64.0-\( \Omega \) resistor is

\[
P = I^2 R = (3.00 \, \text{A})^2 (640 \, \Omega) = 576 \, \text{W}
\]

Therefore the total power consumed by the two resistors is \( 877 \, \text{W} + 576 \, \text{W} = 1450 \, \text{W} \).

56. **REASONING**

a. The two identical resistors, each with a resistance \( R \), are connected in parallel across the battery, so the potential difference across each resistor is \( V \), the potential difference provided by the battery. The power \( P \) supplied to each resistor, then, is \( P = \frac{V^2}{R} \) (Equation 20.6c). The total power supplied by the battery is twice this amount.
b. According to Equation 20.6c, the resistor which is heated until its resistance is $2R$ consumes only half as much power as it did initially. Because the resistance of the other resistor does not change, it consumes the same power as before. The battery supplies less power, therefore, than it did initially. We will use Equation 20.6c to determine the final power supplied.

**SOLUTION**

a. Since the power supplied to each resistor is $P = \frac{V^2}{R}$ (Equation 20.6c), the total power is twice as large: $P_{\text{tot}} = 2P = \frac{2V^2}{R}$. Solving for $R$, we obtain

$$R = \frac{2V^2}{P_{\text{tot}}} = \frac{2(25\,\text{V})^2}{9.6\,\text{W}} = 130\,\Omega$$

b. The resistances of the resistors are now $R_1 = R$ and $R_2 = 2R$. From Equation 20.6c, the total power output $P_{\text{tot}} = P_1 + P_2$ of the resistors is

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{R} + \frac{V^2}{2R} = \frac{3V^2}{2(130\,\Omega)} = 7.2\,\text{W}$$

57. **REASONING** The total power is given by Equation 20.15c as $\bar{P} = \frac{V_{\text{rms}}^2}{R_p}$, where $R_p$ is the equivalent parallel resistance of the heater and the lamp. Since the total power and the rms voltage are known, we can use this expression to obtain the equivalent parallel resistance. This equivalent resistance is related to the individual resistances of the heater and the lamp via Equation 20.17, which is $R_p^{-1} = R_{\text{heater}}^{-1} + R_{\text{lamp}}^{-1}$. Since $R_{\text{heater}}$ is given, $R_{\text{lamp}}$ can be found once $R_p$ is known.

**SOLUTION** According to Equation 20.15c, the equivalent parallel resistance is

$$R_p = \frac{V_{\text{rms}}^2}{\bar{P}}$$

Using this result in Equation 20.17 gives

$$\frac{1}{R_p} = \frac{1}{V_{\text{rms}}^2/\bar{P}} = \frac{1}{R_{\text{heater}}} + \frac{1}{R_{\text{lamp}}}$$

Rearranging this expression shows that

$$\frac{1}{R_{\text{lamp}}} = \frac{\bar{P}}{V_{\text{rms}}^2} - \frac{1}{R_{\text{heater}}} = \frac{111\,\text{W}}{(120\,\text{V})^2} - \frac{1}{4.0 \times 10^2\,\Omega} = 5.2 \times 10^{-3}\,\Omega^{-1}$$

Therefore,

$$R_{\text{lamp}} = \frac{1}{5.2 \times 10^{-3}\,\Omega^{-1}} = 190\,\Omega$$
58. **REASONING**  The series combination has an equivalent resistance of \( R_S = R_1 + R_2 \), as given by Equation 20.16. The parallel combination has an equivalent resistance that can be determined from \( R_p^{-1} = R_1^{-1} + R_2^{-1} \), according to Equation 20.17. In each case the equivalent resistance can be used in Ohm’s law with the given voltage and current. Thus, we can obtain two equations that each contain the unknown resistances. These equations will be solved simultaneously to obtain \( R_1 \) and \( R_2 \).

**SOLUTION** For the series case, Ohm’s law is \( V = I_S (R_1 + R_2) \). Solving for sum of the resistances, we have

\[
R_1 + R_2 = \frac{V}{I_S} = \frac{12.0 \text{ V}}{2.00 \text{ A}} = 6.00 \Omega
\]  

(1)

For the parallel case, Ohm’s law is \( V = I_p R_p \), where \( R_p^{-1} = R_1^{-1} + R_2^{-1} \). Thus, we have

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{I_p}{V} = \frac{9.00 \text{ A}}{12.0 \text{ V}} = 0.750 \Omega^{-1}
\]  

(2)

Solving Equation (1) for \( R_2 \) and substituting the result into Equation (2) gives

\[
\frac{1}{R_1} + \frac{1}{6.00 - R_1} = 0.750 \quad \text{or} \quad R_1^2 - 6.00R_1 + 8.00 = 0
\]

In this result we have suppressed the units in the interest of clarity. Solving the quadratic equation (see Appendix C.4 for the quadratic formula) gives

\[
R_1 = \frac{(-6.00) \pm \sqrt{(-6.00)^2 - 4(1.00)(8.00)}}{2(1.00)} = \frac{6.00 \pm \sqrt{36.0 - 32.0}}{2.00} = 4.00 \text{ or } 2.00
\]

Substituting these values for \( R_1 \) into Equation (1) reveals that

\[
R_2 = 6.00 - R_1 = 2.00 \text{ or } 4.00
\]

Thus, the values for the two resistances are \( 2.00 \Omega \) and \( 4.00 \Omega \).

59. **REASONING AND SOLUTION**  The aluminum and copper portions may be viewed a being connected in parallel since the same voltage appears across them. Using \( a \) and \( b \) to denote the inner and outer radii, respectively, and using Equation 20.3 to express the resistance for each portion, we find for the equivalent resistance that
\[
\frac{1}{R_p} = \frac{1}{R_{Al}} + \frac{1}{R_{Cu}} = \frac{A_{Cu}}{\rho_{Cu}L} + \frac{A_{Al}}{\rho_{Al}L} = \frac{\pi a^2}{\rho_{Cu}L} + \frac{\pi (b^2 - a^2)}{\rho_{Al}L} \\
= \frac{\pi \left(2.00 \times 10^{-3} \text{ m}\right)^2}{\left(1.72 \times 10^{-8} \Omega \cdot \text{m}\right)(1.50 \text{ m})} + \frac{\pi \left[(3.00 \times 10^{-3} \text{ m})^2 - (2.00 \times 10^{-3} \text{ m})^2\right]}{(2.82 \times 10^{-8} \Omega \cdot \text{m})(1.50 \text{ m})} = 0.00116 \Omega
\]

We have taken resistivity values for copper and aluminum from Table 20.1.

60. **REASONING** The total power \( P \) delivered by the battery is related to the equivalent resistance \( R_{eq} \) connected between the battery terminals and to the battery voltage \( V \) according to Equation 20.6c: \( P = V^2 / R_{eq} \).

When two resistors are connected in series, the equivalent resistance \( R_S \) of the combination is greater than the resistance of either resistor alone. This can be seen directly from \( R_S = R_1 + R_2 \) (Equation 20.16).

When two resistors are connected in parallel, the equivalent resistance \( R_P \) of the combination is smaller than the resistance of either resistor alone. This can be seen directly by substituting values in \( R_P^{-1} = R_1^{-1} + R_2^{-1} \) (Equation 20.17) or by reviewing the discussion in Section 20.7 concerning the water flow analogy for electric current in a circuit.

Since the total power delivered by the battery is \( P = V^2 / R_{eq} \) and since the power and the battery voltage are the same in both cases, it follows that the equivalent resistances are also the same. But the parallel combination has an equivalent resistance \( R_p \) that is smaller than \( R_B \), whereas the series combination has an equivalent resistance \( R_S \) that is greater than \( R_A \). This means that \( R_B \) must be greater than \( R_A \), as Diagram 1 at the right shows. If \( R_A \) were greater than \( R_B \), as in Diagram 2, the equivalent resistances \( R_S \) and \( R_p \) would not be equal.

**SOLUTION** As discussed in the **REASONING**, the equivalent resistances in circuits A and B are equal. According to Equations 20.16 and 20.17, the series and parallel equivalent resistances are
\[ R_S = R_A + R_A = 2R_A \]
\[ \frac{1}{R_p} = \frac{1}{R_B} + \frac{1}{R_B} \quad \text{or} \quad R_p = \frac{1}{2} R_B \]

Setting the equivalent resistances equal gives
\[ 2R_A = \frac{1}{2} R_B \quad \text{or} \quad \frac{R_B}{R_A} = 4 \]

As expected, \( R_B \) is greater than \( R_A \).

61. **REASONING** Since the defogger wires are connected in parallel, the total resistance of all thirteen wires can be obtained from Equation 20.17:
\[ \frac{1}{R_p} = \frac{13}{R} \quad \text{or} \quad R_p = R \frac{13}{13} \]

where \( R \) is the individual resistance of one of the wires. The heat required to melt the ice is given by \( Q = mL_f \), where \( m \) is the mass of the ice and \( L_f \) is the latent heat of fusion of the ice (see Section 12.8). Therefore, using Equation 20.6c, we can see that the power or energy dissipated per unit time in the wires and used to melt the ice is

\[ P = \frac{V^2}{R_p} = \frac{mL_f}{t} \quad \text{or} \quad \frac{V^2}{R/13} = \frac{mL_f}{t} \]

According to Equation 20.3, \( R = \rho L/A \), where the length of the wire is \( L \), its cross-sectional area is \( A \) and its resistivity is \( \rho \); therefore, the last expression can be written

\[ \frac{13V^2}{R} = \frac{13V^2}{\rho L/A} = \frac{mL_f}{t} \]

This expression can be solved for the area \( A \).

**SOLUTION** Solving the above expression for \( A \), and substituting given data, and obtaining the latent heat of fusion of water from Table 12.3, we find that

\[ A = \frac{\rho L m L_f}{13V^2t} \]

\[ = \frac{(88.0 \times 10^{-8} \ \Omega \cdot m)(1.30 \ m)(2.10 \times 10^{-2} \ kg)(33.5 \times 10^4 \ J/kg)}{13(12.0 \ V)^2(120 \ s)} = 3.58 \times 10^{-8} \ m^2 \]
62. **REASONING** The circuit diagram is shown at the right. We can find the current in the 120.0-Ω resistor by using Ohm’s law, provided that we can obtain a value for $V_{AB}$, the voltage between points A and B in the diagram. To find $V_{AB}$, we will also apply Ohm’s law, this time by multiplying the current from the battery times $R_{AB}$, the equivalent parallel resistance between A and B. The current from the battery can be obtained by applying Ohm’s law again, now to the entire circuit and using the total equivalent resistance of the series combination of the 20.0-Ω resistor and $R_{AB}$. Once the current in the 120.0-Ω resistor is found, the power delivered to it can be obtained from Equation 20.6b, $P = I^2 R$.

**SOLUTION**

a. According to Ohm’s law, the current in the 120.0-Ω resistor is $I_{120} = V_{AB}/(120.0 \, \Omega)$. To find $V_{AB}$, we note that the equivalent parallel resistance between points A and B can be obtained from Equation 20.17 as follows:

\[ \frac{1}{R_{AB}} = \frac{1}{60.0 \, \Omega} + \frac{1}{120.0 \, \Omega} \quad \text{or} \quad R_{AB} = 40.0 \, \Omega \]

This resistance of 40.0 Ω is in series with the 20.0-Ω resistance, so that according to Equation 20.16, the total equivalent resistance connected across the battery is $40.0 \, \Omega + 20.0 \, \Omega = 60.0 \, \Omega$. Applying Ohm’s law to the entire circuit, we can see that the current from the battery is

\[ I = \frac{15.0 \, \text{V}}{60.0 \, \Omega} = 0.250 \, \text{A} \]

Again applying Ohm’s law, this time to the resistance $R_{AB}$, we find that

\[ V_{AB} = (0.250 \, \text{A})R_{AB} = (0.250 \, \text{A})(40.0 \, \Omega) = 10.0 \, \text{V} \]

Finally, we can see that the current in the 120.0-Ω resistor is

\[ I_{120} = \frac{V_{AB}}{120 \, \Omega} = \frac{10.0 \, \text{V}}{120 \, \Omega} = 8.33 \times 10^{-2} \, \text{A} \]

b. The power delivered to the 120.0-Ω resistor is given by Equation 20.6b as

\[ P = I_{120}^2 R = (8.33 \times 10^{-2} \, \text{A})^2 (120.0 \, \Omega) = 0.833 \, \text{W} \]

63. **SSM REASONING** To find the current, we will use Ohm’s law, together with the proper equivalent resistance. The coffee maker and frying pan are in series, so their equivalent resistance is given by Equation 20.16 as $R_{\text{coffee}} + R_{\text{pan}}$. This total resistance is in parallel with the resistance of the bread maker, so the equivalent resistance of the parallel combination can be obtained from Equation 20.17 as $R_p^{-1} = (R_{\text{coffee}} + R_{\text{pan}})^{-1} + R_{\text{bread}}^{-1}$. 

\[ R_p = \frac{1}{R_{\text{coffee}}} + \frac{1}{R_{\text{pan}}} + \frac{1}{R_{\text{bread}}} \]
**SOLUTION** Using Ohm’s law and the expression developed above for $R_p^{-1}$, we find

$$I = \frac{V}{R_p} = \frac{V}{R_{\text{coffee}} + R_{\text{pan}} + \frac{1}{R_{\text{bread}}}} = \frac{1(200 \text{ V}}{\left(\frac{1}{14 \Omega} + \frac{1}{16 \Omega} + \frac{1}{23 \Omega}\right)} = \frac{9.2 \text{ A}}{1}$$

64. **REASONING** We will approach this problem in parts. The resistors that are in series will be combined according to Equation 20.16, and the resistors that are in parallel will be combined according to Equation 20.17.

**SOLUTION** The 1.00 Ω, 2.00 Ω and 3.00 Ω resistors are in series with an equivalent resistance of $R_s' = 6.00 \Omega$.

This equivalent resistor of 6.00 Ω is in parallel with the 3.00-Ω resistor, so

$$\frac{1}{R_p} = \frac{1}{6.00 \Omega} + \frac{1}{3.00 \Omega}$$

$$R_p = 2.00 \Omega$$

This new equivalent resistor of 2.00 Ω is in series with the 6.00-Ω resistor, so $R_s = 8.00 \Omega$.

$R_s'$ is in parallel with the 4.00-Ω resistor, so

$$\frac{1}{R_p'} = \frac{1}{8.00 \Omega} + \frac{1}{4.00 \Omega}$$

$$R_p' = 2.67 \Omega$$

Finally, $R_p'$ is in series with the 2.00-Ω, so the total equivalent resistance is $4.67 \Omega$. 
65. **REASONING** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16:

\[ R_s = R_1 + R_2 + R_3 + \ldots \]

Likewise, when resistors are in parallel, the expression to be solved to find the equivalent resistance is given by Equation 20.17:

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

We will successively apply these to the individual resistors in the figure in the text beginning with the resistors on the right side of the figure.

**SOLUTION** Since the 4.0-Ω and the 6.0-Ω resistors are in series, the equivalent resistance of the combination of those two resistors is 10.0 Ω. The 9.0-Ω and 8.0-Ω resistors are in parallel; their equivalent resistance is 4.24 Ω. The equivalent resistances of the parallel combination (9.0 Ω and 8.0 Ω) and the series combination (4.0 Ω and the 6.0 Ω) are in parallel; therefore, their equivalent resistance is 2.98 Ω. The 2.98-Ω combination is in series with the 3.0-Ω resistor, so that equivalent resistance is 5.98 Ω. Finally, the 5.98-Ω combination and the 20.0-Ω resistor are in parallel, so the equivalent resistance between the points A and B is 4.6 Ω.

66. **REASONING** Between points a and b there is only one resistor, so the equivalent resistance is \( R_{ab} = R \). Between points b and c the two resistors are in parallel. The equivalent resistance can be found from Equation 20.17:

\[ \frac{1}{R_{bc}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \text{so} \quad R_{bc} = \frac{1}{2} R \]

The equivalent resistance between a and b is in series with the equivalent resistance between b and c, so the equivalent resistance between a and c is

\[ R_{ac} = R_{ab} + R_{bc} = R + \frac{1}{2} R = \frac{3}{2} R \]

Thus, we see that \( R_{ac} > R_{ab} > R_{bc} \).

**SOLUTION** Since the resistance is \( R = 10.0 \) Ω, the equivalent resistances are:

\[ R_{ab} = R = 10.0 \ \Omega \]

\[ R_{bc} = \frac{1}{2} R = 5.00 \ \Omega \]

\[ R_{ac} = \frac{3}{2} R = 15.0 \ \Omega \]

67. **REASONING** The two resistors \( R_1 \) and \( R_2 \) are wired in series, so we can determine their equivalent resistance \( R_{12} \). The resistor \( R_3 \) is wired in parallel with the equivalent resistance \( R_{12} \), so the equivalent resistance \( R_{123} \) can be found. Finally, the resistor \( R_4 \) is wired in series with the equivalent resistance \( R_{123} \). With these observations, we can evaluate the equivalent resistance between the points A and B.
**Solution** Since $R_1$ and $R_2$ are wired in series, the equivalent resistance $R_{12}$ is

$$R_{12} = R_1 + R_2 = 16 \, \Omega + 8 \, \Omega = 24 \, \Omega \quad (20.16)$$

The resistor $R_3$ is wired in parallel with the equivalent resistor $R_{12}$, so the equivalent resistance $R_{123}$ of this combination is

$$\frac{1}{R_{123}} = \frac{1}{R_3} + \frac{1}{R_{12}} = \frac{1}{48 \, \Omega} + \frac{1}{24 \, \Omega} \quad \text{or} \quad R_{123} = 16 \, \Omega \quad (20.17)$$

The resistance $R_4$ is in series with the equivalent resistance $R_{123}$, so the equivalent resistance $R_{AB}$ between the points $A$ and $B$ is

$$R_{AB} = R_4 + R_{123} = 26 \, \Omega + 16 \, \Omega = 42 \, \Omega$$

**Reasoning** The total power $P$ delivered by the battery is related to the equivalent resistance $R_{eq}$ connected between the battery terminals and to the battery voltage $V$ according to Equation 20.6c: $P = V^2 / R_{eq}$. We note that the combination of resistors in circuit A is also present in circuits B and C (see the shaded part of these circuits in the following drawings). In circuit B an additional resistor is in parallel with the combination from A. The equivalent resistance of resistances in parallel is always less than any of the individual resistances alone. Therefore, the equivalent resistance of circuit B is less than that of A. In circuit C an additional resistor is in series with the combination from A. The equivalent resistance of resistances in series is always greater than any of the individual resistances alone. Therefore, the equivalent resistance of circuit C is greater than that of A. We conclude then that the equivalent resistances are ranked C, A, B, with C the greatest and B the smallest.
Since the total power delivered by the battery is \( P = V^2 / R_{\text{eq}} \), it is inversely proportional to the equivalent resistance. The battery voltage \( V \) is the same in all three cases, so the power ranking is the reverse of the ranking deduced previously for \( R_{\text{eq}} \). In other words, we expect that, from greatest to smallest, the total power delivered by the battery is B, A, C.

**SOLUTION** The total power delivered by the battery is \( P = V^2 / R_{\text{eq}} \). The voltage is given, but we must determine the equivalent resistance in each case. In circuit A each branch of the parallel combination consists of two resistances \( R \) in series. Thus, the resistance of each branch is \( R_{\text{eq}} = R + R = 2R \), according to Equation 20.16. The two parallel branches have an equivalent resistance that can be determined from Equation 20.17 as

\[
\frac{1}{R_A} = \frac{1}{2R} + \frac{1}{2R} \quad \text{or} \quad R_A = R
\]

In circuit B the resistance of circuit A is in parallel with an additional resistance \( R \). According to Equation 20.17, the equivalent resistance of this combination is

\[
\frac{1}{R_B} = \frac{1}{R_A} + \frac{1}{R} = \frac{1}{R} + \frac{1}{R} \quad \text{or} \quad R_B = \frac{1}{2} R
\]

In circuit C the resistance of circuit A is in series with an additional resistance \( R \). According to Equation 20.16, the equivalent resistance of this combination is

\[ R_C = R_A + R = R + R = 2R \]

We can now use \( P = V^2 / R_{\text{eq}} \) to find the total power delivered by the battery in each case:

- **Circuit A**
  \[
P = \frac{V^2}{R} = \frac{(6.0 \text{ V})^2}{9.0 \text{ } \Omega} = 4.0 \text{ W}
\]

- **Circuit B**
  \[
P = \frac{V^2}{\frac{1}{2}R} = \frac{(6.0 \text{ V})^2}{\frac{1}{2}(9.0 \text{ } \Omega)} = 8.0 \text{ W}
\]

- **Circuit C**
  \[
P = \frac{V^2}{2R} = \frac{(6.0 \text{ V})^2}{2(9.0 \text{ } \Omega)} = 2.0 \text{ W}
\]

These results are as expected.
69. **REASONING** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16: \[ R_s = R_1 + R_2 + R_3 + \ldots \] When resistors are in parallel, the expression to be solved to find the equivalent resistance is Equation 20.17: \[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

We will use these relations to determine the eight different values of resistance that can be obtained by connecting together the three resistors: 1.00, 2.00, and 3.00 Ω.

**SOLUTION** When all the resistors are connected in series, the equivalent resistance is the sum of all three resistors and the equivalent resistance is \( 6.00 \Omega \). When all three are in parallel, we have from Equation 20.17, the equivalent resistance is \( 0.545 \Omega \).

We can also connect two of the resistors in parallel and connect the parallel combination in series with the third resistor. When the 1.00 and 2.00-Ω resistors are connected in parallel and the 3.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is \( 3.67 \Omega \). When the 1.00 and 3.00-Ω resistors are connected in parallel and the 2.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is \( 2.75 \Omega \). When the 2.00 and 3.00-Ω resistors are connected in parallel and the 1.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is \( 2.20 \Omega \).

We can also connect two of the resistors in series and put the third resistor in parallel with the series combination. When the 1.00 and 2.00-Ω resistors are connected in series and the 3.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is \( 1.50 \Omega \). When the 1.00 and 3.00-Ω resistors are connected in series and the 2.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is \( 1.33 \Omega \). Finally, when the 2.00 and 3.00-Ω resistors are connected in series and the 1.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is \( 0.833 \Omega \).

70. **REASONING** The power \( P \) dissipated in each resistance \( R \) is given by Equation 20.6b as \( P = I^2 R \), where \( I \) is the current. This means that we need to determine the current in each resistor in order to calculate the power. The current in \( R_1 \) is the same as the current in the equivalent resistance for the circuit. Since \( R_2 \) and \( R_3 \) are in parallel and equal, the current in \( R_1 \) splits into two equal parts at the junction \( A \) in the circuit.

**SOLUTION** To determine the equivalent resistance of the circuit, we note that the parallel combination of \( R_2 \) and \( R_3 \) is in series with \( R_1 \). The equivalent resistance of the parallel combination can be obtained from Equation 20.17 as follows:
\[ \frac{1}{R_p} = \frac{1}{576 \, \Omega} + \frac{1}{576 \, \Omega} \quad \text{or} \quad R_p = 288 \, \Omega \]

This 288-Ω resistance is in series with \( R_1 \), so that the equivalent resistance of the circuit is given by Equation 20.16 as

\[ R_{eq} = 576 \, \Omega + 288 \, \Omega = 864 \, \Omega \]

To find the current from the battery we use Ohm’s law:

\[ I = \frac{V}{R_{eq}} = \frac{120.0 \, V}{864 \, \Omega} = 0.139 \, A \]

Since this is the current in \( R_1 \), Equation 20.6b gives the power dissipated in \( R_1 \) as

\[ P_1 = I_1^2 R_1 = (0.139 \, A)^2 (576 \, \Omega) = 11.1 \, W \]

\( R_2 \) and \( R_3 \) are in parallel and equal, so that the current in \( R_1 \) splits into two equal parts at the junction A. As a result, there is a current of \( \frac{1}{2} (0.139 \, A) \) in \( R_2 \) and in \( R_3 \). Again using Equation 20.6b, we find that the power dissipated in each of these two resistors is

\[ P_2 = I_2^2 R_2 = \left[ \frac{1}{2} (0.139 \, A) \right]^2 (576 \, \Omega) = 2.78 \, W \]

\[ P_3 = I_3^2 R_3 = \left[ \frac{1}{2} (0.139 \, A) \right]^2 (576 \, \Omega) = 2.78 \, W \]

71. **SSM REASONING** The power \( P \) delivered to the circuit is, according to Equation 20.6c, \( P = \frac{V^2}{R_{12345}} \), where \( V \) is the voltage of the battery and \( R_{12345} \) is the equivalent resistance of the five-resistor circuit. The voltage and power are known, so that the equivalent resistance can be calculated. We will use our knowledge of resistors wired in series and parallel to evaluate \( R_{12345} \) in terms of the resistance \( R \) of each resistor. In this manner we will find the value for \( R \).

**SOLUTION** First we note that all the resistors are equal, so \( R_1 = R_2 = R_3 = R_4 = R_5 = R \). We can find the equivalent resistance \( R_{12345} \) as follows. The resistors \( R_3 \) and \( R_4 \) are in series, so the equivalent resistance \( R_{34} \) of these two is \( R_{34} = R_3 + R_4 = 2R \). The resistors \( R_2 \), \( R_{34} \), and \( R_5 \) are in parallel, and the reciprocal of the equivalent resistance \( R_{2345} \) is

\[ \frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_{34}} + \frac{1}{R_5} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{R} = \frac{5}{2R} \]

so \( R_{2345} = 2R/5 \). The resistor \( R_1 \) is in series with \( R_{2345} \), and the equivalent resistance of this combination is the equivalent resistance of the circuit. Thus, we have

\[ R_{12345} = R_1 + R_{2345} = R + \frac{2R}{5} = \frac{7R}{5} \]
The power delivered to the circuit is

\[ P = \frac{V^2}{R} = \frac{V^2}{\frac{7R}{5}} \]

Solving for the resistance \( R \), we find that

\[ R = \frac{5V^2}{7P} = \frac{5(45 \text{ V})^2}{7(58 \text{ W})} = 25 \Omega \]

72. **REASONING** The resistance \( R \) of each of the identical resistors determines the equivalent resistance \( R_{eq} \) of the entire circuit, both in its initial form (six resistors) and in its final form (five resistors). We will calculate the initial and final equivalent circuit resistances by replacing groups of resistors that are connected either in series or parallel. The resistances of the replacement resistors are found either from \( R_S = R_1 + R_2 + R_3 + \cdots \) (Equation 20.16) for resistors connected in series, or from \( \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \) (Equation 20.17), for resistors connected in parallel.

Once the equivalent resistance \( R_{eq} \) of the circuit is determined, the current \( I \) supplied by the battery (voltage = \( V \)) is found from \( I = \frac{V}{R_{eq}} \) (Equation 20.2). We are given the decrease \( \Delta I \) in the battery current, which is equal to the final current \( I_f \) minus the initial current \( I_0 \):

\[ \Delta I = I_f - I_0 = -1.9 \text{ A} \]

The algebraic sign of \( \Delta I \) is negative because the final current is smaller than the initial current.

**SOLUTION** Beginning with the circuit in its initial form, we see that resistors 1, 3, and 5 are connected in series (see the drawing). These three resistors, according to Equation 20.16, may be replaced with a single resistor \( R_S \) that has three times the resistance \( R \) of a single resistor:

\[ R_S = 3R \]

The resistor \( R_S \) is connected in parallel with the resistor \( R_4 \) (see the drawing), so these two resistors may be replaced by an equivalent resistor \( R_P \) found from Equation 20.17:
After making this replacement, the three remaining resistors \( R_2, R_p, \) and \( R_6 \) are connected in series across the battery (see the drawing). The initial equivalent resistance \( R_{eq,0} \) of the entire circuit, then, is found from Equation 20.16:

\[
R_{eq,0} = R_p + R_2 + R_4 = \frac{3R}{4} + 2R = \frac{11R}{4}
\]  

(2)

Next, we consider the circuit after the resistor \( R_4 \) has been removed. The remaining five resistors are connected in series (see the drawing). From Equation 20.16, then, the final equivalent resistance \( R_{eq,f} \) of the entire circuit is five times the resistance \( R \) of a single resistor:

\[
R_{eq,f} = 5R
\]  

(3)

From \( I = \frac{V}{R_{eq}} \) (Equation 20.2), the initial and final battery currents are

\[
I_0 = \frac{V}{R_{eq,0}} \quad \text{and} \quad I_f = \frac{V}{R_{eq,f}}
\]  

(4)

Substituting Equations (2), (3), and (4) into Equation (1), we obtain

\[
\Delta I = I_f - I_0 = \frac{V}{R_{eq,f}} - \frac{V}{R_{eq,0}} = \frac{V}{5R} - \frac{V}{\frac{11R}{4}} = \frac{V}{R} \left( \frac{1}{5} - \frac{4}{11} \right) = -9V \frac{V}{55R}
\]  

(5)

Solving Equation (5) for \( R \) yields

\[
R = \frac{9V}{55\Delta I} = \frac{9(35V)}{55(-1.9A)} = 3.0\ \Omega
\]

73. **SSM REASONING** The terminal voltage of the battery is given by \( V_{terminal} = \text{Emf} - Ir \), where \( r \) is the internal resistance of the battery. Since the terminal voltage is observed to be one-half of the emf of the battery, we have \( V_{terminal} = \text{Emf}/2 \) and \( I = \text{Emf}/(2r) \). From Ohm’s law, the equivalent resistance of the circuit is \( R = \text{Emf} / I = 2r \). We can also find the equivalent resistance of the circuit by considering that the identical bulbs are in parallel across the battery terminals, so that the equivalent resistance of the \( N \) bulbs is found from

\[
\frac{1}{R_p} = \frac{N}{R_{bulb}} \quad \text{or} \quad R_p = \frac{R_{bulb}}{N}
\]
This equivalent resistance is in series with the battery, so we find that the equivalent resistance of the circuit is

\[ R = 2r = \frac{R_{\text{bulb}}}{N} + r \]

This expression can be solved for \( N \).

**SOLUTION** Solving the above expression for \( N \), we have

\[ N = \frac{R_{\text{bulb}}}{2r-r} = \frac{15 \Omega}{0.50 \Omega} = 30 \]

74. **REASONING** The following drawing shows the battery (emf = \( \xi \)), its internal resistance \( r \), and the 1.40-\( \Omega \) resistor. The voltage between the terminals of the battery is the voltage \( V_{AB} \) between the points A and B in the drawing. This voltage is not equal to the 9.00-V emf of the battery, because part of the emf is needed to make the current \( I \) go through the internal resistance. Ohm’s law states that this part of the emf is \( Ir \). The 1.40-\( \Omega \) resistor and the internal resistance are in series, so the current can be determined from \( \xi = I(r+R) \), which is Ohm’s law as applied to the entire series circuit.

\[ R = 1.40 \Omega \]

\[ I \]

\[ A \quad - \quad r \quad + \quad B \]

\[ \xi \]

**SOLUTION** The terminal voltage \( V_{AB} \) is equal to the emf \( \xi \) of the battery minus the voltage across the internal resistance \( r \), which is \( Ir \) (Equation 20.2): \( V_{AB} = \xi - Ir \). Solving this equation for \( r \) gives

\[ r = \frac{\xi - V_{AB}}{I} \quad (1) \]

To determine the current \( I \), we resort to Ohm’s law as applied to the entire series circuit, which is \( \xi = I(r+R) \). Solving this expression for \( I \) and substituting the result into Equation (1) reveals that

\[ r = \frac{\xi - V_{AB}}{I} = \frac{\xi - V_{AB}}{\xi / (r+R)} = \frac{(\xi - V_{AB})(r+R)}{\xi} \]

Solving this result for \( r \), we find that

\[ r = \left( \frac{\xi - V_{AB}}{V_{AB}} \right) R = \left[ \frac{(9.00 \text{ V}) - (8.30 \text{ V})}{8.30 \text{ V}} \right] (1.40 \text{ \( \Omega \)}) = 0.12 \text{ \( \Omega \)} \]
75. **REASONING** The following drawing shows the battery (emf = $\xi$), its internal resistance $r$, and the light bulb (represented as a resistor). The voltage between the terminals of the battery is the voltage $V_{AB}$ between the points $A$ and $B$ in the drawing. This voltage is not equal to the emf of the battery, because part of the emf is needed to make the current $I$ go through the internal resistance. Ohm’s law states that this part of the emf is $Ir$. The current can be determined from the relation $P = IV_{AB}$, since the power $P$ delivered to the light bulb and the voltage $V_{AB}$ across it are known.

![Light bulb diagram](image)

**SOLUTION** The terminal voltage $V_{AB}$ is equal to the emf $\xi$ of the battery minus the voltage across the internal resistance $r$, which is $Ir$ (Equation 20.2): $V_{AB} = \xi - Ir$. Solving this equation for the emf gives

$$\xi = V_{AB} + Ir$$

(1)

The current also goes through the light bulb, and it is related to the power $P$ delivered to the bulb and the voltage $V_{AB}$ according to $I = P/V_{AB}$ (Equation 20.6a). Substituting this expression for the current into Equation (1) gives

$$\xi = V_{AB} + Ir = V_{AB} + \left(\frac{P}{V_{AB}}\right)r$$

$$= 11.8 \ V + \left(\frac{24 \ W}{11.8 \ V}\right)(0.10 \ \Omega) = [12.0 \ V]$$

76. **REASONING** When the terminal voltage of the battery (emf = 9.00 V) is 8.90 V, the voltage drop across the internal resistance $r$ is $9.00 \ V - 8.90 \ V = 0.10 \ V$. According to Ohm’s law, this voltage drop is the current $I$ times the internal resistance. Thus, Ohm’s law will allow us to calculate the current.

**SOLUTION** Using Ohm’s law we find that

$$I = \frac{V}{r} = \frac{0.10 \ V}{0.012 \ \Omega} = [8.3 \ A]$$
77. **REASONING** The voltage $V$ across the terminals of a battery is equal to the emf of the battery minus the voltage $V_r$ across the internal resistance of the battery: $V = \text{Emf} - V_r$.

Therefore, we have that

$$\text{Emf} = V + V_r \quad (1)$$

The power $P$ being dissipated by the internal resistance is equal to the product of the voltage $V_r$ across the internal resistance and the current $I$: $P = IV_r$ (Equation 20.6a). Therefore, we can express the voltage $V_r$ across the internal resistance as

$$V_r = \frac{P}{I} \quad (2)$$

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

$$\text{Emf} = V + V_r = V + \frac{P}{I} = 23.4 \text{ V} + \frac{34.0 \text{ W}}{55.0 \text{ A}} = 24.0 \text{ V}$$

78. **REASONING** The power $P$ delivered to a resistor such as the light bulb is found from

$$P = I^2R \quad (20.6b)$$

where $I$ is the current in the resistor and $R$ is the resistance. When either battery is connected to the bulb, the circuit can be modeled as a single-loop circuit with the battery emf in series with two resistors, namely, the internal resistance $r$ and the resistance $R$ of the bulb. As the two resistors are in series, their equivalent resistance $R_s$ is given by $R_s = R_1 + R_2 + R_3 + \ldots$ (Equation 20.16):

$$R_s = r + R \quad (1)$$

The current $I$ supplied by a battery connected to a resistor $R_s$ is, according to Ohm’s law,

$$I = \frac{V}{R_s} \quad (20.2)$$

where $V$ is equal to the emf of the battery. Thus, $V$ is the same for both circuits, since both batteries have the same emf, even though their internal resistances ($r_{\text{wet}}$, $r_{\text{dry}}$) are different.

**SOLUTION** Applying Equation 20.6b separately to each circuit, we obtain

$$P_{\text{wet}} = I_{\text{wet}}^2R \quad \text{and} \quad P_{\text{dry}} = I_{\text{dry}}^2R \quad (2)$$

Taking the ratio of Equations (2) eliminates the bulb resistance $R$, yielding

$$\frac{P_{\text{wet}}}{P_{\text{dry}}} = \frac{I_{\text{wet}}^2}{I_{\text{dry}}^2} = \frac{I_{\text{wet}}}{I_{\text{dry}}}$$

$$P_{\text{wet}} = \frac{I_{\text{wet}}^2}{I_{\text{dry}}^2} \frac{I_{\text{wet}}}{I_{\text{dry}}} \quad (3)$$

Substituting Equation (20.2) for both $I_{\text{wet}}$ and $I_{\text{dry}}$ into Equation (3), we obtain
\[
\frac{P_{\text{wet}}}{P_{\text{dry}}} = \frac{I_{\text{wet}}^2}{I_{\text{dry}}^2} = \left( \frac{\sqrt{V}}{R_{S,\text{wet}}} \right)^2 = \left( \frac{R_{S,\text{dry}}}{R_{S,\text{wet}}} \right)^2
\]  

(4)

Replacing the equivalent series resistances in Equation (4) by Equation (1), we find that

\[
\frac{P_{\text{wet}}}{P_{\text{dry}}} = \left( \frac{R_{S,\text{dry}}}{R_{S,\text{wet}}} \right)^2 = \left( \frac{r_{\text{dry}} + R}{r_{\text{wet}} + R} \right)^2 = \frac{0.33 \Omega + 1.50 \Omega}{0.050 \Omega + 1.50 \Omega} = 1.39
\]

79. **SSM REASONING** The current \( I \) can be found by using Kirchhoff's loop rule. Once the current is known, the voltage between points \( A \) and \( B \) can be determined.

**SOLUTION**

a. We assume that the current is directed clockwise around the circuit. Starting at the upper-left corner and going clockwise around the circuit, we set the potential drops equal to the potential rises:

\[
(5.0 \Omega)I + (27 \Omega)I + 10.0 \text{ V} + (12 \Omega)I + (8.0 \Omega)I = 30.0 \text{ V}
\]

Potential drops

Potential rises

Solving for the current gives \( I = 0.38 \text{ A} \).

b. The voltage between points \( A \) and \( B \) is

\[
V_{AB} = 30.0 \text{ V} - (0.38 \text{ A})(27 \Omega) = 2.0 \times 10^1 \text{ V}
\]

c. **Point B** is at the higher potential.

80. **REASONING** Because all currents in the diagram flow from left to right (see the drawing), currents \( I_1 \) and \( I_2 \) both flow into junction \( A \). Therefore, the current \( I \) in resistor \( R \), which flows out of junction \( A \), is, by Kirchhoff’s junction rule, equal to the sum of the other two currents:

\[
I = I_1 + I_2
\]  

(1)

The current \( I_1 \) is given. To determine \( I_2 \), we note that the resistors \( R_1 \) and \( R_2 \) are connected in parallel, and therefore must have the same potential difference \( V_{12} = V_1 = V_2 \) across them.
We will use Ohm’s law \( V = IR \) (Equation 20.2) to determine the potential difference \( V_{12} \) and then the current \( I_2 \).

**SOLUTION** Using \( V = IR \) (Equation 20.2), we express the voltage \( V_{12} \) across the resistors \( R_1 \) and \( R_2 \) in terms of currents and resistances as

\[
V_{12} = I_1 R_1 = I_2 R_2 \quad (2)
\]

Solving Equation (2) for \( I_2 \) yields

\[
I_2 = \frac{I_1 R_1}{R_2} \quad (3)
\]

Substituting Equation (3) into Equation (1), we obtain

\[
I = I_1 + I_2 = I_1 + \frac{I_1 R_1}{R_2} = I_1 \left( 1 + \frac{R_1}{R_2} \right) = (3.00 \text{ A}) \left( 1 + \frac{2.70 \Omega}{4.40 \Omega} \right) = 4.84 \text{ A}
\]

81. **REASONING AND SOLUTION** Label the currents with the resistor values. Take \( I_3 \) to the right, \( I_2 \) to the left and \( I_1 \) to the right. Applying the loop rule to the top loop (suppressing the units) gives

\[
I_1 + 2.0 I_2 = 1.0 \quad (1)
\]

and to the bottom loop gives

\[
2.0 I_2 + 3.0 I_3 = 5.0 \quad (2)
\]

Applying the junction rule to the left junction gives

\[
I_2 = I_1 + I_3 \quad (3)
\]

Solving Equations (1), (2) and (3) simultaneously, we find \( I_2 = 0.73 \text{ A} \).

The positive sign shows that the assumed direction is correct. That is, to the left.

82. **REASONING** In part \( a \) of the drawing in the text, the current goes from left-to-right through the resistor. Since the current always goes from a higher to a lower potential, the left end of the resistor is + and the right end is –. In part \( b \), the current goes from right-to-left through the resistor. The right end of the resistor is + and the left end is –. The potential drops and rises for the two cases are:

<table>
<thead>
<tr>
<th></th>
<th>Potential drops</th>
<th>Potential rises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part ( a )</td>
<td>( IR )</td>
<td>( V )</td>
</tr>
<tr>
<td>Part ( b )</td>
<td>( V )</td>
<td>( IR )</td>
</tr>
</tbody>
</table>
**SOLUTION**  Since the current $I$ goes from left-to-right through the 3.0-$\Omega$ and 4.0-$\Omega$ resistors, the left end of each resistor is $+$ and the right end is $-$. The current goes through the 5.0-$\Omega$ resistor from right-to-left, so the right end is $+$ and the left end is $-$. Starting at the upper left corner of the circuit, and proceeding clockwise around it, Kirchhoff’s loop rule is written as

$$\left(3.0 \ \Omega\right)I + 12 \ \text{V} + \left(4.0 \ \Omega\right)I + (5.0 \ \Omega)I = 36 \ \text{V}$$

Solving this equation for the current gives $I = 2.0 \ \text{A}$

83. **REASONING**  This problem can be solved by using Kirchhoff’s loop rule. We begin by drawing a current through each resistor. The drawing shows the directions chosen for the currents. The directions are arbitrary, and if any one of them is incorrect, then the analysis will show that the corresponding value for the current is negative.

We mark the two ends of each resistor with plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential ($+$) toward a lower potential ($-$). Thus, given the directions chosen for $I_1$ and $I_2$, the plus and minus signs *must* be those shown in the drawing. We then apply Kirchhoff’s loop rule to the top loop $(ABCF)$ and to the bottom loop $(FCDE)$ to determine values for the currents $I_1$ and $I_2$.

**SOLUTION**  Applying Kirchhoff’s loop rule to the top loop $(ABCF)$ gives

$$\frac{V_1 + I_2R_2}{I_1R_1} =$$

Similarly, for the bottom loop $(FCDE)$,
Solving Equation (2) for $I_2$ gives

$$I_2 = \frac{V_2}{R_2} = \frac{12 \text{ V}}{2.0 \Omega} = 6.0 \text{ A}$$

Since $I_2$ is a positive number, the current in the resistor $R_2$ goes from left to right, as shown in the drawing. Solving Equation (1) for $I_1$ and substituting $I_2 = V_2/R_2$ into the resulting expression yields

$$I_1 = \frac{V_1 + I_2 R_2}{R_1} = \frac{V_1 + \left(\frac{V_2}{R_2}\right) R_2}{R_1} = \frac{V_1 + V_2}{R_1} = \frac{4.0 \text{ V} + 12 \text{ V}}{8.0 \Omega} = 2.0 \text{ A}$$

Since $I_1$ is a positive number, the current in the resistor $R_1$ goes from left to right, as shown in the drawing.

84. **REASONING** We begin by labeling the currents in the three resistors. The drawing below shows the directions chosen for these currents. The directions are arbitrary, and if any of them is incorrect, then the analysis will show that the corresponding value for the current is negative.

![Diagram](image)

We then mark the resistors with the plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (−). Thus, given the directions chosen for $I_1$, $I_2$, and $I_3$, the plus and minus signs must be those shown in the drawing. We can now use Kirchhoff’s rules to find the voltage across the 5.0-Ω resistor.

**SOLUTION** Applying the loop rule to the left loop (and suppressing units for convenience) gives

$$5.0I_1 + 10.0I_3 + 2.0 = 10.0$$

\[ (1) \]
Similarly, for the right loop,
\[10.0I_2 + 10.0I_3 + 2.0 = 15.0\]  \hspace{1cm} (2)

If we apply the junction rule to the upper junction, we obtain
\[I_1 + I_2 = I_3\]  \hspace{1cm} (3)

Subtracting Equation (2) from Equation (1) gives
\[5.0I_1 - 10.0I_2 = -5.0\]  \hspace{1cm} (4)

We now multiply Equation (3) by 10 and add the result to Equation (2); the result is
\[10.0I_1 + 20.0I_2 = 13.0\]  \hspace{1cm} (5)

If we then multiply Equation (4) by 2 and add the result to Equation (5), we obtain
20.0I_1 = 3.0, or solving for I_1, we obtain I_1 = 0.15 A. The fact that I_1 is positive means that the current in the drawing has the correct direction. The voltage across the 5.0-Ω resistor can be found from Ohm's law:
\[V = (0.15 \text{ A})(5.0 \text{ Ω}) = 0.75 \text{ V}\]

Current flows from the higher potential to the lower potential, and the current through the 5.0-Ω flows from left to right, so the left end of the resistor is at the higher potential.

85. **REASONING** In preparation for applying Kirchhoff’s rules, we now choose the currents in each resistor. The directions of the currents are arbitrary, and should they be incorrect, the currents will turn out to be negative quantities. Having chosen the currents, we also mark the ends of the resistors with the plus and minus signs that indicate that the currents are directed from higher (+) toward lower (−) potential. These plus and minus signs will guide us when we apply Kirchhoff’s loop rule.

**SOLUTION** Applying the junction rule to junction B, we find
Applying the loop rule to loop ABCD (going clockwise around the loop), we obtain
\[
I_1 (2.00 \ \Omega) = 6.00 \ \text{V} + I_3 (4.00 \ \Omega) + 3.00 \ \text{V}
\]  
(2)

Applying the loop rule to loop BEFC (going clockwise around the loop), we obtain
\[
I_2 (8.00 \ \Omega) + 9.00 \ \text{V} + I_3 (4.00 \ \Omega) + 6.00 \ \text{V} = 0
\]  
(3)

Substituting \(I_2\) from Equation (1) into Equation (3) gives
\[
(I_1 + I_3) (8.00 \ \Omega) + 9.00 \ \text{V} + I_3 (4.00 \ \Omega) + 6.00 \ \text{V} = 0
\]  
(4)

Solving Equation (2) for \(I_1\) gives
\[
I_1 = 4.50 \ \text{A} + I_3 (2.00)
\]

This result may be substituted into Equation (4) to show that
\[
\left[ 4.50 \ \text{A} + I_3 (2.00) \right] (8.00 \ \Omega) + I_3 (12.00 \ \Omega) + 15.00 \ \text{V} = 0
\]

\[
I_3 (28.00 \ \Omega) + 51.00 \ \text{V} = 0 \quad \text{or} \quad I_3 = \frac{-51.00 \ \text{V}}{28.00 \ \Omega} = -1.82 \ \text{A}
\]

The minus sign indicates that the current in the 4.00-\(\Omega\) resistor is directed downward, rather than upward as selected arbitrarily in the drawing.

86. **REASONING**

a. Currents \(I_5\) and \(I_2\) both go into junction \(C\) (see the drawing). This combined current \(I_5 + I_2\) passes through the battery and splits up again at junction \(A\), with \(I_1 = 9.0 \ \text{A}\) going to the resistor \(R_1\), and \(I_3 = 12.0 \ \text{A}\) going to the resistor \(R_3\). By Kirchhoff’s junction rule, therefore, the sum of the first pair of currents must equal the sum of the second pair of currents.
b. We will apply Kirchhoff’s loop rule to loop CAB (see the drawing) and solve for the resistance $R_2$, which is the only unknown variable that appears in the loop rule. The + and – signs on either end of the resistors indicate that current flows from higher potential (+) to lower potential (−). They do not indicate that the ends of the resistors are charged in any way.

c. We will find $R_3$ by writing out Kirchhoff’s loop rule for the loop CAED (see the drawing).

**SOLUTION**

a. As noted in the REASONING, the sum of the currents flowing into the battery must equal the sum of the currents flowing out of the battery:

$$I_2 + I_5 = I_1 + I_3$$

so

$$I_5 = I_1 + I_3 - I_2 = 9.0 \text{ A} + 12.0 \text{ A} - 6.0 \text{ A} = 15.0 \text{ A}$$

b. To apply Kirchhoff’s loop rule to loop CAB (see the drawing), we imagine traversing the loop counter-clockwise. Observing the + and – signs encountered along the way, we see that the potential rises when we cross the battery, and drops when we cross each resistor. The sum of the potential rises must equal the sum of the potential drops, so we have that

$$V = I_1 R_1 + I_2 R_2$$

Solving Equation (1) for $R_2$, we obtain

$$I_2 R_2 = V - I_1 R_1$$

or

$$R_2 = \frac{V - I_1 R_1}{I_2} = \frac{75.0 \text{ V} - (9.0 \text{ A})(4.0 \Omega)}{6.0 \text{ A}} = 6.5 \Omega$$

c. This time, we apply the loop rule to loop CAED (see the drawing), traversing it clockwise. This procedure yields

$$V = I_3 R_3 + I_5 R_5$$

Solving Equation (2) for $R_3$, we obtain

$$I_3 R_3 = V - I_5 R_5$$

or

$$R_3 = \frac{V - I_5 R_5}{I_3} = \frac{75.0 \text{ V} - (15.0 \text{ A})(2.2 \Omega)}{12.0 \text{ A}} = 3.5 \Omega$$

87. **SSM REASONING** As discussed in Section 20.11, some of the current (6.20 mA) goes directly through the galvanometer and the remainder $I$ goes through the shunt resistor. Since the resistance of the coil $R_C$ and the shunt resistor $R$ are in parallel, the voltage across each is the same. We will use this fact to determine how much current goes through the shunt
resistor. This value, plus the 6.20 mA that goes through the galvanometer, is the maximum current that this ammeter can read.

**SOLUTION** The voltage across the coil resistance is equal to the voltage across the shunt resistor, so

\[
\left(6.20 \times 10^{-3} \text{ A}\right)\left(20.0 \Omega\right) = \left(I\right)\left(24.8 \times 10^{-3} \Omega\right)
\]

So \(I = 5.00\) A. The maximum current is \(5.00\) A + 6.20 mA = \(5.01\) A.

---

88. **REASONING** The following drawing shows a galvanometer that is being used as a nondigital voltmeter to measure the potential difference between the points A and B. The coil resistance is \(R_C\) and the series resistance is \(R\). The voltage \(V_{AB}\) between the points A and B is equal to the voltage across the coil resistance \(R_C\) plus the voltage across the series resistance \(R\).

![Diagram of a galvanometer as a voltmeter](image)

**SOLUTION** Ohm’s law (Equation 20.2) gives the voltage across the coil resistance as \(IR_C\) and that across the series resistor as \(IR\). Thus, the voltage between A and B is

\[V_{AB} = IR_C + IR\]

Solving this equation for \(R\) yields

\[R = \frac{V_{AB} - IR_C}{I} = \frac{10.0 \text{ V} - (0.400 \times 10^{-3} \text{ A})(60.0 \Omega)}{0.400 \times 10^{-3} \text{ A}} = 2.49 \times 10^3 \Omega\]

---

89. **REASONING** According to Ohm’s law and under full-scale conditions, the voltage \(V\) across the equivalent resistance \(R_{eq}\) of the voltmeter is \(V = IR_{eq}\) where I is the full-scale current of the galvanometer. The full-scale current is not given in the problem statement. However, it is the same for both voltmeters. By applying Ohm’s law to each voltmeter, we will be able to eliminate the current algebraically and calculate the full-scale voltage of voltmeter B.

**SOLUTION** Applying Ohm’s law to each voltmeter under full-scale conditions, we obtain

\[V_A = IR_{eq, A} \quad \text{and} \quad V_B = IR_{eq, B}\]
Dividing these two equations and eliminating the full-scale current $I$ give

$$\frac{V_B}{V_A} = \frac{IR_{eq,B}}{IR_{eq,A}} = \frac{R_{eq,B}}{R_{eq,A}} \quad \text{or} \quad V_B = V_A \frac{R_{eq,B}}{R_{eq,A}} = (50.0 \, \text{V}) \frac{1.44 \times 10^5 \, \Omega}{2.40 \times 10^5 \, \Omega} = 30.0 \, \text{V}$$

90. **REASONING** The drawing at the right shows the galvanometer (G), the coil resistance $R_C$ and the shunt resistance $R$. The full-scale current $I_G$ through the galvanometer and the current $I_S$ in the shunt resistor are also shown. Note that $R_C$ and $R$ are in parallel, so that the voltage across each of them is the same. Our solution is based on this fact. The equivalent resistance of the ammeter is the parallel equivalent resistance of $R_C$ and $R$.

**SOLUTION** Expressing voltage as the product of current and resistance and noting that the voltages across $R_C$ and $R$ are the same, we have

$$\frac{I_G R_C}{V} = \frac{I_S R}{V}$$

Solving this equation for the full-scale current $I_G$ through the galvanometer gives

$$I_G = \frac{I_S R}{R_C}$$

In this result $I_S$ and $R_C$ are given, but the shunt resistance $R$ is unknown. However, the equivalent resistance is given as $R_p = 0.40 \, \Omega$, and it is the parallel equivalent resistance of $R_C$ and $R$: $\frac{1}{R} = \frac{1}{R_C} + \frac{1}{R}$ (Equation 20.17). This equation can be solved for $R$ as follows:

$$\frac{1}{R} = \frac{1}{R_C} + \frac{1}{R} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_p} - \frac{1}{R_C} = \frac{R_C - R_p}{R_p R_C} \quad \text{or} \quad R = \frac{R_p R_C}{R_C - R_p}$$

Substituting this result for $R$ into the expression for $I_G$ gives

$$I_G = \frac{I_S R}{R_C} = \frac{I_S}{R_C} \left( \frac{R_p R_C}{R_C - R_p} \right) = I_S \left( \frac{R_p}{R_C - R_p} \right) = \left(3.00 \times 10^{-3} \, \text{A} \right) \left[ \frac{0.40 \, \Omega}{(9.00 \, \Omega) - (0.40 \, \Omega)} \right] = 1.4 \times 10^{-4} \, \text{A}$$
91. **REASONING AND SOLUTION** For the 20.0 V scale

\[ V_1 = I(R_1 + R_c) \]

For the 30.0 V scale

\[ V_2 = I(R_2 + R_c) \]

Subtracting and rearranging yields

\[ I = \frac{V_2 - V_1}{R_2 - R_1} = \frac{30.0 \text{ V} - 20.0 \text{ V}}{2930 \Omega - 1680 \Omega} = 8.00 \times 10^{-3} \text{ A} \]

Substituting this value into either of the equations for \( V_1 \) or \( V_2 \) gives \( R_c = 820 \Omega \).

92. **REASONING AND SOLUTION**

a. According to Ohm's law (Equation 20.2, \( V = IR \)) the current in the circuit is

\[ I = \frac{V}{R + R} = \frac{V}{2R} \]

The voltage across either resistor is \( IR \), so that we find

\[ IR = \left( \frac{V}{2R} \right) R = \frac{V}{2} = \frac{60.0 \text{ V}}{2} = 30.0 \text{ V} \]

b. The voltmeter’s resistance is \( R_v = V/I = (60.0 \text{ V})/(5.00 \times 10^{-3} \text{ A}) = 12.0 \times 10^3 \Omega \), and this resistance is in parallel with the resistance \( R = 1550 \Omega \). The equivalent resistance of this parallel combination can be obtained as follows:

\[ \frac{1}{R_p} = \frac{1}{R_v} + \frac{1}{R} \quad \text{or} \quad R_p = \frac{RR_v}{R + R_v} = \frac{(1550 \Omega)(12.0 \times 10^3 \Omega)}{1550 \Omega + 12.0 \times 10^3 \Omega} = 1370 \Omega \]

The voltage registered by the voltmeter is \( IR_p \), where \( I \) is the current supplied by the battery to the series combination of the other 1550-\( \Omega \) resistor and \( R_p \). According to Ohm’s law

\[ I = \frac{60.0 \text{ V}}{1550 \Omega + 1370 \Omega} = 0.0205 \text{ A} \]

Thus, the voltage registered by the voltmeter is

\[ IR_p = (0.0205 \text{ A})(1370 \Omega) = 28.1 \text{ V} \]

93. **REASONING** A capacitor with a capacitance \( C \) stores a charge \( q \) when connected between the terminals of a battery of voltage \( V \), according to \( q = CV \) (Equation 19.8). This equation can be used to calculate the voltage if \( q \) and \( C \) are known. In this problem we know that the two capacitors together store a total charge of \( 5.4 \times 10^{-5} \text{ C} \). This is also the charge stored by
the equivalent capacitor, which has the equivalent capacitance $C_p$ of the parallel combination, or $C_p = C_1 + C_2$ (Equation 20.18). Thus, we can determine the battery voltage by using Equation 19.8 with the given total charge and the equivalent capacitance obtained from Equation 20.18.

**SOLUTION** With $q = q_{\text{Total}}$ and $C = C_p = C_1 + C_2$ Equation 19.8 becomes

$$q_{\text{Total}} = C_p V = (C_1 + C_2)V \quad \text{or} \quad V = \frac{q_{\text{Total}}}{C_1 + C_2} = \frac{5.4 \times 10^{-5} \text{ C}}{2.0 \times 10^{-6} \text{ F} + 4.0 \times 10^{-6} \text{ F}} = 9.0 \text{ V}$$

94. **REASONING** The capacitance $C$ of a parallel plate capacitor is $C = \kappa \varepsilon_0 A/d$ (Equation 19.10), where $\kappa$ is the dielectric constant of the material between the plates, $\varepsilon_0$ is the permittivity of free space (a constant), $A$ is the area of each plate, and $d$ is the separation between the plates. The equivalent capacitance $C_S$ of three capacitors in series can be determined according to $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ (Equation 20.19), where $C_1$, $C_2$, and $C_3$ are the individual capacitances. Applying Equation 19.10 to the equivalent capacitance, we recognize that the dielectric constant becomes $\kappa_S$, which is the quantity that we seek.

**SOLUTION** Using Equation 20.19 for the series combination and using Equation 19.10 for $C_S$, $C_1$, $C_2$, and $C_3$, we have

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{or} \quad \frac{1}{\kappa_S \varepsilon_0 A/d} = \frac{1}{\kappa_1 \varepsilon_0 A/d} + \frac{1}{\kappa_2 \varepsilon_0 A/d} + \frac{1}{\kappa_3 \varepsilon_0 A/d}$$

In this result $\kappa_S$ is the dielectric constant of the single equivalent capacitor. Note that the same combination of constants $\varepsilon_0 A/d$ appears in every term on each side of the equals sign and may be eliminated algebraically from the result. Thus, we obtain

$$\frac{1}{\kappa_S} = \frac{1}{\kappa_1} + \frac{1}{\kappa_2} + \frac{1}{\kappa_3} = \frac{1}{3.30} + \frac{1}{5.40} + \frac{1}{6.70} = 0.637 \quad \text{or} \quad \kappa_S = 1.57$$

95. **SSM REASONING** The equivalent capacitance $C_S$ of a set of three capacitors connected in series is given by $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ (Equation 20.19). In this case, we know that the equivalent capacitance is $C_S = 3.00 \mu\text{F}$, and the capacitances of two of the individual capacitors in this series combination are $C_1 = 6.00 \mu\text{F}$ and $C_2 = 9.00 \mu\text{F}$. We will use Equation 20.19 to determine the remaining capacitance $C_3$.

**SOLUTION** Solving Equation 20.19 for $C_3$, we obtain
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Therefore, the third capacitance is

\[
C_3 = \frac{1}{\frac{1}{C_S} + \frac{1}{C_1} + \frac{1}{C_2}}
\]

or

\[
C_3 = \frac{1}{\frac{1}{C_S} - \frac{1}{C_1} - \frac{1}{C_2}}
\]

96. **REASONING** When capacitors are connected in parallel, each receives the entire voltage \( V \) of the battery. Thus, the total energy stored in the two capacitors is \( \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \) (see Equation 19.11b). When the capacitors are connected in series, the sum of the voltages across each capacitor equals the battery voltage: \( V_1 + V_2 = V \). Thus, the voltage across each capacitor is series is less than the battery voltage, so the total energy, \( \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \), is less than when the capacitors are wired in parallel.

**SOLUTION**

a. The voltage across each capacitor is the battery voltage, or 60.0 V. The energy stored in both capacitors is

\[
\text{Total energy} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2} (C_1 + C_2) V^2
\]

\[
= \frac{1}{2} (2.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F})(60.0 \text{ V})^2 = 1.08 \times 10^{-2} \text{ J}
\]

b. According to the discussion in Section 20.12, the total energy stored by capacitors in series is \( \text{Total energy} = \frac{1}{2} C_S V^2 \), where \( C_S \) is the equivalent capacitance of the series combination:

\[
\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00 \times 10^{-6} \text{ F}} + \frac{1}{4.00 \times 10^{-6} \text{ F}}
\]

Solving this equation yields \( C_S = 1.33 \times 10^{-6} \text{ F} \). The total energy is

\[
\text{Total energy} = \frac{1}{2} (1.33 \times 10^{-6} \text{ F})(60.0 \text{ V})^2 = 2.39 \times 10^{-3} \text{ J}
\]

97. **REASONING** Our approach to this problem is to deal with the arrangement in parts. We will combine separately those parts that involve a series connection and those that involve a parallel connection.

**SOLUTION** The 24, 12, and 8.0-\( \mu \text{F} \) capacitors are in series. Using Equation 20.19, we can find the equivalent capacitance for the three capacitors:
This 4.0-μF capacitance is in parallel with the 4.0-μF capacitance already shown in the text diagram. Using Equation 20.18, we find that the equivalent capacitance for the parallel group is

\[ C_p = 4.0 \, \mu F + 4.0 \, \mu F = 8.0 \, \mu F \]

This 8.0-μF capacitance is between the 5.0 and the 6.0-μF capacitances and in series with them. Equation 20.19 can be used, then, to determine the equivalent capacitance between A and B in the text diagram:

\[ \frac{1}{C_s} = \frac{1}{5.0 \, \mu F} + \frac{1}{8.0 \, \mu F} + \frac{1}{6.0 \, \mu F} \quad \text{or} \quad C_s = [2.0 \, \mu F] \]

98. **REASONING**

a. When capacitors are wired in parallel, the total charge \( q \) supplied to them is the sum of the charges supplied to the individual capacitors, or \( q = q_1 + q_2 \). The individual charges can be obtained from \( q_1 = C_1V \) and \( q_2 = C_2V \), since the capacitances, \( C_1 \) and \( C_2 \), and the voltage \( V \) are known.

b. When capacitors are wired in series, the voltage \( V \) across them is equal to the sum of the voltages across the individual capacitors, or \( V = V_1 + V_2 \). However, the charge \( q \) on each capacitor is the same. The individual voltages can be obtained from \( V_1 = q/C_1 \) and \( V_2 = q/C_2 \).

**SOLUTION**

a. Substituting \( q_1 = C_1V \) and \( q_2 = C_2V \) (Equation 19.8) into \( q = q_1 + q_2 \), we have

\[ q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V \]

\[ = (2.00 \times 10^{-6} \, F + 4.00 \times 10^{-6} \, F)(60.0 \, V) = 3.60 \times 10^{-4} \, C \]

b. Substituting \( V_1 = q/C_1 \) and \( V_2 = q/C_2 \) (Equation 19.8) into \( V = V_1 + V_2 \) gives

\[ V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} \]

Solving this relation for \( q \), we have

\[ q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{60.0 \, V}{\frac{1}{2.00 \times 10^{-6} \, F} + \frac{1}{4.00 \times 10^{-6} \, F}} = 8.00 \times 10^{-5} \, C \]
99. **REASONING AND SOLUTION** The charges stored on capacitors in series are equal and equal to the charge separated by the battery. The total energy stored in the capacitors is

\[
\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} \\
\text{Energy} = \frac{Q^2}{2} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)
\]

According to Equation 20.19, the quantity in the parentheses is just the reciprocal of the equivalent capacitance \(C\) of the circuit, so

\[
\text{Energy} = \frac{Q^2}{2C}
\]

100. **REASONING AND SOLUTION** The 7.00 and 3.00-\(\mu\)F capacitors are in parallel. According to Equation 20.18, the equivalent capacitance of the two is 7.00 \(\mu\)F + 3.00 \(\mu\)F = 10.0 \(\mu\)F. This 10.0-\(\mu\)F capacitance is in series with the 5.00-\(\mu\)F capacitance. According to Equation 20.19, the equivalent capacitance of the complete arrangement can be obtained as follows:

\[
\frac{1}{C} = \frac{1}{10.0 \ \mu\text{F}} + \frac{1}{5.00 \ \mu\text{F}} = 0.300 \ (\mu\text{F})^{-1} \quad \text{or} \quad C = \frac{1}{0.300 \ (\mu\text{F})^{-1}} = 3.33 \ \mu\text{F}
\]

The battery separates an amount of charge

\[
Q = CV = (3.33 \times 10^{-6} \ \text{F})(30.0 \ \text{V}) = 99.9 \times 10^{-6} \ \text{C}
\]

This amount of charge resides on the 5.00 \(\mu\)F capacitor, so its voltage is

\[
V_5 = (99.9 \times 10^{-6} \ \text{C})/(5.00 \times 10^{-6} \ \text{F}) = 20.0 \ \text{V}
\]

The loop rule gives the voltage across the 3.00 \(\mu\)F capacitor to be

\[
V_3 = 30.0 \ \text{V} - 20.0 \ \text{V} = 10.0 \ \text{V}
\]

This is also the voltage across the 7.00 \(\mu\)F capacitor, since it is in parallel, so \(V_7 = 10.0 \ \text{V}\).

101. **SSM REASONING** When two or more capacitors are in series, the equivalent capacitance of the combination can be obtained from Equation 20.19, \(\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\)

Equation 20.18 gives the equivalent capacitance for two or more capacitors in parallel: \(C_p = C_1 + C_2 + C_3 + \ldots\). The energy stored in a capacitor is given by \(\frac{1}{2} CV^2\), according to Equation 19.11. Thus, the energy stored in the series combination is \(\frac{1}{2} C_s V_s^2\), where
\[ \frac{1}{C_s} = \frac{1}{7.0 \, \mu F} + \frac{1}{3.0 \, \mu F} = 0.476 \, (\mu F)^{-1} \quad \text{or} \quad C_s = \frac{1}{0.476 \, (\mu F)^{-1}} = 2.10 \, \mu F \]

Similarly, the energy stored in the parallel combination is \( \frac{1}{2} C_p V_p^2 \) where
\[ C_p = 7.0 \, \mu F + 3.0 \, \mu F = 10.0 \, \mu F \]

The voltage required to charge the parallel combination of the two capacitors to the same total energy as the series combination can be found by equating the two energy expressions and solving for \( V_p \).

**SOLUTION** Equating the two expressions for the energy, we have
\[ \frac{1}{2} C_s V_s^2 = \frac{1}{2} C_p V_p^2 \]

Solving for \( V_p \), we obtain the result
\[ V_p = V_s \sqrt{\frac{C_s}{C_p}} = (24 \, V) \sqrt{\frac{2.10 \, \mu F}{10.0 \, \mu F}} = 11 \, V \]

102. **REASONING AND SOLUTION** Charge is conserved during the re-equilibrium. Therefore, using \( q_0 \) and \( q_f \) to denote the initial and final charges, respectively, we have
\[ q_{10} + q_{20} = 18.0 \, \mu C = q_{1f} + q_{2f} \quad (1) \]

After equilibrium has been established the capacitors will have equal voltages across them, since they are connected in parallel. Thus, \( V_f = q_{1f}/C_1 = q_{2f}/C_2 \), which leads to
\[ q_{1f} = q_{2f}(C_1/C_2) = q_{2f}(2.00 \, \mu F)/(8.00 \, \mu F) = 0.250 \, q_{2f} \]

Substituting this result into Equation (1) gives
\[ 18.0 \, \mu C = 0.250 \, q_{2f} + q_{2f} \quad \text{or} \quad q_{2f} = 14.4 \, \mu C \]

Hence,
\[ V_f = q_{2f}/C_2 = (14.4 \times 10^{-6} \, C)/(8.00 \times 10^{-6} \, F) = 1.80 \, V \]

103. **SSM REASONING** The charge \( q \) on a discharging capacitor in a \( RC \) circuit is given by Equation 20.22: \( q = q_0 e^{-t/RC} \), where \( q_0 \) is the original charge at time \( t = 0 \) s. Once \( t \) (time for one pulse) and the ratio \( q/q_0 \) are known, this expression can be solved for \( C \).

**SOLUTION** Since the pacemaker delivers 81 pulses per minute, the time for one pulse is
\[ \frac{1 \, \text{min}}{81 \, \text{pulses}} \times \frac{60.0 \, \text{s}}{1.00 \, \text{min}} = 0.74 \, \text{s/pulse} \]
Since one pulse is delivered every time the fully-charged capacitor loses 63.2% of its original charge, the charge remaining is 36.8% of the original charge. Thus, we have \( q = (0.368)q_0 \), or \( q/q_0 = 0.368 \).

From Equation 20.22, we have

\[
\frac{q}{q_0} = e^{-t/RC}
\]

Taking the natural logarithm of both sides, we have,

\[
\ln \left( \frac{q}{q_0} \right) = -\frac{t}{RC}
\]

Solving for \( C \), we find

\[
C = \frac{-t}{R \ln(q/q_0)} = \frac{-(0.74 \text{ s})}{(1.8 \times 10^6 \ \Omega) \ln(0.368)} = 4.1 \times 10^{-7} \text{ F}
\]

104. **REASONING**  The time constant \( \tau \) is related to the resistance \( R \) and capacitance \( C \), according to \( \tau = RC \) (Equation 20.21). The problem deals with two cases. In case A, we have a time constant \( \tau_A = 1.5 \text{ s} \), a resistance \( R_A = 2.0 \times 10^4 \ \Omega \), and a capacitance \( C \). In case B, we have a time constant \( \tau_B \), a resistance \( R_B = 5.2 \times 10^4 \ \Omega \), and a capacitance \( C \). We will apply Equation 20.21 to both cases and take advantage of the fact that the capacitance is the same in both.

**SOLUTION**  Applying Equation 20.21 to both cases, we have

\[
\tau_A = R_A C \quad \text{and} \quad \tau_B = R_B C
\]

Dividing the equation for case B by the equation for case A gives

\[
\frac{\tau_B}{\tau_A} = \frac{R_B C}{R_A C} = \frac{R_B}{R_A}
\]

Note that the unknown capacitance \( C \) has been eliminated algebraically from this result. Solving for the unknown time constant \( \tau_B \) gives

\[
\tau_B = \tau_A \left( \frac{R_B}{R_A} \right) = (1.5 \text{ s}) \left( \frac{5.2 \times 10^4 \ \Omega}{2.0 \times 10^4 \ \Omega} \right) = 3.9 \text{ s}
\]

105. **REASONING**  The time constant of an \( RC \) circuit is given by Equation 20.21 as \( \tau = RC \), where \( R \) is the resistance and \( C \) is the capacitance in the circuit. The two resistors are wired in parallel, so we can obtain the equivalent resistance by using Equation 20.17. The two capacitors are also wired in parallel, and their equivalent capacitance is given by
Equation 20.18. The time constant is the product of the equivalent resistance and equivalent capacitance.

**SOLUTION** The equivalent resistance of the two resistors in parallel is

\[
\frac{1}{R_p} = \frac{1}{2.0 \, \text{k}\Omega} + \frac{1}{4.0 \, \text{k}\Omega} = \frac{3}{4.0 \, \text{k}\Omega} \quad \text{or} \quad R_p = 1.3 \, \text{k}\Omega \tag{20.17}
\]

The equivalent capacitance is

\[
C_p = 3.0 \, \mu\text{F} + 6.0 \, \mu\text{F} = 9.0 \, \mu\text{F} \quad \text{(20.18)}
\]

The time constant for the charge to build up is

\[
\tau = R_p C_p = \left(1.3 \times 10^3 \, \text{\Omega}\right) \left(9.0 \times 10^{-6} \, \text{F}\right) = 1.2 \times 10^{-2} \, \text{s}
\]

106. **REASONING** The charging of a capacitor is described by Equation 20.20, which provides a direct solution to this problem.

**SOLUTION** According to Equation 20.20, in a series RC circuit the charge \( q \) on the capacitor at a time \( t \) is given by

\[
q = q_0 \left(1 - e^{-t/\tau}\right)
\]

where \( q_0 \) is the equilibrium charge that has accumulated on the capacitor after a very long time and \( \tau \) is the time constant. For \( q = 0.800 q_0 \) this equation becomes

\[
q = 0.800 q_0 = q_0 \left(1 - e^{-t/\tau}\right) \quad \text{or} \quad 0.200 = e^{-t/\tau}
\]

Taking the natural logarithm of both sides of this result gives

\[
\ln (0.200) = \ln \left(e^{-t/\tau}\right) \quad \text{or} \quad \ln (0.200) = -\frac{t}{\tau}
\]

Therefore, the number of time constants needed for the capacitor to be charged to 80.0% of its equilibrium charge is

\[
\frac{t}{\tau} = -\ln (0.200) = -(-1.61) = 1.61
\]

107. **REASONING** In either part of the drawing the time constant \( \tau \) of the circuit is \( \tau = R C_{\text{eq}} \), according to Equation 20.21, where \( R \) is the resistance and \( C_{\text{eq}} \) is the equivalent capacitance of the capacitor combination. We will apply this equation to both circuits. To obtain the equivalent capacitance, we will analyze the capacitor combination in parts. For the parallel capacitors \( C_p = C_1 + C_2 + C_3 + ... \) applies (Equation 20.18), while for the series capacitors \( C_s^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + ... \) applies (Equation 20.19).
SOLUTION Using Equation 20.21, we write the time constant of each circuit as follows:

\[ \tau_a = R C_{eq, a} \quad \text{and} \quad \tau_b = R C_{eq, b} \]

Dividing these two equations allows us to eliminate the unknown resistance algebraically:

\[
\frac{\tau_b}{\tau_a} = \frac{R C_{eq, b}}{R C_{eq, a}} \quad \text{or} \quad \tau_b = \tau_a \left( \frac{C_{eq, b}}{C_{eq, a}} \right)
\]

(1)

To obtain the equivalent capacitance in part a of the drawing, we note that the two capacitors in series in each branch of the parallel combination have an equivalent capacitance \( C_S \) that can be determined using Equation 20.19

\[
\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} \quad \text{or} \quad C_S = \frac{1}{2} C
\]

(2)

Using Equation 20.18, we find that the parallel combination in part a of the drawing has an equivalent capacitance of

\[
C_{eq, a} = \frac{1}{2} C + \frac{1}{2} C = C
\]

(3)

To obtain the equivalent capacitance in part b of the drawing, we note that the two capacitors in series have an equivalent capacitance of \( \frac{1}{2} C \), according to Equation (2). The two capacitors in parallel have an equivalent capacitance of \( 2C \), according to Equation 20.18. Finally, then, we have a series combination of \( \frac{1}{2} C \) and \( 2C \), for which Equation 20.19 applies:

\[
\frac{1}{C_{eq, b}} = \frac{1}{\frac{1}{2} C} + \frac{1}{2C} = \frac{5}{2C} \quad \text{or} \quad C_{eq, b} = \frac{2}{5} C
\]

(4)

Using Equations (3) and (4) in Equation (1), we find that

\[
\tau_b = \tau_a \left( \frac{C_{eq, b}}{C_{eq, a}} \right) = \left( 0.72 \text{ s} \right) \frac{\frac{2}{5} C}{C} = 0.29 \text{ s}
\]
108. **REASONING**

a. The power delivered to a resistor is given by Equation 20.6c as \( P = \frac{V^2}{R} \), where \( V \) is the voltage and \( R \) is the resistance. Because of the dependence of the power on \( V^2 \), doubling the voltage has a greater effect in increasing the power than halving the resistance. The following table shows the power for each circuit, given in terms of these variables, and confirms this fact. The table also gives the expected ranking, in decreasing order, of the power.

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( P = \frac{V^2}{R} )</td>
<td>3</td>
</tr>
<tr>
<td>( b )</td>
<td>( P = \frac{V^2}{2R} )</td>
<td>4</td>
</tr>
<tr>
<td>( c )</td>
<td>( P = \frac{(2V)^2}{R} = \frac{4V^2}{R} )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>( P = \frac{(2V)^2}{2R} = \frac{2V^2}{R} )</td>
<td>2</td>
</tr>
</tbody>
</table>

b. The current is given by Equation 20.2 as \( I = \frac{V}{R} \). Note that the current, unlike the power, depends linearly on the voltage. Therefore, either doubling the voltage or halving the resistance has the same effect on the current. The following table shows the current for the four circuits and confirms this fact. The table also gives the expected ranking, in decreasing order, of the current.

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( I = \frac{V}{R} )</td>
<td>2</td>
</tr>
<tr>
<td>( b )</td>
<td>( I = \frac{V}{2R} )</td>
<td>3</td>
</tr>
<tr>
<td>( c )</td>
<td>( I = \frac{2V}{R} )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>( I = \frac{2V}{2R} = \frac{V}{R} )</td>
<td>2</td>
</tr>
</tbody>
</table>
SOLUTION

a. Using the results from the REASONING and the values of $V = 12.0 \text{ V}$ and $R = 6.00 \text{ }\Omega$, we find that the power dissipated in each resistor is

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$P = \frac{V^2}{R} = \frac{(12.0 \text{ V})^2}{6.00 \text{ }\Omega} = 24.0 \text{ W}$</td>
<td>3</td>
</tr>
<tr>
<td>$b$</td>
<td>$P = \frac{V^2}{2R} = \frac{(12.0 \text{ V})^2}{2(6.00 \text{ }\Omega)} = 12.0 \text{ W}$</td>
<td>4</td>
</tr>
<tr>
<td>$c$</td>
<td>$P = \frac{4V^2}{R} = \frac{4(12.0 \text{ V})^2}{(6.00 \text{ }\Omega)} = 96.0 \text{ W}$</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>$P = \frac{2V^2}{R} = \frac{2(12.0 \text{ V})^2}{(6.00 \text{ }\Omega)} = 48.0 \text{ W}$</td>
<td>2</td>
</tr>
</tbody>
</table>

b. Using the results from part (b) and the values of $V = 12.0 \text{ V}$ and $R = 6.00 \text{ }\Omega$, we find that the current in each circuit is

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$I = \frac{V}{R} = \frac{12.0 \text{ V}}{6.00 \text{ }\Omega} = 2.00 \text{ A}$</td>
<td>2</td>
</tr>
<tr>
<td>$b$</td>
<td>$I = \frac{V}{2R} = \frac{12.0 \text{ V}}{2(6.00 \text{ }\Omega)} = 1.00 \text{ A}$</td>
<td>3</td>
</tr>
<tr>
<td>$c$</td>
<td>$I = \frac{2V}{R} = \frac{2(12.0 \text{ V})}{6.00 \text{ }\Omega} = 4.00 \text{ A}$</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>$I = \frac{2V}{2R} = \frac{2(12.0 \text{ V})}{2(6.00 \text{ }\Omega)} = 2.00 \text{ A}$</td>
<td>2</td>
</tr>
</tbody>
</table>

109. SSM REASONING To find the equivalent capacitance of the three capacitors, we must first, following $C_p = C_1 + C_2 + C_3 + \cdots$ (Equation 20.18), add the capacitances of the two parallel capacitors together. We must then combine the result of Equation 20.18 with the remaining capacitance in accordance with $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ (Equation 20.19). As Equation 20.19 shows, combining capacitors in series decreases the overall capacitance, and the resulting equivalent capacitance $C_s$ is smaller than any of the capacitances being added.
Therefore, the way to maximize the overall equivalent capacitance is to choose the largest capacitance \( C_1 \) to be connected in series with the parallel combination \( C_{23} \) of the smaller capacitances.

**SOLUTION** When \( C_2 \) and \( C_3 \) are connected in parallel, their equivalent capacitance \( C_{23} \) is, from \( C_p = C_1 + C_2 + C_3 + \cdots \) (Equation 20.18),

\[
C_{23} = C_2 + C_3 \tag{1}
\]

When the equivalent capacitance \( C_{23} \) is connected in series with \( C_1 \), the resulting equivalent capacitance \( C_S \) is, according to Equation 20.19 and Equation (1),

\[
C_S = \left( \frac{1}{C_S} \right)^{-1} = \left( \frac{1}{C_1} + \frac{1}{C_{23}} \right)^{-1} = \left( \frac{1}{C_1} + \frac{1}{C_2 + C_3} \right)^{-1} = \left( \frac{1}{67 \, \mu F} + \frac{1}{45 \, \mu F + 33 \, \mu F} \right)^{-1} = 36 \, \mu F
\]

110. **REASONING** Electric current is the amount of charge flowing per unit time (see Equation 20.1). Thus, the amount of charge is the current times the time. Furthermore, the potential difference is the difference in electric potential energy per unit charge (see Equation 19.4), so that, once the amount of charge has been calculated, we can determine the energy by multiplying the potential difference by the charge.

**SOLUTION**

a. According to Equation 20.1, the current \( I \) is the amount of charge \( \Delta q \) divided by the time \( \Delta t \), or \( I = \Delta q / \Delta t \). Therefore, the additional charge that passes through the machine in normal mode versus standby mode is

\[
q_{\text{additional}} = \frac{I_{\text{normal}} \Delta t}{\Delta q \text{ in normal mode}} - \frac{I_{\text{standby}} \Delta t}{\Delta q \text{ in standby mode}} = \left( I_{\text{normal}} - I_{\text{standby}} \right) \Delta t
\]

\[
= (0.110 \, \text{A} - 0.067 \, \text{A})(60.0 \, \text{s}) = 2.6 \, \text{C}
\]

b. According to Equation 19.4, the potential difference \( \Delta V \) is the difference \( \Delta (\text{EPE}) \) in the electric potential energy divided by the charge \( q_{\text{additional}} \). As a result, the additional energy used in the normal mode as compared to the standby mode is

\[
\Delta (\text{EPE}) = q_{\text{additional}} \Delta V = (2.6 \, \text{C})(120 \, \text{V}) = 310 \, \text{J}
\]

111. **SSM REASONING AND SOLUTION** Ohm’s law (Equation 20.2), \( V = IR \), gives the result directly:

\[
R = \frac{V}{I} = \frac{9.0 \, \text{V}}{0.11 \, \text{A}} = 82 \, \Omega
\]
112. **REASONING** First, we draw a current $I_1$ (directed to the right) in the 6.00-$\Omega$ resistor. We can express $I_1$ in terms of the other currents in the circuit, $I$ and 3.00 A, by applying the junction rule to the junction on the left; the sum of the currents into the junction must equal the sum of the currents out of the junction.

\[
I = I_1 + 3.00 \text{ A} \quad \text{or} \quad I_1 = I - 3.00 \text{ A}
\]

In order to obtain values for $I$ and $V$ we apply the loop rule to the top and bottom loops of the circuit.

**SOLUTION** Applying the loop rule to the top loop (going clockwise around the loop), we have

\[
(3.00 \text{ A})(4.00 \Omega) + (3.00 \text{ A})(8.00 \Omega) = 24.0 \text{ V} + (I - 3.00 \text{ A})(6.00 \Omega)
\]

This equation can be solved directly for the current; $I = 5.00 \text{ A}$. Applying the loop rule to the bottom loop (going counterclockwise around the loop), we have

\[
(I - 3.00 \text{ A})(6.00 \Omega) + 24.0 \text{ V} + I(2.00 \Omega) = V
\]

Substituting $I = 5.00 \text{ A}$ into this equation and solving for $V$ gives $V = 46.0 \text{ V}$.

113. **REASONING AND SOLUTION** From Equation 20.5 we have that $R = R_0[1 + \alpha (T - T_0)]$. Solving for $T$ gives

\[
T = T_0 + \frac{R}{R_0} - 1 = 20.0 \degree \text{C} + \frac{99.6 \Omega}{125 \Omega} - 1 = -34.6 \degree \text{C}
\]

114. **REASONING** The magnitude $q$ of the charge on one plate of a capacitor is given by Equation 19.8 as $q = CV_1$, where $C = 9.0 \mu\text{F}$ and $V_1$ is the voltage across the capacitor. Since the capacitor and the resistor $R_1$ are in parallel, the voltage across the capacitor is equal to the voltage across $R_1$. From Equation 20.2 we know that the voltage across the 4.0-$\Omega$ resistor is given by $V_1 = IR_1$, where $I$ is the current in the circuit. Thus, the charge can be expressed as

\[
q = CV_1 = C\left(I R_1\right)
\]
The current is equal to the battery voltage $V$ divided by the equivalent resistance of the two resistors in series, so that

$$I = \frac{V}{R} = \frac{V}{R_1 + R_2}$$

Substituting this result for $I$ into the equation for $q$ yields

$$q = C\left(\frac{V}{R_1 + R_2}\right) R_1$$

**SOLUTION** The magnitude of the charge on one of the plates is

$$q = C\left(\frac{V}{R_1 + R_2}\right) R_1 = \left(9.0 \times 10^{-6} \text{ F}\right)\left(\frac{12 \text{ V}}{4.0 \Omega + 2.0 \Omega}\right)(4.0 \Omega) = 7.2 \times 10^{-5} \text{ C}$$

---

**REASONING** Since only 0.100 mA out of the available 60.0 mA is needed to cause a full-scale deflection of the galvanometer, the shunt resistor must allow the excess current of 59.9 mA to detour around the meter coil, as the drawing at the right indicates. The value for the shunt resistance can be obtained by recognizing that the 50.0-Ω coil resistance and the shunt resistance are in parallel, both being connected between points $A$ and $B$ in the drawing. Thus, the voltage across each resistance is the same.

**SOLUTION** Expressing voltage as the product of current and resistance, we find that

$$\left(59.9 \times 10^{-3} \text{ A}\right)(R) = \left(0.100 \times 10^{-3} \text{ A}\right)(50.0 \Omega)$$

$$R = \frac{\left(0.100 \times 10^{-3} \text{ A}\right)(50.0 \Omega)}{59.9 \times 10^{-3} \text{ A}} = 0.0835 \Omega$$

---

**REASONING** Ohm’s law relates the resistance $R$ of either resistor to the current $I$ in it and the voltage $V$ across it:

$$R = \frac{V}{I} \quad (20.2)$$

Because the two resistors are in series, they must have the same current $I$. We will, therefore, apply Equation 20.2 to the 86-Ω resistor to determine the current $I$. Following that, we will use Equation 20.2 again, to obtain the potential difference across the 67-Ω resistor.
**SOLUTION**  Let $R_1 = 86 \, \Omega$ be the resistance of the first resistor, which has a potential difference of $V_1 = 27 \, V$ across it. The current $I$ in this resistor, from Equation 20.2, is

$$ I = \frac{V_1}{R_1} \quad (1) $$

Let $R_2 = 67 \, \Omega$ be the resistance of the second resistor. Again employing Equation 20.2, the potential difference $V_2$ across this resistor is given by

$$ V_2 = IR_2 \quad (2) $$

Since the current in both resistors is the same, substituting Equation (1) into Equation (2) yields

$$ V_2 = IR_2 = \left( \frac{V_1}{R_1} \right) R_2 = \left( \frac{27 \, V}{86 \, \Omega} \right) (67 \, \Omega) = 21 \, V $$

117. [SSM] **REASONING**  Since we know that the current in the 8.00-Ω resistor is 0.500 A, we can use Ohm's law ($V = IR$) to find the voltage across the 8.00-Ω resistor. The 8.00-Ω resistor and the 16.0-Ω resistor are in parallel; therefore, the voltages across them are equal. Thus, we can also use Ohm's law to find the current through the 16.0-Ω resistor. The currents that flow through the 8.00-Ω and the 16.0-Ω resistors combine to give the total current that flows through the 20.0-Ω resistor. Similar reasoning can be used to find the current through the 9.00-Ω resistor.

**SOLUTION**

a. The voltage across the 8.00-Ω resistor is $V_8 = (0.500 \, A)(8.00\Omega) = 4.00 \, V$. Since this is also the voltage that is across the 16.0-Ω resistor, we find that the current through the 16.0-Ω resistor is $I_{16} = (4.00 \, V)/(16.0\Omega) = 0.250 \, A$. Therefore, the total current that flows through the 20.0-Ω resistor is

$$ I_{20} = 0.500 \, A + 0.250 \, A = 0.750 \, A $$

b. The 8.00-Ω and the 16.0-Ω resistors are in parallel, so their equivalent resistance can be obtained from Equation 20.17, $R_p = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$, and is equal to 5.33 Ω. Therefore, the equivalent resistance of the upper branch of the circuit is $R_{upper} = 5.33 \, \Omega + 20.0 \, \Omega = 25.3 \, \Omega$, since the 5.33-Ω resistance is in series with the 20.0-Ω resistance. Using Ohm's law, we find that the voltage across the upper branch must be $V = (0.750 \, A)(25.3 \, \Omega) = 19.0 \, V$. Since the lower branch is in parallel with the upper branch, the voltage across both branches must be the same. Therefore, the current through the 9.00-Ω resistor is, from Ohm's law,

$$ I_9 = \frac{V_{lower}}{R_9} = \frac{19.0 \, V}{9.00 \, \Omega} = 2.11 \, A $$
118. **REASONING AND SOLUTION**

a. In the first case the parallel resistance of the 75.0 Ω and the 45.0 Ω resistors have an equivalent resistance that can be calculated using Equation 20.17:

\[
\frac{1}{R_p} = \frac{1}{75.0 \, \Omega} + \frac{1}{45.0 \, \Omega} \quad \text{or} \quad R_p = 28.1 \, \Omega
\]

Ohm’s law, \( \text{Emf} = IR \) gives \( \text{Emf} = (0.294 \, \text{A})(28.1 \, \Omega + r) \), or

\[
\text{Emf} = 8.26 \, \text{V} + (0.294 \, \text{A})r
\]

In the second case, \( \text{Emf} = (0.116 \, \text{A})(75.0 \, \Omega + r) \), or

\[
\text{Emf} = 8.70 \, \text{V} + (0.116 \, \text{A})r
\]

Multiplying Equation (1) by 0.116 A, Equation (2) by 0.294 A, and subtracting yields

\[
\text{Emf} = 8.99 \, \text{V}
\]

b. Substituting this result into Equation (1) and solving for \( r \) gives \( r = 2.5 \, \Omega \).

119. **SSM REASONING** The resistance of one of the wires in the extension cord is given by

Equation 20.3: \( R = \rho L / A \), where the resistivity of copper is \( \rho = 1.72 \times 10^{-8} \, \Omega \cdot \text{m} \), according to Table 20.1. Since the two wires in the cord are in series with each other, their total resistance is \( R_{\text{cord}} = R_{\text{wire 1}} + R_{\text{wire 2}} = 2 \rho L / A \). Once we find the equivalent resistance of the entire circuit (extension cord + trimmer), Ohm's law can be used to find the voltage applied to the trimmer.

**SOLUTION**

a. The resistance of the extension cord is

\[
R_{\text{cord}} = \frac{2 \rho L}{A} = \frac{2(1.72 \times 10^{-8} \, \Omega \cdot \text{m})(46 \, \text{m})}{1.3 \times 10^{-6} \, \text{m}^2} = 1.2 \, \Omega
\]

b. The total resistance of the circuit (cord + trimmer) is, since the two are in series,

\[
R_s = 1.2 \, \Omega + 15.0 \, \Omega = 16.2 \, \Omega
\]

Therefore from Ohm's law (Equation 20.2: \( V = IR \)), the current in the circuit is

\[
I = \frac{V}{R_s} = \frac{120 \, \text{V}}{16.2 \, \Omega} = 7.4 \, \text{A}
\]

Finally, the voltage applied to the trimmer alone is (again using Ohm's law),
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\[ V_{\text{trimmer}} = (7.4 \, \text{A})(15.0 \, \Omega) = 110 \, \text{V} \]

120. **REASONING** The equivalent resistance of the three devices in parallel is \( R_p \), and we can find the value of \( R_p \) by using our knowledge of the total power consumption of the circuit; the value of \( R_p \) can be found from Equation 20.6c, \( P = \frac{V^2}{R_p} \). Ohm's law (Equation 20.2, \( V = IR \)) can then be used to find the current through the circuit.

**SOLUTION**

a. The total power used by the circuit is \( P = 1650 \, \text{W} + 1090 \, \text{W} + 1250 \, \text{W} = 3990 \, \text{W} \). The equivalent resistance of the circuit is

\[ R_p = \frac{V^2}{P} = \frac{(120 \, \text{V})^2}{3990 \, \text{W}} = 3.6 \, \Omega \]

b. The total current through the circuit is

\[ I = \frac{V}{R_p} = \frac{120 \, \text{V}}{3.6 \, \Omega} = 33 \, \text{A} \]

This current is larger than the rating of the circuit breaker; therefore, the breaker will open.

121. **SSM REASONING** The resistance of a metal wire of length \( L \), cross-sectional area \( A \) and resistivity \( \rho \) is given by Equation 20.3: \( R = \frac{\rho L}{A} \). The volume \( V_2 \) of the new wire will be the same as the original volume \( V_1 \) of the wire, where volume is the product of length and cross-sectional area. Thus, \( V_1 = V_2 \) or \( A_1 L_1 = A_2 L_2 \). Since the new wire is three times longer than the first wire, we can write

\[ A_1 L_1 = A_2 L_2 = A_2 (3L_1) \quad \text{or} \quad A_2 = \frac{A_1}{3} \]

We can form the ratio of the resistances, use this expression for the area \( A_2 \), and find the new resistance.

**SOLUTION** The resistance of the new wire is determined as follows:

\[ \frac{R_2}{R_1} = \frac{\rho L_2/A_2}{\rho L_1/A_1} = \frac{L_2 A_1}{L_1 A_2} = \frac{(3L_1) A_1}{L_1 (A_1 / 3)} = 9 \]

Solving for \( R_2 \), we find that

\[ R_2 = 9R_1 = 9(21.0 \, \Omega) = 189 \, \Omega \]
122. **REASONING** The length $L$ of the wire is related to its resistance $R$ and cross-sectional area $A$ by $L = AR/\rho$ (see Equation 20.3), where $\rho$ is the resistivity of tungsten. The resistivity is known (see Table 20.1), and the cross-sectional area can be determined since the radius of the wire is given. The resistance can be obtained from Ohm’s law as the voltage divided by the current.

**SOLUTION** The length $L$ of the wire is

$$L = \frac{AR}{\rho}$$

Since the cross-section of the wire is circular, its area is $A = \pi r^2$, where $r$ is the radius of the wire. According to Ohm’s law (Equation 20.2), the resistance $R$ is related to the voltage $V$ and current $I$ by $R = V/I$. Substituting the expressions for $A$ and $R$ into Equation (1) gives

$$L = \frac{AR}{\rho} = \frac{\pi r^2 \left(\frac{V}{I}\right)}{\rho} = \frac{\pi \left(0.0030 \times 10^{-3} \text{ m}\right)^2 \left(\frac{120 \text{ V}}{1.24 \text{ A}}\right)}{5.6 \times 10^{-8} \text{ } \Omega \cdot \text{ m}} = 0.049 \text{ m}$$

123. **SSM REASONING** The foil effectively converts the capacitor into two capacitors in series. Equation 19.10 gives the expression for the capacitance of a capacitor of plate area $A$ and plate separation $d$ (no dielectric): $C_0 = \varepsilon_0 A/d$. We can use this expression to determine the capacitance of the individual capacitors created by the presence of the foil. Then using the fact that the "two capacitors" are in series, we can use Equation 20.19 to find the equivalent capacitance of the system.

**SOLUTION** Since the foil is placed one-third of the way from one plate of the original capacitor to the other, we have $d_1 = (2/3)d$, and $d_2 = (1/3)d$. Then

$$C_1 = \frac{\varepsilon_0 A}{(2/3)d} = \frac{3\varepsilon_0 A}{2d}$$

and

$$C_2 = \frac{\varepsilon_0 A}{(1/3)d} = \frac{3\varepsilon_0 A}{d}$$

Since these two capacitors are effectively in series, it follows that

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3\varepsilon_0 A/(2d)} + \frac{1}{3\varepsilon_0 A/d} = \frac{3d}{3\varepsilon_0 A} = \frac{d}{\varepsilon_0 A}$$

But $C_0 = \varepsilon_0 A/d$, so that $d/(\varepsilon_0 A) = 1/C_0$, and we have
\[ \frac{1}{C_s} = \frac{1}{C_0} \quad \text{or} \quad C_s = C_0 \]

124. **REASONING AND SOLUTION** The mass \( m \) of the aluminum wire is equal to the density \( d \) of aluminum times the volume of the wire. The wire is cylindrical, so its volume is equal to the cross-sectional area \( A \) times the length \( L \); \( m = dAL \).

The cross-sectional area of the wire is related to its resistance \( R \) and length \( L \) by Equation 20.3; \( R = \rho L/A \), where \( \rho \) is the resistivity of aluminum (see Table 20.1). Therefore, the mass of the aluminum wire can be written as

\[ m = dAL = d\left( \frac{\rho L}{R} \right) L \]

The resistance \( R \) is given by Ohm’s law as \( R = V/I \), so the mass of the wire becomes

\[ m = d\left( \frac{\rho L}{R} \right)L = \frac{d\rho L^2 I}{V} \]

\[ m = \frac{(2700 \text{ kg/m}^3)(2.82 \times 10^{-8} \ \Omega \cdot \text{m})(175 \text{ m})^2(125 \text{ A})}{0.300 \text{ V}} = 9.7 \times 10^2 \text{ kg} \]
1. (d) Right-Hand Rule No. 1 gives the direction of the magnetic force as $-x$ for both drawings A and B. In drawing C, the velocity is parallel to the magnetic field, so the magnetic force is zero.

2. (b) Using Right-Hand Rule No. 1 (see Section 21.2), we find that the direction of the magnetic force on a positively charged particle is to the west. Reversing this direction because the particle is a negative electron, we see that the magnetic force acting on it points to the east.

3. (a) Using Right-Hand Rule No. 1 (see Section 21.2), we find that the direction of the magnetic force on a positively charged particle is straight down toward the bottom of the screen.

4. $B = 1.1 \times 10^{-1} \text{ T}$, south

5. (c) The electric force points out of the screen, in the direction of the electric field. An application of Right-Hand Rule No. 1 shows that the magnetic force also points out of the screen, parallel to the electric force. When two forces have the same direction, the magnitude of their sum has the largest possible value.

6. (e) In this situation, the centripetal force, $F_c = \frac{mv^2}{r}$ (Equation 5.3), is provided by the magnetic force, $F = qvB \sin 90.0^\circ$ (Equation 21.1), so $\frac{mv^2}{r} = qvB \sin 90.0^\circ$. Thus, $q = \frac{mv}{rB}$, and the charge magnitude $q$ is inversely proportional to the radius $r$. Since the radius of curve 1 is smaller than that of curve 2, and the radius of curve 2 is smaller than that of curve 3, we conclude that $q_1$ is larger than $q_2$, which is larger than $q_3$.

7. (a) The magnetic force that acts on the electron in regions 1 and 2 is always perpendicular to its path, so the force does no work. According to the work-energy theorem, Equation 6.3, the kinetic energy, and hence speed, of the electron does not change when no work is done.

8. (d) According to Equation 21.2, the radius $r$ of the circular path is given by $r = \frac{mv}{qB}$. Since $v$, $q$, and $B$ are the same for the proton and the electron, the more-massive proton travels on the circle with the greater radius. The centripetal force $F_c$ acting on the proton must point toward the center of the circle. In this case, the centripetal force is provided by the magnetic force $F$. According to Right-Hand Rule No. 1, the direction of $F$ is related to the velocity $v$ and the magnetic field $B$. An application of this rule shows that the proton
must travel counterclockwise around the circle in order that the magnetic force point toward the center of the circle.

9. \( \frac{r_{\text{proton}}}{r_{\text{electron}}} = 1835 \)

10. (c) When, for example, a particle moves perpendicular to a magnetic field, the field exerts a force that causes the particle to move on a circular path. Any object moving on a circular path experiences a centripetal acceleration.

11. \( F = 3.0 \text{ N}, \) along the \(-y\) axis

12. (e) The magnetic field is directed from the north pole to the south pole (Section 21.1). According to Right-Hand Rule No. 1 (Section 21.5), the magnetic force in drawing 1 points north.

13. (c) There is no net force. No force is exerted on the top and bottom wires, because the current is either in the same or opposite direction as the magnetic field. According to Right-Hand Rule No. 1 (Section 21.5), the left side of the loop experiences a force that is directed into the screen, and the right side experiences a force that is directed out of the screen (toward the reader). The two forces have the same magnitude, so the net force is zero. The two forces on the left and right sides, however, do exert a net torque on the loop with respect to the axis.

14. (d) According to Right-Hand Rule No. 1 (Section 21.5), all four sides of the loop are subject to forces that are directed perpendicularly toward the opposite side of the square. In addition, the forces have the same magnitude, so the net force is zero. A torque consists of a force and a lever arm. For the axis of rotation through the center of the loop, the lever arm for each of the four forces is zero, so the net torque is also zero.

15. \( N = 86 \) turns

16. (a) Right-Hand Rule No. 2 (Section 21.7) indicates that the magnetic field from the top wire in 2 points into the screen and that from the bottom wire points out of the screen. Thus, the net magnetic field in 2 is zero. Also, the magnetic field from the horizontal wire in 4 points into the screen and that from the vertical wire points out of the screen. Thus, the net magnetic field in 4 is also zero.

17. (b) Two wires attract each other when the currents are in the same direction and repel each other when the currents are in the opposite direction (see Section 21.7). Wire B is attracted to A and repelled by C, but the forces reinforce one another. Therefore, the net force has a magnitude of \( F_{\text{BA}} + F_{\text{BC}} \), where \( F_{\text{BA}} \) and \( F_{\text{BC}} \) are the magnitudes of the forces exerted on wire B by A and on wire B by C. However, \( F_{\text{BA}} = F_{\text{BC}} \), since the wires A and C are equidistance from B. Therefore, the net force on wire B has a magnitude of \( 2F_{\text{BA}} \). The net force exerted on wire A is less than this, because wire A is attracted to B and repelled by C,
the forces partially canceling. The net force expected on wire C is also less than that on A. It is repelled by both A and B, but A is twice as far away as B.

18. (a) The magnetic field in the region inside a solenoid is constant, both in magnitude and in direction (see Section 21.7).

19. $B = 4.7 \times 10^{-6} \text{T}$, out of the screen

20. (d) According to Ampere’s law, $I$ is the net current passing through the surface bounded by the path. The net current is $3 \text{ A} + 4 \text{ A} - 5 \text{ A} = 2 \text{ A}$. 

1. **SSM REASONING** The electron’s acceleration is related to the net force $\Sigma F$ acting on it by Newton’s second law: $a = \Sigma F/m$ (Equation 4.1), where $m$ is the electron’s mass. Since we are ignoring the gravitational force, the net force is that caused by the magnetic force, whose magnitude is expressed by Equation 21.1 as $F = |q_0| vB \sin \theta$. Thus, the magnitude of the electron’s acceleration can be written as $a = \left(\frac{|q_0| vB \sin \theta}{m}\right)$.  

**SOLUTION** We note that $\theta = 90.0^\circ$, since the velocity of the electron is perpendicular to the magnetic field. The magnitude of the electron’s charge is $1.60 \times 10^{-19}$ C, and the electron’s mass is $9.11 \times 10^{-31}$ kg (see the inside of the front cover), so  

$$a = \left(\frac{|q_0| vB \sin \theta}{m}\right) = \frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(2.1 \times 10^6 \text{ m/s}\right)\left(1.6 \times 10^{-5} \text{ T}\right)\sin 90.0^\circ}{9.11 \times 10^{-31} \text{ kg}} = 5.9 \times 10^{12} \text{ m/s}^2$$


2. **REASONING** The magnitude $B$ of the magnetic field is $B = \frac{F}{|q_0|(v \sin \theta)}$ (Equation 21.1), where $F$ is the magnitude of the magnetic force on the charge, whose magnitude is $|q_0|$ and whose velocity has a magnitude $v$ and makes an angle $\theta$ with the direction of the field. Both the proton in part a and the electron in part b have the same charge magnitude of $|q_0| = 1.60 \times 10^{-19}$ C. Therefore, the magnetic field has the same magnitude in both parts of the problem. However, the direction of the field is different for the proton and the electron. This is because the proton charge is positive, whereas the electron charge is negative. Finally, we note that the magnitude of the magnetic force is a maximum, which means that the velocity is perpendicular to the magnetic field, so that $\theta = 90.0^\circ$.  

**SOLUTION**  

a. Using Equation 21.1, we find that the magnitude of the magnetic field for the proton is  

$$B = \frac{F}{|q_0|(v \sin \theta)} = \frac{8.0 \times 10^{-14} \text{ N}}{\left(1.60 \times 10^{-19} \text{ C}\right)\left(4.5 \times 10^6 \text{ m/s}\right)\sin 90.0^\circ} = 0.11 \text{ T}$$
Since the proton is traveling due east and the force points due south, we find from right hand rule no. 1 that the magnetic field points \(\text{upward, perpendicular to the earth’s surface}\).

b. For the electron, the magnitude of the field is the same as for the proton, since the two charges have the same magnitude. Thus, \(B = 0.11 \, \text{T}\). Since the electron is a negative charge, however, right-hand rule no. 1 reveals that the field direction is \(\text{downward, perpendicular to the earth’s surface}\).

3. **SSM REASONING** According to Equation 21.1, the magnitude of the magnetic force on a moving charge is \(F = |q_0| vB \sin \theta\). Since the magnetic field points due north and the proton moves eastward, \(\theta = 90.0^\circ\). Furthermore, since the magnetic force on the moving proton balances its weight, we have \(mg = |q_0| vB \sin \theta\), where \(m\) is the mass of the proton. This expression can be solved for the speed \(v\).

**SOLUTION** Solving for the speed \(v\), we have

\[
v = \frac{mg}{|q_0| B \sin \theta} = \frac{(1.67 \times 10^{-27} \, \text{kg})(9.80 \, \text{m/s}^2)}{(1.6 \times 10^{-19} \, \text{C})(2.5 \times 10^{-5} \, \text{T}) \sin 90.0^\circ} = 4.1 \times 10^{-3} \, \text{m/s}
\]

4. **REASONING** The magnitude \(B\) of the magnetic field is given by \(B = \frac{F}{|q_0| v \sin \theta}\) (Equation 21.1), and we will apply this expression directly to obtain \(B\).

**SOLUTION** The charge \(q_0 = -8.3 \times 10^{-6} \, \text{C}\) travels with a speed \(v = 7.4 \times 10^6 \, \text{m/s}\) at an angle of \(\theta = 52^\circ\) with respect to a magnetic field of magnitude \(B\) and experiences a force of magnitude \(F = 5.4 \times 10^{-3} \, \text{N}\). According to Equation 21.1, the field magnitude is

\[
B = \frac{F}{|q_0| v \sin \theta} = \frac{5.4 \times 10^{-3} \, \text{N}}{-8.3 \times 10^{-6} \, \text{C}(7.4 \times 10^6 \, \text{m/s}) \sin 52^\circ} = 1.1 \times 10^{-4} \, \text{T}
\]

Note in particular that it is only the magnitude \(|q_0|\) of the charge that appears in this calculation. The algebraic sign of the charge does not affect the result.

5. **REASONING** According to Equation 21.1, the magnetic force has a magnitude of \(F = |q| vB \sin \theta\), where \(|q|\) is the magnitude of the charge, \(B\) is the magnitude of the magnetic field, \(v\) is the speed, and \(\theta\) is the angle of the velocity with respect to the field. As \(\theta\) increases from \(0^\circ\) to \(90^\circ\), the force increases, so the angle must lie between \(25^\circ\) and \(90^\circ\).
**SOLUTION** Letting $\theta_1 = 25^\circ$ and $\theta_2$ be the desired angle, we can apply Equation 21.1 to both situations as follows:

\[
\frac{F}{|q|vB \sin \theta_1} \quad \text{and} \quad \frac{2F}{|q|vB \sin \theta_2}
\]

Dividing the equation for situation 2 by the equation for situation 1 gives

\[
\frac{2F}{F} = \frac{|q|vB \sin \theta_2}{|q|vB \sin \theta_1} \quad \text{or} \quad \sin \theta_2 = 2 \sin \theta_1 = 2 \sin 25^\circ = 0.85
\]

\[
\theta_2 = \sin^{-1}(0.85) = 58^\circ
\]

6. **REASONING** A moving charge experiences no magnetic force when its velocity points in the direction of the magnetic field or in the direction opposite to the magnetic field. Thus, the magnetic field must point either in the direction of the $+x$ axis or in the direction of the $-x$ axis. If a moving charge experiences the maximum possible magnetic force when moving in a magnetic field, then the velocity must be perpendicular to the field. In other words, the angle $\theta$ that the charge’s velocity makes with respect to the magnetic field is $\theta = 90^\circ$.

**SOLUTION** The magnitude $B$ of the magnetic field can be determined using Equation 21.1:

\[
B = \frac{F}{|q|v \sin \theta} = \frac{0.48 \text{ N}}{(8.2 \times 10^{-6} \text{ C})(5.0 \times 10^5 \text{ m/s}) \sin 90^\circ} = 0.12 \text{ T}
\]

In this calculation we use $\theta = 90^\circ$, because the 0.48-N force is the maximum possible force. Since the particle experiences no magnetic force when it moves along the $+x$ axis, we can conclude that the magnetic field points

either in the direction of the $+x$ axis or in the direction of the $-x$ axis.

7. **REASONING** The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net field can be obtained using the Pythagorean theorem.

**SOLUTION** According to Equation 21.1, the magnetic force has a magnitude of $F_{\text{magnetic}} = |q|vB \sin \theta$, where $|q|$ is the magnitude of the
charge, \( B \) is the magnitude of the magnetic field, \( v \) is the speed, and \( \theta = 90^\circ \) is the angle of the velocity with respect to the field. Thus, \( F_{\text{magnetic}} = |q|vB \). According to Equation 18.2, the electric force has a magnitude of \( F_{\text{electric}} = |q|E \). Using the Pythagorean theorem, we find the magnitude of the net force to be

\[
F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2} = \sqrt{(|q|vB)^2 + (|q|E)^2} = |q|\sqrt{(vB)^2 + E^2}
\]

\[
= (1.8 \times 10^{-6} \text{ C})\sqrt{[(3.1 \times 10^6 \text{ m/s})(1.2 \times 10^{-3} \text{ T})]^2 + (4.6 \times 10^3 \text{ N/C})^2} = 1.1 \times 10^{-2} \text{ N}
\]

8. **REASONING** According to Equation 21.1, the magnetic force has a magnitude of \( F = |q|vB \sin \theta \). The field \( B \) and the directional angle \( \theta \) are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed \( v \) would lead to a greater force magnitude \( F \). But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

**SOLUTION** Applying Equation 21.1 to each particle, we have

\[
F = \frac{|q_1|v_1B \sin \theta}{\text{Particle 1}} \quad \text{and} \quad F = \frac{|q_2|v_2B \sin \theta}{\text{Particle 2}}
\]

Dividing the equation for particle 1 by the equation for particle 2 and remembering that \( v_1 = 3v_2 \) gives

\[
\frac{F}{F} = \frac{|q_1|v_1B \sin \theta}{|q_2|v_2B \sin \theta} \quad \text{or} \quad \frac{|q_1|}{|q_2|} = \frac{v_1}{v_2} = 3 \quad \text{or} \quad \frac{|q_1|}{|q_2|} = \frac{v_1}{v_2} = \frac{1}{3}
\]

9. **REASONING** The positive plate has a charge \( q \) and is moving downward with a speed \( v \) at right angles to a magnetic field of magnitude \( B \). The magnitude \( F \) of the magnetic force exerted on the positive plate is \( F = |q|vB \sin 90.0^\circ \). The charge on the positive plate is related to the magnitude \( E \) of the electric field that exists between the plates by (see Equation 18.4) \( |q| = \varepsilon_0AE \), where \( A \) is the area of the positive plate. Substituting this expression for \( |q| \) into \( F = |q|vB \sin 90.0^\circ \) gives the answer in terms of known quantities.

**SOLUTION**

\[
F = (\varepsilon_0AE)vB
\]

\[
= \left[ 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \right] \left( 7.5 \times 10^{-4} \text{ m}^2 \right) \left( 170 \text{ N/C} \right) \left( 32 \text{ m/s} \right) \left( 3.6 \text{ T} \right)
\]

\[
= 1.3 \times 10^{-10} \text{ N}
\]
An application of Right-Hand Rule No. 1 shows that the magnetic force is perpendicular to
the plane of the page and directed out of the page, toward the reader.

10. **REASONING** The drawing on the left shows the directions of the two magnetic fields, as
well as the velocity \( \mathbf{v} \) of the particle. Each component of the magnetic field is perpendicular
to the velocity, so each exerts a magnetic force on the particle. The magnitude of the force is
\[
F = q_0 |\mathbf{v}|B \sin \theta \quad \text{(Equation 21.1)},
\]
and the direction can be determined by using Right-Hand Rule No. 1 (RHR-1). The magnitude and direction of the net force can be found by using
trigonometry.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{forces.png}
\caption{Magnetic Forces}
\end{figure}

**SOLUTION**

a. The magnitude \( F_1 \) of the magnetic force due to the 0.048-T magnetic field is
\[
F_1 = q_0 |\mathbf{v}|B_x \sin 90.0^\circ = \left( 2.0 \times 10^{-5} \text{ C} \right) \left( 4.2 \times 10^3 \text{ m/s} \right) \left( 0.048 \text{ T} \right) = 4.0 \times 10^{-3} \text{ N}
\]
The magnitude \( F_2 \) of the magnetic force due to the 0.065-T magnetic field is
\[
F_2 = q_0 |\mathbf{v}|B_y \sin 90.0^\circ = \left( 2.0 \times 10^{-5} \text{ C} \right) \left( 4.2 \times 10^3 \text{ m/s} \right) \left( 0.065 \text{ T} \right) = 5.5 \times 10^{-3} \text{ N}
\]
The directions of the forces are found using RHR-1, and they are indicated in the drawing
on the right. Also shown is the net force \( \mathbf{F} \), as well as the angle \( \theta \) that it makes with respect
to the +x axis. Since the forces are at right angles to each other, we can use the Pythagorean
theorem to find the magnitude \( F \) of the net force:
\[
F = \sqrt{F_1^2 + F_2^2} = \sqrt{(4.0 \times 10^{-3} \text{ N})^2 + (5.5 \times 10^{-3} \text{ N})^2} = 6.8 \times 10^{-3} \text{ N}
\]
b. The angle \( \theta \) can be determined by using the inverse tangent function:
\[
\theta = \tan^{-1} \left( \frac{F_1}{F_2} \right) = \tan^{-1} \left( \frac{4.0 \times 10^{-3} \text{ N}}{5.5 \times 10^{-3} \text{ N}} \right) = 36^\circ
\]
11. **Reasoning** The direction in which the electrons are deflected can be determined using Right-Hand Rule No. 1 and reversing the direction of the force (RHR-1 applies to positive charges, and electrons are negatively charged).

Each electron experiences an acceleration \( a \) given by Newton’s second law of motion, \( a = \frac{F}{m} \), where \( F \) is the net force and \( m \) is the mass of the electron. The only force acting on the electron is the magnetic force, \( F = q_0 v B \sin \theta \), so it is the net force. The speed \( v \) of the electron is related to its kinetic energy \( KE \) by the relation \( KE = \frac{1}{2} mv^2 \). Thus, we have enough information to find the acceleration.

**Solution**

a. According to RHR-1, if you extend your right hand so that your fingers point along the direction of the magnetic field \( B \) and your thumb points in the direction of the velocity \( v \) of a positive charge, your palm will face in the direction of the force \( F \) on the positive charge.

For the electron in question, the fingers of the right hand should be oriented downward (direction of \( B \)) with the thumb pointing to the east (direction of \( v \)). The palm of the right hand points due north (the direction of \( F \) on a positive charge). Since the electron is negatively charged, it will be deflected due south.

b. The acceleration of an electron is given by Newton’s second law, where the net force is the magnetic force. Thus,

\[
a = \frac{F}{m} = \frac{|q_0|vB \sin \theta}{m}
\]

Since the kinetic energy is \( KE = \frac{1}{2} mv^2 \), the speed of the electron is \( v = \sqrt{2(KE)/m} \). Thus, the acceleration of the electron is

\[
a = \frac{|q_0|vB \sin \theta}{m} = \frac{|q_0|\sqrt{\frac{2(KE)}{m}} B \sin \theta}{m}
\]

\[
= \left(1.60 \times 10^{-19} \text{ C}\right) \sqrt{\frac{2(2.40 \times 10^{-15} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} \left(2.00 \times 10^{-5} \text{ T}\right) \sin 90.0^\circ = 2.55 \times 10^{14} \text{ m/s}^2
\]

12. **Reasoning** Since \( e = 1.60 \times 10^{-19} \text{ C} \), we need to determine whether the charge has a magnitude of \( |q| = 1.60 \times 10^{-19} \text{ C} \) or \( |q| = 3.20 \times 10^{-19} \text{ C} \). We can do this by using \( r = \frac{mv}{|q|B} \) (Equation 21.2), which gives the radius \( r \) of the circular path in terms of the mass
m of the charged particle, the particle’s speed \( v \), and the magnitude \( B \) of the magnetic field. This is possible since values are available for \( r \), \( m \), \( v \), and \( B \) in Equation 21.2.

**SOLUTION** Solving Equation 21.2 for \(|q|\), we find that

\[
|q| = \frac{mv}{Br} = \left( \frac{6.6 \times 10^{-27} \text{ kg}}{0.75 \text{ T}} \right) \left( \frac{4.4 \times 10^5 \text{ m/s}}{0.012 \text{ m}} \right) = 3.2 \times 10^{-19} \text{ C}
\]

This charge is \( 2e = 2 \left( 1.60 \times 10^{-19} \text{ C} \right) \). We can see, then, that the charge of the ionized helium atom is \( +2e \).

13. **SSM REASONING** The radius \( r \) of the circular path is given by \( r = \frac{mv}{|q|B} \) (Equation 21.2), where \( m \) and \( v \) are the mass and speed of the particle, respectively, \(|q|\) is the magnitude of the charge, and \( B \) is the magnitude of the magnetic field. This expression can be solved directly for \( B \), since \( r \), \( m \), and \( v \) are given and \( q = +e \), where \( e = 1.60 \times 10^{-19} \text{ C} \).

**SOLUTION** Solving Equation 21.2 for \( B \) gives

\[
B = \frac{mv}{|q|r} = \frac{\left( 3.06 \times 10^{-25} \text{ kg} \right) \left( 7.2 \times 10^3 \text{ m/s} \right)}{1.60 \times 10^{-19} \text{ C} \left( 0.10 \text{ m} \right)} = 0.14 \text{ T}
\]

14. **REASONING** The time \( t \) that it takes the particle to complete one revolution is the time to travel a distance \( d = 2\pi r \) equal to the circumference of a circle of radius \( r \) at a speed \( v \). From Equation 2.1, we know that speed is the ratio of distance to elapsed time \( (v = \frac{d}{t}) \), so the elapsed time is the ratio of distance to speed:

\[
t = \frac{d}{v} = \frac{2\pi r}{v}
\]

Because the particle follows a circular path that is perpendicular to the external magnetic field of magnitude \( B \), the radius of the path is given by \( r = \frac{mv}{|q|B} \) (Equation 21.2), where \( m \) is the mass and \(|q|\) is the magnitude of the charge of the particle. We will use Equation 21.2 to determine the speed of the particle, and then Equation (1) to find the time for one complete revolution.

**SOLUTION** Solving \( r = \frac{mv}{|q|B} \) (Equation 21.2) for \( v \) yields

\[
v = \frac{|q|Br}{m} = \frac{|q|}{m} Br
\]
In the last step of Equation (2), we have expressed the speed \( v \) explicitly in terms of the charge-to-mass ratio \( \frac{|q|}{m} \) of the particle. Substituting Equation (2) into Equation (1), we obtain

\[
\frac{2\pi r}{v} = \frac{2\pi}{\frac{|q|}{m} B} = \frac{2\pi}{\frac{|q|}{m} B} \left( 5.7 \times 10^8 \text{ C/kg} \right) \left( 0.72 \text{ T} \right) = 1.5 \times 10^{-8} \text{ s}
\]

15. **REASONING**

a. The drawing shows the velocity \( v \) of the particle at the top of its path. The magnetic force \( F \), which provides the centripetal force, must be directed toward the center of the circular path. Since the directions of \( v \), \( F \), and \( B \) are known, we can use Right-Hand Rule No. 1 (RHR-1) to determine if the charge is positive or negative.

b. The radius of the circular path followed by a charged particle is given by Equation 21.2 as \( r = \frac{mv}{|q|B} \). The mass \( m \) of the particle can be obtained directly from this relation, since all other variables are known.

**SOLUTION**

a. If the particle were positively charged, an application of RHR-1 would show that the force would be directed straight up, opposite to that shown in the drawing. Thus, the charge on the particle must be **negative**.

b. Solving Equation 21.2 for the mass of the particle gives

\[
m = \frac{|q|Br}{v} = \frac{\left( 8.2 \times 10^{-4} \text{ C} \right) \left( 0.48 \text{ T} \right) \left( 960 \text{ m} \right)}{140 \text{ m/s}} = 2.7 \times 10^{-3} \text{ kg}
\]

16. **REASONING** Equation 21.2 gives the radius \( r \) of the circular path as \( r = \frac{mv}{|q|B} \), where \( m \), \( v \), and \( |q| \) are, respectively, the mass, speed, and charge magnitude of the particle, and \( B \) is the magnitude of the magnetic field. We wish the radius to be the same for both the proton and the electron. The speed \( v \) and the charge magnitude \( |q| \) are the same for the proton and the electron, but the mass of the electron is \( 9.11 \times 10^{-31} \text{ kg} \), while that of the proton is \( 1.67 \times 10^{-27} \text{ kg} \). Therefore, to offset the effect of the smaller electron mass \( m \) in Equation 21.2, the magnitude \( B \) of the field must be reduced for the electron.
SOLUTION Applying Equation 21.2 to the proton and the electron, both of which carry charges of the same magnitude $|q| = e$, we obtain

$$r = \frac{m_p v}{e B_p} \quad \text{and} \quad r = \frac{m_e v}{e B_e}$$

Proton

Electron

Dividing the proton-equation by the electron-equation gives

$$\frac{r}{r} = \frac{m_p v}{e B_p} \quad \text{or} \quad 1 = \frac{m_p B_e}{m_e B_p}$$

Solving for $B_e$, we obtain

$$B_e = \frac{m_e B_p}{m_p} = \frac{(9.11 \times 10^{-31} \text{ kg}) (0.50 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 2.7 \times 10^{-4} \text{ T}$$

17. SSM REASONING As discussed in Section 21.4, the mass $m$ of a singly-ionized particle that has been accelerated through a potential difference $V$ and injected into a magnetic field of magnitude $B$ is given by

$$m = \left(\frac{e r^2}{2V}\right) B^2 \quad (1)$$

where $e = 1.60 \times 10^{-19} \text{ C}$ is the magnitude of the charge of an electron and $r$ is the radius of the particle’s path. If the beryllium-10 ions reach the same position in the detector as the beryllium-7 ions, both types of ions must have the same path radius $r$. Additionally, the accelerating potential difference $V$ is kept constant, so we see that the quantity $\left(\frac{e r^2}{2V}\right)$ in Equation (1) is the same for both types of ions.

SOLUTION All that differs between the two situations are the masses ($m_7, m_{10}$) of the ions and the magnitudes of the magnetic fields ($B_7, B_{10}$). Solving Equation (1) for the constant quantity $\left(\frac{e r^2}{2V}\right)$, we obtain

$$\left(\frac{e r^2}{2V}\right) = \frac{m_{10}}{B_{10}^2} = \frac{m_7}{B_7^2} \quad \text{Same for both ions}$$
Solving Equation (2) for $B_{10}^2$, we find that

$$B_{10}^2 = \frac{m_{10}}{m_7} \text{ or } B_{10} = B_7 \sqrt{\frac{m_{10}}{m_7}} = (0.283 \text{ T}) \sqrt{\frac{16.63 \times 10^{-27} \text{ kg}}{11.65 \times 10^{-27} \text{ kg}}} = 0.338 \text{ T}$$

18. **REASONING** Section 21.4 discusses how a mass spectrometer determines the mass $m$ of an ion that has a charge of $+e$, where $e = 1.60 \times 10^{-19} \text{ C}$. This ion accelerates through a potential difference $V$ and follows a circular path (radius $r$) because of a magnetic field (magnitude $B$). The mass of the ion is $m = \left(\frac{e r^2}{2V}\right) B^2$. If the gold ions in this problem had a charge of $+e$, we could solve this expression directly for the radius $r$. However, the charge of the gold ions is $+2e$, so that before using the expression, we need to replace $e$ by $2e$.

**SOLUTION** Replacing $e$ by $2e$ in the expression from Section 21.4 gives the mass as $m = \left(\frac{2e r^2}{2V}\right) B^2$. Solving this equation for the radius $r$, we find that

$$r = \frac{2mV}{\sqrt{2eB^2}} = \sqrt{\frac{2\left(3.27 \times 10^{-25} \text{ kg}\right)(1.00 \times 10^3 \text{ V})}{2\left(1.60 \times 10^{-19} \text{ C}\right)(0.500 \text{ T})^2}} = 0.0904 \text{ m}$$

19. **REASONING** The speed of the $\alpha$-particle can be obtained by applying the principle of conservation of energy, recognizing that the total energy is the sum of the particle’s kinetic energy and electric potential energy, the gravitational potential energy being negligible in comparison. Once the speed is known, Equation 21.1 can be used to obtain the magnitude of the magnetic force that acts on the particle. Lastly, the radius of its circular path can be obtained directly from Equation 21.2.

**SOLUTION**

a. Using A and B to denote the initial positions, respectively, the principle of conservation of energy can be written as follows:

$$\frac{1}{2}mv_B^2 + \frac{\text{EPE}_B}{\text{Final kinetic energy}} + \frac{\text{EPE}_A}{\text{Initial kinetic energy}} = \frac{1}{2}mv_A^2 + \frac{\text{EPE}_A}{\text{Initial electric potential energy}}$$  \hspace{1cm} (1)

Using Equation 19.3 to express the electric potential energy of the charge $q_0$ as $\text{EPE} = q_0V$, where $V$ is the electric potential, we find from Equation (1) that

$$\frac{1}{2}mv_B^2 + q_0V_B = \frac{1}{2}mv_A^2 + q_0V_A$$  \hspace{1cm} (2)

Since the particle starts from rest, we have that $v_A = 0 \text{ m/s}$, and Equation (2) indicates that
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v_B = \sqrt{\frac{2q_0 (V_A - V_B)}{m}} = \sqrt{\frac{2 \left[2 \left(1.60 \times 10^{-19} \text{ C}\right)\right] \left(1.20 \times 10^6 \text{ V}\right)}{6.64 \times 10^{-27} \text{ kg}}} = 1.08 \times 10^7 \text{ m/s}

b. According to Equation 21.1, the magnitude of the magnetic force that acts on the particle is

\[ F = q|v_B|B \sin \theta = 2 \left(1.60 \times 10^{-19} \text{ C}\right) \left(1.08 \times 10^7 \text{ m/s}\right) \left(2.20 \text{ T}\right) \sin 90.0^\circ = 7.60 \times 10^{-12} \text{ N} \]

where \( \theta = 90.0^\circ \), since the particle travels perpendicular to the field at all times.

c. According to Equation 21.2, the radius of the circular path on which the particle travels is

\[ r = \frac{mv_B}{q|B|} = \frac{6.64 \times 10^{-27} \text{ kg} \left(1.08 \times 10^7 \text{ m/s}\right)}{2 \left(1.60 \times 10^{-19} \text{ C}\right) \left(2.20 \text{ T}\right)} = 0.102 \text{ m} \]

20. **REASONING** Equation 21.2 gives the radius \( r \) of the circular path as \( r = \frac{mv}{|q|B} \), where \( m, v, \) and \( |q| \) are, respectively, the mass, speed, and charge magnitude of the particle, and \( B \) is the magnitude of the magnetic field.

We can determine the speed of each particle by employing the principle of conservation of energy. The electric potential energy lost as the particles accelerate is converted into kinetic energy. Equation 19.4 indicates that the electric potential energy lost is \( |q|V \), where \( |q| \) is the magnitude of the charge and \( V \) is the electric potential difference. Since \( q \) and \( V \) are the same for each particle, each loses the same amount of potential energy. Energy conservation, then, dictates that each gains the same amount of kinetic energy. Since each particle starts from rest, each enters the magnetic field with the same amount of kinetic energy.

**SOLUTION** According to Equation 21.2, \( r = \frac{mv}{|q|B} \). To determine the speed \( v \) with which each particle enters the field, we use Equation 19.4 and the energy-conservation principle as follows:

\[ |q|V = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2|q|V}{m}} \]

Substituting this result into Equation 21.2 gives the radius of the circular motion:

\[ r = \frac{mv}{|q|B} = \frac{m}{|q|B} \sqrt{\frac{2|q|V}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{|q|}} \]

Applying this result to each particle, we obtain
\[ r_1 = \frac{1}{B} \sqrt{\frac{2m_1 \nu}{|q|}} \quad \text{and} \quad r_2 = \frac{1}{B} \sqrt{\frac{2m_2 \nu}{|q|}} \]

Dividing \( r_2 \) by \( r_1 \) gives

\[ \frac{r_2}{r_1} = \frac{1}{B} \sqrt{\frac{2m_2 \nu}{|q|}} \frac{1}{\sqrt{2m_1 \nu / |q|}} = \sqrt{\frac{m_2}{m_1}} \]

Thus,

\[ r_2 = r_1 \sqrt{\frac{m_2}{m_1}} = (12 \text{ cm}) \sqrt{\frac{5.9 \times 10^{-8} \text{ kg}}{2.3 \times 10^{-8} \text{ kg}}} = 19 \text{ cm} \]

21. \textit{REASONING} The drawing shows the velocity \( \nu \) of the carbon atoms as they enter the magnetic field \( B \). The diameter of the circular path followed by the carbon-12 atoms is labeled as \( 2r_{12} \), and that of the carbon-13 atoms as \( 2r_{13} \), where \( r \) denotes the radius of the path. The radius is given by Equation 21.2 as \( r = \frac{m \nu}{(|q| B)} \), where \( q \) is the charge on the ion (\( q = +e \)). The difference \( \Delta d \) in the diameters is \( \Delta d = 2r_{13} - 2r_{12} \) (see the drawing).

\textit{SOLUTION} The spatial separation between the two isotopes after they have traveled through a half-circle is

\[
\Delta d = 2r_{13} - 2r_{12} = 2\left(\frac{m_{13} \nu}{eB}\right) - 2\left(\frac{m_{12} \nu}{eB}\right) = 2\frac{\nu}{eB} (m_{13} - m_{12})
\]

\[
= \frac{2(6.667 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.8500 \text{ T})} (21.59 \times 10^{-27} \text{ kg} - 19.93 \times 10^{-27} \text{ kg}) = 1.63 \times 10^{-2} \text{ m}
\]

22. \textit{REASONING} The radius of the circular path is given by Equation 21.2 as \( r = \frac{m \nu}{(|q| B)} \), where \( m \) is the mass of the species, \( \nu \) is the speed, \( |q| \) is the magnitude of the charge, and \( B \) is the magnitude of the magnetic field. To use this expression, we must know something about the speed. Information about the speed can be obtained by applying the conservation of energy principle. The electric potential energy lost as a charged particle “falls” from a higher to a lower electric potential is gained by the particle as kinetic energy.
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**SOLUTION**  For an electric potential difference $V$ and a charge $q$, the electric potential energy lost is $|q|V$, according to Equation 19.4. The kinetic energy gained is $\frac{1}{2}mv^2$. Thus, energy conservation dictates that

$$|q|V = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2|q|V}{m}}$$

Substituting this result into Equation 21.2 for the radius gives

$$r = \frac{mv}{|q|B} = \frac{m}{|q|B} \sqrt{\frac{2|q|V}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{|q|}}$$

Using $e$ to denote the magnitude of the charge on an electron, we note that the charge for species $X^+$ is $+e$, while the charge for species $X^{2+}$ is $+2e$. With this in mind, we find for the ratio of the radii that

$$\frac{r_1}{r_2} = \frac{1}{B} \sqrt{\frac{2mV}{e}} = \sqrt{2} \approx 1.41$$

23. **SSM REASONING**  When the proton moves in the magnetic field, its trajectory is a circular path. The proton will just miss the opposite plate if the distance between the plates is equal to the radius of the path. The radius is given by Equation 21.2 as $r = \frac{mv}{|q|B}$. This relation can be used to find the magnitude $B$ of the magnetic field, since values for all the other variables are known.

**SOLUTION**  Solving the relation $r = \frac{mv}{|q|B}$ for the magnitude of the magnetic field, and realizing that the radius is equal to the plate separation, we find that

$$B = \frac{mv}{|q|r} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.5 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.23 \text{ m})} = 0.16 \text{ T}$$

The values for the mass and the magnitude of the charge (which is the same as that of the electron) have been taken from the inside of the front cover.
24. **REASONING** The magnitude $F_B$ of the magnetic force acting on the particle is related to its speed $v$ by $F_B = |q_0|vB \sin \theta$ (Equation 21.1), where $B$ is the magnitude of the magnetic field, $q_0$ is the particle’s charge, and $\theta$ is the angle between the magnetic field $B$ and the particle’s velocity $v$. As the drawing shows, the vector $v$ (east, to the right) is perpendicular to the vector $B$ (south, out of the page). Therefore, $\theta = 90^\circ$, and Equation 21.1 becomes

$$F_B = |q_0|vB \sin 90^\circ = |q_0|vB$$

In addition to the magnetic force, there is also an electric force of magnitude $F_E$ acting on the particle. This force magnitude does not depend upon the speed $v$ of the particle, as we see from $F_E = |q_0|E$ (Equation 18.2). The particle is positively charged, so the electric force acting on it points upward in the same direction as the electric field. By Right-Hand Rule No.1, the magnetic force acting on the positively charged particle points down, and is therefore opposite to the electric force. The net force on the particle points upward, so we conclude that the electric force is greater than the magnetic force. Thus, the magnitude $F$ of the net force acting on the particle is equal to the magnitude of the electric force minus the magnitude of the magnetic force:

$$F = F_E - F_B$$

We also note that particles traveling at a speed $v_0 = 6.50 \times 10^3$ m/s experience no net force. Therefore, $F_E = F_B$ for particles moving at the speed $v_0$.

**SOLUTION** Substituting Equation (1) and $F_E = |q_0|E$ (Equation 18.2) into Equation (2) yields

$$F = |q_0|E - |q_0|vB$$

The magnetic field magnitude $B$ is not given, but, as noted above, for particles with speed $v_0 = 6.50 \times 10^3$ m/s, the magnetic force of Equation (1), $F_B = |q_0|v_0B$, is equal to the electric force $F_E = |q_0|E$ (Equation 18.2). Therefore, we have that

$$|q_0|v_0B = |q_0|E \quad \text{or} \quad B = \frac{E}{v_0}$$

Substituting Equation (4) into Equation (3) yields

$$F = |q_0|E - |q_0|vB = |q_0|E - \frac{|q_0|vE}{v_0}$$

$$F = |q_0|E - |q_0|vB = |q_0|E - \frac{|q_0|vE}{v_0}$$
Solving Equation (5) for \( v \), we obtain
\[
\frac{|q_0|v E}{v_0} = |q_0|E - F \quad \text{or} \quad \frac{v}{v_0} = 1 - \frac{F}{|q_0|E} \quad \text{or} \quad v = v_0 \left(1 - \frac{F}{|q_0|E}\right) \quad (6)
\]
Substituting the given values into Equation (6), we find that
\[
v = v_0 \left(1 - \frac{F}{|q_0|E}\right) = \left(6.50 \times 10^3 \text{ m/s}\right) \left[1 - \frac{1.90 \times 10^{-9} \text{ N}}{4.00 \times 10^{-12} \text{ C}(2470 \text{ N/C})}\right] = 5.25 \times 10^3 \text{ m/s}
\]

25. **SSM REASONING** The particle travels in a semicircular path of radius \( r \), where \( r \) is given by Equation 21.2 \( r = \frac{mv}{|q|B} \). The time spent by the particle in the magnetic field is given by \( t = \frac{s}{v} \), where \( s \) is the distance traveled by the particle and \( v \) is its speed. The distance \( s \) is equal to one-half the circumference of a circle \( s = \pi r \).

**SOLUTION** We find that
\[
t = \frac{s}{v} = \frac{\pi r}{v} = \frac{\pi \left(m v^2 / |q|B\right)}{v} = \pi m \frac{\pi(6.0 \times 10^{-8} \text{ kg})}{(7.2 \times 10^{-6} \text{ C})(3.0 \text{ T})} = 8.7 \times 10^{-3} \text{ s}
\]

26. **REASONING** When the electron travels perpendicular to a magnetic field, its path is a circle. The radius of the circle is given by Equation 21.2 as \( r = \frac{mv}{|q|B} \). All the variables in this relation are known, except the speed \( v \). However, the speed is related to the electron’s kinetic energy \( KE \) by \( KE = \frac{1}{2}mv^2 \) (Equation 6.2). By combining these two relations, we will be able to find the radius of the path.

**SOLUTION** Solving the relation \( KE = \frac{1}{2}mv^2 \) for the speed and substituting the result into \( r = \frac{mv}{|q|B} \) give
\[
r = \frac{mv}{|q|B} \frac{\sqrt{2} \sqrt{\frac{KE}{m}}}{|q|B} = \sqrt{\frac{2 \left(9.11 \times 10^{-31} \text{ kg}\right) \left(2.0 \times 10^{-17} \text{ J}\right)}{1.60 \times 10^{-19} \text{ C}(5.3 \times 10^{-5} \text{ T})}} = 0.71 \text{ m}
\]

Values for the mass and charge of the electron have been taken from the inside of the front cover.
27. **REASONING**

a. When the particle moves in the magnetic field, its path is circular. To keep the particle moving on a circular path, it must experience a centripetal force, the magnitude of which is given by Equation 5.3 as \( F_c = \frac{mv^2}{r} \). In the present situation, the magnetic force \( F \) furnishes the centripetal force, so \( F_c = F \). The mass \( m \) and speed \( v \) of the particle are known, but the radius \( r \) of the path is not. However, the particle travels at a constant speed, so in a time \( t \) the distance \( s \) it travels is \( s = vt \). But the distance is one-quarter of the circumference \( (2\pi r) \) of a circle, so \( s = \frac{1}{4}(2\pi r) \). By combining these three relations, we can determine the magnitude of the magnetic force.

b. The magnitude of the magnetic force is given by Equation 21.1 as \( F = |q|vB\sin\theta \). Since \( F, \, v, \, B, \, \text{and } \theta \) are known, this relation can be used to determine the magnitude \(|q|\) of the charge.

**SOLUTION**

a. The magnetic force, which provides the centripetal force, is \( F = \frac{mv^2}{r} \). Solving the relation \( s = \frac{1}{4}(2\pi r) \) for the radius and substituting \( s = vt \) into the result gives

\[
\frac{2s}{\pi} = \frac{2(vt)}{\pi}
\]

Using this expression for \( r \) in Equation 5.3, we find that the magnitude of the magnetic force is

\[
F = \frac{mv^2}{r} = \frac{mv^2}{2vt} = \frac{\pi mv}{2(2.2 \times 10^{-3} \text{ s})} = 4.4 \times 10^{-3} \text{ N}
\]

b. Solving the relation \( F = |q|vB\sin\theta \) for the magnitude \(|q|\) of the charge and noting that \( \theta = 90.0^\circ \) (since the velocity of the particle is perpendicular to the magnetic field), we find that

\[
|q| = \frac{F}{vB\sin90.0^\circ} = \frac{4.4 \times 10^{-3} \text{ N}}{(85 \text{ m/s})(0.31 \text{ T})\sin90.0^\circ} = 1.7 \times 10^{-4} \text{ C}
\]

28. **REASONING AND SOLUTION** The magnitudes of the magnetic and electric forces must be equal. Therefore,

\[
F_B = F_E \quad \text{or} \quad |q|vB = |q|E
\]

This relation can be solved to give the speed of the particle, \( v = E/B \). We also know that when the electric field is turned off, the particle travels in a circular path of radius \( r = mv/(|q|B) \). Substituting \( v = E/B \) into this equation and solving for \(|q|/m\) gives
29. **REASONING AND SOLUTION** The following drawings show the two circular paths leading to the target T when the proton is projected from the origin O. In each case, the center of the circle is at C. Since the target is located at \(x = -0.10 \text{ m}\) and \(y = -0.10 \text{ m}\), the radius of each circle is \(r = 0.10 \text{ m}\). The speed with which the proton is projected can be obtained from Equation 21.2, if we remember that the charge and mass of a proton are \(q = +1.60 \times 10^{-19} \text{ C}\) and \(m = 1.67 \times 10^{-27} \text{ kg}\), respectively:

\[
\frac{|q|}{m} = \frac{E}{rB^2} = \frac{3.80 \times 10^3 \text{ N/C}}{(4.30 \times 10^{-2} \text{ m})(0.360 \text{ T})^2} = 6.8 \times 10^5 \text{ C/kg}
\]

\[
v = \frac{r|q|B}{m} = \frac{(0.10 \text{ m})(1.60 \times 10^{-19} \text{ C})(0.010 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 9.6 \times 10^4 \text{ m/s}
\]

30. **REASONING** A magnetic field exerts no force on a current-carrying wire that is directed along the same direction as the field. This follows directly from \(F = ILB \sin \theta\) (Equation 21.3), which gives the magnitude \(F\) of the magnetic force that acts on a wire of length \(L\) that is directed at an angle \(\theta\) with respect to a magnetic field of magnitude \(B\) and carries a current \(I\). When \(\theta = 0^\circ\) or \(180^\circ\), \(F = 0 \text{ N}\). Therefore, we need only apply Equation 21.3 to the horizontal component of the earth’s magnetic field in this problem. The direction of the magnetic force can be determined with the aid of RHR-1 (fingers point in direction of the field, thumb points in the direction of the current, palm faces in the direction of the magnetic force).

**SOLUTION** According to Equation 21.3, the magnitude of the magnetic force exerted on the wire by the horizontal component of the earth’s field is

\[
F = ILB \sin \theta = (28 \text{ A})(6.0 \text{ m})(1.8 \times 10^{-5} \text{ T}) \sin 90.0^\circ = 3.0 \times 10^{-3} \text{ N}
\]
Note that $\theta = 90.0^\circ$ because the field component points toward the geographic north and the current is directed perpendicularly into the ground. The application of RHR-1 (fingers point due north, thumb points perpendicularly into the ground, palm faces due east) reveals that the direction of the magnetic force is due east.

31. **REASONING** The magnitude $F$ of the magnetic force experienced by the wire is given by $F = ILB\sin \theta$ (Equation 21.3), where $I$ is the current, $L$ is the length of the wire, $B$ is the magnitude of the earth’s magnetic field, and $\theta$ is the angle between the direction of the current and the magnetic field. Since all the variables are known except $B$, we can use this relation to find its value.

**SOLUTION** Solving $F = ILB\sin \theta$ for the magnitude of the magnetic field, we have

$$B = \frac{F}{IL\sin \theta} = \frac{0.15 \text{ N}}{(75 \text{ A})(45 \text{ m})\sin 60.0^\circ} = 5.1 \times 10^{-5} \text{ T}$$

32. **REASONING** The magnitude $F$ of the force on a current $I$ is given by Equation 21.3 as $F = ILB\sin \theta$ (Equation 21.3), where $L$ is the length of the wire and $\theta$ is the angle between the wire and a magnetic field that has a magnitude $B$. We can apply this equation to both situations. The key to the solution is the fact that $L$, $B$, and $\theta$, although unknown, have the same values in both cases. This fact will allow us to eliminate them algebraically from the calculation of the unknown current.

**SOLUTION** Initially we have $F_1 = 0.030 \text{ N}$ and $I_1 = 2.7 \text{ A}$. In the second case, we have $F_2 = 0.047 \text{ N}$ and $I_2$. Applying this Equation 21.3 to both situations we have

$$F_1 = I_1LB\sin \theta \quad \text{and} \quad F_2 = I_2LB\sin \theta$$

Dividing the right-hand equation by the left-hand equation gives

$$\frac{F_2}{F_1} = \frac{I_2LB\sin \theta}{I_1LB\sin \theta} = \frac{I_2}{I_1} \quad \text{or} \quad I_2 = I_1\left(\frac{F_2}{F_1}\right) = (2.7 \text{ A})\left(\frac{0.047 \text{ N}}{0.030 \text{ N}}\right) = 4.2 \text{ A}$$

33. **REASONING** The magnitude $B$ of the external magnetic field is proportional to the magnitude $F$ of the magnetic force exerted on the wire, according to $F = ILB\sin \theta$ (Equation 21.3), where $L$ is the length of the wire, $I$ is the current it carries, and $\theta$ is the angle between the directions of the current and the magnetic field. When the wire is horizontal, the magnetic force is zero, indicating that $\sin \theta = 0$. The only angles $\theta$ for which this holds are $\theta = 0^\circ$ and $\theta = 180^\circ$. Therefore, the external magnetic field must be horizontal, and when the wire is tilted upwards at an angle of $19^\circ$, the angle between the directions of the current and the magnetic field must be $\theta = 19^\circ$. 
**SOLUTION** Solving $F = ILB \sin \theta$ (Equation 21.3) for $B$ yields

$$B = \frac{F}{IL\sin \theta} = \frac{4.4 \times 10^{-3} \text{ N}}{(7.5 \text{ A})(0.53 \text{ m}) \sin 19^\circ} = 3.4 \times 10^{-3} \text{ T}$$

34. **REASONING** We begin by noting that segments AB and BC are both perpendicular to the magnetic field. Therefore, they experience magnetic forces. However, segment CD is parallel to the field. As a result no magnetic force acts on it. According to Equation 21.3, the magnitude $F$ of the magnetic force on a current $I$ is $F = ILB \sin \theta$, where $L$ is the length of the wire segment and $\theta$ is the angle that the current makes with respect to the magnetic field. For both segments AB and BC the value of the current is the same and the value of $\theta$ is 90°. The length of segment AB is greater, however. Because of its greater length, segment AB experiences the greater force.

**SOLUTION** Using $F = ILB \sin \theta$ (Equation 21.3), we find that the magnitudes of the magnetic forces acting on the segments are:

- **Segment AB** $F = ILB \sin \theta = (2.8 \text{ A})(1.1 \text{ m})(0.26 \text{ T}) \sin 90^\circ = 0.80 \text{ N}$
- **Segment BC** $F = ILB \sin \theta = (2.8 \text{ A})(0.55 \text{ m})(0.26 \text{ T}) \sin 90^\circ = 0.40 \text{ N}$
- **Segment CD** $F = ILB \sin \theta = (2.8 \text{ A})(0.55 \text{ m})(0.26 \text{ T}) \sin 0^\circ = 0 \text{ N}$

35. **SSM REASONING** According to Equation 21.3, the magnetic force has a magnitude of $F = ILB \sin \theta$, where $I$ is the current, $B$ is the magnitude of the magnetic field, $L$ is the length of the wire, and $\theta = 90^\circ$ is the angle of the wire with respect to the field.

**SOLUTION** Using Equation 21.3, we find that

$$L = \frac{F}{IB \sin \theta} = \frac{7.1 \times 10^{-5} \text{ N}}{(0.66 \text{ A})(4.7 \times 10^{-5} \text{ T}) \sin 58^\circ} = 2.7 \text{ m}$$

36. **REASONING** Each wire experiences a force due to the magnetic field. The magnitude of the force is given by $F = ILB \sin \theta$ (Equation 21.3), where $I$ is the current, $L$ is the length of the wire, $B$ is the magnitude of the magnetic field, and $\theta$ is the angle between the direction of the current and the magnetic field. Since the currents in the two wires are in opposite directions, the magnetic force acting on one wire is opposite to that acting on the other. Thus, the net force acting on the two-wire unit is the difference between the magnitudes of the forces acting on each wire.

**SOLUTION** The length $L$ of each wire, the magnetic field $B$, and the angle $\theta$ are the same for both wires. Denoting the current in one of the wires as $I_1 = 7.00 \text{ A}$ and the current in the other as $I$, the magnitude $F_{\text{net}}$ of the net magnetic force acting on the two-wire unit is
\[ F_{\text{net}} = I_1LB \sin \theta - ILB \sin \theta = (I_1 - I)LB \sin \theta \]

Solving for the unknown current \( I \) gives

\[ I = I_1 - \frac{F_{\text{net}}}{LB \sin \theta} = 7.00 \text{ A} - \frac{3.13 \text{ N}}{(2.40 \text{ m})(0.360 \text{ T}) \sin 65.0^\circ} = 3.00 \text{ A} \]

37. \textit{REASONING}  The magnitude of the magnetic force exerted on a long straight wire is given by Equation 21.3 as \( F = ILB \sin \theta \). The direction of the magnetic force is predicted by Right-Hand Rule No. 1. The net force on the triangular loop is the vector sum of the forces on the three sides.

\textbf{SOLUTION}

a. The direction of the current in side \( AB \) is opposite to the direction of the magnetic field, so the angle \( \theta \) between them is \( \theta = 180^\circ \). The magnitude of the magnetic force is

\[ F_{AB} = ILB \sin \theta = ILB \sin 180^\circ = 0 \text{ N} \]

For the side \( BC \), the angle is \( \theta = 55.0^\circ \), and the length of the side is

\[ L = \frac{2.00 \text{ m}}{\cos 55.0^\circ} = 3.49 \text{ m} \]

The magnetic force is

\[ F_{BC} = ILB \sin \theta = (4.70 \text{ A})(3.49 \text{ m})(1.80 \text{ T}) \sin 55.0^\circ = 24.2 \text{ N} \]

An application of Right-Hand No. 1 shows that the magnetic force on side \( BC \) is directed perpendicularly out of the paper, toward the reader.

For the side \( AC \), the angle is \( \theta = 90.0^\circ \). We see that the length of the side is

\[ L = (2.00 \text{ m}) \tan 55.0^\circ = 2.86 \text{ m} \]

The magnetic force is

\[ F_{AC} = ILB \sin \theta = (4.70 \text{ A})(2.86 \text{ m})(1.80 \text{ T}) \sin 90.0^\circ = 24.2 \text{ N} \]

An application of Right-Hand No. 1 shows that the magnetic force on side \( AC \) is directed perpendicularly into the paper, away from the reader.

b. The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper, the net force is

\[ \sum F = 0 \text{ N} + 24.2 \text{ N} + (-24.2 \text{ N}) = 0 \text{ N} \]
38. **REASONING AND SOLUTION**

a. From Right-Hand Rule No. 1, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows \textbf{left - to - right} in the copper rod.

b. The downward magnetic force exerted on the copper rod is, according to Equation 21.3

\[ F = ILB \sin \theta = (12 \text{ A})(0.85 \text{ m})(0.16 \text{ T}) \sin 90.0^\circ = 1.6 \text{ N} \]

According to Equation 10.1, the force \( F_x^{\text{Applied}} \) required to stretch each spring is \( F_x^{\text{Applied}} = kx \), where \( k \) is the spring constant. Since there are two springs, we know that the magnetic force \( F \) exerted on the current must equal \( 2F_x^{\text{Applied}} \), so that \( F = 2F_x^{\text{Applied}} = 2kx \).

Solving for \( x \), we find that

\[ x = \frac{F}{2k} = \frac{1.6 \text{ N}}{2(75 \text{ N/m})} = 1.1 \times 10^{-2} \text{ m} \]

39. **SSM REASONING** Since the rod does not rotate about the axis at \( P \), the net torque relative to that axis must be zero; \( \Sigma \tau = 0 \) (Equation 9.2). There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's center of gravity, which is at the geometrical center of the rod (length = \( L \)), because the rod is uniform. According to Right-Hand Rule No. 1, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counterclockwise (positive) torque. Equation 21.3 gives the magnitude \( F \) of the magnetic force as \( F = ILB \sin 90.0^\circ \), since the current is perpendicular to the magnetic field. The weight is \( mg \) and acts downward, producing a clockwise (negative) torque. The magnitude of each torque is the magnitude of the force times the lever arm (Equation 9.1). Thus, we have for the torques:

\[ \tau_{\text{magnetic}} = +\left( \frac{ILB}{\text{force}} \right) \left( \frac{L/2}{\text{lever arm}} \right) \text{ and } \tau_{\text{weight}} = -(mg) \left( \frac{L/2}{\text{force}} \right) \left( \frac{\cos \theta}{\text{lever arm}} \right) \]

Setting the sum of these torques equal to zero will enable us to find the angle \( \theta \) that the rod makes with the ground.
SOLUTION Setting the sum of the torques equal to zero gives $\Sigma \tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0$, and we have

$$+ (ILB)(L/2) - (mg)[(L/2) \cos \theta] = 0 \quad \text{or} \quad \cos \theta = \frac{ILB}{mg}$$

$$\theta = \cos^{-1} \left[ \frac{(4.1 \text{ A})(0.45 \text{ m})(0.36 \text{ T})}{(0.094 \text{ kg})(9.80 \text{ m/s}^2)} \right] = 44^\circ$$

40. REASONING

There are four forces that act on the wire: the magnetic force (magnitude $F$), the weight $mg$ of the wire, and the tension in each of the two strings (magnitude $T$ in each string). Since there are two strings, the following drawing shows the total tension as $2T$. The magnitude $F$ of the magnetic force is given by $F = ILB \sin \theta$ (Equation 21.3), where $I$ is the current, $L$ is the length of the wire, $B$ is the magnitude of the magnetic field, and $\theta$ is the angle between the wire and the magnetic field. In this problem $\theta = 90.0^\circ$.

The direction of the magnetic force is given by Right-Hand Rule No. 1 (see Section 21.5). The drawing shows an end view of the wire, where it can be seen that the magnetic force (magnitude $= F$) points to the right, in the $+x$ direction.

In order for the wire to be in equilibrium, the net force $\Sigma F_x$ in the $x$-direction must be zero, and the net force $\Sigma F_y$ in the $y$-direction must be zero: $\Sigma F_x = 0$ (Equation 4.9a) and $\Sigma F_y = 0$ (Equation 4.9b). These equations will allow us to determine the angle $\phi$ and the tension $T$.

SOLUTION

Since the wire is in equilibrium, the sum of the forces in the $x$ direction is zero:

$$\Sigma F_x = -2T \sin \phi + F = 0$$

Substituting in $F = ILB \sin 90.0^\circ$ for the magnitude of the magnetic force, this equation becomes
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\[ -2T \sin \phi + I LB \sin 90.0^\circ = 0 \]  \tag{1}

The sum of the forces in the y direction is also zero:

\[ +2T \cos \phi - mg = 0 \]  \tag{2}

Since these two equations contain two unknowns, \( \phi \) and \( T \), we can solve for each of them.

a. To obtain the angle \( \phi \), we solve Equation (2) for the tension \( T = \frac{mg}{2 \cos \phi} \) and substitute the result into Equation (1). This gives

\[ -2 \left( \frac{mg}{2 \cos \phi} \right) \sin \phi + I LB \sin 90.0^\circ = 0 \quad \text{or} \quad \frac{\sin \phi}{\cos \phi} = \frac{I LB}{mg} \tan \phi \]

Thus,

\[ \phi = \tan^{-1} \left( \frac{I LB}{mg} \right) = \tan^{-1} \left[ \left( \frac{42 \text{ A} \cdot 0.20 \text{ m} \cdot 0.070 \text{ T}}{(0.080 \text{ kg}) \cdot (9.80 \text{ m/s}^2)} \right) \right] = 37^\circ \]

b. The tension in each wire can be found directly from Equation (2):

\[ T = \frac{mg}{2 \cos \phi} = \frac{(0.080 \text{ kg}) \cdot (9.80 \text{ m/s}^2)}{2 \cos 37^\circ} = 0.49 \text{ N} \]

41. **REASONING** The following drawing shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight \( mg \), (2) the magnetic force \( F \), and the normal force \( F_N \). An application of Right-Hand Rule No. 1 shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, Newton’s second law states that the net force along the x-direction is zero: \( \Sigma F_x = ma_x = 0 \). Using the components of \( F \) and \( mg \) that are along the x-axis, Newton’s second law becomes

\[ -F \cos 30.0^\circ + mg \sin 30.0^\circ = 0 \]

\[ \Sigma F_x \]
The magnetic force is given by Equation 21.3 as $F = ILB \sin \theta$, where $\theta = 90.0^\circ$ is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.

**SOLUTION** Substituting the expression $F = ILB \sin 90.0^\circ$ into Newton’s second law and solving for the current $I$, we obtain

$$I = \frac{mg \sin 30.0^\circ}{(LB \sin 90.0^\circ) \cos 30.0^\circ} = \frac{(0.20 \text{ kg})(9.80 \text{ m/s}^2) \sin 30.0^\circ}{(1.6 \text{ m})(0.050 \text{ T}) \sin 90.0^\circ \cos 30.0^\circ} = 14 \text{ A}$$

42. **REASONING** According to Equation 21.4, the maximum torque is $	au_{\text{max}} = NIAB$, where $N$ is the number of turns in the coil, $I$ is the current, $A = \pi r^2$ is the area of the circular coil, and $B$ is the magnitude of the magnetic field. We can apply the maximum-torque expression to each coil, noting that $\tau_{\text{max}}, N,$ and $I$ are the same for each.

**SOLUTION** Applying Equation 21.4 to each coil, we have

$$\tau_{\text{max}}^1 = NI_1\pi r_1^2 B_1 \quad \text{and} \quad \tau_{\text{max}}^2 = NI_2\pi r_2^2 B_2$$

Dividing the expression for coil 2 by the expression for coil 1 gives

$$\frac{\tau_{\text{max}}^2}{\tau_{\text{max}}^1} = \frac{NI_2\pi r_2^2 B_2}{NI_1\pi r_1^2 B_1} \quad \text{or} \quad 1 = \frac{r_2^2 B_2}{r_1^2 B_1}$$

Solving for $r_2$, we obtain

$$r_2 = r_1 \sqrt{\frac{B_1}{B_2}} = (5.0 \text{ cm}) \sqrt{\frac{0.18 \text{ T}}{0.42 \text{ T}}} = 3.3 \text{ cm}$$
43. **REASONING** The magnitude of the torque that acts on a current-carrying coil placed in a magnetic field is specified by $\tau = NIAB \sin \phi$ (Equation 21.4), where $N$ is the number of loops in the coil, $I$ is the current, $A$ is the area of one loop, $B$ is the magnitude of the magnetic field, and $\phi$ is the angle between the normal to the coil and the magnetic field. All the variables in this relation are known except for the current, which can, therefore, be obtained.

**SOLUTION** Solving the equation $\tau = NIAB \sin \phi$ for the current $I$ and noting that $\phi = 90.0^\circ$ since $\tau$ is specified to be the maximum torque, we have

$$I = \frac{\tau}{NAB\sin \phi} = \frac{5.8 \text{ N} \cdot \text{m}}{(1200)(1.1 \times 10^{-2} \text{ m}^2)(0.20 \text{ T})\sin 90.0^\circ} = 2.2 \text{ A}$$

44. **REASONING** The magnetic moment of a current-carrying coil is discussed in Section 21.6, where it is given as

$$\text{Magnetic moment} = NIA \quad (1)$$

In Equation (1), $N$ is the number of turns in the coil, $I$ is the current it carries, and $A$ is its area. Both coils in this problem are circular, so their areas are calculated from their radii via $A = \pi r^2$.

**SOLUTION** Because the magnetic moments of the two coils are equal, Equation (1) yields

$$N_2 I_2 A_2 = N_1 I_1 A_1 \quad (2)$$

Substituting $A = \pi r^2$ into Equation (2) and solving for $r_2$, we obtain

$$N_2 I_2 \left( \pi r_2^2 \right) = N_1 I_1 \left( \pi r_1^2 \right) \quad \text{or} \quad r_2^2 = r_1^2 \left( \frac{N_1 I_1}{N_2 I_2} \right) \quad \text{or} \quad r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} \quad (3)$$

Therefore, the radius of the second coil is

$$r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} = (0.088 \text{ m}) \sqrt{\frac{(140)(4.2 \text{ A})}{(170)(9.5 \text{ A})}} = 0.053 \text{ m}$$

45. **REASONING** According to Equation 21.4, the torque $\tau$ that the circular coil experiences is $\tau = NIAB \sin \phi$, where $N$ is the number of turns, $I$ is the current, $A$ is the area of the circle, $B$ is the magnetic field strength, and $\phi$ is the angle between the normal to the coil and the magnetic field. To use this expression, we need the area of the circle, which is $\pi r^2$, where $r$ is the radius. We do not know the radius, but we know the length $L$ of the wire, which must equal the circumference of the single turn. Thus, $L = 2\pi r$, which can be solved for the radius.
SOLUTION Using Equation 21.4 and the fact that the area $A$ of a circle is $A = \pi r^2$, we have that

$$\tau = NIAB\sin\phi = NI\left(\pi r^2\right)B\sin\phi \quad (1)$$

Since the length of the wire is the circumference of the circle, or $L = 2\pi r$, it follows that the radius of the circle is $r = \frac{L}{2\pi}$. Substituting this result into Equation (1) gives

$$\tau = NI\left(\pi \left(\frac{L}{2\pi}\right)^2\right)B\sin\phi = \frac{NIL^2B}{4\pi}\sin\phi$$

The maximum torque $\tau_{\text{max}}$ occurs when $\phi = 90.0^\circ$, so that

$$\tau_{\text{max}} = \frac{NIL^2B}{4\pi}\sin90.0^\circ = \frac{(1)(4.30 \text{ A})(7.00 \times 10^{-2} \text{ m})^2(2.50 \text{ T})}{4\pi} \sin90.0^\circ = 4.19 \times 10^{-3} \text{ N} \cdot \text{m}$$

46. REASONING According to the discussion in Section 21.6, the magnetic moment of the current-carrying triangle is $NIA$, where $N = 1$ is the number of loops in the coil, $I$ is the current in the coil, and $A$ is the area of the triangle. The magnitude $\tau$ of the net torque exerted on the triangle by the magnetic field is $\tau = NIA(B\sin\phi)$ (Equation 21.4), where $B$ is the magnitude of the magnetic field and $\phi = 90.0^\circ$ is the angle between the magnetic field and the normal to the plane of the triangle.

SOLUTION
a. Using the fact that the area of a triangle is one-half the base times the height of the triangle, we find that the magnetic moment is

$$\text{Magnetic moment} = NIA = (1)(4.70 \text{ A}) \left(\frac{1}{2}(2.00 \text{ m})\left[(2.00 \text{ m}) \tan 55.0^\circ\right]\right) = 13.4 \text{ A} \cdot \text{m}^2$$

b. The magnitude of the net torque exerted on the triangle is

$$\tau = NIA \left( B\sin\phi \right) = \left(13.4 \text{ A} \cdot \text{m}^2\right)(1.80 \text{ T}) \sin 90.0^\circ = 24.1 \text{ N} \cdot \text{m} \quad (21.4)$$

47. REASONING The magnitude $\tau$ of the torque that acts on a current-carrying coil placed in a magnetic field is given by $\tau = NIAB \sin\phi$ (Equation 21.4), where $N$ is the number of loops in the coil ($N = 1$ in this problem), $I$ is the current, $A$ is the area of one loop, $B$ is the magnitude of the magnetic field (the same for each coil), and $\phi$ is the angle (the same for each coil) between the normal to the coil and the magnetic field. Since we are given that the torque for the square coil is the same as that for the circular coil, we can write
\[ \frac{(1) I_{\text{square}} A_{\text{square}} B \sin \phi}{\tau_{\text{square}}} = \frac{(1) I_{\text{circle}} A_{\text{circle}} B \sin \phi}{\tau_{\text{circle}}} \]

This relation can be used directly to find the ratio of the currents.

**SOLUTION** Solving the equation above for the ratio of the currents yields

\[ \frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{A_{\text{circle}}}{A_{\text{square}}} \]

If the length of each wire is \( L \), the length of each side of the square is \( \frac{1}{4} L \), and the area of the square coil is \( A_{\text{square}} = \left( \frac{1}{4} L \right)^2 = \frac{1}{16} L^2 \). The area of the circular coil is \( A_{\text{circle}} = \pi r^2 \), where \( r \) is the radius of the coil. Since the circumference \( (2\pi r) \) of the circular coil is equal to the length \( L \) of the wire, we have \( 2\pi r = L \), or \( r = L / (2\pi) \). Substituting this value for \( r \) into the expression for the area of the circular coil gives \( A_{\text{circle}} = \pi \left( \frac{L}{2\pi} \right)^2 \). Thus, the ratio of the currents is

\[ \frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{A_{\text{circle}}}{A_{\text{square}}} = \frac{\pi \left( \frac{L}{2\pi} \right)^2}{\frac{1}{16} L^2} = \frac{4}{\pi} = 1.27 \]

48. **REASONING** When the wire is used to make a single-turn square coil, each side of the square has a length of \( \frac{1}{4} L \). When the wire is used to make a two-turn coil, each side of the square has a length of \( \frac{1}{8} L \). The drawing shows these two options and indicates that the total effective area of \( NA \) is greater for the single-turn option. Hence, more torque is obtained by using the single-turn option.

**SOLUTION** According to Equation 21.4 the maximum torque experienced by the coil is \( \tau_{\text{max}} = (NIAB) \sin 90^\circ = NIAB \), where \( N \) is the number of turns, \( I \) is the current, \( A \) is the area of each turn, and \( B \) is the magnitude of the magnetic field. Applying this expression to each option gives

**Single-turn** \( \tau_{\text{max}} = NIAB = NI \left( \frac{1}{4} L \right)^2 B \)

\[ = (1)(1.7 \text{ A}) \left( \frac{1}{4} (1.00 \text{ m}) \right)^2 (0.34 \text{ T}) = 0.036 \text{ N} \cdot \text{m} \]

**Two-turn** \( \tau_{\text{max}} = NIAB = NI \left( \frac{1}{8} L \right)^2 B \)

\[ = (2)(1.7 \text{ A}) \left( \frac{1}{8} (1.00 \text{ m}) \right)^2 (0.34 \text{ T}) = 0.018 \text{ N} \cdot \text{m} \]
49. **SSM REASONING** The torque on the loop is given by Equation 21.4, \( \tau = NIAB \sin \phi \).
From the drawing in the text, we see that the angle \( \phi \) between the normal to the plane of the loop and the magnetic field is \( 90^\circ - 35^\circ = 55^\circ \). The area of the loop is \( 0.70 \text{ m} \times 0.50 \text{ m} = 0.35 \text{ m}^2 \).

**SOLUTION**

a. The magnitude of the net torque exerted on the loop is

\[
\tau = NIAB \sin \phi = (75)(4.4 \text{ A})(0.35 \text{ m}^2)(1.8 \text{ T}) \sin 55^\circ = 170 \text{ N} \cdot \text{m}
\]

b. As discussed in the text, when a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of \( 55^\circ \) with respect to the magnetic field. Since this angle decreases as the loop rotates, the \( 35^\circ \) angle increases.

50. **REASONING AND SOLUTION** According to Equation 21.4, the maximum torque for a single turn is \( \tau_{\text{max}} = IAB \). When the length \( L \) of the wire is used to make the square, each side of the square has a length \( L/4 \). The area of the square is \( A_{\square} = (L/4)^2 \). For the rectangle, since two sides have a length \( d \), while the other two sides have a length \( 2d \), it follows that \( L = 6d \), or \( d = L/6 \). The area is \( A_{\text{rectangle}} = 2d^2 = 2(L/6)^2 \). Using Equation 21.4 for the square and the rectangle, we obtain

\[
\frac{\tau_{\text{square}}}{\tau_{\text{rectangle}}} = \frac{IA_{\text{square}} B}{IA_{\text{rectangle}} B} = \frac{A_{\text{square}}}{A_{\text{rectangle}}} = \frac{(L/4)^2}{2(L/6)^2} = 1.13
\]

51. **REASONING** The coil in the drawing is oriented such that the normal to the surface of the coil is perpendicular to the magnetic field (\( \phi = 90^\circ \)). The magnetic torque is a maximum, and Equation 21.4 gives its magnitude as \( \tau = NIAB \sin \phi \). In this expression \( N \) is the number of loops in the coil, \( I \) is the current, \( A \) is the area of one loop, and \( B \) is the magnitude of the magnetic field. The torque from the brake balances this magnetic torque. The brake torque is \( \tau_{\text{brake}} = F_{\text{brake}} r \), where \( F_{\text{brake}} \) is the brake force, and \( r \) is the radius of the shaft and also the lever arm. The maximum value for the brake force available from static friction is \( F_{\text{brake}} = \mu_s F_N \) (Equation 4.7), where \( F_N \) is the normal force pressing the brake shoe against the shaft. The maximum brake torque, then, is \( \tau_{\text{brake}} = \mu_s F_N r \). By setting \( \tau_{\text{brake}} = \tau_{\text{max}} \), we will be able to determine the magnitude of the normal force.

**SOLUTION** Setting the torque produced by the brake equal to the maximum torque produced by the coil gives
\[
\tau_{\text{brake}} = \tau \quad \text{or} \quad \mu_s F_N r = NIAB \sin \phi
\]

\[
F_N = \frac{NIAB \sin \phi}{\mu_s r} = \frac{(410)(0.26 \text{ A})(3.1 \times 10^{-3} \text{ m}^2)(0.23 \text{ T})\sin 90^\circ}{(0.76)(0.012 \text{ m})} = 8.3 \text{ N}
\]

52. **REASONING** The magnetic moment of the orbiting electron can be found from the expression \( \text{Magnetic moment} = NIA \). For this situation, \( N = 1 \). Thus, we need to find the current and the area for the orbiting charge.

**SOLUTION** The current for the orbiting charge is, by definition (see Equation 20.1),

\[
I = \frac{\Delta q}{\Delta t}, \quad \text{where} \quad \Delta q \text{ is the amount of charge that passes a given point during a time interval } \Delta t. \text{ Since the charge} (\Delta q = e) \text{ passes by a given point once per revolution, we can find the current by dividing the total orbiting charge by the period } T \text{ of revolution.}
\]

\[
I = \frac{\Delta q}{T} = \frac{\Delta q}{2\pi r / v} = \frac{(1.6 \times 10^{-19} \text{ C})(2.2 \times 10^6 \text{ m/s})}{2\pi(5.3 \times 10^{-11} \text{ m})} = 1.06 \times 10^{-3} \text{ A}
\]

The area of the orbiting charge is

\[
A = \pi r^2 = \pi(5.3 \times 10^{-11} \text{ m})^2 = 8.82 \times 10^{-21} \text{ m}^2
\]

Therefore, the magnetic moment is

\[
\text{Magnetic moment} = NIA = (1)(1.06 \times 10^{-3} \text{ A})(8.82 \times 10^{-21} \text{ m}^2) = 9.3 \times 10^{-24} \text{ A} \cdot \text{m}^2
\]

53. **SSM REASONING AND SOLUTION**

a. In Figure 21.27a the magnetic field that exists at the location of each wire points upward. Since the current in each wire is the same, the fields at the locations of the wires also have the same magnitudes. Therefore, a single external field that points \( \text{downward} \) will cancel the mutual repulsion of the wires, if this external field has a magnitude that equals that of the field produced by either wire.

b. Equation 21.5 gives the magnitude of the field produced by a long straight wire. The external field must have this magnitude:

\[
B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(25 \text{ A})}{2\pi(0.016 \text{ m})} = 3.1 \times 10^{-4} \text{ T}
\]
54. **REASONING** The magnitude $B$ of the magnetic field in the interior of a long solenoid is $B = \mu_0 n I$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7}$ T⋅m/A is the permeability of free space, $n$ is the number of turns per unit length of the solenoid, and $I$ is the current.

**SOLUTION** Using Equation 21.7, we find that

$$B = \mu_0 n I = \left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right) \left(\frac{1400 \text{ turns}}{0.65 \text{ m}}\right)(4.7 \text{ A}) = 1.3 \times 10^{-2} \text{ T}$$

55. **SSM REASONING** The magnitude $B$ of the magnetic field in the interior of a solenoid that has a length much greater than its diameter is given by $B = \mu_0 n I$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7}$ T⋅m/A is the permeability of free space, $n$ is the number of turns per meter of the solenoid’s length, and $I$ is the current in the wire of the solenoid. Since $B$ and $I$ are given, we can solve Equation 21.7 for $n$.

**SOLUTION** Solving Equation 21.7 for $n$, we find that the number of turns per meter of length is

$$n = \frac{B}{\mu_0 I} = \left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right) \left(\frac{7.0 \text{ T}}{2.0 \times 10^2 \text{ A}}\right) = 2.8 \times 10^4 \text{ turns/m}$$

56. **REASONING** The torque $\tau$ exerted on a coil with a current $I_{\text{coil}}$ is given by $\tau = NI_{\text{coil}}AB\sin \phi$ (Equation 21.4), where $N$ is the number of turns in the coil, $A$ is its area, $B$ is the magnitude of the external magnetic field causing the torque, and $\phi$ is the angle between the normal to the surface of the coil and the direction of the external magnetic field. The external magnetic field $B$, in this case, is the magnetic field of the solenoid. We will use $B = \mu_0 n I$ (Equation 21.7) to determine the magnetic field, where $\mu_0 = 4\pi \times 10^{-7}$ T⋅m/A is the permeability of free space, $n$ is the number of turns per meter of length of the solenoid, and $I$ is the current in the solenoid. The magnetic field in the interior of a solenoid is parallel to the solenoid’s axis. Because the normal to the surface of the coil is perpendicular to the axis of the solenoid, the angle $\phi$ is equal to 90.0°.

**SOLUTION** Substituting the expression $B = \mu_0 n I$ (Equation 21.7) for the magnetic field of the solenoid into $\tau = NI_{\text{coil}}AB\sin \phi$ (Equation 21.4), we obtain

$$\tau = NI_{\text{coil}}AB\sin \phi = NI_{\text{coil}}A(\mu_0 n I)\sin \phi = \mu_0 n NI_{\text{coil}}I \sin \phi$$

Therefore, the torque exerted on the coil is

$$\tau = \left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(1400 \text{ m}^{-1})(50)(1.2 \times 10^{-3} \text{ m}^2)(0.50 \text{ A})(3.5 \text{ A})\sin 90.0°$$

$$= 1.8 \times 10^{-4} \text{ N}\cdot\text{m}$$
57. **REASONING** The magnitude of the magnetic field at the center of a circular loop of current is given by Equation 21.6 as \( B = N \mu_0 I / (2R) \), where \( N \) is the number of turns, \( \mu_0 \) is the permeability of free space, \( I \) is the current, and \( R \) is the radius of the loop. The field is perpendicular to the plane of the loop. Magnetic fields are vectors, and here we have two fields, each perpendicular to the plane of the loop producing it. Therefore, the two field vectors are perpendicular, and we must add them as vectors to get the net field. Since they are perpendicular, we can use the Pythagorean theorem to calculate the magnitude of the net field.

**SOLUTION** Using Equation 21.6 and the Pythagorean theorem, we find that the magnitude of the net magnetic field at the common center of the two loops is

\[
B_{\text{net}} = \sqrt{\left(\frac{N \mu_0 I}{2R}\right)^2 + \left(\frac{N \mu_0 I}{2R}\right)^2} = \sqrt{2} \left(\frac{N \mu_0 I}{2R}\right)
\]

\[
= \sqrt{2} \left(1\right) \left(4\pi \times 10^{-7} \, \text{T} \cdot \text{m} / \text{A}\right) \left(1.7 \, \text{A}\right) \left(\frac{2(0.040 \, \text{m})}{0.073 \, \text{kg}}\right) = 3.8 \times 10^{-5} \, \text{T}
\]

58. **REASONING** The two rods attract each other because they each carry a current \( I \) in the same direction. The bottom rod floats because it is in equilibrium. The two forces that act on the bottom rod are the downward force of gravity \( mg \) and the upward magnetic force of attraction to the upper rod. If the two rods are a distance \( s \) apart, the magnetic field generated by the top rod at the location of the bottom rod is (see Equation 21.5) \( B = \mu_0 I / (2\pi s) \). According to Equation 21.3, the magnetic force exerted on the bottom rod is \( F = 1L B \sin \theta = \mu_0 I^2 L \sin \theta / (2\pi s) \), where \( \theta \) is the angle between the magnetic field at the location of the bottom rod and the direction of the current in the bottom rod. Since the rods are parallel, the magnetic field is perpendicular to the direction of the current (RHR-2), and \( \theta = 90.0^\circ \), so that \( \sin \theta = 1.0 \).

**SOLUTION** Taking upward as the positive direction, the net force on the bottom rod is

\[
\mu_0 I^2 L \sin \theta \frac{2\pi s}{2\pi s} - mg = 0
\]

Solving for \( I \), we find

\[
I = \sqrt{\frac{2\pi gs}{\mu_0 L}} = \sqrt{\frac{2\pi (0.073 \, \text{kg})(9.80 \, \text{m/s}^2)(8.2 \times 10^{-3} \, \text{m})}{(4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A})(0.85 \, \text{m})}} = 190 \, \text{A}
\]
59. **REASONING AND SOLUTION** Let the current in the left-hand wire be labeled $I_1$ and that in the right-hand wire $I_2$.

a. At point $A$: $B_1$ is up and $B_2$ is down, so we subtract them to get the net field. We have

$$B_1 = \frac{\mu_0 I_1}{2\pi d_1} = \frac{\mu_0 (8.0 \text{ A})}{2\pi (0.030 \text{ m})}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi d_2} = \frac{\mu_0 (8.0 \text{ A})}{2\pi (0.150 \text{ m})}$$

So the net field at point $A$ is

$$B_A = B_1 - B_2 = 4.3 \times 10^{-5} \text{ T}$$

b. At point $B$: $B_1$ and $B_2$ are both down so we add the two. We have

$$B_1 = \frac{\mu_0 (8.0 \text{ A})}{2\pi (0.060 \text{ m})}$$

$$B_2 = \frac{\mu_0 (8.0 \text{ A})}{2\pi (0.060 \text{ m})}$$

So the net field at point $B$ is

$$B_B = B_1 + B_2 = 5.3 \times 10^{-5} \text{ T}$$

60. **REASONING** The net magnetic field is the sum of the uniform field and the field produced by the current in the wire. In order for the net field to be zero, the two field contributions must have the same magnitude and opposite directions. The current $I$ in the wire creates a field that has a magnitude $B$ that is given by $B = \frac{\mu_0 I}{2\pi r}$ (Equation 21.5), where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space and $r$ is the perpendicular distance from the wire. We can solve this equation for $r$, in order to locate the point where the field produced by the current has a magnitude of $7.00 \times 10^{-3} \text{ T}$. Right-hand rule no. 2 indicates that the field of the current has opposite directions on opposite sides of the wire. Therefore, since the wire is perpendicular to the uniform field, we can be confident that this value for $r$ will locate a place on one side or the other of the wire where the net field is zero.

**SOLUTION** Solving Equation 21.5 for $r$, we find that

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(305 \text{ A})}{2\pi(7.00 \times 10^{-3} \text{ T})} = 8.71 \times 10^{-3} \text{ m}$$

61. **REASONING** The magnitude $B_i$ of the magnetic field at the center of the inner coil is given by Equation 21.6 as $B_i = \mu_0 I_i N_i / (2R_i)$, where $I_i$, $N_i$, and $R_i$ are, respectively, the current, the number of turns, and the radius of the inner coil. The magnitude $B_o$ of the magnetic field at the center of the outer coil is $B_o = \mu_0 I_o N_o / (2R_o)$. In order that the net magnetic field at the common center of the two coils be zero, the individual magnetic fields must have the same
magnitude, but opposite directions. Equating the magnitudes of the magnetic fields produced by the inner and outer coils will allow us to find the current in the outer coil.

**SOLUTION** Setting $B_i = B_o$ gives

$$\frac{\mu_0 I_i N_i}{2R_i} = \frac{\mu_0 I_o N_o}{2R_o}$$

Solving this expression for the current in the outer coil, we have

$$I_o = I_i \left( \frac{N_i}{N_o} \right) \left( \frac{R_o}{R_i} \right) = (7.2 \, \text{A}) \left( \frac{140 \, \text{turns}}{180 \, \text{turns}} \right) \left( \frac{0.023 \, \text{m}}{0.015 \, \text{m}} \right) = 8.6 \, \text{A}$$

In order that the two magnetic fields have opposite directions, the current in the outer coil must have an opposite direction to the current in the inner coil.

62. **REASONING**

   a. The compass needle lines up with the net horizontal magnetic field $B$ that is the vector sum of the magnetic fields $B_I$ (the field at the location of the compass due to the current $I$ in the wire) and $B_E$ (the horizontal component of the earth’s magnetic field): $B = B_I + B_E$. The field lines of the magnetic field created by the current $I$ are circles centered on the wire. Because the wire is perpendicular to the earth’s surface, and the compass is directly north of the wire, the magnetic field $B_I$ due to the wire must point either due east or due west at the location of the compass, depending on the direction of the current. The magnetic field $B_I$ causes the compass needle to deflect east of north, so we conclude that $B_I$ points due east (see the drawing, which shows the situation as viewed from above). We will use Right-Hand Rule No. 2 to determine the direction of the current $I$ from the direction of $B_I$.

   b. The horizontal component $B_E$ of the earth’s magnetic field points north, so it is perpendicular to the magnetic field $B_I$ created by the current in the wire, which points east. Therefore, the vectors $B$, $B_E$, and $B_I$ form a right triangle with $B$ serving as the hypotenuse (see the drawing). The angle $\theta$ between the vectors $B_E$ and $B$, then, is given by $\tan \theta = \frac{B_I}{B_E}$ (Equation 1.3). We will use $B = \frac{\mu_0 I}{2\pi r}$ (Equation 21.5) to determine $B_I$, where $r = 0.280 \, \text{m}$ is
the radial distance between the wire and the center of the compass needle and \( \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \). Then we will use Equation 1.3 to find the magnitude \( B_E \) of the horizontal component of earth’s magnetic field.

**SOLUTION**

a. Because the magnetic field \( B_1 \) due to the current in the wire points east at the location of the compass, the magnetic field lines must circulate clockwise around the wire, as viewed from above. Right-Hand Rule No. 2, then, indicates that the current \( I \) flows into the page. Since the drawing shows a top view of the situation, the current flows toward the earth’s surface.

b. Solving \( \tan \theta = \frac{B_1}{B_E} \) (Equation 1.3) for \( B_E \), we obtain

\[
B_E = \frac{B_1}{\tan \theta}
\]  

(Substituting \( B_1 = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5) into Equation (1), we find that

\[
B_E = \frac{B_1}{\tan \theta} = \frac{\left( \frac{\mu_0 I}{2\pi r} \right)}{\tan \theta} = \frac{\mu_0 I}{2\pi r \tan \theta} = \frac{4\pi \times 10^{-7} \text{T} \cdot \text{m/A}}{2\pi \left(0.280 \text{ m} \right) \tan 23.0^\circ} = 4.21 \times 10^{-5} \text{T}
\]

63. **REASONING** The drawing shows an end-on view of the two wires, with the currents directed out of the plane of the paper toward you. \( B_1 \) and \( B_2 \) are the individual fields produced by each wire. Applying RHR-2 (thumb points in the direction of the current, fingers curl into the shape of a half-circle and the finger tips point in the direction of the field), we obtain the field directions shown in the drawing for each of the three regions mentioned in the problem statement. Note that it is only in the region between the wires that \( B_1 \) and \( B_2 \) have opposite directions. Hence, the spot where the net magnetic field (the vector sum of the individual fields) is zero must lie between the wires. At this spot the magnitudes of the fields \( B_1 \) and \( B_2 \) from the wires are equal and each is given by \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( \mu_0 \) is the permeability of free space, \( I \) is the current in the wire, and \( r \) is the distance from the wire.

**SOLUTION** The spot we seek is located at a distance \( d \) from wire 1 (see the drawing). Note that the wires are separated by a distance of one meter. Applying Equation 21.5 to each wire, we have that
\[ B_1 = B_2 \quad \text{or} \quad \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I_2}{2\pi(1.00 \text{ m} - d)} \]

Since \( I_1 = 4I_2 \), this result becomes

\[ \frac{\mu_0 (4I_2)}{2\pi d} = \frac{\mu_0 I_2}{2\pi(1.00 \text{ m} - d)} \quad \text{or} \quad \frac{4}{d} = \frac{1}{1.00 \text{ m} - d} \quad \text{or} \quad 4(1.00 \text{ m} - d) = d \]

Solving for \( d \), we find that \( d = 0.800 \text{ m} \)

64. **REASONING** Using Right-Hand Rule No. 2, we can see that at point \( A \) the magnetic field due to the horizontal current points perpendicularly out of the plane of the paper. Similarly, the magnetic field due to the vertical current points perpendicularly into the plane of the paper. Point \( A \) is closer to the horizontal wire, so that the effect of the horizontal current dominates and the net field is directed out of the plane of the paper.

Using Right-Hand Rule No. 2, we can see that at point \( B \) the magnetic field due to the horizontal current points perpendicularly into the plane of the paper. Similarly, the magnetic field due to the vertical current points perpendicularly out of the plane of the paper. Point \( B \) is closer to the horizontal wire, so that the effect of the horizontal current dominates and the net field is directed into the plane of the paper.

Points \( A \) and \( B \) are the same distance of 0.20 m from the horizontal wire. They are also the same distance of 0.40 m from the vertical wire. Therefore, the magnitude of the field contribution from the horizontal current is the same at both points, although the directions of the field contributions are opposite. Likewise, the magnitude of the field contribution from the vertical current is the same at both points, although the directions of the field contributions are opposite. At either point the magnitude of the net field is the magnitude of the difference between the two contributions, and this is the same at points \( A \) and \( B \).

**SOLUTION** According to Equation 21.5, the magnitude of the magnetic field from the current in a long straight wire is \( B = \mu_0 I/(2\pi r) \), where \( \mu_0 \) is the permeability of free space, \( I \) is the current, and \( r \) is the distance from the wire. Applying this equation to each wire at each point, we see that the magnitude of the net field \( B_{\text{net}} \) is

\[
\text{Point A} \quad B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{A,H}} - \frac{\mu_0 I}{2\pi r_{A,V}} = \frac{\mu_0 I}{2\pi} \left( \frac{1}{r_{A,H}} - \frac{1}{r_{A,V}} \right) = \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 5.6 \text{ A} \right) \left( \frac{1}{0.20 \text{ m}} - \frac{1}{0.40 \text{ m}} \right) = 2.8 \times 10^{-6} \text{ T}
\]

MAGNETIC FORCES AND MAGNETIC FIELDS

\[ \text{Point B } B_{\text{net}} = \frac{\mu_0 I}{r_{B, H}} - \frac{\mu_0 I}{r_{B, V}} = \frac{\mu_0 I}{2\pi} \left( \frac{1}{r_{B, H}} - \frac{1}{r_{B, V}} \right) \]

\[
= \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) (5.6 \text{ A}) \left( \frac{1}{0.20 \text{ m}} - \frac{1}{0.40 \text{ m}} \right) = 2.8 \times 10^{-6} \text{ T}
\]

65. **REASONING** According to Equation 21.6 the magnetic field at the center of a circular, current-carrying loop of \( N \) turns and radius \( r \) is \( B = N\mu_0 I / (2r) \). The number of turns \( N \) in the coil can be found by dividing the total length \( L \) of the wire by the circumference after it has been wound into a circle. The current in the wire can be found by using Ohm's law, \( I = V / R \).

**SOLUTION** The number of turns in the wire is

\[ N = \frac{L}{2\pi r} \]

The current in the wire is

\[ I = \frac{V}{R} = \frac{12.0 \text{ V}}{(5.90 \times 10^{-3} \Omega / \text{m}) L} = \frac{2.03 \times 10^3}{L} \text{ A} \]

Therefore, the magnetic field at the center of the coil is

\[ B = N \left( \frac{\mu_0 I}{2r} \right) = \frac{L}{2\pi r} \left( \frac{\mu_0 I}{2r} \right) = \frac{\mu_0 LI}{4\pi r^2} \]

\[ = \frac{\left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) L \left( \frac{2.03 \times 10^3}{L} \text{ A} \right)}{4\pi (0.140 \text{ m})^2} = 1.04 \times 10^{-2} \text{ T} \]

66. **REASONING** In the drawing at the right, we have labeled the current on the left as \( I_1 \) and its distance from point \( P \) as \( r_1 \). Similarly, we have labeled the current on the right as \( I_2 \) and its distance from point \( P \) as \( r_2 \). In fact, however, as the drawing in the text indicates, \( I_1 = I_2 \) and \( r_1 = r_2 \) (the dashed triangle is an equilateral triangle). It is important to note that within the triangle the angle between \( r_1 \) and the \(-y\) axis is 30°, as is the angle between \( r_2 \) and the \(-y\) axis, because the \(-y\) axis bisects the 60° apex angle of the triangle. Moreover, it follows from right-hand rule no. 2 that the magnetic field \( B_1 \) (from current \( I_1 \)) is perpendicular to \( r_1 \) and also that the magnetic field \( B_2 \) (from current \( I_2 \)) is perpendicular to \( r_2 \). This, in turn, means that the fields \( B_1 \) and \( B_2 \) make
60° angles with the \(-y\) axis, as indicated in the drawing. We will use this geometry in dealing with the components of the two magnetic fields. It is necessary to use components to find the net field at point \(P\), since the fields are vectors.

**SOLUTION** The current \(I\) in each wire creates a field that has a magnitude \(B\) that is given by \(B = \frac{\mu_0 I}{2\pi r}\) (Equation 21.5), where \(\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}\) is the permeability of free space and \(r\) is the perpendicular distance from the wire. Using Equation 21.5 for the magnetic fields \(B_1\) and \(B_2\), we list the components of the fields at point \(P\) and the net components as follows:

<table>
<thead>
<tr>
<th>Field</th>
<th>(x) component</th>
<th>(y) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_1)</td>
<td>(B_{1x} = B_1 \sin 60.0° = \left(\frac{\mu_0 I}{2\pi r}\right) \sin 60.0°)</td>
<td>(B_{1y} = -B_1 \cos 60.0° = -\left(\frac{\mu_0 I}{2\pi r}\right) \cos 60.0°)</td>
</tr>
<tr>
<td>(B_2)</td>
<td>(B_{2x} = -B_2 \sin 60.0° = -\left(\frac{\mu_0 I}{2\pi r}\right) \sin 60.0°)</td>
<td>(B_{2y} = -B_2 \cos 60.0° = -\left(\frac{\mu_0 I}{2\pi r}\right) \cos 60.0°)</td>
</tr>
<tr>
<td>Net</td>
<td>0</td>
<td>(-2\left(\frac{\mu_0 I}{2\pi r}\right) \cos 60.0°)</td>
</tr>
</tbody>
</table>

The net field at point \(P\) has a zero \(x\) component, so its magnitude is just the magnitude of the \(y\) component:

\[
\text{Magnitude of net field at point } P = 2 \left(\frac{\mu_0 I}{2\pi r}\right) \cos 60.0° = 2 \left[\frac{(4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(85.0 \text{ A})}{2\pi(0.150 \text{ m})}\right] \cos 60.0° = 1.13 \times 10^{-4} \text{T}
\]

Since the net field has only a \(y\) component that is negative, the net field points downward and is perpendicular to the dashed line that is the base of the triangle in our drawing.

**REASONING AND SOLUTION** The forces acting on each wire are the magnetic force \(F\), the gravitational force \(mg\), and the tension \(T\) in the strings. Each string makes an angle of 7.5° with respect to the vertical. From the drawing below at the right we can relate the magnetic force to the gravitational force. Since the wire is in equilibrium, Newton’s second law requires that \(\Sigma F_x = 0\) and \(\Sigma F_y = 0\). These equations become

\[
\begin{align*}
-T \sin 7.5° + F &= 0 \\
\sum F_x &= 0 \\
T \cos 7.5° - mg &= 0 \\
\sum F_y &= 0
\end{align*}
\]

![Diagram of forces and equilibrium](image)
Solving the first equation for $T$, and then substituting the result into the second equation gives (after some simplification)

$$\tan 7.5^\circ = \frac{F}{mg}$$  \hspace{1cm} (1)

The magnetic force $F$ exerted on one wire by the other is $F = \frac{\mu_0 I^2 L}{2\pi d}$, where $d$ is the distance between the wires [$d/2 = (1.2 \text{ m}) \sin 7.5^\circ$, so that $d = 0.31 \text{ m}$], $I$ is the current (which is the same for each wire), and $L$ is the length of each wire. Substituting this relation for $F$ into Equation (1) and then solving for the current, gives

$$I = \sqrt{\frac{m}{L}} g \tan 7.5^\circ \left(\frac{2\pi d}{\mu_0}\right)$$

$$\tan 7.5^\circ \left(\frac{2\pi (0.31 \text{ m})}{\mu_0}\right) = 320 \text{ A}$$

68. **REASONING AND SOLUTION** Ampère's law in the form of Equation 21.8 indicates that $\Sigma B_\parallel \Delta \ell = \mu_0 I$. Since the magnetic field is everywhere perpendicular to the plane of the paper, it is everywhere perpendicular to the circular path and has no component $B_\parallel$ that is parallel to the circular path. Therefore, Ampère's law reduces to $\Sigma B_\parallel \Delta \ell = 0 = \mu_0 I$, so that the net current passing through the circular surface is zero.

69. **REASONING** Since the two wires are next to each other, the net magnetic field is everywhere parallel to $\Delta \ell$ in Figure 21.38. Moreover, the net magnetic field $B$ has the same magnitude $B$ at each point along the circular path, because each point is at the same distance from the wires. Thus, in Ampère's law (Equation 21.8), $B_\parallel = B$, $I = I_1 + I_2$, and we have

$$\Sigma B_\parallel \Delta \ell = B(\Sigma \Delta \ell) = \mu_0 \left(I_1 + I_2\right)$$

But $\Sigma \Delta \ell$ is just the circumference ($2\pi r$) of the circle, so Ampère's law becomes

$$B(2\pi r) = \mu_0 \left(I_1 + I_2\right)$$

This expression can be solved for $B$.

**SOLUTION**
a. When the currents are in the same direction, we find that

\[ B = \frac{\mu_0 (I_1 + I_2)}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(28 \text{ A} + 12 \text{ A})}{2\pi (0.72 \text{ m})} = 1.1 \times 10^{-5} \text{ T} \]

b. When the currents have opposite directions, a similar calculation shows that

\[ B = \frac{\mu_0 (I_1 - I_2)}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(28 \text{ A} - 12 \text{ A})}{2\pi (0.72 \text{ m})} = 4.4 \times 10^{-6} \text{ T} \]

70. **REASONING** Both parts of this problem can be solved using Ampère's law with the circular closed paths suggested in the *Hint* given with the problem statement. The circular closed paths are used because of the symmetry in the way the current is distributed on the copper cylinder.

**SOLUTION**

a. An end-on view of the copper cylinder is a circle, as the drawing at the right shows. The dots around the circle represent the current coming out of the paper toward you. The larger dashed circle of radius \( r \) is the closed path used in Ampère's law and is centered on the axis of the cylinder. Equation 21.8 gives Ampère's law as \( \Sigma B \Delta \ell = \mu_0 I \). Because of the symmetry of the arrangement in the drawing, we have \( B_\parallel = B \) for all \( \Delta \ell \) on the circular path, so that Ampère's law becomes

\[ \Sigma B_\parallel \Delta \ell = B(\Sigma \Delta \ell) = \mu_0 I \]

In this result, \( I \) is the net current through the circular surface bounded by the dashed path. In other words, it is the current \( I \) in the copper tube. Furthermore, \( \Sigma \Delta \ell \) is the circumference of the circle, so we find that

\[ B(\Sigma \Delta \ell) = B(2\pi r) = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r} \]
b. The setup here is similar to that in part a, except that the smaller dashed circle of radius \( r \) is now the closed path used in Ampère's law (see the drawing at the right). With this change, the derivation then proceeds exactly as in part a. Now, however, there is no current through the circular surface bounded by the dashed path, because all of the current is outside the path. Therefore, \( I = 0 \) A, and

\[
B = \frac{\mu_0 I}{2\pi r} = 0 \text{ T}
\]

71. **SSM REASONING AND SOLUTION** The drawing at the right shows an end-on view of the solid cylinder. The dots represent the current in the cylinder coming out of the paper toward you. The dashed circle of radius \( r \) is the closed path used in Ampère's law and is centered on the axis of the cylinder. Equation 21.8 gives Ampère's law as

\[
\sum B_l \Delta \ell = \mu_0 I
\]

Because of the symmetry of the arrangement in the drawing, we have \( B_l = B \) for all \( \Delta \ell \) on the circular path, so that Ampère's law becomes

\[
\sum B_l \Delta \ell = B (\sum \Delta \ell) = \mu_0 I
\]

In this result, \( \sum \Delta \ell = 2\pi r \), the circumference of the circle. The current \( I \) is the part of the total current that comes through the area \( \pi r^2 \) bounded by the dashed path. We can calculate this current by using the current per unit cross-sectional area of the solid cylinder. This current per unit area is called the current density. The current \( I \) is the current density times the area \( \pi r^2 \):

\[
I = \left( \frac{I_0}{\pi R^2} \right) (\pi r^2) = \frac{I_0 r^2}{R^2}
\]

Thus, Ampère's law becomes

\[
B (\sum \Delta \ell) = \mu_0 I \quad \text{or} \quad B (2\pi r) = \mu_0 \left( \frac{I_0 r^2}{R^2} \right) \quad \text{or} \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}
\]
72. **REASONING** The magnitude $|q_0|$ of the electric charge of the bullet is related to the magnitude $F$ of the magnetic force exerted on the bullet according to $B = \frac{F}{|q_0|(v \sin \theta)}$ (Equation 21.1).

Here, $\theta$ is the angle between the direction of the bullet’s velocity $v$ and the earth’s magnetic field $B$. (See the drawing, which depicts the situation as seen from the side.) We will find the magnitude of the bullet’s charge from Equation 21.1, and Right-Hand Rule No. 1 will allow us to determine the algebraic sign of the charge.

**SOLUTION** Solving $B = \frac{F}{|q_0|(v \sin \theta)}$ (Equation 21.1) for $|q_0|$, we obtain

$$|q_0| = \frac{F}{B(v \sin \theta)}$$  \hspace{1cm} (1)

The angle $\theta$ is the angle between the directions of the bullet’s velocity $v$ and the direction of earth’s magnetic field $B$. Therefore, $\theta$ is the sum of the angles that these vectors make with the horizontal (see the drawing):

$$\theta = 58^\circ + 11^\circ = 69^\circ$$

Applying Right-Hand Rule No. 1 to the vectors $v$ and $B$ (see the drawing), we see that the force on a positively charged bullet would point into the page, which is west. Because the force on this bullet points to the east, out of the page, the bullet’s charge must be negative. When removing the absolute value brackets from Equation (1), therefore, we must insert a minus sign on the right hand side. Making this change, we find that

$$q_0 = -\frac{F}{B(v \sin \theta)} = -\frac{2.8 \times 10^{-10} \text{ N}}{(5.4 \times 10^{-5} \text{ T})(670 \text{ m/s}) \sin 69^\circ} = -8.3 \times 10^{-9} \text{ C}$$

73. **SSM REASONING** The angle $\theta$ between the electron’s velocity and the magnetic field can be found from Equation 21.1,

$$\sin \theta = \frac{F}{|q|vB}$$

According to Newton’s second law, the magnitude $F$ of the force is equal to the product of the electron’s mass $m$ and the magnitude $a$ of its acceleration, $F = ma$. 
SOLUTION  The angle $\theta$ is

$$\theta = \sin^{-1}\left(\frac{ma}{q|vB|}\right) = \sin^{-1}\left[\frac{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(3.50 \times 10^{14} \text{ m/s}^2\right)}{\left(1.60 \times 10^{-19} \text{ C}\right)\left(6.80 \times 10^6 \text{ m/s}\right)\left(8.70 \times 10^{-4} \text{ T}\right)}\right] = 19.7^\circ$$

74. REASONING  At the center of the loop, the magnitude $B$ of the magnetic field due to the current $I$ in the long, straight wire is equal to the magnitude $B_{\text{loop}}$ of the magnetic field due to the current $I_{\text{loop}}$ in the circular wire loop. We know this because the vector sum of the two magnetic fields at that location is zero. We will determine the magnitude of the magnetic field due to the current in the circular loop from

$$B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R} \quad \text{(Equation 21.6, with } N = 1), \text{ where } \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A is the permeability of free space, and } R \text{ is the radius of the loop. The magnitude } B \text{ of the magnetic field of the long, straight wire is given by}$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{(Equation 21.5), where } r \text{ is the radial distance from the wire to the center of the loop. Because the wire is tangent to the loop, the radial distance } r \text{ is equal to the radius } R \text{ of the loop, and we have that}$$

$$B = \frac{\mu_0 I}{2\pi R} \quad \text{(1)}$$

SOLUTION  The two magnetic fields have equal magnitudes at the center of the circular loop, so $B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R}$ (Equation 21.6) and Equation (1) together yield

$$B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R} = \frac{\mu_0 I}{2\pi R} = B \quad \text{(2)}$$

Solving Equation (2) for $I_{\text{loop}}$, we obtain

$$\frac{\mu_0 I_{\text{loop}}}{2\pi R} = \frac{\mu_0 I}{2\pi R} \quad \text{or} \quad I_{\text{loop}} = \frac{I}{\pi} = 0.12 \frac{\text{A}}{\pi} = 0.038 \frac{\text{A}}{\pi}$$

75. SSM REASONING  According to Equation 21.4, the maximum torque is $\tau_{\text{max}} = NIAB$, where $N$ is the number of turns in the coil, $I$ is the current, $A = \pi r^2$ is the area of the circular coil, and $B$ is the magnitude of the magnetic field. Since the coil contains only one turn, the length $L$ of the wire is the circumference of the circle, so that $L = 2\pi r$ or $r = L/(2\pi)$. Since $N, I,$ and $B$ are known we can solve for $L$.

SOLUTION  According to Equation 21.4 and the fact that $r = L/(2\pi)$, we have

$$\tau_{\text{max}} = NI\pi r^2 B = NI\pi \left(\frac{L}{2\pi}\right)^2 B$$
Solving this result for \( L \) gives

\[
L = \sqrt{\frac{4\pi \tau_{\text{max}}}{NIB}} = \sqrt{\frac{4\pi (8.4 \times 10^{-4} \, \text{N} \cdot \text{m})}{(1)(3.7 \, \text{A})(0.75 \, \text{T})}} = 0.062 \, \text{m}
\]

76. **REASONING** The magnitude of the magnetic force acting on the particle is \( F = |q_0|vB \sin \theta \) (Equation 21.1), where \( |q_0| \) and \( v \) are the charge magnitude and speed of the particle, respectively, \( B \) is the magnitude of the magnetic field, and \( \theta \) is the angle between the particle’s velocity and the magnetic field. The magnetic field is produced by a very long, straight wire, so its value is given by Equation 21.5 as \( B = \mu_0 I / (2\pi r) \). By combining these two relations, we can determine the magnitude of the magnetic force.

**SOLUTION** The direction of the magnetic field \( B \) produced by the current-carrying wire can be found by using Right-Hand Rule No. 2. At the location of the charge, this field points perpendicularly into the page, as shown in the drawing. Since the direction of the particle’s velocity is perpendicular to the magnetic field, \( \theta = 90.0^\circ \). Substituting \( B = \mu_0 I / (2\pi r) \) into \( F = |q_0|vB \sin \theta \) gives

\[
F = |q_0|vB \sin \theta = |q_0|v\left(\frac{\mu_0 I}{2\pi r}\right) \sin \theta
\]

\[
= \left(\frac{6.00 \times 10^{-6} \, \text{C}}{(4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A})(67.0 \, \text{A})}\right)7.50 \times 10^4 \, \text{m/s} \times \sin 90.0^\circ
\]

\[
= \frac{2\pi (5.00 \times 10^{-2} \, \text{m})}{1.21 \times 10^{-4} \, \text{N}}
\]

The direction of the magnetic force \( F \) exerted on the particle can be determined by using Right-Hand Rule No. 1. This direction, which is shown in the drawing, is perpendicular to the wire and is directed away from it.

77. **REASONING** A maximum magnetic force is exerted on the wire by the field components that are perpendicular to the wire, and no magnetic force is exerted by field components that are parallel to the wire. Thus, the wire experiences a force only from the \( x \)- and \( y \)-components of the field. The \( z \)-component of the field may be ignored, since it is parallel to the wire. We can use the Pythagorean theorem to find the net field in the \( x, y \)-plane. This net field, then, is perpendicular to the wire and makes an angle of \( \theta = 90^\circ \) with respect to the wire. Equation 21.3 can be used to calculate the magnitude of the magnetic force that this net field applies to the wire.
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SOLUTION  According to Equation 21.3, the magnetic force has a magnitude of  \( F = ILB \sin \theta \), where \( I \) is the current, \( B \) is the magnitude of the magnetic field, \( L \) is the length of the wire, and \( \theta \) is the angle of the wire with respect to the field. Using the Pythagorean theorem, we find that the net field in the \( x, y \) plane is

\[
B = \sqrt{B_x^2 + B_y^2}
\]

Using this field in Equation 21.3, we calculate the magnitude of the magnetic force to be

\[
F = ILB \sin \theta = IL \sqrt{B_x^2 + B_y^2} \sin \theta
\]

\[
= (4.3 \text{ A})(0.25 \text{ m})\sqrt{(0.10 \text{ T})^2 + (0.15 \text{ T})^2} \sin 90^\circ = 0.19 \text{ N}
\]

78. REASONING  The current \( I \) is the rate of flow of charge and is  \( I = \frac{\Delta q}{\Delta t} \)  (Equation 20.1), where \( \Delta q \) is the amount of charge that flows in a time period \( \Delta t \). We can solve this equation for \( \Delta q \). The current is not given. However, since we are assuming that the bolt can be represented as a long, straight line of current, we can obtain the current from the information provided about the magnetic field \( B \). The field is  \( B = \frac{\mu_0 I}{2\pi r} \)  (Equation 21.5), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space and \( r \) is the perpendicular distance from the current.

SOLUTION  Solving Equation 20.1 for the charge \( \Delta q \), we have

\[
I = \frac{\Delta q}{\Delta t} \quad \text{or} \quad \Delta q = I \Delta t
\]

Solving Equation 21.5 for the current \( I \), we have

\[
B = \frac{\mu_0 I}{2\pi r} \quad \text{or} \quad I = \frac{B2\pi r}{\mu_0}
\]

Substituting this result for \( I \) into the expression for \( \Delta q \), we find that

\[
\Delta q = I \Delta t = \left( \frac{B2\pi r}{\mu_0} \right) \Delta t = \left[ \left( 8.0 \times 10^{-5} \text{ T} \right)\frac{2\pi (27 \text{ m})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \right] (1.8 \times 10^{-3} \text{ s}) = 19 \text{ C}
\]

79. REASONING  When a charge \( q_0 \) travels at a speed \( v \) and its velocity makes an angle \( \theta \) with respect to a magnetic field of magnitude \( B \), the magnetic force acting on the charge has a magnitude \( F \) that is given by  \( F = |q_0|vB \sin \theta \)  (Equation 21.1). We will solve this problem by applying this expression twice, first to the motion of the charge when it moves perpendicular to the field so that \( \theta = 90.0^\circ \) and then to the motion when \( \theta = 38^\circ \).
**SOLUTION** When the charge moves perpendicular to the field so that \( \theta = 90.0^\circ \), Equation 21.1 indicates that

\[
F_{90.0^\circ} = |q_0| vB \sin 90.0^\circ
\]

When the charge moves so that \( \theta = 38^\circ \), Equation 21.1 shows that

\[
F_{38^\circ} = |q_0| vB \sin 38^\circ
\]

Dividing the second expression by the first expression gives

\[
\frac{F_{38^\circ}}{F_{90.0^\circ}} = \frac{|q_0| vB \sin 38^\circ}{|q_0| vB \sin 90.0^\circ}
\]

\[
F_{38^\circ} = F_{90.0^\circ} \left( \frac{\sin 38^\circ}{\sin 90.0^\circ} \right) = \left( 2.7 \times 10^{-3} \text{ N} \right) \left( \frac{\sin 38^\circ}{\sin 90.0^\circ} \right) = 1.7 \times 10^{-3} \text{ N}
\]

80. **REASONING** Each of the four wires makes a contribution to the net magnetic field \( B \) at the center of the square. The magnitude of each wire’s magnetic field is given by \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( I \) is the current in a wire, \( r \) is the radial distance from the wire to the center of the square, and \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space. For all four wires, the radial distance \( r \) is half of the length \( s \) of one of the sides of the square \( \left( r = \frac{1}{2} s \right) \).

In order to add up the four magnetic fields, we must determine the direction of each one. Right-Hand Rule No. 2 predicts that, at the center of the square, the currents \( I_1 = 3.9 \text{ A} \), \( I_2 = 8.5 \text{ A} \), and \( I_3 = 4.6 \text{ A} \) all produce fields pointing into the page, while the magnetic field \( B \) of the unknown current \( I \) points out of the page. Therefore, the magnitude \( B_{\text{net}} \) of the net magnetic field is given by

\[
B_{\text{net}} = B_1 + B_2 + B_3 - B
\]

**SOLUTION** Applying \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5) to the magnetic fields on the right side of Equation (1), we obtain

\[
B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_2}{2\pi r} + \frac{\mu_0 I_3}{2\pi r} - \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r} \left( I_1 + I_2 + I_3 - I \right)
\]

Solving Equation (2) for \( I \) yields

\[
\frac{2\pi r B_{\text{net}}}{\mu_0} = I_1 + I_2 + I_3 - I \quad \text{or} \quad I = I_1 + I_2 + I_3 - \frac{2\pi r B_{\text{net}}}{\mu_0}
\]
The length $s$ of a side of the square is $s = 0.050$ m, so the radial distance $r$ from each wire to the center of the square is half as large: $r = \frac{1}{2} s = 0.025$ m. Therefore, Equation (3) yields

\[
I = 3.9 \text{ A} + 8.5 \text{ A} + 4.6 \text{ A} - \frac{2 \pi (0.025 \text{ m})(61 \times 10^{-6} \text{ T})}{4 \pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 9.4 \text{ A}
\]

81. **REASONING** From the discussion in Section 21.3, we know that when a charged particle moves perpendicular to a magnetic field, the trajectory of the particle is a circle. The drawing at the right shows a particle moving in the plane of the paper (the magnetic field is perpendicular to the paper). If the particle is moving initially through the coordinate origin and to the right (along the $+x$ axis), the subsequent circular path of the particle will intersect the $y$ axis at the greatest possible value, which is equal to twice the radius $r$ of the circle.

**SOLUTION**

a. From the drawing above, it can be seen that the largest value of $y$ is equal to the diameter $(2r)$ of the circle. When the particle passes through the coordinate origin its velocity must be parallel to the $+x$ axis. Thus, the angle is $\theta = 0^\circ$.

b. The maximum value of $y$ is twice the radius $r$ of the circle. According to Equation 21.2, the radius of the circular path is $r = \frac{mv}{|q|B}$. The maximum value $y_{\text{max}}$ is, therefore,

\[
y_{\text{max}} = 2r = 2 \left( \frac{mv}{|q|B} \right) = 2 \left[ \frac{3.8 \times 10^{-8} \text{ kg} \times (44 \text{ m/s})}{(7.3 \times 10^{-6} \text{ C})(1.6 \text{ T})} \right] = 0.29 \text{ m}
\]

82. **REASONING**

a. If the particle is stationary, only the electric field $E_x$ exerts a force on it; this force is $F_E = q E_x$ (Equation 18.2), where $q$ is the charge. A magnetic field does not exert a force on a stationary particle.
b. If the particle is moving along the +x axis, the electric field exerts a force on it. This force is the same as that in part (a) above. The magnetic field \( B_x \) does not exert a force on the particle, because the particle’s velocity is parallel to the field. The magnetic field \( B_y \) does exert a force on the particle, because the particle’s velocity is perpendicular to this field.

c. The particle experiences a force from each of the three fields. The electric field exerts a force on it, and this force is the same as that in part (a) above. It does not matter in which direction the particle travels, for the electric force is independent of the particle’s velocity. The particle experiences magnetic forces from both \( B_x \) and \( B_y \). When the particle moves along the +z axis, its velocity is perpendicular to both \( B_x \) and \( B_y \), so each field exerts a force on the particle.

**SOLUTION**

a. The electric force exerted on the particle is \( F_E = qE_x \) (Equation 18.2), so

\[
F_E = qE_x = \left( +5.60 \times 10^{-6} \text{ C} \right) \left( +245 \text{ N/C} \right) = +1.37 \times 10^{-3} \text{ N}
\]

where the plus sign indicates that the force points along the \(+x\) axis. The magnitude \( F\) of the magnetic force is given by Equation 21.1 as \( F = |q|vB\sin\theta\). Since \( v = 0 \text{ m/s} \), the magnetic forces exerted by \( B_x \) and \( B_y \) are zero:

\[
F_{B_x} = 0 \text{ N} \quad F_{B_y} = 0 \text{ N}
\]

b. The electric force is the same as that computed in part (a), because this force does not depend on the velocity of the particle: \( F_E = +1.37 \times 10^{-3} \text{ N} \), where the plus sign indicates that the force points along the \(+x\) axis.

Since the velocity of the particle and \( B_x \) are along the +x axis \( (\theta = 0^\circ \) in Equation 21.1), the magnitude of the magnetic force is

\[
F_{B_x} = |q|vB_x \sin\theta = |q|vB_x \sin 0^\circ = 0 \text{ N}
\]

The magnetic force exerted by the magnetic field \( B_y \) on the charge has a magnitude of

\[
F_{B_y} = |q|vB_y \sin\theta = \left( 5.60 \times 10^{-6} \text{ C} \right) \left( 375 \text{ m/s} \right) \left( 1.40 \text{ T} \right) \sin 90.0^\circ = 2.94 \times 10^{-3} \text{ N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \(+z\) axis.
c. The electric force is the same as that computed in part (a), because this force does not depend on the velocity of the particle: \( F_E = +1.37 \times 10^{-3} \text{ N} \), where the plus sign indicates that the force points along the \( +x \text{ axis} \).

When the particle moves along the \(+z\) axis, the magnetic field \( B_x \) exerts a force on the charge that has a magnitude of

\[
F_{Bx} = |q_0| v B_x \sin \theta = \left( 5.60 \times 10^{-6} \text{ C} \right) \left( 375 \text{ m/s} \right) \left( 1.80 \text{ T} \right) \sin 90.0^\circ = 3.78 \times 10^{-3} \text{ N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \( +y \text{ axis} \).

When the particle moves along the \(+z\) axis, the magnetic field \( B_y \) exerts a force on the charge that has a magnitude of

\[
F_{By} = |q_0| v B_y \sin \theta = \left( 5.60 \times 10^{-6} \text{ C} \right) \left( 375 \text{ m/s} \right) \left( 1.40 \text{ T} \right) \sin 90.0^\circ = 2.94 \times 10^{-3} \text{ N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \( -x \text{ axis} \).

83. **REASONING** Two wires that are parallel and carry current in the same direction exert attractive magnetic forces on one another, as Section 21.7 discusses. This attraction between the wires causes the spring to compress. When compressed, the spring exerts an elastic restoring force on each wire, as Section 10.1 discusses. For each wire, this restoring force acts to push the wires apart and balances the magnetic force, thus keeping the separation between the wires from decreasing to zero. Equation 10.2 (without the minus sign) gives the magnitude of the restoring force as \( F_x = kx \), where \( k \) is the spring constant and \( x \) is the magnitude of the displacement of the spring from its unstrained length. By setting the magnitude of the magnetic force equal to the magnitude of the restoring force, we will be able to find the separation between the rods when the current is present.

**SOLUTION** According to Equation 21.3, the magnetic force has a magnitude of \( F = ILB \sin \theta \), where \( I \) is the current, \( B \) is the magnitude of the magnetic field, \( L \) is the length of the wire, and \( \theta \) is the angle of the wire with respect to the field. Using RHR-2 reveals that the magnetic field produced by either wire is perpendicular to the other wire, so that \( \theta = 90^\circ \) in Equation 21.3, which becomes \( F = ILB \). According to Equation 21.5 the magnitude of the magnetic field produced by a long straight wire is \( B = \mu_0 I/(2\pi r) \). Substituting this expression into Equation 21.3 gives the magnitude of the magnetic force as
Equating this expression to the magnitude of the restoring force from the spring gives

$$\frac{\mu_0 I^2 L}{2\pi r} = kx$$

Solving for the separation $r$, we find

$$r = \frac{\mu_0 I^2 L}{2\pi kx} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A})(950 \text{ A})^2 (0.50 \text{ m})}{2\pi (150 \text{ N} / \text{m})(0.020 \text{ m})} = 0.030 \text{ m}$$

84. **REASONING** The turns of the coil are as close together as possible without overlapping, meaning that there are no gaps between the turns. Therefore, the width of a single turn is equal to the diameter $D = 2r$ of the wire, where $r$ is the wire’s radius. The number $N$ of turns, therefore, is $N = L/(2r)$, where $L$ is the length of the solenoid. The number of turns per unit length is $n$, where

$$n = \frac{N}{L} = \frac{L/(2r)}{L} = \frac{1}{2r} \quad (1)$$

The magnitude $B$ of the magnetic field inside a solenoid with $n$ turns per meter and a current $I$ is given by $B = \mu_0 nI$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$ is the permeability of free space. We will use Ohm’s law, $R = \frac{V}{I}$ (Equation 20.2), to determine the current $I$ in the solenoid when the battery (Emf = $V$) is connected to it. The resistance $R$ of the silver wire will be found from $R = \rho \frac{L}{A}$ (Equation 20.3), where $\rho = 1.59 \times 10^{-8} \text{ \Omega} \cdot \text{m}$ is the resistivity of silver (see Table 20.1), $L$ is the length of the wire, and $A$ is its cross-sectional area.

**SOLUTION** Substituting Equation (1) into $B = \mu_0 nI$ (Equation 21.7) and solving for the radius $r$ yields

$$B = \mu_0 nI = \frac{\mu_0 I}{2r} \quad \text{or} \quad r = \frac{\mu_0 I}{2B} \quad (2)$$

Solving $R = \frac{V}{I}$ (Equation 20.2) for $I$, we obtain $I = \frac{V}{R}$. Substituting this result into Equation (2), we find that

$$r = \frac{\mu_0 I}{2B} = \frac{\mu_0 V}{2BR} \quad (3)$$

Substituting $R = \rho \frac{L}{A}$ (Equation 20.3) into Equation (3), we obtain
\[ r = \frac{\mu_0 V}{2BR} = \frac{\mu_0 V}{2B \left( \frac{\rho L}{A} \right)} = \frac{\mu_0 VA}{2B \rho L} \tag{4} \]

The wire has a circular cross-section, so we may express the area \( A \) in Equation (4) as \( A = \pi r^2 \). Substituting this expression into Equation (4) and simplifying the resulting expression, we find that

\[ f = \frac{\mu_0 \pi r^2 V}{2B \rho L} \quad \text{or} \quad 1 = \frac{\mu_0 \pi r V}{2B \rho L} \tag{5} \]

Solving Equation (5) for \( r \) yields

\[ r = \frac{2B \rho L}{\mu_0 \pi V} = \frac{2 \left( 6.48 \times 10^{-3} \, \text{T} \right) \left( 1.59 \times 10^{-8} \, \Omega \cdot \text{m} \right) \left( 25.0 \, \text{m} \right)}{\left( 4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A} \right) \left( 3.00 \, \text{V} \right) \pi} = 4.35 \times 10^{-4} \, \text{m} \]

85. **SSM REASONING** The magnetic moment of the rotating charge can be found from the expression \( \text{Magnetic moment} = NIA \), as discussed in Section 21.6. For this situation, \( N = 1 \). Thus, we need to find the current and the area for the rotating charge. This can be done by resorting to first principles.

**SOLUTION** The current for the rotating charge is, by definition (see Equation 20.1), \( I = \frac{\Delta q}{\Delta t} \), where \( \Delta q \) is the amount of charge that passes by a given point during a time interval \( \Delta t \). Since the charge passes by once per revolution, we can find the current by dividing the total rotating charge by the period \( T \) of revolution.

\[ I = \frac{\Delta q}{T} = \frac{\omega \Delta q}{2\pi} = \frac{(150 \, \text{rad/s})(4.0 \times 10^{-6} \, \text{C})}{2\pi} = 9.5 \times 10^{-5} \, \text{A} \]

The area of the rotating charge is \( A = \pi r^2 = \pi (0.20 \, \text{m})^2 = 0.13 \, \text{m}^2 \)

Therefore, the magnetic moment is

\[ \text{Magnetic moment} = NIA = (1)(9.5 \times 10^{-5} \, \text{A})(0.13 \, \text{m}^2) = 1.2 \times 10^{-5} \, \text{A} \cdot \text{m}^2 \]
1. 3.5 m/s

2. (e) The work done by the hand equals the energy dissipated in the bulb. The energy dissipated in the bulb equals the power used by the bulb times the time. Since the time is the same in each case, more work is done when the power used is greater. The power, however, is the voltage squared divided by the resistance of the bulb, according to Equation 20.6c, so that a smaller resistance corresponds to a greater power. Thus, more work is done when the resistance of the bulb is smaller.

3. (c) The magnetic flux $\Phi$ that passes through a surface is $\Phi = BA\cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the field and the normal to the surface. Knowing $\Phi$ and $A$, we can calculate $B\cos \phi = \Phi / A$, which is the component of the field parallel to the normal or perpendicular to the surface.

4. (b) The magnetic flux $\Phi$ that passes through a surface is $\Phi = BA\cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the field and the normal to the surface. It has the greatest value when the field strikes the surface perpendicularly ($\phi = 0^\circ$) and a value of zero when the field is parallel to a surface ($\phi = 90^\circ$). The field is more nearly perpendicular to face 1 ($\phi = 20^\circ$) than to face 3 ($\phi = 70^\circ$) and is parallel to face 2.

5. (d) Faraday’s law of electromagnetic induction states that the average emf $\xi$ induced in a coil of $N$ loops is $\xi = -N(\Delta \Phi / \Delta t)$ (Equation 22.3), where $\Delta \Phi$ is the change in magnetic flux through one loop and $\Delta t$ is the time interval during which the change occurs. Reducing the time interval $\Delta t$ during which the field magnitude increases means that the rate of change of the flux will increase, which will increase (not reduce) the induced emf.

6. 3.2 V

7. (c) According to Faraday’s law, the magnitude of the induced emf is the magnitude of the change in magnetic flux divided by the time interval over which the change occurs (see Equation 22.3). In each case the field is perpendicular to the coil, and the initial flux is zero since the coil is outside the field region. Therefore, the changes in flux are as follows: $\Delta \Phi_A = BL^2$, $\Delta \Phi_B = \Delta \Phi_C = B2L^2$ (see Equation 22.2). The corresponding time intervals are
\[ \Delta t_A = \Delta t_B = \Delta t, \quad \Delta t_C = 2\Delta t. \]

Dividing gives the following results for the magnitudes of the emfs:

\[ |\xi_A| = \frac{BL^2}{\Delta t}, \quad |\xi_B| = \frac{B2L^2}{\Delta t}, \quad |\xi_C| = \frac{B2L^2}{2\Delta t} = \frac{BL^2}{\Delta t}. \]

8. (a) An induced current appears only when there is an induced emf to drive it around the loop. According to Faraday’s law, an induced emf exists only when the magnetic flux through the loop changes as time passes. Here, however, there is no magnetic flux through the loop. The magnetic field lines produced by the current are circular and centered on the wire, with the planes of the circles perpendicular to the wire. Therefore, the magnetic field is always parallel to the plane of the loop as the loop falls and never penetrates the loop. In other words, no magnetic flux passes through the loop. No magnetic flux, no induced emf, no induced current.

9. (d) When the switch is closed, current begins to flow counterclockwise in the larger coil, and the field that it creates appears inside the smaller coil. Using RHR-2 reveals that this field points out of the screen toward you. According to Lenz’s law, the induced current in the smaller coil flows in such a direction that it creates an induced field that opposes the growth of the field from the larger coil. Thus, the induced field must point into the screen away from you. Using RHR-2 reveals that the induced current must, then, flow clockwise. The induced current exists only for the short period following the closing of the switch, when the field from the larger coil is growing from zero to its equilibrium value. Once the field from the larger coil reaches its equilibrium value and ceases to change, the induced current in the smaller coil becomes zero.

10. (b) The peak emf is proportional to the area \( A \) of the coil, according to Equation 22.4. Thus, we need to consider the areas of the coils. The length of the wire is \( L \) and is the same for each of the coil shapes. For the circle, the circumference is \( 2\pi r = L \), so that the area is

\[ A_{\text{circle}} = \pi r^2 = \pi \left( \frac{L}{2\pi} \right)^2 = \frac{L^2}{4\pi}. \]

For the square, the area is

\[ A_{\text{square}} = \left( \frac{L}{4} \right)^2 = \frac{L^2}{16}. \]

For the rectangle, the perimeter is \( 2(D + 2D) = L \), so that the area is

\[ A_{\text{rectangle}} = \left( \frac{L}{6} \right) \left( \frac{2L}{6} \right) = \frac{L^2}{18}. \]

The circle has the largest area, while the rectangle has the smallest area, corresponding to answer b.

11. 5.3 cm

12. (d) The back emf is proportional to the motor speed, so it decreases when the speed decreases. The current \( I \) drawn by the motor is given by Equation 22.5 as

\[ I = \frac{V - \xi}{R}, \]

where \( V \) is the voltage at the socket, \( \xi \) is the back emf, and \( R \) is the resistance of the motor coil. As \( \xi \) decreases, \( I \) increases.
13. (c) According to Equation 22.7, the mutual inductance is \( M = \frac{-\xi_s \Delta I}{\Delta I_p} \). If the time interval is cut in half and the change in the primary current is doubled, while the induced emf remains the same, the mutual inductance must be reduced by a factor of four.

14. (b) The energy stored in an inductor is given by Equation 22.10 as \( \text{Energy} = \frac{1}{2} L I^2 \). Since the two inductors store the same amount of energy, we have \( \frac{1}{2} L_1 I_1^2 = \frac{1}{2} L_2 I_2^2 \). Thus, \( \frac{I_1}{I_2} = \frac{\sqrt{L_2}}{\sqrt{L_1}} = \sqrt{2} = 1.414 \).

15. (e) According to Equation 22.8, we have \( N = \frac{LI}{\Phi} \). Since \( \Phi \) is the same for each coil, the number of turns is proportional to the product \( LI \) of the inductance and the current. For the coils specified in the table, this product is \( (LI)_A = L_0 I_0 \), \( (LI)_B = L_0 I_0 / 2 \), \( (LI)_C = 4L_0 I_0 \).

16. (c) The current in the primary is proportional to the current in the secondary according to Equation 22.13: \( I_p = I_s N_s / N_p \). The current in the secondary is the secondary voltage divided by the resistance, according to Ohm’s law. Thus, when the resistance increases, the current in the secondary decreases and so does the current in the primary. The wall socket delivers to the primary the same power that the secondary delivers to the resistance, assuming that no power is lost within the transformer. The power delivered to the resistance is given by Equation 20.15c as the square of the secondary voltage divided by the resistance. When the resistance increases, the power decreases. Hence, the power delivered to the primary by the wall socket also decreases.

17. 0.31 W

18. (a) The current in resistor 2 (without the transformer) is the same as the current in resistor 1 (with the transformer). In either event, the current \( I \) is \( I = V/R \), where \( V \) is the voltage across the resistance \( R \). Since the transformer is a step-up transformer, the voltage applied across resistor 2 is smaller than the voltage applied across resistor 1. The smaller voltage across resistor 2 can lead to the same current as does the greater voltage across resistor 1 only if \( R_2 \) is less than \( R_1 \).
1. **REASONING** During its fall, both the length of the bar and its velocity \( v \) are perpendicular to the horizontal component \( B_h \) of the earth’s magnetic field (see the drawing for an overhead view). Therefore, the emf \( \xi \) induced across the length \( L \) of the rod is given by \( \xi = v B_h L \) (Equation 22.1), where \( v \) is the speed of the rod. We will use Equation 22.1 to determine the magnitude \( B_h \) of the horizontal component of the earth’s magnetic field, and Right-Hand Rule No.1 from Section 21.2 to determine which end of the rod is positive.

**SOLUTION**

a. Solving \( \xi = v B_h L \) (Equation 22.1) for \( B_h \), we find that

\[
B_h = \frac{\xi}{vL} = \frac{6.5 \times 10^{-4} \text{ V}}{(22 \text{ m/s})(0.80 \text{ m})} = 3.7 \times 10^{-5} \text{ T}
\]

b. Consider a hypothetical positive charge that is free to move inside the falling rod. The bar is falling downward, carrying the positive charge with it, so that the velocity \( v \) of the charge is downward. In the drawing, which shows the situation as seen from above, downward is into the page. Applying Right-Hand Rule No.1 to the vectors \( v \) and \( B_h \), the magnetic force \( F \) on the charge points to the east. Therefore, positive charges in the rod would accelerate to the east, and negative charges would accelerate to the west. As a result, the east end of the rod acquires a positive charge.

2. **REASONING** We can treat the car as if it were a straight rod of length \( L = 2.0 \text{ m} \) between the driver’s side and the passenger’s side. This “rod” and the vertically downward component of the earth’s magnetic field are perpendicular. The emf \( \xi \) induced between the ends of this moving “rod” is \( \xi = vB_L \) (Equation 22.1), where \( v = 25 \text{ m/s} \) is the speed of the car, and \( B = 4.8 \times 10^{-5} \text{ T} \) is the vertically downward component of the earth’s magnetic field. To determine which side of the car is positive, we can use right-hand rule no. 1 (See Section 21.2), which will tell us the end of the rod at which positive charge will accumulate.
SOLUTION
a. Using Equation 22.1, we find that the emf is

\[ \xi = vBL = (25 \text{ m/s}) \left( 4.8 \times 10^{-5} \text{ T} \right) (2.0 \text{ m}) = 2.4 \times 10^{-3} \text{ V} \]

b. Right-hand rule no. 1 shows that positive charge accumulates on the driver’s side.

3. **REASONING AND SOLUTION**  The motional emf \( \xi \) generated by a conductor moving perpendicular to a magnetic field is given by Equation 22.1 as \( \xi = vBL \), where \( v \) and \( L \) are the speed and length, respectively, of the conductor, and \( B \) is the magnitude of the magnetic field. The emf would have been

\[ \xi = vBL = \left( 7.6 \times 10^3 \text{ m/s} \right) \left( 5.1 \times 10^{-5} \text{ T} \right) \left( 2.0 \times 10^4 \text{ m} \right) = 7800 \text{ V} \]

4. **REASONING** The situation in the drawing given with the problem statement is analogous to that in Figure 22.4b in the text. The blood flowing at a speed \( v \) corresponds to the moving rod, and the diameter of the blood vessel corresponds to the length \( L \) of the rod in the figure. The magnitude of the magnetic field is \( B \), and the measured voltage is the emf \( \xi \) induced by the motion. Thus, we can apply \( \xi = BvL \) (Equation 22.1).

**SOLUTION** Using Equation 22.1, we find that

\[ \xi = BvL = (0.60 \text{ T}) (0.30 \text{ m/s}) \left( 5.6 \times 10^{-3} \text{ m} \right) = 1.0 \times 10^{-3} \text{ V} \]

5. **SSM REASONING AND SOLUTION** For the three rods in the drawing in the text, we have the following:

**Rod A:** The motional emf is zero, because the velocity of the rod is parallel to the direction of the magnetic field, and the charges do not experience a magnetic force.

**Rod B:** The motional emf \( \xi \) is, according to Equation 22.1,

\[ \xi = vBL = (2.7 \text{ m/s})(0.45 \text{ T})(1.3 \text{ m}) = 1.6 \text{ V} \]

The positive end of Rod B is end 2.

**Rod C:** The motional emf is zero, because the magnetic force \( F \) on each charge is directed perpendicular to the length of the rod. For the ends of the rod to become charged, the magnetic force must be directed parallel to the length of the rod.
6. **REASONING**  
   a. The motional emf generated by the moving metal rod depends only on its speed, its length, and the magnitude of the magnetic field (see Equation 22.1). The motional emf does not depend on the resistance in the circuit. Therefore, the emfs for the circuits are the same.

   b. According to Equation 20.2, the current \( I \) is equal to the emf divided by the resistance \( R \) of the circuit. Since the emfs in the two circuits are the same, the circuit with the smaller resistance has the larger current. Since circuit 1 has one-half the resistance of circuit 2, the current in circuit 1 is twice as large.

   c. The power \( P \) is \( P = \frac{\xi^2}{R} \) (Equation 20.6c), where \( \xi \) is the emf (or voltage) and \( R \) is the resistance. The emf produced by the moving bar is directly proportional to its speed (see Equation 22.1). Thus, the bar in circuit 1 produces twice the emf, since it’s moving twice as fast. Moreover, the resistance in circuit 1 is half that in circuit 2. As a result, the power delivered to the bulb in circuit 1 is \( 2^2 / \left(\frac{1}{2}\right) = 8 \) times greater than in circuit 2.

**SOLUTION**  
   a. The ratio of the emfs is, according to Equation 22.1
   \[
   \frac{\xi_1}{\xi_2} = \frac{vBL}{vBL} = 1
   \]

   b. Equation 20.2 states that the current is equal to the emf divided by the resistance. The ratio of the currents is
   \[
   \frac{I_1}{I_2} = \frac{\xi_1/R_1}{\xi_2/R_2} = \frac{R_2}{R_1} = \frac{110 \, \Omega}{55 \, \Omega} = 2
   \]

   c. The power, according to Equation 20.6c, is \( P = \frac{\xi^2}{R} \). The motional emf is given by Equation 22.1 as \( \xi = vBL \). The ratio of the powers is
   \[
   \frac{P_1}{P_2} = \frac{\frac{\xi_1^2}{R_1}}{\frac{\xi_2^2}{R_2}} = \left(\frac{v_1}{v_2}\right)^2 \left(\frac{R_2}{R_1}\right) = \left(\frac{v_1}{v_2}\right)^2 = 8
   \]

7. **REASONING**  
   Once the switch is closed, there is a current in the rod. The magnetic field applies a force to this current and accelerates the rod to the right. As the rod begins to move, however, a motional emf appears between the ends of the rod. This motional emf depends on the speed of the rod and increases as the speed increases. Equally important is the fact that the motional emf opposes the emf of the battery. The net emf causing the current in the
rod is the algebraic sum of the two emf contributions. Thus, as the speed of the rod increases and the motional emf increases, the net emf decreases. As the net emf decreases, the current in the rod decreases and so does the force that field applies to the current. Eventually, the speed reaches the point when the motional emf has the same magnitude as the battery emf, and the net emf becomes zero. At this point, there is no longer a net force acting on the rod and the speed remains constant from this point onward, according to Newton’s second law. This maximum speed can be determined by using Equation 22.1 for the motional emf, with a value of the motional emf that equals the battery emf.

**SOLUTION** Using Equation 22.1 \( \xi = vBL \) and a value of 3.0 V for the emf, we find that the maximum speed of the rod is

\[
v = \frac{\xi}{BL} = \frac{3.0 \text{ V}}{(0.60 \text{ T})(0.20 \text{ m})} = 25 \text{ m/s}
\]

8. **REASONING** The average power \( \bar{P} \) delivered by the hand is given by \( \bar{P} = W/t \), where \( W \) is the work done by the hand and \( t \) is the time interval during which the work is done. The work done by the hand is equal to the product of the magnitude \( F_{\text{hand}} \) of the force exerted by the hand, the magnitude \( x \) of the rod’s displacement, and the cosine of the angle between the force and the displacement.

Since the rod moves to the right at a constant speed, it has no acceleration and is, therefore, in equilibrium. Thus, the force exerted by the hand must be equal to the magnitude \( F \) of the magnetic force that the current exerts on the rod. The magnitude of the magnetic force is given by \( F = ILB\sin \theta \) (Equation 21.3), where \( I \) is the current, \( L \) is the length of the moving rod, \( B \) is the magnitude of the magnetic field, and \( \theta \) is the angle between the direction of the current and that of the magnetic field.

**SOLUTION** The average power \( \bar{P} \) delivered by the hand is

\[
\bar{P} = \frac{W}{t}
\]

(6.10a)

The work \( W \) done by the hand in Figure 22.5 is given by \( W = F_{\text{hand}}x\cos \theta' \) (Equation 6.1). In this equation \( F_{\text{hand}} \) is the magnitude of the force that the hand exerts on the rod, \( x \) is the magnitude of the rod’s displacement, and \( \theta' \) is the angle between the force and the displacement. The force and displacement point in the same direction, so \( \theta' = 0^\circ \). Since the magnitude of the force exerted by the hand equals the magnitude \( F \) of the magnetic force, \( F_{\text{hand}} = F \). Substituting \( W = F_{\text{hand}}x\cos 0^\circ \) into Equation 6.10a and using the fact that \( F_{\text{hand}} = F \), we have that

\[
\bar{P} = \frac{W}{t} = \frac{F_{\text{hand}}x\cos 0^\circ}{t} = \frac{Fx\cos 0^\circ}{t}
\]

(1)

The magnitude \( F \) of the magnetic force is given by \( F = ILB\sin \theta \) (Equation 21.3). In this case, the current and magnetic field are perpendicular to each other, so \( \theta = 90^\circ \) (see Figure 22.5). Substituting this expression for \( F \) into Equation 1 gives
The term \( x/t \) in Equation 2 is the speed \( v \) of the rod. Thus, the average power delivered by the hand is

\[
\bar{P} = \frac{F x \cos 0^\circ}{t} = \frac{(ILB \sin 90^\circ) x \cos 0^\circ}{t}
\]

(2)

The minimum length \( d \) of the rails is the speed \( v \) of the rod times the time \( t \), or \( d = vt \). We can obtain the speed from the expression for the motional emf given in Equation 22.1. Solving this equation for the speed gives

\[
v = \frac{\xi}{BL}
\]

where \( \xi \) is the motional emf, \( B \) is the magnitude of the magnetic field, and \( L \) is the length of the rod. Thus, the length of the rails is

\[
d = vt = \left( \frac{\xi}{BL} \right) t
\]

While we have no value for the motional emf, we do know that the bulb dissipates a power of \( P = 60.0 \text{ W} \), and has a resistance of \( R = 240 \Omega \). Power is related to the emf and the resistance according to

\[
P = \frac{\xi^2}{R}
\]

(Equation 20.6c), which can be solved to show that

\[
\xi = \sqrt{PR}
\]

Substituting this expression into the equation for \( d \) gives

\[
d = \left( \frac{\xi}{BL} \right) t = \left( \frac{\sqrt{PR}}{BL} \right) t
\]

**SOLUTION** Using the above expression for the minimum necessary length of the rails, we find that

\[
d = \left( \frac{\sqrt{PR}}{BL} \right) t = \left[ \sqrt{(60.0 \text{ W})(240 \Omega)} \right] (0.50 \text{ s}) = 250 \text{ m}
\]

10. **REASONING AND SOLUTION**

a. Newton's second law gives the magnetic retarding force to be

\[
F = mg = IBL
\]

Now the current, \( I \), is

\[
I = \frac{\xi}{R} = \frac{vBL}{R}
\]

So

\[
m = \frac{v(BL)^2}{Rg} = \frac{(4.0 \text{ m/s})(0.50 \text{ T})^2 (1.3 \text{ m})^2}{(0.75 \Omega)(9.80 \text{ m/s}^2)} = 0.23 \text{ kg}
\]
b. The change in height in a time $\Delta t$ is $\Delta h = -v\Delta t$. The change in gravitational potential energy is

$$\Delta PE = mg\Delta h = -mgv\Delta t = -(0.23 \text{ kg})(9.80 \text{ m/s}^2)(4.0 \text{ m/s})(0.20 \text{ s}) = \boxed{-1.8 \text{ J}}$$

c. The energy dissipated in the resistor is the amount by which the gravitational potential energy decreases or $\boxed{1.8 \text{ J}}$.

11. **REASONING** The definition of magnetic flux $\Phi$ is $\Phi = BA\cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the magnetic field vector and the normal to the surface. The values of $B$ and $A$ are the same for each of the surfaces, while the values for the angle $\phi$ are different. The $z$ axis is the normal to the surface lying in the $x$, $y$ plane, so that $\phi_{xz} = 35^\circ$. The $y$ axis is the normal to the surface lying in the $x$, $z$ plane, so that $\phi_{xy} = 55^\circ$. We can apply the definition of the flux to obtain the desired ratio directly.

**SOLUTION** Using Equation 22.2, we find that

$$\frac{\Phi_{xz}}{\Phi_{xy}} = \frac{BA\cos \phi_{xz}}{BA\cos \phi_{xy}} = \frac{\cos 55^\circ}{\cos 35^\circ} = \boxed{0.70}$$

12. **REASONING**

a. The magnetic flux $\Phi$ though a surface of area $A$ due to a uniform magnetic field of magnitude $B$ is given by $\Phi = BA\cos \phi$ (Equation 22.2) where $\phi$ is the angle between the direction of the magnetic field and the normal to the surface (see the drawing, which shows an edge-on view of the situation). The magnetic field $\mathbf{B}$ is horizontal, and the surface makes an angle of $12^\circ$ with the horizontal, so the normal to the surface is $\phi = 90.0^\circ - 12^\circ = 78^\circ$. We will use Equation 22.2 to determine the surface area $A$.

b. When exposed to a uniform magnetic field, the magnetic flux through a flat surface is greatest when the surface is perpendicular to the direction of the magnetic field. When this occurs, the normal to the surface is parallel to the direction of the magnetic field, and the angle $\phi$ is zero. Therefore, to find the smallest surface area that has same amount of magnetic flux passing through it as the surface in part (a), we will take $\phi = 0.0^\circ$ in Equation 22.2.
SOLUTION

a. Solving $\Phi = BA \cos \phi$ (Equation 22.2) for $A$ yields $A = \frac{\Phi}{B \cos \phi}$. Substituting $\phi = 78^\circ$, we find that

$$A = \frac{\Phi}{B \cos \phi} = \frac{8.4 \times 10^{-3} \text{ Wb}}{(0.47 \text{ T}) \cos 78^\circ} = 0.086 \text{ m}^2$$

b. From $A = \frac{\Phi}{B \cos \phi}$, with $\phi = 0.0^\circ$, the minimum possible area of the second surface is

$$A = \frac{8.4 \times 10^{-3} \text{ Wb}}{(0.47 \text{ T}) \cos 0.0^\circ} = 0.018 \text{ m}^2$$

13. **SSM REASONING** The general expression for the magnetic flux through an area $A$ is given by Equation 22.2: $\Phi = BA \cos \phi$, where $B$ is the magnitude of the magnetic field and $\phi$ is the angle of inclination of the magnetic field $\mathbf{B}$ with respect to the normal to the area.

The magnetic flux through the door is a maximum when the magnetic field lines are perpendicular to the door and $\phi_1 = 0.0^\circ$ so that $\Phi_1 = \Phi_{\max} = BA(\cos 0.0^\circ) = BA$.

**SOLUTION** When the door rotates through an angle $\phi_2$, the magnetic flux that passes through the door decreases from its maximum value to one-third of its maximum value. Therefore, $\Phi_2 = \frac{1}{3} \Phi_{\max}$, and we have

$$\Phi_2 = BA \cos \phi_2 = \frac{1}{3} BA \quad \text{or} \quad \cos \phi_2 = \frac{1}{3} \quad \text{or} \quad \phi_2 = \cos^{-1} \left( \frac{1}{3} \right) = 70.5^\circ$$

14. **REASONING** At any given moment, the flux $\Phi$ that passes through the loop is given by $\Phi = BA \cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the loop, and $\phi = 0^\circ$ is the angle between the normal to the loop and the direction of the magnetic field (both directed into the page). As the handle turns, the area $A$ of the loop changes, causing a change $\Delta \Phi$ in the flux passing through the loop. We can think of the loop as being divided into a rectangular portion and a semicircular portion. Initially, the area $A_0$ of the loop is equal to the rectangular area $A_{\text{rec}}$ plus the area $A_{\text{semi}} = \frac{1}{2} \pi r^2$ of the semicircle, where $r$ is the radius of the semicircle: $A_0 = A_{\text{rec}} + \frac{1}{2} \pi r^2$. After half a revolution, the semicircle is once again within the plane of the loop, but now as a reduction of the area of the rectangular portion. Therefore, the final area $A$ of the loop is equal to the area of the rectangular portion minus the area of the semicircle: $A = A_{\text{rec}} - \frac{1}{2} \pi r^2$.

**SOLUTION** The change $\Delta \Phi$ in the flux that passes through the loop is the difference between the final flux $\Phi = BA \cos \phi$ (Equation 22.2) and the initial flux $\Phi_0 = BA_0 \cos \phi$: 
\[ \Delta \Phi = \Phi - \Phi_0 = BA \cos \phi - BA_0 \cos \phi = B \cos \phi (A - A_0) \]  
(1)

Substituting \( A_0 = A_{\text{rec}} + \frac{1}{2} \pi r^2 \) and \( A = A_{\text{rec}} - \frac{1}{2} \pi r^2 \) into Equation (1) yields

\[ \Delta \Phi = B \cos \phi (A - A_0) = B \cos \phi \left[ (A_{\text{rec}} - \frac{1}{2} \pi r^2) - (A_{\text{rec}} + \frac{1}{2} \pi r^2) \right] = -\pi r^2 B \cos \phi \]

Therefore the change in the flux passing through the loop during half a revolution of the semicircle is

\[ \Delta \Phi = -\pi r^2 B \cos \phi = -\pi (0.20 \text{ m})^2 (0.75 \text{ T}) \cos 0^\circ = -0.094 \text{ Wb} \]

15. **REASONING** According to Equation 22.2, the magnetic flux \( \Phi \) is the product of the magnitude \( B \) of the magnetic field, the area \( A \) of the surface, and the cosine of the angle \( \phi \) between the direction of the magnetic field and the normal to the surface. The area of a circular surface is \( A = \pi r^2 \), where \( r \) is the radius.

**SOLUTION** The magnetic flux \( \Phi \) through the surface is

\[ \Phi = BA \cos \phi = B (\pi r^2) \cos \phi = (0.078 \text{ T}) \pi (0.10 \text{ m})^2 \cos 25^\circ = 2.2 \times 10^{-3} \text{ Wb} \]

16. **REASONING** The magnetic flux \( \Phi \) that passes through a flat single-turn loop of wire is \( \Phi = BA \cos \phi \) (Equation 22.2), where \( B \) is the magnitude of the magnetic field (the same for each loop since the field is uniform), \( A \) is the area of the loop, and \( \phi \) is the angle between the magnetic field and the normal to the plane of the loop. Since both the square and the circle are perpendicular to the field, we know that \( \phi = 0^\circ \) for both loops. We will apply Equation 22.2 to both the square and the circle.

**SOLUTION** Using \( L \) to denote the length of each side of the square and \( R \) to denote the radius of the circle and recognizing that the areas of the square and circle are, respectively, \( L^2 \) and \( \pi R^2 \), we have from Equation 22.2 that

\[ \Phi_{\text{square}} = BA_{\text{square}} \cos 0^\circ = BL^2 \quad \text{and} \quad \Phi_{\text{circle}} = BA_{\text{circle}} \cos 0^\circ = B\pi R^2 \]

Dividing the right-hand equation by the left-hand equation, we obtain

\[ \frac{\Phi_{\text{circle}}}{\Phi_{\text{square}}} = \frac{B\pi R^2}{BL^2} = \frac{\pi R^2}{L^2} \]  
(1)

We do have a value for either \( L \) or \( R \), but we do know that the square and the circle both contain the same length of wire. Therefore, it follows that

\[ \frac{4L}{\text{Length of wire in square}} = \frac{2\pi R}{\text{Circumference of circle}} \quad \text{or} \quad R = \frac{2L}{\pi} \]
Substituting this result for the radius of the circle into Equation (1) gives

\[
\frac{\Phi_{\text{circle}}}{\Phi_{\text{square}}} = \frac{\pi R^2}{L^2} = \frac{\pi (2L/\pi)^2}{L^2} = \frac{4}{\pi}
\]

\[
\Phi_{\text{circle}} = \frac{4}{\pi} \Phi_{\text{square}} = \frac{4}{\pi} (7.0 \times 10^{-3} \text{ Wb}) = 8.9 \times 10^{-3} \text{ Wb}
\]

17. **REASONING** The general expression for the magnetic flux through an area \( A \) is given by Equation 22.2: \( \Phi = BA \cos \phi \), where \( B \) is the magnitude of the magnetic field and \( \phi \) is the angle of inclination of the magnetic field \( B \) with respect to the normal to the surface.

**SOLUTION** Since the magnetic field \( B \) is parallel to the surface for the triangular ends and the bottom surface, the flux through each of these three surfaces is \( 0 \text{ Wb} \).

The flux through the 1.2 m by 0.30 m face is

\[
\Phi = BA \cos \phi = (0.25 \text{ T})(1.2 \text{ m})(0.30 \text{ m}) \cos 0.0^\circ = 0.090 \text{ Wb}
\]

For the 1.2 m by 0.50 m side, the area makes an angle \( \phi \) with the magnetic field \( B \), where

\[
\phi = 90^\circ - \tan^{-1} \left( \frac{0.30 \text{ m}}{0.40 \text{ m}} \right) = 53^\circ
\]

Therefore,

\[
\Phi = BA \cos \phi = (0.25 \text{ T})(1.2 \text{ m})(0.50 \text{ m}) \cos 53^\circ = 0.090 \text{ Wb}
\]

18. **REASONING** An emf is induced during the first and third intervals, because the magnetic field is changing in time. The time interval is the same (3.0 s) for the two cases. However, the magnitude of the field changes more during the first interval. Therefore, the magnetic flux is changing at a greater rate in that interval, which means that the magnitude of the induced emf is greatest during the first interval.

The induced emf is zero during the second interval, 3.0 – 6.0 s. According to Faraday’s law of electromagnetic induction, Equation 22.3, an induced emf arises only when the magnetic flux changes. During this interval, the magnetic field, the area of the loop, and the orientation of the field relative to the loop are constant. Thus, the magnetic flux does not change, so there is no induced emf.

During the first interval the magnetic field is increasing with time. During the third interval, the field is decreasing with time. As a result, the induced emfs will have opposite polarities during these intervals. If the direction of the induced current is clockwise during the first interval, it will be counterclockwise during the third interval.

**SOLUTION**

a. The induced emf is given by Equations 22.3 and 22.3:
0–3.0 s:
\[ \xi = -N \frac{\Delta \Phi}{\Delta t} = -N \left( \frac{BA \cos \phi - B_0 A \cos \phi}{t - t_0} \right) \]
\[ = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.40 \text{ T} - 0 \text{ T}}{3.0 \text{ s} - 0 \text{ s}} \right) = -1.0 \text{ V} \]

3.0–6.0 s:
\[ \xi = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.40 \text{ T} - 0.40 \text{ T}}{6.0 \text{ s} - 3.0 \text{ s}} \right) = 0 \text{ V} \]

6.0–9.0 s:
\[ \xi = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.20 \text{ T} - 0.40 \text{ T}}{9.0 \text{ s} - 6.0 \text{ s}} \right) = +0.50 \text{ V} \]

b. The induced current is given by Equation 20.2 as \( I = \frac{\xi}{R} \).

0–3.0 s:
\[ I = \frac{\xi}{R} = \frac{-1.0 \text{ V}}{0.50 \Omega} = -2.0 \text{ A} \]

6.0–9.0 s:
\[ I = \frac{\xi}{R} = \frac{+0.50 \text{ V}}{0.50 \Omega} = +1.0 \text{ A} \]

As expected, the currents are in opposite directions.

19. **REASONING** The magnitude \( |\xi| \) of the emf induced in the loop can be found using Faraday’s law of electromagnetic induction:

\[ |\xi| = \left| -N \frac{\Phi - \Phi_0}{t - t_0} \right| \quad (22.3) \]

where \( N \) is the number of turns, \( \Phi \) and \( \Phi_0 \) are, respectively, the final and initial fluxes, and \( t - t_0 \) is the elapsed time. The magnetic flux is given by \( \Phi = BA \cos \phi \) (Equation 22.2), where \( B \) is the magnitude of the magnetic field, \( A \) is the area of the surface, and \( \phi \) is the angle between the direction of the magnetic field and the normal to the surface.

**SOLUTION** Setting \( N = 1 \) since there is only one turn, noting that the final area is \( A = 0 \text{ m}^2 \) and the initial area is \( A_0 = 0.20 \text{ m} \times 0.35 \text{ m} \), and noting that the angle \( \phi \) between the magnetic field and the normal to the surface is \( 0^\circ \), we find that the magnitude of the emf induced in the coil is
\[ |\xi| = -N \frac{BA \cos \phi - BA_0 \cos \phi}{t - t_0} \]

\[ = \left( -1 \right) \frac{(0.65 \text{ T})(0 \text{ m}^2)(\cos 0^\circ - (0.65 \text{ T})(0.20 \text{ m} \times 0.35 \text{ m})(\cos 0^\circ)}{0.18 \text{ s}} = 0.25 \text{ V} \]

20. **Reasoning** An emf is induced in the body because the magnetic flux is changing in time. According to Faraday’s law, as given by Equation 22.3, the magnitude of the emf is

\[ |\xi| = -N \frac{\Delta \Phi}{\Delta t} \]

This expression can be used to determine the time interval \( \Delta t \) during which the magnetic field goes from its initial value to zero. The magnetic flux \( \Phi \) is obtained from Equation 22.2 as \( \Phi = BA \cos \phi \), where \( \phi = 0^\circ \) in this problem.

**Solution** The magnitude \( |\xi| \) of the induced emf is

\[ |\xi| = -N \left( \frac{\Delta \Phi}{\Delta t} \right) = -N \left( \frac{BA \cos \phi - BA_0 \cos \phi}{\Delta t} \right) \]

Solving this relation for \( \Delta t \) gives

\[ \Delta t = \frac{-N A \cos \phi (B - B_0)}{|\xi|} = \frac{-1(0.032 \text{ m}^2)(\cos 0^\circ)(0 - 1.5 \text{ T})}{0.010 \text{ V}} = 4.8 \text{ s} \]

21. **SSM Reasoning** According to Equation 22.3, the average emf induced in a coil of \( N \) loops is \( \xi = -N \Delta \Phi / \Delta t \).

**Solution** For the circular coil in question, the flux through a single turn changes by

\[ \Delta \Phi = B A \cos 45^\circ - B A \cos 90^\circ = B A \cos 45^\circ \]

during the interval of \( \Delta t = 0.010 \text{ s} \). Therefore, for \( N \) turns, Faraday’s law gives the magnitude of the emf as

\[ |\xi| = -N \frac{B A \cos 45^\circ}{\Delta t} \]

Since the loops are circular, the area \( A \) of each loop is equal to \( \pi r^2 \). Solving for \( B \), we have

\[ B = \frac{|\xi| \Delta t}{N \pi r^2 \cos 45^\circ} = \frac{(0.065 \text{ V})(0.010 \text{ s})}{(950)\pi(0.060 \text{ m})^2 \cos 45^\circ} = 8.6 \times 10^{-5} \text{ T} \]
22. **REASONING** According to Ohm’s law (see Section 20.2), the resistance of the wire is equal to the emf divided by the current. The emf can be obtained from Faraday’s law of electromagnetic induction.

**SOLUTION** The resistance $R$ of the wire is

$$ R = \frac{\varepsilon}{I} \tag{20.2} $$

According to Faraday’s law of electromagnetic induction, the induced emf is

$$ \varepsilon = -N \frac{\Delta \Phi}{\Delta t} = -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right) \tag{22.3} $$

where $N$ is the number of loops in the coil, $\Phi$ and $\Phi_0$ are, respectively, the final and initial fluxes, and $t - t_0$ is the elapsed time. Substituting Equation 22.3 into Equation 20.2 yields

$$ R = \frac{\varepsilon}{I} = \frac{-N \left( \frac{\Phi - \Phi_0}{t - t_0} \right)}{I} = \frac{-12 \left( \frac{4.0 \text{ Wb} - 9.0 \text{ Wb}}{0.050 \text{ s}} \right)}{230 \text{ A}} = 5.2 \Omega $$

23. **REASONING** We will use Faraday’s law of electromagnetic induction, Equation 22.3, to find the emf induced in the loop. Once this value has been determined, we can employ Equation 22.3 again to find the rate at which the area changes.

**SOLUTION**

a. The magnitude $|\varepsilon|$ of the induced emf is given by Equation 22.3 as

$$ |\varepsilon| = \left| -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right) \right| = \left| -N \left( \frac{BA \cos \phi - B_0 A \cos \phi}{t - t_0} \right) \right| = \left| -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) \right| $$

$$ = (1)(0.35 \text{ m} \times 0.55 \text{ m}) \cos 65^\circ \left( \frac{2.1 \text{ T} - 0 \text{ T}}{0.45 \text{ s} - 0 \text{ s}} \right) = 0.38 \text{ V} $$

b. When the magnetic field is constant and the area is changing in time, Faraday’s law can be written as

$$ |\varepsilon| = \left| -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right) \right| = \left| -N \left( \frac{BA \cos \phi - BA_0 \cos \phi}{t - t_0} \right) \right| $$

$$ = \left| -NB \cos \phi \left( \frac{A - A_0}{t - t_0} \right) \right| = \left| -NB \cos \phi \left( \frac{\Delta A}{\Delta t} \right) \right| $$
Solving this equation for $\frac{\Delta A}{\Delta t}$ and substituting in the value of 0.38 V for the magnitude of the emf, we find that

$$\frac{\Delta A}{\Delta t} = \frac{\xi}{NB \cos \phi} = \frac{0.38 \text{ V}}{(1)(2.1 \text{ T}) \cos 65^\circ} = 0.43 \text{ m}^2/\text{s}$$

24. **REASONING** According to Faraday’s law the emf induced in either single-turn coil is given by Equation 22.3 as

$$\xi = -N \frac{\Delta \Phi}{\Delta t} = -\frac{\Delta \Phi}{\Delta t}$$

since the number of turns is $N = 1$. The flux is given by Equation 22.2 as

$$\Phi = BA \cos \phi = BA \cos 0^\circ = BA$$

where the angle between the field and the normal to the plane of the coil is $\phi = 0^\circ$, because the field is perpendicular to the plane of the coil and, hence, parallel to the normal. With this expression for the flux, Faraday’s law becomes

$$\xi = -\frac{\Delta \Phi}{\Delta t} = -\frac{\Delta (BA)}{\Delta t} = -A \frac{\Delta B}{\Delta t}$$

In this expression we have recognized that the area $A$ does not change in time and have separated it from the magnitude $B$ of the magnetic field. We will apply this form of Faraday’s law to each coil. The current induced in either coil depends on the resistance $R$ of the coil, as well as the emf. According to Ohm’s law, the current $I$ induced in either coil is given by

$$I = \frac{\xi}{R}$$

**SOLUTION** Applying Faraday’s law in the form of Equation 1 to both coils, we have

$$\xi_{\text{square}} = -A_{\text{square}} \frac{\Delta B}{\Delta t} = -L^2 \frac{\Delta B}{\Delta t} \quad \text{and} \quad \xi_{\text{circle}} = -A_{\text{circle}} \frac{\Delta B}{\Delta t} = -\pi r^2 \frac{\Delta B}{\Delta t}$$

The area of a square of side $L$ is $L^2$, and the area of a circle of radius $r$ is $\pi r^2$. The rate of change $\Delta B/\Delta t$ of the field magnitude is the same for both coils. Dividing these two expressions gives

$$\frac{\xi_{\text{square}}}{\xi_{\text{circle}}} = \frac{-L^2 \frac{\Delta B}{\Delta t}}{-\pi r^2 \frac{\Delta B}{\Delta t}} = \frac{L^2}{\pi r^2}$$

The same wire is used for both coils, so we know that the perimeter of the square equals the circumference of the circle:
4L = 2\pi r \quad \text{or} \quad \frac{L}{r} = \frac{2\pi}{4} = \frac{\pi}{2}

Substituting this result into Equation (3) gives

$$\frac{\varepsilon_{\text{square}}}{\varepsilon_{\text{circle}}} = \frac{L^2}{\pi r^2} = \frac{\pi^2}{\pi 2^2} = \frac{\pi}{4} \quad \text{or} \quad \frac{\varepsilon_{\text{square}}}{\varepsilon_{\text{circle}}} = \frac{\pi}{4} \frac{\varepsilon_{\text{circle}}}{\varepsilon_{\text{circle}}} = \frac{\pi}{4} (0.80 \text{ V}) = 0.63 \text{ V} \quad (4)$$

Using Ohm’s law as given in Equation (2), we find for the induced currents that

$$\frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{\varepsilon_{\text{square}}}{\varepsilon_{\text{circle}}} = \frac{\pi}{4} \quad \text{or} \quad I_{\text{square}} = \frac{\pi}{4} I_{\text{circle}} = \frac{\pi}{4} (3.2 \text{ A}) = 2.5 \text{ A}$$

25. \textit{REASONING} The magnitude $|\varepsilon|$ of the average emf induced in the triangle is given by

$$|\varepsilon| = \left| -N \frac{\Delta \Phi}{\Delta t} \right| \quad \text{(see Equation 22.3), which is Faraday’s law. This expression can be used directly to calculate the magnitude of the average emf. Since the triangle is a single-turn coil, the number of turns is } N = 1. \text{ According to Equation 22.2, the magnetic flux } \Phi \text{ is}$$

$$\Phi = BA \cos \phi = BA \cos 0^\circ = BA \quad (1)$$

where $B$ is the magnitude of the field, $A$ is the area of the triangle, and $\phi = 0^\circ$ is the angle between the field and the normal to the plane of the triangle (the magnetic field is perpendicular to the plane of the triangle). It is the change $\Delta \Phi$ in the flux that appears in Faraday’s law, so that we use Equation (1) as follows:

$$\Delta \Phi = BA - BA_0 = BA$$

where $A_0 = 0 \text{ m}^2$ is the initial area of the triangle just as the bar passes point $A$, and $A$ is the area after the time interval $\Delta t$ has elapsed. The area of a triangle is one-half the base ($d_{AC}$) times the height ($d_{CB}$) of the triangle. Thus, the change in flux is

$$\Delta \Phi = BA = B \left( \frac{1}{2} d_{AC} d_{CB} \right)$$

The base and the height of the triangle are related, according to $d_{CB} = d_{AC} \tan \theta$, where $\theta = 19^\circ$. Furthermore, the base of the triangle becomes longer as the rod moves. Since the rod moves with a speed $v$ during the time interval $\Delta t$, the base is $d_{AC} = v\Delta t$. With these substitutions the change in flux becomes
\[ \Delta \Phi = B \left( \frac{1}{2} d_{AC} d_{CB} \right) = B \left[ \frac{1}{2} d_{AC} \left( d_{AC} \tan \theta \right) \right] = \frac{1}{2} B (v \Delta t)^2 \tan \theta \]  

**SOLUTION** Substituting Equation (2) for the change in flux into Faraday’s law, we find that the magnitude of the induced emf is

\[ |\xi| = \left| -N \frac{\Delta \Phi}{\Delta t} \right| = \left| -N \frac{1}{2} B (v \Delta t)^2 \tan \theta \right| = N \frac{1}{2} B v^2 \Delta t \tan \theta \]

\[ = (1) \frac{1}{2} (0.38 \text{ T})(0.60 \text{ m/s})^2(6.0 \text{ s}) \tan 19^\circ = 0.14 \text{ V} \]

26. **REASONING** An emf is induced in the coil because the magnetic flux through the coil is changing in time. The flux is changing because the angle \( \phi \) between the normal to the coil and the magnetic field is changing.

The amount of induced current is equal to the induced emf divided by the resistance of the coil (see Equation 20.2).

According to Equation 20.1, the amount of charge \( \Delta q \) that flows is \( \Delta q = I \Delta t \) during which the coil rotates, or \( \Delta q = I(t - t_0) \).

**SOLUTION** According to Equation 20.1, the amount of charge that flows is \( \Delta q = I \Delta t \). The current is related to the emf \( \xi \) in the coil and the resistance \( R \) by Equation 20.2 as \( I = \xi / R \). The amount of charge that flows can, therefore, be written as

\[ \Delta q = I \Delta t = \left( \frac{\xi}{R} \right) \Delta t \]

The emf is given by Faraday’s law of electromagnetic induction as

\[ \xi = -N \frac{\Delta \Phi}{\Delta t} = -N \frac{BA \cos \phi - BA \cos \phi_0}{\Delta t} \]

where we have also used Equation 22.2, which gives the definition of magnetic flux as \( \Phi = BA \cos \phi \). With this emf, the expression for the amount of charge becomes

\[ \Delta q = I \Delta t = \left[ -N \frac{BA \cos \phi - BA \cos \phi_0}{\Delta t} \right] \Delta t = \frac{-NBA(\cos \phi - \cos \phi_0)}{R} \]

Solving for the magnitude of the magnetic field yields

\[ B = \frac{-R \Delta q}{NA(\cos \phi - \cos \phi_0)} = \frac{-140 \Omega \left( 8.5 \times 10^{-5} \text{ C} \right)}{(50) \left( 1.5 \times 10^{-3} \text{ m}^2 \right) \left( \cos 90^\circ - \cos 0^\circ \right)} = 0.16 \text{ T} \]
27. **REASONING** According to Equation 22.3, the average emf $\xi$ induced in a single loop ($N = 1$) is $\xi = -\Delta \Phi / \Delta t$. Since the magnitude of the magnetic field is changing, the area of the loop remains constant, and the direction of the field is parallel to the normal to the loop, the change in flux through the loop is given by $\Delta \Phi = (\Delta B)A$. Thus the magnitude $|\xi|$ of the induced emf in the loop is given by $|\xi| = |-(\Delta B)A/\Delta t|$.

Similarly, when the area of the loop is changed and the field $B$ has a given value, we find the magnitude of the induced emf to be $|\xi| = -B(\Delta A)/\Delta t$.

**SOLUTION**

a. The magnitude of the induced emf when the field changes in magnitude is

$$|\xi| = -A \left( \frac{\Delta B}{\Delta t} \right) = (0.018 \text{ m}^2)(0.20 \text{ T/s}) = 3.6 \times 10^{-3} \text{ V}$$

b. At a particular value of $B$ (when $B$ is changing), the rate at which the area must change can be obtained from

$$|\xi| = -\frac{B \Delta A}{\Delta t} \quad \text{or} \quad \frac{\Delta A}{\Delta t} = \frac{|\xi|}{B} = \frac{3.6 \times 10^{-3}}{1.8} \text{ m}^2/\text{s} = 2.0 \times 10^{-3} \text{ m}^2/\text{s}$$

In order for the induced emf to be zero, the magnitude of the magnetic field and the area of the loop must change in such a way that the flux remains constant. Since the magnitude of the magnetic field is increasing, the area of the loop must decrease, if the flux is to remain constant. Therefore, the area of the loop must be shrunk.

28. **REASONING** The magnitude $B_1$ of the magnetic field at the center of a circular coil with $N$ turns and a radius $r$, carrying a current $I$ is given by

$$B_1 = N \frac{\mu_0 I}{2r} \quad (21.6)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space. We use the symbol $r$ to denote the radius of the coil in order to distinguish it from the coil’s resistance $R$. The coil’s induced current $I$ is caused by the induced emf $\xi$. We will determine the induced current with Ohm’s law (Equation 20.2), the magnitude $|\xi|$ of the induced emf, and the resistance $R$ of the coil:

$$I = \frac{|\xi|}{R} \quad (20.2)$$

The magnitude of the induced emf is found from Faraday’s law: $|\xi| = -N \frac{\Delta \Phi}{\Delta t}$

(Equation 22.3), where $\Delta \Phi/\Delta t$ is the rate of change of the flux caused by the external
magnetic field \( B \). At any instant, the flux through each turn of the coil due to the external magnetic field is given by

\[
\Phi = B A \cos \phi = B A \cos 0^\circ = BA
\]  

(22.2)

where \( A \) is the cross-sectional area of the coil. The angle \( \phi \) between the direction of the external magnetic field and the normal to the plane of the coil is zero, because the external magnetic field is perpendicular to the plane of the coil.

**SOLUTION** In this situation, the cross-sectional area \( A \) of the coil is constant, but the magnitude \( B \) of the external magnetic field is changing. Therefore, substituting Equation 22.2 into \( \Delta \Phi \) yields

\[
|\varepsilon| = -N \frac{\Delta \Phi}{\Delta t} = N \frac{\Delta (BA)}{\Delta t} = NA \frac{\Delta B}{\Delta t}
\]  

(1)

Substituting Equation (1) into Equation 20.2, we obtain

\[
I = \frac{|\varepsilon|}{R} = \frac{N A \frac{\Delta B}{\Delta t}}{R} = \frac{NA}{R} \left( \frac{\Delta B}{\Delta t} \right)
\]  

(2)

Substituting Equation (2) into Equation (21.6) yields

\[
B_1 = N \frac{\mu_0 I}{2r} = N \frac{\mu_0}{2r} \left( \frac{NA}{R} \right) \left( \frac{\Delta B}{\Delta t} \right) = N^2 \frac{\mu_0 A}{2rR} \left( \frac{\Delta B}{\Delta t} \right)
\]  

(3)

Lastly, we substitute \( A = \pi r^2 \) into Equation (3), because the coil is circular. This yields

\[
B_1 = N^2 \frac{\mu_0 \pi r^2}{2fR} \left( \frac{\Delta B}{\Delta t} \right) = N^2 \frac{\mu_0 \pi r^2}{2R} \left( \frac{\Delta B}{\Delta t} \right)
\]  

(4)

\[
= \left(105\right)^2 \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}}{2 \left(0.480 \Omega\right)} \left(0.783 \text{ T/s}\right) = 1.42 \times 10^{-3} \text{ T}
\]

29. **REASONING** A greater magnetic flux passes through the coil in part \( b \) of the drawing. The flux is given as \( \Phi = BA \cos \phi \) (Equation 22.2), where \( B \) is the magnitude of the magnetic field, \( A \) is the area of the surface through which the field passes, and \( \phi \) is the angle between the normal to the plane of the surface and the field. In part \( b \) the coil area has two parts, and so does the flux. There is the larger semicircular area \( \frac{1}{2} \pi r^2_{\text{larger}} \) on the horizontal surface and the smaller semicircular \( \frac{1}{2} \pi r^2_{\text{smaller}} \) area perpendicular to the horizontal surface. The field is parallel to the normal for the larger area and perpendicular to the normal for the smaller area.
Hence, the flux is

\[ \Phi_b = BA_{\text{larger}} \cos \phi_{\text{larger}} + BA_{\text{smaller}} \cos \phi_{\text{smaller}} \]

\[ = B \left( \frac{1}{2} \pi r_{\text{larger}}^2 \right) \cos 0^\circ + B \left( \frac{1}{2} \pi r_{\text{smaller}}^2 \right) \cos 90^\circ = B \left( \frac{1}{2} \pi r_{\text{larger}}^2 \right) \]  

(1)

In part \(a\) the entire area of the coil lies on the horizontal surface and is the area between the two semicircles. The plane of the coil is perpendicular to the magnetic field. The flux, therefore, is

\[ \Phi_a = BA \cos \phi = B \left( \frac{1}{2} \pi r_{\text{larger}}^2 - \frac{1}{2} \pi r_{\text{smaller}}^2 \right) \cos 0^\circ = B \left( \frac{1}{2} \pi r_{\text{larger}}^2 - \frac{1}{2} \pi r_{\text{smaller}}^2 \right) \]  

(2)

which is smaller than the flux in part \(b\) of the drawing.

According to Lenz’s law the induced magnetic field associated with the induced current opposes the change in flux. To oppose the increase in flux from part \(a\) to part \(b\) of the drawing, the induced field must point downward. In this way, it will reduce the effect of the increasing coil area available to the upward-pointing external field. An application of RHR-2 shows that the induced current must flow clockwise in the larger semicircle of wire (when viewed from above) in order to create a downward-pointing induced field. The average induced current can be determined according to Ohm’s law as the average induced emf divided by the resistance of the coil.

The period \(T\) of the rotational motion is related to the angular frequency \(\omega\) according to \(\omega = \frac{2\pi}{T}\) (Equation 10.6). The shortest time interval \(\Delta t\) that elapses between parts \(a\) and \(b\) of the drawing is the time needed for one quarter of a turn or \(\frac{1}{4} T\).

**SOLUTION** According to Ohm’s law, the average induced current is

\[ I = \frac{\xi}{R} \]

where \(\xi\) is the average induced emf and \(R\) is the resistance of the coil. According to Faraday’s law, Equation 22.3, the average induced emf is \(\xi = -\frac{\Delta \Phi}{\Delta t}\), where we have set \(N = 1\). We can determine the change \(\Delta \Phi\) in the flux as \(\Delta \Phi = \Phi_b - \Phi_a\), where the fluxes in
parts \(a\) and \(b\) of the drawing are available from Equations (1) and (2). The time interval \(\Delta t\) is \(\frac{1}{4}T\), as discussed in the REASONING. Using Equation 10.6, we have for the period that 

\[ T = \frac{2\pi}{\omega}. \]

Thus, we find for the current that

\[ I = \frac{\xi}{R} = \frac{-\Delta\Phi}{\Delta t} = \frac{-\left(\Phi_b - \Phi_a\right)}{\frac{1}{4}T} \]

\[ = -\frac{B\left(\frac{1}{2}\pi r_{\text{larger}}^2\right) - B\left(\frac{1}{2}\pi r_{\text{larger}}^2 - \frac{1}{2}\pi r_{\text{smaller}}^2\right)}{R\left(\frac{1}{4}T\right)} = -\frac{B\left(\frac{1}{2}\pi r_{\text{smaller}}^2\right)}{R\left(\frac{1}{4}2\pi / \omega\right)} \]

\[ = -\frac{(0.35 T)\frac{1}{2}\pi (0.20 \text{ m})^2}{(0.025 \Omega)\frac{1}{4}2\pi / (1.5 \text{ rad/s})} = -0.84 A \]

30. **REASONING** We can obtain the magnitude \(B\) of the magnetic field with the aid of Faraday’s law, which is \(\xi = -N \frac{\Delta\Phi}{\Delta t}\) (Equation 22.3), where \(\xi\) is the emf induced in the coil, \(N = 1850\) is the number of turns in the coil, \(\Delta\Phi\) is the change in magnetic flux, and \(\Delta t\) is the time interval during which the flux changes. The flux is \(\Phi = BA\cos\phi\) (Equation 22.3), where \(A = 4.70 \times 10^{-4} \text{ m}^2\) is the area of each turn of the coil and \(\phi = 0^\circ\) is the angle between the normal to the coil and the field. The emf can be related to the induced charge that flows in the coil by using Ohm’s law \(\xi = IR\) (Equation 20.2), where \(R = 45.0 \Omega\) is the total resistance of the circuit and \(I\) is the current. The current is given by \(I = \frac{\Delta q}{\Delta t}\) (Equation 20.1), where \(\Delta q = 8.87 \times 10^{-3} \text{ C}\) is the amount of charge that flows in the time interval \(\Delta t\). We have no value for \(\Delta t\). However, it will be eliminated algebraically from our calculation, as we will see.

**SOLUTION** Using Equation 22.3, we can write the change in flux as

\[ \Delta\Phi = B\frac{A\cos\phi}{\text{Final flux when } B \text{ is present}} - 0 = B\frac{A\cos0^\circ}{\text{Initial flux when } B \text{ is zero}} = BA \]

With this change in flux, Faraday’s law (without the minus sign, since we seek only the magnitude of the field) becomes

\[ \xi = N \frac{\Delta\Phi}{\Delta t} = N \frac{BA}{\Delta t} \quad \text{or} \quad B = \frac{\xi (\Delta t)}{NA} \]
Combining Ohm’s law and Equation 20.1 for the current, we obtain \( \xi = IR = \left( \frac{\Delta q}{\Delta t} \right) R \).

Substituting this result for \( \xi \) into the expression for \( B \), we obtain

\[
B = \frac{\xi (\Delta t)}{NA} = \frac{(\Delta q) R}{NA} = \frac{(8.87 \times 10^{-3} \text{ C})(45.0 \Omega)}{1850(4.70 \times 10^{-4} \text{ m}^2)} = 0.459 \text{ T}
\]

31. **REASONING AND SOLUTION** Consider one revolution of either rod. The magnitude \( |\xi| \) of the emf induced across the rod is

\[
|\xi| = \left| -B \frac{\Delta A}{\Delta t} \right| = \frac{B \left( \pi L^2 \right)}{\Delta t}
\]

The angular speed of the rods is \( \omega = \frac{2 \pi}{\Delta t} \), so \( |\xi| = \frac{1}{2} BL^2 \omega \). The rod tips have opposite polarity since they are rotating in opposite directions. Hence, the difference in potentials of the tips is

\[
\Delta V = BL^2 \omega
\]

so

\[
\omega = \frac{\Delta V}{BL^2} = \frac{4.5 \times 10^3 \text{ V}}{(4.7 \text{ T})(0.68 \text{ m})^2} = 2100 \text{ rad/s}
\]

32. **REASONING** Our solution is based on Lenz’s law. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. We will also have need of right-hand rule no. 2 (RHR-2) as we determine which end of the resistor is the positive end.

**SOLUTION** As the loop rotates through one-half a revolution, the area through which the magnetic field passes decreases. The magnetic flux, being proportional to the area, also decreases. The induced magnetic field must oppose this decrease in flux, so the induced magnetic field must strengthen the original magnetic field. Thus, the induced magnetic field points in the same direction as the original magnetic field, or into the plane of the page. According to RHR-2, the induced current flows clockwise around the loop (and right-to-left through the resistor) so that its magnetic field will be directed into the page. Since conventional current flows through a resistor from higher potential toward lower potential, the right end of the resistor must be the positive end.
33. **Reasoning** The external magnetic field is perpendicular to the plane of the horizontal loop, so it must point either upward or downward. We will use Lenz's law to decide whether the external magnetic \( B \) field points up or down. This law predicts that the direction of the induced magnetic field \( B_{\text{ind}} \) opposes the change in the magnetic flux through the loop due to the external field.

**Solution** The external magnetic field \( B \) is increasing in magnitude, so that the magnetic flux through the loop also increases with time. In order to oppose the increase in magnetic flux, the induced magnetic field \( B_{\text{ind}} \) must be directed opposite to the external magnetic field \( B \). The drawing shows the loop as viewed from above, with an induced current \( I_{\text{ind}} \) flowing clockwise. According to Right-Hand Rule No. 2 (see Section 21.7), this induced current creates an induced magnetic field \( B_{\text{ind}} \) that is directed into the page at the center of the loop (and all other points of the loop's interior). Therefore, the external magnetic field \( B \) must be directed out of the page. Because we are viewing the loop from above, “out of the page” corresponds to upward toward the viewer.

---

34. **Reasoning** The magnetic field produced by \( I \) extends throughout the space surrounding the loop. Using RHR-2, it can be shown that the magnetic field is parallel to the normal to the loop. Thus, the magnetic field penetrates the loop and generates a magnetic flux.

According to Faraday's law of electromagnetic induction, an emf is induced when the magnetic flux through the loop is changing in time. If the current \( I \) is constant, the magnetic flux is constant, and no emf is induced in the loop. However, if the current is decreasing in time, the magnetic flux is decreasing and an induced current exists in the loop.

Lenz’s law states that the induced magnetic field opposes the change in the magnetic field produced by the current \( I \). The induced magnetic field does not necessarily oppose the magnetic field itself. Thus, the induced magnetic field does not always have a direction that is opposite to the direction of the field produced by \( I \).

**Solution** At the location of the loop, the magnetic field produced by the current \( I \) is directed into the page (this can be verified by using RHR-2). The current is decreasing, so the magnetic field is decreasing. Therefore, the magnetic flux that penetrates the loop is decreasing. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes this flux change. The induced magnetic field will oppose this decrease in flux by pointing into the page, in the same direction as the field produced by \( I \). According to RHR-2, the induced current must flow clockwise around the loop in order to produce such an induced field. The current then flows from left-to-right through the resistor.
35. **REASONING** The current $I$ produces a magnetic field, and hence a magnetic flux, that passes through the loops A and B. Since the current decreases to zero when the switch is opened, the magnetic flux also decreases to zero. According to Lenz’s law, the current induced in each coil will have a direction such that the induced magnetic field will oppose the original flux change.

**SOLUTION**

a. The drawing in the text shows that the magnetic field at coil A is perpendicular to the plane of the coil and points down (when viewed from above the table top). When the switch is opened, the magnetic flux through coil A decreases to zero. According to Lenz's law, the induced magnetic field produced by coil A must oppose this change in flux. Since the magnetic field points down and is decreasing, the induced magnetic field must also point down. According to Right-Hand Rule No. 2 (RHR-2), the induced current must be **clockwise** around loop A.

b. The drawing in the text shows that the magnetic field at coil B is perpendicular to the plane of the coil and points up (when viewed from above the table top). When the switch is opened, the magnetic flux through coil B decreases to zero. According to Lenz's law, the induced magnetic field produced by coil B must oppose this change in flux. Since the magnetic field points up and is decreasing, the induced magnetic field must also point up. According to RHR-2, the induced current must be **counterclockwise** around loop B.

36. **REASONING** According to Lenz’s law, the induced current in the triangular loop flows in such a direction so as to create an induced magnetic field that opposes the original flux change.

**SOLUTION**

a. As the triangle is crossing the $+y$ axis, the magnetic flux down into the plane of the paper is increasing, since the field now begins to penetrate the loop. To offset this increase, an induced magnetic field directed up and out of the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a **counterclockwise induced current**.

b. As the triangle is crossing the $-x$ axis, there is no flux change, since all parts of the triangle remain in the magnetic field, which remains constant. Therefore, there is no induced magnetic field, and **no induced current appears**.

c. As the triangle is crossing the $-y$ axis, the magnetic flux down into the plane of the paper is decreasing, since the loop now begins to leave the field region. To offset this decrease, an induced magnetic field directed down and into the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a **clockwise induced current**.

d. As the triangle is crossing the $+x$ axis, there is no flux change, since all parts of the triangle remain in the field-free region. Therefore, there is no induced magnetic field, and **no induced current appears**.
37. **REASONING** The current \( I \) in the straight wire produces a circular pattern of magnetic field lines around the wire. The magnetic field at any point is tangent to one of these circular field lines. Thus, the field points perpendicular to the plane of the table. Furthermore, according to Right-Hand Rule No. 2, the field is directed up out of the table surface in region 1 above the wire and is directed down into the table surface in region 2 below the wire (see the drawing at the right). To deduce the direction of any induced current in the circular loop, we consider Faraday’s law and the change that occurs in the magnetic flux through the loop due to the field of the straight wire.

**SOLUTION** As the current \( I \) decreases, the magnitude of the field that it produces also decreases. However, the directions of the fields in regions 1 and 2 do not change and remain as discussed in the reasoning. Since the fields in these two regions always have opposite directions and equal magnitudes at any given radial distance from the straight wire, the flux through the regions add up to give zero for any value of the current. With the flux remaining constant as time passes, Faraday’s law indicates that there is no induced emf in the coil. Since there is no induced emf in the coil, there is no induced current.

38. **REASONING** Our solution is based on Lenz’s law. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. As we refer to the drawing at the right, we will also have need of right-hand rule no. 2 (RHR-2). Note that the horizontal arrows in the drawing indicate induced current that exists in the ring.

**SOLUTION**

a. When the magnet is above the ring its magnetic field points down through the ring and is increasing in strength as the magnet falls. According to Lenz’s law, an induced magnetic field appears that attempts to reduce the increasing field. Therefore, the induced field must point up. Using RHR-2, we can see that the induced current in the ring is as shown in the drawing. Because of the induced current, the ring looks like a magnet with its north pole at the top (use RHR-2). The north pole of the loop repels the falling magnet and retards its motion. When the magnet is below the ring, its magnetic field still points down through the ring but is decreasing as the magnet falls. According to Lenz’s law, the induced magnetic field attempts to bolster the decreasing field and, therefore, must point down. The induced current in the ring is as shown in the drawing, and the ring then looks like a magnet with its...
north pole at the bottom, attracting the south pole of the falling magnet and retarding its motion.

b. The motion of the magnet is unaffected, since no induced current can flow in the cut ring. No induced current means that no induced magnetic field can be produced to repel or attract the falling magnet.

39. **REASONING AND SOLUTION**

a. **Location I**

As the loop swings downward, the normal to the loop makes a smaller angle with the applied field. Hence, the flux through the loop is increasing. The induced magnetic field must point generally to the left to counteract this increase. The induced current flows

\[ x \rightarrow y \rightarrow z \]

**Location II**

The angle between the normal to the loop and the applied field is now increasing, so the flux through the loop is decreasing. The induced field must now be generally to the right, and the current flows

\[ z \rightarrow y \rightarrow x \]

b. **Location I**

The argument is the same as for location II in part a.

\[ z \rightarrow y \rightarrow x \]

**Location II**

The argument is the same as for location I in part a.

\[ x \rightarrow y \rightarrow z \]

40. **REASONING** When the motor is running at normal speed, the current is the net emf divided by the resistance \( R \) of the armature wire. The net emf is the applied voltage \( V \) minus the back emf developed by the rotating coil. We can use this relation to find the back emf. When the motor is just turned on, there is no back emf, so the current is just the applied voltage divided by the resistance of the wire. To limit the starting current to 15.0 A, a resistor \( R_1 \) is placed in series with the resistance of the wire, so the equivalent resistance is \( R + R_1 \). The current to the motor is equal to the applied voltage divided by the equivalent resistance.

**SOLUTION**

a. According to Equation 22.5, the back emf \( \xi \) generated by the motor is

\[ \xi = V - IR = 120.0 \text{ V} - (7.00 \text{ A})(0.720 \text{ \Omega}) = 115 \text{ V} \]

b. When the motor has been just turned on, the back emf is zero, so the current is
\[ I = \frac{V - \xi}{R} = \frac{120.0 \text{ V} - 0 \text{ V}}{0.720 \Omega} = 167 \text{ A} \]

c. When a resistance \( R_1 \) is placed in series with the resistance \( R \) of the wire, the equivalent resistance is \( R_1 + R \). The current to the motor is

\[ I = \frac{V - \xi}{R_1 + R} \]

Solving this expression for \( R_1 \) gives

\[ R_1 = \frac{V - \xi}{I} - R = \frac{120.0 \text{ V} - 0 \text{ V}}{15.0 \text{ A}} - 0.720 \Omega = 7.28 \Omega \]

41. SSM REASONING We can use the information given in the problem statement to determine the area of the coil \( A \). Since it is square, the length of one side is \( \ell = \sqrt{A} \).

**SOLUTION** According to Equation 22.4, the maximum emf \( \xi_0 \) induced in the coil is \( \xi = NAB \omega \). Therefore, the length of one side of the coil is

\[ \ell = \sqrt{A} = \sqrt{\frac{\xi_0}{NB \omega}} = \frac{75.0 \text{ V}}{(248)(0.170 \text{ T})(79.1 \text{ rad/s})} = 0.150 \text{ m} \]

42. REASONING The emf \( \xi \) of the generator is \( \xi = NAB \omega \sin \omega t \) (Equation 22.4), where \( N = 150 \) is the number of turns in the coil, \( A = 0.85 \text{ m}^2 \) is the area per turn, \( B \) is the magnitude of the magnetic field, \( \omega \) is the angular speed in rad/s, and \( t \) is the time. The angular speed is \( \omega = 2\pi f \) (Equation 10.6), where \( f = 60.0 \text{ Hz} \) is the frequency in cycles per second or Hertz. Equation 22.4 indicates that the maximum emf is \( \xi_{\text{max}} = NAB \omega \), since \( \sin \omega t \) has a maximum value of 1.

**SOLUTION** Substituting \( \omega = 2\pi f \) into \( \xi_{\text{max}} = NAB \omega \) and solving for the result for \( B \) reveals that

\[ B = \frac{\xi_{\text{max}}}{NA(2\pi f)} = \frac{5500 \text{ V}}{150(0.85 \text{ m}^2)(2\pi)(60.0 \text{ Hz})} = 0.11 \text{ T} \]

43. REASONING The number \( N \) of turns in the coil of a generator is given by \( N = \frac{\xi_0}{AB\omega} \) (Equation 22.4), where \( \xi_0 \) is the peak emf, \( A \) is the area per turn, \( B \) is the magnitude of the magnetic field, and \( \omega \) is the angular speed in rad/s. We have values for \( A \) and \( B \). Although we are not given the peak emf \( \xi_0 \), we know that it is related to the rms emf, which is known:
\[ \xi_0 = \sqrt{2} \xi_{\text{rms}} \] (Equation 20.13). We are also not given the angular speed \( \omega \), but we know that it is related to the frequency \( f \) in hertz according to \( \omega = 2\pi f \) (Equation 10.6).

**SOLUTION** Substituting Equation 20.13 for \( \xi_0 \) and Equation 10.6 for \( \omega \) into Equation 22.4 gives

\[
N = \frac{\xi_0}{AB\omega} = \frac{\sqrt{2} \xi_{\text{rms}}}{AB2\pi f} = \frac{\sqrt{2}(120 \text{ V})}{(0.022 \text{ m}^2)(6.9 \times 10^{-5} \text{ T})2\pi(60.0 \text{ Hz})} = 3.0 \times 10^5
\]

44. **REASONING AND SOLUTION** Using Equation 22.5 to take the back emf into account, we find

\[
R = \frac{V - \xi}{I} = \frac{(120.0 \text{ V}) - (72.0 \text{ V})}{3.0 \text{ A}} = 16 \Omega
\]

45. **REASONING** The length of the wire is the number \( N \) of turns times the length per turn. Since each turn is a square that is a length \( L \) on a side, the length per turn is \( 4L \), and the total length is \( N(4L) \). Since the area of a square is \( A = L^2 \), the length of a side of the square can be obtained as \( L = \sqrt{A} \), so that the total length is \( N(4\sqrt{A}) \). The area can be found from the peak emf, which is \( \xi_0 = NAB\omega \), according to Equation 22.4. Solving this expression for \( A \) and substituting the result into the expression for the total length gives

\[
\text{Total length} = N\left(4\sqrt{\frac{\xi_0}{NB\omega}}\right) = 4\sqrt{\frac{N\xi_0}{NB\omega}} \tag{1}
\]

Although we are not given the peak emf \( \xi_0 \), we know that it is related to the rms emf, which is known: \( \xi_0 = \sqrt{2} \xi_{\text{rms}} \) (Equation 20.13). We are also not given the angular speed \( \omega \), but we know that it is related to the frequency \( f \) in hertz according to \( \omega = 2\pi f \) (Equation 10.6).

**SOLUTION** According to Equation 20.13, the peak emf is

\[ \xi_0 = \sqrt{2} \xi_{\text{rms}} = \sqrt{2}(120 \text{ V}) = 170 \text{ V} \]

Substituting \( \omega = 2\pi f \) and the value for the peak emf into Equation (1) gives

\[
\text{Total length} = 4\sqrt{\frac{N\xi_0}{B\omega}} = 4\sqrt{\frac{N\xi_0}{B2\pi f}} = 4\sqrt{\frac{100(170 \text{ V})}{(0.50 \text{ T})2\pi(60.0 \text{ Hz})}} = 38 \text{ m}
\]
46. **REASONING** The emf $\xi$ of the generator is $\xi = NAB\omega \sin \omega t$ (Equation 22.4), where $N$ is the number of turns in the coil, $A$ is the area per turn, $B = 0.20$ T is the magnitude of the magnetic field, $\omega = 25$ rad/s is the angular speed of the coil, and $t$ is the time. Equation 22.4 indicates that the peak or maximum emf is $\xi_{\text{peak}} = NAB\omega$, since $\sin \omega t$ has a maximum value of 1. Recognizing that the area of a circle can be calculated from its radius, we can apply this equation directly to determine $\xi_{\text{peak}}$. However, we do not have a value for the number of turns $N$. To determine it, we will utilize the length $L = 5.7$ m of wire in the coil and the coil’s circumference, which can be calculated from its radius $R = 0.14$ m.

**SOLUTION** Using $A = \pi R^2$ for the area of a circle, we have

$$\xi_{\text{peak}} = NAB\omega = N\left(\pi R^2\right)B\omega$$

The number of turns is the length $L$ of the wire divided by the circumference $2\pi R$ of the circular coil or $N = L / (2\pi R)$. Substituting this expression for $N$ into the equation for $\xi_{\text{peak}}$, we obtain

$$\xi_{\text{peak}} = N\left(\pi R^2\right)B\omega = \left(\frac{L}{2\pi R}\right)\left(\pi R^2\right)B\omega$$

$$= \frac{LBR\omega}{2} = \frac{(5.7 \text{ m})(0.14 \text{ m})(0.20 \text{ T})(25 \text{ rad/s})}{2} = 2.0 \text{ V}$$

47. **SSM REASONING** The peak emf $\xi_0$ produced by a generator is related to the number $N$ of turns in the coil, the area $A$ of the coil, the magnitude $B$ of the magnetic field, and the angular speed $\omega_{\text{coil}}$ of the coil by $\xi_0 = NAB\omega_{\text{coil}}$ (see Equation 22.4). We are given that the angular speed $\omega_{\text{coil}}$ of the coil is 38 times as great as the angular speed $\omega_{\text{tire}}$ of the tire. Since the tires roll without slipping, the angular speed of a tire is related to the linear speed $v$ of the bike by $\omega_{\text{tire}} = v/r$ (see Section 8.6), where $r$ is the radius of a tire. The speed of the bike after 5.1 s can be found from its acceleration and the fact that the bike starts from rest.

**SOLUTION** The peak emf produced by the generator is

$$\xi_0 = NAB\omega_{\text{coil}}$$

(22.4)

Since the angular speed of the coil is 38 times as great as the angular speed of the tire, $\omega_{\text{coil}} = 38\omega_{\text{tire}}$. Substituting this expression into Equation 22.4 gives $\xi_0 = NAB\omega_{\text{coil}} = NAB\left(38\omega_{\text{tire}}\right)$. Since the tire rolls without slipping, the angular speed of the tire is related to the linear speed $v$ (the speed at which its axle is moving forward) by $\omega_{\text{tire}} = v/r$ (Equation 8.12), where $r$ is the radius of the tire. Substituting this result into the expression for $\xi_0$ yields
\[ \xi_0 = NAB\left(38\omega_{tire}\right) = NAB\left(38\frac{v}{r}\right) \]  \hspace{1cm} \text{(1)}

The velocity of the car is given by \( v = v_0 + at \) (Equation 2.4), where \( v_0 \) is the initial velocity, \( a \) is the acceleration and \( t \) is the time. Substituting this relation into Equation (1), and noting that \( v_0 = 0 \) m/s since the bike starts from rest, we find that

\[ \xi_0 = NAB\left(38\frac{v}{r}\right) = NAB\left[38\left(v_0 + at\right)\right] \]

\[ = (125)(3.86\times10^{-3} \text{ m}^2)(0.0900 \text{ T})(38) \left[ 0 \text{ m/s} + \left(0.550 \text{ m/s}^2\right)(5.10 \text{ s}) \right] = 15.4 \text{ V} \]

48. \textit{REASONING AND SOLUTION}

a. On startup, the back emf of the generator is zero. Then,

\[ R = \frac{V}{I} = \frac{117 \text{ V}}{12.2 \text{ A}} = 9.59 \Omega \]

b. At normal speed

\[ \xi = V - IR = 117 \text{ V} - (2.30 \text{ A})(9.59 \Omega) = 95 \text{ V} \]

c. The back emf of the motor is proportional to the rotational speed, so at \( \frac{1}{3} \) the normal speed, the back emf is

\[ \frac{1}{3}(95 \text{ V}) = 32 \text{ V} \]

The voltage applied to the resistor is then \( V = 117 \text{ V} - 32 \text{ V} = 85 \text{ V} \), so the current is

\[ I = \frac{V}{R} = \frac{85 \text{ V}}{9.59 \Omega} = 8.9 \text{ A} \]

49. \textbf{SSM REASONING} The energy density is given by Equation 22.11 as

\[ \text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2\mu_0}B^2 \]

The energy stored is the energy density times the volume.

\textit{SOLUTION} The volume is the area \( A \) times the height \( h \). Therefore, the energy stored is

\[ \text{Energy} = \frac{B^2Ah}{2\mu_0} = \frac{(7.0\times10^{-5} \text{ T})^2(5.0\times10^8 \text{ m}^2)(1500 \text{ m})}{2(4\pi\times10^{-7} \text{ T} \cdot \text{m/A})} = 1.5\times10^9 \text{ J} \]
50. **REASONING** When the current through an inductor changes, the induced emf $\xi$ is given by Equation 22.9 as

$$\xi = -L \frac{\Delta I}{\Delta t}$$

where $L$ is the inductance, $\Delta I$ is the change in the current, and $\Delta t$ is the time interval during which the current changes. For each interval, we can determine $\Delta I$ and $\Delta t$ from the graph.

**SOLUTION**

a. $$\xi = -L \frac{\Delta I}{\Delta t} = -(3.2 \times 10^{-3} \text{ H}) \left( \frac{4.0 \text{ A} - 0 \text{ A}}{2.0 \times 10^{-3} \text{ s} - 0 \text{ s}} \right) = -6.4 \text{ V}$$

b. $$\xi = -L \frac{\Delta I}{\Delta t} = -(3.2 \times 10^{-3} \text{ H}) \left( \frac{4.0 \text{ A} - 4.0 \text{ A}}{5.0 \times 10^{-3} \text{ s} - 2.0 \times 10^{-3} \text{ s}} \right) = 0 \text{ V}$$

c. $$\xi = -L \frac{\Delta I}{\Delta t} = -(3.2 \times 10^{-3} \text{ H}) \left( \frac{0 \text{ A} - 4.0 \text{ A}}{9.0 \times 10^{-3} \text{ s} - 5.0 \times 10^{-3} \text{ s}} \right) = +3.2 \text{ V}$$

51. **REASONING** We will designate the coil containing the current as the primary coil, and the other as the secondary coil. The emf $\xi_s$ induced in the secondary coil due to the changing current in the primary coil is given by $\xi_s = -M \left( \frac{\Delta I_p}{\Delta t} \right)$ (Equation 22.7), where $M$ is the mutual inductance of the two coils, $\Delta I_p$ is the change in the current in the primary coil, and $\Delta t$ is the change in time. This equation can be used to find the mutual inductance.

**SOLUTION** Solving Equation 22.7 for the mutual inductance gives

$$M = -\frac{\xi_s \Delta t}{\Delta I_p} = -\frac{(1.7 \text{ V})(3.7\times10^{-2} \text{ s})}{(0 \text{ A} - 2.5 \text{ A})} = 2.5\times10^{-2} \text{ H}$$

52. **REASONING** According to $\xi_s = -M \frac{\Delta I_p}{\Delta t}$ (Equation 22.7), a change $\Delta I_p$ in the current in the primary coil induces an emf $\xi_s$ in the secondary, where $M$ is the mutual inductance of the two coils and $\Delta t$ is the time interval of the current change. We are interested only in the magnitude of the current change $\Delta I_p$, so we will omit the minus sign in Equation 22.7. The induced emf $\xi_s$ in the secondary coil will drive a current $I_s$, as we see from Ohm’s law: $\xi_s = I_s R$ (Equation 20.2), where $R$ is the resistance of the circuit that includes the secondary coil.
**SOLUTION**  Omitting the minus sign in $\xi_s = -M \frac{\Delta I_p}{\Delta t}$ (Equation 22.7) and solving for $\Delta I_p$ yields

$$\Delta I_p = \frac{\xi_s \Delta t}{M}$$

Substituting $\xi_s = I_s R$ (Equation 20.2) into Equation (1), we obtain

$$\Delta I_p = \frac{I_s R \Delta t}{M} = \frac{(6.0 \times 10^{-3} \text{ A})(12 \Omega)(72 \times 10^{-3} \text{ s})}{3.2 \times 10^{-3} \text{ H}} = 1.6 \text{ A}$$

53. **REASONING AND SOLUTION**  The induced emf in the secondary coil is proportional to the mutual inductance. If the primary coil is assumed to be unaffected by the metal, that is $\Delta I_1/\Delta t$ is the same for both cases, then

New emf $= 3(0.46 \text{ V}) = 1.4 \text{ V}$

54. **REASONING**  According to Faraday’s law of electromagnetic induction, expressed as $\xi = -L(\Delta I/\Delta t)$ (Equation 22.9), an emf is induced in the solenoid as long as the current is changing in time.

The amount of electrical energy $E$ stored by an inductor is $E = \frac{1}{2} LI^2$ (Equation 22.10), where $L$ is the inductance and $I$ is the current.

The power $P$ is equal to the energy removed divided by the time $t$ or $P = \frac{E}{t} = \frac{1}{2} LI^2$ (Equation 6.10b).

**SOLUTION**

a. The emf induced in the solenoid is

$$\text{Emf} = -L \left( \frac{\Delta I}{\Delta t} \right) = -(3.1 \text{ H}) \left( \frac{0 \text{ A} - 15 \text{ A}}{75 \times 10^{-3} \text{ s}} \right) = +620 \text{ V}$$

(22.9)

b. The energy stored in the solenoid is

$$E = \frac{1}{2} LI^2 = \frac{1}{2} (3.1 \text{ H})(15 \text{ A})^2 = 350 \text{ J}$$

(22.10)

c. The rate (or power, $P$) at which the energy is removed is

$$P = \frac{E}{t} = \frac{\frac{1}{2} LI^2}{t} = \frac{\frac{1}{2} (3.1 \text{ H})(15 \text{ A})^2}{75 \times 10^{-3} \text{ s}} = 4700 \text{ W}$$

(6.10b)
55. **REASONING AND SOLUTION** From the results of Example 13, the self-inductance \( L \) of a long solenoid is given by \( L = \mu_0 n^2 A \ell \). Solving for the number of turns \( n \) per unit length gives

\[
n = \sqrt{\frac{L}{\mu_0 A \ell}} = \sqrt{\frac{1.4 \times 10^{-3} \text{ H}}{(4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(1.2 \times 10^{-3} \text{ m}^2)(0.052 \text{ m})}} = 4.2 \times 10^3 \text{ turns/m}
\]

Therefore, the total number of turns \( N \) is the product of \( n \) and the length \( \ell \) of the solenoid:

\[
N = n \ell = (4.2 \times 10^3 \text{ turns/m})(0.052 \text{ m}) = 220 \text{ turns}
\]

56. **REASONING** Let us arbitrarily label the long solenoid as the primary (p) and the coil as the secondary (s). The mutual inductance \( M \) is

\[
M = \frac{N_s \Phi_s}{I_p}
\]

where \( N_s = 125 \) is the number of turns in the coil, \( \Phi_s \) is the flux through one loop of the coil due to the current \( I_p \) in the solenoid. Since the flux \( \Phi_s \) is due to the current in the solenoid, the flux is \( \Phi_s = B_{\text{solenoid}} A \cos \phi \) (Equation 22.3), where \( A = \pi R^2 \) is the area of a turn of the coil (or the solenoid) and \( \phi = 0^\circ \) is the angle between the normal to the coil and the field of the solenoid. Note that a turn of the solenoid and a turn of the coil have the virtually the same radius \( R = 0.0180 \text{ m} \). The field of the solenoid is \( B_{\text{solenoid}} = \mu_0 n I_p \) (Equation 21.7), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space, \( n = 1750 \text{ m}^{-1} \) is the number of turns per meter in the solenoid, and \( I_p \) is the current in the solenoid. We do not have a value for the current \( I_p \) in the solenoid. However, we will see that it will be eliminated algebraically from the solution.

**SOLUTION** Substituting \( \Phi_s = B_{\text{solenoid}} A \cos \phi \) (Equation 22.3) into Equation 22.6 for the mutual inductance gives

\[
M = \frac{N_s \Phi_s}{I_p} = \frac{N_s \left( B_{\text{solenoid}} A \cos \phi \right)}{I_p} = \frac{N_s \left( B_{\text{solenoid}} \pi R^2 \cos 0^\circ \right)}{I_p} = \frac{N_s \left( B_{\text{solenoid}} \pi R^2 \right)}{I_p}
\]

Substituting \( B_{\text{solenoid}} = \mu_0 n I_p \) (Equation 21.7) into this result for \( M \), we obtain

\[
M = \frac{N_s \left( B_{\text{solenoid}} \pi R^2 \right)}{I_p} = \frac{N_s \left( \mu_0 n I_p \right) \pi R^2}{I_p} = N_s \mu_0 n \pi R^2 = 125 \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) (1750 \text{ m}^{-1}) \pi (0.0180 \text{ m})^2 = 2.80 \times 10^{-4} \text{ H}
\]
57. **REASONING** As demonstrated in Example 13, the inductance $L$ of a solenoid of length $\ell$ and cross-sectional area $A$ is given by

$$L = \mu_0 n^2 \ell A$$

(1)

where $\mu_0 = 4\pi \times 10^{-7}$ T·m/A is the permeability of free space and $n$ is the number of turns per unit length. It may appear that solving Equation (1) for the length $\ell$ of the solenoid would determine $\ell$ as a function of the inductance $L$, and therefore solve the problem. However, the number $n$ of turns per unit length is the ratio of the total number $N$ of turns to the length $\ell$:

$$n = \frac{N}{\ell}$$

(2)

Therefore, changing the length $\ell$ of the solenoid also changes the number $n$ of turns per unit length, since the total number $N$ of turns is unchanged. Using Equations (1) and (2), we will derive an expression for the length of the solenoid in terms of its inductance $L$ and the constant quantities $N$, $\mu_0$, and $A$.

**SOLUTION** Substituting Equation (2) into Equation (1) and simplifying, we obtain

$$L = \mu_0 n^2 \ell A = \mu_0 \left( \frac{N}{\ell} \right)^2 \ell A = \frac{\mu_0 N^2 A}{\ell}$$

(3)

Solving Equation (3) for the length of the solenoid yields

$$\ell = \frac{\mu_0 N^2 A}{L}$$

(4)

When the solenoid’s length is $\ell_1$, its inductance is $L_1 = 5.40 \times 10^{-5}$ H. Reducing its length to $\ell_2$ increases its inductance to $L_2 = 8.60 \times 10^{-5}$ H. The amount $\Delta \ell = \ell_1 - \ell_2$ by which the solenoid’s length decreases is, from Equation (4),

$$\Delta \ell = \ell_1 - \ell_2 = \frac{\mu_0 N^2 A}{L_1} - \frac{\mu_0 N^2 A}{L_2} = \mu_0 N^2 A \left( \frac{1}{L_1} - \frac{1}{L_2} \right)$$

$$= \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 65 \right)^2 \left( 9.0 \times 10^{-4} \text{ m}^2 \right) \left( \frac{1}{5.40 \times 10^{-5} \text{ H}} - \frac{1}{8.60 \times 10^{-5} \text{ H}} \right) = 0.033 \text{ m}$$

58. **REASONING** The emf due to self-induction is given by $\xi = -L(\Delta I / \Delta t)$ (Equation 22.9), where $L$ is the self-inductance, and $\Delta I / \Delta t$ is the rate at which the current changes. Both $\Delta I$ and $\Delta t$ are known. We can find the self-inductance of a toroid by starting with that of a long solenoid, which we obtained in Example 13.

**SOLUTION** The emf induced in the toroid is given by

$$\xi = -L \left( \frac{\Delta I}{\Delta t} \right)$$

(22.9)
As obtained in Example 13, the self-inductance of a long solenoid is \( L = \mu_0 n^2 A \ell \), where \( \ell \) is the length of the solenoid, \( n \) is the number of turns per unit length, and \( A \) is the cross-sectional area of the solenoid. A toroid is a solenoid that is bent to form a circle of radius \( R \). The length \( \ell \) of the toroid is the circumference of the circle, \( \ell = 2\pi R \). Substituting this expression for \( \ell \) into the equation for \( L \), we obtain

\[
L = \mu_0 n^2 A \ell = \mu_0 n^2 A (2\pi R)
\]

Substituting this relation for \( L \) into Equation (22.9) yields

\[
\xi = -L \left( \frac{\Delta I}{\Delta t} \right) = -\mu_0 n^2 A (2\pi R) \left( \frac{\Delta I}{\Delta t} \right)
\]

\[
= -\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right) \left(2400 \text{ m}^{-1}\right)^2 \left(1.0 \times 10^{-6} \text{ m}^2\right) 2\pi (0.050 \text{ m}) \left(\frac{1.1 \text{ A} - 2.5 \text{ A}}{0.15 \text{ s}}\right)
\]

\[
= 2.1 \times 10^{-5} \text{ V}
\]

59. **REASONING AND SOLUTION** The mutual inductance is

\[
M = \frac{N_2 \Phi_2}{I_1}
\]

The flux through loop 2 is

\[
\Phi_2 = B_1 A_2 = \left( \frac{\mu_0 N_1 I_1}{2R_1} \right) \left( \pi R_2^2 \right)
\]

Then

\[
M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2}{I_1} \left( \frac{\mu_0 N_1 I_1}{2R_1} \right) \left( \pi R_2^2 \right) = \frac{\mu_0 \pi N_1 N_2 R_2^2}{2R_1}
\]

60. **REASONING** To solve this problem, we can use the transformer equation:

\[
\frac{V_s}{V_p} = \frac{N_s}{N_p}
\]

(Equation 22.12), where \( V_s \) is the voltage provided by the secondary coil, \( V_p = 120 \text{ V} \) is the voltage applied across the primary coil, and \( \frac{N_s}{N_p} = \frac{1}{32} \) is the turns ratio.

**SOLUTION** Solving the transformer equation for the voltage \( V_s \) provided by the secondary coil gives

\[
\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{or} \quad V_s = V_p \frac{N_s}{N_p} = (120 \text{ V}) \left( \frac{1}{32} \right) = 3.8 \text{ V}
\]
61. **REASONING** The air filter is connected to the secondary, so that the power used by the air filter is the power provided by the secondary. However, the power provided by the secondary comes from the primary, so the power used by the air filter is also the power delivered by the wall socket to the primary. This power is \( \overline{P} = I_P V_P \) (Equation 20.15a), where \( I_P \) is the current in the primary and \( V_P \) is the voltage provided by the socket, which we know. Although we do not have a value for \( I_P \), we do have a value for \( I_S \), which is the current in the secondary. We will take advantage of the fact that \( I_P \) and \( I_S \) are related according to \( \frac{I_S}{I_P} = \frac{N_P}{N_S} \) (Equation 22.13).

**SOLUTION** The power used by the filter is

\[
\overline{P} = I_P V_P
\]

Solving Equation 22.13 for \( I_P \) shows that \( I_P = I_S \left( \frac{N_S}{N_P} \right) \). Substituting this result into the expression for the power gives

\[
\overline{P} = I_P V_P = I_S \left( \frac{N_S}{N_P} \right) V_P = \left( 1.7 \times 10^{-3} \text{ A} \right) \left( \frac{50}{1} \right) (120 \text{ V}) = 1.0 \times 10^1 \text{ W}
\]

62. **REASONING** Since the secondary voltage (the voltage to charge the batteries) is less than the primary voltage (the voltage at the wall socket), the transformer is a step-down transformer.

In a step-down transformer, the voltage across the secondary coil is less than the voltage across the primary coil. However, the current in the secondary coil is greater than the current in the primary coil. Thus, the current that goes through the batteries is greater than the current from the wall socket.

If the transformer has negligible resistance, the power delivered to the batteries is equal to the power coming from the wall socket.

**SOLUTION**

a. The turns ratio \( N_S / N_P \) is equal to the ratio of secondary voltage to the primary voltage:

\[
\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{9.0 \text{ V}}{120 \text{ V}} = 1:13 \quad (22.12)
\]

b. The current from the wall socket is given by Equation 22.13:

\[
I_P = I_S \left( \frac{N_S}{N_P} \right) = \left( 225 \times 10^{-3} \text{ A} \right) \left( \frac{1}{13} \right) = 1.7 \times 10^{-2} \text{ A} \quad (22.13)
\]
c. The average power delivered by the wall socket is the product of the primary current and voltage:

\[ P_p = I_p V_p = \left(17 \times 10^{-3} \text{ A}\right)(120 \text{ V}) = 2.0 \text{ W} \]  

(20.15a)

The average power delivered to the batteries is the same as that coming from the wall socket, so \( P_s = 2.0 \text{ W} \).

63. **REASONING AND SOLUTION** The resistance of the primary is (see Equation 20.3)

\[ R_p = \frac{\rho L_p}{A} \]  

(1)

The resistance of the secondary is

\[ R_s = \frac{\rho L_s}{A} \]  

(2)

In writing Equations (1) and (2) we have used the fact that both coils are made of the same wire, so that the resistivity \( \rho \) and the cross-sectional area \( A \) is the same for each. Division of the equations gives

\[ \frac{R_s}{R_p} = \frac{L_s}{L_p} = \frac{14 \Omega}{56 \Omega} = \frac{1}{4} \]

Since the diameters of the coils are the same, the lengths of the wires are proportional to the number of turns. Therefore,

\[ \frac{N_s}{N_p} = \frac{L_s}{L_p} = \frac{1}{4} \]

64. **REASONING** The generator drives a fluctuating current in the primary coil, and the changing magnetic flux that results from this current induces a fluctuating voltage in the secondary coil, attached to the resistor. The peak emf of the generator is equal to the peak voltage \( V_p \) of the primary coil. Given a peak voltage in the secondary coil of \( V_s = 67 \text{ V} \), the peak voltage \( V_p \) can be found from

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \]  

(22.12)

In Equation 22.12, \( N_p \) and \( N_s \) are, respectively, the number of turns in the primary coil and the number of turns in the secondary coil.

**SOLUTION** Solving Equation 22.12 for \( V_p \), we obtain

\[ V_p = V_s \left(\frac{N_p}{N_s}\right) \]  

(1)
Since there are \( N_p = 11 \) turns in the primary coil and \( N_s = 18 \) turns in the secondary coil, Equation (1) gives

\[
V_p = (67 \text{ V}) \left( \frac{11}{18} \right) = 41 \text{ V}
\]

65. **REASONING** The ratio \( \left( I_s / I_p \right) \) of the current in the secondary coil to that in the primary coil is equal to the ratio \( \left( N_p / N_s \right) \) of the number of turns in the primary coil to that in the secondary coil. This relation can be used directly to find the current in the primary coil.

**SOLUTION** Solving the relation \( \left( I_s / I_p \right) = \left( N_p / N_s \right) \) (Equation 22.13) for \( I_p \) gives

\[
I_p = I_s \left( \frac{N_s}{N_p} \right) = (1.6 \text{ A}) \left( \frac{1}{8} \right) = 0.20 \text{ A}
\]

66. **REASONING** The turns ratio is related to the current \( I_p \) in the primary and the current \( I_s \) in the secondary according to \( \frac{I_p}{I_s} = \frac{N_s}{N_p} \) (Equation 22.13).

The turns ratio is related to the voltage \( V_p \) in the primary and the voltage \( V_s \) in the secondary according to the transformer equation, which is \( \frac{V_s}{V_p} = \frac{N_s}{N_p} \) (Equation 22.12).

Given a value for the power \( P \) used by the picture tube means that we know the power provided by the secondary. Since power is current times voltage (Equation 20.15a), we know that \( P = I_s V_s \). This expression can be solved for \( V_s \), and the result can be substituted into Equation 22.12 to give the turns ratio:

\[
\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{P/I_s}{V_p} = \frac{P}{I_s V_p}
\]

**SOLUTION** Using Equation (1), we find that the turns ratio is

\[
\frac{N_s}{N_p} = \frac{P}{I_s V_p} = \frac{91 \text{ W}}{(5.5 \times 10^{-3} \text{ A})(120 \text{ V})} = 140
\]

67. **REASONING** The power used to heat the wires is given by Equation 20.6b:

\[
P = I^2 R
\]

Before we can use this equation, however, we must determine the total resistance \( R \) of the wire and the current that flows through the wire.
SOLUTION

a. The total resistance of one of the wires is equal to the resistance per unit length multiplied by the length of the wire. Thus, we have

\[(5.0\times10^{-2} \ \Omega/km)(7.0 \ km) = 0.35 \ \Omega\]

and the total resistance of the transmission line is twice this value or 0.70 \ \Omega. According to Equation 20.6a \((P = IV)\), the current flowing into the town is

\[I = \frac{P}{V} = \frac{1.2\times10^6 \ W}{1200 \ V} = 1.0\times10^3 \ A\]

Thus, the power used to heat the wire is

\[P = I^2R = (1.0\times10^3 \ A)^2 (0.70 \ \Omega) = 7.0\times10^5 \ W\]

b. According to the transformer equation (Equation 22.12), the stepped-up voltage is

\[V_s = V_p \left(\frac{N_s}{N_p}\right) = (1200 \ V) \left(\frac{100}{1}\right) = 1.2\times10^5 \ V\]

According to Equation 20.6a \((P = IV)\), the current in the wires is

\[I = \frac{P}{V} = \frac{1.2\times10^6 \ W}{1.2\times10^5 \ V} = 1.0\times10^1 \ A\]

The power used to heat the wires is now

\[P = I^2R = (1.0\times10^1 \ A)^2 (0.70 \ \Omega) = 7.0\times10^1 \ W\]

68. REASONING The power that your house is using can be determined from \(\bar{P} = I_{rms}\xi_{rms}\) (Equation 20.15a), where \(I_{rms}\) is the current in your house. We know that \(\xi_{rms}\) is 240 V. To find \(I_{rms}\), we must apply \(\frac{I_S}{I_P} = \frac{N_P}{N_S}\) (Equation 22.13) to convert the current given for the primary of the substation transformer into the current in the secondary of the substation transformer. This secondary current then becomes the current in the primary of the transformer on the telephone pole. We will use Equation 22.13 again to find the current in the secondary of the transformer on the telephone pole. This secondary current is \(I_{rms}\).

SOLUTION According to Equation 20.15a, the power that your house is using is

\[\bar{P} = I_{rms}\xi_{rms}\]  \hspace{1cm} (1)

Applying Equation 22.13 to the substation transformer, we find
\[
\frac{I_S}{I_p} = \frac{N_p}{N_s} \quad \text{or} \quad I_s = I_p \left( \frac{N_p}{N_s} \right) = \left( 48 \times 10^{-3} \, \text{A} \right) \left( \frac{29}{1} \right) = 1.4 \, \text{A}
\]

Thus, the current in the primary of the transformer on the telephone pole is 1.4 A. Applying Equation 22.3 to the telephone-pole transformer, we obtain

\[
\frac{I_S}{I_p} = \frac{N_p}{N_s} \quad \text{or} \quad I_s = I_p \left( \frac{N_p}{N_s} \right) = (1.4 \, \text{A}) \left( \frac{32}{1} \right) = 45 \, \text{A}
\]

Telephone pole

Using this value of 45 A for the current in Equation (1) gives

\[
\bar{P} = I_{\text{rms}} \bar{\xi}_{\text{rms}} = (45 \, \text{A})(240 \, \text{V}) = 1.1 \times 10^4 \, \text{W}
\]

69. **REASONING AND SOLUTION** Ohm’s law written for the secondary is \( V_s = I_s R_2 \). We know that

\[
V_s = (N/N_p) V_p \quad \text{and} \quad I_s = (N/N_p) I_p
\]

Substituting these expressions for \( V_s \) and \( I_s \) into \( V_s = I_s R_2 \) and recognizing that \( R_1 = V_p/I_p \), we find that

\[
R_1 = \left( \frac{N_p}{N_s} \right)^2 R_2
\]

70. **REASONING** According to Equation 22.3, Faraday’s law specifies the emf induced in a coil of \( N \) loops as

\[
\bar{\xi} = -N \frac{\Delta \Phi}{\Delta t}
\]

where \( \Delta \Phi/\Delta t \) is the rate of change of the magnetic flux in a single loop. Recognizing that \( \Delta \Phi/\Delta t \) is the same for each of the coils, we will apply Faraday’s law to each coil to obtain our solution.

**SOLUTION** Applying Faraday’s law to each coil gives

\[
\bar{\xi}_1 = -N_1 \frac{\Delta \Phi}{\Delta t} \quad \text{and} \quad \bar{\xi}_2 = -N_2 \frac{\Delta \Phi}{\Delta t}
\]

Dividing these equations and remembering that the rate of change of the flux is the same for each coil, we find that

\[
\frac{\bar{\xi}_2}{\bar{\xi}_1} = -\frac{N_2}{N_1} \frac{\Delta \Phi}{\Delta t} = \frac{N_2}{N_1} \quad \text{or} \quad N_2 = N_1 \frac{\bar{\xi}_2}{\bar{\xi}_1} = 184 \left( \frac{4.23 \, \text{V}}{2.82 \, \text{V}} \right) = 276
\]
71. **SSM REASONING** The peak emf $\xi_0$ of a generator is found from $\xi_0 = NAB\omega$ (Equation 22.4), where $N$ is the number of turns in the generator coil, $A$ is the coil’s cross-sectional area, $B$ is the magnitude of the uniform magnetic field in the generator, and $\omega$ is the angular frequency of rotation of the coil. In terms of the frequency $f$ (in Hz), the angular frequency is given by $\omega = 2\pi f$ (Equation 10.6). Substituting Equation 10.6 into Equation 22.4, we obtain

$$\xi_0 = NAB(2\pi f) = 2\pi NABf$$

(1)

When the rotational frequency $f$ of the coil changes, the peak emf $\xi_0$ also changes. The quantities $N$, $A$, and $B$ remain constant, however, because they depend on how the generator is constructed, not on how rapidly the coil rotates. We know the peak emf of the generator at one frequency, so we will use Equation (1) to determine the peak emf for a different frequency in part (a), and the frequency needed for a different peak emf in part (b).

**SOLUTION**

a. Solving Equation (1) for the quantities that do not change with frequency, we find that

$$\frac{\xi_0}{f} = \frac{2\pi NAB}{\text{Same for all frequencies}}$$

(2)

The peak emf is $\xi_{0,1} = 75$ V when the frequency is $f_1 = 280$ Hz. We wish to find the peak emf $\xi_{0,2}$ when the frequency is $f_2 = 45$ Hz. From Equation (2), we have that

$$\frac{\xi_{0,2}}{f_2} = \frac{2\pi NAB}{\text{Same for all frequencies}} = \frac{\xi_{0,1}}{f_1}$$

(3)

Solving Equation (3) for $\xi_{0,2}$, we obtain

$$\xi_{0,2} = \left(\frac{f_2}{f_1}\right)\xi_{0,1} = \left(\frac{45 \text{ Hz}}{280 \text{ Hz}}\right)(75 \text{ V}) = 12 \text{ V}$$

b. Letting $\xi_{0,3} = 180$ V, Equation (2) yields

$$\frac{\xi_{0,3}}{f_3} = \frac{\xi_{0,1}}{f_1}$$

(4)

Solving Equation (4) for $f_3$, we find that

$$f_3 = \left(\frac{\xi_{0,3}}{\xi_{0,1}}\right)f_1 = \left(\frac{180 \text{ V}}{75 \text{ V}}\right)(280 \text{ Hz}) = 670 \text{ Hz}$$
72. **REASONING** According to Faraday’s law, as given in Equation 22.3, the magnitude of the emf is 

\[ |\varepsilon| = \left| - (1) \frac{\Delta \Phi}{\Delta t} \right|, \]

where we have set \( N = 1 \) for a single turn. Since the normal is parallel to the magnetic field, the angle \( \phi \) between the normal and the field is \( \phi = 0^\circ \) when calculating the flux \( \Phi \) from Equation 22.2: \( \Phi = BA \cos 0^\circ = BA \). We will use this expression for the flux in Faraday’s law.

**SOLUTION** Representing the flux as \( \Phi = BA \), we find that the magnitude of the induced emf is

\[ |\varepsilon| = - \frac{\Delta \Phi}{\Delta t} = - \frac{\Delta (BA)}{\Delta t} = - \frac{B \Delta A}{\Delta t}. \]

In this result we have used the fact that the field magnitude \( B \) is constant. Rearranging this equation gives

\[ \frac{\Delta A}{\Delta t} = \frac{|\varepsilon|}{B} = \frac{2.6 \text{ V}}{1.7 \text{ T}} = \frac{1.5 \text{ m}^2/\text{s}}{1.7 \text{ T}}. \]

73. **SSM REASONING** In solving this problem, we apply Lenz's law, which essentially says that the change in magnetic flux must be opposed by the induced magnetic field.

**SOLUTION**

a. The magnetic field due to the wire in the vicinity of position 1 is directed out of the paper. The coil is moving closer to the wire into a region of higher magnetic field, so the flux through the coil is increasing. Lenz’s law demands that the induced field counteract this increase. The direction of the induced field, therefore, must be into the paper. The current in the coil must be **clockwise**.

b. At position 2 the magnetic field is directed into the paper and is decreasing as the coil moves away from the wire. The induced magnetic field, therefore, must be directed into the paper, so the current in the coil must be **clockwise**.

74. **REASONING AND SOLUTION** According to the transformer equation (Equation 22.12), we have

\[ N_s = \left( \frac{V_s}{V_p} \right) N_p = \left( \frac{4320 \text{ V}}{120.0 \text{ V}} \right) (21) = 756 \]

75. **SSM REASONING AND SOLUTION** The energy stored in a capacitor is given by Equation 19.11b as \( \frac{1}{2} CV^2 \). The energy stored in an inductor is given by Equation 22.10 as \( \frac{1}{2} LI^2 \). Setting these two equations equal to each other and solving for the current \( I \), we get
\[
I = \sqrt{\frac{C}{L}} V = \sqrt{\frac{3.0 \times 10^{-6} \text{ F}}{5.0 \times 10^{-3} \text{ H}}} (35 \text{ V}) = 0.86 \text{ A}
\]

76. **REASONING** When the motor has just started turning the fan blade, there is no back emf, and the voltage across the resistance \( R \) of the motor is equal to the voltage \( V_0 \) of the outlet. Under this condition the resulting current in the motor is

\[
I_0 = \frac{V_0}{R} \quad (1)
\]

according to Ohm's law. When the fan blade is turning at its normal operating speed, the back emf \( \xi \) in the motor reduces the voltage across the resistance \( R \) of the motor to \( V_0 - \xi \), so that, the current drawn by the motor is

\[
I = \frac{V_0 - \xi}{R} \quad (22.5)
\]

The current \( I \) drawn at the normal operating speed is only 15.0% of the current \( I_0 \) drawn when the fan blade just begins to turn, so we have that

\[
I = 0.150I_0 \quad (2)
\]

**SOLUTION** Substituting Equation (2) into Equation (22.5) yields

\[
0.150I_0 = \frac{V_0 - \xi}{R} \quad (3)
\]

Substituting Equation (1) into Equation (3) and solving for \( \xi \), we find that

\[
0.150\left(\frac{V_0}{R}\right) = \frac{V_0 - \xi}{R} \quad \text{or} \quad \xi = V_0 - 0.150V_0 = 0.850V_0 = 0.850(120.0 \text{ V}) = 102 \text{ V}
\]

77. **REASONING** Using Equation 22.3 (Faraday’s law) and recognizing that \( N = 1 \), we can write the magnitude of the emfs for parts \( a \) and \( b \) as follows:

\[
|\xi_a| = \left| -\left(\frac{\Delta \Phi}{\Delta t}\right)_a \right| \quad (1) \quad \text{and} \quad |\xi_b| = \left| -\left(\frac{\Delta \Phi}{\Delta t}\right)_b \right| \quad (2)
\]

To solve this problem, we need to consider the change in flux \( \Delta \Phi \) and the time interval \( \Delta t \) for both parts of the drawing in the text.

**SOLUTION** The change in flux is the same for both parts of the drawing and is given by

\[
(\Delta \Phi)_a = (\Delta \Phi)_b = \Phi_\text{inside} - \Phi_\text{outside} = \Phi_\text{inside} = BA \quad (3)
\]
In Equation (3) we have used the fact that initially the coil is outside the field region, so that \( \Phi_{\text{outside}} = 0 \) Wb for both cases. Moreover, the field is perpendicular to the plane of the coil and has the same magnitude \( B \) over the entire area \( A \) of the coil, once it has completely entered the field region. Thus, \( \Phi_{\text{inside}} = BA \) in both cases, according to Equation 22.2.

The time interval required for the coil to enter the field region completely can be expressed as the distance the coil travels divided by the speed at which it is pushed. In part \( a \) of the drawing the distance traveled is \( W \), while in part \( b \) it is \( L \). Thus, we have

\[
(\Delta t)_a = \frac{W}{v} \quad (4) \\
(\Delta t)_b = \frac{L}{v} \quad (5)
\]

Substituting Equations (3), (4), and (5) into Equations (1) and (2), we find

\[
\left| \xi_a \right| = -\left( \frac{\Delta \Phi}{\Delta t} \right)_a = \frac{BA}{W/v} \quad (6) \\
\left| \xi_b \right| = -\left( \frac{\Delta \Phi}{\Delta t} \right)_b = \frac{BA}{L/v} \quad (7)
\]

Dividing Equation (6) by Equation (7) gives

\[
\left| \frac{\xi_a}{\xi_b} \right| = \frac{BA}{W/v} \cdot \frac{L/v}{W} = 3.0 \quad \text{or} \quad \left| \xi_b \right| = \left| \xi_a \right| = \frac{0.15 \text{ V}}{3.0} = 0.050 \text{ V}
\]

78. **REASONING AND SOLUTION** If the applied magnetic field is decreasing in time, then the flux through the circuit is decreasing. Lenz's law requires that an induced magnetic field be produced which attempts to counteract this decrease; hence its direction is out of the paper. The sense of the induced current in the circuit must be CCW. Therefore, the lower plate of the capacitor is positive while the upper plate is negative. The electric field between the plates of the capacitor points from positive to negative so the electric field points upward.

79. **SSM REASONING** The energy dissipated in the resistance is given by Equation 6.10b as the power \( P \) dissipated multiplied by the time \( t \), Energy = \( Pt \). The power, according to Equation 20.6c, is the square of the induced emf \( \xi \) divided by the resistance \( R \), \( P = \xi^2/R \). The induced emf can be determined from Faraday’s law of electromagnetic induction, Equation 22.3.

**SOLUTION** Expressing the energy consumed as Energy = \( Pt \), and substituting in \( P = \xi^2/R \), we find
Energy = $P_t = \frac{\xi^2 t}{R}$

The induced emf is given by Faraday’s law as $\xi = -N \frac{\Delta\Phi}{\Delta t}$, and the resistance $R$ is equal to the resistance per unit length ($3.3 \times 10^{-2} \, \Omega/m$) times the length of the circumference of the loop, $2\pi r$. Thus, the energy dissipated is

$$\text{Energy} = \frac{\left(-N \frac{\Delta\Phi}{\Delta t}\right)^2 t}{(3.3 \times 10^{-2} \, \Omega/m)2\pi r} = \frac{\left[-N \left(\frac{\Phi - \Phi_0}{t - t_0}\right)\right]^2 t}{(3.3 \times 10^{-2} \, \Omega/m)2\pi r}$$

$$= \frac{\left[-N \left(\frac{BA \cos \phi - B_0 \cos \phi}{t - t_0}\right)\right]^2 t}{(3.3 \times 10^{-2} \, \Omega/m)2\pi r} = \frac{\left[-NA \cos \phi \left(\frac{B - B_0}{t - t_0}\right)\right]^2 t}{(3.3 \times 10^{-2} \, \Omega/m)2\pi r}$$

$$= \frac{\left[\left(-1\right)\pi \left(0.12 \, m\right)^2 \left(\cos 0^\circ\right)\left(\frac{0.60 \, T - 0 \, T}{0.45 \, s - 0 \, s}\right)\right]^2 \left(0.45 \, s\right)}{(3.3 \times 10^{-2} \, \Omega/m)2\pi \left(0.12 \, m\right)} = 6.6 \times 10^{-2} \, J$$

---

80. **REASONING AND SOLUTION** If the rectangle is made $\Delta I = I_t$ wide, then the top of the rectangle will intersect the line at $LI_t$. The work is one-half the area of the rectangle, so

$$W = \frac{1}{2} (LI_t) I_t = \frac{1}{2} LI_t^2$$

---

81. **REASONING** According to Ohm’s law, the average current $I$ induced in the coil is given by $I = |\xi|/R$, where $|\xi|$ is the magnitude of the induced emf and $R$ is the resistance of the coil. To find the induced emf, we use Faraday’s law of electromagnetic induction

**SOLUTION** The magnitude of the induced emf can be found from Faraday’s law of electromagnetic induction and is given by Equation 22.3:

$$|\xi| = -N \frac{\Delta\Phi}{\Delta t} = -N \frac{\Delta \left(BA \cos 0^\circ\right)}{\Delta t} = NA \frac{\Delta B}{\Delta t}$$

We have used the fact that the field within a long solenoid is perpendicular to the cross-sectional area $A$ of the solenoid and makes an angle of $0^\circ$ with respect to the normal to the
area. The field is given by Equation 21.7 as $B = \mu_0 n I$, so the change $\Delta B$ in the field is $\Delta B = \mu_0 n \Delta I$, where $\Delta I$ is the change in the current. The induced current is, then,

$$I = \frac{N A \Delta B}{RT} = \frac{N \mu_0 n \Delta I}{RT}$$

$$= \frac{(10)(6.0 \times 10^{-4} \text{ m}^2)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(400 \text{ turns/m})(0.40 \text{ A})}{1.5 \Omega} = 1.6 \times 10^{-5} \text{ A}$$

82. **REASONING** The generator coil rotates in an external magnetic field $B$, inducing a varying emf with a peak value $\xi_0$ and a varying current $I$ with a peak value $I_0$. Once the current is flowing, the external magnetic field $B$ exerts a torque $\tau$ on the coil, according to

$$\tau = NIAB \sin \phi$$

In Equation 21.4, $N$ is the number of turns in the generator coil, $A$ is its cross-sectional area, and $\phi$ is the angle between the external magnetic field and the normal to the plane of the coil. Following Lenz’s law, this torque opposes the rotation that induces the emf and the current, hence it is labeled a countertorque. The induced emf is given by

$$\xi = NAB\omega \sin \alpha t = \xi_0 \sin \alpha t$$

where $\omega$ is the angular frequency of the rotation of the coil.

In terms of the peak current $I_0$ and peak voltage $V_0$ across the light bulb, the average power $\bar{P}$ that the generator delivers to the light bulb is $\bar{P} = \frac{1}{2} I_0 V_0$ (Equation 20.10). The peak voltage $V_0$ across the bulb is equal to the peak emf $\xi_0$ of the generator, and we have that

$$\bar{P} = \frac{1}{2} I_0 V_0 = \frac{1}{2} I_0 \xi_0$$

**SOLUTION** The peak value $\xi_0$ of the induced emf occurs when $\sin \alpha t = 1$ in Equation 22.4, which is the instant when the plane of the coil is parallel to the external magnetic field. At this instant, $\phi = 90^\circ$ in Equation 21.4. Because the peak value $I_0$ of the current occurs at the same instant as the peak value $\xi_0 = NAB\omega$ of the emf, the maximum value $\tau_0$ of the countertorque is

$$\tau_0 = NI_0 AB \sin 90^\circ = NI_0 AB$$

(1)
Solving Equation (20.10) for $I_0$, we obtain

$$I_0 = \frac{2 \bar{P}}{\varepsilon_0} \quad (2)$$

Substituting Equation (2) into Equation (1) yields

$$\tau_0 = NAB \left( \frac{2 \bar{P}}{\varepsilon_0} \right) = 2 \bar{P} \frac{NAB}{\varepsilon_0} \quad (3)$$

Substituting $\varepsilon_0 = NAB \omega$ from Equation 22.4 into Equation (3), we obtain

$$\tau_0 = \frac{2 \bar{P} NAB}{NAB \omega} = \frac{2 \bar{P}}{\omega} \quad (4)$$

We are given the frequency $f = 60.0 \text{ Hz}$, which is related to the angular frequency by $\omega = 2\pi f$ (Equation 10.6). Making this substitution into Equation (4) gives the maximum countertorque:

$$\tau_0 = \frac{2 \bar{P}}{\omega} = \frac{\bar{P}}{2 \pi f} = \frac{\bar{P}}{\pi f (60.0 \text{ Hz})} = \frac{75 \text{ W}}{0.40 \text{ N} \cdot \text{m}} = 0.40 \text{ N} \cdot \text{m}$$
1. (d) According to $P = V_{\text{rms}}^2 / R$ (Equation 20.15c), the average power is proportional to the square of the rms voltage. Tripling the voltage causes the power to increase by a factor of $3^2 = 9$.

2. $I_{\text{rms}} = 1.9$ A

3. (b) The current $I_{\text{rms}}$ through a capacitor depends inversely on the capacitive reactance $X_C$, as expressed by the relation $I_{\text{rms}} = V_{\text{rms}} / X_C$ (Equation 23.1). The capacitive reactance becomes infinitely large as the frequency goes to zero (see Equation 23.2), so the current goes to zero.

4. (e) According to $X_C = 1/(2\pi f C)$ (Equation 23.2) and $X_L = 2\pi f L$ (Equation 23.4), doubling the frequency $f$ causes $X_C$ to decrease by a factor of 2 and $X_L$ to increase by a factor of 2.

5. $I_{\text{rms}} = 1.3$ A

6. (a) The component of the phasor along the vertical axis is $V_0 \sin 2\pi ft$ (see the drawing that accompanies this problem), which is the instantaneous value of the voltage.

7. (b) The instantaneous value of the voltage is the component of the phasor that lies along the vertical axis (see Sections 23.1 and 23.2). This vertical component is greatest in B and least in A, so the ranking is (largest to smallest) B, C, A.

8. (d) In a resistor the voltage and current are in phase. This means that the two phasors are colinear.

9. (c) Power is dissipated by the resistor, as discussed in Section 20.5. On the other hand, the average power dissipated by a capacitor is zero (see Section 23.1).

10. $I_{\text{rms}} = 2.00$ A

11. (a) When the rms voltage across the inductor is greater than that across the capacitor, the voltage across the RCL combination leads the current (see Section 23.3).
12. (d) Since \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) (Equation 23.6), the current is a maximum when the impedance \( Z \) is a minimum. The impedance is 
\[ Z = \sqrt{R^2 + \left( X_L - X_C \right)^2} \] (Equation 23.7), and it has a minimum value when \( X_C = X_L = 50 \, \Omega \).

13. (c) The inductor has a very small reactance at low frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, \( R_1 \) and \( R_2 \), connected in parallel. The inductor has a very large reactance at high frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. The circuit behaves as a single resistance \( R_2 \) connected across the generator. The situation at low frequency gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance \( R_2 \).

14. (a) The capacitor has a very small reactance at high frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, \( R_1 \) and \( R_2 \), connected in parallel. The capacitor has a very large reactance at low frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. Therefore, the circuit behaves as a single resistor \( R_1 \) connected across the generator. The situation at high frequencies gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance \( R_1 \).

15. (e) At low frequencies, the capacitor has a very large reactance. In the series circuit, this large reactance gives rise to a large impedance and, hence, a small current. The parallel circuit has the larger current, because current can flow through the inductor, which has a small reactance at low frequencies.

16. (b) The resonant frequency \( f_0 \) is given by 
\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \] (Equation 23.10). It depends only on \( C \) and \( L \), and not on \( R \).

17. \( f_0 = 1.3 \times 10^3 \) Hz

18. (d) The resonant frequency \( f_0 \) is given by 
\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \] (Equation 23.10). When a second capacitor is added in parallel, the equivalent capacitance increases (see Section 20.12). Therefore, the resonant frequency decreases.
1. **REASONING** As the frequency \( f \) of the generator increases, the capacitive reactance \( X_C \) of the capacitor decreases, according to \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( C \) is the capacitance of the capacitor. The decreasing capacitive reactance leads to an increasing rms current \( I_{\text{rms}} \), as we see from \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} \) (Equation 23.1), where \( V_{\text{rms}} \) is the constant rms voltage across the capacitor. We know that \( V_{\text{rms}} \) is constant, because it is equal to the constant rms generator voltage. The fuse is connected in series with the capacitor, so both have the same current \( I_{\text{rms}} \). We will use Equations 23.1 and 23.2 to determine the frequency \( f \) at which the rms current is 15.0 A.

**SOLUTION** Solving Equation 23.2 for \( f \), we obtain

\[
  f = \frac{1}{2\pi C X_C}
\]

In terms of the rms current and voltage, Equation 23.1 gives the capacitive reactance as \( X_C = \frac{V_{\text{rms}}}{I_{\text{rms}}} \). Substituting this relation into Equation (1) yields

\[
  f = \frac{1}{2\pi C} \left( \frac{V_{\text{rms}}}{I_{\text{rms}}} \right) = \frac{I_{\text{rms}}}{2\pi C V_{\text{rms}}} = \frac{15.0 \text{ A}}{2\pi \left( 63.0 \times 10^{-6} \text{ F} \right) \left( 4.00 \text{ V} \right)} = 9470 \text{ Hz}
\]

2. **REASONING** When two capacitors of capacitance \( C \) are connected in parallel, their equivalent capacitance \( C_p \) is given by \( C_p = C + C = 2C \) (Equation 20.18). We will find the capacitive reactance \( X_C \) of the equivalent capacitance from \( X_C = \frac{1}{2\pi f C_p} \) (Equation 23.2).

Because the capacitors are the only devices connected to the generator, the rms voltage \( V_{\text{rms}} \) across the capacitors is equal to the rms voltage of the generator. Therefore, the capacitive reactance \( X_C \) is related to the rms voltage of the generator and the rms current \( I_{\text{rms}} \) in the circuit by \( V_{\text{rms}} = I_{\text{rms}} X_C \) (Equation 23.1).
**SOLUTION** Substituting \( C_p = 2C \) into \( X_C = \frac{1}{2\pi f C_p} \) (Equation 23.2) and solving for \( C \), we obtain

\[
X_C = \frac{1}{2\pi f C_p} = \frac{1}{2\pi f (2C)} = \frac{1}{4\pi f C} \quad \text{or} \quad C = \frac{1}{4\pi f X_C} \quad (1)
\]

Solving \( V_{\text{rms}} = I_{\text{rms}} X_C \) (Equation 23.1) for \( X_C \) yields \( X_C = \frac{V_{\text{rms}}}{I_{\text{rms}}} \). Substituting this result into Equation (1), we obtain

\[
C = \frac{1}{4\pi f X_C} = \frac{1}{4\pi f} \left( \frac{V_{\text{rms}}}{I_{\text{rms}}} \right) = \frac{0.16 \text{ A}}{4\pi (610 \text{ Hz}) (24 \text{ V})} = 8.7 \times 10^{-7} \text{ F}
\]

3. **REASONING** The reactance \( X_C \) of a capacitor is given as \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( f \) is the frequency in hertz (Hz) and \( C \) is the capacitance of the capacitor. We note that \( X_C \) is inversely proportional to \( f \) for a given value of \( C \). Therefore, we will be able to solve this problem by applying Equation 23.2 twice, once for each value of the frequency and each time with the same value of the capacitance.

**SOLUTION** Applying Equation 23.2 for each value of the frequency, we obtain

\[
X_{C, 870} = \frac{1}{2\pi f_{870} C} \quad \text{and} \quad X_{C, 460} = \frac{1}{2\pi f_{460} C}
\]

Dividing the equation on the left by the equation on the right and noting that the unknown capacitance \( C \) is eliminated algebraically, we find that

\[
\frac{X_{C, 870}}{X_{C, 460}} = \frac{2\pi f_{870} C}{1} = \frac{f_{460}}{f_{870}}
\]

Solving for the reactance at a frequency of 870 Hz gives

\[
X_{C, 870} = X_{C, 460} \frac{f_{460}}{f_{870}} = (68 \Omega) \left( \frac{460 \text{ Hz}}{870 \text{ Hz}} \right) = 36 \Omega
\]

4. **REASONING** The rms voltage \( V_{\text{rms}} \) and current \( I_{\text{rms}} \) in a capacitor are related according to \( V_{\text{rms}} = I_{\text{rms}} X_C \) (Equation 23.1). \( X_C \) is the capacitive reactance \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( f \) is the frequency in hertz (Hz) and \( C \) is the capacitance of the capacitor. We will apply these equations to the capacitor when connected to each of the
SOLUTION Substituting Equation 23.2 for the capacitive reactance into Equation 23.1, we obtain

\[ V_{\text{rms}} = I_{\text{rms}} X_C = I_{\text{rms}} \left( \frac{1}{2\pi f C} \right) \]  

(1)

Applying Equation (1) to the capacitor when connected to generator 1 and generator 2, we have

\[ \left( V_{\text{rms}} \right)_1 = \left( I_{\text{rms}} \right)_1 \left( \frac{1}{2\pi f_1 C} \right) \quad \text{and} \quad \left( V_{\text{rms}} \right)_2 = \left( I_{\text{rms}} \right)_2 \left( \frac{1}{2\pi f_2 C} \right) \]

Dividing the equation on the right by the equation on the left, we note that the capacitance \( C \) is eliminated algebraically and find that

\[ \frac{\left( V_{\text{rms}} \right)_2}{\left( V_{\text{rms}} \right)_1} = \frac{\left( I_{\text{rms}} \right)_2 / \left( 2\pi f_2 C \right)}{\left( I_{\text{rms}} \right)_1 / \left( 2\pi f_1 C \right)} = \frac{\left( I_{\text{rms}} \right)_2 f_1}{\left( I_{\text{rms}} \right)_1 f_2} \]  

(2)

Solving Equation (2) for the voltage of the second generator gives

\[ \left( V_{\text{rms}} \right)_2 = \frac{\left( V_{\text{rms}} \right)_1 \left( I_{\text{rms}} \right)_2 f_1}{f_2 \left( I_{\text{rms}} \right)_1} = \frac{(2.0 \text{ V}) \left( 85 \times 10^{-3} \text{ A} \right) \left( 3.4 \times 10^3 \text{ Hz} \right)}{(5.0 \times 10^3 \text{ Hz}) \left( 35 \times 10^{-3} \text{ A} \right)} = 3.3 \text{ V} \]

5. **SSM REASONING** The rms current in a capacitor is \( I_{\text{rms}} = V_{\text{rms}} / X_C \), according to Equation 23.1. The capacitive reactance is \( X_C = 1 / (2\pi f C) \), according to Equation 23.2. For the first capacitor, we use \( C = C_1 \) in these expressions. For the two capacitors in parallel, we use \( C = C_p \), where \( C_p \) is the equivalent capacitance from Equation 20.18 \( (C_p = C_1 + C_2) \). Taking the difference between the currents and using the given data, we can obtain the desired value for \( C_2 \). The capacitance \( C_1 \) is unknown, but it will be eliminated algebraically from the calculation.

**SOLUTION** Using Equations 23.1 and 23.2, we find that the current in a capacitor is

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{1 / (2\pi f C)} = V_{\text{rms}} \left( 2\pi f C \right) \]

Applying this result to the first capacitor and the parallel combination of the two capacitors, we obtain

\[ I_1 = \frac{V_{\text{rms}}}{2\pi f C_1} \quad \text{and} \quad I_{\text{Combination}} = \frac{V_{\text{rms}}}{2\pi f (C_1 + C_2)} \]

Subtracting \( I_1 \) from \( I_{\text{Combination}} \) reveals that
\[ I_{\text{Combination}} - I_1 = V_{\text{rms}} 2\pi f (C_1 + C_2) - V_{\text{rms}} 2\pi f C_1 = V_{\text{rms}} 2\pi f C_2 \]

Solving for \( C_2 \) gives

\[ C_2 = \frac{I_{\text{Combination}} - I_1}{V_{\text{rms}} 2\pi f} = \frac{0.18 \text{ A}}{(24 \text{ V})2\pi(440 \text{ Hz})} = 2.7 \times 10^{-6} \text{ F} \]

6. **REASONING**  The rms current in the circuit is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} \) (Equation 23.1), where \( V_{\text{rms}} \) is the rms voltage of the generator, and \( X_C = 1/(2\pi f C) \) is the capacitive reactance (see Equation 23.2). For a given voltage, smaller reactances lead to greater currents.

When two capacitors are connected in parallel, the equivalent capacitance \( C_P \) is given by \( C_P = C_1 + C_2 \) (Equation 20.18), where \( C_1 \) and \( C_2 \) are the individual capacitances. Therefore, \( C_P \) is greater than either \( C_1 \) or \( C_2 \). Thus, when the capacitors are connected in parallel, the greater capacitance leads to a smaller reactance (\( C \) is in the denominator in Equation 23.2), which in turn leads to a greater current. As a result, the current delivered by the generator increases when the second capacitor is connected in parallel with the first capacitor.

The capacitance of a parallel plate capacitor is given by \( C = \kappa \varepsilon_0 A/d \) (Equation 19.10), where \( \kappa \) is the dielectric constant of the material between the plates, \( \varepsilon_0 \) is the permittivity of free space, \( A \) is the area of each plate, and \( d \) is the separation between the plates. When the capacitor is empty, \( \kappa = 1 \), so that \( C = \kappa C_{\text{empty}} \). Thus, the capacitance increases when the dielectric material is inserted.

**SOLUTION** Using Equation 23.1 to express the current as \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} \) and Equation 23.2 to express the capacitive reactance as \( X_C = 1/(2\pi f C) \), we have for the current that

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{1/(2\pi f C)} = V_{\text{rms}} 2\pi f C \]

Applying this result to the case where the empty capacitor \( C_1 \) is connected alone to the generator and to the case where the “full” capacitor \( C_2 \) (which contains the dielectric material) is connected in parallel with \( C_1 \), we obtain

\[ I_{1, \text{rms}} = V_{\text{rms}} 2\pi f C_1 \quad \text{and} \quad I_{P, \text{rms}} = V_{\text{rms}} 2\pi f C_P \]

Dividing the two expressions gives

\[ \frac{I_{P, \text{rms}}}{I_{1, \text{rms}}} = \frac{V_{\text{rms}} 2\pi f C_P}{V_{\text{rms}} 2\pi f C_1} = \frac{C_P}{C_1} \]
According to Equation 20.18, the equivalent capacitance of the two capacitors in parallel is

\[ C_p = C_1 + C_2, \]

so that the result for the current ratio becomes

\[ \frac{I_{p,\text{rms}}}{I_{1,\text{rms}}} = \frac{C_1 + C_2}{C_1} = 1 + \frac{C_2}{C_1} \]

Since the capacitance of a filled capacitor is given by Equation 19.10 as

\[ C = \kappa \varepsilon_0 A/d, \]

we find

\[ \frac{I_{p,\text{rms}}}{I_{1,\text{rms}}} = 1 + \frac{\kappa \varepsilon_0 A/d}{\varepsilon_0 A/d} = 1 + \kappa \]

Solving for \( I_{p,\text{rms}} \) gives

\[ I_{p,\text{rms}} = I_{1,\text{rms}} (1 + \kappa) = (0.22 \text{ A})(1 + 4.2) = 1.1 \text{ A} \]

7. **REASONING** The capacitance \( C \) is related to the capacitive reactance \( X_C \) and the frequency \( f \) via Equation 23.2 as \( C = 1/(2\pi f X_C) \). The capacitive reactance, in turn is related to the rms-voltage \( V_{\text{rms}} \) and the rms-current \( I_{\text{rms}} \) by \( X_C = V_{\text{rms}}/I_{\text{rms}} \) (see Equation 23.1). Thus, the capacitance can be written as \( C = I_{\text{rms}}/(2\pi f V_{\text{rms}}) \). The magnitude of the maximum charge \( q \) on one plate of the capacitor is, from Equation 19.8, the product of the capacitance \( C \) and the peak voltage \( V \).

**SOLUTION**

a. Recall that the rms-voltage \( V_{\text{rms}} \) is related to the peak voltage \( V \) by \( V_{\text{rms}} = \frac{V}{\sqrt{2}} \). The capacitance is, then,

\[ C = \frac{I_{\text{rms}}}{2\pi f V_{\text{rms}}} = \frac{3.0 \text{ A}}{2\pi (750 \text{ Hz}) \left( \frac{140 \text{ V}}{\sqrt{2}} \right)} = 6.4 \times 10^{-6} \text{ F} \]

b. The maximum charge on one plate of the capacitor is

\[ q = CV = (6.4 \times 10^{-6} \text{ F})(140 \text{ V}) = 9.0 \times 10^{-4} \text{ C} \]

8. **REASONING AND SOLUTION** Equations 23.1 and 23.2 indicate that the rms current in a capacitor is \( I = V/X_c \), where \( V \) is the rms voltage and \( X_c = 1/(2\pi f C) \). Therefore, the current is \( I = V 2\pi f C \). For a single capacitor \( C = C_1 \), and we have

\[ I = V 2\pi f C_1 \]

For two capacitors in series, Equation 20.19 indicates that the equivalent capacitance can be
obtained from \( C^{-1} = C_1^{-1} + C_2^{-1} \), which can be solved to show that \( C = C_1 C_2 / (C_1 + C_2) \). The total series current is, then,

\[
I_{\text{series}} = V 2 \pi f C = V 2 \pi f \left( \frac{C_1 C_2}{C_1 + C_2} \right)
\]

The series current is one-third of the current \( I \). It follows, therefore, that

\[
I_{\text{series}} = \frac{V 2 \pi f C}{3} = \frac{V 2 \pi f C_1}{3} = \frac{C_2}{C_1 + C_2} = \frac{1}{3} \quad \text{or} \quad \frac{C_1}{C_2} = 2
\]

For two capacitors in parallel, Equation 20.18 indicates that the equivalent capacitance is \( C = C_1 + C_2 \). The total current in this case is

\[
I_{\text{parallel}} = V 2 \pi f C = V 2 \pi f (C_1 + C_2)
\]

The ratio of \( I_{\text{parallel}} \) to the current \( I \) in the single capacitor is

\[
\frac{I_{\text{parallel}}}{I} = \frac{V 2 \pi f (C_1 + C_2)}{V 2 \pi f C_1} = \frac{C_1 + C_2}{C_1} = 1 + \frac{C_2}{C_1} = \frac{3}{2}
\]

9. **REASONING** The rms current can be calculated from Equation 23.3, \( I_{\text{rms}} = V_{\text{rms}} / X_L \), provided that the inductive reactance is obtained first. Then the peak value of the current \( I_0 \) supplied by the generator can be calculated from the rms current \( I_{\text{rms}} \) by using Equation 20.12, \( I_0 = \sqrt{2} I_{\text{rms}} \).

**SOLUTION** At the frequency of \( f = 620 \text{ Hz} \), we find, using Equations 23.4 and 23.3, that

\[
X_L = 2 \pi f L = 2 \pi (620 \text{ Hz})(8.2 \times 10^{-3} \text{ H}) = 32 \Omega
\]

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{10.0 \text{ V}}{32 \Omega} = 0.31 \text{ A}
\]

Therefore, from Equation 20.12, we find that the peak value \( I_0 \) of the current supplied by the generator must be

\[
I_0 = \sqrt{2} I_{\text{rms}} = \sqrt{2} (0.31 \text{ A}) = 0.44 \text{ A}
\]

10. **REASONING** The rms voltage \( V_{\text{rms}} \) and current \( I_{\text{rms}} \) in an inductor are related according to \( V_{\text{rms}} = I_{\text{rms}} X_L \) (Equation 23.3). \( X_L \) is the capacitive reactance \( X_L = 2 \pi f L \) (Equation 23.4), where \( f \) is the frequency in hertz (Hz) and \( L \) is the inductance of the inductor. Since we have values for \( V_{\text{rms}} \), \( f \), and \( L \), we can use these equations to calculate the unknown current \( I_{\text{rms}} \).
**SOLUTION** Substituting Equation 23.4 for the inductive reactance into Equation 23.3, we obtain

\[ V_{\text{rms}} = I_{\text{rms}} X_L = I_{\text{rms}} (2\pi f L) \]  

(1)

Solving Equation (1) for \( I_{\text{rms}} \), we find that

\[ V_{\text{rms}} = I_{\text{rms}} (2\pi f L) \quad \text{or} \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{2\pi f L} = \frac{55 \text{ V}}{2\pi (650 \text{ Hz})(0.080 \text{ H})} = 0.17 \text{ A} \]

11. **REASONING** The rms voltage \( V_{\text{rms}} \) across the inductor is given by \( V_{\text{rms}} = I_{\text{rms}} X_L \) (Equation 23.3), where \( I_{\text{rms}} \) is the rms current in the circuit, and \( X_L \) is the inductive reactance. The inductor is the only circuit element connected to the generator, so the rms voltage across the inductor is equal to the rms generator voltage: \( V_{\text{rms}} = 15.0 \text{ V} \).

**SOLUTION** Solving Equation 23.3 for \( X_L \), we obtain

\[ X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{15.0 \text{ V}}{0.610 \text{ A}} = 24.6 \text{ \Omega} \]

12. **REASONING** The inductance \( L \) of the inductor determines its inductive reactance \( X_L \) according to \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the frequency of the generator. When the inductor is connected to the terminals of the generator, the rms voltage \( V_{\text{rms}} \) of the generator drives an rms current \( I_{\text{rms}} \) that depends upon the inductive reactance via \( X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 23.3). We note that the generator frequency is given in kHz, where 1 kHz = \( 10^3 \) Hz, and the rms current is given in mA, where 1 mA = \( 10^{-3} \) A.

**SOLUTION** Solving \( X_L = 2\pi f L \) (Equation 23.4) for \( L \) yields

\[ L = \frac{X_L}{2\pi f} \]  

(2)

Substituting \( X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 23.3) into Equation (1), we find that

\[ L = \frac{X_L}{2\pi f} = \frac{\left( \frac{V_{\text{rms}}}{I_{\text{rms}}} \right)}{2\pi f} = \frac{V_{\text{rms}}}{2\pi f I_{\text{rms}}} = \frac{39 \text{ V}}{2\pi (7.5 \times 10^3 \text{ Hz})(42 \times 10^{-3} \text{ A})} = 0.020 \text{ H} \]
13. **REASONING** Since the capacitor and the inductor are connected in parallel, the voltage across each of these elements is the same or \( V_L = V_C \). Using Equations 23.3 and 23.1, respectively, this becomes \( I_{rms} L = I_{rms} C \). Since the currents in the inductor and capacitor are equal, this relation simplifies to \( X_L = X_C \). Therefore, we can find the value of the inductance by equating the expressions (Equations 23.4 and 23.2) for the inductive reactance and the capacitive reactance, and solving for \( L \).

**SOLUTION** Since \( X_L = X_C \), we have

\[
2\pi f L = \frac{1}{2\pi f C}
\]

Therefore, the value of the inductance is

\[
L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60.0 \text{ Hz})^2 (40.0 \times 10^{-6} \text{ F})} = 0.176 \text{ H} = 176 \text{ mH}
\]

14. **REASONING** The current in \( L_1 \) is given by Equation 23.3 as \( I_{rms} = V_{rms}/X_L \), where \( X_L = 2\pi f L_1 \) (Equation 23.4) is the inductive reactance of \( L_1 \). This current does not depend in any way on \( L_2 \) and exists whether or not \( L_2 \) is present.

The current delivered to the parallel combination is the sum of the currents delivered to each inductance and is, therefore, greater than either individual current. The current in \( L_2 \) is given by \( I_{rms} = V_{rms}/X_L \) (Equation 23.3), where \( X_L = 2\pi f L_2 \) is the inductive reactance of \( L_2 \) according to Equation 23.4. This current does not depend in any way on \( L_1 \) and exists whether or not \( L_1 \) is present.

**SOLUTION** Using Equation 23.3 to express the current as \( I_{rms} = V_{rms}/X_L \) and Equation 23.4 to express the inductive reactance as \( X_L = 2\pi f L \), we have for the current that

\[
I_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L}
\]

Applying this result to the case where \( L_1 \) or \( L_2 \) is connected alone to the generator, we obtain

\[
I_{1, \text{ rms}} = V_{rms} \frac{L_1}{X_L} = \frac{V_{rms}}{2\pi f L_1} \quad \text{and} \quad I_{2, \text{ rms}} = V_{rms} \frac{L_2}{X_L} = \frac{V_{rms}}{2\pi f L_2}
\]

The current delivered to \( L_1 \) alone is

\[
I_{1, \text{ rms}} = \frac{V_{rms}}{2\pi f L_1} = \frac{240 \text{ V}}{2\pi (2200 \text{ Hz})(6.0 \times 10^{-3} \text{ H})} = 2.9 \text{ A}
\]
The current delivered to the parallel combination of $L_1$ and $L_2$ is the sum of that delivered individually to each inductor and is

\[ I_{P,\text{rms}} = I_{1, \text{rms}} + I_{2, \text{rms}} = \frac{V_{\text{rms}}}{2\pi f L_1} + \frac{V_{\text{rms}}}{2\pi f L_2} = \frac{V_{\text{rms}}}{2\pi f} \left( \frac{1}{L_1} + \frac{1}{L_2} \right) \]

\[ = \frac{240 \text{ V}}{2\pi (2200 \text{ Hz})} \left( \frac{1}{6.0 \times 10^{-3} \text{ H}} + \frac{1}{9.0 \times 10^{-3} \text{ H}} \right) = 4.8 \text{ A} \]

15. **SSM REASONING**

a. The inductive reactance $X_L$ depends on the frequency $f$ of the current and the inductance $L$ through the relation $X_L = 2\pi f L$ (Equation 23.4). This equation can be used directly to find the frequency of the current.

b. The capacitive reactance $X_C$ depends on the frequency $f$ of the current and the capacitance $C$ through the relation $X_C = 1/(2\pi f C)$ (Equation 23.2). By setting $X_C = X_L$ as specified in the problem statement, the capacitance can be found.

c. Since the inductive reactance is directly proportional to the frequency, tripling the frequency triples the inductive reactance.

d. The capacitive reactance is inversely proportional to the frequency, so tripling the frequency reduces the capacitive reactance by a factor of one-third.

**SOLUTION**

a. The frequency of the current is

\[ f = \frac{X_L}{2\pi L} = \frac{2.10 \times 10^3 \Omega}{2\pi \left(30.0 \times 10^{-3} \text{ H} \right)} = 1.11 \times 10^4 \text{ Hz} \]  

(23.4)

b. The capacitance is

\[ C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi (1.11 \times 10^4 \text{ Hz}) (2.10 \times 10^3 \Omega)} = 6.83 \times 10^{-9} \text{ F} \]

(23.2)

c. Since $X_L = 2\pi f L$, tripling the frequency $f$ causes $X_L$ to also triple:

\[ X_L = 3 \left(2.10 \times 10^3 \Omega \right) = 6.30 \times 10^3 \Omega \]

d. Since $X_C = 1/(2\pi f C)$, tripling the frequency $f$ causes $X_C$ to decrease by a factor of 3:

\[ X_C = \frac{1}{3} \left(2.10 \times 10^3 \Omega \right) = 7.00 \times 10^2 \Omega \]
16. **REASONING AND SOLUTION** Equations 23.3 and 23.4 indicate that the rms current in a single inductance \( L_1 \) is \( I_1 = V / X_{L1} \), where \( V \) is the rms voltage and \( X_{L1} = 2\pi f L_1 \). Therefore, the current is \( I_1 = V / \left(2\pi f L_1\right) \). Similarly, the current in the second inductor connected across the terminals of the generator is \( I_2 = V / \left(2\pi f L_2\right) \). The total current delivered by the generator is the sum of these two values:

\[
I_{\text{total}} = I_1 + I_2 = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2}
\]

But this same total current is delivered to the single inductance \( L \), so it also follows that \( I_{\text{total}} = V / \left(2\pi f L\right) \). Equating the two expressions for \( I_{\text{total}} \) shows that

\[
\frac{V}{2\pi f L} = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}
\]

Using this result, we determine the value of \( L \) as follows:

\[
\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L = \frac{L_1 L_2}{L_1 + L_2} = \frac{(0.030 \text{ H})(0.060 \text{ H})}{0.030 \text{ H} + 0.060 \text{ H}} = 0.020 \text{ H}
\]

17. **SSM REASONING** The voltage supplied by the generator can be found from Equation 23.6, \( V_{\text{rms}} = I_{\text{rms}} Z \). The value of \( I_{\text{rms}} \) is given in the problem statement, so we must obtain the impedance of the circuit.

**SOLUTION** The impedance of the circuit is, according to Equation 23.7,

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(275 \Omega)^2 + (648 \Omega - 415 \Omega)^2} = 3.60 \times 10^2 \Omega
\]

The rms voltage of the generator is

\[
V_{\text{rms}} = I_{\text{rms}} Z = (0.233 \text{ A})(3.60 \times 10^2 \Omega) = 83.9 \text{ V}
\]

18. **REASONING** As discussed in Section 23.3, the power factor is

\[
\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}},
\]

where \( R \) is the resistance, \( X_L \) is the inductive reactance, and \( X_C \) is the capacitive reactance of the circuit. The inductive reactance is given by \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the frequency in hertz (Hz) and \( L \) is the inductance. The capacitive reactance is given by \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( C \) is the capacitance.
SOLUTION Using Equation 23.4 and Equation 23.2 to substitute for the reactances, we find that the power factor is

$$\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

$$= \frac{47.0 \, \Omega}{\sqrt{(47.0 \, \Omega)^2 + \left[2\pi (2550 \, \text{Hz}) (4.00 \times 10^{-3} \, \text{H}) - \frac{1}{2\pi (2550 \, \text{Hz}) (2.00 \times 10^{-6} \, \text{F})}\right]^2}}$$

$$= 0.819$$

19. SSM REASONING We can use the equations for a series RCL circuit to solve this problem provided that we set $X_C = 0 \, \Omega$ since there is no capacitor in the circuit. The current in the circuit can be found from Equation 23.6, $V_{\text{rms}} = I_{\text{rms}}Z$, once the impedance of the circuit has been obtained. Equation 23.8, $\tan \phi = (X_L - X_C)/R$, can then be used (with $X_C = 0 \, \Omega$) to find the phase angle between the current and the voltage.

SOLUTION The inductive reactance is (Equation 23.4)

$$X_L = 2\pi f L = 2\pi (106 \, \text{Hz}) (0.200 \, \text{H}) = 133 \, \Omega$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_L^2} = \sqrt{(215 \, \Omega)^2 + (133 \, \Omega)^2} = 253 \, \Omega$$

a. The current through each circuit element is, using Equation 23.6,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{234 \, \text{V}}{253 \, \Omega} = 0.925 \, \text{A}$$

b. The phase angle between the current and the voltage is, according to Equation 23.8 (with $X_C = 0 \, \Omega$),

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{X_L}{R} = \frac{133 \, \Omega}{215 \, \Omega} = 0.619 \quad \text{or} \quad \phi = \tan^{-1}(0.619) = 31.8^\circ$$
20. **REASONING** The phase angle is given by \( \tan \phi = \frac{(X_L - X_C)}{R} \) (Equation 23.8). When a series circuit contains only a resistor and a capacitor, the inductive reactance \( X_L \) is zero, and the phase angle is negative, signifying that the current leads the voltage of the generator. The impedance is given by Equation 23.7 with \( X_L = 0 \Omega \), or \( Z = \sqrt{R^2 + X_C^2} \). Since the phase angle \( \phi \) and the impedance \( Z \) are given, we can use these relations to find the resistance \( R \) and \( X_C \).

**SOLUTION** Since the phase angle is negative, we can conclude that only a resistor and a capacitor are present. Using Equations 23.8, then, we have

\[
\tan \phi = \frac{-X_C}{R} \quad \text{or} \quad X_C = -R \tan(-75.0^\circ) = 3.73R
\]

According to Equation 23.7, the impedance is

\[
Z = 192 \Omega = \sqrt{R^2 + X_C^2}
\]

Substituting \( X_C = 3.73R \) into this expression for \( Z \) gives

\[
192 \Omega = \sqrt{R^2 + (3.73R)^2} = \sqrt{14.9 R^2} \quad \text{or} \quad (192 \Omega)^2 = 14.9 R^2
\]

\[
R = \sqrt{\frac{(192 \Omega)^2}{14.9}} = 49.7 \Omega
\]

Using the fact that \( X_C = 3.73R \), we obtain

\[
X_C = 3.73(49.7 \Omega) = 185 \Omega
\]

21. **REASONING** For a series RCL circuit the total impedance \( Z \) and the phase angle \( \phi \) are given by

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (23.7) \\
\tan \phi = \frac{X_L - X_C}{R} \quad (23.8)
\]

where \( R \) is the resistance, \( X_L \) is the inductive reactance, and \( X_C \) is the capacitive reactance. In the present case, there is no capacitance, so that \( X_C = 0 \Omega \). Therefore, these equations simplify to the following:

\[
Z = \sqrt{R^2 + X_L^2} \quad (1) \\
\tan \phi = \frac{X_L}{R} \quad (2)
\]

We are given neither \( R \) nor \( X_L \). However, we do know the current and voltage when only the resistor is connected and can determine \( R \) from these values using \( R = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 20.14). In addition, we know the current and voltage when only the inductor is
connected and can determine \( X_L \) from these values using \( X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 23.3). Since the generator frequency is fixed, this value for \( X_L \) also applies for the series combination of the resistor and the inductor.

**SOLUTION** With only the resistor connected, Equation 20.14 indicates that the resistance is

\[
R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112 \text{ V}}{0.500 \text{ A}} = 224 \Omega
\]

With only the inductor connected, Equation 23.3 indicates that the inductive reactance is

\[
X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112 \text{ V}}{0.400 \text{ A}} = 2.80 \times 10^2 \Omega
\]

a. Using these values for \( R \) and \( X_L \) in Equations (1) and (2), we find that the impedance is

\[
Z = \sqrt{R^2 + X_L^2} = \sqrt{(224 \Omega)^2 + (2.80 \times 10^2 \Omega)^2} = 359 \Omega
\]

b. The phase angle \( \phi \) between the current and the voltage of the generator is

\[
\tan \phi = \frac{X_L}{R} \quad \text{or} \quad \phi = \tan^{-1} \left( \frac{X_L}{R} \right) = \tan^{-1} \left( \frac{2.80 \times 10^2 \Omega}{224 \Omega} \right) = 51.3^\circ
\]

22. **REASONING** Since, on the average, only the resistor consumes power, the average power dissipated in the circuit is \( \bar{P} = I_{\text{rms}}^2 R \) (Equation 20.15b), where \( I_{\text{rms}} \) is the rms current and \( R \) is the resistance. The current is related to the voltage \( V \) of the generator and the impedance \( Z \) of the circuit by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) (Equation 23.6). Thus, the average power can be written as \( \bar{P} = \frac{V^2 R}{Z^2} \). The impedance of the circuit is (see Equation 23.7) \( Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_C^2} \), since there is no inductor in the circuit. Therefore, the expression for the average power becomes

\[
\bar{P} = \frac{V^2 R}{Z^2} = \frac{V^2 R}{R^2 + X_C^2}
\]

**SOLUTION** The capacitive reactance is

\[
X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (60.0 \text{ Hz}) (1.1 \times 10^{-6} \text{ F})} = 2400 \Omega \quad \text{(23.2)}
\]

The average power dissipated is
23. **REASONING** From Figure 23.11 we see that $V_0^2 = (V_L - V_C)^2 + V_R^2$. Since $V_L = 0$ V ($L = 0$ H), and we know $V_0$ and $V_R$, we can use this equation to find $V_C$.

**SOLUTION** Solving the equation above for $V_C$ gives

$$V_C = \sqrt{V_0^2 - V_R^2} = \sqrt{(45 \text{ V})^2 - (24 \text{ V})^2} = 38 \text{ V}$$

24. **REASONING** The rms current $I_{\text{rms}}$ in the circuit in part c of the drawing is equal to the rms voltage $V_{\text{rms}}$ divided by the impedance $Z$ or $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$ (Equation 23.6). The impedance $Z$ of a series RC circuit is given by Equation 23.7 with $X_L = 0$ Ω, since there is no inductance in the circuit; $Z = \sqrt{R^2 + X_C^2}$. The resistance $R$ is known and the capacitive reactance $X_C$ can be obtained from the relation $X_C = 1/(2\pi f C)$ (Equation 23.2), where $C$ is the capacitance and $f$ is the frequency. The capacitance, however, is related to the time constant $\tau$ of the circuit in part a of the drawing. The time constant of an RC circuit is the time for the capacitor to lose 63.2% of its initial charge (see the discussion in Section 20.13), and it is equal to the product of the resistance $R$ and the capacitance $C$; $\tau = RC$ (Equation 20.21).

**SOLUTION** Substituting $Z = \sqrt{R^2 + X_C^2}$ into $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$ gives

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}}$$

(1)

Substituting $X_C = 1/(2\pi f C)$ (Equation 23.2) into Equation (1) allows us to write the rms current in the circuit in part c of the drawing as follows:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$$

Substituting $C = \tau/R$ (Equation 20.21) into Equation (1), we arrive at an expression for the rms current:
25. **SSM REASONING** We can use the equations for a series RCL circuit to solve this problem, provided that we set $X_L = 0$ since there is no inductance in the circuit. Thus, according to Equations 23.6 and 23.7, the current in the circuit is $I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$. When the frequency $f$ is very large, the capacitive reactance is zero, or $X_C = 0$, in which case the current becomes $I_{\text{rms}}(\text{large } f) = \frac{V_{\text{rms}}}{R}$. When the current $I_{\text{rms}}$ in the circuit is one-half the value of $I_{\text{rms}}(\text{large } f)$ that exists when the frequency is very large, we have

$$\frac{I_{\text{rms}}}{I_{\text{rms}}(\text{large } f)} = \frac{1}{2}$$

We can use these expressions to write the ratio above in terms of the resistance and the capacitive reactance. Once the capacitive reactance is known, the frequency can be determined.

**SOLUTION** The ratio of the currents is

$$\frac{I_{\text{rms}}}{I_{\text{rms}}(\text{large } f)} = \frac{\frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}}}{\frac{V_{\text{rms}}}{R}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{1}{2} \quad \text{or} \quad \frac{R^2}{R^2 + X_C^2} = \frac{1}{4}$$

Taking the reciprocal of this result gives

$$\frac{R^2 + X_C^2}{R^2} = 4 \quad \text{or} \quad 1 + \frac{X_C^2}{R^2} = 4$$

Therefore,

$$\frac{X_C}{R} = \sqrt{3}$$

According to Equation 23.2, $X_C = \frac{1}{2\pi f C}$, so it follows that

$$\frac{X_C}{R} = \frac{1/(2\pi f C)}{R} = \sqrt{3}$$

Thus, we find that
\[
f = \frac{1}{2\pi RC\sqrt{3}} = \frac{1}{2\pi(85\Omega)(4.0 \times 10^{-6} \text{ F})\sqrt{3}} = 270 \text{ Hz}
\]

26. **REASONING** For a series RCL circuit the total impedance \(Z\) and the phase angle \(\phi\) are given by

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{(23.7)}
\]

\[
\tan \phi = \frac{X_L - X_C}{R} \quad \text{(23.8)}
\]

where \(R\) is the resistance, \(X_L\) is the inductive reactance, and \(X_C\) is the capacitive reactance. In the present case, there is no inductance, so \(X_L = 0\ \Omega\). Therefore, these equations simplify to the following:

\[
Z = \sqrt{R^2 + X_C^2} \quad \text{(1)}
\]

\[
\tan \phi = -\frac{X_C}{R} \quad \text{(2)}
\]

Values are given for \(Z\) and \(\phi\), so we can solve for \(R\) and \(X_C\).

**SOLUTION** Using the value given for the phase angle in Equation (2), we find that

\[
\tan \phi = \tan (-9.80^\circ) = -0.173 = -\frac{X_C}{R} \quad \text{or} \quad X_C = 0.173R \quad \text{(3)}
\]

Substituting this result for \(X_C\) into Equation (1) gives

\[
Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (0.173 R)^2} = \left[\sqrt{1 + (0.173)^2}\right]R
\]

Solving for \(R\) reveals that

\[
R = \frac{Z}{\sqrt{1 + (0.173)^2}} = \frac{4.50 \times 10^2 \ \Omega}{\sqrt{1 + (0.173)^2}} = 443 \ \Omega
\]

Using this value for \(R\) in Equation (3), we find that

\[
X_C = 0.173R = 0.173(443 \ \Omega) = 76.6 \ \Omega
\]

27. **REASONING** The instantaneous value of the generator voltage is \(V(t) = V_0 \sin 2\pi ft\), where \(V_0\) is the peak voltage and \(f\) is the frequency. We will see that the inductive reactance is greater than the capacitive reactance, \(X_L > X_C\), so that the current in the circuit lags the voltage by \(\pi/2\) radians, or 90°. Thus, the current in the circuit obeys the relation \(I(t) = I_0 \sin (2\pi ft - \pi/2)\), where \(I_0\) is the peak current.

**SOLUTION**

a. The instantaneous value of the voltage at a time of \(1.20 \times 10^{-4}\) s is
\[ V(t) = V_0 \sin 2\pi ft = (32.0 \text{ V}) \sin \left[ 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 1.20 \times 10^{-4} \text{ s} \right) \right] = 29.0 \text{ V} \]

Note: When evaluating the sine function in the expression above, be sure to set your calculator to the \textit{radian} mode.

b. The inductive and capacitive reactances are

\[ X_L = 2\pi fL = 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 7.20 \times 10^{-3} \text{ H} \right) = 67.9 \text{ } \Omega \]  \hspace{1cm} (23.4)

\[ X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 6.60 \times 10^{-6} \text{ F} \right)} = 16.1 \text{ } \Omega \]  \hspace{1cm} (23.2)

Since \( X_L \) is greater than \( X_C \), the current lags the voltage by \( \pi/2 \) radians. Thus, the instantaneous current in the circuit is

\[ I(t) = I_0 \sin (2\pi ft - \pi/2) \]

where \( I_0 = V_0/Z \). The impedance \( Z \) of the circuit is

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(0 \text{ } \Omega)^2 + (67.9 \text{ } \Omega - 16.1 \text{ } \Omega)^2} = 51.8 \text{ } \Omega \]  \hspace{1cm} (23.7)

The instantaneous current is

\[ I = \frac{V_0}{Z} \sin \left( 2\pi ft - \frac{\pi}{2} \right) \]

\[ = \left( \frac{32.0 \text{ V}}{51.8 \text{ } \Omega} \right) \sin \left[ 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 1.20 \times 10^{-4} \text{ s} \right) - \frac{\pi}{2} \right] = -0.263 \text{ } \text{A} \]

28. \textit{REASONING} The rms voltages across the inductor \( L \) and the capacitor \( C \) are given, respectively, by \( V_{L,\text{rms}} = I_{\text{rms}}X_L \) (Equation 23.3) and \( V_{C,\text{rms}} = I_{\text{rms}}X_C \) (Equation 23.1), where \( I_{\text{rms}} \) is the rms current in the circuit, \( X_L = 2\pi fL \) (Equation 23.4) is the inductive reactance of the inductor, and \( X_C = \frac{1}{2\pi fC} \) (Equation 23.2) is the capacitive reactance of the capacitor. We know the frequency \( f \) of the generator, but we are not given the rms current in the circuit. We will make use of Equations 23.3 and 23.1 to eliminate the unknown current \( I_{\text{rms}} \), and then solve for the rms voltage across the inductor.

\textit{SOLUTION} Solving Equation 23.1 for \( I_{\text{rms}} \) gives \( I_{\text{rms}} = \frac{V_{C,\text{rms}}}{X_C} \). Substituting this relation into Equation 23.3, we obtain

\[ V_{L,\text{rms}} = I_{\text{rms}}X_L = \frac{V_{C,\text{rms}}X_L}{X_C} \]  \hspace{1cm} (1)
Substituting Equations 23.2 and 23.4 for the reactances into Equation (1) yields

\[ V_{L,\text{rms}} = \frac{V_{C,\text{rms}} X_L}{X_C} = \frac{V_{C,\text{rms}} 2\pi f L}{1 - \frac{1}{2\pi f C}} = V_{C,\text{rms}} \left(2\pi f\right)^2 LC \]

Therefore, when the rms voltage across the capacitor is \( V_{C,\text{rms}} = 2.20 \) V, the rms voltage across the inductor is

\[ V_{L,\text{rms}} = V_{C,\text{rms}} \left(2\pi f\right)^2 LC \]

\[ V_{L,\text{rms}} = (2.20 \text{ V}) \left(2\pi \left(375 \text{ Hz}\right)\right)^2 \left(84.0 \times 10^{-3} \text{ H}\right) \left(5.80 \times 10^{-6} \text{ F}\right) = 5.95 \text{ V} \]

**29. REASONING** A resistance \( R \) and an inductance \( L \) are connected in series to the generator, but there is no capacitance in the circuit. Therefore, the impedance \( Z \) of the circuit is given by \( Z = \sqrt{R^2 + (X_L - X_C)^2} \) (Equation 23.7), where \( X_L \) is the inductive reactance of the inductor, and the capacitive reactance \( X_C \) is zero:

\[ \sqrt{R^2 + (X_L - 0)^2} = Z \quad \text{or} \quad \sqrt{R^2 + X_L^2} = Z \quad (1) \]

The impedance \( Z \) is related to the rms current \( I_{\text{rms}} \) and the rms generator voltage \( V_{\text{rms}} \) by \( V_{\text{rms}} = I_{\text{rms}} Z \) (Equation 23.6). The rms voltage \( V_{L,\text{rms}} \) across the inductor is given by \( I_{\text{rms}} = \frac{V_{L,\text{rms}}}{X_L} \) (Equation 23.3). We will determine the inductive reactance from the generator frequency \( f \) and the inductance by means of \( X_L = 2\pi f L \) (Equation 23.4).

**SOLUTION** Solving \( V_{\text{rms}} = I_{\text{rms}} Z \) (Equation 23.6) for \( Z \), we obtain

\[ Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} \quad (2) \]

Substituting Equation (2) into Equation (1) yields

\[ \sqrt{R^2 + X_L^2} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \quad (3) \]

Substituting \( I_{\text{rms}} = \frac{V_{L,\text{rms}}}{X_L} \) (Equation 23.3) into Equation (3), we obtain

\[ \sqrt{R^2 + \frac{X_L^2}{X_L}} = \frac{V_{\text{rms}}}{V_{L,\text{rms}}} \quad \text{or} \quad \sqrt{R^2 + \frac{X_L^2}{X_L}} = \frac{V_{\text{rms}} X_L}{V_{L,\text{rms}}} \quad (4) \]
Squaring both sides of Equation (4) and solving for $R^2$, we find that

$$R^2 + X^2_L = \left(\frac{V_{\text{rms}} X_L}{V_{L,\text{rms}}}\right)^2$$
or

$$R^2 = \left(\frac{V_{\text{rms}} X_L}{V_{L,\text{rms}}}\right)^2 - X^2_L$$
or

$$R^2 = X^2_L \left(\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1\right) \quad (5)$$

Taking the square root of both sides of Equation (5) yields

$$R = X_L \sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1} \quad (6)$$

Substituting $X_L = 2\pi f L$ (Equation 23.4) into Equation (6), we obtain the resistance $R$:

$$R = 2\pi f L \sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1} = 2\pi (130 \text{ Hz})(0.032 \text{ H}) \sqrt{(8.0 \text{ V})^2 - 1} = 76 \Omega$$

### 30. REASONING

The inductance $L$ and the capacitance $C$ of a series RCL circuit determine the resonant frequency $f_0$ according to

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (23.10)$$

As we see from Equation 23.10, the smaller the inductance $L$, the larger the resonant frequency, and the larger the inductance, the smaller the resonant frequency. Therefore, in part (a) we will use the largest frequency to determine the minimum inductance, and in part (b) we will use the smallest frequency to find the maximum inductance. We note that 1 MHz = $1 \times 10^6$ Hz.

### SOLUTION

a. Squaring both sides of Equation 23.10 and solving for $L$, we obtain

$$f_0^2 = \frac{1}{4\pi^2 LC} \quad \text{or} \quad L = \frac{1}{4\pi^2 f_0^2 C} \quad (1)$$

The minimum inductance is obtained when the resonant frequency is greatest, so Equation (1) gives

$$L = \frac{1}{4\pi^2 \left(9.0 \times 10^6 \text{ Hz}\right)^2 \left(1.8 \times 10^{-11} \text{ F}\right)} = 1.7 \times 10^{-5} \text{ H}$$

b. Using Equation (1) once more, this time with the smallest resonant frequency, yields the maximum inductance:

$$L = \frac{1}{4\pi^2 \left(4.0 \times 10^6 \text{ Hz}\right)^2 \left(1.8 \times 10^{-11} \text{ F}\right)} = 8.8 \times 10^{-5} \text{ H}$$
31. **REASONING** The resonant frequency \( f_0 \) of a series RCL circuit depends on the inductance \( L \) and capacitance \( C \) through the relation \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (Equation 23.10). Since all the variables are known except \( L \), we can use this relation to find the inductance.

**SOLUTION** Solving Equation 23.10 for the inductance gives

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 (690 \times 10^3 \text{ Hz})^2 (2.0 \times 10^{-9} \text{ F})} = 2.7 \times 10^{-5} \text{ H}
\]

32. **REASONING** The average power \( \bar{P} \) dissipated in the circuit is \( \bar{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi \) (Equation 23.9), where \( V_{\text{rms}} \) is rms voltage of the generator, \( I_{\text{rms}} \) is the rms current in the circuit, and \( \cos \phi \) is the power factor. The angle \( \phi \) is the angle between the current and voltage phasors and can be determined from \( \tan \phi = \frac{X_L - X_C}{R} \) (Equation 23.8), where \( X_L \) is the inductive reactance, \( X_C \) is the capacitive reactance, and \( R \) is the resistance of the circuit. In Equation 23.9, we have a value for the average power \( \bar{P} \), the current \( I_{\text{rms}} \) and we can obtain a value for the angle \( \phi \) from Equation 23.8 and the fact that the circuit is at resonance. Therefore, Equation 23.9 can be used to determine the voltage \( V_{\text{rms}} \).

**SOLUTION** Solving Equation 23.9 for the voltage, we have

\[
\bar{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi \quad \text{or} \quad V_{\text{rms}} = \frac{\bar{P}}{I_{\text{rms}} \cos \phi}
\]

At resonance in a series RCL circuit we know that \( X_L = X_C \). Therefore, Equation 23.8 becomes

\[
\tan \phi = \frac{X_L - X_C}{R} = 0.00 \quad \text{or} \quad \phi = \tan^{-1}(0.00) = 0.00^\circ
\]

With this value for \( \phi \) Equation (1) reveals that

\[
V_{\text{rms}} = \frac{\bar{P}}{I_{\text{rms}} \cos \phi} = \frac{65.0 \text{ W}}{(0.530 \text{ A}) \cos 0.00^\circ} = 123 \text{ V}
\]

33. **REASONING** The current in an RCL circuit is given by Equation 23.6, \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where the impedance \( Z \) of the circuit is given by Equation 23.7 as \( Z = \sqrt{R^2 + (X_L - X_C)^2} \). The current is a maximum when the impedance is a minimum for a given generator voltage. The minimum impedance occurs when the frequency is \( f_0 \), corresponding to the condition that \( X_L = X_C \), or \( 2\pi f_0 L = 1/(2\pi f_0 C) \). Solving for the frequency \( f_0 \), called the resonant frequency, we find that
Note that the resonant frequency depends on the inductance and the capacitance, but does not depend on the resistance.

**SOLUTION**

a. The frequency at which the current is a maximum is

\[
f_0 = \frac{1}{2\pi \sqrt{LC}}
\]

This occurs when \( \omega = \omega_0 \).

\[
\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2\pi \sqrt{(17.0 \times 10^{-3}) \text{ H}(12.0 \times 10^{-6}) \text{ F}}} = 352 \text{ Hz}
\]

b. The maximum value of the current occurs when \( f = f_0 \). This occurs when \( X_L = X_c \), so that \( Z = R \). Therefore, according to Equation 23.6, we have

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{155 \text{ V}}{10.0 \Omega} = 15.5 \text{ A}
\]

34. **REASONING** The resonant frequency is given by Equation 23.10 as \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) and is inversely proportional to the square root of the circuit capacitance \( C \). Therefore, to reduce the resonant frequency, it is necessary to increase the circuit capacitance. The equivalent capacitance \( C_p \) of two capacitors in parallel is \( C_p = C_1 + C_2 \) (Equation 20.18), which is greater than either capacitance individually. Therefore, to increase the circuit capacitance, \( C_2 \) should be added in parallel with \( C_1 \).

**SOLUTION** The initial resonant frequency is \( f_{01} \). The resonant frequency that results after \( C_2 \) is added in parallel with \( C_1 \) is \( f_{0p} \). Using Equation 23.10, we can express both of these frequencies as follows:

\[
f_{01} = \frac{1}{2\pi \sqrt{LC_1}} \quad \text{and} \quad f_{0p} = \frac{1}{2\pi \sqrt{LC_p}}
\]

Here, \( C_p \) is the equivalent parallel capacitance. Dividing the expression for \( f_{01} \) by the expression for \( f_{0p} \) yields

\[
\frac{f_{01}}{f_{0p}} = \frac{1}{\sqrt{\frac{2\pi \sqrt{LC_1}}{2\pi \sqrt{LC_p}}}} = \sqrt{\frac{C_p}{C_1}}
\]

According to Equation 20.18, the equivalent capacitance is \( C_p = C_1 + C_2 \), so that this frequency ratio becomes

\[
\frac{f_{01}}{f_{0p}} = \sqrt{\frac{C_1 + C_2}{C_1}} = \sqrt{1 + \frac{C_2}{C_1}}
\]
Squaring both sides of this result and solving for $C_2$, we find

$$\left(\frac{f_{01}}{f_{0P}}\right)^2 = 1 + \frac{C_2}{C_1}$$

$$C_2 = C_1 \left[\left(\frac{f_{01}}{f_{0P}}\right)^2 - 1\right] = (2.60 \ \mu F) \left[\left(\frac{7.30 \ kHz}{5.60 \ kHz}\right)^2 - 1\right] = 1.8 \ \mu F$$

35. **REASONING** As discussed in Section 23.4, a RCL series circuit is at resonance when the current is a maximum and the impedance $Z$ of the circuit is a minimum. This happens when the inductive reactance $X_L$ equals the capacitive reactance $X_C$. When $X_L = X_C$, the impedance of the circuit (see Equation 23.7) becomes $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$, so the impedance is due solely to the resistance $R$. The average power dissipated in the circuit is $\bar{P} = \frac{V_{rms}^2}{R}$ (Equation 20.15c). This relation can be used to find the power when the variable resistor is set to another value.

**SOLUTION** The average power $\bar{P}_1$ dissipated when the resistance is $R_1 = 175 \ \Omega$ is $\bar{P}_1 = \frac{V_{rms}^2}{R_1}$. Likewise, the average power $\bar{P}_2$ dissipated when the resistance is $R_2 = 562 \ \Omega$ is $\bar{P}_2 = \frac{V_{rms}^2}{R_2}$. Solving the first equation for $V_{rms}^2$ and substituting the result into the second equation gives

$$\bar{P}_2 = \frac{V_{rms}^2}{R_2} = \frac{\bar{P}_1 R_1}{R_2} = \frac{(2.6 \ W)(175 \ \Omega)}{562 \ \Omega} = 0.81 \ W$$

36. **REASONING** The resonant frequency of an RCL circuit is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$ (Equation 23.10), where $L$ is inductance and $C$ is the capacitance. Because only the inductance of this circuit changes, from $L_1 = 7.0 \ \text{mH}$ to $L_2 = 1.5 \ \text{mH}$, we obtain the initial and final resonant frequencies from Equation 23.10:

$$f_{01} = \frac{1}{2\pi\sqrt{L_1 C}} \quad \text{and} \quad f_{02} = \frac{1}{2\pi\sqrt{L_2 C}} \quad (1)$$

We will solve the first of Equations (1) for the capacitance $C$, and substitute the result into the second of Equations (1).

**SOLUTION** Squaring both sides of the first of Equations (1) and solving for $C$, we obtain

$$\left(f_{01}\right)^2 = \frac{1}{(2\pi)^2 L_1 C} \quad \text{or} \quad C = \frac{1}{\left(2\pi f_{01}\right)^2 L_1} \quad (2)$$
Substituting Equation (2) into the second of Equations (1) yields

$$f_{02} = \frac{1}{2\pi \sqrt{L_2C}} = \frac{1}{2\pi \sqrt{L_2\left[\frac{1}{(2\pi f_{01})^2 L_1}\right]}} = \frac{2\pi f_{01}}{2\pi \sqrt{L_2 L_1}} = f_{01} \sqrt{\frac{L_1}{L_2}}$$  (3)

In Equation (3) the initial resonant frequency is multiplied by the square root of the ratio of the inductances, so that if we express the initial resonant frequency in kHz, the final resonant frequency will also be expressed in kHz, as requested. Similarly, we do not need to convert the inductances from millihenries to henries, since their units will cancel out. From Equation (3), then, the final resonant frequency of the circuit is

$$f_{02} = f_{01} \sqrt{\frac{L_1}{L_2}} = (1.3 \text{ kHz}) \sqrt{\frac{7.0 \text{ mH}}{1.5 \text{ mH}}} = 2.8 \text{ kHz}$$

37. **REASONING**  Since the resonant frequency $f_0$ is known, we may use Equation 23.10, $f_0 = \frac{1}{2\pi \sqrt{LC}}$ to find the inductance $L$, provided the capacitance $C$ can be determined. The capacitance can be found by using the definitions of capacitive and inductive reactances.

**SOLUTION**

a. Solving Equation 23.10 for the inductance, we have

$$L = \frac{1}{4\pi^2 f_0^2 C}$$  (1)

where $f_0$ is the resonant frequency. From Equations 23. 2 and 23.4, the capacitive and inductive reactances are

$$X_C = \frac{1}{2\pi f C} \quad \text{and} \quad X_L = 2\pi f L$$

where $f$ is any frequency. Solving the first of these equations for $f$, substituting the result into the second equation, and solving for $C$ yields $C = \frac{L}{X_L X_C}$. Substituting this result into Equation (1) above and solving for $L$ gives

$$L = \frac{1}{2\pi f_0} \sqrt{X_L X_C} = \frac{1}{2\pi (1500 \text{ Hz})} \sqrt{(30.0 \Omega)(5.0 \Omega)} = 1.3 \times 10^{-3} \text{ H}$$

b. The capacitance is

$$C = \frac{L}{X_L X_C} = \frac{1.3 \times 10^{-3} \text{ H}}{(30.0 \Omega)(5.0 \Omega)} = 8.7 \times 10^{-6} \text{ F}$$
38. **REASONING** The inductive reactance \( X_L \) is given by \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the nonresonant frequency in hertz (Hz) and \( L \) is the inductance. The capacitive reactance \( X_C \) is given by \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( C \) is the capacitance. The resonant frequency \( f_0 \) is \( f_0 = \frac{1}{2\pi\sqrt{LC}} \) (23.10). From the values given for the ratio \( \left( \frac{X_L}{X_C} = 5.36 \right) \) and the resonant frequency \( (f_0 = 225 \text{ Hz}) \) we will determine the nonresonant frequency \( f \).

**SOLUTION** Using Equation 23.4 for \( X_L \) and Equation 23.2 for \( X_C \), we can write the ratio of the two reactances as follows:

\[
\frac{X_L}{X_C} = \frac{2\pi f L}{1/(2\pi f C)} = 4\pi^2 f^2 LC
\]

Solving Equation (1) for the nonresonant frequency \( f \) shows that

\[
\frac{X_L}{X_C} = 4\pi^2 f^2 LC \quad \text{or} \quad f = \sqrt{\frac{X_L}{X_C}} \left( \frac{1}{4\pi^2 LC} \right) = \sqrt{\frac{X_L}{X_C}} \left( \frac{1}{2\pi \sqrt{LC}} \right)
\]

We note in Equation (2) that the term in parentheses is the resonant frequency \( f_0 \) as given by Equation 23.10, so that we have

\[
f = \sqrt{\frac{X_L}{X_C}} (f_0) = \sqrt{5.36} (225 \text{ Hz}) = 521 \text{ Hz}
\]

39. **SSM REASONING AND SOLUTION** At the resonant frequency \( f_0 \), we have \( C = 1/(4\pi^2 f_0^2 L) \). We want to determine some series combination of capacitors whose equivalent capacitance \( C'_s \) is such that \( f'_0 = 3f_0 \). Thus,

\[
C'_s = \frac{1}{4\pi^2 f_0'^2 L} = \frac{1}{4\pi^2 (3f_0)^2 L} = \frac{1}{9} \left( \frac{1}{4\pi^2 f_0^2 L} \right) = \frac{1}{9} C
\]

The equivalent capacitance of a series combination of capacitors is \( 1/C'_s = 1/C_1 + 1/C_2 + ... \). If we require that all the capacitors have the same capacitance \( C \), the equivalent capacitance is

\[
\frac{1}{C'_s} = \frac{1}{C} + \frac{1}{C} + ... = \frac{n}{C}
\]

where \( n \) is the total number of identical capacitors. Using the result above, we find that
\[
\frac{1}{C'_s} = \frac{1}{\frac{n}{9} C} = \frac{n}{C} \quad \text{or} \quad n = 9
\]

Therefore, the number of additional capacitors that must be inserted in series in the circuit so that the frequency triples is \( n' = n - 1 = 8 \).

40. **REASONING**
   a. When a capacitor stores charge, it also stores electrical energy. The energy stored by the capacitor can be expressed as 
   \[
   \text{Energy} = \frac{1}{2} \left( \frac{q^2}{C} \right),
   \]
   according to Equation 19.11c.

   b. There is no resistance in the circuit, so no energy is lost as it shuttles back and forth between the capacitor and the inductor. The energy removed from the capacitor when it is completely discharged is \( \frac{1}{2} \left( \frac{q^2}{C} \right) \). This energy is gained by the inductor. The energy stored by an inductor is given by 
   \[
   \text{Energy} = \frac{1}{2} L I^2,
   \]
   (Equation 22.10), where \( L \) is the inductance and \( I \) is the current. The maximum energy stored by the inductor is \( \frac{1}{2} L I_{\text{max}}^2 \), where \( I_{\text{max}} \) is the maximum current in the inductor.

**SOLUTION**
   a. The electrical energy stored in the fully charged capacitor is
   \[
   \text{Energy} = \frac{1}{2} \left( \frac{q^2}{C} \right) = \frac{1}{2} \left[ \frac{(2.90 \times 10^{-6} \text{ C})^2}{3.60 \times 10^{-6} \text{ F}} \right] = 1.17 \times 10^{-6} \text{ J}
   \]  
   (19.11c)

   b. Since the energy stored by the capacitor is equal to the maximum energy stored by the inductor, we can write
   \[
   \frac{1}{2} \left( \frac{q^2}{C} \right) = \frac{1}{2} L I_{\text{max}}^2 \quad \text{or} \quad I_{\text{max}} = \frac{q}{\sqrt{LC}}
   \]
   The maximum current in the inductor is
   \[
   I_{\text{max}} = \frac{q}{\sqrt{LC}} = \frac{2.90 \times 10^{-6} \text{ C}}{\sqrt{(75.0 \times 10^{-3} \text{ H})(3.60 \times 10^{-6} \text{ F})}} = 5.58 \times 10^{-3} \text{ A}
   \]
REASONING  We will find the desired percentage from the ratio $L_B / L_A$. The beat frequency that is heard is $|f_{OB} - f_{0A}|$, and the resonant frequencies are $f_{0B} = 1/\left(2\pi\sqrt{L_B C}\right)$ and $f_{0A} = 1/\left(2\pi\sqrt{L_A C}\right)$, according to Equation 23.10. By expressing the beat frequency in terms of these expressions, we will be able to obtain $L_B / L_A$.

SOLUTION  Using Equation 23.10 to express each resonant frequency, we find that the beat frequency is

$$|f_{OB} - f_{0A}| = \left| \frac{1}{2\pi\sqrt{L_B C}} - \frac{1}{2\pi\sqrt{L_A C}} \right|$$

Factoring out the term $1/\left(2\pi\sqrt{L_A C}\right)$ gives

$$|f_{OB} - f_{0A}| = \frac{1}{2\pi\sqrt{L_A C}} \left| \frac{2\pi\sqrt{L_B C}}{2\pi\sqrt{L_A C}} - 1 \right| = \frac{1}{2\pi\sqrt{L_A C}} \left| \sqrt{\frac{L_A}{L_B}} - 1 \right| = \frac{1}{2\pi\sqrt{L_A C}} \left( 1 - \sqrt{\frac{L_A}{L_B}} \right)$$

Note that $L_B$ is greater than $L_A$, so that $\sqrt{\frac{L_A}{L_B}} - 1$ is a negative quantity. Therefore, we have written $\sqrt{\frac{L_A}{L_B}} - 1$ as $\left(1 - \sqrt{\frac{L_A}{L_B}}\right)$. Solving for $\sqrt{\frac{L_A}{L_B}}$, we obtain

$$\sqrt{\frac{L_A}{L_B}} = 1 - \frac{|f_{OB} - f_{0A}|}{\frac{1}{2\pi\sqrt{L_A C}}}$$

Remembering that $f_{0A} = 1/\left(2\pi\sqrt{L_A C}\right)$, we see that this result becomes

$$\sqrt{\frac{L_A}{L_B}} = 1 - \frac{|f_{OB} - f_{0A}|}{f_{0A}} \quad \text{or} \quad \frac{L_A}{L_B} = \left(1 - \frac{|f_{OB} - f_{0A}|}{f_{0A}}\right)^2$$

Taking the reciprocal of this expression reveals that

$$\frac{L_B}{L_A} = \left(1 - \frac{|f_{OB} - f_{0A}|}{f_{0A}}\right)^2 = \frac{1}{\left(1 - \frac{7.30 \text{ kHz}}{630.0 \text{ kHz}}\right)^2} = 1.024$$

Thus, the percentage increase of $L_B$ is $(1.024 - 1.000) \times 100\% = 2.4\%$. 
42. **REASONING AND SOLUTION** The current in an RCL-circuit is

\[ I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \]

Rearranging terms

\[ (X_L - X_C)^2 = (\frac{V}{I})^2 - R^2 \]

Using Equations 23.2 and 23.4 for \( X_C \) and \( X_L \), respectively, we obtain

\[ 2\pi f L - \frac{1}{2\pi f C} = \sqrt{\left(\frac{V}{I}\right)^2 - R^2} = \sqrt{\left(\frac{26.0 \text{ V}}{0.141 \text{ A}}\right)^2 - (108 \Omega)^2} = 149 \Omega \]

Multiplying by \( f \) leads to

\[ 2\pi f^2 L - (149 \Omega)f - 1/(2\pi C) = 0 \]

or

\[ 2\pi f^2 (5.42 \times 10^{-3} \text{ H}) - (149 \Omega)f - 1/[(2\pi(0.200 \times 10^{-6} \text{ F})] = 0 \]

We can solve this quadratic equation for the frequencies. We obtain

\[ f_1 = 3.11 \times 10^3 \text{ Hz} \quad \text{and} \quad f_2 = 7.50 \times 10^3 \text{ Hz} \]

43. **SSM REASONING** The voltage across the capacitor reaches its maximum instantaneous value when the generator voltage reaches its maximum instantaneous value. The maximum value of the capacitor voltage first occurs one-fourth of the way, or one-quarter of a period, through a complete cycle (see the voltage curve in Figure 23.4).

**SOLUTION** The period of the generator is \( T = 1/f = 1/(5.00 \text{ Hz}) = 0.200 \text{ s} \). Therefore, the least amount of time that passes before the instantaneous voltage across the capacitor reaches its maximum value is \( \frac{1}{4} T = \frac{1}{4} (0.200 \text{ s}) = 5.00 \times 10^{-2} \text{ s} \).

44. **REASONING** The average power dissipated is that dissipated in the resistor and is \( \bar{P} = I_{\text{rms}}^2 R \), according to Equation 20.15b. We are given the current \( I_{\text{rms}} \) but need to find the resistance \( R \). Since the inductive reactance \( X_L \) is known, we can find the resistance from the impedance, which is \( Z = \sqrt{R^2 + X_L^2} \), according to Equation 23.7. Since the voltage and the current are known, we can obtain the impedance from Equation 23.6 as \( Z = V_{\text{rms}}/I_{\text{rms}} \).

**SOLUTION** From Equation 23.7, we can determine the resistance as \( R = \sqrt{Z^2 - X_L^2} \). With this expression for the resistance, Equation 20.15b for the power becomes
\[
\bar{P} = I_{\text{rms}}^2 R = I_{\text{rms}}^2 \sqrt{Z^2 - X_L^2}
\]

Using Equation 23.6 to express the impedance, we obtain the following value for the dissipated power

\[
\bar{P} = I_{\text{rms}}^2 \sqrt{Z^2 - X_L^2} = I_{\text{rms}}^2 \left( \frac{V_{\text{rms}}}{I_{\text{rms}}} \right)^2 - X_L^2
\]

\[
= (1.75 \text{ A})^2 \sqrt{\left( \frac{115 \text{ V}}{1.75 \text{ A}} \right)^2 - (52.0 \text{ } \Omega)^2} = 123 \text{ W}
\]

**45. REASONING AND SOLUTION** We begin by calculating the impedance of the circuit using \( Z = \sqrt{R^2 + (X_L - X_C)^2} \). We have

\[
X_C = \frac{1}{2 \pi f C} = \frac{1}{2 \pi (1350 \text{ Hz})(4.10 \times 10^{-6} \text{ F})} = 28.8 \text{ } \Omega
\]

\[
X_L = 2 \pi f L = 2 \pi (1350 \text{ Hz})(5.30 \times 10^{-3} \text{ H}) = 45.0 \text{ } \Omega
\]

\[
Z = \sqrt{(16.0 \text{ } \Omega)^2 + (45.0 \text{ } \Omega - 28.8 \text{ } \Omega)^2} = 22.8 \text{ } \Omega
\]

The current is therefore,

\[
I = \frac{V}{Z} = \frac{15.0 \text{ V}}{22.8 \text{ } \Omega} = 0.658 \text{ A}
\]

Since the circuit elements are in series, the current through each element is the same. The voltage across each element is

\[
V_R = IR = (0.658 \text{ A})(16.0 \text{ } \Omega) = 10.5 \text{ V}
\]

\[
V_C = IX_C = (0.658 \text{ A})(28.8 \text{ } \Omega) = 19.0 \text{ V}
\]

\[
V_L = IX_L = (0.658 \text{ A})(45.0 \text{ } \Omega) = 29.6 \text{ V}
\]

**46. REASONING AND SOLUTION** At very high frequencies the capacitors behave as if they were replaced with wires that have zero resistance, while the inductors behave as if they were cut out of the circuit. The drawings below show the circuits under this condition. Circuit I behaves as if the two resistors are in parallel, and the equivalent resistance can be obtained from \( R_p^{-1} = R^{-1} + R^{-1} \) as \( R_p = R/2 \). Circuit II behaves as if the two resistors are in series, and the equivalent resistance is \( R_s = R + R = 2R \). In either case, the current is the voltage divided by the resistance. Therefore, the ratio of the currents in the two circuits is
47. SSM REASONING The individual reactances are given by Equations 23.2 and 23.4, respectively,

**Capacitive reactance** \[ X_C = \frac{1}{2\pi f C} \]

**Inductive reactance** \[ X_L = 2\pi f L \]

When the reactances are equal, we have \( X_C = X_L \), from which we find

\[
\frac{1}{2\pi f C} = 2\pi f L \quad \text{or} \quad 4\pi^2 f^2 LC = 1
\]

The last expression may be solved for the frequency \( f \).

**SOLUTION** Solving for \( f \) with \( L = 52 \times 10^{-3} \) H and \( C = 76 \times 10^{-6} \) F, we obtain

\[
f = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{(52 \times 10^{-3} \text{ H})(76 \times 10^{-6} \text{ F})}} = 8.0 \times 10^1 \text{ Hz}
\]

48. REASONING Only the resistor, on average, consumes power. Therefore, the average power delivered to the circuit is equal to the average power delivered to the resistor. The average power is given by \( \overline{P} = I_{\text{rms}}^2 R \) (Equation 20.15b), where \( I_{\text{rms}} \) is the rms current in the circuit and \( R \) is the resistance. According to Equation 23.6, the current is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( V_{\text{rms}} \) is the rms voltage of the generator and \( Z \) is the circuit impedance.

The impedance of the circuit is given by \( Z = \sqrt{R^2 + (X_L - X_C)^2} \) (Equation 23.7). At resonance the inductive reactance \( X_L \) and the capacitive reactance \( X_C \) are equal, so that \( Z = R \).
**SOLUTION** Substituting \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) into \( P = I_{\text{rms}}^2 R \), the average power delivered to the circuit can be written as

\[
P = I_{\text{rms}}^2 R = \left( \frac{V_{\text{rms}}}{Z} \right)^2 R
\]

(1)

Substituting \( Z = R \) into Equation (1) yields

\[
P = \left( \frac{V_{\text{rms}}}{R} \right)^2 R = \frac{V_{\text{rms}}^2}{R} = \frac{(3.0 \text{ V})^2}{92 \Omega} = 0.098 \text{ W}
\]

49. **REASONING** The rms current in an inductor is \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} \), according to Equation 23.3. The inductive reactance is \( X_L = 2\pi f L \), according to Equation 23.4. Applying these expressions to both generators will allow us to obtain the desired current.

**SOLUTION** Using Equations 23.3 and 23.4, we find that the current in an inductor is

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L}
\]

Applying this result to the two generators gives

\[
I_1 = \frac{V_{\text{rms}}}{2\pi f_1 L} \quad \text{and} \quad I_2 = \frac{V_{\text{rms}}}{2\pi f_2 L}
\]

Generator 1

Generator 2

Dividing the equation for generator 2 by the equation for generator 1, we obtain

\[
\frac{I_2}{I_1} = \frac{2\pi f_2 L}{2\pi f_1 L} = \frac{f_1}{f_2} \quad \text{or} \quad I_2 = I_1 \frac{f_1}{f_2} = (0.30 \text{ A}) \frac{1.5 \text{ kHz}}{6.0 \text{ kHz}} = 0.075 \text{ A}
\]

50. **REASONING** To find the frequency at which the current is one-half its value at zero frequency, we first determine the value of the current when \( f = 0 \text{ Hz} \). We note that at zero frequency the reactive inductance is zero \( (X_L = 0 \Omega) \), since \( X_L = 2\pi f L \) (Equation 23.4). The current at zero frequency is \( I_{\text{rms}}^0 = \frac{V_{\text{rms}}}{R} \) (Equation 20.14), since the inductor does not play a role in determining the current at this frequency. When the frequency is not zero, the current is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) (Equation 23.6), where \( Z \) is the impedance of the circuit.
These last two relations will allow us to find the frequency at which the current is one-half its value at zero frequency.

**SOLUTION** We are given that \( I_{\text{rms}} = \frac{1}{2} I^0_{\text{rms}} \), so

\[
\frac{V_{\text{rms}}}{Z} = \frac{1}{2} \left( \frac{V_{\text{rms}}}{R} \right) \quad \text{or} \quad Z = 2R
\]

Since the impedance of the circuit is \( Z = \sqrt{R^2 + X_L^2} \) (Equation 23.7) and \( X_L = 2\pi f L \) (Equation 23.4), the relation \( Z = 2R \) becomes

\[
\sqrt{R^2 + (2\pi f L)^2} = 2R
\]

Solving for the frequency gives

\[
f = \frac{\sqrt{3}R}{2\pi L} = \frac{\sqrt{3}(16 \text{ } \Omega)}{2\pi \left(4.0 \times 10^{-3} \text{ } \text{H}\right)} = 1.1 \times 10^3 \text{ Hz}
\]

51. **SSM REASONING** Since we know the values of the resonant frequency of the circuit, the capacitance, and the generator voltage, we can find the value of the inductance from Equation 23.10, the expression for the resonant frequency. The resistance can be found from energy considerations at resonance; the power factor is given by \( \cos \phi \), where the phase angle \( \phi \) is given by Equation 23.8, \( \tan \phi = \left( X_L - X_C \right) / R \).

**SOLUTION**

a. Solving Equation 23.10 for the inductance \( L \), we find that

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 (1.30 \times 10^3 \text{ } \text{Hz})^2 (5.10 \times 10^{-6} \text{ } \text{F})} = 2.94 \times 10^{-3} \text{ H}
\]

b. At resonance, \( f = f_0 \), and the current is a maximum. This occurs when \( X_L = X_C \), so that \( Z = R \). Thus, the average power \( \bar{P} \) provided by the generator is \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \), and solving for \( R \) we find

\[
R = \frac{V_{\text{rms}}^2}{\bar{P}} = \frac{(11.0 \text{ V})^2}{25.0 \text{ W}} = 4.84 \text{ } \Omega
\]

c. When the generator frequency is 2.31 kHz, the individual reactances are
\[ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (2.31 \times 10^3 \text{Hz}) (5.10 \times 10^{-6} \text{F})} = 13.5 \Omega \]

\[ X_L = 2\pi f L = 2\pi (2.31 \times 10^3 \text{Hz}) (2.94 \times 10^{-3} \text{H}) = 42.7 \Omega \]

The phase angle \( \phi \) is, from Equation 23.8,

\[ \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) = \tan^{-1} \left( \frac{42.7 \Omega - 13.5 \Omega}{4.84 \Omega} \right) = 80.6^\circ \]

The power factor is then given by

\[ \cos \phi = \cos 80.6^\circ = 0.163 \]

52. **REASONING AND SOLUTION**  With only the resistor in the circuit, the power dissipated is \( P_1 = V_0^2/R = 1.000 \text{W} \). Therefore, \( V_0^2 = (1.000 \text{W}) R \). When the capacitor is added in series with the resistor, the power dissipated is given by \( P_2 = I_2 V_0 \cos \phi = 0.500 \text{W} \), where \( \cos \phi \) is the power factor, with \( \cos \phi = R/Z_2 \), and \( I_2 = V_0/Z_2 \). The impedance \( Z_2 \) is \( Z_2 = \sqrt{R^2 + X_C^2} \). Substituting yields,

\[ P_2 = V_0^2 R/Z_2^2 = (1.000 \text{W}) R^2/(R^2 + X_C^2) = 0.500 \text{W} \]

Solving for \( X_C \) gives, \( X_C = R \). When the inductor is added in series with the resistor, we have \( P_3 = V_0 I_3 \cos \phi = 0.250 \text{W} \), where \( I_3 = V_0/Z_3 \) and \( \cos \phi = R/Z_3 \). The impedance \( Z_3 \) is \( Z_3 = \sqrt{R^2 + X_L^2} \). Thus,

\[ P_3 = (1.000 \text{W}) R^2/(R^2 + X_L^2) = 0.250 \text{W} \]

Solving for \( X_L \), we find that \( X_L = R \sqrt{3} \). Finally, when both the inductor and capacitor are added in series with the resistor we have

\[ P_4 = \frac{V_0^2 R}{R^2 + (X_L - X_C)^2} = \frac{(1.000 \text{W}) R^2}{R^2 + (R\sqrt{3} - R)^2} = 0.651 \text{W} \]
1. (b) The loop can only detect the wave if the wave’s magnetic field has a component perpendicular to the plane of the loop, that is, along the $y$ axis. Only then will there be a changing magnetic flux through the loop. The changing flux is needed, so that an induced emf will arise in the loop according to Faraday’s law of electromagnetic induction. The electric and magnetic fields of an electromagnetic wave are mutually perpendicular and are both perpendicular to the direction in which the wave travels. Thus, when the wave travels along the $z$ axis with its electric field along the $x$ axis, the magnetic field will be along the $y$ axis as needed.

2. (c) The wavelength $\lambda$, frequency $f$, and speed $c$ of an electromagnetic wave are related according to $c = \lambda f$, where $c$ is the same for any electromagnetic wave traveling in a vacuum and is independent of $\lambda$ and $f$. Since $c$ is constant, $\lambda$ and $f$ are inversely proportional. When $f$ is reduced by a factor of three, $\lambda$ increases by a factor of three.

3. (b) The magnitudes of the electric and magnetic fields of the wave are proportional to each other, according to $E = cB$ (Equation 24.3). As Section 24.4 discusses, the wave carries equal amounts of electric and magnetic energy.

4. (a) The magnitudes of the electric and magnetic fields of the wave are proportional, according to $E = cB$ (Equation 24.3). Thus, when $E$ doubles, so does $B$. The total energy density and the intensity are each proportional to the square of the electric field magnitude, according to $u = \varepsilon_0 E^2$ (Equation 24.2b) and $S = c\varepsilon_0 E^2$ (Equation 24.5b). Therefore, when $E$ doubles, $u$ and $S$ both increase by a factor of $2^2 = 4$.

5. 698 J/(s·m$^2$)

6. (d) The observed frequency is $f_o = f_s \left(1 \pm \frac{v_{rel}}{c}\right)$ according to Equation 24.6. The frequency $f_s$ emitted by the source is the same in each case, so that only the relative speed $v_{rel}$ and the direction of the relative motion determine the observed frequency. In each case either the source or the observer is moving, so the relative speed is just the magnitude of the velocity vector shown in the drawing. Since the velocity vector has the same magnitude in each case, the relative speed is the same in each case. Thus, it is only the direction of the relative motion that needs to be considered here. In A and C the source and observer are moving apart at the same relative speed, the minus sign applies in Equation 24.6, and the observed frequencies are the same. In B and D they are coming together at the same relative speed, the plus sign applies in Equation 24.6, and the observed frequencies are the same, but greater than that in A and C.
7. \(3.0 \times 10^6 \text{ m/s}\)

8. (c) When the incident light is completely unpolarized, half of its intensity is absorbed by the polarizer on the left, and half passes through. The half that passes through is completely polarized along the vertical direction, which is the same as the transmission axis of the second polarizer. Thus, the second polarizer absorbs none of the light, and the intensity of the exiting light is half that of the incident light. When the incident light is completely polarized along the vertical direction to begin with, it passes through both sheets of material with none of its intensity being absorbed. In this case the exiting light has the same intensity as the incident light. Thus, the exiting light has a greater intensity when the incident light is polarized.

9. (d) When the unpolarized light strikes the first polarizer, the light that passes through it is polarized in the vertical direction. When this polarized light strikes the second polarizer, all of it is absorbed, since the two polarizers are crossed. When the polarized light strikes the first polarizer, all of it passes through, since the direction of polarization and the transmission axis are both vertical. When this polarized light strikes the second polarizer, all of it is absorbed, since the two polarizers are crossed. Thus, no light exits the polarizer on the right in either case.

10. (e) When the light is incident from the left, Malus’ law (Equation 24.7) indicates that the transmitted light has an average intensity that is reduced relative to the incident intensity by a factor of \(\cos^2 45^\circ = \frac{1}{2}\). When the light is incident from the right, Malus’ law also applies. Now, however, polarizer 2 has its transmission axis at an angle of 45º with respect to the polarization direction of the incident light. Malus’ law indicates that the intensity is reduced by a factor of \(\frac{1}{2}\). But the light leaving polarizer 2 is polarized at an angle of 45º with respect to the transmission axis of polarizer 1. Malus’ law again applies and indicates that the intensity is reduced by a second factor of \(\frac{1}{2}\). The transmitted light, therefore, has an intensity that is reduced by a factor of \(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\) relative to the initial intensity.
CHAPTER 24  | ELECTROMAGNETIC WAVES

PROBLEMS

1. **REASONING** The distance $d$ between earth and the probe is determined by $d = ct$ (Equation 2.1), where $c$ is the speed of light in a vacuum and $t$ is the time for the radio signal to reach earth.

**SOLUTION** The elapsed time $t$ is given in hours, so it must be converted to seconds:

$$t = (2.53 \text{ h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 9110 \text{ s} \quad (1)$$

The distance, then, between earth and the probe is

$$d = ct = \left( 3.00 \times 10^8 \text{ m/s} \right) (9110 \text{ s}) = 2.73 \times 10^{12} \text{ m}$$

2. **REASONING** Radio waves are electromagnetic waves that travel at the speed of light. In the vacuum of space light travels at a speed of $c = 3.00 \times 10^8 \text{ m/s}$. The constant speed $c$ is given by $c = \frac{d}{t}$ (Equation 2.1), where $d$ is the distance traveled in a time $t$. We can use this equation to determine $t$, since values are known for $c$ and $d$.

**SOLUTION**

a. Solving Equation 2.1 for the communication time $t$ between the moon and the earth reveals that

$$t = \frac{d}{c} = \frac{3.85 \times 10^8 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \text{ s}$$

b. A calculation similar to that in part a shows that the minimum communication time between Mars and the earth is

$$t = \frac{d}{c} = \frac{5.6 \times 10^{10} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 190 \text{ s}$$

3. **SSM REASONING** This is a standard exercise in the conversion of units. However, we will first need to determine the number of meters that light travels in one year, which is what we call a light-year. In the vacuum of space light travels at a speed of $c = 3.00 \times 10^8 \text{ m/s}$. The constant speed $c$ is given by $c = \frac{d}{t}$ (Equation 2.1), where $d$ is the distance traveled in a time $t$. We can use this equation to determine $d$, since values are known for $c$ and $t$. Then,
armed with this value for one light-year, we will follow the usual procedure for converting
units, as discussed in Section 1.3.

**SOLUTION** Solving Equation 2.1 for the distance \( d \) that light travels in one year, we find

\[
d = ct = \left( 3.00 \times 10^8 \text{ m/s} \right) (1.00 \text{ year}) \left( \frac{365.25 \text{ days}}{1 \text{ year}} \right) \left( \frac{24 \text{ hours}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ hour}} \right) = 9.47 \times 10^{15} \text{ m}
\]

Thus, 1 light-year = \( 9.47 \times 10^{15} \text{ m} \). To find the distance to Alpha Centauri in meters, we
follow the usual procedure and multiply the distance of 4.3 light-years by

\[
1 = \frac{9.47 \times 10^{15} \text{ m}}{1 \text{ light-year}}
\]

In so doing, we obtain

\[
(4.3 \text{ light-years}) \left( \frac{9.47 \times 10^{15} \text{ m}}{1 \text{ light-year}} \right) = 4.1 \times 10^{16} \text{ m}
\]

---

4. **REASONING** In order to pick up radio waves, the circuit must have a resonant frequency
\( f_0 \) that matches the frequency of the radio waves. The resonant frequency depends upon the
capacitance \( C \) and inductance \( L \) of the circuit via

\[
f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{(Equation 23.10)}
\]

In order to pick up the entire range of FM waves, the circuit must be able to attain the lowest
\( f_{\text{low}} = 88 \text{ MHz} \) and highest \( f_{\text{high}} = 108 \text{ MHz} \) necessary resonant frequency. We will use
Equation 23.10 to determine the corresponding minimum and maximum capacitance values.

**SOLUTION** Squaring both sides of

\[
f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{(Equation 23.10)}
\]

and solving for \( C \), we obtain

\[
(f_0)^2 = \frac{1}{(2\pi)^2 LC} \quad \text{or} \quad C = \frac{1}{(2\pi f_0)^2 L} \quad \text{(1)}
\]

As we see from Equation (1), the greater the frequency, the smaller the value of the
capacitance. So the highest frequency \( f_{\text{high}} = 108 \text{ MHz} \) corresponds to the minimum value of
the capacitance \( C_{\text{min}} \). From Equation (1), we obtain

\[
C_{\text{min}} = \frac{1}{(2\pi f_{\text{high}})^2 L} = \frac{1}{(2\pi)^2 \left( 108 \times 10^6 \text{ Hz} \right)^2 \left( 6.00 \times 10^{-7} \text{ H} \right)} = 3.62 \times 10^{-12} \text{ F}
\]

On the other end of the FM frequency range, matching the lowest frequency of
\( f_{\text{low}} = 88.0 \text{ MHz} \) requires a maximum capacitance value \( C_{\text{max}} \) of
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\[
C_{\text{max}} = \frac{1}{(2\pi f_{\text{low}})^2 L} = \frac{1}{(2\pi)^2 (88.0 \times 10^6 \, \text{Hz})^2 (6.00 \times 10^{-7} \, \text{H})} = 5.45 \times 10^{-12} \, \text{F}
\]

Therefore, the capacitance values should range from \(3.62 \times 10^{-12} \, \text{F}\) to \(5.45 \times 10^{-12} \, \text{F}\).

5. **REASONING** According to Equation 16.3, the displacement \(y\) of a wave that travels in the \(+x\) direction and has amplitude \(A\), frequency \(f\), and wavelength \(\lambda\) is given by

\[
y = A \sin \left( 2\pi f t - \frac{2\pi x}{\lambda} \right)
\]

This equation, with \(y = E\), applies to the traveling electromagnetic wave in the problem, which is represented mathematically as

\[
E = E_0 \sin \left( \left(1.5 \times 10^{10} \, \text{s}^{-1}\right)t - \left(5.0 \times 10^1 \, \text{m}^{-1}\right)x \right)
\]

As \(E_0\) is the maximum field strength, it represents the amplitude \(A\) of the wave. We can find the frequency and wavelength of this electromagnetic wave by comparing the mathematical form of the electric field with Equation 16.3.

**SOLUTION**

a. By inspection, we see that \(2\pi f = 1.5 \times 10^{10} \, \text{s}^{-1}\). Therefore, the frequency of the wave is

\[
f = \frac{1.5 \times 10^{10} \, \text{s}^{-1}}{2\pi} = 2.4 \times 10^9 \, \text{Hz}
\]

b. As shown in Figure 17.15, the separation between adjacent nodes in any standing wave is one-half of a wavelength. By inspection of the mathematical form of the electric field and comparison with Equation 16.3, we infer that \(2\pi / \lambda = 5.0 \times 10^1 \, \text{m}^{-1}\). Therefore,

\[
\lambda = \frac{2\pi}{5.0 \times 10^1 \, \text{m}^{-1}} = 0.126 \, \text{m}
\]

Therefore, the nodes in the standing waves formed by this electromagnetic wave are separated by \(\lambda / 2 = 0.063 \, \text{m}\).

6. **REASONING AND SOLUTION** The average flux change through the coil in one fourth of the wave period is, according to Faraday's law, \(\Delta \Phi = NAB = NB_0A\). The magnitude of the average emf is then \(\text{emf} = \Delta \Phi / \Delta t = NB_0A / \Delta t\). Now \(\Delta t = T/4 = 1/(4f)\), so

\[
\text{Emf} = 4NfB_0A = 4(450)(1.2 \times 10^6 \, \text{Hz})(2.0 \times 10^{-13} \, \text{T})(\pi)(0.25 \, \text{m})^2 = 8.5 \times 10^{-5} \, \text{V}
\]
7. **REASONING AND SOLUTION** Using Equation 16.1, we obtain

\[ \lambda = \frac{c}{f} = \frac{2.9979 \times 10^8 \text{ m/s}}{26.965 \times 10^6 \text{ Hz}} = 11.118 \text{ m} \]

8. **REASONING** According to Equation 16.1, the wavelength \( \lambda \) (in vacuum) is the speed of light \( c \) in a vacuum divided by the frequency \( f \) of the X-rays: \( \lambda = \frac{c}{f} \).

**SOLUTION** Using Equation 16.1, we find that

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.05 \times 10^8 \text{ Hz}} = 4.96 \times 10^{-11} \text{ m} \]

9. **SSM REASONING** The frequency \( f \) of the UHF wave is related to its wavelength by \( c = f \lambda \) (Equation 16.1), where \( c \) is the speed of light in a vacuum and \( \lambda \) is the wavelength. The electric and magnetic fields are both zero at the same positions, which are separated by a distance \( d \) equal to half a wavelength (see Figure 24.3). Therefore, we can express the wavelength in terms of the distance between adjacent positions of zero field as

\[ \lambda = 2d \]  

(1)

**SOLUTION** Solving \( c = f \lambda \) (Equation 16.1) for \( f \) yields

\[ f = \frac{c}{\lambda} \]  

(2)

Substituting Equation (1) into Equation (2), we obtain

\[ f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{2(0.34 \text{ m})} = 4.4 \times 10^8 \text{ Hz} \]

10. **REASONING** According to Equation 16.1, the wavelength \( \lambda \) (in vacuum) is the speed of light \( c \) in a vacuum divided by the frequency \( f \) of the radio waves: \( \lambda = \frac{c}{f} \).

**SOLUTION** Using Equation 16.1, we find that the longest FM radio wavelength is

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{88.0 \times 10^6 \text{ Hz}} = 3.41 \text{ m} \]
The shortest FM radio wavelength is

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{108.0 \times 10^6 \text{ Hz}} = 2.78 \text{ m} \]

11. **REASONING AND SOLUTION** According to Equation 16.1, the wavelength of these waves is \( \lambda = c/f \). Therefore,

\[ \frac{\lambda_{\text{MRI}}}{\lambda_{\text{PET}}} = \frac{c/f_{\text{MRI}}}{c/f_{\text{PET}}} = \frac{f_{\text{PET}}}{f_{\text{MRI}}} = \frac{1.23 \times 10^{20} \text{ Hz}}{6.38 \times 10^7 \text{ Hz}} = 1.93 \times 10^{12} \]

12. **REASONING** The length of each pulse is equal to the product of its speed and the time, or \( x = ct_0 \). According to Equation 16.1, the wavelength \( \lambda \) in a vacuum is related to the frequency \( f \) by \( \lambda = c/f \), where \( c \) is the speed of light in a vacuum. When the light travels in water, the speed is no longer \( c \), but its value \( v \) is given. The wavelength in water, then, is \( \lambda = v/f \). The frequency in a vacuum and in water is the same.

**SOLUTION**

a. The number of wavelengths in one pulse is equal to the length of the pulse divided by the wavelength. The length of each pulse is \( x = ct_0 \) and the wavelength is \( \lambda = c/f \), so

\[ \text{Number of wavelengths} = \frac{x}{\lambda} = \frac{ct_0}{c/f} \]

\[ = f t_0 = (5.2 \times 10^{14} \text{ Hz})(2.7 \times 10^{-11} \text{ s}) = 1.4 \times 10^4 \]

b. When the light is traveling in water, its speed is \( v \), which is less than the speed of light in a vacuum. The length of each pulse is now \( x = vt_0 \) and the wavelength is \( \lambda = v/f \), so

\[ \text{Number of wavelengths} = \frac{x}{\lambda} = \frac{vt_0}{v/f} \]

\[ = f t_0 = (5.2 \times 10^{14} \text{ Hz})(2.7 \times 10^{-11} \text{ s}) = 1.4 \times 10^4 \]

13. **REASONING** To determine the difference in frequencies, we will calculate each frequency and subtract one from the other. Each frequency \( f \) is related to the wavelength \( \lambda \) and the speed of light \( c \) according to \( f = c/\lambda \) (Equation 16.1).

**SOLUTION** Using Equation 16.1 to calculate each frequency, we find that

\[ f_2 - f_1 = \frac{c}{\lambda_2} - \frac{c}{\lambda_1} = \left(2.9979 \times 10^8 \text{ m/s}\right)\left(\frac{1}{0.34339 \text{ m}} - \frac{1}{0.36205 \text{ m}}\right) = 4.500 \times 10^7 \text{ Hz} \]
14. **REASONING** According to \( \omega = \sqrt{\frac{k}{m}} \) (Equation 10.11), the angular oscillation frequency \( \omega \) depends upon the mass \( m \) of the oscillator and the spring constant \( k \). The angular frequency \( \omega \) (in rad/s) of the motion of the oscillating mass is related to the frequency \( f \) (in Hz) of the resulting ELF radio waves by \( \omega = 2\pi f \) (Equation 10.6). We will use \( c = f\lambda \) (Equation 16.1) to determine the frequency of the ELF radio waves, where \( c \) is the speed of light in a vacuum and \( \lambda \) is the wavelength.

**SOLUTION** Squaring both sides of \( \omega = \sqrt{\frac{k}{m}} \) (Equation 10.11) and solving for \( k \), we obtain

\[
\frac{k}{m} = \omega^2 \quad \text{or} \quad k = m\omega^2 \quad (1)
\]

Substituting \( \omega = 2\pi f \) (Equation 10.6) into Equation (1) yields

\[
k = m(2\pi f)^2 = 4\pi^2 mf^2 \quad (2)
\]

Solving \( c = f\lambda \) (Equation 16.1) for \( f \) gives \( f = \frac{c}{\lambda} \). Substituting this result into Equation (2), we find that

\[
k = 4\pi^2 m \left( \frac{c}{\lambda} \right)^2 = 4\pi^2 \left( \frac{0.115 \text{ kg}}{4.80 \times 10^7 \text{ m}} \right)^2 = 177 \text{ N/m}
\]

15. **REASONING** The distance between Polaris and earth is equal to the speed of the light multiplied by the time it takes for the light to make the journey. The time is given. Since light is an electromagnetic wave, and all electromagnetic waves travel through a vacuum at the speed of light \( c \), the speed of the light is also known.

**SOLUTION** The distance \( s \) between Polaris and earth is \( s = ct \), where \( t \) is the time for the light to travel this distance. Using the fact that \( 1 \text{ yr} = 3.156 \times 10^7 \text{ s} \) (see the table of conversion factors at the front of the book), we find that

\[
s = ct = \left( 3.00 \times 10^8 \text{ m/s} \right) \left( \frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) = 6.4 \times 10^{18} \text{ m}
\]

16. **REASONING** The mirror must rotate with a sufficient angular speed \( \omega \) so that, after reflecting light from the source toward the fixed mirror, one of its faces is in the correct position to intercept light returning from the fixed mirror and reflect it toward the observer. Since the rotating mirror in Michelson’s setup has eight sides, the minimum angular displacement \( \Delta \theta \) meeting this condition is one eighth of a revolution: \( \Delta \theta = 0.125 \text{ rev} \). The mirror must rotate at least this far in the time \( \Delta t \) it takes the light to travel to the fixed mirror.
and back. The minimum, constant, angular speed of the mirror, then, can be found from
\[ \omega = \frac{\Delta \theta}{\Delta t} \] (Equation 8.2). The time \( \Delta t \) it takes the light to travel from the rotating mirror to
the fixed mirror and back is given by \( c = \frac{2d}{\Delta t} \) (Equation 2.1), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the
speed of light in a vacuum, and \( d = 35 \text{ km} \) is the distance between the rotating mirror and
the fixed mirror. Together, Equations 8.2 and 2.1 will allow us to determine the minimum
angular speed \( \omega \) of the rotating mirror. The angles between the rays of light shown in Figure
24.12 are exaggerated. In reality, the diameter of the rotating mirror is so much smaller than
the distance \( d \) to the fixed mirror that these two rays may be considered to be parallel.

**SOLUTION** Solving \( c = \frac{2d}{\Delta t} \) (Equation 2.1) for \( \Delta t \) yields
\[ \Delta t = \frac{2d}{c} \] (1)
Substituting Equation (1) into \( \omega = \frac{\Delta \theta}{\Delta t} \) (Equation 8.2), we obtain
\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{c(\Delta \theta)}{2d} = \frac{(3.00 \times 10^8 \text{ m/s})(0.125 \text{ rev})}{2(35 \times 10^3 \text{ m})} = 540 \text{ rev/s} \]

17. **SSM REASONING** We proceed by first finding the time \( t \) for sound waves to travel
between the astronauts. Since this is the same time it takes for the electromagnetic waves to
travel to earth, the distance between earth and the spaceship is \( d_{\text{earth-ship}} = ct \).

**SOLUTION** The time it takes for sound waves to travel at 343 m/s through the air between
the astronauts is
\[ t = \frac{d_{\text{astronaut}}}{v_{\text{sound}}} = \frac{1.5 \text{ m}}{343 \text{ m/s}} = 4.4 \times 10^{-3} \text{ s} \]
Therefore, the distance between the earth and the spaceship is
\[ d_{\text{earth-ship}} = ct = (3.0 \times 10^8 \text{ m/s})(4.4 \times 10^{-3} \text{ s}) = 1.3 \times 10^6 \text{ m} \]

18. **REASONING** Let \( R \) denote the average rate at which the laptop downloads information,
measured in bits per second (bps). This average rate is equal to the number \( N \) of bits
downloaded in a time \( t \) divided by the time: \( R = \frac{N}{t} \). Therefore, the number \( N \) of bits that the
laptop downloads is given by
\[ N = R \cdot t \] (1)
We note that 1 Mbps (megabit per second) is equal to \( 10^6 \) bps. The time \( t \) is the time it takes
the wireless signal to travel the distance \( d = 8.1 \text{ m} \) between the router and the laptop. This
time is determined by Equation 2.1 as
\[ t = \frac{d}{c} \] 

(2)

where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum.

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

\[ N = R \frac{d}{c} \] 

(3)

To convert the download rate \( R \) into bits per second, we use the equivalence 1 Mbps = 10^6 bps, and rewrite “bps” as “bits/s”

\[ R = \left( \frac{260 \text{ Mbps}}{1 \text{ Mbps}} \right) \times 10^6 \text{ bps} = 260 \times 10^6 \text{ bps} = 260 \times 10^6 \text{ bit/s} \]

Therefore, from Equation (3), the average number \( N \) of bits downloaded is

\[ N = R \frac{d}{c} = \left( 260 \times 10^6 \text{ bits/s} \right) \times \frac{8.1 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 7.0 \text{ bits} \]

19. **REASONING** Since the speed at which an electromagnetic wave travels is known, the round-trip travel time \( t \) of a single pulse from the lidar gun can be used to determine the distance \( d \) of the speeding vehicle from the gun as \( d = ct/2 \). Here, we have divided by a factor of two in order to account for the fact that the time given is that for the electromagnetic wave to travel out to the vehicle and return. Thus, in effect, the two pulses are used to measure the distances of the vehicle from the gun at two different instants. The difference in the distances is the distance that the speeder travels in the interval between the pulses. We can determine the vehicle’s speed by dividing the travel distance by the time interval of 0.450 s.

**SOLUTION** Applying the expression \( d = ct/2 \) for each pulse, we obtain the distance \( D \) traveled by the speeding vehicle between the two pulses as

\[ D = d_2 - d_1 = c \left( \frac{1}{2} t_2 - \frac{1}{2} t_1 \right) \]

Dividing this distance by the interval \( t_{\text{pulses}} \) between the pulses gives the speed \( v \) of the vehicle as

\[ v = \frac{D}{t_{\text{pulses}}} = \frac{c \left( \frac{1}{2} t_2 - \frac{1}{2} t_1 \right)}{t_{\text{pulses}}} = \frac{1}{2} c \left( \frac{t_2 - t_1}{t_{\text{pulses}}} \right) = \frac{1}{2} \left( 3.00 \times 10^8 \text{ m/s} \right) \left( \frac{1.27 \times 10^{-7} \text{ s}}{0.450 \text{ s}} \right) = 42.3 \text{ m/s} \]

20. **REASONING** The press conference is being broadcast via electromagnetic waves that travel at the speed of light \( c = 3.00 \times 10^8 \text{ m/s} \). According to the stated assumptions and to Equation 2.1, the maximum distance \( d \) that these waves travel between the politician and
viewer’s television set is \( d = ct_{\text{em}} \), where \( t_{\text{em}} \) is the travel time of the waves in reaching the television set. We are given no value for \( t_{\text{em}} \). However, we know that the total time required for the sound emitted by the politician to reach the viewer’s ears consists of \( t_{\text{em}} \) plus the travel time \( t_{\text{view}} \) of the sound waves emitted by the television set to reach the viewer’s ears. We also know that the television viewer and the reporter hear the politician’s words at exactly the same instant. Therefore, \( t_{\text{em}} + t_{\text{view}} = t_{\text{rep}} \), where \( t_{\text{rep}} \) is the travel time of the sound waves emitted by the politician to reach the reporter’s ears. We can rearrange this equation in order to obtain the necessary value of \( t_{\text{em}} \), namely \( t_{\text{em}} = t_{\text{rep}} - t_{\text{view}} \). To calculate the travel times \( t_{\text{rep}} \) and \( t_{\text{view}} \) of the sound waves through the air, we will use Equation 2.1 in the forms \( d_{\text{rep}} = v_{\text{sound}} t_{\text{rep}} \) and \( d_{\text{view}} = v_{\text{sound}} t_{\text{view}} \), where \( v_{\text{sound}} = 343 \text{ m/s} \), \( d_{\text{rep}} = 4.1 \text{ m} \), and \( d_{\text{view}} = 2.3 \text{ m} \).

SOLUTION As discussed in the REASONING, the maximum distance \( d \) between the television set and the politician is

\[
d = ct_{\text{em}}
\]

As also discussed in the REASONING, the travel time \( t_{\text{em}} \) of the electromagnetic waves being used to broadcast the press conference is

\[
t_{\text{em}} = t_{\text{rep}} - t_{\text{view}}
\]

Using Equation 2.1 in the forms \( d_{\text{rep}} = v_{\text{sound}} t_{\text{rep}} \) and \( d_{\text{view}} = v_{\text{sound}} t_{\text{view}} \), we can calculate the travel times \( t_{\text{rep}} \) and \( t_{\text{view}} \) of the sound waves through the air as follows:

\[
t_{\text{rep}} = \frac{d_{\text{rep}}}{v_{\text{sound}}} \quad \text{and} \quad t_{\text{view}} = \frac{d_{\text{view}}}{v_{\text{sound}}}
\]

Using Equation (2) and Equations (3), we find that Equation (1) becomes

\[
d = ct_{\text{em}} = c (t_{\text{rep}} - t_{\text{view}}) = c \left( \frac{d_{\text{rep}}}{v_{\text{sound}}} - \frac{d_{\text{view}}}{v_{\text{sound}}} \right) = \left( 3.00 \times 10^8 \text{ m/s} \right) \left( \frac{4.1 \text{ m}}{343 \text{ m/s}} - \frac{2.3 \text{ m}}{343 \text{ m/s}} \right) = 1.6 \times 10^6 \text{ m}
\]

21. REASONING Because the flash from the gunshot travels at the speed \( c = 3.00 \times 10^8 \text{ m/s} \) of light in a vacuum, it can make \( N \) round trips between the two mirrors in the time \( \Delta t_s \) that it takes the sound of the gunshot to make one round trip, returning as an echo. Therefore, in terms of the time \( \Delta t_f \) for one round-trip of the light flash, the number \( N \) of round trips of the flash is given by
\[ N = \frac{\Delta t_s}{\Delta t_f} \]

The time \( \Delta t \) it takes either the sound or the flash to travel the round-trip distance \( d \) between the gun and the cliff at a constant speed \( v \) is given by \( \Delta t = \frac{d}{v} \) (Equation 2.1). The sound of the gunshot travels at a speed \( v = 343 \text{ m/s} \), so Equation 2.1 yields both

\[
\Delta t_s = \frac{d}{v} \quad \text{Sound of echo} \quad \text{and} \quad \Delta t_f = \frac{d}{c} \quad \text{Light flash}
\]

**SOLUTION** Substituting Equations (2) into Equation (1) yields

\[
N = \frac{\Delta t_s}{\Delta t_f} = \left( \frac{d}{v} \right) \left( \frac{c}{d} \right) = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{343 \text{ m/s}} = 8.75 \times 10^5
\]

22. **REASONING AND SOLUTION** According to Equation 16.8, we have

\[
S = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.2 \times 10^{-3} \text{ W}}{\pi \left( 1.0 \times 10^{-3} \text{ m} \right)^2} = 3.8 \times 10^2 \text{ W/m}^2
\]

23. **REASONING AND SOLUTION** Using Equations 24.5b and 24.3, we find the following results:

a. \( E_{\text{rms}} = \sqrt{\frac{S}{c \varepsilon_0}} = \sqrt{\frac{1.23 \times 10^9 \text{ W/m}^2}{\left( 3.00 \times 10^8 \text{ m/s} \right) \left( 8.85 \times 10^{-12} \text{ C}^2 / \left( \text{N} \cdot \text{m}^2 \right) \right)}} = 6.81 \times 10^5 \text{ N/C} \)

b. \( B_{\text{rms}} = E_{\text{rms}} / c = 2.27 \times 10^{-3} \text{ T} \)

24. **REASONING** The magnitude \( E \) of the electric field in an electromagnetic wave is related to the magnitude \( B \) of the magnetic field according to \( E = cB \) (Equation 24.3), where \( c \) is the speed of light.

**SOLUTION** Using Equation 24.3, we find that

\[
E = cB = (3.00 \times 10^8 \text{ m/s}) (3.3 \times 10^{-6} \text{ T}) = 990 \text{ N/C}
\]
25. **SSM REASONING** The rms value \( E_{\text{rms}} \) of the electric field is related to the average energy density \( \bar{u} \) of the microwave radiation according to \( \bar{u} = \varepsilon_0 E_{\text{rms}}^2 \) (Equation 24.2b).

**SOLUTION** Solving for \( E_{\text{rms}} \) gives

\[
E_{\text{rms}} = \sqrt{\frac{\bar{u}}{\varepsilon_0}} = \sqrt{\frac{4 \times 10^{-14} \text{ J/m}^3}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)}} = 0.07 \text{ N/C}
\]

26. **REASONING** Since the energy density \( u \) is the energy per unit volume of space, the electromagnetic energy contained in a volume \( V \) is the product of \( u \) and \( V \). We are not given a value for \( u \). However, the energy density is related to the intensity \( S \) of the electromagnetic wave by \( u = \frac{S}{c} \) (Equation 24.4), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum.

**SOLUTION** Expressing the energy as the product of the energy density and the volume and using Equation 24.4 to relate the energy density to the intensity, we find that

\[
\text{Energy} = uV = \left( \frac{S}{c} \right) V = \left( \frac{1.0 \times 10^3 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} \right) (5.5 \text{ m}^3) = 1.8 \times 10^{-5} \text{ J}
\]

27. **SSM REASONING** The average intensity of a wave is the average power per unit area that passes perpendicularly through a surface. Thus, the average power \( \bar{P} \) of the wave is the product of the average intensity \( \bar{S} \) and the area \( A \). The average intensity is related to the rms-value \( E_{\text{rms}} \) of the electric field by \( \bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \).

**SOLUTION** The average power is \( \bar{P} = \bar{S} A \). Since \( \bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \) (Equation 24.5b), we have

\[
\bar{P} = \bar{S} A = \left(c\varepsilon_0 E_{\text{rms}}^2 \right) A
\]

\[
= (3.00 \times 10^8 \text{ m/s}) \left[ 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \right] \left( 2.0 \times 10^9 \text{ N/C} \right)^2 (1.6 \times 10^{-5} \text{ m}^2)
\]

\[
= 1.7 \times 10^{11} \text{ W}
\]

28. **REASONING** The average intensity \( \bar{S} \) is related to the distance \( r \) from the bulb by \( \bar{S} = \bar{P}/(4\pi r^2) \) (Equation 16.9), where \( \bar{P} \) is the average power radiated by the bulb.

The intensity of an electromagnetic wave is related to the magnitude \( E \) of its electric field by \( S = c\varepsilon_0 E^2 \) (Equation 24.5b). According to the discussion in Section 24.4, if the intensity is
an average intensity, then the value for the electric field must be an rms value, not a peak value. The peak value $E_0$ and the rms value $E_{\text{rms}}$ are related by $E_{\text{rms}} = E_0 / \sqrt{2}$.

**SOLUTION**

a. The average intensity of the wave is

$$\bar{S} = \frac{P}{4\pi r^2} = \frac{150.0 \text{ W}}{4\pi (5.00 \text{ m})^2} = 0.477 \text{ W/m}^2$$

(16.9)

b. The average intensity $\bar{S}$ is related to the rms value $E_{\text{rms}}$ of the electric field by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$ (Equation 24.5b). Solving for the electric field gives

$$E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{0.477 \text{ W/m}^2}{(3.00 \times 10^8 \text{ m/s})[8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)]}} = 13.4 \text{ N/C}$$

c. The rms value $E_{\text{rms}}$ of the electric field is related to the peak value $E_0$ by $E_{\text{rms}} = E_0 / \sqrt{2}$. The peak electric field is, therefore,

$$E_0 = \sqrt{2}E_{\text{rms}} = \sqrt{2}(13.4 \text{ N/C}) = 19.0 \text{ N/C}$$

29. **REASONING AND SOLUTION** Since the sun emits radiation uniformly in all directions, at a distance $r$ from the sun's center, the energy spreads out over a sphere of surface area $4\pi r^2$. Therefore, according to $S = P/(4\pi r^2)$ (Equation 16.9), the total power radiated by the sun is

$$P = S(4\pi r^2) = (1390 \text{ W/m}^2)(4\pi)(1.50 \times 10^{11} \text{ m})^2 = 3.93 \times 10^{26} \text{ W}$$

30. **REASONING** When a stationary charge is placed in an electric field, it experiences an electric force. The magnitude $F$ of the electric force is given by Equation 18.2 as $F = |q|E$, where $|q|$ is the magnitude of the charge and $E$ is the magnitude of the electric field.

When a stationary charge is placed in a magnetic field, it does not experience a magnetic force, because the charge is not moving. According to Equation 21.1, the magnitude of the magnetic force is related to the magnitude $B$ of the magnetic field by $F = |q|vB\sin \theta$, where $v$ is the speed of the charge and $\theta$ is the angle between the velocity of the charge and the magnetic field. Since the charge is stationary, $v = 0 \text{ m/s}$ and the magnetic force is zero.

When a moving charge is placed in an electric field, it experiences an electric force that is given by Equation 18.2. It does not matter whether the charge is stationary or moving.
When a charge moves \( (v \neq 0 \text{ m/s}) \) and its velocity is perpendicular to the magnetic field \((\theta = 90^\circ)\), it experiences a magnetic force, as specified by Equation 21.1.

**SOLUTION**

a. The magnitude of the electric force is \( F = |q|E \), where the magnitude of the electric field is related to the intensity \( S \) of the laser beam by \( S = c\varepsilon_0E^2 \) (Equation 24.5b). Therefore, the magnitude of the electric force is

\[
F = |q|E = |q|\sqrt{\frac{S}{c\varepsilon_0}}
\]

\[
= \left(2.6 \times 10^{-8} \text{ C}\right)\sqrt{\frac{2.5 \times 10^1 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}}} \left[8.85 \times 10^{-12} \text{ C}^2 / \left(\text{N} \cdot \text{m}^2\right)\right] = 2.5 \times 10^{-5} \text{ N}
\]

b. Since the particle is not moving, the magnetic force on it is zero, \( F = 0 \text{ N} \).

c. The electric force on the particle is the same whether it is moving or not, so the answer is the same as in part (a); \( F = 2.5 \times 10^{-5} \text{ N} \).

d. The magnitude of the magnetic force is given by Equation 21.1 as \( F = |q|vB\sin \theta \). The magnitude \( B \) of the magnetic field is related to the intensity \( S \) of the laser beam by \( S = cB^2/\mu_0 \) (Equation 24.5c). Thus, the magnetic force is

\[
F = |q|vB\sin \theta = |q|v\sqrt{\frac{\mu_0S}{c}} \sin \theta
\]

\[
= \left(2.6 \times 10^{-8} \text{ C}\right)\left(3.7 \times 10^4 \text{ m/s}\right)\sqrt{\frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{3.00 \times 10^8 \text{ m/s}}} \left(2.5 \times 10^1 \text{ W/m}^2\right) \sin 90.0^\circ
\]

\[
= 3.1 \times 10^{-3} \text{ N}
\]

31. **REASONING AND SOLUTION** The sun radiates sunlight (electromagnetic waves) uniformly in all directions, so the intensity at a distance \( r \) from the sun is given by Equation 16.9 as \( S = P/(4\pi r^2) \), where \( P \) is the power radiated by the sun. The power that strikes an area \( A_\perp \) oriented perpendicular to the direction in which the sunlight is radiated is \( P' = SA_\perp \), according to Equation 16.8. The 0.75-m\(^2\) patch of flat land on the equator at point \( Q \) is not perpendicular to the direction of the sunlight, however.
The figure at the right shows that

\[ A_{\perp} = (0.75 \, \text{m}^2) \cos 27^\circ \]

Therefore, the power striking the patch of land is

\[ P' = S A_{\perp} = \left( \frac{P}{4\pi r^2} \right) (0.75 \, \text{m}^2) \cos 27^\circ \]

\[ = \left[ \frac{3.9 \times 10^{26} \, \text{W}}{4\pi (1.5 \times 10^{11} \, \text{m})^2} \right] (0.75 \, \text{m}^2) \cos 27^\circ = 920 \, \text{W} \]

32. **REASONING** The average power \( \bar{P} \) delivered to the wall is equal to the energy delivered divided by the time \( t \). Thus, the time is equal to the energy divided by the average power, or \( t = \frac{\text{Energy}}{\bar{P}} \). The energy is given. The average power is equal to the average intensity \( \bar{S} \) times the area \( A \), so \( \bar{P} = \bar{S} A \). The area is known, and the average intensity is related to the rms-value \( B_{\text{rms}} \) of the wave’s magnetic field by \( \bar{S} = c B_{\text{rms}}^2 / \mu_0 \).

**SOLUTION** The time required to deliver the energy to the wall is

\[ t = \frac{\text{Energy}}{\bar{P}} \quad (6.10b) \]

Since \( \bar{P} = \bar{S} A \) (Equation 16.8), the expression for the time can be written as

\[ t = \frac{\text{Energy}}{\bar{S} A} = \frac{\text{Energy}}{\bar{S} A} \quad (1) \]

The average intensity is related to \( B_{\text{rms}} \) by \( \bar{S} = c B_{\text{rms}}^2 / \mu_0 \) (Equation 24.5c), where \( c \) is the speed of light in a vacuum and \( \mu_0 \) is the permeability of free space. Substituting this expression into Equation (1) and noting that \( A = 1.30 \, \text{cm}^2 = 1.30 \times 10^{-4} \, \text{m}^2 \) gives

\[ t = \frac{\text{Energy}}{\bar{S} A} = \frac{\text{Energy}}{c B_{\text{rms}}^2 / \mu_0} A \]

\[ = \frac{1850 \, \text{J}}{(3.00 \times 10^8 \, \text{m/s})(6.80 \times 10^{-4} \, \text{T})^2} \frac{1}{(4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A})(1.30 \times 10^{-4} \, \text{m}^2)} = 0.129 \, \text{s} \]
33. **REASONING** According to Equation 24.5b, the average intensity $\bar{S}$ of the infrared radiation is related to the rms value of the electric field $E_{\text{rms}}$ by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$. According to Equation 16.8, the average power $\bar{P}$ is equal to the average intensity times the area $A$ to which the power is being delivered, the area being that of a circle or $A = \pi r^2$. Thus, $\bar{P} = \bar{S}A = \bar{S}(\pi r^2)$. The average power is given by Equation 6.10b as the energy $Q$ absorbed by the leg divided by the time $t$, so that $t = Q/\bar{P}$. The energy absorbed by the leg is related to the rise in temperature $\Delta T$ by Equation 12.4 as $Q = cm \Delta T$, where $c$ is the specific heat capacity and $m$ is the mass.

**SOLUTION**

a. The average intensity of the infrared radiation is

$$\bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \quad (24.5b)$$

$$= \left(3.0 \times 10^8 \text{ m/s}\right)\left[8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)\right] (2800 \text{ N/C})^2 = 2.1 \times 10^4 \text{ W/m}^2$$

b. The average power delivered to the leg is

$$\bar{P} = \bar{S}A = \bar{S}(\pi r^2) = \left(2.1 \times 10^4 \text{ W/m}^2\right)\pi (4.0 \times 10^{-2} \text{ m})^2 = 1.1 \times 10^2 \text{ W} \quad (16.8)$$

c. Combining the relations $t = Q/\bar{P}$ and $Q = cm \Delta T$, the time required to raise the temperature by $2.0 \text{ C}^\circ$ is

$$t = \frac{Q}{\bar{P}} = \frac{cm \Delta T}{\bar{P}} = \frac{[3500 \text{ J/(kg} \cdot \text{C}^\circ)(0.28 \text{ kg})](2.0 \text{ C}^\circ)}{1.1 \times 10^2 \text{ W}} = 18 \text{ s}$$

34. **REASONING** The rms magnetic field $B_{\text{rms}}$ and the average intensity $\bar{S}_1$ of the gamma radiation emitted from the surface of the magnetar are related by

$$\bar{S}_1 = \frac{c}{\mu_0} B_{\text{rms}}^2 \quad (24.5c)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space and $c$ is the speed of light in a vacuum. The average intensity $\bar{S}_1$ of the gamma radiation is defined as the average power that passes perpendicularly through an area $A_1$. Therefore, if the radius of the magnetar is $r_1$, then the average intensity must be

$$\bar{S}_1 = \frac{\bar{P}}{A_1} \quad \bar{P} = \frac{\bar{P}}{4\pi r_1^2} \quad (1)$$

In Equation (1), $A_1 = 4\pi r_1^2$ is the surface area of the magnetar (assumed to be spherical). When the pulse of gamma radiation reaches earth, the average power $\bar{P}_1$ is spread out
uniformly over the surface of a spherical surface of radius \( R \) equal to the distance between earth and the magnetar. The area \( A \) of this sphere is \( A = 4\pi R^2 \), and the average intensity \( \overline{S}_2 \) of the pulse when it reaches the telescope is given by \( \overline{S}_2 = \frac{\overline{P}_1}{A} \), so we have that

\[
\overline{P}_1 = A\overline{S}_2 = 4\pi R^2 \overline{S}_2 \tag{2}
\]

The average power \( \overline{P}_2 \) intercepted by the telescope detector is proportional to the average intensity \( \overline{S}_2 \) and the surface area \( A_2 \) of the detector: \( \overline{P}_2 = A_2\overline{S}_2 \). Rearranging this relation, we obtain

\[
\overline{S}_2 = \frac{\overline{P}_2}{A_2} \tag{3}
\]

We will determine the average power \( \overline{P}_2 \) from the energy delivered to the detector and the duration \( \Delta t \) of the pulse via Equation 6.10b:

\[
\overline{P}_2 = \frac{\text{Energy}}{\Delta t} \tag{6.10b}
\]

**SOLUTION** Solving Equation 24.5c for \( B_{\text{rms}} \), we obtain

\[
B_{\text{rms}}^2 = \frac{\mu_0 \overline{S}_1}{c} \quad \text{or} \quad B_{\text{rms}} = \sqrt{\frac{\mu_0 \overline{S}_1}{c}} \tag{4}
\]

Substituting Equation (1) into Equation (4) yields

\[
B_{\text{rms}} = \sqrt{\frac{\mu_0 \overline{P}_1}{4\pi c r_1^2}} \tag{5}
\]

Substituting Equation (2) into Equation (5), we find that

\[
B_{\text{rms}} = \sqrt{\frac{\mu_0 \overline{P}_1}{4\pi c r_1^2}} \sqrt{A_2\overline{S}_2} = \frac{R}{r_1} \sqrt{\frac{\mu_0 \overline{S}_2}{c}} \tag{6}
\]

Substituting Equation (3) into Equation (6) gives

\[
B_{\text{rms}} = \frac{R}{r_1} \sqrt{\frac{\mu_0 \overline{S}_2}{c}} = \frac{R}{r_1} \sqrt{cA_2} \tag{7}
\]

Replacing the average power \( \overline{P}_2 \) in Equation (7) with Equation 6.10b, we finally obtain

\[
B_{\text{rms}} = \frac{R}{r_1} \sqrt{\frac{\mu_0 (\text{Energy})}{cA_2\Delta t}} = \frac{4.5\times10^{20} \text{ m}}{9.0\times10^3 \text{ m}} \frac{1}{\sqrt{\left(4\pi\times10^{-7} \text{ T} \cdot \text{m/A}\right)\left(8.4\times10^{-6} \text{ J}\right)\left(3.00\times10^8 \text{ m/s}\right)\left(75 \text{ m}^2\right)\left(0.24 \text{ s}\right)}} = 2.2\times10^6 \text{ T}
\]
35. **REASONING** The Doppler effect for electromagnetic radiation is given by Equation 24.6;

\[ f_o = f_s \left( 1 \pm \frac{v_{rel}}{c} \right) \quad \text{if} \quad v_{rel} \ll c \]

where \( f_o \) is the observed frequency, \( f_s \) is the frequency emitted by the source, and \( v_{rel} \) is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. According to Equation 16.1, the wavelength of these waves is \( \lambda = \frac{c}{f} \). Therefore, the Doppler shift can be written in terms of wavelengths:

\[ \frac{1}{\lambda_o} = \frac{1}{\lambda_s} \left( 1 \pm \frac{v_{rel}}{c} \right) \quad \text{if} \quad v_{rel} \ll c \]

**SOLUTION**

a. The wavelength \( \lambda_o \) of the light observed on earth is greater than the wavelength \( \lambda_s \) of the light when it is emitted from the distant galaxy (the source). Therefore, the frequency of the light observed on earth is less than the frequency of the light when it is emitted from the distant galaxy. Thus, the quantity in the brackets in Equation 24.6 must be less than one; it must be equal to \( \left[ 1 - \frac{v_{rel}}{c} \right] \). Since the minus sign applies, we can conclude that

[the galaxy must be receding from the earth].

b. We can find the speed of the galaxy relative to the earth by solving the wavelength version of Equation 24.6 for \( v_{rel} \):

\[ v_{rel} = c \left( 1 - \frac{\lambda_s}{\lambda_o} \right) = (3.0 \times 10^8 \text{ m/s}) \left( 1 - \frac{434.1 \text{ nm}}{438.6 \text{ nm}} \right) = 3.1 \times 10^6 \text{ m/s} \]

36. **REASONING** Using the Doppler effect, we will find the relative speed between the speeding car and the police car. Since we know the speed of the police car relative to the ground, we can determine the speed of the car relative to the ground, once the relative speed \( v_{rel} \) is found.

There are two Doppler frequency changes in this situation. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency \( f_o \) that is different from the emitted frequency \( f_s \). The second Doppler shift occurs after the wave reflects from the speeder's car and returns to the police car.

The Doppler frequency for electromagnetic radiation is given by \( f_o = f_s [1 \pm (v_{rel}/c)] \) (Equation 24.6), where \( v_{rel} \) is the relative speed between the source and the observer of the radiation, and the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. Since the distance
between the police car and the speeder's car is increasing, they are moving apart, and according to Equation 24.6, the first Doppler frequency change is given by \( f_o - f_s = -f_s (v_{rel} / c) \). After the wave reflects from the speeder's car it returns to the police car where it is observed to have a frequency \( f'_o \) that is different from its frequency \( f_o \) at the instant of reflection. Equation 24.6 may again be used, this time to determine the second Doppler frequency shift: \( f'_o - f_o = -f_s (v_{rel} / c) \). We can use these two equations for the frequency shifts to determine an expression for the total Doppler change in frequency. Adding the two equations, we have

\[
(f'_o - f_o) + (f_o - f_s) = -f_o \left( \frac{v_{rel}}{c} \right) - f_s \left( \frac{v_{rel}}{c} \right)
\]

where we have assumed that \( f_s \) and \( f_o \) differ only by a negligibly small amount, so that \( f_o \approx f_s \). Rearranging, we have

\[
f_s - f'_o \approx 2 f_s \left( \frac{v_{rel}}{c} \right)
\]

**SOLUTION** Solving for the relative speed \( v_{rel} \) gives

\[
v_{rel} \approx \left( \frac{f_s - f'_o}{2 f_s} \right) c = \left[ \frac{320 \text{ Hz}}{2(7.0 \times 10^9 \text{ Hz})} \right] (3.0 \times 10^8 \text{ m/s}) = 6.9 \text{ m/s}
\]

The relative speed \( v_{rel} \) is related to the speeds of the vehicles with respect to the ground by \( v_{rel} = v_{speeder} - v_{police} \). Therefore, the speeder's speed with respect to the ground is

\[
v_{speeder} = v_{rel} + v_{police} = 6.9 \text{ m/s} + 25 \text{ m/s} = 32 \text{ m/s}
\]

**37. SSM REASONING** The Doppler effect for electromagnetic radiation is given by Equation 24.6, \( f_o = f_s \left( 1 \pm \frac{v_{rel}}{c} \right) \), where \( f_o \) is the observed frequency, \( f_s \) is the frequency emitted by the source, and \( v_{rel} \) is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart.

**SOLUTION**

a. At location A, the galaxy is moving away from the earth with a relative speed of

\[
v_{rel} = (1.6 \times 10^6 \text{ m/s}) - (0.4 \times 10^6 \text{ m/s}) = 1.2 \times 10^6 \text{ m/s}
\]
Therefore, the minus sign in Equation 24.6 applies and the observed frequency for the light from region \( A \) is

\[
f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = (6.200 \times 10^{14} \text{ Hz}) \left( 1 - \frac{1.2 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = \text{[corrected]} \]

b. Similarly, at location \( B \), the galaxy is moving away from the earth with a relative speed of

\[
v_{rel} = (1.6 \times 10^6 \text{ m/s}) + (0.4 \times 10^6 \text{ m/s}) = 2.0 \times 10^6 \text{ m/s}
\]

The observed frequency for the light from region \( B \) is

\[
f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = (6.200 \times 10^{14} \text{ Hz}) \left( 1 - \frac{2.0 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = \text{6.159} \times 10^{14} \text{ Hz}
\]

38. **REASONING** The observed frequency is \( f_o = f_s \left( 1 \pm \frac{v_{rel}}{c} \right) \) according to Equation 24.6. The frequency \( f_s \) emitted by the source is the same in each case, so that only the direction of the relative motion and the relative speed \( v_{rel} \) determine the observed frequency. In situations A and B the observer and the source move away from each other, and the minus sign in Equation 24.6 applies. In situation C the observer and the source move toward each other, and the plus sign applies. Thus, the observed frequency is largest in C. To distinguish between A and B, we note that the relative speed in A is \( 2v - v = v \), whereas in B the relative speed is \( 2v + v = 3v \). The greater relative speed means that the term \( v_{rel}/c \) is greater in B than in A, and since the minus sign applies, the observed frequency is more reduced in B than in A. We conclude, then, that the situations are ranked in descending order according to the observed frequencies as follows: C (largest), A, B

**SOLUTION** To apply Equation 24.6 to calculate the observed frequencies, we need the relative speed in each situation. The relative speeds and the observed frequencies are:

**[Situation A, minus sign in Equation 24.6]**

\[
v_{rel} = 2v - v = v
\]

\[
f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = f_s \left( 1 - \frac{v}{c} \right) = (4.57 \times 10^{14} \text{ Hz}) \left( 1 - \frac{1.50 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = \text{4.55} \times 10^{14} \text{ Hz}
\]

**[Situation B, minus sign in Equation 24.6]**

\[
v_{rel} = 2v + v = 3v
\]

\[
f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = f_s \left( 1 - \frac{3v}{c} \right) = (4.57 \times 10^{14} \text{ Hz}) \left[ 1 - \frac{3(1.50 \times 10^6 \text{ m/s})}{3.0 \times 10^8 \text{ m/s}} \right] = \text{4.50} \times 10^{14} \text{ Hz}
\]
[Situation C, plus sign in Equation 24.6]

\[ v_{\text{rel}} = v + v = 2v \]

\[ f_o = f_s \left( 1 + \frac{v_{\text{rel}}}{c} \right) = f_s \left( 1 + \frac{2v}{c} \right) = \left( 4.57 \times 10^{14} \text{ Hz} \right) \left[ 1 + \frac{2 \left( 1.50 \times 10^6 \text{ m/s} \right)}{3.00 \times 10^8 \text{ m/s}} \right] = 4.62 \times 10^{14} \text{ Hz} \]

39. **REASONING AND SOLUTION**

a. The polarizer reduces the intensity of the light by a factor of two or to \( 0.55 \text{ W/m}^2 \).

b. The intensity of the light leaving the analyzer is given by Malus’ law.

\[ S = (0.55 \text{ W/m}^2) \cos^2 75^\circ = 3.7 \times 10^{-2} \text{ W/m}^2 \]

40. **REASONING**

**Drawing A** The transmission axes of the polarizer and analyzer are parallel to each other, so all the light transmitted by the polarizer is completely transmitted by the analyzer.

**Drawing B** The transmission axes of the polarizer and analyzer are perpendicular to each other, so no light is transmitted through the analyzer.

**Drawing C** The transmission axes of the polarizer and analyzer make an angle of 30.0° with respect to each other. Thus, some of the light transmitted by the polarizer, but not all, is transmitted through the analyzer.

Therefore, we expect the transmitted intensities to be in the following decreasing order (largest first): A, C, B.

**SOLUTION** Since the incident light is unpolarized, the average intensity \( \overline{S}_1 \) of the light transmitted by the polarizer is one-half the average intensity \( \overline{S}_0 \) of the incident light, or \( \overline{S}_1 = \frac{1}{2} \overline{S}_0 = \frac{1}{2} (48 \text{ W/m}^2) = 24 \text{ W/m}^2 \). The average intensity \( \overline{S}_2 \) of the light transmitted by the analyzer is given by Malus’ law, Equation 24.7, as \( \overline{S}_2 = \overline{S}_1 \cos^2 \theta \), where \( \theta \) is the angle between the direction of polarization and the transmission axis. The average intensity of the transmitted beams for each of the three cases is

A \( \overline{S}_2 = \overline{S}_1 \cos^2 \theta = \left( 24 \text{ W/m}^2 \right) \cos^2 0^\circ = 24 \text{ W/m}^2 \)

B \( \overline{S}_2 = \overline{S}_1 \cos^2 \theta = \left( 24 \text{ W/m}^2 \right) \cos^2 90^\circ = 0 \text{ W/m}^2 \)

C \( \overline{S}_2 = \overline{S}_1 \cos^2 \theta = \left( 24 \text{ W/m}^2 \right) \cos^2 (60.0^\circ - 30.0^\circ) = 18 \text{ W/m}^2 \)
41. **REASONING** Malus’ law, \( \overline{S} = \overline{S}_0 \cos^2 \theta \) (Equation 24.7), relates the average intensity \( \overline{S}_0 \) of polarized light incident on the polarizing sheet to the average intensity \( \overline{S} \) of light transmitted by the sheet, where \( \theta \) is the angle between the polarization axis of the incident light and the transmission axis of the polarizing sheet. The incident light is horizontally polarized, so the angle \( \theta \) is measured from the horizontal, and is, therefore, the angle we seek.

**SOLUTION** Solving \( \overline{S} = \overline{S}_0 \cos^2 \theta \) (Equation 24.7) for \( \theta \), we obtain

\[
\cos^2 \theta = \frac{\overline{S}}{\overline{S}_0} \quad \text{or} \quad \cos \theta = \sqrt{\frac{\overline{S}}{\overline{S}_0}} \quad \text{or} \quad \theta = \cos^{-1}\left(\sqrt{\frac{\overline{S}}{\overline{S}_0}}\right)
\]  

(2)

Substituting the given values of the average incident and transmitted intensities yields

\[
\theta = \cos^{-1}\left(\sqrt{\frac{0.764 \text{ W/m}^2}{0.883 \text{ W/m}^2}}\right) = 21.5^\circ
\]

42. **REASONING** Since no light passes through the second sheet, it must be in a crossed configuration with the first sheet. In other words, its transmission axis must be perpendicular to the transmission axis of the first sheet. Therefore, to find its orientation with respect to the vertical, we will determine the angle that the axis of the first sheet makes with respect to the vertical and add 90.0º to it. The angle that the axis of the first sheet makes with respect to the vertical can be obtained directly from Malus’ law.

**SOLUTION** The angle \( \theta_2 \) that the second sheet makes with respect to the vertical is

\[
\theta_2 = \theta_1 + 90.0^\circ
\]

where \( \theta_1 \) is the angle giving the orientation of the axis of the first sheet. Using Malus’ law (Equation 24.7) and recognizing that the ratio of the intensity \( \overline{S} \) passing through the first sheet to the incident intensity \( \overline{S}_0 \) is \( \overline{S} / \overline{S}_0 = 0.94 \), we find that

\[
\overline{S} = \overline{S}_0 \cos^2 \theta_1 \quad \text{or} \quad \frac{\overline{S}}{\overline{S}_0} = 0.94 = \cos^2 \theta_1 \quad \text{or} \quad \theta_1 = \cos^{-1}\sqrt{0.94} = 14^\circ
\]

The angle of the axis of the second sheet with respect to the vertical is, then,

\[
\theta_2 = \theta_1 + 90.0^\circ = 14^\circ + 90.0^\circ = 104^\circ
\]
43. **REASONING** If the intensity of the unpolarized light is \( I_0 \), the intensity of the polarized light leaving the polarizer is \( \frac{1}{2} I_0 \). By Malus’ law, the intensity of the light leaving the insert is \( \frac{1}{2} I_0 \cos^2 \theta \). From the results of Conceptual Example 8, the intensity of light leaving the analyzer is \( \frac{1}{2} I_0 \cos^2 \theta \sin^2 \theta \).

**SOLUTION** The intensity \( I \) of light that reaches the photocell is

\[
I = \frac{1}{2} I_o \cos^2 \theta \sin^2 \theta = \frac{1}{2} (150 \text{ W/m}^2) \cos^2 30.0^\circ \sin^2 30.0^\circ = 14 \text{ W/m}^2
\]

44. **REASONING** If the incident light is unpolarized, the average intensity of the transmitted light is one-half the average intensity of the incident light, independent of the angle of the transmission axis. Thus, the average intensity of the transmitted light remains the same as the polarizing material is rotated.

If the incident light is polarized along the \( z \) axis, the direction of polarization and the transmission axis are initially parallel to each other, and the maximum amount of light is transmitted. As the polarizing material is rotated, the intensity of the transmitted light decreases in accord with Malus’ law.

If the incident light is polarized along the \( y \) axis, the direction of polarization and the transmission axis are initially perpendicular to each other, and no light is transmitted. As the polarizing material is rotated, the average intensity of the transmitted light increases.

**SOLUTION**

a. Since the incident light is unpolarized, the average intensity of the transmitted light is one-half the average intensity of the incident light. Therefore, for both \( \alpha = 0^\circ \) and \( 35^\circ \), we have

\[
\bar{S} = \frac{1}{2} \bar{S}_0 = \frac{1}{2} \left( 7.0 \text{ W/m}^2 \right) = 3.5 \text{ W/m}^2
\]

b. When the incident light is polarized along the \( z \) axis, the direction of polarization and the transmission axis are initially parallel to each other. Therefore, the angle \( \alpha \) is the same as the angle \( \theta \) between the transmission axis of the polarizer and the direction of the polarization. According to Malus’ law (Equation 24.7), the average intensity of the transmitted light is given by

\[
\bar{S} = \bar{S}_0 \cos^2 \theta = \left( 7.0 \text{ W/m}^2 \right) \cos^2 0^\circ = 7.0 \text{ W/m}^2
\]

\[
\bar{S} = \bar{S}_0 \cos^2 \theta = \left( 7.0 \text{ W/m}^2 \right) \cos^2 35^\circ = 4.7 \text{ W/m}^2
\]
c. When the incident light is polarized along the $y$ axis, the direction of polarization and the transmission axis are initially perpendicular to each other. The angle $\theta$ in Malus’ law is the angle between the direction of polarization (along the $y$ axis) and the transmission axis (measured relative to the $z$ axis). It is related to the angle $\alpha$ according to $\theta = 90.0^\circ - \alpha$. The average intensity of the transmitted light is, therefore,

$$\bar{S} = \bar{S}_0 \cos^2 \theta = \left(7.0\ \text{W/m}^2 \right) \cos^2 (90.0^\circ - 0^\circ) = 0\ \text{W/m}^2$$

$$\bar{S} = \bar{S}_0 \cos^2 \theta = \left(7.0\ \text{W/m}^2 \right) \cos^2 (90.0^\circ - 35^\circ) = 2.3\ \text{W/m}^2$$

The table below summarizes the results:

<table>
<thead>
<tr>
<th>Incident Light</th>
<th>Average Intensity of Transmitted Light</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = 0^\circ$</td>
</tr>
<tr>
<td>(a) Unpolarized</td>
<td>3.5 W/m²</td>
</tr>
<tr>
<td>(b) Polarized parallel to $z$ axis</td>
<td>7.0 W/m²</td>
</tr>
<tr>
<td>(c) Polarized parallel to $y$ axis</td>
<td>0 W/m²</td>
</tr>
</tbody>
</table>

45. **REASONING** Since the incident beam is unpolarized, the intensity of the light transmitted by the first sheet of polarizing material is one-half the intensity of the incident beam. The beams striking the second and third sheets of polarizing material are polarized, so the average intensity $\bar{S}$ of the light transmitted by each sheet is given by Malus’ law, $\bar{S} = \bar{S}_0 \cos^2 \theta$, where $\bar{S}_0$ is the average intensity of the light incident on each sheet.

**SOLUTION** The average intensity $\bar{S}_1$ of the light leaving the first sheet is one-half the intensity of the incident beam, so $\bar{S}_1 = \frac{1}{2} \left(1260.0\ \text{W/m}^2 \right) = 630.0\ \text{W/m}^2$. The intensity $\bar{S}_2$ of the light leaving the second sheet of polarizing material is given by Malus’ law, Equation 24.7, $\bar{S}_2 = \bar{S}_1 \cos^2 \theta$, where $\theta$ is the angle between the polarization of the incident beam and the transmission axis of the second sheet:

$$\bar{S}_2 = \left(630.0\ \text{W/m}^2 \right) \cos^2 (55.0^\circ - 19.0^\circ) = 412\ \text{W/m}^2$$

The intensity $\bar{S}_3$ of the light leaving the third sheet of polarizing material is $\bar{S}_3 = \bar{S}_2 \cos^2 \theta$, where $\theta$ is the angle between the polarization of the incident beam and the transmission axis of the third sheet:

$$\bar{S}_3 = \left(412\ \text{W/m}^2 \right) \cos^2 (100.0^\circ - 55.0^\circ) = 206\ \text{W/m}^2$$
46. **REASONING** The polarizer, the insert and the analyzer in the set-up in Figure 24.24(a) all reduce the intensity of the light that reaches the photocell. The polarizer reduces the intensity by a factor of one-half, as described in Section 24.6 of the text; if the average intensity of the incident unpolarized light is $\bar{T}$, the average intensity of the polarized light that leaves the polarizer and strikes the insert is $\bar{S}_0 = \bar{T}/2$. According to Malus’ law (see Equation 24.7), the average intensity $\bar{S}_{\text{insert}}$ of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta$, where $\theta$ is the relative angle between the transmission axes of the polarizer and the insert. The intensity of the light is further reduced as the polarized light passes through the analyzer. Malus’ law can be used in succession at each piece of polarizing material to determine the intensity that reaches the photocell, both with and without the presence of the analyzer.

**SOLUTION** When the analyzer is present, the average intensity reaching the photocell is equal to the average intensity that leaves the analyzer. The average intensity leaving the analyzer is, according to Malus’ law, $\bar{S}_{\text{insert}} \cos^2 \phi$, where $\bar{S}_{\text{insert}}$ is the average intensity that leaves the insert and reaches the analyzer, and $\phi$ is the relative angle between the transmission axes of the analyzer and insert. From Figure 24.24(a) we see that $\phi = 90^\circ - \theta$. The average intensity of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta$, according to Malus’ law. Therefore, when the analyzer is present as shown in Figure 24.24(a), the average intensity leaving the analyzer and reaching the photocell is

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} \cos^2 \phi = (\bar{S}_0 \cos^2 \theta) \cos^2 (90^\circ - \theta)$$

This expression can be solved for $\bar{S}_0$ to determine the average intensity leaving the polarizer:

$$\bar{S}_0 = \frac{\bar{S}_{\text{photocell}}}{\cos^2 \theta \cos^2 (90^\circ - \theta)} = \frac{110 \text{ W/m}^2}{(\cos^2 23^\circ) \cos^2 (90^\circ - 23^\circ)} = 850 \text{ W/m}^2$$

If the analyzer were removed from the setup, everything else remaining the same, the intensity reaching the photocell would be equal to the intensity that leaves the insert. Therefore, if the analyzer were removed, the intensity reaching the photocell would be

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta = (850 \text{ W/m}^2) \cos^2 23^\circ = 720 \text{ W/m}^2$$

47. **SSM REASONING** The average intensity of light leaving each analyzer is given by Malus’ Law (Equation 24.7). Thus, intensity of the light transmitted through the first analyzer is

$$\bar{S}_1 = \bar{S}_0 \cos^2 27^\circ$$

Similarly, the intensity of the light transmitted through the second analyzer is

$$\bar{S}_2 = \bar{S}_1 \cos^2 27^\circ = \bar{S}_0 \cos^4 27^\circ$$
And the intensity of the light transmitted through the third analyzer is

\[ S_3 = S_2 \cos^2 27^\circ = S_0 \cos^6 27^\circ \]

If we generalize for the \( N \)th analyzer, we deduce that

\[ S_N = S_{N-1} \cos^2 27^\circ = S_0 \cos^{2N} 27^\circ \]

Since we want the light reaching the photocell to have an intensity that is reduced by a least a factor of one hundred relative to the first analyzer, we want \( S_N / S_0 = 0.010 \). Therefore, we need to find \( N \) such that \( \cos^{2N} 27^\circ = 0.010 \). This expression can be solved for \( N \).

**SOLUTION**

Taking the common logarithm of both sides of the last expression gives

\[
2N \log(\cos 27^\circ) = \log 0.010 \quad \text{or} \quad N = \frac{\log 0.010}{2 \log (\cos 27^\circ)} = 20
\]

48. **REASONING** No light intensity will pass through two adjacent polarizers that are in a crossed configuration, that is, whose transmission axes are oriented perpendicular to one another. With this in mind, let’s remove the polarizers one by one (see the following drawing). When A is removed, no two of the remaining adjacent polarizers are crossed. When B is removed, A and C are left in a crossed configuration. When C is removed, B and D are left in a crossed configuration. When D is removed, no two of the remaining adjacent polarizers are crossed. Thus, when sheet B or C is removed, the intensity transmitted on the right is zero, and when sheet A or D is removed, the intensity transmitted on the right is greater than zero.

We can anticipate that the greater intensity is transmitted on the right when sheet D is removed. To begin with, we note that the polarization directions of the light striking B and C are the same, no matter whether A or D is removed, with the result that B and C absorb the same fraction of the intensity in either situation. When A is removed, however, D is a third sheet that absorbs light intensity. In contrast, when D is removed, A is present as a third sheet, but it absorbs none of the light intensity. This is because the transmission axis of A is vertical and matches the direction in which the incident light is polarized. We conclude, therefore, that the greater light intensity is transmitted when D is removed.
**SOLUTION** When light with an average intensity $S_0$ is polarized at an angle $\theta$ with respect to the polarization axis of a polarizer, the average intensity $S$ that is transmitted through the polarizer is given by Malus’ law as $S = S_0 \cos^2 \theta$ (Equation 24.7). The light that passes through is polarized in the direction of the transmission axis. In this problem, each of the polarizers, therefore, transmits a light intensity that is smaller than the incident light by a factor of $\cos^2 \theta$. We use this insight now to determine the transmitted light intensity in the two situations that result when A is removed and when D is removed.

[A is removed; B, C, and D remain]

$S = \left(27 \text{ W/m}^2\right) \cos^2 30.0^\circ \cos^2 60.0^\circ \cos^2 30.0^\circ = 3.8 \text{ W/m}^2$

[B is removed; A, C, and D remain]

$S = 0 \text{ W/m}^2$

[C is removed; A, B, and D remain]

$S = 0 \text{ W/m}^2$

[D is removed; A B, and C remain]

$S = \left(27 \text{ W/m}^2\right) \cos^2 0.0^\circ \cos^2 30.0^\circ \cos^2 60.0^\circ = 5.1 \text{ W/m}^2$

49. **REASONING** The wavelength $\lambda$ of a wave is related to its speed $v$ and frequency $f$ by $\lambda = \frac{v}{f}$ (Equation 16.1). Since blue light and orange light are electromagnetic waves, they travel through a vacuum at the speed of light $c$; thus, $v = c$.

**SOLUTION**

a. The wavelength of the blue light is

$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.34 \times 10^{14} \text{ Hz}} = 4.73 \times 10^{-7} \text{ m}$

Since 1 nm = $10^{-9}$ m,

$\lambda = \left(4.73 \times 10^{-7} \text{ m}\right) \left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 473 \text{ nm}$

b. In a similar manner, we find that the wavelength of the orange light is

$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{4.95 \times 10^{14} \text{ Hz}} = 6.06 \times 10^{-7} \text{ m} = 606 \text{ nm}$
50. **REASONING** The relationship between the intensity $S$ of an electromagnetic wave and its electric field $E$ is given by Equation 24.5b as $S = c\varepsilon_0 E^2$. For example, if the magnitude of the electric field triples, the intensity increases by a factor of $3^2 = 9$.

The magnitude of the magnetic field is given by Equation 24.3 as $B = E/c$. Even though the magnitude of the magnetic field is much smaller than that of the electric field, tripling the magnetic field also causes the intensity to increase by a factor of $3^2 = 9$. This can be seen by examining Equation 24.5c, $S = \frac{c}{\mu_0} B^2$.

**SOLUTION**

a. When the magnitude of the electric field is 315 N/C, the intensity of the electromagnetic wave is

$$S = c\varepsilon_0 E^2 = \left(3.00 \times 10^8 \text{ m/s}\right)\left[8.85 \times 10^{-12} \text{ C}^2 / \left(\text{N} \cdot \text{m}^2\right)\right](315 \text{ N/C})^2 = 263 \text{ W/m}^2$$

When the magnitude of the electric field is 945 N/C, the intensity of the electromagnetic wave is

$$S = c\varepsilon_0 E^2 = \left(3.00 \times 10^8 \text{ m/s}\right)\left[8.85 \times 10^{-12} \text{ C}^2 / \left(\text{N} \cdot \text{m}^2\right)\right](945 \text{ N/C})^2 = 2370 \text{ W/m}^2$$

b. The magnitudes of the magnetic fields associated with each electric field are

$$B = \frac{E}{c} = \frac{315 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 1.05 \times 10^{-6} \text{ T}$$

$$B = \frac{E}{c} = \frac{945 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 3.15 \times 10^{-6} \text{ T}$$

c. The intensities of the waves associated with each value of the magnetic field are

$$S = \frac{c}{\mu_0} B^2 = \frac{3.00 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \left(1.05 \times 10^{-6} \text{ T}\right)^2 = 263 \text{ W/m}^2$$

$$S = \frac{c}{\mu_0} B^2 = \frac{3.00 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \left(3.15 \times 10^{-6} \text{ T}\right)^2 = 2370 \text{ W/m}^2$$

51. **SSM REASONING** The electromagnetic wave will be picked up by the radio when the resonant frequency $f_0$ of the circuit in Figure 24.4 is equal to the frequency of the broadcast wave, or $f_0 = 1400$ kHz. This frequency, in turn, is related to the capacitance $C$ and inductance $L$ of the circuit via $f_0 = 1/(2\pi\sqrt{LC})$ (Equation 23.10). Since $C$ is known, we can use this relation to find the inductance.
**SOLUTION** Solving the relation \( f_0 = \frac{1}{2\pi\sqrt{LC}} \) for the inductance \( L \), we find that

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \left(1400 \times 10^3 \text{ Hz}\right)^2 \left(8.4 \times 10^{-11} \text{ F}\right)} = 1.5 \times 10^{-4} \text{ H}
\]

---

52. **REASONING AND SOLUTION** The number of wavelengths that can fit across the width \( W \) of your thumb is \( W/\lambda \). From Equation 16.1, we know that \( \lambda = c/f \), so

\[
\text{No. of wavelengths} = \frac{W}{\lambda} = \frac{Wf}{c} = \frac{(2.0 \times 10^{-2} \text{ m})(5.5 \times 10^{14} \text{ Hz})}{3.0 \times 10^8 \text{ m/s}} = 3.7 \times 10^4
\]

---

53. **SSM REASONING AND SOLUTION**

a. According to Equation 24.5b, the average intensity is \( \bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \). In addition, the average intensity is the average power \( \bar{P} \) divided by the area \( A \). Therefore,

\[
E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{\bar{P}}{c\varepsilon_0 A}} = \sqrt{\frac{1.20 \times 10^4 \text{ W}}{(3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))(135 \text{ m}^2)}} = 183 \text{ N/C}
\]

b. Then, from \( E_{\text{rms}} = cB_{\text{rms}} \) (Equation 24.3), we have

\[
B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{183 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 6.10 \times 10^{-7} \text{ T}
\]

---

54. **REASONING** There are two Doppler frequency changes in the emitted wave in this case. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency \( f_0 \) that is different from the emitted (source) frequency \( f_s \). The wave then reflects and returns to the police car, where it is observed to have a frequency \( f'_0 \) that is different than its frequency \( f_0 \) at the instant of reflection. Although the police car is now moving, the relative motion of the two vehicles is one of approach. In Example 6, it is shown that, when the source and the observer of the radar are approaching each other, the magnitude of the difference between frequency of the emitted wave and the wave that returns to the police car after reflecting from the speeder's car is

\[
f'_0 - f_s \approx 2f_s \left(\frac{v_{\text{rel}}}{c}\right)
\]

where \( v_{\text{rel}} \) is the relative speed between the speeding car and the police car.
**SOLUTION** Since the police car is moving to the right at 27 m/s, while the speeder is coming from behind at 39 m/s, the relative speed \( v_{\text{rel}} \) is \( 39 \text{ m/s} - 27 \text{ m/s} = 12 \text{ m/s} \). The total Doppler change in frequency is, therefore,

\[
f'_0 - f_s = 2(8.0 \times 10^9 \text{ Hz}) \left( \frac{12 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = 640 \text{ Hz}
\]

55. **SSM REASONING** In experiment 1, the light falling on the polarizer is unpolarized, so the average intensity \( \bar{S} \) transmitted by it is \( \bar{S} = \frac{1}{2} \bar{S}' \), where \( \bar{S}' \) is the intensity of the incident light. The intensity of the light passing through the analyzer and reaching the photocell can be found by using Malus’ law.

In the second experiment, all of the polarized light passes through the polarizer, so the average intensity reaching the analyzer is \( \bar{S}' \). The intensity of the light passing through the analyzer and reaching the photocell can again be found by using Malus’ law. By setting the intensities of the light reaching the photocells in experiments 1 and 2 equal, we can determine the number of additional degrees that the analyzer in experiment 2 must be rotated relative to that in experiment 1.

**SOLUTION** In experiment 1 the light intensity incident on the analyzer is \( \bar{S}_0 = \frac{1}{2} \bar{S}' \). Malus’ law (Equation 24.7) gives the average intensity \( \bar{S}_1 \) of the light reaching the photocell as

\[
\bar{S}_1 = \bar{S}_0 \cos^2 60.0^\circ = \frac{1}{2} \bar{S}' \cos^2 60.0^\circ
\]

In experiment 2 the incident light is polarized along the axis of the polarizer, so all the light is transmitted by the polarizer. Thus, the light intensity incident on the analyzer is \( \bar{S}_0 = \bar{S}' \). Malus’ law again gives the average intensity \( \bar{S}_2 \) of the light reaching the photocell as

\[
\bar{S}_2 = \bar{S}_0 \cos^2 \theta = \bar{S}' \cos^2 \theta
\]

Setting \( \bar{S}_1 = \bar{S}_2 \), we have

\[
\frac{1}{2} \bar{S}' \cos^2 60.0^\circ = \bar{S}' \cos^2 \theta
\]

Algebraically eliminating \( \bar{S}' \) from this equation, we have that

\[
\frac{1}{2} \cos^2 60.0^\circ = \cos^2 \theta \quad \text{or} \quad \theta = \cos^{-1} \left( \sqrt{\frac{1}{2} \cos^2 60.0^\circ} \right) = 69.3^\circ
\]

The number of additional degrees that the analyzer must be rotated is \( 69.3^\circ - 60.0^\circ = 9.3^\circ \). The angle \( \theta \) is increased by the additional rotation.
56. **REASONING AND SOLUTION**  The intensity $S$ of a wave is the power passing perpendicularly through a surface divided by the area $A$ of the surface. But power is the total energy $U$ per unit time $t$, so the intensity can be written as

$$S = \frac{\text{Total energy}}{\text{Time} \cdot \text{Area}} = \frac{U}{tA}$$

Equation 24.5c relates the intensity $S$ of the electromagnetic wave to the magnitude $B$ of its magnetic field; namely $S = (c / \mu_0)B^2$. Combining these two results, we have

$$U = cB^2 \mu_0$$

If the rms value for the magnetic field is used, the energy becomes the average energy $\bar{U}$. Thus, the average energy that this wave carries through the window in a 45 s phone call is

$$\bar{U} = \frac{c}{\mu_0}B_{\text{rms}}^2 tA = \left( \frac{3.0 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \right) (1.5 \times 10^{-10} \text{ T})^2 (45 \text{ s})(0.20 \text{ m}^2) = 4.8 \times 10^{-5} \text{ J}$$

57. **REASONING**  The equation that represents the wave mathematically is $y = A \sin(2\pi ft - 2\pi x / \lambda)$. In this expression the amplitude is $A = 156$ N/C. The wavelength $\lambda$ can be calculated using Equation 16.1, and we obtain

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{1.50 \times 10^8 \text{ Hz}} = 2.00 \text{ m}$$

**SOLUTION**

a. For $t = 0$ s, the wave expression becomes

$$y = A \sin(2\pi f t - 2\pi x / \lambda) = 156 \sin \left[ 2\pi f \left( 0 \right) - \frac{2\pi x}{2.00} \right] = -156 \sin \left( \frac{2\pi x}{2.00} \right) = -156 \sin (\pi x)$$

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position $x$. The term $\pi x$ is in radians when $x$ is in meters, and conversion from radians to degrees is accomplished using the fact that $2\pi \text{ rad} = 360^\circ$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -156 \sin (\pi x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>$-156 \sin (0) = -156 \sin (0^\circ) = 0$</td>
</tr>
<tr>
<td>0.50 m</td>
<td>$-156 \sin (0.50 \pi ) = -156 \sin (90^\circ) = -156$</td>
</tr>
<tr>
<td>1.00 m</td>
<td>$-156 \sin (1.00 \pi ) = -156 \sin (180^\circ) = 0$</td>
</tr>
<tr>
<td>1.50 m</td>
<td>$-156 \sin (1.50 \pi ) = -156 \sin (270^\circ) = +156$</td>
</tr>
<tr>
<td>2.00 m</td>
<td>$-156 \sin (2.00 \pi ) = -156 \sin (360^\circ) = 0$</td>
</tr>
</tbody>
</table>
These values for the electric field are plotted in the graph shown at the right.

b. For \( t = T/4 \), we use the fact that \( f = 1/T \), and the wave expression becomes

\[
y = A \sin \left( 2\pi f t - 2\pi x / \lambda \right) = 156 \sin \left[ 2\pi \left( \frac{1}{T} \right) \left( \frac{T}{4} \right) - \frac{2\pi x}{2.00} \right] = 156 \sin \left( \frac{\pi}{2} - \pi x \right) = 156 \cos (\pi x)
\]

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position \( x \). The term \( \pi x \) is in radians when \( x \) is in meters, and conversion from radians to degrees is accomplished using the fact that \( 2\pi \text{ rad} = 360^\circ \).

<table>
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<th>( x ) (m)</th>
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</tr>
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</tr>
</tbody>
</table>

These values for the electric field are plotted in the graph shown at the right.

58. **REASONING** The rms value \( E_{\text{rms}} \) of the electric field is related to the average intensity \( \bar{S} \) of the light by \( \bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \) (Equation 24.5b). The average intensity \( \bar{S} \) of the light transmitted by the polarizer is related to the incident intensity \( \bar{S}_0 \) by Malus’ law,
$\vec{S} = S_0 \cos^2 \theta$, where $\theta$ is the angle between the transmission axis and the direction of polarization. These two relations will allow us to determine the rms value of the electric field.

**SOLUTION** Combining the two equations given above and solving for the rms value of the electric field, we have

$$E_{\text{rms}} = \sqrt{\frac{S_0 \cos^2 \theta}{c \varepsilon_0}} = \sqrt{\frac{15 \text{ W/m}^2 \cos^2 25^\circ}{(3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}))}} = 68 \text{ N/C}$$

59. **REASONING AND SOLUTION** The intensity of the laser light is $S = P/A = cu$, where $u$ is the energy density of the light. The energy in a section of length $L$ of the cylindrical beam is $U = uAL$ or

$$U = \frac{PL}{c} = \frac{(0.750 \text{ W})(2.50 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 6.25 \times 10^{-9} \text{ J}$$

60. **REASONING** The fraction of the sun’s power that is intercepted by Mercury is the power intercepted by Mercury divided by the power $P_{\text{sun}}$ radiated by the sun, or Fraction = $P_{\text{intercepted}} / P_{\text{sun}}$. According to Equation 16.8, the power intercepted by Mercury is the intensity $S$ of the radiation at Mercury’s location times the area $A$ that Mercury presents to the radiation. Since the sun radiates uniformly in all directions, we can use Equation 16.9 to evaluate the intensity at Mercury’s location. The planet Mercury looks like a circular disk to the radiation (just like the moon looks like a disk to anyone viewing it); this area is $A = \pi r_{\text{Mercury}}^2$, where $r_{\text{Mercury}}$ is the radius of Mercury.

**SOLUTION** The fraction of the sun’s power that is intercepted by Mercury is

$$\text{Fraction} = \frac{\text{Power intercepted by Mercury}}{\text{Power radiated by sun}} = \frac{P_{\text{intercepted}}}{P_{\text{sun}}} = \frac{P_{\text{intercepted}}}{P_{\text{sun}}} = \frac{S A}{P_{\text{sun}}} = \frac{S A}{P_{\text{sun}}}$$

According to Equation 16.8, the power $P_{\text{intercepted}}$ by Mercury is equal to the intensity $S$ of the radiation at Mercury times the area $A$ of the Mercury disk, or $P_{\text{intercepted}} = SA$. Substituting this expression into Equation (1) gives

$$\text{Fraction} = \frac{P_{\text{intercepted}}}{P_{\text{sun}}} = \frac{SA}{P_{\text{sun}}}$$

Since the sun radiates uniformly in all directions, the intensity of the radiation at Mercury’s location is $S = P_{\text{sun}} / 4\pi r^2$ (Equation 16.9), where $r$ is the mean distance from the sun to
Mercury. As mentioned in the REASONING section, the area of the Mercury disk is 
\[ A = \pi r_{\text{Mercury}}^2 \]. Substituting these expressions for \( S \) and \( A \) into Equation (2) yields

\[
\text{Fraction} = \frac{SA}{P_{\text{sun}}} = \frac{P_{\text{sun}}}{4 \pi r^2} \left( \frac{\pi}{4} r_{\text{Mercury}}^2 \right) = \frac{r_{\text{Mercury}}^2}{4 r^2}
\]

\[
= \frac{(2.44 \times 10^6 \text{ m})^2}{4 (5.79 \times 10^{10} \text{ m})^2} = \frac{4.44 \times 10^{-10}}
\]

61. **SSM REASONING** The electromagnetic solar power that strikes an area \( A_{\perp} \) oriented perpendicular to the direction in which the sunlight is radiated is \( P = SA_{\perp} \), where \( S \) is the intensity of the sunlight. In the problem, the solar panels are not oriented perpendicular to the direction of the sunlight, because it strikes the panels at an angle \( \theta \) with respect to the normal. We wish to find the solar power that impinges on the solar panels when \( \theta = 25^\circ \), given that the incident power is 2600 W when \( \theta = 65^\circ \).

**SOLUTION** When the angle that the sunlight makes with the normal to the solar panel is \( \theta \), the power that strikes the solar panel is given by \( P = SA\cos\theta \), where the area perpendicular to the sunlight is \( A_{\perp} = A \cos\theta \) (see the drawing). Therefore we can write

\[
\frac{P_2}{P_1} = \frac{SA \cos \theta_2}{SA \cos \theta_1}
\]

where the intensity \( S \) of the sunlight that reaches the panel, as well as the area \( A \), are the same in both cases. Therefore, we have

\[
\frac{P_2}{P_1} = \frac{\cos \theta_2}{\cos \theta_1}
\]

Solving for \( P_2 \), we find that when \( \theta_2 = 35^\circ \), the solar power impinging on the panel is

\[
P_2 = P_1 \left( \frac{\cos \theta_2}{\cos \theta_1} \right) = (2600 \text{ W}) \left( \frac{\cos 25^\circ}{\cos 65^\circ} \right) = 5600 \text{ W}
\]
62. **REASONING AND SOLUTION** The polarizer will transmit a maximum intensity of \( \frac{1}{2} \bar{S}_U + \bar{S}_P \), when its axis is parallel to the polarization direction of the polarized component of the incident light. Then the light intensity at the photocell is

\[
\bar{S}_{\text{max}} = \left( \frac{1}{2} \bar{S}_U + \bar{S}_P \right) \cos^2 \theta
\]  

(1)

The polarizer transmits minimum light intensity of \( \frac{1}{2} \bar{S}_U \) when its axis is perpendicular to the polarization direction of the polarized incident light, so

\[
\bar{S}_{\text{min}} = \frac{1}{2} \bar{S}_U \cos^2 \theta
\]  

(2)

Solving Equation (2) for \( \bar{S}_U \) gives

\[
\bar{S}_U = \frac{2 \bar{S}_{\text{min}}}{\cos^2 \theta}
\]  

(3)

Using Equation (3) in Equation (1) and solving give

\[
\bar{S}_P = \frac{\bar{S}_{\text{max}} - \bar{S}_{\text{min}}}{\cos^2 \theta}
\]  

(4)

Using Equations (3) and (4), we find that the percent polarization is

\[
\frac{100 \bar{S}_P}{\bar{S}_P + \bar{S}_U} = \frac{100 \left( \bar{S}_{\text{max}} - \bar{S}_{\text{min}} \right)}{\frac{\bar{S}_{\text{max}} - \bar{S}_{\text{min}} + 2 \bar{S}_{\text{min}}}{\cos^2 \theta}} = \frac{100 \left( \bar{S}_{\text{max}} - \bar{S}_{\text{min}} \right)}{\bar{S}_{\text{max}} + \bar{S}_{\text{min}}}
\]
1. (e) This is the definition of a wave front (see Section 25.1).

2. (b) Rays are radial lines pointing outward from the source and perpendicular to the wave fronts. They point in the direction of the velocity of the wave.

3. (c) When diffuse reflection occurs, the surface reflects different light rays in different directions.

4. (c) The ray of light strikes the mirror four units down from the top of the mirror with a $45^\circ$ angle of incidence. The ray reflects from the mirror at an angle of $45^\circ$ and passes through point C.

5. (a) The image is as far behind the mirror as the object is in front of the mirror. In addition, the image and the object lie on the horizontal line that is perpendicular to the mirror.

6. (d) The image of your friend is 2 m behind the mirror. The distance between you and the mirror is 5 m. Thus, the distance between you and your friend’s image is 7 m.

7. (b) Letters and words held up to a mirror are reversed left-to-right and right-to-left.

8. (d) As discussed in Section 25.4, rays that are parallel and near the principal axis of a concave mirror converge at the focal point after reflecting from the mirror.

9. (a) Parallel rays that are near the principal axis converge at the focal point after reflecting from a concave mirror. The radius of curvature is twice the focal length (see Equation 25.1), so $R = 2f = 36$ cm.

10. (d) This is how real and virtual images are defined. See Sections 25.3 and 25.5.

11. (a) Any ray that leaves the object and reflects from the mirror can be used in the method of ray tracing to locate the image.

12. (c) According to the discussion in Section 25.5, a concave mirror can produce an enlarged image, provided the object distance is less than the radius of curvature. A convex mirror cannot produce an enlarged image, regardless of where the object is located.

13. (b) A convex mirror always produces a virtual, upright image (see Section 25.5).
14. (e) A negative image distance means that the image is behind the mirror and, hence, is a virtual image. See the Reasoning Strategy at the end of Section 25.6.

15. (c) A convex lens always produces an upright image that is smaller than the object.

16. \( f = 4.0 \text{ cm} \)

17. (b) The image distance is \( d_i = -md_o = -2(25 \text{ cm}) = -50 \text{ cm} \) (Equation 25.4).

18. \( f = 90.0 \text{ cm} \)
1. **REASONING AND SOLUTION** Referring to Figure 25.9b and Conceptual Example 2, we find the following locations for the three images:

<table>
<thead>
<tr>
<th>Image</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image 1</td>
<td>$x = -2.0$ m</td>
<td>$y = +1.0$ m</td>
</tr>
<tr>
<td>Image 2</td>
<td>$x = +2.0$ m</td>
<td>$y = -1.0$ m</td>
</tr>
<tr>
<td>Image 3</td>
<td>$x = +2.0$ m</td>
<td>$y = +1.0$ m</td>
</tr>
</tbody>
</table>

2. **REASONING** The drawing shows the situation described. The law of reflection indicates that the angle of incidence $\theta_i$ is equal to the angle of reflection $\theta_r$.

**SOLUTION** Referring to the drawing, we see that for the incident and reflected light

\[
\tan \theta_i = \frac{d}{3.0 \text{ cm}} \quad \text{and} \quad \tan \theta_r = \frac{\ell}{6.0 \text{ cm}}
\]

But $\theta_i = \theta_r$, according to the law of reflection, so that $\tan \theta_i = \tan \theta_r$, and we have

\[
\frac{d}{3.0 \text{ cm}} = \frac{\ell}{6.0 \text{ cm}} \quad \text{or} \quad \ell = 2d
\]

From the drawing we see that $d + \ell = 9.0$ cm. Using the fact that $\ell = 2.0d$, we obtain

\[
d + \ell = d + 2.0d = 9.0 \text{ cm} \quad \text{or} \quad d = 3.0 \text{ cm}
\]

Therefore, the laser should be aimed at the point at $x = +3.0$ cm.
3. **SSM REASONING** The drawing shows the image of the bird in the plane mirror, as seen by the camera. Note that the image is as far behind the mirror as the bird is in front of it. We can find the distance \( d \) between the camera and the image by noting that this distance is the hypotenuse of a right triangle. The base of the triangle has a length of 3.7 m + 2.1 m and the height of the triangle is 4.3 m.

**SOLUTION** The distance \( d \) from the camera to the image of the bird can be obtained by using the Pythagorean theorem:

\[
d = \sqrt{(3.7 \text{ m} + 2.1 \text{ m})^2 + (4.3 \text{ m})^2} = 7.2 \text{ m}
\]

4. **REASONING** According to the discussion about relative velocity in Section 3.4, the velocity \( v_{\text{IV}} \) of your image relative to you is the vector sum of the velocity \( v_{\text{IM}} \) of your image relative to the mirror and the velocity \( v_{\text{MY}} \) of the mirror relative to you: \( v_{\text{IV}} = v_{\text{IM}} + v_{\text{MY}} \). As you walk perpendicularly toward the stationary mirror, you perceive the mirror moving toward you in the opposite direction, so that \( v_{\text{MY}} = -v_{\text{YM}} \). The velocities \( v_{\text{YM}} \) and \( v_{\text{IM}} \) have the same magnitude. This is because the image in a plane mirror is always just as far behind the mirror as the object is in front of it. For instance, if you move 1 meter perpendicularly toward the mirror in 1 second, the magnitude of your velocity relative to the mirror is 1 m/s. But your image also moves 1 meter toward the mirror in the same time interval, so that the magnitude of its velocity relative to the mirror is also 1 m/s. The two velocities, however, have opposite directions.

**SOLUTION** According to the discussion in the **REASONING**,

\[
v_{\text{IV}} = v_{\text{IM}} + v_{\text{MY}} = v_{\text{IM}} - v_{\text{YM}}
\]

Remembering that the magnitudes of both velocities \( v_{\text{YM}} \) and \( v_{\text{IM}} \) are the same and that the direction in which you walk is positive, we have

\[
v_{\text{YM}} = +0.90 \text{ m/s} \quad \text{and} \quad v_{\text{IM}} = -0.90 \text{ m/s}
\]

The velocity \( v_{\text{IM}} \) is negative, because its direction is opposite to the direction in which you walk. Substituting these values into Equation (1), we obtain

\[
v_{\text{IV}} = (-0.90 \text{ m/s}) - (+0.90 \text{ m/s}) = -1.80 \text{ m/s}
\]
5. **REASONING** The geometry is shown below. According to the law of reflection, the incident ray, the reflected ray, and the normal to the surface all lie in the same plane, and the angle of reflection $\theta_r$ equals the angle of incidence $\theta_i$. We can use the law of reflection and the properties of triangles to determine the angle $\theta$ at which the ray leaves $M_2$.

![Diagram of reflection](image)

**SOLUTION** From the law of reflection, we know that $\phi = 65^\circ$. We see from the figure that $\phi + \alpha = 90^\circ$, or $\alpha = 90^\circ - \phi = 90^\circ - 65^\circ = 25^\circ$. From the figure and the fact that the sum of the interior angles in any triangle is $180^\circ$, we have $\alpha + \beta + 120^\circ = 180^\circ$. Solving for $\beta$, we find that $\beta = 180^\circ - (120^\circ + 25^\circ) = 35^\circ$. Therefore, since $\beta + \gamma = 90^\circ$, we find that the angle $\gamma$ is given by $\gamma = 90^\circ - \beta = 90^\circ - 35^\circ = 55^\circ$. Since $\gamma$ is the angle of incidence of the ray on mirror $M_2$, we know from the law of reflection that $\theta = 55^\circ$.

6. **REASONING** The drawing at the right shows the laser beam after reflecting from the plane mirror. The angle of reflection is $\alpha$, and it is equal to the angle of incidence, which is $33.0^\circ$. Note that the angles labeled $\beta$ in the drawing are also $33.0^\circ$, since they are angles formed by a line that intersects two parallel lines. Knowing these angles, we can use trigonometry to determine the distance $d_{DC}$, which locates the spot where the beam strikes the floor.

![Diagram of laser beam](image)

**SOLUTION** Applying trigonometry to triangle DBC, we see that

$$\tan \beta = \frac{d_{BC}}{d_{DC}} \quad \text{or} \quad d_{DC} = \frac{d_{BC}}{\tan \beta} \quad (1)$$
The distance \( d_{BC} \) can be determined from \( d_{BC} = 1.80 \, \text{m} - d_{AB} \), which can be substituted into Equation (1) to show that

\[
d_{DC} = \frac{d_{BC}}{\tan \beta} = \frac{1.80 \, \text{m} - d_{AB}}{\tan \beta}
\]  

(2)

The distance \( d_{AB} \) can be found by applying trigonometry to triangle LAB, which shows that

\[
\tan \beta = \frac{d_{AB}}{1.10 \, \text{m}} \quad \text{or} \quad d_{AB} = (1.10 \, \text{m}) \tan \beta
\]

Substituting this result into Equation (2), gives

\[
d_{DC} = \frac{1.80 \, \text{m} - d_{AB}}{\tan \beta} = \frac{1.80 \, \text{m} - (1.10 \, \text{m}) \tan 33.0^\circ}{\tan 33.0^\circ} = 1.67 \, \text{m}
\]

7. **REASONING AND SOLUTION** The two arrows, A and B are located in front of a plane mirror, and a person at point \( P \) is viewing the image of each arrow. As discussed in Conceptual Example 1, light emanating from the arrow is reflected from the mirror and is reflected toward the observer at \( P \). In order for the observer to see the arrow in its entirety, both rays, the one from the top of the arrow and the one from the bottom of the arrow, must pass through the point \( P \).

According to the law of reflection, all rays will be reflected so that the angle of reflection is equal to the angle of incidence. The ray from the top of arrow \( A \) strikes the mirror and reflects so that it passes through point \( P \). Likewise, the ray from the bottom of the arrow is reflected such that it too passes through point \( P \). Therefore, the observer at \( P \) sees the arrow at \( A \) in its entirety.

Similar reasoning shows that the ray from the top of arrow \( B \) passes through point \( P \). However, as the drawing shows, the ray from the bottom of the arrow does not pass through \( P \). This conclusion is true no matter where the bottom ray strikes the mirror. The observer does not see the arrow at \( B \) in its entirety.
8. **REASONING** The drawing shows the ray of light reflecting from the mirror and striking the floor. The angle of reflection $\theta$ is the same as the angle of incidence. The angle $\theta$ is also the angle that the light ray makes with the floor (see the drawing). Therefore, we can use the inverse tangent function to find $\theta$ as a function of $y$ and $x$;

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$. Since the mirror is facing east and the sun is rising, the angle of incidence $\theta$ becomes larger (see the drawing). As the angle $\theta$ becomes larger, the distance $x$ becomes smaller.

**SOLUTION** When the horizontal distance is $x_1 = 3.86$ m, the angle of incidence $\theta_1$ is

$$\theta_1 = \tan^{-1}\left(\frac{y}{x_1}\right) = \tan^{-1}\left(\frac{1.80\text{ m}}{3.86\text{ m}}\right) = 25.0^\circ$$

When the horizontal distance is $x_2 = 1.26$ m, the angle of incidence $\theta_2$ is

$$\theta_2 = \tan^{-1}\left(\frac{y}{x_2}\right) = \tan^{-1}\left(\frac{1.80\text{ m}}{1.26\text{ m}}\right) = 55.0^\circ$$

As the sun rises, the change in the angle of incidence is $55.0^\circ - 25.0^\circ = 30.0^\circ$. Since the earth rotates at a rate of $15.0^\circ$ per hour, the elapsed time between the two observations is

$$\text{Elapsed time} = \left(30.0^\circ\right)\left(\frac{1\text{ h}}{15.0^\circ}\right) = 2.00\text{ h}$$

9. **REASONING**

a. The incident ray from the laser travels a distance of $L = 50.0$ km due north to the mirror (see the drawing, which is not to scale). The angle $\theta$ we seek, that between the normal to the surface of the mirror and due south, is the angle of incidence. The angle of incidence must equal the angle of reflection, so the angle $\alpha$ between the incident and reflected rays is bisected by the normal to the surface of the mirror. Therefore, we have that

$$\theta = \frac{1}{2} \alpha$$  \hspace{1cm} (1)
Because the laser, the mirror, and the detector sit at the corners of a right triangle, we will use \( \alpha = \tan^{-1}\left(\frac{d}{L}\right) \) (Equation 1.6) to determine the angle \( \alpha \), where \( d = 117 \) m is the distance between the laser and the detector.

b. Turning the surface normal too far from due south increases both the angle of incidence and the angle of reflection, in this case by 0.004° each. Thus, the angle \( \alpha \) between the incident and reflected rays increases by twice that amount. We will use the new angle \( \alpha \), with \( \tan \alpha = \frac{D}{L} \) (Equation 1.3), to determine the distance \( D \) between the laser and the reflected beam (see the drawing). The distance \( x \) by which the reflected beam misses the detector is the distance \( D \) minus the distance \( d \) between the laser and the detector:

\[
x = D - d
\]

We note that the distance \( d \) is given in meters, while the distance \( L \) is given in kilometers, where \( 1 \) km = \( 10^3 \) m.

**SOLUTION**

a. Substituting \( \alpha = \tan^{-1}\left(\frac{d}{L}\right) \) (Equation 1.6) into Equation (1) yields

\[
\theta = \frac{1}{2} \alpha = \frac{1}{2} \tan^{-1}\left(\frac{d}{L}\right) = \frac{1}{2} \tan^{-1}\left(\frac{117 \text{ m}}{50.0 \times 10^3 \text{ m}}\right) = 0.0670^\circ
\]

b. When the mirror is misaligned, the angle of incidence is larger than the result found in part (a) by 0.004°. From Equation (1), then, the angle \( \alpha \) between the incident and reflected rays becomes

\[
\alpha = 2\theta = 2\left(0.0670^\circ + 0.004^\circ\right) = 0.142^\circ
\]

Solving \( \tan \alpha = \frac{D}{L} \) (Equation 1.3) for the distance \( D \) between the laser and the reflected beam, we obtain

\[
D = L \tan \alpha
\]

Substituting Equation (3) into Equation (2), and using \( \alpha = 0.142^\circ \), we find that

\[
x = D - d = L \tan \alpha - d = \left(50.0 \times 10^3 \text{ m}\right) \tan 0.142^\circ - 117 \text{ m} = 7 \text{ m}
\]
10. **REASONING** The drawing shows two plane mirrors that intersect at an angle of 50°. An incident light ray reflects from one mirror and then the other. The various angles are labeled \( \theta, \alpha, \beta, \gamma, \varepsilon \) and \( \phi \). We wish to find the numerical value of the angle \( \theta \). We will do this using the law of reflection, which states that when light is reflected from a surface, the angle of incidence is equal to the angle of reflection. We will also use the fact that the sum of the interior angles of a triangle is 180°.

**SOLUTION** We can see in the drawing that \( \theta + \phi = 180^\circ \), and solving for \( \theta \) we find that

\[
\theta = 180^\circ - \phi \quad (1)
\]

Since the sum of the interior angles of a triangle is 180°, we can see in the drawing that \( 2\alpha + 2\beta + \phi = 180^\circ \), and solving for \( \phi \), we find that

\[
\phi = 180^\circ - 2(\alpha + \beta) \quad (2)
\]

Substituting Equation (2) into Equation (1) gives

\[
\theta = 180^\circ - \phi = 180^\circ - [180^\circ - 2(\alpha + \beta)] = 2(\alpha + \beta) \quad (3)
\]

To determine the quantity \( \alpha + \beta \) in Equation (3), we refer to the triangle formed by the intersection of the mirrors and the first reflected ray. For this triangle we know that

\[
50^\circ + \gamma + \varepsilon = 180^\circ \quad (4)
\]

In the drawing, the normal to the mirror that the incident ray strikes is the dashed line and makes a 90° angle with that mirror. Therefore, we know that \( \gamma = 90^\circ - \alpha \). Using the same reasoning for the normal to the other mirror, we can also conclude that \( \varepsilon = 90^\circ - \beta \). Substituting these expressions into Equation (4) gives

\[
50^\circ + (90^\circ - \alpha) + (90^\circ - \beta) = 180^\circ \quad \text{or} \quad \alpha + \beta = 50^\circ
\]

Substituting this value for \( \alpha + \beta \) into Equation (3) gives

\[
\theta = 2(\alpha + \beta) = 2(50^\circ) = 100^\circ
\]
11. **REASONING**
   a. The smallest angle of incidence such that the laser beam hits only one of the mirrors is shown in the left drawing. Here the laser beam strikes the top mirror at an angle of incidence $\theta_1$ and just passes the right edge of the lower mirror without striking it. The angle $\theta_1$ can be obtained by using trigonometry.

   b. The smallest angle such that the laser beam hits each mirror only once is shown in the right drawing. The laser beam strikes the top mirror at an angle of incidence $\theta_2$, reflects from the top and bottom mirrors, and just passes the right edge of the upper mirror without striking it. The angle $\theta_2$ can also be obtained by using trigonometry.

![Left drawing](image1)

**SOLUTION**

a. Note from the left drawing that the length of the side opposite the angle $\theta_1$ is $\frac{1}{3}(17.0 \text{ cm})$, because the angle of reflection equals the angle of incidence. The length of the side adjacent to $\theta_1$ is 3.00 cm. Using the inverse tangent function, we find that

$$\theta_1 = \tan^{-1}\left(\frac{\frac{1}{3}(17.0 \text{ cm})}{3.00 \text{ cm}}\right) = 70.6^\circ$$

b. From the right drawing we see that the length of the side opposite the angle $\theta_2$ is $\frac{1}{3}(17.0 \text{ cm})$, because the angle of reflection equals the angle of incidence. The length of the side adjacent to $\theta_2$ is 3.00 cm. Again, using the inverse tangent function, we obtain

$$\theta_2 = \tan^{-1}\left(\frac{\frac{1}{3}(17.0 \text{ cm})}{3.00 \text{ cm}}\right) = 62.1^\circ$$

12. **REASONING AND SOLUTION** We can see from the upper triangle in the drawing that

$$\tan \theta = \frac{L - x}{L}$$

We also see from the lower triangle that

$$\tan \theta = \frac{x}{(L/2)}$$

Equating these two expressions gives

$$\frac{L - x}{L} = \frac{x}{(L/2)} \quad \text{or} \quad x = \frac{1}{3}L$$
Therefore,
\[ \theta = \tan^{-1} \left( \frac{x}{\frac{1}{2}L} \right) = \tan^{-1} \left( \frac{\frac{1}{3}L}{\frac{1}{2}L} \right) = 33.7^\circ \]

13. **REASONING** When an object is located very far away from a spherical mirror (concave or convex), the image is located at the mirror’s focal point. Here, the image of the distant object is located 18 cm behind the convex mirror, so that the focal length \( f \) of the mirror is \( f = -18 \text{ cm} \) and is negative since the mirror is convex.

**SOLUTION** In constructing a ray diagram, we will need to know the radius \( R \) of the mirror.

The focal length of a convex mirror is related to the radius by \( f = -\frac{1}{2}R \) (Equation 25.2). We can use this expression to determine the radius:

\[ f = -\frac{1}{2}R \quad \text{or} \quad R = -2f = -2(-18 \text{ cm}) = 36 \text{ cm} \]

In the ray diagram that follows, we denote the focal point by \( F \) and the center of curvature by \( C \). Note that the horizontal and vertical distances in this drawing are to scale. This means that the mirror is represented by a circular arc that is also drawn to scale. Note that we have used only rays 1 and 3 in constructing this diagram. Only two of the three rays discussed in the text are needed.

From the drawing, we see that the image is located 6.0 cm behind the mirror.

14. **REASONING** Since the car is “very distant,” we can assume that it is infinitely far from the mirror. Therefore, when paraxial rays leave the car and travel parallel to the principal axis, they appear to come from the focal point of the mirror after reflection. In effect, then, the given location of the image tells us the focal length of the convex mirror. The focal length \( f \) is related to the radius \( R \) of the mirror according to \( f = -\frac{1}{2}R \) (Equation 25.2).

**SOLUTION**

a. Since the image of the very distant car is located 12 cm behind the mirror, we know that the focal length is \( f = -12 \text{ cm} \), the minus sign being required because the mirror is a convex mirror. Using Equation 25.2, we find that
\[ f = -\frac{1}{2}R \quad \text{or} \quad R = -2f = -2(-12 \text{ cm}) = 24 \text{ cm} \]

b. The ray diagram is similar to that shown in Figure 25.16.

15. **SSM REASONING** The object distance \((d_o = 11 \text{ cm})\) is shorter than the focal length \((f = 18 \text{ cm})\) of the mirror, so we expect the image to be virtual, appearing behind the mirror. Taking Figure 25.18a as our model, we will trace out: three rays from the tip of the object to the surface of the mirror, then three reflected rays, and finally three virtual rays extending behind the mirror and meeting at the tip of the image. The scale of the ray tracing will determine the location and height of the image. The three sets of rays are:

1. An incident ray from the object to the mirror, parallel to the principal axis and then reflected through the focal point \(F\).

2. An incident ray from the object to the mirror, directly away from the focal point \(F\) and then reflected parallel to the principal axis. (The incident ray cannot pass from the object through the focal point, as this would take it away from the mirror, and it would not be reflected.)

3. An incident ray from the object to the mirror, directly away from the center of curvature \(C\), then reflected back through \(C\).

**SOLUTION**

![Ray diagram showing image formation for a virtual image]

a. The ray diagram indicates that the image is 28 cm behind the mirror.

b. We see from the ray diagram that the image is 7.6 cm tall.
16. **REASONING AND SOLUTION** The ray diagram is shown in the figure (Note: $f = 5.0 \text{ cm}$ and $d_o = 15.0 \text{ cm}$).

   a. The ray diagram indicates that the image distance is $7.5 \text{ cm}$ in front of the mirror.

   b. The image height is $1.0 \text{ cm}$, and the image is inverted relative to the object.

17. **REASONING AND SOLUTION**
   a. A ray diagram, which will look similar, but not identical, to that in Figure 25.21a, reveals that
      the image distance is $20.0 \text{ cm}$ behind the mirror, or $d_i = -20.0 \text{ cm}$.

   b. The ray diagram also shows that the image height is $6.0 \text{ cm}$, and the image is upright relative to the object.

18. **REASONING AND SOLUTION** A plane mirror faces a concave mirror ($f = 8.0 \text{ cm}$). The following is a ray diagram of an object placed $10.0 \text{ cm}$ in front of the plane mirror.

   The ray diagram shows the light that is first reflected from the plane mirror and then the concave mirror. The scale is shown in the figure. For the reflection from the plane mirror, as discussed in the text, the image is upright, the same size as the object, and is located as far behind the mirror as the object is in front of it. When the reflected ray reaches the
concave mirror, the ray that is initially parallel to the principal axis passes through the focal point \( F \) after reflection from the concave mirror. The ray that passes directly through the focal point emerges parallel to the principal axis after reflection from the concave surface. The point of intersection of these two rays locates the position of the image. By inspection, we see that the image is located at \( 10.9 \text{ cm} \) from the concave mirror.

19. **SSM REASONING** This problem can be solved using the mirror equation, Equation 25.3.

**SOLUTION** Using the mirror equation with \( d_i = +26 \text{ cm} \) and \( f = 12 \text{ cm} \), we find

\[
\frac{1}{d_o} = \frac{1}{f} + \frac{1}{d_i} = \frac{1}{12 \text{ cm}} - \frac{1}{26 \text{ cm}} \quad \text{or} \quad d_o = +22 \text{ cm}
\]

20. **REASONING** We have seen that a convex mirror always forms a *virtual image* as shown in Figure 25.21a of the text, where the image is *upright* and *smaller* than the object. These characteristics should bear out in the results of our calculations.

**SOLUTION** The radius of curvature of the convex mirror is 68 cm. Therefore, the focal length is, from Equation 25.2, \( f = -(1/2)R = -34 \text{ cm} \). Since the image is virtual, we know that \( d_i = -22 \text{ cm} \).

a. With \( d_i = -22 \text{ cm} \) and \( f = -34 \text{ cm} \), the mirror equation gives

\[
\frac{1}{d_o} = \frac{1}{f} + \frac{1}{d_i} = \frac{1}{-34 \text{ cm}} - \frac{1}{-22 \text{ cm}} \quad \text{or} \quad d_o = +62 \text{ cm}
\]

b. According to the magnification equation, the magnification is

\[
m = -\frac{d_i}{d_o} = -\frac{-22 \text{ cm}}{62 \text{ cm}} = +0.35
\]

c. Since the magnification \( m \) is positive, the image is *upright*.

d. Since the magnification \( m \) is less than one, the image is *smaller* than the object.

21. **REASONING**

a. We are dealing with a concave mirror whose radius of curvature is 56.0 cm. Thus, the focal length of the mirror is \( f = \frac{1}{2}R = 28.0 \text{ cm} \) (Equation 25.1). The object distance is \( d_o = 31.0 \text{ cm} \). With known values for \( f \) and \( d_o \), we can use the mirror equation to (Equation 25.3) find the image distance.
b. To determine the image height \( h_i \), recall that it is related to the object height \( h_o \) by the magnification \( m \); \( h_i = mh_o \). The magnification is related to the image and object distances by the magnification equation, \( m = -d_i / d_o \) (Equation 25.4). Since we know values for \( h_o \), \( d_i \), and \( d_o \), we can find the image height \( h_i \).

**SOLUTION**

a. The image distance is given by the mirror equation as follows:

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{28.0 \text{ cm}} - \frac{1}{31.0 \text{ cm}} \quad \text{so} \quad d_i = 290 \text{ cm}
\]  

(b) Using the magnification equation, we find that the image height is

\[
h_i = mh_o = \left( -\frac{d_i}{d_o} \right) h_o = \left( -\frac{290 \text{ cm}}{31.0 \text{ cm}} \right)(0.95 \text{ cm}) = -8.9 \text{ cm}
\]  

c. Since \( h_i \) is negative, the image is inverted relative to the object. Thus, to make the picture on the wall appear normal, the slide must be oriented **upside down** in the projector.

22. **REASONING** For an image that is in front of a mirror, the image distance is positive. Since the image is inverted, the image height is negative. Given the image distance, the mirror equation can be used to determine the focal length, but to do so a value for the object distance is also needed. The object and image heights, together with the knowledge that the image is inverted, allows us to calculate the magnification \( m \). The magnification \( m \) is given by \( m = -d_i/d_o \) (Equation 25.4), where \( d_i \) and \( d_o \) are the image and object distances, respectively.

**SOLUTION** According to Equation 25.4, the magnification is

\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{or} \quad d_o = -\frac{d_i h_o}{h_i}
\]

Substituting this result into the mirror equation, we obtain

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = -\frac{h_i}{d_i h_o} + \frac{1}{d_i} \left( -\frac{h_i}{h_o} + 1 \right)
\]

\[
= \left( \frac{1}{13 \text{ cm}} \right) \left[ -\frac{(-1.5 \text{ cm})}{(3.5 \text{ cm})} + 1 \right] = 0.11 \text{ cm}^{-1} \quad \text{or} \quad f = 9.1 \text{ cm}
\]

23. **SSM REASONING** Since the image is behind the mirror, the image is virtual, and the image distance is negative, so that \( d_i = -34.0 \text{ cm} \). The object distance is given as \( d_o = 7.50 \text{ cm} \). The mirror equation relates these distances to the focal length \( f \) of the mirror.
If the focal length is positive, the mirror is concave. If the focal length is negative, the mirror is convex.

**SOLUTION** According to the mirror equation (Equation 25.3), we have

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{or} \quad f = \frac{1}{\frac{1}{d_o} + \frac{1}{d_i}} = \frac{1}{\frac{1}{7.50 \text{ cm}} + \frac{1}{-34.0 \text{ cm}}} = 9.62 \text{ cm}
\]

Since the focal length is positive, the mirror is **concave**.

24. **REASONING**

a. We are given that the focal length of the mirror is \(f = 45 \text{ cm}\) and that the image distance is one-third the object distance, or \(d_i = \frac{1}{3} d_o\). These two pieces of information, along with the mirror equation, will allow us to find the object distance.

b. Once the object distance has been determined, the image distance is one-third that value, since we are given that \(d_i = \frac{1}{3} d_o\).

**SOLUTION**

a. The mirror equation indicates that

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  \hspace{1cm} (25.3)

Substituting \(d_i = \frac{1}{3} d_o\) and solving for \(d_o\) gives

\[
\frac{1}{d_o} + \frac{1}{\frac{1}{3} d_o} = \frac{1}{f} \quad \text{or} \quad \frac{4}{d_o} = \frac{1}{f} \quad \text{or} \quad d_o = 4 f = 4(45 \text{ cm}) = 180 \text{ cm}
\]

b. The image distance is

\[
d_i = \frac{1}{3} d_o = \frac{1}{3}(180 \text{ cm}) = 6.0 \times 10^1 \text{ cm}
\]

25. **REASONING** Since the focal length \(f\) and the object distance \(d_o\) are known, we will use the mirror equation to determine the image distance \(d_i\). Then, knowing the image distance as well as the object distance, we will use the magnification equation to find the magnification \(m\).

**SOLUTION**

a. According to the mirror equation (Equation 25.3), we have

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \quad \text{or} \quad d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{1}{\frac{1}{11 \text{ m}} - \frac{1}{-7.0 \text{ m}}} = -4.3 \text{ m}
\]

The image distance is negative because the image is a virtual image behind the mirror.
b. According to the magnification equation (Equation 25.4), the magnification is

\[ m = \frac{-d_i}{d_o} = \frac{-(-4.3 \text{ m})}{11 \text{ m}} = 0.39 \]

26. **REASONING** The magnification \( m \) is given by \( m = -d_i/d_o \) (Equation 25.4), where \( d_i \) and \( d_o \) are the image and object distances, respectively. The object distance is known, and we can obtain the image distance from the mirror equation:

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]  

(25.3)

**SOLUTION** Solving the mirror equation (Equation 25.3) for the image distance \( d_i \) gives

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

or

\[ \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{fd_o} \]

or

\[ d_i = \frac{fd_o}{d_o - f} \]

Substituting this result into the magnification equation (Equation 25.4) gives

\[ m = -\frac{d_i}{d_o} = -\frac{fd_o}{(d_o - f)} = \frac{f}{f - d_o} \]

Using this result with the given values for the focal length and object distances, we find

- **Smaller object distance**
  \[ m = \frac{f}{f - d_o} = \frac{-27.0 \text{ cm}}{(-27.0 \text{ cm}) - (9.0 \text{ cm})} = 0.750 \]

- **Greater object distance**
  \[ m = \frac{f}{f - d_o} = \frac{-27.0 \text{ cm}}{(-27.0 \text{ cm}) - (18.0 \text{ cm})} = 0.600 \]

27. **SSM REASONING** When paraxial light rays that are parallel to the principal axis strike a convex mirror, the reflected rays diverge after being reflected, and appear to originate from the focal point \( F \) behind the mirror (see Figure 25.16). We can treat the sun as being infinitely far from the mirror, so it is reasonable to treat the incident rays as paraxial rays that are parallel to the principal axis.

**SOLUTION**

a. Since the sun is infinitely far from the mirror and its image is a virtual image that lies behind the mirror, we can conclude that the mirror is a **convex mirror**.

b. With \( d_i = -12.0 \text{ cm} \) and \( d_o = \infty \), the mirror equation (Equation 25.3) gives

\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{d_i} \]
Therefore, the focal length $f$ lies 12.0 cm behind the mirror (this is consistent with the reasoning above that states that, after being reflected, the rays appear to originate from the focal point behind the mirror). In other words, $f = -12.0$ cm. Then, according to Equation 25.2, $f = -\frac{1}{2} R$, and the radius of curvature is

$$R = -2f = -2(-12.0\text{ cm}) = 24.0\text{ cm}$$

28. **REASONING** The magnification of the mirror is related to the image and object distances via the magnification equation. The image distance is given, and the object distance is unknown. However, we can obtain the object distance by using the mirror equation, which relates the image distances to the focal length, which is also given.

**SOLUTION** According to the magnification equation, the magnification $m$ is related to the image distance $d_i$ and the object distance $d_o$ according to

$$m = -\frac{d_i}{d_o} \quad (25.4)$$

According to the mirror equation, the image distance $d_i$ and the object distance $d_o$ are related to the focal length $f$ as follows:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \quad (25.3)$$

Substituting this expression for $1/d_o$ into Equation 25.4 gives

$$m = -\frac{d_i}{d_o} = -d_i \left( \frac{1}{f} - \frac{1}{d_i} \right) = -\frac{d_i}{f} + 1$$

$$= -\left(\frac{36\text{ cm}}{12\text{ cm}}\right) + 1 = -2.0$$

29. **REASONING** We need to know the focal length of the mirror and can obtain it from the mirror equation, Equation 25.3, as applied to the first object:

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{14.0\text{ cm}} + \frac{1}{-7.00\text{ cm}} = \frac{1}{f} \quad \text{or} \quad f = -14.0\text{ cm}$$

According to the magnification equation, Equation 25.4, the image height $h_i$ is related to the object height $h_o$ as follows: $h_i = mh_o = \left(-\frac{d_i}{d_o}\right)h_o$.

**SOLUTION** Applying this result to each object, we find that $h_{i2} = h_{i1}$, or

$$\left(-\frac{d_{i2}}{d_{o2}}\right)h_{o2} = \left(-\frac{d_{i1}}{d_{o1}}\right)h_{o1}$$
Therefore,

\[ d_{i2} = d_{o2} \left( \frac{d_{i1}}{d_{o1}} \right) \left( \frac{h_{o1}}{h_{o2}} \right) \]

Using the fact that \( h_{o2} = 2h_{o1} \), we have

\[ d_{i2} = d_{o2} \left( \frac{d_{i1}}{d_{o1}} \right) \left( \frac{h_{o1}}{2h_{o1}} \right) = d_{o2} \left( \frac{-7.00 \text{ cm}}{14.0 \text{ cm}} \right) \left( \frac{h_{o1}}{2h_{o1}} \right) = -0.250 d_{o2} \]

Using this result in the mirror equation, as applied to the second object, we find that

\[ \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f} \]

or

\[ \frac{1}{d_{o2}} + \frac{1}{-0.250 d_{o2}} = \frac{1}{-14.0 \text{ cm}} \]

Therefore,

\[ d_{o2} = +42.0 \text{ cm} \]

30. **REASONING** We will determine the type of mirror from the fact that the image is enlarged. To determine the focal length \( f \), we will use the mirror equation, which is

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]  

(Equation 25.3), where \( d_o \) is the object distance and \( d_i \) is the image distance.

To determine the magnification \( m \), we will use the magnification equation, which is

\[ m = -\frac{d_i}{d_o} \]  

(Equation 25.4). The algebraic sign of the value for \( m \) will tell us whether the image is upright (+) or inverted (−) with respect to the object.

**SOLUTION**

a. Since the image of the tooth is enlarged, it cannot be a plane mirror, for which the object and the image would have the same size. Convex mirrors produce smaller images in all cases. Therefore, the enlarged image means that the mirror must be [concave].

b. Using the mirror equation (Equation 25.3), we find for the focal length that

\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{2.0 \text{ cm}} + \frac{1}{-5.6 \text{ cm}} = 0.32 \text{ cm}^{-1} \]

or \[ f = \frac{1}{0.32 \text{ cm}^{-1}} = +3.1 \text{ cm} \]

Note that the value of the image distance is −5.6 cm, which is negative because the image is behind the mirror (virtual).

c. Using the magnification equation (Equation 25.4), we find for the magnification that
\[ m = \frac{-d_i}{d_o} = \frac{-5.6 \text{ cm}}{2.0 \text{ cm}} = +2.8 \]

d. Since \( m \) is positive, the image is \textit{upright} relative to the object.

31. \textit{REASONING}

a. You need a concave mirror to make the measurement. Since you must measure the image distance and image height of the tree, the image must be a real image. Only a concave mirror can produce a real image. The image distance of the sun is equal to the focal length of the mirror. The sun is extremely far away, so the light rays from it are nearly parallel when they reach the mirror. According to the discussion in Section 25.5, light rays near and parallel to the principal axis are reflected from a concave mirror and converge at the focal point.

With numerical values for the focal length \( f \) of the mirror and the image distance \( d_i \), the mirror equation (Equation 25.3) can be used to find the object distance \( d_o \):

\[ \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \]

b. The height \( h_o \) of the tree is related to the height \( h_i \) of its image by the magnification \( m \);

\[ h_o = \frac{h_i}{m} \text{ (Equation 25.4).} \]

However, the magnification is given by the magnification equation (Equation 25.4) as

\[ m = -\frac{d_i}{d_o}, \]

where \( d_i \) is the image distance and \( d_o \) is the object distance, both of which we know.

\textit{SOLUTION}

a. The distance to the tree is given by the mirror equation as

\[ \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{0.9000 \text{ m}} - \frac{1}{0.9100 \text{ m}} \quad \text{so} \quad d_o = 82 \text{ m} \]

b. Since \( h_o = \frac{h_i}{m} \) and \( m = -\frac{d_i}{d_o} \), we have that

\[ h_o = \frac{h_i}{m} = \frac{h_i}{-\frac{d_i}{d_o}} = h_i \left( -\frac{d_o}{d_i} \right) \]

Now \( h_i = -0.12 \text{ m} \), where the minus sign has been used since the image is inverted relative to the tree (see Figure 25.18b). Thus, the height of the tree is

\[ h_o = h_i \left( -\frac{d_o}{d_i} \right) = (-0.12 \text{ m}) \left( -\frac{82 \text{ m}}{0.9100 \text{ m}} \right) = 11 \text{ m} \]
32. **REASONING** The mirror equation, which is \(
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\) (Equation 25.3), and the magnification equation, which is \(m = -\frac{d_i}{d_o}
\) (Equation 25.4), apply to both types of mirrors. In these equations \(d_o\) is the object distance, \(d_i\) is the image distance, \(f\) is the focal length, and \(m\) is the magnification. The focal lengths of the two types of mirrors are \(f_{\text{concave}} = \frac{1}{2}R\) (Equation 25.1) and \(f_{\text{convex}} = -\frac{1}{2}R\) (Equation 25.2). We will apply these equations to both types of mirrors, taking particular advantage of the facts that the radius \(R\) and the object distance \(d_o\) are the same for each type.

**SOLUTION** We are given information about the magnification \(m\). In addition, we know that the object distance \(d_o\) is the same for each type of mirror. We also know that the radius \(R\) is the same for each type of mirror, so that we can substitute \(f_{\text{concave}} = \frac{1}{2}R\) (Equation 25.1) into \(f_{\text{convex}} = -\frac{1}{2}R\) (Equation 25.2) and see that

\[
f_{\text{convex}} = -\frac{1}{2}R = -f_{\text{concave}} \quad \text{or} \quad \frac{1}{f_{\text{convex}}} = -\frac{1}{f_{\text{concave}}} \quad (1)
\]

Thus, in the mirror equation and the magnification equation we have information about all of the variables except the image distance \(d_i\). To eliminate it from the problem, we solve the magnification equation for \(d_i\):

\[
m = -\frac{d_i}{d_o} \quad \text{or} \quad d_i = -m d_o \quad (2)
\]

Substituting Equation (2) into the mirror equation gives

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \left(-m d_o\right) = \frac{1}{d_o} \left(1 - \frac{1}{m}\right) \quad (3)
\]

Applying Equation (3) to each type of focal length in Equation (1), we have

\[
\frac{1}{d_o} \left(1 - \frac{1}{m_{\text{convex}}}\right) = -\frac{1}{d_o} \left(1 - \frac{1}{m_{\text{concave}}}\right) \quad \text{or} \quad m_{\text{convex}} = \frac{m_{\text{concave}}}{2m_{\text{concave}} - 1} = +2.0 \quad (1)
\]

33. **REASONING** The radius of curvature \(R\) of a concave mirror is related to the focal length \(f\) of the mirror by \(f = \frac{1}{2}R\) (Equation 25.1), so we have that

\[
R = 2f \quad (1)
\]
The mirror’s focal length is given by the mirror equation
\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \] (Equation 25.3),
where \(d_o = 14 \text{ cm}\) is the object distance and \(d_i\) is the image distance. The image distance is related to the object distance via the magnification equation
\[ m = -\frac{d_i}{d_o} \] (Equation 25.4).
Because the image is virtual, the image distance \(d_i\) is negative. The object distance is positive, so we conclude from Equation 25.4 that the magnification is positive: \(m > 0\). The image is twice the size of the object, so we have that the magnification is \(m = +2.0\).

**SOLUTION** Taking the reciprocal of both sides of
\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \] (Equation 25.3) yields
\[ f = \frac{1}{\frac{1}{d_o} + \frac{1}{d_i}} \cdot \] Substituting this result into Equation (1), we find that
\[ R = 2f = \frac{2}{\frac{1}{d_o} + \frac{1}{d_i}} \] (2)
Solving \(m = -\frac{d_i}{d_o}\) (Equation 25.4) for \(d_i\) and substituting \(m = +2.0\) yields
\[ d_i = -md_o = -(+2.0)d_o = -2d_o \] (3)
Substituting Equation (3) into Equation (2), we obtain
\[ R = \frac{2}{\frac{1}{d_o} + \frac{1}{d_i}} = \frac{2}{\frac{1}{d_o} + \frac{1}{(-2d_o)}} = \frac{2}{\frac{1}{2d_o}} = 4d_o = 4(14 \text{ cm}) = 56 \text{ cm} \]

34. **REASONING AND SOLUTION** We know that \(d_o - d_i = 45.0 \text{ cm}\). We also have \(1/d_o + 1/d_i = 1/f\). Solving the first equation for \(d_i\) and substituting the result into the second equation yields,
\[ \frac{1}{d_o} + \frac{1}{d_o - 45.0 \text{ cm}} = \frac{1}{30.0 \text{ cm}} \]
Cross multiplying gives \(d_o^2 - 105d_o + 1350 = 0\), which we can solve using the quadratic equation to yield two roots, \(d_o = (105 \pm 75)/2\).

a. When the object lies beyond the center of curvature we have
\[ d_{o+} = (1.80 \times 10^2 \text{ cm})/2 = 9.0 \times 10^1 \text{ cm} \] and \(d_{i+} = 45 \text{ cm}\)
b. When the object lies within the focal point
\[ d_{o-} = \frac{3.0 \times 10^1 \text{ cm}}{2} = 15 \text{ cm}, \quad \text{and} \quad d_{i-} = -3.0 \times 10^1 \text{ cm} \]

35. **SSM REASONING**

a. The image of the spacecraft appears a distance \( d_i \) beneath the surface of the moon, which we assume to be a convex spherical mirror of radius \( R = 1.74 \times 10^6 \text{ m} \). The image distance \( d_i \) is related to the focal length \( f \) of the moon and the object distance \( d_o \) by the mirror equation

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{25.3}
\]

The spacecraft is the object, so the object distance \( d_o \) is equal to the altitude of the spacecraft: \( d_o = 1.22 \times 10^5 \text{ m} \). The moon is assumed to be a convex mirror of radius \( R \), so its focal length is given by

\[
f = -\frac{1}{2} R \tag{25.2}
\]

b. Both the spacecraft and its image would have the same angular speed \( \omega \) about the center of the moon because both have the same orbital period. The linear speed \( v \) of an orbiting body is related to its angular speed \( \omega \) by \( v = r \omega \) (Equation 8.9), where \( r \) is the radius of the orbit. Therefore, the speeds of the spacecraft and its image are, respectively,

\[
v_o = r_o \omega \quad \text{and} \quad v_i = r_i \omega \tag{1}
\]

We have used \( v_o \) and \( r_o \) in Equation (1) to denote the orbital speed and orbital radius of the spacecraft, because it is the object. The symbols \( v_i \) and \( r_i \) denote orbital speed and orbital radius of the spacecraft’s image. The orbital radius \( r_o \) of the spacecraft is the sum of the radius \( R \) of the moon and the spacecraft’s altitude \( d_o = 1.20 \times 10^5 \text{ m} \) above the moon’s surface:

\[
r_o = R + d_o \tag{2}
\]

The “orbital radius” \( r_i \) of the image is its distance from the center of the moon. The image is below the surface, and the image distance \( d_i \) is negative. Therefore, the distance between the center of the moon and the image is found from

\[
r_i = R + d_i \tag{3}
\]

**SOLUTION**

a. Solving Equation 25.3 for \( \frac{1}{d_i} \) and then taking the reciprocal of both sides, we obtain

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{d_o f} \quad \text{or} \quad d_i = \frac{d_o f}{d_o - f} \tag{4}
\]
Substituting Equation 25.2 into Equation (4), we find that

\[
d_i = \frac{d_o f}{d_o - f} = \frac{d_o \left(-\frac{1}{2} R\right)}{d_o - \left(-\frac{1}{2} R\right)} = -\frac{1}{2} \frac{d_o R}{d_o + \frac{1}{2} R} = -\frac{1}{2} \left(\frac{1.22 \times 10^5 \text{ m}}{1.74 \times 10^6 \text{ m}}\right) = -1.07 \times 10^5 \text{ m}
\]

Therefore, the image of the spacecraft would appear \[1.07 \times 10^5 \text{ m}\] below the surface of the moon.

b. Solving the first of Equations (1) for \( \omega \) yields \( \omega = \frac{v_o}{r_o} \). Substituting this result into the second of Equations (1), we find that

\[
v_i = r_i \omega = r_i \left(\frac{v_o}{r_o}\right) = \left(\frac{r_i}{r_o}\right) v_o
\]

Substituting Equations (2) and (3) into Equation (5) yields

\[
v_i = \left(\frac{r_i}{r_o}\right) v_o = \left(\frac{R + d_i}{R + d_o}\right) v_o = \left[\frac{1.74 \times 10^6 \text{ m} + (-1.07 \times 10^5 \text{ m})}{1.74 \times 10^6 \text{ m} + 1.22 \times 10^5 \text{ m}}\right] (1620 \text{ m/s}) = 1420 \text{ m/s}
\]

36. **REASONING** We will start by drawing the two situations in which the object is 25 cm and 5 cm from the mirror, making sure that all distances (including the radius of curvature of the mirror) and heights are to scale. For each location of the object, we will draw several rays to locate the image (see the Reasoning Strategy for convex mirrors in Section 25.5). Once the images have been located, we can readily answer the questions regarding their positions and heights.

**SOLUTION** The following two ray diagrams illustrate the situations where the objects are at different distances from the convex mirror.
a. As the object moves closer to the mirror, it can be seen that the magnitude of the image distance becomes [smaller].

b. As the object moves closer to the mirror, the magnitude of the image height becomes [larger].

c. By measuring the image heights, we find that the ratio of the image height when the object distance is 5 cm to that when the object distance is 25 cm is 3.

37. **SSM REASONING** The mirror equation relates the object and image distances to the focal length. Thus, we can apply the mirror equation once with the given object and image distances to determine the focal length. Then, we can use the mirror equation again with the focal length and the second object distance to determine the unknown image distance.

**SOLUTION** According to the mirror equation, we have

\[
\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f} \quad \text{and} \quad \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f}
\]

First position of object

Second position of object

Since the focal length is the same in both cases, it follows that

\[
\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}}
\]

\[
\frac{1}{d_{i2}} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} - \frac{1}{d_{o2}} = \frac{1}{25 \text{ cm}} + \frac{1}{(-17 \text{ cm})} - \frac{1}{19 \text{ cm}} = -0.071 \text{ cm}^{-1}
\]

\[
d_{i2} = -14 \text{ cm}
\]

The negative value for \( d_{i2} \) indicates that the image is located 14 cm behind the mirror.
38. **REASONING** The object distance \( d_o \) is the distance between the object and the mirror. It is found from \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 25.3), where \( d_i \) is the distance between the mirror and the image, and \( f \) is the focal length of the mirror. We are told that the image appears in front of the mirror, so, according to the sign conventions for spherical mirrors, the image distance must be positive: \( d_i = +97 \text{ cm} \).

**SOLUTION** Solving \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 25.3) for \( d_o \) yields

\[
\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \quad \text{or} \quad d_o = \frac{1}{\frac{1}{f} - \frac{1}{d_i}} = \frac{1}{\frac{1}{1} - \frac{1}{97 \text{ cm}}} = 74 \text{ cm}
\]

39. **REASONING AND SOLUTION**
   a. The height of the shortest mirror would be one-half the height of the person. Therefore,
   \[
h = H/2 = (1.70 \text{ m} + 0.12 \text{ m})/2 = 0.91 \text{ m}
\]
   b. The bottom edge of the mirror should be above the floor by
   \[
h' = (1.70 \text{ m})/2 = 0.85 \text{ m}
\]

40. **REASONING** The focal length \( f \) of the water drop is given by the mirror equation \( \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \) (Equation 25.3), where \( d_o \) and \( d_i \) are, respectively, the object distance and the image distance. The object distance \( (d_o = 3.0 \text{ cm}) \) is given, and we will determine the image distance \( d_i \) from the magnification equation \( m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \) (Equation 25.4), where \( h_o \) is the diameter of the flower and \( h_i \) is the diameter of its image. The water drop acts as a convex spherical mirror, so the image is upright. Therefore, the image height \( h_i \) is positive, and we expect the focal length \( f \) to be negative.

**SOLUTION** Solving \( \frac{h_i}{h_o} = -\frac{d_i}{d_o} \) (Equation 25.4) for \( d_i \) and taking the reciprocal, we obtain

\[
d_i = -\frac{d_o h_i}{h_o} \quad \text{or} \quad \frac{1}{d_i} = -\frac{h_o}{d_o h_i}
\]
Substituting Equation (1) into \( \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \) (Equation 25.3) yields

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{h_o}{d_i h_i} = \frac{1}{d_o} \left( 1 - \frac{h_o}{h_i} \right)
\]

(Equation 25.3) yields

\[
f = d_o \left( \frac{1}{1 - \frac{h_o}{h_i}} \right) = \frac{d_o}{1 - \frac{h_o}{h_i}} = \frac{3.0 \text{ cm}}{1 - \frac{2.0 \text{ cm}}{0.10 \text{ cm}}} = -0.16 \text{ cm}
\]

Taking the reciprocal of Equation (2), we find that the focal length of the water drop is

\[
f = d_o \left( \frac{1}{1 - \frac{h_o}{h_i}} \right) = \frac{d_o}{1 - \frac{h_o}{h_i}} = \frac{3.0 \text{ cm}}{1 - \frac{2.0 \text{ cm}}{0.10 \text{ cm}}} = -0.16 \text{ cm}
\]

41. SSM REASONING This problem can be solved by using the mirror equation, Equation 25.3, and the magnification equation, Equation 25.4.

**SOLUTION**

a. Using the mirror equation with \( d_i = d_o \) and \( f = R/2 \), we have

\[
\frac{1}{d_o} - \frac{1}{d_i} = \frac{1}{R/2} - \frac{1}{d_o} \quad \text{or} \quad \frac{2}{d_o} = \frac{2}{R}
\]

Therefore, we find that \( d_o = R \).

b. According to the magnification equation, the magnification is

\[
m = -\frac{d_i}{d_o} = -\frac{d_o}{d_o} = -1
\]

c. Since the magnification \( m \) is negative, the image is inverted.

42. REASONING The mirror equation relates the object and image distances to the focal length. The magnification equation relates the magnification to the object and image distances. The problem neither gives nor asks for information about the image distance. Therefore, we can solve the magnification equation for the image distance and substitute the result into the mirror equation to obtain an expression relating the object distance, the magnification, and the focal length. This expression can be applied to both mirrors A and B to obtain the ratio of the focal lengths.
SOLUTION The magnification equation gives the magnification as \( m = -\frac{d_i}{d_o} \). Solving for \( d_i \), we obtain \( d_i = -md_o \). Substituting this result into the mirror equation, we obtain

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} \left( \frac{1 - \frac{1}{m}}{1} \right) = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_o} \left( \frac{1 - \frac{1}{m}}{1} \right) = \frac{1}{f} \quad \text{or} \quad f = \frac{d_o m}{m-1}
\]

Applying this result for the focal length \( f \) to each mirror gives

\[
f_A = \frac{d_o m_A}{m_A - 1} \quad \text{and} \quad f_B = \frac{d_o m_B}{m_B - 1}
\]

Dividing the expression for \( f_A \) by the expression for \( f_B \), we find

\[
\frac{f_A}{f_B} = \frac{d_o m_A / (m_A - 1)}{d_o m_B / (m_B - 1)} = \frac{m_A (m_B - 1)}{m_B (m_A - 1)} = \frac{4.0(2.0 - 1)}{2.0(4.0 - 1)} = 0.67
\]

43. REASONING According to the discussion about relative velocity in Section 3.4, the velocity \( v_{\text{IV}} \) of your image relative to you is the vector sum of the velocity \( v_{\text{IM}} \) of your image relative to the mirror and the velocity \( v_{\text{MY}} \) of the mirror relative to you:

\[
v_{\text{IV}} = v_{\text{IM}} + v_{\text{MY}}
\]

As you walk toward the stationary mirror, you perceive the mirror moving toward you in the opposite direction. Thus, \( v_{\text{MY}} = -v_{\text{YM}} \). The velocity \( v_{\text{YM}} \) has the components \( v_{\text{YM},x} \) and \( v_{\text{YM},y} \), while the velocity \( v_{\text{IM}} \) has the components \( v_{\text{IM},x} \) and \( v_{\text{IM},y} \). The \( x \) direction is perpendicular to the mirror, and the two \( x \) components have the same magnitude. This is because the image in a plane mirror is always just as far behind the mirror as the object is in front of it. For instance, if an object moves 1 meter perpendicularly toward the mirror in 1 second, the magnitude of its velocity relative to the mirror is 1 m/s. But the image also moves 1 meter toward the mirror in the same time interval, so that the magnitude of its velocity relative to the mirror is also 1 m/s. The two \( x \) components, however, have opposite directions. The two \( y \) components have the same magnitude and the same direction. This is because an object moving parallel to a plane mirror has an image that remains at the same distance behind the mirror as the object is in front of it and moves in the same direction as the object.

SOLUTION According to the discussion in the REASONING,

\[
v_{\text{IV}} = v_{\text{IM}} + v_{\text{MY}} = v_{\text{IM}} - v_{\text{YM}}
\]
This vector equation is equivalent to two equations, one for the \( x \) components and one for the \( y \) components. For the \( x \) direction, we note that \( v_{YM,x} = -v_{IM,x} \)

\[
v_{IY,x} = v_{IM,x} - v_{YM,x} = -(0.90 \text{ m/s}) \cos 50.0^\circ - (0.90 \text{ m/s}) \cos 50.0^\circ = -1.2 \text{ m/s}
\]

For the \( y \) direction, we note that \( v_{YM,y} = v_{IM,y} \)

\[
v_{IY,y} = v_{IM,y} - v_{YM,y} = (0.90 \text{ m/s}) \sin 50.0^\circ - (0.90 \text{ m/s}) \sin 50.0^\circ = 0 \text{ m/s}
\]

Since the \( y \) component of the velocity \( v_{IV} \) is zero, the velocity of your image relative to you points in the \(-x \) direction and has a magnitude of 1.2 m/s.

44. **REASONING** The focal length \( f \) of the convex mirror determines the first image distance \( d_{i1} \) via

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_{i1}} \quad \text{(Equation 25.3)}
\]

where \( d_o = 15.0 \text{ cm} \) is the distance between the candle and the convex mirror. The second image, formed by the plane mirror, is located as far behind the mirror as the object is in front of the mirror. Therefore, when the plane mirror replaces the convex mirror, the second image distance \( d_{i2} \) is given by

\[
d_{i2} = -d_o = -15.0 \text{ cm}
\]

The negative sign in Equation (1) arises because the image lies behind the plane mirror and, therefore, is a virtual image.

When the plane mirror replaces the convex mirror, the image moves a distance \( x \) farther away from the mirror, so the initial and final image distances are related by

\[
d_{i2} = d_{i1} - x
\]

The negative sign in Equation (2) occurs because the image moves farther from the mirror, thus making the second image distance \( d_{i2} \) a negative number of greater magnitude than the first image distance \( d_{i1} \).

**SOLUTION** Substituting Equation (2) into Equation (1) and solving for \( d_{i1} \) yields

\[
d_{i1} - x = -d_o \quad \text{or} \quad d_{i1} = x - d_o
\]
Substituting Equation (3) into \( \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_{i1}} \) (Equation 25.3), we obtain

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_{i1}} = \frac{1}{d_o} + \frac{1}{x-d_o} \quad (4)
\]

Taking the reciprocal of both sides of Equation (4), we find that the focal length of the convex mirror is

\[
f = \frac{1}{\frac{1}{d_o} + \frac{1}{x-d_o}} = \frac{1}{\frac{1}{15.0\text{ cm}} + \frac{1}{7.0\text{ cm}-15.0\text{ cm}}} = \left[-17 \text{ cm}\right]
\]

45. **SSM REASONING** Since the size \( h_i \) of the image is one-fourth the size \( h_o \) of the object, we know from the magnification equation that

\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{so that} \quad d_i = -\frac{1}{4}d_o \quad (25.4)
\]

We can substitute this relation into the mirror equation to find the ratio \( d_o/f \).

**SOLUTION** Substituting \( d_i = -\frac{1}{4}d_o \) into the mirror equation gives

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_o} - \frac{1}{4d_o} = \frac{1}{f} \quad \text{or} \quad \frac{-3}{d_o} = \frac{1}{f} \quad (25.3)
\]

Solving this equation for the ratio \( d_o/f \) yields \( d_o/f = \left[-3\right] \).

46. **REASONING** In both cases, the image and object distances \( (d_i \text{ and } d_o) \) are related to the focal length \( f \) of the mirror by the mirror equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 25.3). Letting the subscript 1 denote the first situation (convex side of the mirror) and the subscript 2 denote the second situation (concave side of the mirror), we have that

\[
\frac{1}{d_o} + \frac{1}{d_{i1}} = \frac{1}{f_2} \quad \text{and} \quad \frac{1}{d_o} + \frac{1}{d_{i2}} = \frac{1}{f_1} \quad (1)
\]
In Equations (1), we have made use of the fact that the man maintains the same distance of \( d_0 = 45 \) cm between his face and the mirror in both cases. Because the mirror is double-sided, the focal lengths of each side differ only by sign: that of the convex side is negative, and that of the concave side is positive. Therefore, we have that \( f_2 = -f_1 \), where \( f_1 \) is the focal length of the convex side and \( f_2 \) is the focal length of the concave side. Given this relation, we can rewrite the first of Equations (1) as

\[
\frac{1}{d_0} + \frac{1}{d_{i2}} = -\frac{1}{f_1} \tag{2}
\]

We will determine the image distance \( d_{i1} \) from the magnification \( m_1 \) of the first image by means of \( m = -\frac{d_i}{d_o} \) (Equation 25.4), the magnification equation:

\[
m_1 = -\frac{d_{i1}}{d_o} \tag{3}
\]

After using Equation 25.4 to determine \( d_{i1} \), we will use mirror equation to determine the image distance \( d_{i2} \) when the man looks into the concave side of the mirror. If the image distance is positive, it will appear in front of the mirror; if the image distance is negative, the image will appear behind the mirror.

**SOLUTION** Solving Equation (2) for \( \frac{1}{d_{i2}} \), we obtain

\[
\frac{1}{d_{i2}} = -\frac{1}{f_1} - \frac{1}{d_o} = -\left( \frac{1}{f_1} + \frac{1}{d_o} \right) \tag{4}
\]

In this result, we can replace the term \( 1/f_1 \) by using the second of Equations (1). In this way we find that

\[
\frac{1}{d_{i2}} = -\left( \frac{1}{f_1} + \frac{1}{d_o} \right) = -\left( \frac{1}{d_o} + \frac{1}{d_{i1}} + \frac{1}{d_o} \right) = -\left( \frac{2}{d_o} + \frac{1}{d_{i1}} \right) \tag{5}
\]

Taking the reciprocal of Equation (5), we obtain

\[
d_{i2} = -\frac{1}{\left( \frac{2}{d_o} + \frac{1}{d_{i1}} \right)} \tag{6}
\]
Solving Equation (3) for \( d_{i1} \) yields 
\[ d_{i1} = -m_{i}d_{o} = -0.20d_{o} \]. Substituting this result into Equation (6) yields

\[
d_{i2} = -\frac{1}{\left( \frac{2}{d_{o}} + \frac{1}{d_{i1}} \right)} = -\frac{1}{\left( \frac{2d_{o}}{d_{o}} - \frac{1}{0.20d_{o}} \right)} = -\frac{1}{d_{o}(2-5.0)} = \sqrt{d_{o}/3.0} = +45 \text{ cm} - 3.0 \text{ cm} = +15 \text{ cm}
\]

In the preceding equation, we have used the fact that \( \frac{1}{0.20} = 5.0 \).

---

47. **SSM REASONING** The time of travel is proportional to the total distance for each path. Therefore, using the distances identified in the drawing, we have

\[
\frac{t_{\text{reflected}}}{t_{\text{direct}}} = \frac{d_{LM} + d_{MP}}{d_{LP}} \quad (1)
\]

We know that the angle of incidence is equal to the angle of reflection. We also know that the lamp, L, is a distance 2s from the mirror, while the person, P, is a distance s from the mirror.

Therefore, it follows that if \( d_{MP} = d \), then \( d_{LM} = 2d \). We may use the law of cosines (see Appendix E) to express the distance \( d_{LP} \) as

\[
d_{LP} = \sqrt{(2d)^2 + d^2 - 2(2d)(d)\cos 2(30.0^\circ)}
\]

**SOLUTION** Substituting the expressions for \( d_{MP}, d_{LM}, \) and \( d_{LP} \) into Equation (1), we find that the ratio of the travel times is

\[
\frac{t_{\text{reflected}}}{t_{\text{direct}}} = \frac{2d + d}{\sqrt{(2d)^2 + d^2 - 2(2d)(d)\cos 2(30.0^\circ)}} = \frac{3}{\sqrt{5 - 4 \cos 60.0^\circ}} = 1.73
\]
CHAPTER 26  
THE REFRACTION OF LIGHT: LENSES AND OPTICAL INSTRUMENTS

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (c) When the light is refracted into liquid B it is bent away from the normal, so that \( n_A > n_B \). When the light is refracted into liquid C it is bent toward the normal, so that \( n_C > n_A \). Therefore, we conclude that \( n_C > n_A > n_B \).

2. 1.41

3. (b) When the light is refracted into liquid B it is bent away from the normal, so that \( n_A > n_B \). When the light is refracted into liquid C it is also bent away from the normal, so that \( n_A > n_C \). However, the bending is less than that in liquid B. Thus, the index of refraction of liquid C is closer to that of liquid A than the index of refraction of liquid B is. In fact, if \( n_A \) and \( n_C \) were equal, the ray would not be bent at all upon entering liquid C. This means that \( n_C > n_B \). Therefore, we conclude that \( n_A > n_C > n_B \).

4. 4.41 cm

5. (d) The apparent depth \( d' \) is given by \( d' = d \left( \frac{1.00}{n} \right) \), according to Equation 26.3, where \( d \) is the actual depth and \( n \) is the refractive index of the liquid. Thus, the apparent depth and the refractive index are inversely proportional, which means that the ranking in descending order is \( n_C, n_B, n_A \).

6. (c) Material B has the smaller refractive index. Therefore, a light ray that begins in material B and travels toward material A, with its greater refractive index, is bent toward the normal (not away from it) when it crosses the interface.

7. (a) When light travels from a material with a greater refractive index toward a material with a smaller refractive index, total internal reflection occurs at the interface when the angle of incidence is greater than the critical angle. Here, Equation 26.4 reveals that the critical angle for the glass-air interface is \( \theta_c = \sin^{-1} \left( \frac{1.00}{1.52} \right) = 41.1^\circ \). At point A the angle of incidence is 35º and is less than the critical angle. Therefore, some light passes into the air at point A. At point B, however, total internal reflection occurs, because the angle of incidence is 55º and exceeds the critical angle.
8. 67.5 degrees

9. (d) The displacement occurs because of the refraction or bending of the ray that occurs when the ray enters and leaves the glass. The bending is greater when the refractive index is greater. Hence, the displacement is greatest for violet light.

10. (b) Equation 26.4 reveals that the critical angles for the glass-air interface are

\[ \theta_{c, \text{red}} = \sin^{-1} \left( \frac{1.000}{1.520} \right) = 41.14^\circ \quad \text{and} \quad \theta_{c, \text{violet}} = \sin^{-1} \left( \frac{1.000}{1.538} \right) = 40.56^\circ. \]

The angle of incidence shown is 40.85º, which is greater than the critical angle for violet light, but less than the critical angle for red light. Therefore, only red light enters the air, the violet light being totally internally reflected.

11. (e) For a converging lens, rays that are parallel to and close to the principal axis are bent toward the axis by the lens and pass through the focal point on the far side of the lens. For a diverging lens, rays that are parallel to and close to the principal axis are bent away from the axis by the lens. After being bent they appear as if they originated at the focal point on the near side of the lens. They do not actually pass through that focal point, however.

12. (b) A converging lens can produce a virtual image if the object is within the focal point of the lens. However, the image is upright. A diverging lens produces only upright virtual images, no matter where the object is located with respect to the lens.

13. (d) A converging lens can produce an upright virtual image, if the object is within the focal point of the lens. However, this image is larger (not smaller) than the object (see Section 26.7).

14. 2.5 cm

15. 9.17 cm

16. 417 cm

17. (c) (Statement A) The refractive power in diopters is the reciprocal of the focal length in meters, according to Equation 26.8. Thus, a positive refractive power means that the focal length is positive, which means that the lens is a converging lens. Converging lenses can produce images that are reduced in size. (Statement D) Two lenses with different refractive powers cannot have the same focal length. This follows directly from Equation 26.8. (Statement E) Since refractive power and focal length are inversely proportional according to Equation 26.8, it follows that lens A, with twice the refractive power of lens B, must have a focal length that is one-half that of lens B.
18. (a) For small angles the angular size in radians is given by the object height divided by the
distance from the eye. Thus, when an object has the greatest angular size and is located the
farthest away, it must have the greatest height. This means that object B has the greatest
height. Objects A and C have the same angular size, but object C is twice as far away, which
means that object C must have a greater height than object A. The ranking in descending
order is, then, B, C, A.

19. $8.00 \times 10^{-3}$

20. (d) Both dispersion and chromatic aberration occur because the refractive index depends on
the wavelength of light, as discussed in Sections 26.5 and 26.14.


**PROBLEMS**

1. **REASONING** Since the light will travel in glass at a constant speed \( v \), the time it takes to pass perpendicularly through the glass is given by \( t = \frac{d}{v} \), where \( d \) is the thickness of the glass. The speed \( v \) is related to the vacuum value \( c \) by Equation 26.1: \( n = \frac{c}{v} \).

   **SOLUTION** Substituting for \( v \) from Equation 26.1 and substituting values, we obtain
   \[
   t = \frac{d}{v} = \frac{nd}{c} = \frac{(1.5)(4.0 \times 10^{-3} \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 2.0 \times 10^{-11} \text{ s}
   \]

2. **REASONING** The speed \( v \) of light in any medium is found from \( n = \frac{c}{v} \) (Equation 26.1), where \( n \) is the index of refraction of the medium, and \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum. Equation 26.1 is sufficient to determine the speed of light in this collection of atoms.

   **SOLUTION** Solving Equation 26.1 for \( v \), we obtain
   \[
   v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.57 \times 10^7} = 19.1 \text{ m/s}
   \]

3. **REASONING** The refractive index \( n \) is defined by Equation 26.1 as \( n = \frac{c}{v} \), where \( c \) is the speed of light in a vacuum and \( v \) is the speed of light in a material medium. We will apply this definition to both materials A and B, and then form the ratio of the refractive indices. This will allow us to determine the unknown speed.

   **SOLUTION** Applying Equation 26.1 to both materials, we have
   \[
   n_A = \frac{c}{v_A} \quad \text{and} \quad n_B = \frac{c}{v_B}
   \]

   Dividing the equation for material A by that for material B gives
   \[
   \frac{n_A}{n_B} = \frac{c/v_A}{c/v_B} = \frac{v_B}{v_A}
   \]
Solving for $v_B$, we find that

$$v_B = v_A \left( \frac{n_A}{n_B} \right) = \left( 1.25 \times 10^8 \text{ m/s} \right) \left( 1.33 \right) = 1.66 \times 10^8 \text{ m/s}$$

4. **REASONING** The wavelength $\lambda$ is related to the frequency $f$ and speed $v$ of the light in a material by Equation 16.1 ($\lambda = v/f$). The speed of the light in each material can be expressed using Equation 26.1 ($v = c/n$) and the refractive indices $n$ given in Table 26.1. With these two equations, we can obtain the desired ratio.

**SOLUTION** Using Equations 16.1 and 26.1, we find

$$\lambda = \frac{v}{f} = \frac{c}{n} = \frac{c}{fn}$$

Using this result and recognizing that the frequency $f$ and the speed $c$ of light in a vacuum do not depend on the material, we obtain the ratio of the wavelengths as follows:

$$\frac{\lambda_{\text{alcohol}}}{\lambda_{\text{disulfide}}} = \frac{\left( \frac{c}{fn} \right)_{\text{alcohol}}}{\left( \frac{c}{fn} \right)_{\text{disulfide}}} = \frac{c}{f} \frac{\left( \frac{1}{n} \right)_{\text{alcohol}}}{c}{f} \frac{\left( \frac{1}{n} \right)_{\text{disulfide}}} = \frac{n_{\text{disulfide}}}{n_{\text{alcohol}}} = \frac{1.632}{1.362} = 1.198$$

5. **SSM REASONING** The substance can be identified from Table 26.1 if its index of refraction is known. The index of refraction $n$ is defined as the speed of light $c$ in a vacuum divided by the speed of light $v$ in the substance (Equation 26.1), both of which are known.

**SOLUTION** Using Equation 26.1, we find that

$$n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{2.201 \times 10^8 \text{ m/s}} = 1.362$$

Referring to Table 26.1, we see that the substance is **ethyl alcohol**.

6. **REASONING** We can identify the substance in Table 26.1 if we can determine its index of refraction. The index of refraction $n$ is equal to the speed of light $c$ in a vacuum divided by the speed of light $v$ in the substance, or $n = c/v$. According to Equation 16.1, however, the speed of light is related to its wavelength $\lambda$ and frequency $f$ via $v = f \lambda$. Combining these two equations by eliminating the speed $v$ yields $n = c/(f \lambda)$. 


SOLUTION The index of refraction of the substance is
\[
n = \frac{c}{f \lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{(5.403 \times 10^{14} \text{ Hz})(340.0 \times 10^{-9} \text{ m})} = 1.632
\]

An examination of Table 26.1 shows that the substance is carbon disulfide.

7. REASONING The refractive index \( n \) is defined by Equation 26.1 as \( n = \frac{c}{v} \), where \( c \) is the speed of light in a vacuum and \( v \) is the speed of light in a material medium. The speed in a vacuum or in the liquid is the distance traveled divided by the time of travel. Thus, in the definition of the refractive index, we can express the speeds \( c \) and \( v \) in terms of the distances and the time. This will allow us to calculate the refractive index.

SOLUTION According to Equation 26.1, the refractive index is
\[
n = \frac{c}{v}
\]

Using \( d_{\text{vacuum}} \) and \( d_{\text{liquid}} \) to represent the distances traveled in a time \( t \), we find the speeds to be
\[
c = \frac{d_{\text{vacuum}}}{t} \quad \text{and} \quad v = \frac{d_{\text{liquid}}}{t}
\]

Substituting these expressions into the definition of the refractive index shows that
\[
n = \frac{c}{v} = \frac{d_{\text{vacuum}}/t}{d_{\text{liquid}}/t} = \frac{d_{\text{vacuum}}}{d_{\text{liquid}}} = \frac{6.20 \text{ km}}{3.40 \text{ km}} = 1.82
\]

8. REASONING Distance traveled is the speed times the travel time. Assuming that \( t \) is the time it takes for the light to travel through the two sheets, it would travel a distance of \( ct \) in a vacuum, where its speed is \( c \). Thus, to find the desired distance, we need to determine the travel time \( t \). This time is the sum of the travel times in each sheet. The travel time in each sheet is determined by the thickness of the sheet and the speed of the light in the material. The speed in the material is less than the speed in a vacuum and depends on the refractive index of the material.

SOLUTION In the ice of thickness \( d_i \), the speed of light is \( v_i \), and the travel time is \( t_i = d_i/v_i \). Similarly, the travel time in the quartz sheet is \( t_q = d_q/v_q \). Therefore, the desired distance \( ct \) is
\[
ct = c(t_i + t_q) = c \left( \frac{d_i}{v_i} + \frac{d_q}{v_q} \right) = d_i \frac{c}{v_i} + d_q \frac{c}{v_q}
\]

Since Equation 26.1 gives the refractive index as \( n = c/v \) and since Table 26.1 gives the indices of refraction for ice and quartz as \( n_i = 1.309 \) and \( n_q = 1.544 \), the result just obtained can be written as follows:
\[
ct = d_i \frac{c}{v_i} + d_q \frac{c}{v_q} = d_i n_i + d_q n_q = (2.0 \text{ cm})(1.309) + (1.1 \text{ cm})(1.544) = 4.3 \text{ cm}
\]

9. **REASONING**
   a. The refracted ray is shown correctly. When light goes from a medium of lower index of refraction \( n = 1.4 \) to one of higher index of refraction \( n = 1.6 \), the refracted ray is bent toward the normal, as it does in part (a).

   b. The refracted ray is shown incorrectly. When light goes from a medium of lower index of refraction \( n = 1.5 \) to one of higher index of refraction \( n = 1.6 \), the refracted ray must bend toward the normal, not away from it, as part (b) of the drawing shows.

   c. The refracted ray is shown correctly. When light goes from a medium of higher index of refraction \( n = 1.6 \) to one of lower index of refraction \( n = 1.4 \), the refracted ray bends away from the normal, as it does part (c) of the drawing.

   d. The refracted ray is shown incorrectly. When the angle of incidence is \( 0^\circ \), the angle of refraction is also \( 0^\circ \), regardless of the indices of refraction.

**SOLUTION**
   a. The angle of refraction \( \theta_2 \) is given by Snell’s law, Equation 26.2, as
   \[
   \theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) = \sin^{-1} \left[ \frac{(1.4) \sin 55^\circ}{1.6} \right] = 46^\circ
   \]

   b. The actual angle of refraction is
   \[
   \theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) = \sin^{-1} \left[ \frac{(1.5) \sin 55^\circ}{1.6} \right] = 50^\circ
   \]

   c. The angle of refraction is
   \[
   \theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) = \sin^{-1} \left[ \frac{(1.6) \sin 55^\circ}{1.4} \right] = 69^\circ
   \]

   d. The actual angle of refraction is
   \[
   \theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right) = \sin^{-1} \left[ \frac{(1.6) \sin 0^\circ}{1.4} \right] = 0^\circ
   \]

10. **REASONING** The index of refraction of the oil is given, as are the angles of incidence and refraction. The incident ray originates in the oil, and the refracted ray is in the unknown liquid. As always, we label all variables associated with the incident ray with subscript 1 and all variables associated with the refracted ray with subscript 2. Therefore, we know that
\( n_1 = 1.45 \) and \( \theta_1 = 64.0^\circ \), whereas \( \theta_2 = 53.0^\circ \). We can use these data in Snell’s law \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) (Equation 26.2) to determine the unknown index of refraction \( n_2 \).

**SOLUTION** Using Snell’s law we find that

\[
\frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \quad \text{or} \quad n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{1.45 \sin 64.0^\circ}{\sin 53.0^\circ} = 1.63 \quad (26.2)
\]

11. **REASONING** The angle of refraction \( \theta_2 \) is related to the angle of incidence \( \theta_1 \) by Snell’s law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) (Equation 26.2), where \( n_1 \) and \( n_2 \) are, respectively the indices of refraction of the incident and refracting media. For each case (ice and water), the variables \( \theta_1, n_1 \) and \( n_2 \), are known, so the angles of refraction can be determined.

**SOLUTION** The ray of light impinges from air \((n_1 = 1.00)\) onto either the ice or water at an angle of incidence of \( \theta_1 = 60.0^\circ \). Using \( n_2 = 1.309 \) for ice and \( n_2 = 1.333 \) for water, we find that the angles of refraction are

\[
\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{n_1 \sin \theta_1}{n_2}\right)
\]

*Ice*

\[
\theta_{2, \text{ice}} = \sin^{-1}\left[\frac{(1.00) \sin 60.0^\circ}{1.309}\right] = 41.4^\circ
\]

*Water*

\[
\theta_{2, \text{water}} = \sin^{-1}\left[\frac{(1.00) \sin 60.0^\circ}{1.333}\right] = 40.5^\circ
\]

The difference in the angles of refraction is \( \theta_{2, \text{ice}} - \theta_{2, \text{water}} = 41.4^\circ - 40.5^\circ = 0.9^\circ \)

12. **REASONING** If refraction did not occur, the light would travel straight ahead, as indicated by the dotted path in the drawing at the right. It would then strike the lake-bottom at a distance \( d \) from point B. Because of refraction, however, the light ray is bent toward the dashed normal to the air-water surface, the angle of refraction being \( \theta_2 \).

With refraction, the light strikes the lake-bottom at a distance \( d' \) from point B. We will use trigonometry to find the distances \( d \) and \( d' \). To find the angle of refraction \( \theta_2 \), we will use Snell’s law \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) (Equation 26.2).
**SOLUTION**

a. In the absence of refraction the light would travel straight ahead, the angle between the dotted line and the dashed normal being 55°. Therefore, we can use the tangent function (see Equation 1.3) to find the distance \( d \) in the following way:

\[
\tan 55^\circ = \frac{d}{3.0 \text{ m}} \quad \text{or} \quad d = (3.0 \text{ m}) \tan 55^\circ = 4.3 \text{ m}
\]

b. In the presence of refraction we also use the tangent function to find the distance \( d' \). This time, however, the angle between the refracted light and the dashed normal is \( \theta_2 \) (see the drawing) instead of 55°:

\[
\tan \theta_2 = \frac{d'}{3.0 \text{ m}} \quad \text{or} \quad d' = (3.0 \text{ m}) \tan \theta_2 \quad \text{(1)}
\]

To determine the angle of refraction \( \theta_2 \) we resort to Snell’s law, noting that \( n_1 = 1.00 \) for air and \( n_2 = 1.33 \) for water and \( \theta_1 = 55^\circ \) for the angle of incidence:

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{or} \quad \sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} = \frac{1.00 \sin 55^\circ}{1.33} = 0.62
\]

We find, then, that \( \theta_2 = \sin^{-1}(0.62) = 38^\circ \), which we can substitute into Equation (1) to determine the distance \( d' \):

\[
d' = (3.0 \text{ m}) \tan \theta_2 = (3.0 \text{ m}) \tan 38^\circ = 2.3 \text{ m}
\]

13. **SSM REASONING** We will use the geometry of the situation to determine the angle of incidence. Once the angle of incidence is known, we can use Snell's law to find the index of refraction of the unknown liquid. The speed of light \( v \) in the liquid can then be determined.

**SOLUTION** From the drawing in the text, we see that the angle of incidence at the liquid-air interface is

\[
\theta_1 = \tan^{-1}\left(\frac{5.00 \text{ cm}}{6.00 \text{ cm}}\right) = 39.8^\circ
\]

The drawing also shows that the angle of refraction is 90.0°. Thus, according to Snell's law (Equation 26.2: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)), the index of refraction of the unknown liquid is

\[
n_1 = \frac{n_2 \sin \theta_2}{\sin \theta_1} = \frac{(1.000) \sin 90.0^\circ}{\sin 39.8^\circ} = 1.56
\]

From Equation 26.1 (\( n = c/v \)), we find that the speed of light in the unknown liquid is

\[
v = \frac{c}{n_1} = \frac{3.00 \times 10^8 \text{ m/s}}{1.56} = 1.92 \times 10^8 \text{ m/s}
\]
14. **REASONING AND SOLUTION** Using Equation 26.3, we find

\[ d = \left( \frac{n_1}{n_2} \right) d' = \left( \frac{1.546}{1.000} \right) 2.5 \text{ cm} = 3.9 \text{ cm} \]

15. **SSM REASONING** We begin by using Snell's law (Equation 26.2: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)) to find the index of refraction of the material. Then we will use Equation 26.1, the definition of the index of refraction \( (n = c/v) \) to find the speed of light in the material.

**SOLUTION** From Snell's law, the index of refraction of the material is

\[ n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{(1.000) \sin 63.0^\circ}{\sin 47.0^\circ} = 1.22 \]

Then, from Equation 26.1, we find that the speed of light \( v \) in the material is

\[ v = \frac{c}{n_2} = \frac{3.00 \times 10^8 \text{ m/s}}{1.22} = 2.46 \times 10^8 \text{ m/s} \]

16. **REASONING** When the light ray passes from \( a \) into \( b \), it is bent toward the normal. According to the discussion in Section 26.2, this happens when the index of refraction of \( b \) is greater than that of \( a \), or \( n_b > n_a \). When the light passes from \( b \) into \( c \), it is bent away from the normal. This means that the index of refraction of \( c \) is less than that of \( b \), or \( n_c < n_b \). The smaller the value of \( n_c \), the greater is the angle of refraction. As can be seen from the drawing, the angle of refraction in material \( c \) is greater than the angle of incidence at the \( a-b \) interface. Applying Snell’s law to the \( a-b \) and \( b-c \) interfaces gives \( n_a \sin \theta_a = n_b \sin \theta_b = n_c \sin \theta_c \). Since \( \theta_c \) is greater than \( \theta_a \), the equation \( n_a \sin \theta_a = n_c \sin \theta_c \) shows that the index of refraction of \( a \) must be greater than that of \( c \), \( n_a > n_c \). Thus, the ordering of the indices of refraction, highest to lowest, is \( n_b, n_a, n_c \).

**SOLUTION** The index of refraction for each medium can be evaluated from Snell’s law, Equation 26.2:

\[ a-b \text{ interface} \quad n_b = \frac{n_a \sin \theta_a}{\sin \theta_b} = \frac{(1.20) \sin 50.0^\circ}{\sin 45.0^\circ} = 1.30 \]

\[ b-c \text{ interface} \quad n_c = \frac{n_b \sin \theta_b}{\sin \theta_c} = \frac{(1.30) \sin 45.0^\circ}{\sin 56.7^\circ} = 1.10 \]

As expected, the ranking of the indices of refraction, highest to lowest, is \( n_b = 1.30, n_a = 1.20, n_c = 1.10 \)
17. **REASONING** When the incident light is in a vacuum, Snell’s law, Equation 26.2, can be used to express the relation between the angle of incidence (35.0°), the (unknown) index of refraction \( n_2 \) of the glass and the (unknown) angle \( \theta_2 \) of refraction for the light entering the slab: 
\[
(1.00)\sin 35.0^\circ = n_2 \sin \theta_2
\]
When the incident light is in the liquid, we can again use Snell’s law to express the relation between the index of refraction \( n_1 \) of the liquid, the angle of incidence (20.3°), the index of refraction \( n_2 \) of the glass, and the (unknown) angle of refraction \( \theta_2 \): 
\[
n_1 \sin 20.3^\circ = n_2 \sin \theta_2
\]
By equating these two equations, we can determine the index of refraction of the liquid.

**SOLUTION** Setting the two equations above equal to each other and solving for the index of refraction of the liquid gives
\[
n_1 \sin 20.3^\circ = (1.00) \sin 35.0^\circ \quad \text{and} \quad n_1 = \frac{(1.00) \sin 35.0^\circ}{\sin 20.3^\circ} = 1.65
\]

18. **REASONING** According to Equation 2.1, when an object moves with a constant speed \( v \) and travels a distance \( d \) in an elapsed time \( t \), these three quantities are related by \( v = \frac{d}{t} \). In this situation, the stone sinks a distance \( d \) from the surface to the bottom, and the elapsed time is \( t \). When viewed from above, the stone still sinks to the bottom in an elapsed time \( t \), but the apparent depth \( d' \) to which it sinks is not equal to the actual depth \( d \). Therefore, the apparent speed \( v' \) of the stone is not the same as its actual speed \( v \), and we have that
\[
v = \frac{d}{t} \quad \text{and} \quad v' = \frac{d'}{t}
\]
(1)

The apparent depth \( d' \) is given by 
\[
d' = d \left( \frac{n_2}{n_1} \right) \quad \text{(Equation 26.3)},
\]
where \( n_2 = 1.000 \) is the index of refraction of air and \( n_1 = 1.333 \) (See Table 26.1) is the index of refraction of water.

**SOLUTION** Solving the first of Equations (1) for \( t \) yields \( t = \frac{d}{v} \). Substituting this expression into the second of Equations (1), we obtain
\[
v' = \frac{d'}{t} = \frac{d'}{\frac{d}{v}} = v \left( \frac{d'}{d} \right)
\]
(2)

Substituting \( d' = d \left( \frac{n_2}{n_1} \right) \) (Equation 26.3) into Equation (2), we find that
\[
v' = v \left( \frac{d'}{d} \right) = v \left( \frac{n_2}{n_1} \right) = v \left( \frac{1.000}{1.333} \right) = (0.48 \text{ m/s}) \left( \frac{1.000}{1.333} \right) = 0.36 \text{ m/s}
\]
19. **REASONING** Following the discussion in Conceptual Example 4, we have the drawing at the right to use as a guide. In this drawing the symbol \( d \) refers to depths in the water, while the symbol \( h \) refers to heights in the air above the water. Moreover, symbols with a prime denote apparent distances, and unprimed symbols denote actual distances. We will use Equation 26.3 to relate apparent distances to actual distances. In so doing, we will use the fact that the refractive index of air is essentially \( n_{\text{air}} = 1 \) and denote the refractive index of water by \( n_w = 1.333 \) (see Table 26.1).

**SOLUTION** To the fish, the man appears to be a distance above the air-water interface that is given by Equation 26.3 as

\[
\frac{h'}{1} = \frac{h}{n_w}
\]

Thus, measured above the eyes of the fish, the man appears to be located at a distance of

\[
h' + d = h\left(\frac{n_w}{1}\right) + d
\]

To the man, the fish appears to be a distance below the air-water interface that is given by Equation 26.3 as \( d' = d\left(\frac{1}{n_w}\right) \). Thus, measured below the man’s eyes, the fish appears to be located at a distance of

\[
h + d' = h + d\left(\frac{1}{n_w}\right)
\]

Dividing Equation (1) by Equation (2) and using the fact that \( h = d \), we find

\[
\frac{h' + d}{h + d'} = \frac{h\left(\frac{n_w}{1}\right) + d}{h + d\left(\frac{1}{n_w}\right)} = \frac{n_w + 1}{1 + \frac{1}{n_w}} = n_w
\]

In Equation (3), \( h' + d \) is the distance we seek, and \( h + d' \) is given as 2.0 m. Thus, we find

\[
h' + d = n_w (h + d') = (1.333)(2.0 \text{ m}) = 2.7 \text{ m}
\]

20. **REASONING** Snell’s law will allow us to calculate the angle of refraction \( \theta_{2, B} \) with which the ray leaves the glass at point B, provided that we have a value for the angle of incidence \( \theta_{1, B} \) at this point (see the drawing). This angle of incidence is not given, but we can obtain it by considering what happens to the incident ray at point A. This ray is incident at an angle...
\( \theta_{1, A} \) and refracted at an angle \( \theta_{2, A} \). Snell’s law can be used to obtain \( \theta_{2, A} \), the value for which can be combined with the geometry at points A and B to provide the needed value for \( \theta_{1, B} \). Since the light ray travels from a material (carbon disulfide) with a higher refractive index toward a material (glass) with a lower refractive index, it is bent away from the normal at point A, as the drawing shows.

**SOLUTION** Using Snell’s law at point B, we have

\[
(1.52) \sin \theta_{1, B} = (1.63) \sin \theta_{2, B} \quad \text{or} \quad \sin \theta_{2, B} = \left( \frac{1.52}{1.63} \right) \sin \theta_{1, B} \tag{1}
\]

To find \( \theta_{1, B} \) we note from the drawing that

\[
\theta_{1, B} + \theta_{2, A} = 90.0^\circ \quad \text{or} \quad \theta_{1, B} = 90.0^\circ - \theta_{2, A} \tag{2}
\]

We can find \( \theta_{2, A} \), which is the angle of refraction at point A, by again using Snell’s law:

\[
(1.63) \sin \theta_{1, A} = (1.52) \sin \theta_{2, A} \quad \text{or} \quad \sin \theta_{2, A} = \left( \frac{1.63}{1.52} \right) \sin \theta_{1, A}
\]

Thus, we have

\[
\sin \theta_{2, A} = \left( \frac{1.63}{1.52} \right) \sin \theta_{1, A} = \left( \frac{1.63}{1.52} \right) \sin 30.0^\circ = 0.536 \quad \text{or} \quad \theta_{2, A} = \sin^{-1}(0.536) = 32.4^\circ
\]

Using Equation (2), we find that

\[
\theta_{1, B} = 90.0^\circ - \theta_{2, A} = 90.0^\circ - 32.4^\circ = 57.6^\circ
\]

With this value for \( \theta_{1, B} \) in Equation (1) we obtain

\[
\sin \theta_{2, B} = \left( \frac{1.52}{1.63} \right) \sin \theta_{1, B} = \left( \frac{1.52}{1.63} \right) \sin 57.6^\circ = 0.787 \quad \text{or} \quad \theta_{2, B} = \sin^{-1}(0.787) = 51.9^\circ
\]
21. **REASONING** The drawing at the right shows the geometry of the situation using the same notation as that in Figure 26.6. In addition to the text's notation, let \( t \) represent the thickness of the pane, let \( L \) represent the length of the ray in the pane, let \( x \) (shown twice in the figure) equal the displacement of the ray, and let the difference in angles \( \theta_1 - \theta_2 \) be given by \( \phi \).

We wish to find the amount \( x \) by which the emergent ray is displaced relative to the incident ray. This can be done by applying Snell's law at each interface, and then making use of the geometric and trigonometric relations in the drawing.

**SOLUTION** If we apply Snell's law (see Equation 26.2) to the bottom interface we obtain
\[
\sin \theta_1 = n_2 \sin \theta_2.
\]
Similarly, if we apply Snell's law at the top interface where the ray emerges, we have
\[
\sin \theta_2 = n_3 \sin \theta_3 = n_1 \sin \theta_3.
\]
Comparing this with Snell's law at the bottom face, we see that \( n_1 \sin \theta_1 = n_1 \sin \theta_3 \), from which we can conclude that \( \theta_3 = \theta_1 \).

Therefore, the emerging ray is parallel to the incident ray.

From the geometry of the ray and thickness of the pane, we see that \( L \cos \theta_2 = t \), from which it follows that \( L = \frac{t}{\cos \theta_2} \). Furthermore, we see that \( x = L \sin \phi = L \sin (\theta_1 - \theta_2) \).

Substituting for \( L \), we find
\[
x = L \sin (\theta_1 - \theta_2) = \frac{t \sin (\theta_1 - \theta_2)}{\cos \theta_2}.
\]

Before we can use this expression to determine a numerical value for \( x \), we must find the value of \( \theta_2 \). Solving the expression for Snell's law at the bottom interface for \( \theta_2 \), we have
\[
\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} = \frac{(1.000) (\sin 30.0^\circ)}{1.52} = 0.329 \quad \text{or} \quad \theta_2 = \sin^{-1} 0.329 = 19.2^\circ.
\]

Therefore, the amount by which the emergent ray is displaced relative to the incident ray is
\[
x = \frac{t \sin (\theta_1 - \theta_2)}{\cos \theta_2} = \frac{(6.00 \text{ mm}) \sin (30.0^\circ - 19.2^\circ)}{\cos 19.2^\circ} = 1.19 \text{ mm}.
\]

22. **REASONING** The fish appears to be a distance less than \( \frac{1}{2} L \) from the front wall of the aquarium, where \( L \) is the distance between the back and front walls. The phenomenon of **apparent depth** is at play here. According to Equation 26.3, the apparent depth or distance \( d' \)
is related to the actual depth \( \frac{1}{2} L \) by \( d' = \frac{1}{2} L \left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) \), where \( n_{\text{air}} \) and \( n_{\text{water}} \) are the refractive indices of air and water. Referring to Table 26.1, we see that \( n_{\text{air}} < n_{\text{water}} \), so that \( d' < \frac{1}{2} L \).

Since the fish is a distance \( \frac{1}{2} L \) in front of the plane mirror, the image of the fish is a distance \( \frac{1}{2} L \) behind the plane mirror. Thus, the image is a distance \( \frac{3}{2} L \) from the front wall.

The image of the fish appears to be at a distance less than \( \frac{3}{2} L \) from the front wall, because of the phenomenon of *apparent depth*. The explanation given above applies here also, except that the actual depth or distance is \( \frac{3}{2} L \) instead of \( \frac{1}{2} L \).

**SOLUTION**

a. Using Equation 26.3, we find that the apparent distance \( d' \) between the fish and the front wall is

\[
\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) = \left( \frac{1.000}{1.333} \right) = \left( \frac{40.0 \text{ cm}}{15.0 \text{ cm}} \right) = 15.0 \text{ cm}
\]

\[
\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) = \left( \frac{1.000}{1.333} \right) = \left( \frac{40.0 \text{ cm}}{15.0 \text{ cm}} \right) = 15.0 \text{ cm}
\]

b. Again using Equation 26.3, we find that the apparent distance between the image of the fish and the front wall is

\[
\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) = \left( \frac{3}{2} \right) \left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) = \left( \frac{3}{2} \right) \left( \frac{40.0 \text{ cm}}{15.0 \text{ cm}} \right) = 45.0 \text{ cm}
\]

23. **REASONING AND SOLUTION** The horizontal distance of the chest from the normal is found from Figure 26.4b to be \( x = d \tan \theta_1 \) and \( x = d' \tan \theta_2 \), where \( \theta_1 \) is the angle from the dashed normal to the solid rays and \( \theta_2 \) is the angle from the dashed normal to the dashed rays. Hence,

\[
d' = d \left( \frac{\tan \theta_1}{\tan \theta_2} \right)
\]

Snell's law applied at the interface gives

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

For small angles, \( \sin \theta_1 \approx \tan \theta_1 \) and \( \sin \theta_2 \approx \tan \theta_2 \), so

\[
\tan \theta_1 / \tan \theta_2 \approx \sin \theta_1 / \sin \theta_2 = \left( \frac{n_2}{n_1} \right)
\]

Now \( d' = d \left( \frac{\tan \theta_1}{\tan \theta_2} \right) \). Therefore,

\[
d' \approx d \left( \frac{n_2}{n_1} \right)
\]
24. **REASONING** As applied to this problem, the apparent depth \( d' \) of the paper is given by

\[
d' = d \left( \frac{n_{\text{air}}}{n_{\text{plastic}}} \right) \tag{26.3}
\]

where \( d \) is the actual depth and \( n_{\text{air}} \) and \( n_{\text{plastic}} \) are, respectively, the indices of refraction of air and the plastic. We will apply this equation to the view of the paper from the top and from the bottom. In so doing, we will also make use of the fact that \( d_{\text{top}} + d_{\text{bottom}} = L \), where \( d_{\text{top}} \) and \( d_{\text{bottom}} \) are the actual distances of the paper from opposite faces of the cube and \( L = 9.00 \text{ cm} \) is the thickness of the cube. See the edge-on view of the paper and the cube in the drawing.

**SOLUTION**

We begin by applying Equation 26.3 to the view of the paper from the top and solving it for \( n_{\text{plastic}} \):

\[
d'_{\text{top}} = d_{\text{top}} \left( \frac{n_{\text{air}}}{n_{\text{plastic}}} \right) \quad \text{or} \quad n_{\text{plastic}} = \frac{d_{\text{top}} n_{\text{air}}}{d'} \tag{1}
\]

We do not have a value for \( d_{\text{top}} \), so we eliminate it from Equation (1) by making use of the fact that \( d_{\text{top}} + d_{\text{bottom}} = L \); we substitute \( d_{\text{top}} = L - d_{\text{bottom}} \) into Equation (1) and obtain

\[
n_{\text{plastic}} = \frac{d_{\text{top}} n_{\text{air}}}{d'} = \frac{(L - d_{\text{bottom}}) n_{\text{air}}}{d'} = \frac{L n_{\text{air}} - d_{\text{bottom}} n_{\text{air}}}{d'} \tag{2}
\]

We also do not have a value for \( d_{\text{bottom}} \). In order to eliminate it from Equation (2), we apply Equation 26.3 to the view of the paper from the bottom and solve it for \( d_{\text{bottom}} n_{\text{air}} \):

\[
d'_{\text{bottom}} = d_{\text{bottom}} \left( \frac{n_{\text{air}}}{n_{\text{plastic}}} \right) \quad \text{or} \quad d_{\text{bottom}} n_{\text{air}} = d'_{\text{bottom}} n_{\text{plastic}} \tag{3}
\]

Substituting Equation (3) into Equation (2) gives

\[
n_{\text{plastic}} = \frac{L n_{\text{air}} - d_{\text{bottom}} n_{\text{air}}}{d'} = \frac{L n_{\text{air}} - d'_{\text{bottom}} n_{\text{plastic}}}{d'} \tag{4}
\]

Equation (4) contains only variables for which we have values, so we can solve it for \( n_{\text{plastic}} \) and obtain
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25. **REASONING** The drawing at the right shows the situation. As discussed in the text, when the observer is directly above, the apparent depth \( d' \) of the object is related to the actual depth by Equation 26.3:

\[
d' = d \left( \frac{n_2}{n_1} \right)
\]

In this case, we must apply Equation 26.3 twice; once for the rays in the glass, and once again for the rays in the water.

**SOLUTION** We refer to the drawing for our notation and begin at the logo. To an observer in the water directly above the logo, the apparent depth of the logo is

\[
d'_w = d_g \left( \frac{n_w}{n_g} \right)
\]

When viewed directly from above in air, the logo’s apparent depth is

\[
d'_w = \left( d_w + d'_g \right) \left( \frac{n_w}{n_g} \right)
\]

Substituting the expression for \( d'_g \) into the expression for \( d'_w \), we obtain

\[
d'_w = (d_w + d'_g) \left( \frac{n_{air}}{n_w} \right) = d_w \left( \frac{n_{air}}{n_w} \right) + d_g \left( \frac{n_w}{n_g} \right) \left( \frac{n_{air}}{n_w} \right) = d_w \left( \frac{n_{air}}{n_w} \right) + d_g \left( \frac{n_{air}}{n_g} \right)
\]

\[
= (1.50 \text{ cm}) \left( \frac{1.000}{1.333} \right) + (3.20 \text{ cm}) \left( \frac{1.000}{1.52} \right) = 3.23 \text{ cm}
\]

26. **REASONING** For this problem, the critical angle \( \theta_c \) is specified by \( \sin \theta_c = \frac{n_{air}}{n_{liquid}} \) (Equation 26.4), where \( n_{air} \) and \( n_{liquid} \) are, respectively, the indices of refraction of air and the liquid. The index of refraction of the liquid appears in the denominator on the right side.
of this equation. Therefore, the liquid with the largest index of refraction has the smallest value of \( \sin \theta_c \) and, correspondingly, the smallest value of the critical angle.

**SOLUTION** In Table 26.1 the liquid with the largest index of refraction is carbon disulfide. Using Equation 26.4 and taking the refractive index for carbon disulfide from Table 26.1, we obtain

\[
\sin \theta_c = \frac{n_{\text{air}}}{n_{\text{liquid}}} = \frac{1.000}{1.632} \quad \text{or} \quad \theta_c = \sin^{-1} \left( \frac{1.000}{1.632} \right) = 37.79^\circ
\]

27. **SSM REASONING** The light ray traveling in the oil can only penetrate into the water if it does not undergo total internal reflection at the boundary between the oil and the water. Total internal reflection will occur if the angle of incidence \( \theta = 71.4^\circ \) is greater than the critical angle \( \theta_c \) for these two media. The critical angle is found from

\[
\sin \theta_c = \frac{n_2}{n_1}
\]

where \( n_2 = 1.333 \) is the index of refraction of water (see Table 26.1), and \( n_1 = 1.47 \) is the index of refraction of the oil.

**SOLUTION** Solving Equation 26.4 for \( \theta_c \), we obtain

\[
\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = \sin^{-1} \left( \frac{1.333}{1.47} \right) = 65.1^\circ
\]

Comparing this result to \( \theta = 71.4^\circ \), we see that the angle of incidence is greater than the critical angle (\( \theta > \theta_c \)). Therefore, the ray of light will not enter the water; it will instead undergo total internal reflection within the oil.

28. **REASONING AND SOLUTION** Only the light which has an angle of incidence less than or equal \( \theta_c \) can escape. This light leaves the source in a cone whose apex angle is \( 2 \theta_c \). The radius of this cone at the surface of the water \((n = 1.333, \text{ see Table 26.1})\) is \( R = d \tan \theta_c \).

Now

\[
\theta_c = \sin^{-1} \left( \frac{1.000}{1.333} \right) = 48.6^\circ
\]

so

\[
R = (2.2 \text{ m}) \tan 48.6^\circ = 2.5 \text{ m}
\]

29. **REASONING AND SOLUTION**

a. The index of refraction \( n_2 \) of the liquid must match that of the glass, or \( n_2 = 1.50 \).
b. When none of the light is transmitted into the liquid, the angle of incidence must be equal to or greater than the critical angle. According to Equation 26.4, the critical angle $\theta_c$ is given by $\sin \theta_c = n_2/n_1$, where $n_2$ is the index of refraction of the liquid and $n_1$ is that of the glass. Therefore,

$$n_2 = n_1 \sin \theta_c = (1.50) \sin 58.0^\circ = 1.27$$

If $n_2$ were larger than 1.27, the critical angle would also be larger, and light would be transmitted from the glass into the liquid. Thus, $n_2 = 1.27$ represents the largest index of refraction of the liquid such that none of the light is transmitted into the liquid.

30. **REASONING** Total internal reflection can occur only when light is traveling from a higher index material toward a lower index material. Thus, total internal reflection is possible when the material above or below a layer has a smaller index of refraction than the layer itself. With this criterion in mind, the following table indicates the various possibilities:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Is total internal reflection possible?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top surface of layer</td>
</tr>
<tr>
<td>$a$</td>
<td>Yes</td>
</tr>
<tr>
<td>$b$</td>
<td>Yes</td>
</tr>
<tr>
<td>$c$</td>
<td>No</td>
</tr>
</tbody>
</table>

**SOLUTION** The critical angle for each interface at which total internal reflection is possible is obtained from Equation 26.4:

**Layer a, top surface**

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_a}\right) = \sin^{-1}\left(\frac{1.00}{1.30}\right) = 50.3^\circ$$

$$\Delta \theta = 75.0^\circ - 50.3^\circ = 24.7^\circ$$

**Layer b, top surface**

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_b}\right) = \sin^{-1}\left(\frac{1.30}{1.50}\right) = 60.1^\circ$$

$$\Delta \theta = 75.0^\circ - 60.1^\circ = 14.9^\circ$$
**Layer b, bottom surface**

\[ \theta_c = \sin^{-1}\left(\frac{n_c}{n_b}\right) = \sin^{-1}\left(\frac{1.40}{1.50}\right) = 69.0^\circ \]

\[ \Delta \theta = 75.0^\circ - 69.0^\circ = 6.0^\circ \]

**Layer c, bottom surface**

\[ \theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_c}\right) = \sin^{-1}\left(\frac{1.00}{1.40}\right) = 45.6^\circ \]

\[ \Delta \theta = 75.0^\circ - 45.6^\circ = 29.4^\circ \]

31. **REASONING AND SOLUTION**

a. Using Equation 26.4 and the refractive index for crown glass given in Table 26.1, we find that the critical angle for a crown glass-air interface is

\[ \theta_c = \sin^{-1}\left(\frac{1.00}{1.523}\right) = 41.0^\circ \]

The light will be totally reflected at point A since the incident angle of 60.0° is greater than \(\theta_c\). The incident angle at point B, however, is 30.0° and smaller than \(\theta_c\). Thus, the light will exit first at point B.

b. The critical angle for a crown glass-water interface is

\[ \theta_c = \sin^{-1}\left(\frac{1.333}{1.523}\right) = 61.1^\circ \]

The incident angle at point A is less than this, so the light will first exit at point A.

32. **REASONING** Total internal reflection occurs only when light goes from a higher index material toward a lower index material (see Section 26.3). Since total internal reflection occurs at both the a-b and a-c interfaces, the index of refraction of material a is larger than that of either material b or c: \(n_a > n_b\) and \(n_a > n_c\). We now need to determine which index of refraction, \(n_b\) or \(n_c\), is larger. The critical angle is given by Equation 26.4 as \(\sin \theta_c = n_2/n_1\), where \(n_2\) is the smaller index of refraction. Therefore, the larger the value of \(n_2\), the larger the critical angle. It is evident from the drawing that the critical angle for the a-c interface is larger than the critical angle for the a-b interface. Therefore \(n_c\) must be larger than \(n_b\). The ranking of the indices of refraction, largest to smallest, is: \(n_a, n_c, n_b\).

**SOLUTION** For the a-b interface, the critical angle is given by Equation 26.4 as \(\sin \theta_c = n_b/n_a\). Therefore, the index of refraction for material b is
For the $a$-$c$ interface, we note that the angle of incidence is $90.0^\circ - 40.0^\circ = 50.0^\circ$. The index of refraction for material $c$ is

$$n_c = n_a \sin \theta_c = (1.80) \sin 50.0^\circ = 1.38$$

As expected, the ranking of the indices of refraction, highest-to-lowest, is $n_a = 1.80$, $n_c = 1.38$, $n_b = 1.16$.

33. **SSM REASONING** Total internal reflection will occur at point $P$ provided that the angle $\alpha$ in the drawing at the right exceeds the critical angle. This angle is determined by the angle $\theta_2$ at which the light rays enter the quartz slab. We can determine $\theta_2$ by using Snell’s law of refraction and the incident angle, which is given as $\theta_1 = 34^\circ$.

**SOLUTION** Using $n$ for the refractive index of the fluid that surrounds the crystalline quartz slab and $n_q$ for the refractive index of quartz and applying Snell’s law give

$$n \sin \theta_1 = n_q \sin \theta_2 \quad \text{or} \quad \sin \theta_2 = \frac{n}{n_q} \sin \theta_1 \quad (1)$$

But when $\alpha$ equals the critical angle, we have from Equation 26.4 that

$$\sin \alpha = \sin \theta_c = \frac{n}{n_q} \quad (2)$$

According to the geometry in the drawing above, $\alpha = 90^\circ - \theta_2$. As a result, Equation (2) becomes

$$\sin \left(90^\circ - \theta_2\right) = \cos \theta_2 = \frac{n}{n_q} \quad (3)$$

Squaring Equation (3), using the fact that $\sin^2 \theta_2 + \cos^2 \theta_2 = 1$, and substituting from Equation (1), we obtain

$$\cos^2 \theta_2 = 1 - \sin^2 \theta_2 = 1 - \frac{n^2}{n_q^2} \sin^2 \theta_1 = \frac{n^2}{n_q^2} \quad (4)$$
Solving Equation (4) for \( n \) and using the value given in Table 26.1 for the refractive index of crystalline quartz, we find

\[
\frac{n}{\sqrt{1+\sin^2 \theta}} = \frac{1.544}{\sqrt{1+\sin^2 34^\circ}} = 1.35
\]

34. **REASONING** The time it takes for the light to travel from \( A \) to \( B \) is equal to the distance divided by the speed of light in the substance. The distance is known, and the speed of light \( v \) in the substance is equal to the speed of light \( c \) in a vacuum divided by the index of refraction \( n_1 \) (Equation 26.1). The index of refraction can be obtained by noting that the light is incident at the critical angle \( \theta_c \) (which is known). According to Equation 26.4, the index of refraction \( n_1 \) is related to the critical angle and the index of refraction \( n_2 \) by

\[
n_1 = \frac{n_2}{\sin \theta_c}
\]

**SOLUTION** The time \( t \) it takes for the light to travel from \( A \) to \( B \) is

\[
t = \frac{\text{Distance}}{\text{Speed of light in the substance}} = \frac{d}{v}
\]

The speed of light \( v \) in the substance is related to the speed of light \( c \) in a vacuum and the index of refraction \( n_1 \) of the substance by \( v = c/n_1 \) (Equation 26.1). Substituting this expression into Equation (1) gives

\[
t = \frac{d}{v} = \frac{d}{c/n_1} = \frac{d n_1}{c}
\]

Since the light is incident at the critical angle \( \theta_c \), we know that \( n_1 \sin \theta_c = n_2 \) (Equation 26.4). Solving this expression for \( n_1 \) and substituting the result into Equation (2) yields

\[
t = \frac{d n_1}{c} = \frac{d \left( \frac{n_2}{\sin \theta_c} \right)}{c} = \frac{(4.60 \text{ m}) \left( \frac{1.63}{\sin 48.1^\circ} \right)}{3.00 \times 10^8 \text{ m/s}} = 3.36 \times 10^{-8} \text{ s}
\]

35. **REASONING** In the ratio \( n_B/n_C \) each refractive index can be related to a critical angle for total internal reflection according to Equation 26.4. By applying this expression to the A-B interface and again to the A-C interface, we will obtain expressions for \( n_B \) and \( n_C \) in terms of the given critical angles. By substituting these expressions into the ratio, we will be able to obtain a result from which the ratio can be calculated.
**SOLUTION** Applying Equation 26.4 to the A-B interface, we obtain

\[
\sin \theta_{c, AB} = \frac{n_B}{n_A} \quad \text{or} \quad n_B = n_A \sin \theta_{c, AB}
\]

Applying Equation 26.4 to the A-C interface gives

\[
\sin \theta_{c, AC} = \frac{n_C}{n_A} \quad \text{or} \quad n_C = n_A \sin \theta_{c, AC}
\]

With these two results, the desired ratio can now be calculated:

\[
\frac{n_B}{n_C} = \frac{n_A \sin \theta_{c, AB}}{n_A \sin \theta_{c, AC}} = \frac{\sin 36.5^\circ}{\sin 47.0^\circ} = 0.813
\]

36. **REASONING** Using the value given for the critical angle in Equation 26.4 (\(\sin \theta_c = \frac{n_2}{n_1}\)), we can obtain the ratio of the refractive indices. Then, using this ratio in Equation 26.5 (Brewster’s law), we can obtain Brewster’s angle \(\theta_B\).

**SOLUTION** From Equation 26.4, with \(n_2 = n_{\text{air}} = 1\) and \(n_2 = n_{\text{liquid}}\), we have

\[
\sin \theta_c = \sin 39^\circ = \frac{1}{n_{\text{liquid}}}
\]

According to Brewster’s law,

\[
\tan \theta_B = \frac{n_2}{n_1} = \frac{1}{n_{\text{liquid}}}
\]

Substituting Equation (2) into Equation (1), we find

\[
\tan \theta_B = \frac{1}{n_{\text{liquid}}} = \sin 39^\circ = 0.63 \quad \text{or} \quad \theta_B = \tan^{-1}(0.63) = 32^\circ
\]

37. **SSM REASONING** Since the light reflected from the coffee table is completely polarized parallel to the surface of the glass, the angle of incidence must be the Brewster angle (\(\theta_B = 56.7^\circ\)) for the air-glass interface. We can use Brewster's law (Equation 26.5: \(\tan \theta_B = \frac{n_2}{n_1}\)) to find the index of refraction \(n_2\) of the glass.

**SOLUTION** Solving Brewster's law for \(n_2\), we find that the refractive index of the glass is

\[
n_2 = n_1 \tan \theta_B = (1.000)(\tan 56.7^\circ) = 1.52
\]
38. **REASONING** The reflected light is 100% polarized when the angle of incidence is equal to the Brewster angle $\theta_B$. The Brewster angle is given by $\tan \theta_B = \frac{n_{\text{liquid}}}{n_{\text{air}}}$ (Equation 26.5), where $n_{\text{liquid}}$ and $n_{\text{air}}$ are the refractive indices of the liquid and air (neither of which is known). However, $n_{\text{liquid}}$ and $n_{\text{air}}$ are related by Snell’s law (Equation 26.2), $n_{\text{air}} \sin \theta_1 = n_{\text{liquid}} \sin \theta_2$, where $\theta_1$ and $\theta_2$ are, respectively, the angles of incidence and refraction. These two relations will allow us to determine the Brewster angle.

**SOLUTION** The Brewster angle is given by

$$\tan \theta_B = \frac{n_{\text{liquid}}}{n_{\text{air}}}$$

Snell’s law is

$$n_{\text{air}} \sin \theta_1 = n_{\text{liquid}} \sin \theta_2$$

from which we obtain $n_{\text{liquid}}/n_{\text{air}} = \sin \theta_1 / \sin \theta_2$. Substituting this result into Equation 26.5 yields

$$\tan \theta_B = \frac{n_{\text{liquid}}}{n_{\text{air}}} = \frac{\sin \theta_1}{\sin \theta_2}$$

Thus, the Brewster angle is

$$\theta_B = \tan^{-1} \left( \frac{\sin \theta_1}{\sin \theta_2} \right) = \tan^{-1} \left( \frac{\sin 53.0^\circ}{\sin 34.0^\circ} \right) = 55.0^\circ$$

39. **SSM REASONING** Brewster's law (Equation 26.5: $\tan \theta_B = n_2 / n_1$) relates the angle of incidence $\theta_B$ at which the reflected ray is completely polarized parallel to the surface to the indices of refraction $n_1$ and $n_2$ of the two media forming the interface. We can use Brewster's law for light incident from above to find the ratio of the refractive indices $n_2/n_1$. This ratio can then be used to find the Brewster angle for light incident from below on the same interface.

**SOLUTION** The index of refraction for the medium in which the incident ray occurs is designated by $n_1$. For the light striking from above $n_2/n_1 = \tan \theta_B = \tan 65.0^\circ = 2.14$. The same equation can be used when the light strikes from below if the indices of refraction are interchanged

$$\theta_B = \tan^{-1} \left( \frac{n_1}{n_2} \right) = \tan^{-1} \left( \frac{1}{n_2/n_1} \right) = \tan^{-1} \left( \frac{1}{2.14} \right) = 25.0^\circ$$
40. **REASONING** Given the height \( h \) of the laser above the glass pane, the distance \( d \) between the edge of the pane and the point where the laser reflects depends upon the angle \( \theta \) between the incident ray and the vertical (see the drawing). The height \( h \) is adjacent to the angle \( \theta \), and the distance \( d \) is opposite, so from \( \tan \theta = \frac{d}{h} \) (Equation 1.3) we have that

\[
d = h \tan \theta
\]  
(Equation 1.3)

The glass pane is horizontal, so the normal to the surface is vertical, and we see that \( \theta \) is also the angle of incidence when the incident ray strikes the glass pane. The reflected beam is 100% polarized, so we conclude that the angle \( \theta \) of incidence is equal to Brewster’s angle \( \theta_B \) for the glass-air interface: \( \theta = \theta_B \). Brewster’s angle is given by \( \tan \theta_B = \frac{n_2}{n_1} \) (Equation 26.5), where \( n_2 = 1.523 \) and \( n_1 = 1.000 \) are the indices of refraction for crown glass and air, respectively (see Table 26.1). Therefore,

\[
\tan \theta = \tan \theta_B = \frac{n_2}{n_1}
\]  
(Equation 2)

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

\[
d = h \tan \theta = h \left( \frac{n_2}{n_1} \right) = (0.476 \text{ m}) \left( \frac{1.523}{1.000} \right) = 0.725 \text{ m}
\]

41. **REASONING** When light is incident at the Brewster angle, we know that the angle between the refracted ray and the reflected ray is 90°. This relation will allow us to determine the Brewster angle. By applying Snell’s law to the incident and refracted rays, we can find the index of refraction of the glass.

**SOLUTION** The drawing shows the incident, reflected, and refracted rays.
a. We see from the drawing that $\theta_B + 90^\circ + \theta_2 = 180^\circ$, so that $\theta_B = 90^\circ - 29.9^\circ = 60.1^\circ$.

b. Applying Snell’s law at the vacuum/glass interface gives

$$n_{\text{vacuum}} \sin \theta_B = n_{\text{glass}} \sin \theta_2 \quad \text{or} \quad n_{\text{glass}} = \frac{(1.00) \sin 60.1^\circ}{\sin 29.9^\circ} = 1.74$$

42. **REASONING** We can use Snell’s law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ (Equation 26.2) to determine the index of refraction $n_2$ of the liquid. This is possible since we know that the index of refraction of air is $n_1 = 1.00$ and the angle of refraction is $\theta_2 = 33.7^\circ$. Furthermore, the light in Figure 26.7 strikes the liquid surface at the Brewster angle $\theta_B$. Therefore, we also know that the reflected and refracted rays are perpendicular, as the figure shows. This means that $\theta_B + \theta_2 = 90.0^\circ$ in the figure. Since we know that $\theta_2 = 33.7^\circ$, we can use this equation to determine the Brewster angle, which is also the angle of incidence $\theta_1$.

**SOLUTION** Using Snell’s with $\theta_1 = \theta_B$, we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{or} \quad n_2 = \frac{n_1 \sin \theta_B}{\sin \theta_2}$$

Since $\theta_B + \theta_2 = 90.0^\circ$, we know that $\theta_B = 90.0^\circ - \theta_2$, which we can substitute into the expression for $n_2$:

$$n_2 = \frac{n_1 \sin \theta_B}{\sin \theta_2} = \frac{n_1 \sin (90.0^\circ - \theta_2)}{\sin \theta_2} = (1.00) \frac{\sin (90.0^\circ - 33.7^\circ)}{\sin 33.7^\circ} = 1.50$$

43. **REASONING** The angle of each refracted ray in the crown glass can be obtained from Snell’s law (Equation 26.2) as $n_{\text{diamond}} \sin \theta_1 = n_{\text{crown glass}} \sin \theta_2$, where $\theta_1$ is the angle of incidence and $\theta_2$ is the angle of refraction.

**SOLUTION** The angles of refraction for the red and blue rays are:

**Blue ray**

$$\theta_2 = \sin^{-1} \left( \frac{n_{\text{diamond}} \sin \theta_1}{n_{\text{crown glass}}} \right) = \sin^{-1} \left[ \frac{(2.444) \sin 35.00^\circ}{1.531} \right] = 66.29^\circ$$

**Red ray**

$$\theta_2 = \sin^{-1} \left( \frac{n_{\text{diamond}} \sin \theta_1}{n_{\text{crown glass}}} \right) = \sin^{-1} \left[ \frac{2.410 \sin 35.00^\circ}{1.520} \right] = 65.43^\circ$$

The angle between the blue and red rays is

$$\theta_{\text{blue}} - \theta_{\text{red}} = 66.29^\circ - 65.43^\circ = 0.86^\circ$$
44. **REASONING** When light goes from air into the plastic, the light is refracted. Snell’s law relates the incident and refracted angles ($\theta_1$ and $\theta_2$) to the indices of refraction ($n_1$ and $n_2$) of the incident and refracting media by:

\[
\begin{align*}
\text{Violet light} & \quad n_1 \sin \theta_1 = n_{2, \text{Violet}} \sin \theta_{2, \text{Violet}} \\
\text{Red light} & \quad n_1 \sin \theta_1 = n_{2, \text{Red}} \sin \theta_{2, \text{Red}}
\end{align*}
\]  

(26.2)

By using these relations, and the fact that $n_{2, \text{Violet}} - n_{2, \text{Red}} = 0.0400$, we will be able to determine $n_{2, \text{Violet}}$.

**SOLUTION** Since the angle of incidence $\theta_1$ is the same for both colors and since $n_1 = n_{\text{air}}$ for both colors, the left-hand sides of the two equations above are equal. Thus, the right-hand sides of these equations must also be equal:

\[
\begin{align*}
n_{2, \text{Violet}} \sin \theta_{2, \text{Violet}} &= n_{2, \text{Red}} \sin \theta_{2, \text{Red}} \\
\Rightarrow \quad n_{2, \text{Violet}} &= \frac{n_{2, \text{Red}} \sin \theta_{2, \text{Red}}}{\sin \theta_{2, \text{Violet}}}
\end{align*}
\]  

(1)

We are given that $n_{2, \text{Violet}} - n_{2, \text{Red}} = 0.0400$, or $n_{2, \text{Red}} = n_{2, \text{Violet}} - 0.0400$. Substituting this expression for $n_{2, \text{Red}}$ into Equation (1), we have that

\[
\begin{align*}
n_{2, \text{Violet}} \sin \theta_{2, \text{Violet}} &= (n_{2, \text{Violet}} - 0.0400) \sin \theta_{2, \text{Red}} \\
\Rightarrow \quad n_{2, \text{Violet}} &= \frac{(0.0400) \sin \theta_{2, \text{Red}}}{\sin \theta_{2, \text{Violet}} - \sin \theta_{2, \text{Red}}}
\end{align*}
\]

Solving this equation for $n_{2, \text{Violet}}$ gives

\[
\begin{align*}
n_{2, \text{Violet}} &= \frac{-0.0400 \sin \theta_{2, \text{Red}}}{\sin \theta_{2, \text{Violet}} - \sin \theta_{2, \text{Red}}} = \frac{-0.0400 \sin 31.200^\circ}{\sin 30.400^\circ - \sin 31.200^\circ} = 1.73
\end{align*}
\]

45. **SSM REASONING** Because the refractive index of the glass depends on the wavelength (i.e., the color) of the light, the rays corresponding to different colors are bent by different amounts in the glass. We can use Snell’s law (Equation 26.2: $n_1 \sin \theta_1 = n_2 \sin \theta_2$) to find the angle of refraction for the violet ray and the red ray. The angle between these rays can be found by the subtraction of the two angles of refraction.

**SOLUTION** In Table 26.2 the index of refraction for violet light in crown glass is 1.538, while that for red light is 1.520. According to Snell’s law, then, the sine of the angle of refraction for the violet ray in the glass is $\sin \theta_2 = (1.000/1.538) \sin 45.00^\circ = 0.4598$, so that

$\theta_2 = \sin^{-1}(0.4598) = 27.37^\circ$

Similarly, for the red ray, $\sin \theta_2 = (1.000/1.520) \sin 45.00^\circ = 0.4652$, from which it follows that

$\theta_2 = \sin^{-1}(0.4652) = 27.72^\circ$
Therefore, the angle between the violet ray and the red ray in the glass is
\[27.7^\circ - 27.3^\circ = 0.35^\circ\]

46. **REASONING** To determine the angle of refraction we will use Snell’s law
\[n_1 \sin \theta_1 = n_2 \sin \theta_2\]  
(Equation 26.2), where, as always, we label all variables associated with the incident ray with subscript 1 and all variables associated with the refracted ray with subscript 2. We note that light is refracted upon entering the vertical face of the prism and then again upon leaving the slanted face. At the slanted face, since the index of refraction of the glass exceeds the index of refraction of the air, we need to consider the possibility of total internal reflection, depending on whether or not the angle of incidence exceeds the critical angle, as specified by
\[\sin \theta_c = n_{\text{air}} / n_{\text{flint glass}}\]  
(Equation 26.4). For later reference in our solution we determine the critical angles here:
\[
\sin \theta_c, \text{ red} = \frac{n_{\text{air}}}{n_{\text{red}}} = 1.00 \quad \text{or} \quad \theta_c, \text{ red} = \sin^{-1}\left(\frac{1.00}{1.662}\right) = 37.0^\circ
\]
\[
\sin \theta_c, \text{ violet} = \frac{n_{\text{air}}}{n_{\text{violet}}} = 1.00 \quad \text{or} \quad \theta_c, \text{ violet} = \sin^{-1}\left(\frac{1.00}{1.698}\right) = 36.1^\circ
\]

**SOLUTION** As the drawing at the right indicates, the light rays incident on the vertical face of the prism are perpendicular to it, as is the normal. Therefore, the angle of incidence is \(\theta_1 = 0.00^\circ\). Substituting this value for \(\theta_1\) into Snell’s law (Equation 26.2), we obtain
\[n_1 \sin 0.00^\circ = n_2 \sin \theta_2 \quad \text{or} \quad \theta_2 = 0.00^\circ\]
The angle of refraction at the vertical face of the prism is also 0.00°, and the incident rays penetrate the prism without having their directions changed.

At the slanted face of the prism, the angle of incidence is \(\theta_1 = 25.0^\circ\). The reason for this can be seen from the drawing, which indicates that \(25.0^\circ + \alpha = 90.0^\circ\), since the sum of the angles in a triangle is 180°. As a result, we see that \(\alpha = 65.0^\circ\). However, it is also true that \(\alpha + \theta_1 = 90.0^\circ\), so that \(\theta_1 = 90.0^\circ - \alpha = 90.0^\circ - 65.0^\circ = 25.0^\circ\). Since this angle of incidence is less than the critical angles for both the red and the violet rays (see the **REASONING**), total internal reflection does not occur, and the rays are refracted into the air at the slanted face of the prism. We can now use Snell’s law to determine the angles of refraction at the slanted face for both the red and violet rays:
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\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{or} \quad \sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} \]

\[ \sin \theta_{2, \text{red}} = \frac{n_{1, \text{red}} \sin \theta_1}{n_2} = \frac{(1.662) \sin 25.0^\circ}{1.000} = 0.702 \quad \text{or} \quad \theta_{2, \text{red}} = \sin^{-1}(0.702) = 44.6^\circ \]

\[ \sin \theta_{2, \text{violet}} = \frac{n_{1, \text{violet}} \sin \theta_1}{n_2} = \frac{(1.698) \sin 25.0^\circ}{1.000} = 0.718 \quad \text{or} \quad \theta_{2, \text{violet}} = \sin^{-1}(0.718) = 45.9^\circ \]

47. [SSM] **REASONING** We can use Snell's law (Equation 26.2: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)) at each face of the prism. At the first interface where the ray enters the prism, \( n_1 = 1.000 \) for air and \( n_2 = n_g \) for glass. Thus, Snell's law gives

\[ (1) \sin 60.0^\circ = n_g \sin \theta_2 \quad \text{or} \quad \sin \theta_2 = \frac{\sin 60.0^\circ}{n_g} \quad (1) \]

We will represent the angles of incidence and refraction at the second interface as \( \theta_1' \) and \( \theta_2' \), respectively. Since the triangle is an equilateral triangle, the angle of incidence at the second interface, where the ray emerges back into air, is \( \theta_1' = 60.0^\circ - \theta_2 \). Therefore, at the second interface, where \( n_1 = n_g \) and \( n_2 = 1.000 \), Snell’s law becomes

\[ n_g \sin (60.0^\circ - \theta_2) = (1) \sin \theta_2' \quad (2) \]

We can now use Equations (1) and (2) to determine the angles of refraction \( \theta_2' \) at which the red and violet rays emerge into the air from the prism.

**SOLUTION**

[Red Ray] The index of refraction of flint glass at the wavelength of red light is \( n_g = 1.662 \). Therefore, using Equation (1), we can find the angle of refraction for the red ray as it enters the prism:

\[ \sin \theta_2 = \frac{\sin 60.0^\circ}{1.662} = 0.521 \quad \text{or} \quad \theta_2 = \sin^{-1} 0.521 = 31.4^\circ \]

Substituting this value for \( \theta_2 \) into Equation (2), we can find the angle of refraction at which the red ray emerges from the prism:

\[ \sin \theta_2' = 1.662 \sin (60.0^\circ - 31.4^\circ) = 0.796 \quad \text{or} \quad \theta_2' = \sin^{-1} 0.796 = 52.7^\circ \]
For violet light, the index of refraction for glass is \( n_g = 1.698 \). Again using Equation (1), we find

\[
\sin \theta_2 = \frac{\sin 60.0^\circ}{1.698} = 0.510 \quad \text{or} \quad \theta_2 = \sin^{-1} 0.510 = 30.7^\circ
\]

Using Equation (2), we find

\[
\sin \theta'_2 = 1.698 \sin (60.0^\circ - 30.7^\circ) = 0.831 \quad \text{or} \quad \theta'_2 = \sin^{-1} 0.831 = 56.2^\circ
\]

48. **REASONING** At the slanted face, the refracted ray(s) emerge into the surrounding material, which has an index of refraction \( n_2 \). Together, the index of refraction \( n_2 \) of the surrounding material and the index of refraction \( n_1 \) of crown glass (which varies with the color of the light; see Table 26.2) determine the critical angle \( \theta_c \) for total internal reflection, as we see from \( \sin \theta_c = \frac{n_2}{n_1} \) (Equation 26.4). To guarantee that the colors that are to emerge from the prism at the slanted face do not undergo total internal reflection there, we must choose \( n_2 \) such that the angle of incidence \( \theta_1 \) is less than the critical angle \( \theta_c \) for each such color. The incident ray and the totally- internally- reflected ray are perpendicular to one another, and the 90.00° angle between them is bisected by the normal to the slanted surface (see the drawing). Therefore, the angle of incidence is \( \theta_1 = 45.00^\circ \). Thus, all colors of light for which the critical angle is greater than 45.00° will not undergo total internal reflection and will emerge from the slanted face of the prism.

**SOLUTION**

a. Solving \( \sin \theta_c = \frac{n_2}{n_1} \) (Equation 26.4) for \( n_2 \), we obtain

\[
n_2 = n_1 \sin \theta_c
\]

Since only red light is to emerge from the slanted face of the prism, we will use \( n_1 = 1.520 \), the refractive index for red light in crown glass (see Table 26.2). The indices of refraction for the remaining colors are all larger than 1.520, so all other colors of light will have critical angles less than 45.00° and will undergo total internal reflection at the slanted face of the prism. Thus, we have

\[
n_2 = (1.520) \sin 45.00^\circ = 1.075
\]
b. All colors except violet are to emerge from the slanted face. Therefore, we will use 
\( n_1 = 1.531 \), the index of refraction of blue light in crown glass (see Table 26.2), in 
Equation (1):

\[
n_2 = (1.531) \sin 45.00' = 1.083
\]

49. **SSM REASONING** The ray diagram is constructed by drawing the paths of two rays
from a point on the object. For convenience, we choose the top of the object. The ray that is
parallel to the principal axis will be refracted by the lens and pass through the focal point on
the right side. The ray that passes through the center of the lens passes through undeflected.
The image is formed at the intersection of these two rays on the right side of the lens.

**SOLUTION** The following ray diagram (to scale) shows that \( d_i = 18 \text{ cm} \) and reveals a
real, inverted, and enlarged image.

![Ray Diagram](image)

50. **REASONING**

a. Given the focal length \( f = -0.300 \text{ m} \) of the lens and the image distance \( d_i \), we will
employ the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6) to determine how far from the
van the person is actually standing, which is the object distance \( d_o \). The image and the
person are both behind the van, so the image is virtual, and the image distance is negative:
\( d_i = -0.240 \text{ m} \).

b. Once we have determined the object distance \( d_o \), we will use the magnification equation
\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}
\]
(Equation 26.7) to calculate the true height \( h_o \) of the person from the height
\( h_i = 0.34 \text{ m} \) of the image.
SOLUTION

a. Solving Equation 26.6 for $d_o$, we obtain

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \quad \text{or} \quad d_o = \frac{1}{\frac{1}{f} - \frac{1}{d_i}} = \frac{1}{\frac{1}{-0.300 \text{ m}} - \frac{1}{-0.240 \text{ m}}} = 1.2 \text{ m}$$

b. Taking the reciprocal of both sides of $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$ (Equation 26.7) and solving for $h_o$ yields

$$\frac{h_o}{h_i} = -\frac{d_o}{d_i} \quad \text{or} \quad h_o = -h_i \left( \frac{d_o}{d_i} \right) = -\left(0.34 \text{ m} \right) \left( \frac{1.2 \text{ m}}{-0.240 \text{ m}} \right) = 1.7 \text{ m}$$

51. REASONING We can use the magnification equation (Equation 26.7) to determine the image height $h_i$. This equation is

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{or} \quad h_i = h_o \left( -\frac{d_i}{d_o} \right)$$  \hspace{1cm} (26.7)

We are given the object height $h_o$ and the object distance $d_o$. Thus, we need to begin by finding the image distance $d_i$, for which we use the thin-lens equation (Equation 26.6):

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{fd_o} \quad \text{or} \quad d_i = \frac{fd_o}{d_o - f}$$  \hspace{1cm} (26.6)

Substituting this result into Equation 26.7 gives

$$h_i = h_o \left( -\frac{d_i}{d_o} \right) = h_o \left( -\frac{1}{d_o} \right) \left( \frac{fd_o}{d_o - f} \right) = h_o \left( \frac{f}{f - d_o} \right)$$  \hspace{1cm} (1)

SOLUTION

a. Using Equation (1), we find that the image height for the 35.0-mm lens is

$$h_i = h_o \left( \frac{f}{f - d_o} \right) = (1.60 \text{ m}) \left[ \frac{35.0 \times 10^{-3} \text{ m}}{(35.0 \times 10^{-3} \text{ m}) - 9.00 \text{ m}} \right] = -0.00625 \text{ m}$$

b. Using Equation (1), we find that the image height for the 150.0-mm lens is

$$h_i = h_o \left( \frac{f}{f - d_o} \right) = (1.60 \text{ m}) \left[ \frac{150.0 \times 10^{-3} \text{ m}}{(150.0 \times 10^{-3} \text{ m}) - 9.00 \text{ m}} \right] = -0.0271 \text{ m}$$

Both heights are negative because the images are inverted with respect to the object.
52. **REASONING** The focal length $f$ can be found with the aid of the thin-lens equation
\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]
(Equation 26.6), where $d_o$ is the object distance and $d_i$ is the image distance.

**SOLUTION**

a. The diverging lens produces a virtual image.

b. We know that $d_o = 13.0 \text{ cm}$ and $d_i = -5.0 \text{ cm}$, where the image distance is negative because the image is virtual. Using the thin lens equation, we find

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{d_i + d_o}{d_o d_i} = \frac{1}{f} \quad \text{or} \quad f = \frac{d_o d_i}{d_i + d_o} = \frac{(13.0 \text{ cm})(-5.0 \text{ cm})}{(-5.0 \text{ cm}) + (13.0 \text{ cm})} = -8.1
\]

53. **REASONING** The distance from the lens to the screen, the image distance, can be obtained directly from the thin-lens equation, Equation 26.6, since the object distance and focal length are known. The width and height of the image on the screen can be determined by using Equation 26.7, the magnification equation.

**SOLUTION**

a. The distance $d_i$ to the screen is

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{105.00 \text{ mm}} - \frac{1}{108.00 \text{ mm}} = 2.646 \times 10^{-4} \text{ mm}^{-1}
\]

so that $d_i = 3.78 \times 10^3 \text{ mm} = 3.78 \text{ m}$.

b. According to the magnification equation, the width and height of the image on the screen are

**Width**

\[
h_i = h_o \left( -\frac{d_i}{d_o} \right) = (24.0 \text{ mm}) \left( -\frac{3.78 \times 10^3 \text{ mm}}{108 \text{ mm}} \right) = -8.40 \times 10^2 \text{ mm}
\]

The width is $8.40 \times 10^2 \text{ mm}$.

**Height**

\[
h_i = h_o \left( -\frac{d_i}{d_o} \right) = (36.0 \text{ mm}) \left( -\frac{3.78 \times 10^3 \text{ mm}}{108 \text{ mm}} \right) = -1.26 \times 10^3 \text{ mm}
\]

The height is $1.26 \times 10^3 \text{ mm}$.
54. **REASONING AND SOLUTION** We note that the object is placed 20.0 cm from the lens. Since the focal point of the lens is \( f = -20.0 \) cm, the object is situated at the focal point. In the scale drawing that follows, we locate the image using the two rays labeled a and b, which originate at the top of the object.

![Diagram of light rays and lens](image)

- a. Measuring according to the scale used in the drawing, we find that the image is located 10.0 cm to the left of the lens. The lens is a diverging lens and forms a virtual image, so the image distance is \( d_i = -10.0 \) cm.

- b. Measuring the heights of the image and the object in the drawing, we find that the magnification is \( m = +0.500 \).

55. **REASONING** The height of the mountain’s image is given by the magnification equation as \( h_i = -h_o d_i / d_o \). To use this expression, however, we will need to know the image distance \( d_i \), which can be determined using the thin-lens equation. Knowing the image distance, we can apply the expression for the image height directly to calculate the desired ratio.

**SOLUTION** According to the thin-lens equation, we have

\[
\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}
\]

For both pictures, the object distance \( d_o \) is very large compared to the focal length \( f \). Therefore, \( 1/d_o \) is negligible compared to \( 1/f \), and the thin-lens equation indicates that \( d_i = f \). As a result, the magnification equation indicates that the image height is given by

\[
h_i = -\frac{h_o d_i}{d_o} = -\frac{h_o f}{d_o}
\]
Applying Equation (2) for the two pictures and noting that in each case the object height $h_o$ and the focal length $f$ are the same, we find

$$\left( \frac{h_i}{h_o} \right)_{5 \text{ km}} = \left( \frac{-h_o f}{d_o} \right)_{5 \text{ km}} = \left( \frac{d_o}{h_o} \right)_{14 \text{ km}} = \frac{14 \text{ km}}{5.0 \text{ km}} = 2.8$$

56. **REASONING** A converging lens must be used, because a diverging lens cannot produce a real image.

Since the image is one-half the size of the object and inverted relative to it, the image height $h_i$ is related to the object height $h_o$ by $h_i = -\frac{1}{2} h_o$, where the minus sign indicates that the image is inverted.

According to the magnification equation, Equation 26.7, the image distance $d_i$ is related to the object distance $d_o$ by $d_i / d_o = -h_i / h_o$. But we know that $h_i / h_o = -\frac{1}{2}$, so $d_i / d_o = -\left( -\frac{1}{2} \right) = \frac{1}{2}$.

**SOLUTION**

a. Let $d$ be the distance between the object and image, so that $d = d_o + d_i$. However, we know from the **REASONING** that $d_i = \frac{1}{2} d_o$, so $d = d_o + \frac{1}{2} d_o = \frac{3}{2} d_o$. The object distance is, therefore,

$$d_o = \frac{2}{3} d = \frac{2}{3} (90.0 \text{ cm}) = 60.0 \text{ cm}$$

b. The thin-lens equation, Equation 26.6, can be used to find the focal length $f$ of the lens:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{\frac{1}{2} d_o} = \frac{3}{d_o}$$

$$f = \frac{d_o}{3} = \frac{60.0 \text{ cm}}{3} = 20.0 \text{ cm}$$

57. **REASONING** Since we are given the focal length and the object distance, we can use the thin-lens equation to calculate the image distance. From the algebraic sign of the image distance we can tell if the image is real (image distance is positive) or virtual (image distance is negative). Knowing the image distance and the object distance will enable us to use the magnification equation to determine the height of the image.
SOLUTION

a. Using the thin-lens equation to obtain the image distance \( d_i \) from the focal length \( f \) and the object distance \( d_o \), we find

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{88.00 \text{ cm}} - \frac{1}{155.0 \text{ cm}} = 0.00491 \text{ cm}^{-1} \quad \text{or} \quad d_i = 204 \text{ cm}
\]

b. The fact that the image distance is positive indicates that the image is real.

c. The magnification equation indicates that the magnification \( m \) is

\[
m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}
\]

where \( h_o \) and \( h_i \) are the object height and image height, respectively. Solving for the image height gives

\[
h_i = h_o \left( -\frac{d_i}{d_o} \right) = (13.0 \text{ cm}) \left( -\frac{204 \text{ cm}}{155.0 \text{ cm}} \right) = -17.1 \text{ cm}
\]

The negative value indicates that the image is inverted with respect to the object.

58. REASONING A diverging lens always produces a virtual image, so that the image distance \( d_i \) is negative. Moreover, the object distance \( d_o \) is positive. Therefore, the distance between the object and the image is \( d_o + d_i = 49.0 \text{ cm} \), rather than \( d_o - d_i = 49.0 \text{ cm} \). The equation \( d_o + d_i = 49.0 \text{ cm} \) and the thin-lens equation constitute two equations in two unknowns, and we will solve them simultaneously to obtain values for \( d_i \) and \( d_o \).

SOLUTION

a. Solving the equation \( d_o + d_i = 49.0 \text{ cm} \) for \( d_o \), substituting the result into the thin-lens equation, and suppressing the units give

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{49.0-d_i} + \frac{1}{d_i} = \frac{1}{-233.0}
\]

(1)

Grouping the terms on the left of Equation (1) over a common denominator, we have

\[
\frac{d_i + 49.0 - d_i}{d_i (49.0 - d_i)} = \frac{49.0}{d_i (49.0 - d_i)} = \frac{1}{-233.0}
\]

(2)

Cross-multiplying and rearranging in Equation (2) gives

\[
ds_i (49.0 - d_i) = -11417 \quad \text{or} \quad d_i^2 - 49.0d_i - 11417 = 0
\]

(3)

Using the quadratic formula to solve Equation (3), we obtain
\[ d_i = \frac{-(-49.0) \pm \sqrt{(-49.0)^2 - 4(1.00)(-11417)}}{2(1.00)} = -85.1 \text{ cm} \]

We have discarded the positive root, because we know that \( d_i \) must be negative for the virtual image.

b. Using the fact that \( d_o + d_i = 49.0 \text{ cm} \), we find that the object distance is

\[ d_o = 49.0 \text{ cm} - d_i = (49.0 \text{ cm}) - (-85.1 \text{ cm}) = 134.1 \text{ cm} \]

59. **SSM Reasoning** The optical arrangement is similar to that in Figure 26.26. We begin with the thin-lens equation, [Equation 26.6: \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \)]. Since the distance between the moon and the camera is so large, the object distance \( d_o \) is essentially infinite, and \( \frac{1}{d_o} = \frac{1}{\infty} = 0 \). Therefore the thin-lens equation becomes \( \frac{1}{d_i} = \frac{1}{f} \) or \( d_i = f \). The diameter of the moon's image on the slide film is equal to the image height \( h_i \), as given by the magnification equation (Equation 26.7: \( \frac{h_i}{h_o} = -\frac{d_i}{d_o} \)).

When the slide is projected onto a screen, the situation is similar to that in Figure 26.27. In this case, the thin-lens and magnification equations can be used in their usual forms.

**Solution**

a. Solving the magnification equation for \( h_i \) gives

\[ h_i = -h_o \frac{d_i}{d_o} = (-3.48 \times 10^6 \text{ m}) \left( \frac{50.0 \times 10^{-3} \text{ m}}{3.85 \times 10^8 \text{ m}} \right) = -4.52 \times 10^{-4} \text{ m} \]

The diameter of the moon's image on the slide film is, therefore, \( 4.52 \times 10^{-4} \text{ m} \).

b. From the magnification equation, \( h_i = -h_o \left( \frac{d_i}{d_o} \right) \). We need to find the ratio \( d_i/d_o \).

Beginning with the thin-lens equation, we have

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \quad \text{which leads to} \quad \frac{d_i}{d_o} = \frac{d_i}{f} - \frac{d_i}{d_i} = \frac{d_i}{f} - 1 \]

Therefore,

\[ h_i = -h_o \left( \frac{d_i}{f} - 1 \right) = -\left( 4.52 \times 10^{-4} \text{ m} \right) \left( \frac{15.0 \text{ m}}{110.0 \times 10^{-3} \text{ m}} - 1 \right) = -6.12 \times 10^{-2} \text{ m} \]

The diameter of the image on the screen is \( 6.12 \times 10^{-2} \text{ m} \).
60. **REASONING AND SOLUTION** The focal length of the lens can be obtained from the thin-lens equation as follows:

\[
\frac{1}{f} = \frac{1}{4.00 \text{ m}} + \frac{1}{0.210 \text{ m}} \quad \text{or} \quad f = 0.200 \text{ m}
\]

The same equation applied to the projector gives

\[
\frac{1}{d_o} = \frac{1}{0.200 \text{ m}} - \frac{1}{0.500 \text{ m}} \quad \text{or} \quad d_o = 0.333 \text{ m}
\]

61. **SSM REASONING** The magnification equation (Equation 26.7) relates the object and image distances \(d_o\) and \(d_i\), respectively, to the relative size of the of the image and object: \(m = -(d_i / d_o)\). We consider two cases: in case 1, the object is placed 18 cm in front of a diverging lens. The magnification for this case is given by \(m_1\). In case 2, the object is moved so that the magnification \(m_2\) is reduced by a factor of 2 compared to that in case 1. In other words, we have \(m_2 = \frac{1}{2} m_1\). Using Equation 26.7, we can write this as

\[
-\frac{d_{i2}}{d_{o2}} = -\frac{1}{2} \left(\frac{d_{i1}}{d_{o1}}\right)
\]

This expression can be solved for \(d_{o2}\). First, however, we must find a numerical value for \(d_{i1}\), and we must eliminate the variable \(d_{i2}\).

**SOLUTION**

The image distance for case 1 can be found from the thin-lens equation [Equation 26.6: \((1/d_o) + (1/d_i) = (1/f)\)]. The problem statement gives the focal length as \(f = -12 \text{ cm}\). Since the object is 18 cm in front of the diverging lens, \(d_{o1} = 18 \text{ cm}\). Solving for \(d_{i1}\), we find

\[
\frac{1}{d_{i1}} = \frac{1}{f} - \frac{1}{d_{o1}} = \frac{1}{-12 \text{ cm}} - \frac{1}{18 \text{ cm}} \quad \text{or} \quad d_{i1} = -7.2 \text{ cm}
\]

where the minus sign indicates that the image is virtual. Solving Equation (1) for \(d_{o2}\), we have

\[
d_{o2} = 2d_{i2} \left(\frac{d_{o1}}{d_{i1}}\right)
\]

To eliminate \(d_{i2}\) from this result, we note that the thin-lens equation applied to case 2 gives

\[
\frac{1}{d_{i2}} = \frac{1}{f} - \frac{1}{d_{o2}} = \frac{d_{o2} - f}{fd_{o2}} \quad \text{or} \quad d_{i2} = \frac{fd_{o2}}{d_{o2} - f}
\]
Substituting this expression for \( d_{i2} \) into Equation (2), we have

\[
d_{o2} = \left( \frac{2f d_{o2}}{d_{o2} - f} \right) \left( \frac{d_{o1}}{d_{i1}} \right)
\]

or

\[
d_{o2} - f = 2f \left( \frac{d_{o1}}{d_{i1}} \right)
\]

Solving for \( d_{o2} \), we find

\[
d_{o2} = f \left[ 2 \left( \frac{d_{o1}}{d_{i1}} \right) + 1 \right] = (-12 \text{ cm}) \left[ 2 \left( \frac{18 \text{ cm}}{-7.2 \text{ cm}} \right) + 1 \right] = 48 \text{ cm}
\]

62. **REASONING** The image distance \( d_{i2} \) produced by the 2nd lens is related to the object distance \( d_{o2} \) and the focal length \( f_2 \) by the thin-lens equation (Equation 26.6). The focal length is known, but the object distance is not. However, the problem states that the object distance is equal to that \( (d_{o1}) \) of the 1st lens, so \( d_{o2} = d_{o1} \). Since the final image distance \( d_{i1} \) and the focal length \( f_1 \) of the 1st lens are known, we can determine the object distance for this lens by employing the thin-lens equation.

**SOLUTION** The image distance \( d_{i2} \) produced by the 2nd lens is related to the object distance \( d_{o2} \) and the focal length \( f_2 \) by the thin-lens equation:

\[
\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}}
\]

(26.6)

Since \( d_{o2} = d_{o1} \) (the image distance for the 1st lens), Equation 26.6 can be written as

\[
\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o1}}
\]

(1)

The object distance for the first lens can be obtained from the thin-lens equation:

\[
\frac{1}{d_{o1}} = \frac{1}{f_1} - \frac{1}{d_{i1}}
\]

(2)

Substituting Equation (2) into Equation (1) gives

\[
\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o1}} = \frac{1}{f_2} - \left( \frac{1}{f_1} - \frac{1}{d_{i1}} \right) = \frac{1}{16.0 \text{ cm}} - \left( \frac{1}{12.0 \text{ cm}} - \frac{1}{21.0 \text{ cm}} \right)
\]

Solving for \( d_{i2} \) gives \( d_{i2} = 37.3 \text{ cm} \).
63. **REASONING AND SOLUTION** Let $d$ represent the distance between the object and the screen. Then, $d_o + d_i = d$. Using this expression in the thin-lens equation gives

$$\frac{1}{d_o} + \frac{1}{d - d_o} = \frac{1}{f} \quad \text{or} \quad d_o^2 - d d_o + df = 0$$

With $d = 125$ cm and $f = 25.0$ cm, the quadratic formula yields solutions of

$$d_o = +35 \text{ cm} \quad \text{and} \quad d_o = +90.5 \text{ cm}$$

64. **REASONING AND SOLUTION** From the drawing in the text we see that $d_o = x + f$ and $d_i = x' + f$. Substituting these two expressions into the thin-lens equation, we obtain

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{x + f} + \frac{1}{x' + f} = \frac{1}{f}$$

Combining the terms on the left over a common denominator gives

$$\frac{x + f + x' + f}{(x + f)(x' + f)} = \frac{x + x' + 2f}{(x + f)(x' + f)} = \frac{1}{f}$$

Cross-multiplying shows that

$$f (x + x' + 2f) = (x + f)(x' + f)$$

Expanding and simplifying this result, we obtain

$$fx + fx' + 2f^2 = xx' + fx' + xf + f^2 \quad \text{or} \quad xx' = f^2$$

65. **REASONING** We will consider one lens at a time, using the thin-lens equation for each. The key to the solution is the fact that the image formed by the first lens serves as the object for the second lens.

**SOLUTION** Using the thin-lens equation, we find the image distance for the first lens:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0 \text{ cm}} - \frac{1}{4.0 \text{ cm}} \quad \text{or} \quad d_i = -2.7 \text{ cm}$$

The negative value for $d_i$ indicates that the image is virtual and located 2.7 cm to the left of the first lens. The lenses are 16 cm apart, so this image is located $2.7 \text{ cm} + 16 \text{ cm} = 18.7 \text{ cm}$ from the second lens. Since this image serves as the object for the second lens, we can locate the image formed by the second lens with the aid of the thin-lens equation, with $d_o = 18.7 \text{ cm}$:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0 \text{ cm}} - \frac{1}{18.7 \text{ cm}} \quad \text{or} \quad d_i = -5.6 \text{ cm}$$
66. **REASONING** In part *a* of the drawing, the object lies inside the focal point of the converging (#1) lens. According to Figure 26.28, such an object produces a virtual image that lies to the left of the lens. This image act as the object for the diverging (#2) lens. Since a diverging lens always produces a virtual image that lies to the left of the lens, the final image lies to the left of the diverging lens. In part (b), the diverging (#1) lens produces a virtual image that lies to the left of the lens. This image act as the object for the converging (#2) lens. Since the object lies outside the focal point of the converging lens, the converging lens produces a real image that lies to the right of the lens (see Figure 26.26). Thus, the final image lies to the right of the converging lens.

**SOLUTION**

a. The focal length of lens #1 is $f_1 = 15.00$ cm, and the object distance is $d_{o1} = 10.0$ cm. The image distance $d_{i1}$ produced by the first lens can be obtained from the thin-lens equation, Equation 26.6:

$$\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = \frac{1}{15.00\text{ cm}} - \frac{1}{10.00\text{ cm}} = -3.33 \times 10^{-2}\text{ cm}^{-1} \quad \text{or} \quad d_{i1} = -30.0\text{ cm}$$

This image is located to the left of lens #1 and serves as the object for lens #2. Thus, the object distance for lens #2 is $d_{i2} = 30.0 \text{ cm} + 50.0 \text{ cm} = 80.0 \text{ cm}$. The image distance produced by lens #2 is

$$\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}} = \frac{1}{-20.00\text{ cm}} - \frac{1}{80.0\text{ cm}} = -6.25 \times 10^{-2}\text{ cm}^{-1} \quad \text{or} \quad d_{i2} = -16.0\text{ cm}$$

The negative value for $d_{i2}$ indicates that, as expected, the final image is to the left of lens #2.

b. The focal length of the lens #1 is $f_1 = -20.0$ cm, and the object distance is $d_{o1} = 10.00$ cm. The image distance $d_{i1}$ produced by the first lens can be obtained from the thin-lens equation, Equation 26.6.

$$\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = \frac{1}{-20.0\text{ cm}} - \frac{1}{10.00\text{ cm}} = -1.50 \times 10^{-1}\text{ cm}^{-1} \quad \text{or} \quad d_{i1} = -6.67\text{ cm}$$

This image is located to the left of lens #1 and serves as the object for lens #2. Thus the object distance for lens #2 is $d_{i2} = 6.67 \text{ cm} + 50.0 \text{ cm} = 56.7 \text{ cm}$. The image distance produced by lens #2 is

$$\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}} = \frac{1}{15.00\text{ cm}} - \frac{1}{56.7\text{ cm}} = 4.90 \times 10^{-2}\text{ cm}^{-1} \quad \text{or} \quad d_{i2} = 20.4\text{ cm}$$

The positive value for $d_{i2}$ indicates that, as expected, the final image is to the right of lens #2.

---

67. **REASONING** The thin-lens equation can be used to find the image distance of the first image (the image produced by the first lens). This image, in turn, acts as the object for the second lens. The thin-lens equation can be used again to determine the image distance for the final image (the image produced by the second lens).
**SOLUTION** For the first lens, the object and image distances, $d_{o1}$ and $d_{i1}$, are related to the focal length $f$ of the lens by the thin-lens equation

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f} \quad (26.6)$$

Solving this expression for the image distance produced by the first lens, we find that

$$\frac{1}{d_{i1}} = \frac{1}{f} - \frac{1}{d_{o1}} = \frac{1}{12.00 \text{ cm}} - \frac{1}{36.00 \text{ cm}} \quad \text{or} \quad d_{i1} = 18.0 \text{ cm}$$

This image distance indicates that the first image lies between the lenses. Since the lenses are separated by 24.00 cm, the distance between the image produced by the first lens and the second lens is $24.00 \text{ cm} - 18.0 \text{ cm} = 6.0 \text{ cm}$. Since the image produced by the first lens acts as the object for the second lens, we have that $d_{o2} = 6.0 \text{ cm}$. Applying the thin-lens equation to the second lens gives

$$\frac{1}{d_{i2}} = \frac{1}{f} - \frac{1}{d_{o2}} = \frac{1}{12.00 \text{ cm}} - \frac{1}{6.0 \text{ cm}} \quad \text{or} \quad d_{i2} = -12 \text{ cm}$$

The fact that this image distance is negative means that the final image is virtual and lies to the left of the second lens.

---

68. **REASONING** In a two-lens situation, the image produced by the first (converging) lens serves as the object for the second (diverging) lens. We will use the thin-lens equation

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f} \quad (\text{Equation 26.6})$$

to determine the object distance $d_{o2}$ for the diverging lens from the focal length $f_2 = -28.0 \text{ cm}$ and the image distance $d_{i2}$. This object is the image produced by the first (converging) lens. A second application of Equation 26.6 will then permit us to find the distance $d_{o1}$ between the converging lens and the object.

**SOLUTION** Because the final image appears to the left of the second (diverging) lens, the image distance $d_{i2}$ is negative: $d_{i2} = -20.7 \text{ cm}$. Solving

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2}$$

for $d_{o2}$ yields

$$\frac{1}{d_{o2}} = \frac{1}{f_2} - \frac{1}{d_{i2}} = \frac{1}{-28.0 \text{ cm}} - \frac{1}{-20.7 \text{ cm}} = 0.0126 \text{ cm}^{-1} \quad \text{or} \quad d_{o2} = 79.4 \text{ cm}$$

This is a positive object distance, so our sign convention indicates that the object for the diverging lens is located 79.4 cm to the left of the diverging lens. Because this distance is
greater than the distance $L = 56.0 \text{ cm}$ between the lenses, the object for the diverging lens is located $d_{o2} - L = 79.4 \text{ cm} - 56.0 \text{ cm} = 23.4 \text{ cm}$ left of the converging lens (see the drawing).

This object is the image produced by the first (converging) lens. However, as the image appears left of the converging lens, the image distance $d_{i1}$ is negative: $d_{i1} = -23.4 \text{ cm}$.

Solving $\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1}$ (Equation 26.6) for $d_{o1}$, we find that

$$\frac{1}{d_{o1}} = \frac{1}{f_1} - \frac{1}{d_{i1}} = \frac{1}{24.0 \text{ cm}} - \frac{1}{-23.4 \text{ cm}} = 0.0844 \text{ cm}^{-1} \quad \text{or} \quad d_{o1} = 11.8 \text{ cm}$$

69. **REASONING** The problem can be solved using the thin-lens equation [Equation 26.6: $(1/d_o) + (1/d_i) = (1/f)$] twice in succession. We begin by using the thin lens-equation to find the location of the image produced by the converging lens; this image becomes the object for the diverging lens.

**SOLUTION**

a. The image distance for the converging lens is determined as follows:

$$\frac{1}{d_{i1}} = \frac{1}{f} - \frac{1}{d_{o1}} = \frac{1}{12.0 \text{ cm}} - \frac{1}{36.0 \text{ cm}} \quad \text{or} \quad d_{i1} = 18.0 \text{ cm}$$

This image acts as the object for the diverging lens. Therefore,

$$\frac{1}{d_{i2}} = \frac{1}{f} - \frac{1}{d_{o2}} = \frac{1}{-6.00 \text{ cm}} - \frac{1}{(30.0 \text{ cm} - 18.0 \text{ cm})} \quad \text{or} \quad d_{i2} = -4.00 \text{ cm}$$

Thus, the final image is located $4.00 \text{ cm}$ to the left of the diverging lens.

b. The magnification equation (Equation 26.7: $h_i/h_o = -d_i/d_o$) gives
Therefore, the overall magnification is given by the product \( m_c m_d = -0.167 \).

c. Since the final image distance is negative, we can conclude that the image is virtual.

d. Since the overall magnification of the image is negative, the image is inverted.

e. The magnitude of the overall magnification is less than one; therefore, the final image is smaller.

---

**SOLUTION**

a. Using the thin-lens equation to obtain the distance \( d_i \) of the first image from the diverging lens, which has a focal length \( f \) and an object distance \( d_o \), we find

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = -0.200 \text{ cm}^{-1} \quad \text{or} \quad d_i = -5.00 \text{ cm}
\]

The minus sign indicates that this first image is a virtual image located to the left of the diverging lens. This first image is also the object for the converging lens and is located within its focal point. From the drawing, we can see that the corresponding object distance is...
\( d_o = 30.0 \text{ cm} - 5.00 \text{ cm} = 25.0 \text{ cm}. \) To determine the final image distance, we again use the thin-lens equation:

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{30.0 \text{ cm}} - \frac{1}{25.0 \text{ cm}} = -0.0067 \text{ cm}^{-1} \quad \text{or} \quad d_i = -150 \text{ cm}
\]

The minus sign means that the final image is virtual and located to the left of the converging lens. Furthermore, the size of the image distance indicates that the final image is located to the left of both lenses.

b. Using the magnification equation we can determine the size of the first image:

\[
\frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{or} \quad h_i = h_o \left( -\frac{d_i}{d_o} \right) = (3.00 \text{ cm}) \left[ -\frac{(-5.00 \text{ cm})}{10.0 \text{ cm}} \right] = 1.50 \text{ cm}
\]

The fact that the image height (which is also the object height for the converging lens) is positive means that the image is upright with respect to the original object. Using the magnification equation again, we find that the height of the final image is

\[
h_f = h_o \left( -\frac{d_f}{d_o} \right) = (1.50 \text{ cm}) \left[ -\frac{(-150 \text{ cm})}{25.0 \text{ cm}} \right] = 9.0 \text{ cm}
\]

Since the final image height is positive, we conclude that the final image is upright with respect to the original object.

71. **SSM REASONING** We begin by using the thin-lens equation [Equation 26.6: \((1/d_o) + (1/d_i) = (1/f)\)] to locate the image produced by the lens. This image is then treated as the object for the mirror.

**SOLUTION**

a. The image distance from the diverging lens can be determined as follows:

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.00 \text{ cm}} - \frac{1}{20.0 \text{ cm}} \quad \text{or} \quad d_i = -5.71 \text{ cm}
\]

The image produced by the lens is 5.71 cm to the left of the lens. The distance between this image and the concave mirror is 5.71 cm + 30.0 cm = 35.7 cm. The mirror equation [Equation 25.3: \((1/d_o) + (1/d_i) = (1/f)\)] gives the image distance from the mirror:

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{12.0 \text{ cm}} - \frac{1}{35.7 \text{ cm}} \quad \text{or} \quad d_i = 18.1 \text{ cm}
\]

b. The image is **real**, because \(d_i\) is a positive number, indicating that the final image lies to the left of the concave mirror.
c. The image is **inverted**, because a diverging lens always produces an upright image, and the concave mirror produces an inverted image when the object distance is greater than the focal length of the mirror.

72. **REASONING** In dealing with this combination of two lenses, we will follow the standard procedure of treating the image produced by the first lens as the object for the next lens. In so doing, we will employ the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6), where \( d_o \) is the object distance, \( d_i \) is the image distance, and \( f \) is the focal length of the lenses. The overall magnification of the combination is the product of the individual magnifications of the lenses. The magnification \( m \) of a single lens is given by the magnification equation \( \frac{d_i}{d_o} \) (Equation 26.7).

**SOLUTION**

a. The thin-lens equation reveals the image distance for the first lens as follows:

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_1} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f_1} - \frac{1}{d_o} = \frac{1}{9.00 \, \text{cm}} - \frac{1}{12.0 \, \text{cm}} \quad \text{or} \quad d_i = +36 \, \text{cm}
\]

Since the lenses are separated by 18.0 cm, the value of \( d_i = +36 \, \text{cm} \) places the image 18 cm to the right of the second lens. This image serves as the object for the second lens. This is a case in which the object, being to the right of the lens, has a negative object distance, as indicated in the Reasoning Strategy given in Section 26.8. The thin-lens equation reveals image distance for the second lens as follows:

\[
\frac{1}{d_o'} + \frac{1}{d_i'} = \frac{1}{f_2} \quad \text{or} \quad \frac{1}{d_i'} = \frac{1}{f_2} - \frac{1}{d_o'} = \frac{1}{6.00 \, \text{cm}} - \frac{1}{-18 \, \text{cm}} \quad \text{or} \quad d_i' = +4.50 \, \text{cm}
\]

The positive sign indicates that the final image lies **4.50 cm to the right of the second lens**.

b. According to the magnification equation, the magnification of the first lens is

\[
m = -\frac{d_i}{d_o} = -\frac{36 \, \text{cm}}{12.0 \, \text{cm}} = -3.0
\]

Similarly, the magnification of the second lens is

\[
m' = -\frac{d_i'}{d_o'} = -\frac{4.50 \, \text{cm}}{-18 \, \text{cm}} = +0.25
\]

The overall magnification is the product of the magnifications of the individual lens, so that \( mm' = (-3.0)(0.25) = -0.75 \).

c. The final image is **real** since its distance is positive.
d. The final image is inverted since the overall magnification is negative.

e. The final image is smaller than the original object, since the magnitude of the overall magnification is less than one.

73. **REASONING**

a. We will use the thin-lens equation, \[ \frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \] (Equation 26.6), where \( d_{o1} = 2.00 \text{ m} \) is the object distance and \( f_1 = -3.00 \text{ m} \) is the focal length of the diverging lens, to determine the image distance \( d_{i1} \) when the friend is viewed through the diverging lens only. Because this image is the image of a real object formed by a diverging lens, the image is virtual, appearing to the left of the diverging lens. The magnification \( m_1 \) of the first image is given by the magnification equation, \[ m_1 = -\frac{d_{i1}}{d_{o1}} \] (Equation 26.7)

b. When the converging lens is placed a distance \( L = 1.60 \text{ m} \) to the right of the diverging lens, the image of the friend formed by the diverging lens becomes the object for the converging lens (see the drawing). The object distance \( d_{o2} \) is, therefore, the distance between the converging lens and the image formed by the diverging lens. This object is to the left of the converging lens, so the object distance is positive, and equal to the sum of the distance \( L \) and the magnitude \( |d_{i1}| \) of the first image distance:

\[ d_{o2} = L + |d_{i1}| \] (1)

We will use the thin-lens equation (Equation 26.6) to determine the second image distance \( d_{i2} \), and then the magnification equation (Equation 26.7) to determine the magnification \( m_2 \) of the second image relative to the first image. The overall magnification \( m \), that of the final image relative to the friend, is equal to the product of the two magnifications:

\[ m = m_1 m_2 \] (2)

For example, if the first image is upright and one-half the size of the friend, then \( m_1 = +0.50 \). If the second image is inverted and is three times the size of the first image, then \( m_2 = -3.0 \). The overall magnification provided by the two-lens system is, then, equal to \( m = m_1 m_2 = (+0.50)(-3.0) = -1.5 \). This means that the second image is inverted, and one-and-a-half times as tall as the visitor’s friend.
SOLUTION

a. Solving the thin-lens equation (Equation 26.6) for \(1/d_{1i}\) and taking the reciprocal of both sides yields

\[
\frac{1}{d_{1i}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = \frac{d_{o1} - f_1}{d_{o1}f_1}
\]

or

\[
d_{1i} = \frac{d_{o1}f_1}{d_{o1} - f_1} = \frac{(2.00 \text{ m})(-3.00 \text{ m})}{2.00 \text{ m} - (-3.00 \text{ m})} = -1.20 \text{ m}
\]

(3)

Substituting this result into Equation 26.7, we find that

\[
m_1 = -\frac{d_{1i}}{d_{o1}} = -\frac{(-1.20 \text{ m})}{2.00 \text{ m}} = +0.600
\]

b. Adapting Equation (3) to the formation of the second image produced by the converging lens, we see that

\[
d_{2i} = \frac{d_{o2}f_2}{d_{o2} - f_2}
\]

(4)

Substituting Equation (4) into the magnification equation (Equation 26.7), we obtain

\[
m_2 = -\frac{d_{2i}}{d_{o2}} = -\frac{d_{o2} - f_2}{d_{o2}f_2} = -\frac{f_2}{d_{o2} - f_2}
\]

(5)

Substituting Equation (5) into Equation (2) yields

\[
m = m_1m_2 = -\frac{m_1f_2}{d_{o2} - f_2}
\]

(6)

Substituting Equation (1) into Equation (6), we obtain the overall magnification:

\[
m = -\frac{m_1f_2}{d_{o2} - f_2} = -\frac{m_1f_2}{1.60 \text{ m} + |d_{1i}| - f_2} = -\frac{(+0.600)(+4.00 \text{ m})}{1.60 \text{ m} + 1.20 \text{ m} - 4.00 \text{ m}} = +2.00
\]

74. REASONING The thin-lens equation, Equation 26.6, can be used to find the distance from the blackboard to her eyes (the object distance). The distance from her eye lens to the retina is the image distance, and the focal length of her lens is the reciprocal of the refractive power (see Equation 26.8). The magnification equation, Equation 26.7, can be used to find the height of the image on her retina.

SOLUTION

a. The thin-lens equation can be used to find the object distance \(d_o\). However, we note from Equation 26.8 that \(1/f = 57.50 \text{ m}^{-1}\) and \(d_i = 0.01750 \text{ m}\), so that

\[
\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = 57.50 \text{ m}^{-1} - \frac{1}{0.01750 \text{ m}} = 0.36 \text{ m}^{-1}
\]

or

\[
d_i = 2.8 \text{ m}
\]
b. The magnification equation can be used to find the height \( h_i \) of the image on the retina

\[
h_i = h_o \left( -\frac{d_i}{d_o} \right) = (5.00 \text{ cm}) \left( -\frac{0.01750 \text{ m}}{2.8 \text{ m}} \right) = -3.1 \times 10^{-2} \text{ cm}
\]  

(26.7)

75. **REASONING** Nearsightedness is corrected using diverging lenses to form a virtual image at the far point of the eye, as Section 26.10 discusses. The far point is given as 5.2 m, so we know that the image distance for the contact lenses is \( d_i = -5.2 \text{ m} \). The minus sign indicates that the image is virtual. The thin-lens equation can be used to determine the focal length.

**SOLUTION** According to the thin-lens equation, we have

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

or

\[
f = -9.2 \text{ m}
\]

76. **REASONING** According to Equation 26.8, the refractive power is the reciprocal of the focal length in meters. Therefore, we need to determine the reciprocal of the focal length, which we will do by using the thin-lens equation

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

(Equation 26.6), where \( d_o \) is the object distance, \( d_i \) is the image distance, and \( f \) is the focal length.

**SOLUTION**

a. The woman sees distant objects clearly, so she must be farsighted. The lenses in her glasses must be converging lenses in order to correct her eyesight, as shown in Figure 26.36.

b. The refractive power (in diopters) of the glasses is the reciprocal of the focal length \( f \) in meters:

\[
\text{Refractive power in diopters} = \frac{1}{f \ (\text{in meters})}
\]  

(26.8)

To use the thin-lens equation to find the reciprocal of the focal length, we need to know the object distance \( d_o \) and the image distance \( d_i \). The object distance is the distance of the newspaper from her glasses and is \( d_o = (25 \text{ cm}) - (2.0 \text{ cm}) = 23 \text{ cm} \), where we have taken into account the fact that the glasses are 2.0 cm from her eyes. In order for her to see the image clearly, the image formed by the glasses must be located at her near point, which is 65 cm from her eyes or 63 cm from her glasses. The image distance, then, is \( d_i = -63 \text{ cm} \), the negative sign arising because the image formed by the converging lenses in the glasses is a virtual image, as shown in Figure 26.36. In using the thin-lens equation to find the reciprocal of the focal length, we must express these distances in meters, since Equation 26.8 requires it:

\[
\text{Refractive power in diopters} = \frac{1}{f \ (\text{in meters})} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.23 \text{ m}} + \frac{1}{-0.63 \text{ m}} = 2.8 \text{ diopters}
\]
77. **REASONING** The closest she can read the magazine is when the image formed by the contact lens is at the near point of her eye, or \( d_i = -138 \text{ cm} \). The image distance is negative because the image is a virtual image (see Section 26.10). Since the focal length is also known, the object distance can be found from the thin-lens equation.

**SOLUTION** The object distance \( d_o \) is related to the focal length \( f \) and the image distance \( d_i \) by the thin-lens equation:

\[
\frac{1}{d_o} - \frac{1}{d_i} = \frac{1}{f}
\]

or \( d_o = \frac{d_i}{1 - \frac{d_i}{f}} \) or \( d_o = \frac{1}{f} \cdot d_i \) (Equation 26.6)

78. **REASONING** When reading the newspaper with her glasses on, the woman focuses on the virtual image appearing at her near point, which is a distance \( N \) from her eyes. The image is a distance \( d_i \) from the glasses, and because the image is virtual, the image distance \( d_i \) is negative. Therefore, we can express the near point distance \( N \) as a positive quantity in the following fashion:

\[ N = x - d_i \]  

where \( x = 2.00 \text{ cm} \) is the distance between her eyes and the glasses. [For example, if the image distance is \( d_i = -55 \text{ cm} \), then her near point is \( N = 2.00 \text{ cm} - (-55 \text{ cm}) = 57 \text{ cm} \) from her eyes.] We will determine the image distance \( d_i \) from the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6), where \( d_o \) is the distance between the newspaper and her glasses. The focal length \( f \) of the glasses (in meters) is the reciprocal of the refractive power (in diopters), according to Equation 26.8:

\[ f = \frac{1}{1.660 \text{ diopters}} = 0.6024 \text{ m} = 60.24 \text{ cm} \]

We are given the distance \( D = 42.00 \text{ cm} \) between her eyes and the newspaper, so that the distance \( d_o \) between the glasses and the newspaper is given by

\[ d_o = D - x \]

**SOLUTION** Solving \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6) for \( d_i \) yields

\[ d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} \]

Substituting Equation (3) into Equation (4), we obtain
\[ d_i = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{f} - \frac{1}{D - x} \]  

Substituting Equation (5) into Equation (1), we find that

\[ N = x - d_i = x - \frac{1}{f} - \frac{1}{D - x} = 2.00 \text{ cm} - \frac{1}{f} - \frac{1}{60.24 \text{ cm} - 42.00 \text{ cm} - 2.00 \text{ cm}} = 121 \text{ cm} \]

79. **SSM REASONING** The far point is 5.0 m from the right eye, and 6.5 m from the left eye. For an object infinitely far away \((d_0 = \infty)\), the image distances for the corrective lenses are then \(-5.0 \text{ m}\) for the right eye and \(-6.5 \text{ m}\) for the left eye, the negative sign indicating that the images are virtual images formed to the left of the lenses. The thin-lens equation [Equation 26.6: \((1/d_o) + (1/d_i) = (1/f)\)] can be used to find the focal length. Then, Equation 26.8 can be used to find the refractive power for the lens for each eye.

**SOLUTION** Since the object distance \(d_o\) is essentially infinite, \(1/d_o = 1/\infty = 0\), and the thin-lens equation becomes \(1/d_i = 1/f\), or \(d_i = f\). Therefore, for the right eye, \(f = -5.0 \text{ m}\), and the refractive power is (see Equation 26.8)

\[
\text{Refractive power (in diopters)} = \frac{1}{f} = \frac{1}{(-5.0 \text{ m})} = -0.20 \text{ diopters}
\]

Similarly, for the left eye, \(f = -6.5 \text{ m}\), and the refractive power is

\[
\text{Refractive power (in diopters)} = \frac{1}{f} = \frac{1}{(-6.5 \text{ m})} = -0.15 \text{ diopters}
\]

80. **REASONING** The required refractive power of the contact lenses (in diopters) is the reciprocal of the focal length \(f_c\) (in meters) of the contact lenses, according to Equation 26.8:

\[
(\text{Refractive power})_c = \frac{1}{f_c}
\]

The focal length \(f_c\) of the contact lenses must be such that, when a book is a distance of 0.280 m from his eyes, the image is formed at the same distance \(d_i\), as it is when the man is
wearing his glasses. From the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6), then, we have that

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_C} \quad \text{and} \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_G} \tag{2}
\]

where \( f_G \) is the focal length of the glasses and \( d_{oG} \) is the distance between the book and the glasses. Because the glasses are a distance \( s = 0.025 \text{ m} \) from his eyes, the object distance \( d_{oG} \) is shorter than the object distance \( d_{oC} = 0.280 \text{ m} \) by \( 0.025 \text{ m} \): \( d_{oG} = 0.280 \text{ m} - 0.025 \text{ m} = 0.255 \text{ m} \). The focal length \( f_G \) is related to the refractive power of the glasses (4.20 diopters) by Equation 26.8:

\[
(\text{Refractive power})_G = \frac{1}{f_G} \tag{3}
\]

**SOLUTION**  According to Equation (1), \( \frac{1}{f_C} \) is the refractive power of the contact lenses, so the first of Equations (2) becomes

\[
(\text{Refractive power})_C = \frac{1}{f_C} = \frac{1}{d_{oC}} + \frac{1}{d_i} \tag{4}
\]

Solving the second of Equations (2) for \( 1/d_i \) yields \( \frac{1}{d_i} = \frac{1}{f_G} - \frac{1}{d_{oG}} \). Substituting this result into Equation (4) gives

\[
(\text{Refractive power})_C = \frac{1}{d_{oC}} + \frac{1}{d_i} = \frac{1}{d_{oC}} + \frac{1}{d_{oG}} - \frac{1}{d_{oG}} \tag{5}
\]

Substituting Equation (3) into Equation (5), we find that

\[
(\text{Refractive power})_C = \frac{1}{d_{oC}} + \frac{1}{f_G} - \frac{1}{d_{oG}} = \frac{1}{d_{oC}} + (\text{Refractive power})_G - \frac{1}{d_{oG}}
\]

Therefore, the refractive power of the contact lenses should be

\[
(\text{Refractive power})_C = \frac{1}{0.280 \text{ m}} + 3.80 \text{ diopters} - \frac{1}{0.255 \text{ m}} = 3.45 \text{ diopters}
\]

81. **REASONING AND SOLUTION**
a. The far point of 6.0 m tells us that the focal length of the lens is \( f = -6.0 \text{ m} \). The image distance can be found using

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-6.0 \text{ m}} - \frac{1}{18.0 \text{ m}} \quad \text{or} \quad d_i = -4.5 \text{ m}
\]

b. The image size as obtained from the magnification is
82. **REASONING AND SOLUTION** If the near point is 79.0 cm, then \( d_i = -79.0 \text{ cm} \), and \( d_o = 25.0 \text{ cm} \). Using the thin-lens equation, we find that the focal length of the correcting lens is

\[
f = \frac{d_o d_i}{d_o + d_i} = \frac{(25.0 \text{ cm})(-79.0 \text{ cm})}{25.0 \text{ cm} + (-79.0 \text{ cm})} = 36.6 \text{ cm}
\]

a. The distance \( d'_o \) to the poster can be obtained as follows:

\[
\frac{1}{d'_o} = \frac{1}{f} - \frac{1}{d'_i} = \frac{1}{36.6 \text{ cm}} - \frac{1}{-217 \text{ cm}} \quad \text{or} \quad d'_o = 31.3 \text{ cm}
\]

b. The image size is

\[
h'_i = h_o \left( -\frac{d'_i}{d'_o} \right) = (0.350 \text{ m}) \left( -\frac{217 \text{ cm}}{31.3 \text{ cm}} \right) = 2.43 \text{ m}
\]

83. **SSM REASONING** The angular magnification \( M \) of a magnifying glass is given by

\[
M \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) N \tag{26.10}
\]

where \( f \) is the focal length of the lens, \( d_i \) is the image distance, and \( N \) is the near point of the eye. The focal length and the image distance are related to the object distance \( d_o \) by the thin-lens equation:

\[
\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o} \tag{26.6}
\]

These two relations will allow us to determine the angular magnification.

**SOLUTION** Substituting Equation 26.6 into Equation 26.10 yields

\[
M \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) N = \frac{N}{d_o} = \frac{72 \text{ cm}}{4.0 \text{ cm}} = 18
\]
84. **REASONING** As discussed in Section 26.11, the angular size $\theta$ (in radians) of an object perceived by the eye is $\theta \approx \frac{h_o}{d_o}$, where $h_o$ is the height of the object and $d_o$ is the distance of the object from the eye. This approximation is good to within 1% for angles of 9º or smaller.

**SOLUTION**

a. For the spectator watching the game live, we find

$$\theta \approx \frac{h_o}{d_o} = \frac{1.9 \text{ m}}{75 \text{ m}} = 0.025 \text{ rad}$$

b. Similarly for the TV viewer, we find

$$\theta \approx \frac{h_o}{d_o} = \frac{0.12 \text{ m}}{3.0 \text{ m}} = 0.040 \text{ rad}$$

c. Since the angular size of the player seen on TV is greater than the angular size seen by the spectator, the player looks larger on television.

85. **REASONING** The distance between the work and the magnifying glass is the object distance $d_o$. This distance can be calculated by using the thin-lens equation, since the image distance and the focal length are known. The angular magnification of the magnifying glass is given by Equation 26.10.

**SOLUTION**

a. The object distance $d_o$ is

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{9.50 \text{ cm}} - \frac{1}{-25.0 \text{ cm}} \quad \text{or} \quad d_o = 6.88 \text{ cm} \quad (26.6)$$

Note that $d_i = -25.0 \text{ cm}$, since the image falls to the left of the lens; see Figure 26.39b.

b. The angular magnification $M$ of the magnifying glass is

$$M = \left( \frac{1}{f} - \frac{1}{d_i} \right) N = \left( \frac{1}{9.50 \text{ cm}} - \frac{1}{-25.0 \text{ cm}} \right) (25.0 \text{ cm}) = 3.63 \quad (26.10)$$

86. **REASONING** The angular size $\theta'$ of the image is the angular magnification $M$ times the reference angular size $\theta$ of the object: $\theta' = M\theta$ (Equation 26.9). The reference angular size is that seen by the naked eye when the object is located at the near point and is given in the problem statement as $\theta = 0.060 \text{ rad}$. To determine the angular magnification, we can utilize Equation 26.10, assuming that the angles involved are small:
where $f$ is the focal length of the magnifying glass, $d_i$ is the image distance, and $N$ is the distance between the eye and the near point. In this expression we note that the image distance is negative since the image in a magnifying glass is virtual ($d_i = -64$ cm).

**SOLUTION** Substituting the expression for the angular magnification into Equation 26.9 gives

$$\theta' = M\theta \approx \left(\frac{1}{f} - \frac{1}{d_i}\right)N\theta = \left[\frac{1}{16\text{ cm}} - \frac{1}{(-64\text{ cm})}\right](32\text{ cm})(0.060\text{ rad}) = 0.15\text{ rad}$$

87. **REASONING** The angular magnification $M$ of a magnifying glass is given by Equation 26.10 as

$$M = \frac{\theta'}{\theta} \approx \left(\frac{1}{f} - \frac{1}{d_i}\right)N$$

where $\theta' = 0.0380$ rad is the angular size of the final image produced by the magnifying glass, $\theta = 0.0150$ rad is the reference angular size of the object seen at the near point without the magnifying glass, and $N$ is the near point of the eye. The largest possible angular magnification occurs when the image is at the near point of the eye, or $d_i = -N$, where the minus sign denotes that the image lies on the left side of the lens (the same side as the object). This equation can be solved to find the focal length of the magnifying glass.

**SOLUTION** Letting $d_i = -N$, and solving Equation 26.10 for the focal length $f$ gives

$$f = \frac{N}{\frac{\theta'}{\theta} - 1} = \frac{21.0\text{ cm}}{\frac{0.0380\text{ rad}}{0.0150\text{ rad}} - 1} = 13.7\text{ cm}$$

88. **REASONING** The angular magnification of a magnifying glass is given by Equation 26.10: $M \approx \left[\frac{(1/f) - (1/d_i)}{N}\right]$, where $N$ is the distance from the eye to the near-point. For maximum magnification, the closest to the eye that the image can be is at the near point, with $d_i = -N$ (where the minus sign indicates that the image lies to the left of the lens and is virtual). In this case, Equation 26.10 becomes $M_{\text{max}} \approx N/f + 1$. At minimum magnification, the image is as far from the eye as it can be ($d_i = -\infty$); this occurs when the object is placed at the focal point of the lens. In this case, Equation 26.10 simplifies to $M_{\text{min}} \approx N/f$.

Since the woman observes that for clear vision, the maximum angular magnification is 1.25 times larger than the minimum angular magnification, we have $M_{\text{max}} = 1.25 M_{\text{min}}$. 
This equation can be written in terms of $N$ and $f$ using the above expressions, and then solved for $f$.

**SOLUTION** We have
\[
\frac{N}{f} + 1 = 1.25 \frac{N}{f}
\]

Solving for $f$, we find that
\[
f = 0.25N = (0.25)(25 \text{ cm}) = 6.3 \text{ cm}
\]

89. [SSM] **REASONING AND SOLUTION** The information given allows us to determine the near point for this farsighted person. With $f = 45.4 \text{ cm}$ and $d_o = 25.0 \text{ cm}$, we find from the thin-lens equation that
\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{45.4 \text{ cm}} - \frac{1}{25.0 \text{ cm}} \quad \text{or} \quad d_i = -55.6 \text{ cm}
\]

Therefore, this person's near point, $N$, is 55.6 cm. We now need to find the focal length of the magnifying glass based on the near point for a normal eye, i.e., $M = N/f + 1$ (Equation 26.10, with $d_i = -N$), where $N = 25.0 \text{ cm}$. Solving for the focal length gives
\[
f = \frac{N}{M - 1} = \frac{25.0 \text{ cm}}{7.50 - 1} = 3.85 \text{ cm}
\]

We can now determine the maximum angular magnification for the farsighted person
\[
M = \frac{N}{f} + 1 = \frac{55.6 \text{ cm}}{3.85 \text{ cm}} + 1 = 15.4
\]

90. **REASONING AND SOLUTION** According to Equation 26.11, the angular magnification of the microscope with the 100-diopter objective is
\[
M_1 \approx -\frac{(L - f_e)N}{f_{o1} f_e}
\]

and with the 300-diopter objective is
\[
M_2 \approx -\frac{(L - f_e)N}{f_{o2} f_e}
\]

Dividing Equation (2) by Equation (1) gives
\[
\frac{M_2}{M_1} = \frac{f_{o1}}{f_{o2}} = \frac{300\ \text{diopters}}{100\ \text{diopters}} = 3
\]

Since the angular magnification of the microscope with the 300-diopter objective is three times greater than that with the 100-diopter objective, the angle will be

\[
3(3 \times 10^{-3}\ \text{rad}) = 9 \times 10^{-3}\ \text{rad}
\]

91. **REASONING** The angular magnification of a compound microscope is given by Equation 26.11:

\[
M \approx -\frac{(L - f_e)N}{f_o f_e}
\]

where \( f_o \) is the focal length of the objective, \( f_e \) is the focal length of the eyepiece, and \( L \) is the separation between the two lenses. This expression can be solved for \( f_o \), the focal length of the objective.

**SOLUTION** Solving for \( f_o \), we find that the focal length of the objective is

\[
\frac{f_o}{f_e} = -\frac{(L - f_e)N}{f_o f_e} \Rightarrow f_o = -\frac{(L - f_e)N}{f_e M} = -\frac{(16.0 \ \text{cm} - 1.4 \ \text{cm})(25 \ \text{cm})}{(1.4 \ \text{cm})(-320)} = 0.81 \ \text{cm}
\]

92. **REASONING** The angular magnification \( M \) of a compound microscope is given by

\[
M \approx -\frac{(L - f_e)N}{f_o f_e} \quad \text{(Equation 26.11)}
\]

where \( L \) is the distance between the objective and the eyepiece, \( N \) is the distance between the eye and the near point, \( f_e \) is the focal length of the eyepiece, and \( f_o \) is the focal length of the objective. Values for all of the variables in this expression are known, except for the focal length of the eyepiece. Therefore, we can solve Equation 26.11 for the unknown focal length \( f_e \).

**SOLUTION** Using Equation 26.11, we find

\[
M \approx -\frac{(L - f_e)N}{f_o f_e} \quad \text{or} \quad M f_o f_e \approx (f_e - L)N \quad \text{or} \quad (M f_o - N) f_e \approx -L N
\]

or

\[
f_e = \frac{-LN}{M f_o - N} = \frac{- (18 \ \text{cm})(25 \ \text{cm})}{(-83)(1.5 \ \text{cm}) - (25 \ \text{cm})} = 3.0 \ \text{cm}
\]

93. **REASONING** The focal length \( f_e \) of the eyepiece can be determined by using Equation 26.11:

\[
M \approx -\frac{(L - f_e)N}{f_o f_e} \quad \text{(26.11)}
\]
where $M$ is the angular magnification, $L$ is the distance between the objective and the eyepiece, $N$ is the distance between the eye and the near point, and $f_o$ is the focal length of the objective. We can calculate $f_e$ from this equation, provided that we have a value for $M$, since values for all of the other variables are given in the problem statement. Although $M$ is not given directly, we do have a value for the angular size of the image ($\theta' = -8.8 \times 10^{-3}$ rad) and the reference angular size seen by the naked eye when the object is located at the near point ($\theta = 5.2 \times 10^{-5}$ rad). From these two angular sizes we can determine the angular magnification using the definition in Equation 26.10:

$$M = \frac{\theta'}{\theta} \quad (26.10)$$

**SOLUTION** Substituting Equation 26.10 into Equation 26.11 and solving for $f_e$ shows that

$$M = \frac{\theta'}{\theta} \approx -\frac{(L-f_e)N}{f_o f_e} \quad \text{or} \quad \theta' f_o f_e \approx -\theta (L-f_e)N \quad \text{or} \quad \theta' f_o f_e - \theta f_e N \approx -\theta L N$$

$$f_e \approx -\frac{\theta LN}{\theta' f_o - \theta N} = \frac{-\left(5.2 \times 10^{-5} \text{ rad}\right)\left(16 \text{ cm}\right)\left(25 \text{ cm}\right)}{-\left(8.8 \times 10^{-3} \text{ rad}\right)\left(2.6 \text{ cm}\right) - \left(5.2 \times 10^{-5} \text{ rad}\right)\left(25 \text{ cm}\right)} = 0.86 \text{ cm}$$

94. **REASONING** Equation 26.11 gives the angular magnification of a compound microscope. We can apply this expression to the microscopes of length $L$ and $L'$ and then set the two angular magnifications equal to one another. From the resulting equation, we will be able to obtain a value for $L'$.

**SOLUTION** According to Equation 26.11, the angular magnification $M$ of a microscope of length $L$ is

$$M \approx -\frac{(L-f_o)N}{f_o f_e} \quad (26.11)$$

where $N$ is the near point of the viewer’s eye. For a microscope of length $L'$, Equation 26.11 also applies, but with $L$ replaced by $L'$ and the focal lengths $f_o$ and $f_e$ interchanged. The angular magnification $M'$ of this microscope is

$$M' \approx -\frac{(L'-f_o)N}{f_e f_o} \quad (1)$$

Since $M' = M$, we have from Equations 26.11 and (1) that

$$-\frac{(L'-f_o)N}{f_e f_o} = -\frac{(L-f_e)N}{f_o f_e} \quad \text{or} \quad L' = L - f_e + f_o \quad (2)$$
Using Equation (2), we obtain

\[ L' = L - f_e + f_o = (12.0 \text{ cm}) - (2.0 \text{ cm}) + (0.60 \text{ cm}) = 10.6 \text{ cm} \]

95. **REASONING** The angular magnification of a compound microscope is given by Equation 26.11. All the necessary data are given in the statement of the problem, so the angular magnification can be calculated directly. In order to find how far the object is from the objective, examine Figure 26.32a. The object distance \( d_{o1} \) for the first lens is related to its focal length \( f_1 \) and image distance \( d_{i1} \) by the thin-lens equation (Equation 26.6). From the drawing we see that the image distance is approximately equal to the distance \( L \) between the lenses minus the focal length \( f_e \) of the eyepiece, or \( d_{i1} \approx L - f_e \). Thus, the object distance is given by

\[
\frac{1}{d_{o1}} = \frac{1}{f_o} - \frac{1}{f_1} \approx \frac{1}{f_o} - \frac{1}{L - f_e}
\]

The magnification due to the objective is given by Equation 26.7 as

\[ m_{\text{objective}} = -\frac{d_{i1}}{d_{o1}} \]

Since both \( d_{i1} \) and \( d_{o1} \) are now known, the magnification can be evaluated.

**SOLUTION**

a. According to Equation 26.11, the angular magnification of the compound microscope is

\[
M \approx -\frac{(L - f_e)N}{f_o f_e} = -\frac{(26.0 \text{ cm} - 6.50 \text{ cm})(35.0 \text{ cm})}{(3.50 \text{ cm})(6.50 \text{ cm})} = -30.0
\]

b. Using the thin-lens equation, we can determine the object distance from the objective as follows:

\[
\frac{1}{d_{o1}} = \frac{1}{f_o} - \frac{1}{L - f_e} = \frac{1}{3.50 \text{ cm}} - \frac{1}{26.0 \text{ cm} - 6.50 \text{ cm}} = 0.234 \text{ cm}^{-1} \quad \text{or} \quad d_{o1} = 4.27 \text{ cm}
\]

c. The magnification \( m \) of the objective is given by Equation 26.7 as

\[
m_{\text{objective}} = -\frac{d_{i1}}{d_{o1}} = -\frac{26.0 \text{ cm} - 6.50 \text{ cm}}{4.27 \text{ cm}} = -4.57
\]

96. **REASONING** The refractive power of the eyepiece (in diopters) is the reciprocal of its focal length \( f_e \) (in meters), according to the definition in Equation 26.8:

\[
(\text{Refractive power})_e = \frac{1}{f_e}
\]

To obtain the focal length of the eyepiece, we consider the angular magnification of the telescope. According to Equation 26.12, the angular magnification \( M \) is
To use this expression to determine \( f_e \), however, we need a value for \( f_o \), which is the focal length of the objective. Although a value for \( f_o \) is not given directly, the value of the refractive power of the objective is given, and it can be used in Equation 26.8 to obtain \( f_o \):

\[
(\text{Refractive power})_o = \frac{1}{f_o}
\]

**SOLUTION** Substituting \( f_e \) and \( f_o \) from Equations (1) and (2) into Equation 26.12, we find

\[
M \approx -\frac{f_o}{f_e} = -\frac{1/(\text{Refractive power})_o}{1/(\text{Refractive power})_e} = \frac{(\text{Refractive power})_e}{(\text{Refractive power})_o}
\]

Solving for the refractive power of the eyepiece gives

\[
(\text{Refractive power})_e \approx -M (\text{Refractive power})_o = -(-132)(1.50 \text{ diopters}) = 198 \text{ diopters}
\]

97. **SSM REASONING** Knowing the angles subtended at the unaided eye and with the telescope will allow us to determine the angular magnification of the telescope. Then, since the angular magnification is related to the focal lengths of the eyepiece and the objective, we will use the known focal length of the eyepiece to determine the focal length of the objective.

**SOLUTION** From Equation 26.12, we have

\[
M = \frac{\theta'}{\theta} = -\frac{f_o}{f_e}
\]

where \( \theta \) is the angle subtended by the unaided eye and \( \theta' \) is the angle subtended when the telescope is used. We note that \( \theta' \) is negative, since the telescope produces an inverted image. Thus, using Equation 26.12, we find

\[
f_o = -\frac{f_e \theta'}{\theta} = -\frac{(0.032 \text{ m})(-2.8 \times 10^{-3} \text{ rad})}{8.0 \times 10^{-5} \text{ rad}} = 1.1 \text{ m}
\]

98. **REASONING** The angular magnification \( M \) of the telescope is given by \( M \approx -\frac{f_o}{f_e} \) (Equation 26.12), where \( f_o \) is the focal length of the objective and \( f_e \) is the focal length of the eyepiece. We are not given the focal lengths. However, we are given the refractive powers of the two lenses, and the refractive power in diopters is the reciprocal of the focal length \( f \) in meters according to Equation 26.8.
**SOLUTION** Using Equation 26.12 for the angular magnification, we have

\[ M \approx -\frac{f_o}{f_e} \]  

(26.12)

Since the refractive power (in diopters) of a lens is the reciprocal of the focal length \( f \) in meters according to Equation 26.8, we know that

\[ \text{Refractive power} = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{\text{Refractive power}} \]  

(1)

Expressing the focal lengths in Equation 26.12 with the aid of Equation (1), we obtain

\[ M \approx -\frac{f_o}{f_e} = \frac{1}{\text{(Refractive power) }_o} - \frac{1}{\text{(Refractive power) }_e} = \frac{250 \text{ diopters}}{1.25 \text{ diopters}} = -20 \times 10^2 \]

99. **SSM REASONING** The angular magnification \( M \) of an astronomical telescope is given by \( M \approx -\frac{f_o}{f_e} \) (Equation 26.12), where \( f_o \) is the focal length of the objective lens, and \( f_e \) is the focal length of the eyepiece. The length of the barrel must be adjusted so that the image formed by the objective appears very close to the focal point of the eyepiece. The result is that the length \( L \) of the barrel is approximately equal to the sum of the focal lengths of the telescope’s two lenses: \( L \approx f_o + f_e \). Therefore, if the barrel can only be shortened by 0.50 cm, then the replacement eyepiece can have a focal length \( f_{e2} \) that is only 0.50 cm shorter than the focal length \( f_{e1} \) of the current eyepiece.

**SOLUTION** The focal length of the replacement eyepiece is

\[ f_{e2} = f_{e1} - 0.50 \text{ cm} = 1.20 \text{ cm} - 0.50 \text{ cm} = 0.70 \text{ cm} \]

Therefore, from Equation 26.12, the angular magnification with the second eyepiece in place is

\[ M \approx -\frac{f_o}{f_e} = -\frac{180 \text{ cm}}{0.70 \text{ cm}} = -260 \]

100. **REASONING AND SOLUTION**

a. The lens with the largest focal length should be used for the objective of the telescope. Since the refractive power is the reciprocal of the focal length (in meters), the lens with the smallest refractive power is chosen as the objective, namely, the 1.3-diopter lens.

b. According to Equation 26.8, the refractive power is related to the focal length \( f \) by \( \text{Refractive power (in diopters)} = \frac{1}{f \text{ (in meters)}} \). Since we know the refractive powers of
the two lenses, we can solve Equation 26.8 for the focal lengths of the objective and the eyepiece. We find that \( f_o = 1/(1.3 \text{ diopters}) = 0.77 \text{ m} \). Similarly, for the eyepiece, \( f_e = 1/(11 \text{ diopters}) = 0.091 \text{ m} \). Therefore, the distance between the lenses should be
\[
L \approx f_o + f_e = 0.77 \text{ m} + 0.091 \text{ m} = 0.86 \text{ m}
\]

c. The angular magnification of the telescope is given by Equation 26.12 as
\[
M \approx \frac{f_o}{f_e} = \frac{0.77 \text{ m}}{0.091 \text{ m}} = 8.5
\]

101. **REASONING AND SOLUTION**

a. According to Equation 26.12, the magnification is
\[
M = \frac{f_o}{f_e} = \frac{19.4 \text{ m}}{0.100 \text{ m}} = 194
\]

b. The angular size of the crater is
\[
\theta = \frac{h_o}{d_o} = \frac{1500 \text{ m}}{3.77 \times 10^8 \text{ m}} = 4.0 \times 10^{-6} \text{ rad}
\]

The angular magnification is, \( M = \theta'/\theta \) (Equation 26.12), so that
\[
\theta' = M\theta = (-194)(4.0 \times 10^{-6} \text{ rad}) = -7.8 \times 10^{-4} \text{ rad}
\]

Since \( h'_i = \theta' f_e \), we have
\[
h'_i = \theta' f_e = (-7.8 \times 10^{-4} \text{ rad})(0.100 \text{ m}) = -7.8 \times 10^{-5} \text{ m}
\]

c. The apparent distance is shorter by a factor of 194, so
\[
\text{Apparent distance} = \frac{3.77 \times 10^8 \text{ m}}{194} = 1.94 \times 10^6 \text{ m}
\]

102. **REASONING** The angular magnification \( M \) of a telescope is given by Equation 26.12:
\[
M \approx \frac{f_o}{f_e}
\]  

(26.12)

To achieve large values for \( M \), the focal length \( f_o \) of the objective needs to be greater than the focal length \( f_e \) of the eyepiece.

In an astronomical telescope one of the focal points of the objective falls virtually at the same place as one of the focal points of the eyepiece (see Section 26.13). Therefore, the length \( L \) of the telescope is approximately equal to the sum of the focal lengths:
\[ L \approx f_o + f_e \]  

Solving Equation (1) for \( f_o \) and substituting the result into Equation 26.12 gives

\[ M \approx -\frac{f_o}{f_e} \approx -\frac{L - f_e}{f_e} \]

**SOLUTION**

Applying Equation (2) to each telescope, we find

\[
M_A \approx -\frac{L_A - f_e}{f_e} = -\frac{(455 \text{ mm} - 3.00 \text{ mm})}{3.00 \text{ mm}} = -151
\]

\[
M_B \approx -\frac{L_B - f_e}{f_e} = -\frac{(615 \text{ mm} - 3.00 \text{ mm})}{3.00 \text{ mm}} = -204
\]

\[
M_C \approx -\frac{L_C - f_e}{f_e} = -\frac{(824 \text{ mm} - 3.00 \text{ mm})}{3.00 \text{ mm}} = -274
\]

103. **REASONING**

From the discussion of the telescope in Section 26.13, we know that the length \( L \) of the barrel is approximately equal to the focal length \( f_o \) of the objective plus the focal length \( f_e \) of the eyepiece; \( L \approx f_o + f_e \). In addition, the angular magnification \( M \) of a telescope is given by \( M \approx -\frac{f_o}{f_e} \) (Equation 26.12). These two relations will permit us to determine the focal lengths of the objective and the eyepiece.

**SOLUTION**

a. Since \( L \approx f_o + f_e \), the focal length of the objective can be written as

\[ f_o \approx L - f_e \]

Solving the expression for the angular magnification (Equation 26.12) for \( f_e \) gives \( f_e \approx -\frac{f_o}{M} \). Substituting this result into Equation (1) gives

\[
f_o \approx L - \left( -\frac{f_o}{M} \right) \quad \text{or} \quad f_o \approx \frac{L}{1 - \frac{1}{M}} = \frac{1.500 \text{ m}}{1 - \frac{1}{83.00}} = 1.482 \text{ m}
\]

b. Using Equation (1), we have

\[
f_e \approx L - f_o = (1.500 \text{ m}) - (1.482 \text{ m}) = 0.018 \text{ m}
\]
104. **REASONING AND SOLUTION** Using the thin–lens equation, we can find the first image distance \(d_i\) with respect to the objective:

\[
\frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o} = \frac{1}{1.500\ m} - \frac{1}{114.00\ m} \quad \text{or} \quad d_i = 1.520\ m \tag{1}
\]

Using the magnification equation, we can find the linear magnification \(m\) achieved by the objective:

\[
m = \frac{-d_i}{d_o} = \frac{-1.520\ m}{114.00\ m} = -0.01333 \tag{2}
\]

We then use this "first image" as the object for the second lens (the eyepiece). With the aid of the thin-lens equation, we can determine the distance \(d_i'\) of the final image with respect to the eyepiece:

\[
\frac{1}{d_i'} = \frac{1}{f_e} - \frac{1}{d_o'} = \frac{1}{0.070\ m} - \frac{1}{0.050\ m} \quad \text{or} \quad d_i' = -0.18\ m \tag{3}
\]

In Equation (3) we have used \(d_o' = 0.050\ m\). This follows because we know that the separation between the objective and the eyepiece is \(L \approx f_o + f_e\) (see Example 17 in the text). Therefore, we know that \(L \approx 1.500\ m + 0.070\ m = 1.570\ m\), and we can determine that the distance of the "first image" from the second lens (the eyepiece) is \(L - 1.520\ m \approx 1.570\ m - 1.520\ m = 0.050\ m\).

Using the magnification equation, we can find the linear magnification \(m'\) achieved by the eyepiece:

\[
m' = \frac{-d_i'}{d_o'} = \frac{-0.18\ m}{0.050\ m} = +3.6 \tag{4}
\]

Using Equations (2) and (4), we find that the total linear magnification achieved by both the objective and the eyepiece is

\[
m_{\text{Total}} = \frac{h_i'}{h_o} = m \times m' = (-0.01333)(+3.6) = -0.048 \tag{5}
\]

where \(h_i'\) is the height of the final image and \(h_o\) is the height of the initial object.

However, we need the angular magnification, which, according to Equation 26.9, is

\[
M = \frac{\theta'}{\theta} = \frac{-h_i'}{\frac{d_i'}{h_o}} = \left( -\frac{h_i'}{h_o} \right) \left( \frac{d_o + f_o + f_e}{d_i'} \right) \tag{6}
\]

In Equation (6) \(\theta' = \frac{-h_i'}{d_i'}\) is the angular size of the final image and \(\theta = \frac{h_o}{d_o + f_o + f_e}\) is the angular size of the initial object.
Substituting Equations (3) and (5) into Equation (6) and using the given data for \( d_o, f_o, \) and \( f_e, \) we find that

\[
M = \left( -\frac{h'_i}{h_o} \left( \frac{d_o + f_o + f_e}{d_i} \right) \right) = \left( -\frac{0.048}{0.18} \right) \left( \frac{114.00 \text{ m} + 1.500 \text{ m} + 0.070 \text{ m}}{-0.18} \right) = -31
\]

105. **SSM REASONING** The ray diagram is constructed by drawing the paths of two rays from a point on the object. For convenience, we will choose the top of the object. The ray that is parallel to the principal axis will be refracted by the lens so that it passes through the focal point on the right of the lens. The ray that passes through the center of the lens passes through undeflected. The image is formed at the intersection of these two rays. In this case, the rays do not intersect on the right of the lens. However, if they are extended backwards they intersect on the left of the lens, locating a virtual, upright, and enlarged image.

**SOLUTION**

a. The ray-diagram, drawn to scale, is shown below.

![Ray Diagram](image)

From the diagram, we see that the image distance is \( d_i = -75 \text{ cm} \) and the magnification is \( +2.5 \). The negative image distance indicates that the image is virtual. The positive magnification indicates that the image is larger than the object.

b. From the thin-lens equation [Equation 26.6: \( \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \)], we obtain

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{50.0 \text{ m}} - \frac{1}{30.0 \text{ cm}} \quad \text{or} \quad d_i = -75.0 \text{ cm}
\]

The magnification equation (Equation 26.7) gives the magnification to be

\[
m = -\frac{d_i}{d_o} = -\frac{-75.0 \text{ cm}}{30.0 \text{ cm}} = +2.50
\]
The camera records a clear image on its image sensor only when the distance between the sensor and the lens equals the image distance \( d_i \) for a given object in front of the lens. The image distance is determined by the object distance \( d_o \) and the focal length \( f \) of the lens, according to the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6). To determine the amount by which the distance between the sensor and the lens needs to be increased, we will use the thin-lens equation once to determine \( d_i \) for an object distance of \( d_o = 0.500 \text{ m} \) and then again for \( d_o = 0.200 \text{ m} \).

**SOLUTION** The amount \( \Delta d_i \) by which the distance between the sensor and the lens needs to be increased is

\[
\Delta d_i = (d_i)_{d_o=0.200 \text{ m}} - (d_i)_{d_o=0.500 \text{ m}}
\]

Using the thin-lens equation to calculate the image distance, we find

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{d_o f} \quad \text{or} \quad d_i = \frac{d_o f}{d_o - f}
\]

Using Equation (2), we obtain the two image distances needed in Equation (1) as follows:

\[
(d_i)_{d_o=0.500 \text{ m}} = \left( \frac{d_o f}{d_o - f} \right)_{d_o=0.500 \text{ m}} = \frac{(0.500 \text{ m})(0.0500 \text{ m})}{(0.500 \text{ m}) - (0.0500 \text{ m})} = 0.0556 \text{ m}
\]

\[
(d_i)_{d_o=0.200 \text{ m}} = \left( \frac{d_o f}{d_o - f} \right)_{d_o=0.200 \text{ m}} = \frac{(0.200 \text{ m})(0.0500 \text{ m})}{(0.200 \text{ m}) - (0.0500 \text{ m})} = 0.0667 \text{ m}
\]

Substituting these two values into Equation (1) reveals that

\[
\Delta d_i = (d_i)_{d_o=0.200 \text{ m}} - (d_i)_{d_o=0.500 \text{ m}} = (0.0667 \text{ m}) - (0.0556 \text{ m}) = 0.0111 \text{ m}
\]

**SSM REASONING** The refractive index \( n_{\text{Liquid}} \) of the liquid can be less than the refractive index of the glass \( n_{\text{Glass}} \). However, we must consider the phenomenon of total internal reflection. Some of the light will enter the liquid as long as the angle of incidence is less than or equal to the critical angle. At incident angles greater than the critical angle, total internal reflection occurs, and no light enters the liquid. Since the angle of incidence is 75.0°, the critical angle cannot be allowed to fall below 75.0°. The critical angle \( \theta_c \) is determined according to Equation 26.4:

\[
\sin \theta_c = \frac{n_{\text{Liquid}}}{n_{\text{Glass}}}
\]
As $n_{\text{Liquid}}$ decreases, the critical angle decreases. Therefore, $n_{\text{Liquid}}$ cannot be less than the value calculated from this equation, in which $\theta_c = 75.0^\circ$ and $n_{\text{Glass}} = 1.56$.

**SOLUTION** Using Equation 26.4, we find that

$$\sin \theta_c = \frac{n_{\text{Liquid}}}{n_{\text{Glass}}} \quad \text{or} \quad n_{\text{Liquid}} = n_{\text{Glass}} \sin \theta_c = (1.56) \sin 75.0^\circ = 1.51$$

108. **REASONING AND SOLUTION** The actual height $d$ of the diving board above the water can be obtained by using Equation 26.3. As usual, $n_1$ is the index of refraction of the medium (air) associated with the incident ray, and $n_2$ is that of the medium (water) associated with the refracted ray. Taking the refractive index of water from Table 26.1, we find

$$d = d' \left( \frac{n_1}{n_2} \right) = (4.0 \text{ m}) \left( \frac{1.00}{1.33} \right) = 3.0 \text{ m}$$

109. **SSM REASONING** The glasses form an image of a distant object at her far point, a distance $L = 0.690$ m from her eyes. This distance is equal to the magnitude $|d_i|$ of the image distance, which is measured from the glasses to the image, plus the distance $s$ between her eyes and the glasses:

$$L = |d_i| + s \quad (1)$$

Because distant objects may be considered infinitely distant, the object distance in the thin-lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ (Equation 26.6) is $d_o = \infty$, and we see that the image distance is equal to the focal length $f$ of the eyeglasses:

$$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{d_i} = 0 + \frac{1}{d_i} = \frac{1}{d_i} \quad \text{or} \quad d_i = f \quad (2)$$

The focal length $f$ (in meters) is equal to the reciprocal of the refractive power (in diopters) of the glasses, according to $f = \frac{1}{\text{Refractive power}}$ (Equation 26.8).

**SOLUTION** Solving Equation (1) for $s$ yields $s = L - |d_i|$. Substituting Equation (2) into this result, we obtain

$$s = L - |d_i| = L - |f| \quad (3)$$
Substituting \( f = \frac{1}{\text{Refractive power}} \) (Equation 26.8) into Equation (3), we find that

\[
s = L - |f| = L - \left| \frac{1}{\text{Refractive power}} \right| = 0.690 \text{ m} - \left| \frac{1}{-1.50 \text{ diopters}} \right| = 0.023 \text{ m}
\]

110. **REASONING** The index of refraction of the oil is one of the factors that determine the apparent depth of the bolt. Equation 26.3 gives the apparent depth and can be solved for the index of refraction.

**SOLUTION** According to Equation 26.3, the apparent depth \( d' \) of the bolt is

\[
d' = d \left( \frac{n_2}{n_1} \right)
\]

where \( d \) is the actual depth, \( n_1 \) is the refractive index of the medium (oil) in which the object is located, and \( n_2 \) is the medium (air) in which the observer is located directly above the object. Solving for \( n_1 \) and recognizing that the refractive index of air is \( n_2 = 1.00 \), we obtain

\[
n_1 = n_2 \left( \frac{d}{d'} \right) = (1.00) \left( \frac{5.00 \text{ cm}}{3.40 \text{ cm}} \right) = 1.47
\]

111. **REASONING AND SOLUTION** According to Equation 26.11, the angular magnification of the microscope is

\[
M \approx - \left( \frac{L - f_e}{f_o f_e} \right) N = - \left( \frac{14.0 \text{ cm} - 2.5 \text{ cm}}{25.0 \text{ cm}} \right) = -2.3 \times 10^2
\]

Now the new angle is

\[
\theta' = M \theta = \left( -2.3 \times 10^2 \right) (2.1 \times 10^{-5} \text{ rad}) = -4.8 \times 10^{-3} \text{ rad}
\]

The magnitude of the angle is \( 4.8 \times 10^{-3} \text{ rad} \).

112. **REASONING AND SOLUTION**

a. The sun is so far from the lens that the incident rays are nearly parallel to the principal axis, so the image distance \( d_i \) is nearly equal to the focal length of the lens. The magnification of the lens is
113. **SSM REASONING** The drawing shows a ray of sunlight reaching the scuba diver (drawn as a black dot). The light reaching the scuba diver makes an angle of 28.0° with respect to the vertical. In addition, the drawing indicates that this angle is also the angle of refraction θ₂ of the light entering the water. The angle of incidence for this light is θ₁. These angles are related by Snell’s law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) (Equation 26.2), where \( n_1 \) and \( n_2 \) are, respectively, the indices of refraction of the air and water. Since \( \theta_2, n_1, \) and \( n_2 \) are known, the angle of incidence can be determined.

**SOLUTION** The angle of incidence of the light is given according to

\[
\sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1} \quad \text{or} \quad \theta_1 = \sin^{-1}\left(\frac{n_2 \sin \theta_2}{n_1}\right) = \sin^{-1}\left(\frac{1.333 \sin 28.0^\circ}{1.000}\right) = 38.7^\circ
\]

where the values \( n_1 = 1.000 \) and \( n_2 = 1.333 \) have been taken from Table 26.1.

114. **REASONING**

a. When the filling is viewed from a distance \( d_o \) without magnifying glasses its angular size (in radians) is given by \( \theta \approx \frac{h_o}{d_o} \), where \( h_o \) is the object height (the diameter of the filling,
in this case). The greatest angular size occurs when the object distance \( d_o \) is as small as possible. Without magnifying glasses, the smallest object distance is the dentist’s near point \( N: d_o = N = 17.0 \text{ cm} \).

b. Wearing the magnifying glasses enables the dentist to get closer to the filling and see an enlarged image of the filling. The increased angular size \( \theta' \) of the filling then depends upon the focal length \( f \) of the lens and the image distance \( d_i \), according to

\[
\theta' \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) N
\]

(Equation 26.10), where \( \theta \) is the angular size of the filling found in part (a). The largest possible angular size \( \theta' \) is obtained when the image appears at the near point \( N \), so we will take \( d_i = -N \), where the negative sign indicates that the image is virtual.

**SOLUTION**

a. The diameter of the filling is \( h_o = 2.4 \text{ mm} \), and the object distance is equal to the dentist’s near point: \( d_o = N = 17.0 \text{ cm} \). One centimeter equals ten millimeters, so the object distance can be expressed as \( d_o = 170 \text{ mm} \). From \( \theta \approx \frac{h_o}{d_o} \), the angular size of the filling is

\[
\theta \approx \frac{2.4 \text{ mm}}{170 \text{ mm}} = 0.014 \text{ rad}
\]

b. Solving \( \theta' \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) N \) (Equation 26.10) for \( \theta' \) yields

\[
\theta' \approx \theta \left( \frac{1}{f} - \frac{1}{d_i} \right) N
\]

To obtain the maximum possible angular size, we take \( d_i = -N \) in Equation (1), which gives

\[
\theta' \approx \theta \left[ \frac{1}{f} - \frac{1}{(-N)} \right] N = \theta \left( \frac{1}{f} - \frac{1}{N} \right) N = \theta \left( \frac{N}{f} + 1 \right) = (0.014 \text{ rad}) \left( \frac{17.0 \text{ cm}}{6.0 \text{ cm}} \right) = 0.054 \text{ rad}
\]

115. **REASONING** We will apply the thin-lens equation to solve this problem. In doing so, we must be careful to take into account the fact that the lenses of the glasses are worn at a distance of 2.2 or 3.3 cm from her eyes.

**SOLUTION**

a. The object distance is 25.0 cm – 2.2 cm, since it is measured relative to the lenses, which are worn 2.2 cm from the eyes. As discussed in the text, the lenses form a virtual image located at the near point. The image distance must be negative for a virtual image, but the value is not –67.0 cm, because the glasses are worn 2.2 cm from the eyes. Instead, the image distance is –67.0 cm + 2.2 cm. Using the thin-lens equation, we can find the focal length as follows:
\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25.0 \text{ cm} - 2.2 \text{ cm}} + \frac{1}{-67.0 \text{ cm} + 2.2 \text{ cm}} \quad \text{or} \quad f = \boxed{35.2 \text{ cm}}
\]

b. Similarly, we find
\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25.0 \text{ cm} - 3.3 \text{ cm}} + \frac{1}{-67.0 \text{ cm} + 3.3 \text{ cm}} \quad \text{or} \quad f = \boxed{32.9 \text{ cm}}
\]

116. **REASONING** Since the angles of refraction are the same, the angles of incidence must be different, because the refractive indices of the red and violet light are different. This follows directly from Snell’s law. We can apply the law for each color, obtaining two equations in the process. By eliminating the common angle of refraction from these equations, we can obtain a single expression from which the angle of incidence of the violet light can be determined.

**SOLUTION** Applying Snell’s law for each color, we obtain
\[
\frac{n_{1, \text{red}} \sin \theta_{1, \text{red}}}{\text{Air}} = \frac{n_{2, \text{red}} \sin \theta_{2, \text{red}}}{\text{Glass}} \quad \text{and} \quad \frac{n_{1, \text{violet}} \sin \theta_{1, \text{violet}}}{\text{Air}} = \frac{n_{2, \text{violet}} \sin \theta_{2, \text{violet}}}{\text{Glass}}
\]

Dividing the equation on the right by the equation on the left and recognizing that the angles of refraction \(\theta_{2, \text{red}}\) and \(\theta_{2, \text{violet}}\) are equal, we find
\[
\frac{n_{1, \text{violet}} \sin \theta_{1, \text{violet}}}{n_{1, \text{red}} \sin \theta_{1, \text{red}}} = \frac{n_{2, \text{violet}} \sin \theta_{2, \text{violet}}}{n_{2, \text{red}} \sin \theta_{2, \text{red}}}
\]

Since both colors are incident in air, the indices of refraction \(n_{1, \text{red}}\) and \(n_{1, \text{violet}}\) are both equal to 1.000, and this expression simplifies to
\[
\frac{\sin \theta_{1, \text{violet}}}{\sin \theta_{1, \text{red}}} = \frac{n_{2, \text{violet}}}{n_{2, \text{red}}}
\]

Solving for the angle of incidence of the violet light gives
\[
\sin \theta_{1, \text{violet}} = \sin \theta_{1, \text{red}} \left( \frac{n_{2, \text{violet}}}{n_{2, \text{red}}} \right) = \left( \sin 30.00^\circ \right) \left( \frac{1.538}{1.520} \right) = 0.5059
\]

\[
\theta_{1, \text{violet}} = \sin^{-1}(0.5059) = 30.39^\circ
\]

117. **REASONING** Since the object distance and the focal length of the lens are given, the thin-lens equation (Equation 26.6) can be used to find the image distance. The height of the image can be determined by using the magnification equation (Equation 26.7).
**SOLUTION**

a. The object distance \(d_o\), the image distance \(d_i\), and the focal length \(f\) of the lens are related by the thin-lens equation:

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  
(26.6)

Solving for the image distance gives

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{12.0 \text{ cm}} - \frac{1}{8.00 \text{ cm}} \quad \text{or} \quad d_i = -24 \text{ cm}
\]

b. The image height \(h_i\) (the height of the magnified print) is related to the object height \(h_o\), the image distance \(d_i\), and the object distance \(d_o\) by the magnification equation:

\[
h_i = -h_o \left( \frac{d_i}{d_o} \right) = -(2.00 \text{ mm}) \left( \frac{-24 \text{ cm}}{8.00 \text{ cm}} \right) = 6.0 \text{ mm}
\]  
(26.7)

118. **REASONING**  In order for a sharply focused image to appear on the image sensor, the distance between the sensor and the lens must be equal to the image distance \(d_i\), which can be found from the thin-lens equation \[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  
(Equation 26.6), where \(f = 200.0 \text{ mm} = 0.2000 \text{ m}\) is the focal length of the lens and \(d_o\) is the distance between the object and the lens. Since there are two object distances \((d_{o1}, d_{o2})\), there will be two image distances \((d_{i1}, d_{i2})\). When the converging lens forms a real image, increasing the object distance decreases the image distance. Therefore, the smaller object distance of \(d_{o1} = 3.5 \text{ m}\) will yield the greater image distance \(d_{i1}\), and the greater object distance of \(d_{o2} = 50.0 \text{ m}\) will yield the smaller image distance \(d_{i2}\). Thus, the distance \(x\) that the lens must move is equal to the difference between the image distances:

\[
x = d_{i1} - d_{i2}
\]  
(1)

**SOLUTION**  Solving \[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  
(Equation 26.6) for \(d_i\) yields \(d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}}\). Using this result with the first and second object distances, we have that

\[
d_{i1} = \frac{1}{\frac{1}{f} - \frac{1}{d_{o1}}} \quad \text{and} \quad d_{i2} = \frac{1}{\frac{1}{f} - \frac{1}{d_{o2}}}
\]  
(2)

Substituting Equations (2) into Equation (1) yields
119. \textit{SSM REASONING AND SOLUTION}

a. A real image must be projected on the drum; therefore, the lens in the copier must be a \textbf{converging lens}.

b. If the document and its copy have the same size, but are inverted with respect to one another, the magnification equation (Equation 26.7) indicates that \(m = -d_i/d_o = -1\). Therefore, \(d_i/d_o = 1\) or \(d_i = d_o\). Then, the thin-lens equation (Equation 26.6) gives

\[
\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \quad \text{or} \quad d_o = d_i = 2f
\]

Therefore the document is located at a distance \(2f\) from the lens.

c. Furthermore, the image is located at a distance of \(2f\) from the lens.

120. \textit{REASONING} According to Equation 26.8, the refractive power is the reciprocal of the focal length in meters. To determine the focal length, we will utilize the value given for the angular magnification, because it is related to the focal length. The angular magnification \(M\) of the magnifying glass is \(M \approx \left(\frac{1}{f} - \frac{1}{d_i}\right)N\) (Equation 26.10), where \(f\) is the focal length of the magnifying glass, \(d_i\) is the image distance for the magnifying glass, and \(N\) is the distance between the eye (the magnifying glass is held next to the eye) and the near point.

\textit{SOLUTION}

a. The refractive power (in diopters) of the magnifying glass is the reciprocal of the focal length \(f\) in meters:

\[
\text{Refractive power in diopters} = \frac{1}{f \text{ (in meters)}} \quad (26.8)
\]

Considering the virtual image at the near point of the eye (so that \(d_i\) is negative, \(d_i = -N\)) and using Equation 26.10 for the angular magnification \((M = 6.0)\), we have

\[
M \approx \left(\frac{1}{f} - \frac{1}{d_i}\right)N = \left(\frac{1}{f} - \frac{1}{-N}\right)N = \frac{N}{f} + 1 \quad \text{or} \quad f = \frac{N}{M - 1} = \frac{25 \text{ cm}}{6.0 - 1} = 5.0 \text{ cm} = 0.050 \text{ m}
\]

Substituting this value for \(f\) into Equation 26.8, we find that
Refractive power in diopters = \( \frac{1}{f} \) (in meters) = \( \frac{1}{0.050 \text{ m}} \) = \( 2.0 \times 10^4 \) diopters

b. When the virtual image of the stamp is at \( d_i = -45 \text{ cm} \), Equation 26.10 indicates that the angular magnification is

\[
M \approx \left( \frac{1}{f} - \frac{1}{d_i} \right) \approx \left( \frac{1}{5.0 \text{ cm}} - \frac{1}{-45 \text{ cm}} \right) (25 \text{ cm}) = 5.6
\]

121. **REASONING** A contact lens is placed directly on the eye. Therefore, the object distance, which is the distance from the book to the lens, is 25.0 cm. The near point can be determined from the thin-lens equation [Equation 26.6: \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \)].

**SOLUTION**

a. Using the thin-lens equation, we have

\[
\frac{1}{d_i} = \frac{1}{f} \left( \frac{1}{d_o} - \frac{1}{d_i} \right) = \frac{1}{65.0 \text{ cm}} - \frac{1}{25.0 \text{ cm}} \quad \text{or} \quad d_i = -40.6 \text{ cm}
\]

In other words, at age 40, the man’s near point is 40.6 cm. Similarly, when the man is 45, we have

\[
\frac{1}{d_i} = \frac{1}{f} \left( \frac{1}{d_o} - \frac{1}{d_i} \right) = \frac{1}{65.0 \text{ cm}} - \frac{1}{29.0 \text{ cm}} \quad \text{or} \quad d_i = -52.4 \text{ cm}
\]

and his near point is 52.4 cm. Thus, the man’s near point has changed by 52.4 cm – 40.6 cm = 11.8 cm.

b. With \( d_o = 25.0 \text{ cm} \) and \( d_i = -52.4 \text{ cm} \), the focal length of the lens is found as follows:

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25.0 \text{ cm}} + \frac{1}{-52.4 \text{ cm}} \quad \text{or} \quad f = 47.8 \text{ cm}
\]

122. **REASONING** To find the distance through which the object must be moved, we must obtain the object distances for the two situations described in the problem. To do this, we combine the thin-lens equation and the magnification equation, since data for the magnification is given.

**SOLUTION**

a. Since the magnification is positive, the image is upright, and the object must be located within the focal point of the lens, as in Figure 26.28. When the magnification is negative and has a magnitude greater than one, the object must be located between the focal point and the point that is at a distance of twice the focal length from the lens, as in Figure 26.27. Therefore, the object should be moved away from the lens.
b. According to the thin-lens equation, we have

\[
\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \quad \text{or} \quad d_i = \frac{d_o f}{d_o - f}
\]  

(1)

According to the magnification equation, with \(d_i\) expressed as in Equation (2), we have

\[
m = -\frac{d_i}{d_o} = -\frac{1}{d_o} \left( \frac{d_o f}{d_o - f} \right) = \frac{f}{f - d_o} \quad \text{or} \quad d_o = \frac{f(m-1)}{m}
\]  

(2)

Applying Equation (2) to the two cases described in the problem, we have

\[
(d_o)_{+m} = \frac{f(m-1)}{m} = \frac{f(+4.0-1)}{+4.0} = \frac{3.0f}{4.0}
\]  

(3)

\[
(d_o)_{-m} = \frac{f(m-1)}{m} = \frac{f(-4.0-1)}{-4.0} = \frac{5.0f}{4.0}
\]  

(4)

Subtracting Equation (3) from Equation (4), we find that the object must be moved away from the lens by an additional distance of

\[
(d_o)_{-m} - (d_o)_{+m} = \frac{5.0f}{4.0} - \frac{3.0f}{4.0} = \frac{2.0f}{4.0} = \frac{0.30 \text{ m}}{2.0} = 0.15 \text{ m}
\]

123. **REASONING** The angular magnification of a refracting telescope is 32 800 times larger when you look through the correct end of the telescope than when you look through the wrong end. We wish to find the angular magnification, \( M = \frac{-f_o}{f_e} \) (see Equation 26.12) of the telescope. Thus, we proceed by finding the ratio of the focal lengths of the objective and the eyepiece and using Equation 26.12 to find \( M \).

**SOLUTION** When you look through the correct end of the telescope, the angular magnification of the telescope is \( M_c = -\frac{f_o}{f_e} \). If you look through the wrong end, the roles of the objective and eyepiece lenses are interchanged, so that the angular magnification would be \( M_w = -\frac{f_e}{f_o} \). Therefore,

\[
\frac{M_c}{M_w} = -\frac{f_o}{f_e} \left( \frac{f_o}{f_e} \right)^2 = 32 800 \quad \text{or} \quad \frac{f_o}{f_e} = \pm \sqrt{32 800} = \pm 181
\]

The angular magnification of the telescope is negative, so we choose the positive root and obtain \( M = -\frac{f_o}{f_e} = -(+181) = -181 \).

124. **REASONING** According to Equation 2.1, the average speed \( \bar{v} \) of a moving object is the distance \( D \) traveled, divided by the elapsed time \( \Delta t \):

\[
\bar{v} = \frac{D}{\Delta t}
\]  

(2.1)
In order to determine the average speed of the image of the horse over each interval of \( \Delta t = 2.0 \) seconds, we will need to find the image distance \( d_{i,0} \) at the beginning of each interval and the image distance \( d_{i,f} \) at the end of each interval. The horse is closer to the lens than the focal point \( F \) of the lens, so the image of the horse is virtual, and the image distances are negative. Therefore, the distance \( D \) traveled by the image of the horse will be equal to the magnitude of the difference between the final and initial image distances

\[
D = |d_{i,f} - d_{i,0}|
\]

(1)

The images distances are determined by the thin-lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6), where \( d_o \) is the object distance between the horse and the lens, and \( f \) is the focal length of the lens. The horse canters at a constant speed of \( v = 7.0 \) m/s, and is initially a distance \( d_{o,0} = 40.0 \) m from the lens. During each two-second interval, the object distance between the horse and the lens will decrease by \( (7.0 \text{ m/s})(2.0 \text{ s}) = 14.0 \text{ m} \).

**SOLUTION**

a. Solving \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Equation 26.6) for \( 1/d_i \) and taking the reciprocal of both sides yields

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{d_o f} \quad \text{or} \quad d_i = \frac{d_o f}{d_o - f}
\]

(2)

Applying Equation (2) to both the final and initial positions of the horse and its image during this interval, we obtain

\[
d_{i,f} = \frac{d_{o,f} f}{d_{o,f} - f} \quad \text{and} \quad d_{i,0} = \frac{d_{o,0} f}{d_{o,0} - f}
\]

(3)

Substituting Equations (3) into Equation (1), we find that

\[
D = \left| d_{i,f} - d_{i,0} \right| = \left| \frac{d_{o,f} f}{d_{o,f} - f} - \frac{d_{o,0} f}{d_{o,0} - f} \right| = \left| \frac{d_{o,f} - d_{o,0}}{d_{o,f} - f} - \frac{d_{o,0}}{d_{o,0} - f} \right|
\]

(4)

Substituting Equation (4) into Equation 2.1, we obtain

\[
\bar{v} = \frac{D}{\Delta t} = \frac{\frac{d_{o,f} - d_{o,0}}{d_{o,f} - f} - \frac{d_{o,0}}{d_{o,0} - f}}{\Delta t}
\]

(5)

During the first two seconds after the camera begins rolling, the horse is initially a distance \( d_{o,0} = 40.0 \) m from the lens, and ends up \( (7.0 \text{ m/s})(2.0 \text{ s}) = 14.0 \text{ m} \) closer to the lens. The final object distance, then, is \( d_{o,f} = 40.0 \text{ m} - 14.0 \text{ m} = 26.0 \text{ m} \). From Equation (5), the average speed of the image during this interval is
\[ f \left( \frac{d_{o,f} - d_{o,0}}{d_{o,f} - f} \right) = \frac{(50.0 \text{ m})}{\Delta t} \left( \frac{26.0 \text{ m}}{26.0 \text{ m} - 50.0 \text{ m}} - \frac{40.0 \text{ m}}{40.0 \text{ m} - 50.0 \text{ m}} \right) = 73 \text{ m/s} \]

b. During the next 2.0 s of filming, the horse’s initial position is \( d_{o,0} = 26.0 \text{ m} \) from the lens, and its final position is \( d_{o,f} = 26.0 \text{ m} - 14.0 \text{ m} = 12.0 \text{ m} \). Once again employing Equation (5), we find that

\[ f \left( \frac{d_{o,f} - d_{o,0}}{d_{o,f} - f} \right) = \frac{(50.0 \text{ m})}{\Delta t} \left( \frac{12.0 \text{ m}}{12.0 \text{ m} - 50.0 \text{ m}} - \frac{26.0 \text{ m}}{26.0 \text{ m} - 50.0 \text{ m}} \right) = 19 \text{ m/s} \]

125. **REASONING AND SOLUTION** We need to determine the focal lengths for Bill's glasses and for Anne's glasses. Using the thin-lens equation we have

**[Bill]**

\[ \frac{1}{f_B} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{23.0 \text{ cm}} + \frac{1}{-123 \text{ cm}} \quad \text{or} \quad f_B = 28.3 \text{ cm} \]

**[Anne]**

\[ \frac{1}{f_A} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{23.0 \text{ cm}} + \frac{1}{-73.0 \text{ cm}} \quad \text{or} \quad f_A = 33.6 \text{ cm} \]

Now find \( d'_o \) for Bill and Anne when they switch glasses.

a. Anne:

\[ \frac{1}{d'_o} = \frac{1}{f_B} - \frac{1}{d_i} = \frac{1}{28.3 \text{ cm}} - \frac{1}{-73 \text{ cm}} \quad \text{or} \quad d'_o = 20.4 \text{ cm} \]

Relative to the eyes, this becomes 20.4 cm + 2.00 cm = 22.4 cm.

b. Bill:

\[ \frac{1}{d'_o} = \frac{1}{f_A} - \frac{1}{d_i} = \frac{1}{33.6 \text{ cm}} - \frac{1}{-123 \text{ cm}} \quad \text{or} \quad d'_o = 26.4 \text{ cm} \]

Relative to the eyes, this becomes 26.4 cm + 2.00 cm = 28.4 cm.
1. (e) The first intensity minimum occurs when the difference in path lengths is \( \frac{1}{2} \lambda \), the second when the difference is \( \frac{3}{2} \lambda \), and the third when the difference is \( \frac{5}{2} \lambda \).

2. (a) The angle \( \theta \) that specifies the \( m \)th bright fringe is given by \( \sin \theta = m \frac{\lambda}{d} \) (Equation 27.1).

   If \( \sin \theta \approx \theta \), then \( \theta = m \frac{\lambda}{d} \). Thus, if \( \lambda \) and \( d \) are both doubled, the angle does not change.

3. (b) According to the discussion in Section 27.2, the difference in path lengths of the light waves increases by one wavelength as one moves from one bright fringe to the next one farther out.

4. \( d = 1.1 \times 10^{-5} \) m

5. (d) The wavelength \( \lambda_{\text{water}} \) of the light in water is related to the wavelength \( \lambda_{\text{vacuum}} \) in a vacuum by \( \lambda_{\text{water}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{water}}} \) (Equation 27.3). Since the index of refraction of water \( n_{\text{water}} \) is greater than one, the wavelength decreases when the apparatus is placed in water.

   The angle \( \theta \) that specifies the \( m \)th bright fringe is given by \( \sin \theta = m \frac{\lambda}{d} \) (Equation 27.1).

   When the wavelength decreases, the angle also decreases.

6. (d) The down-and-back distance traveled by the light wave in the film is 2400 nm. The wavelength of the light within the film is (see Equation 27.3) \( \lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} = \frac{600 \text{ nm}}{1.5} = 400 \text{ nm} \). Thus, the down-and-back distance is equivalent to

   \[
   (2400 \text{ nm}) \left( \frac{\lambda_{\text{film}}}{400 \text{ nm}} \right) = 6 \lambda_{\text{film}}.
   \]

7. (c) In drawings 1 and 2, light travels through a material with a smaller refractive index toward a material with a larger refractive index, so the reflection at the boundary occurs with a phase change that is equivalent to one-half a wavelength in the material. In drawings 3 and 4, light travels through a material with a larger refractive index toward a material with a smaller refractive index, so there is no phase change upon reflection at the boundary.
8. (e) In film 4, the reflection at the top surface occurs with a phase change that is equivalent to one-half a wavelength in the film, since light travels from a material with a smaller refractive index \((n_{\text{air}} = 1.0)\) toward a material with a larger refractive index \((n_{\text{film}} = 1.5)\). The reflection at the bottom surface occurs without a phase change, since light travels from a material with a larger refractive index \((n_{\text{film}} = 1.5)\) toward a material with a smaller refractive index \((n_{\text{air}} = 1.0)\). The down-and-back distance traveled by the light is one-half of a wavelength. Therefore, the net phase change is one wavelength, which leads to constructive interference.

9. (b) A bright fringe occurs at a point where the thickness of the air wedge has a certain value. As the thickness of the air wedge becomes smaller, the fringe moves to the right where the air wedge is thicker.

10. \(t = 2.6 \times 10^{-6} \text{ m}\)

11. (c) The amount of diffraction depends on the angles that locate the dark fringes on either side of the central bright fringe (see Section 27.5). These angles (one for each value of the integer \(m\)) are related to the wavelength \(\lambda\) of the light and the width \(W\) of the slit by \(\sin \theta = m(\lambda/W)\), (Equation 27.4). When \(\lambda\) is much less than \(W\), \(\theta \approx 0^\circ\), and there is very little diffraction.

12. (c) The width of the central bright fringe is determined by the angle \(\theta\) that locates the first \((m = 1)\) dark fringe on either side of the central bright fringe (see Section 27.5). This angle is given by \(\sin \theta = (1)\lambda/W\) (Equation 27.4), where \(\lambda\) is the wavelength and \(W\) is the width of the slit. If the width of the slit decreases, the angle increases.

13. (c) The angles that locates the dark fringes on either side of the central bright fringe are given by \(\sin \theta = m\lambda/W\), where \(\lambda\) is the wavelength, \(W\) is the width of the slit, and \(m = 1, 2, 3, \ldots\) (Equation 27.4). If the wavelength is doubled and the width of the slit is doubled, the angles remain the same. Hence, the diffraction patterns remain the same.

14. \(\theta = 5.44^\circ\)

15. (d) The best possible resolving power occurs when the angle subtended by two objects is a minimum (see Section 27.6) The minimum angle \(\theta_{\text{min}}\) between two objects that are just resolved is \(\theta_{\text{min}} \approx 1.22\lambda/D\) (Equation 27.6), where \(\lambda\) is the wavelength and \(D\) is the diameter of the lens through which the light passes. If the diameter of the lens is fixed, the minimum angle is proportional to the wavelength. Thus, blue light, having the smallest wavelength, gives the smallest angle and, hence, the best resolving power.

16. (c) The minimum angle \(\theta_{\text{min}}\) between two objects that are just resolved is \(\theta_{\text{min}} \approx 1.22\lambda/D\) (Equation 27.6), where \(\lambda\) is the wavelength and \(D\) is the diameter of the lens through which the light passes. When the light intensity decreases, the diameter of your pupil increases so
as to admit more light. Therefore, the minimum angle decreases, and your ability to resolve
the two dots, increases.

17. \( s_{\text{min}} = 24 \, \text{m} \)

18. (d) Because there is always a central bright maximum for each wavelength, there is an
orange (red plus yellow) fringe at \( \theta = 0^\circ \). According to Equation 27.7, \( \sin \theta = m\lambda / d \), the
angle for the \( m = 1 \) bright fringe of red light is greater than that for yellow light, since red
has a longer wavelength.

19. (b) As the number of lines per centimeter increases, the separation \( d \) between adjacent slits
becomes smaller. The angle \( \theta \) that specifies a principal maxima of a diffraction grating is
given by \( \sin \theta = m\lambda / d \), where \( \lambda \) is the wavelength and \( m = 0, 1, 2, 3, \ldots \) (Equation 27.7).
For fixed values of \( m \) and the wavelength, this relation shows that as \( d \) decreases, \( \theta \)
increases. For a fixed screen location, this means that the distance between two adjacent
principal maxima also increases.

20. \( \theta = 9.4^\circ \)
1. **REASONING** The angles \( \theta \) that determine the locations of the dark and bright fringes in a Young’s double-slit experiment are related to the integers \( m \) that identify the fringes, the wavelength \( \lambda \) of the light, and the separation \( d \) between the slits. Since values are given for \( m \), \( \lambda \), and \( d \), the angles can be calculated.

**SOLUTION** The expressions that specify \( \theta \) in terms of \( m \), \( \lambda \), and \( d \) are as follows:

- **Bright fringes**: \( \sin \theta = \frac{m \lambda}{d} \quad m = 0, 1, 2, 3, ... \quad (27.1) \)
- **Dark fringes**: \( \sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d} \quad m = 0, 1, 2, 3, ... \quad (27.2) \)

Applying these expressions gives the answers that we seek.

a. \( \sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[ \left( 0 + \frac{1}{2} \right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = 11^\circ \)

b. \( \sin \theta = \frac{m \lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[ \left( 1 \right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = 22^\circ \)

c. \( \sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[ \left( 1 + \frac{1}{2} \right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = 34^\circ \)

d. \( \sin \theta = \frac{m \lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left[ \left( 2 \right) \frac{520 \times 10^{-9} \text{ m}}{1.4 \times 10^{-6} \text{ m}} \right] = 48^\circ \)

2. **REASONING** In a Young’s double-slit experiment the angle \( \theta \) that locates a dark fringe on either side of the central bright fringe is specified by \( \sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d} \) (Equation 27.2), where \( \lambda \) is the wavelength of the light, \( d \) is the slit separation, and \( m = 0, 1, 2, 3, ... \) Note that \( m = 1 \) for the second dark fringe.
SOLUTION Using Equation 27.2 and the fact that $m = 1$ for the second dark fringe, we find
\[
\sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d} \quad \text{or} \quad \frac{d}{\lambda} = \frac{m + \frac{1}{2}}{\sin \theta} = \frac{1 + \frac{1}{2}}{\sin 5.4^\circ} = 16
\]

3. REASONING Let $\ell_1$ and $\ell_2$ be the distances from source 1 and source 2, respectively. For constructive interference the condition is $\ell_2 - \ell_1 = m\lambda$, where $\lambda$ is the wavelength and $m = 0, 1, 2, 3, \ldots$. For destructive interference the condition is $\ell_2 - \ell_1 = (m + \frac{1}{2})\lambda$, where $m = 0, 1, 2, 3, \ldots$. The fact that the wavelength is greater than the separation between the sources will allow us to decide which type of interference we have and to locate the two places in question.

SOLUTION a. Since the places we seek lie on the line between the two sources and since the separation between the sources is 4.00 m, we know that $\ell_2 - \ell_1$ cannot have a value greater than 4.00 m. Therefore, with a wavelength of 5.00 m, the only way constructive interference can occur is at a place where $m = 0$. But this would mean that $\ell_2 = \ell_1$, and there is only one place where this is true, namely, at the midpoint between the sources. But we know that there are two places. As a result, we conclude that destructive interference is occurring.

b. For destructive interference, the possible values for the integer $m$ are $m = 0, 1, 2, 3, \ldots$, and we must decide which to use. The condition for destructive interference is $\ell_2 - \ell_1 = (m + \frac{1}{2})\lambda$. Since $\lambda = 5.00$ m, all values of $m$ greater than or equal to one result in $\ell_2 - \ell_1$ being greater than 4.00 m, which we already know is not possible. Therefore, we conclude that $m = 0$, and the condition for destructive interference becomes $\ell_2 - \ell_1 = \frac{1}{2} \lambda$. We also know that $\ell_2 + \ell_1 = 4.00$ m, since the separation between the sources is 4.00 m. In other words, we have
\[
\ell_2 - \ell_1 = \frac{1}{2} \lambda = \frac{1}{2} (5.00 \text{ m}) = 2.50 \text{ m} \quad \text{and} \quad \ell_2 + \ell_1 = 4.00 \text{ m}
\]
Adding these two equations gives
\[2\ell_2 = 6.50 \text{ m} \quad \text{or} \quad \ell_2 = 3.25 \text{ m}\]
Since $\ell_2 + \ell_1 = 4.00$ m, it follows that
\[\ell_1 = 4.00 \text{ m} - \ell_2 = 4.00 \text{ m} - 3.25 \text{ m} = 0.75 \text{ m}\]
These results indicate that the two places are 3.25 m and 0.75 m from one of the sources.
4. **REASONING** In a Young’s double-slit experiment a dark fringe is located at an angle $\theta$ that is determined according to

$$\sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \ldots$$

(27.2)

where $\lambda$ is the wavelength of the light and $d$ is the separation of the slits. Here, neither $\lambda$ nor $d$ is known. However, we know the angle for the dark fringe for which $m = 0$. Using this angle and $m = 0$ in Equation 27.2 will allow us to determine the ratio $\lambda/d$, which can then be used to find the angle that locates the dark fringe for $m = 1$.

**SOLUTION** Applying Equation 27.2 to the dark fringes for $m = 0$ and $m = 1$, we have

$$\sin \theta_0 = \left(0 + \frac{1}{2}\right) \frac{\lambda}{d} \quad \text{and} \quad \sin \theta_1 = \left(1 + \frac{1}{2}\right) \frac{\lambda}{d}$$

Dividing the expression for $\sin \theta_1$ by the expression for $\sin \theta_0$ gives

$$\frac{\sin \theta_1}{\sin \theta_0} = \frac{\left(1 + \frac{1}{2}\right) \frac{\lambda}{d}}{\left(0 + \frac{1}{2}\right) \frac{\lambda}{d}} = 3.0$$

The angle that locates the dark fringe for $m = 1$ can now be found:

$$\sin \theta_1 = 3.0 \sin \theta_0 \quad \text{or} \quad \theta_1 = \sin^{-1}(3.0 \sin \theta_0) = \sin^{-1}(3.0 \sin 15^\circ) = 51^\circ$$

5. **REASONING** According to Equation 27.2, the wavelength $\lambda$ of the light is related to the angle $\theta$ between the central bright fringe and the seventh dark fringe according to

$$\lambda = d \sin \theta \quad m + \frac{1}{2}$$

where $d$ is the separation between the slits, and $m = 0, 1, 2, 3, \ldots$ The first dark fringe occurs when $m = 0$, so the seventh dark fringe occurs when $m = 6$. The distance $d$ is given, and we can determine the angle $\theta$ by using the inverse tangent function, $\theta = \tan^{-1}\left(\frac{y}{L}\right)$, since both $y$ and $L$ are known (see the drawing).

**SOLUTION** We will first compute the angle between the central bright fringe and the seventh dark fringe using the geometry shown in the drawing:

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.025 \text{ m}}{1.1 \text{ m}}\right) = 1.3^\circ$$

The wavelength of the light is
\[ \lambda = \frac{d \sin \theta}{m + \frac{1}{2}} = \left(1.4 \times 10^{-4} \, \text{m}\right) \sin 1.3^\circ = 4.9 \times 10^{-7} \, \text{m} \]

6. **REASONING** For a double-slit experiment like this one, the angle \( \theta \) that locates a bright fringe on either side of the central bright fringe is specified by \( \sin \theta = m \frac{\lambda}{d} \) (Equation 27.1), where \( \lambda \) is the wavelength of the light, \( d \) is the slit separation, and \( m = 0, 1, 2, 3, \ldots \) Note that \( m = 3 \) for the third-order bright fringe. In contrast, the angle \( \theta \) that locates a dark fringe on either side of the central bright fringe is specified by \( \sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \) (Equation 27.2), where \( m = 0, 1, 2, 3, \ldots \) Note that \( m = 3 \) for the fourth dark fringe. We will use Equation 27.1 with the wavelength \( \lambda_A = 645 \, \text{nm} \) and Equation 27.2 with the wavelength \( \lambda_B \).

**SOLUTION** Using Equation 27.1 (with \( m = 3 \)) for the bright fringe and Equation 27.2 (with \( m = 3 \)) for the dark fringe, we obtain

\[
\sin \theta_A = 3 \frac{\lambda_A}{d} \quad \text{(27.1)} \quad \text{and} \quad \sin \theta_B = \left(3 + \frac{1}{2}\right) \frac{\lambda_B}{d} \quad \text{(27.2)}
\]

Since the bright fringe and the dark fringe are produced at the same place on the viewing screen, we know that \( \theta_A = \theta_B \). Therefore, it follows that

\[
\sin \theta_A = \sin \theta_B \quad \text{or} \quad 3 \frac{\lambda_A}{d} = \left(3 + \frac{1}{2}\right) \frac{\lambda_B}{d}
\]

Solving this result for \( \lambda_B \) shows that

\[
\lambda_B = \left(3 + \frac{1}{2}\right) \frac{\lambda_A}{3} = \left(3 + \frac{1}{2}\right) \left(645 \, \text{nm}\right) = 553 \, \text{nm}
\]

7. **REASONING** The drawing shows the double slit and the screen. The variable \( y \) denotes one half of the screen width. Each bright fringe is located by an angle \( \theta \) that is determined by \( \sin \theta = m \lambda / d \) (Equation 27.1), where \( \lambda \) is the wavelength, \( d \) is the separation between the slits, and \( m = 0, 1, 2, 3, \ldots \). The maximum number \( m \) of bright fringes that can fall on the screen (on one side of the central bright fringe) occurs when \( \tan \theta = y / L \).

**SOLUTION** For version A of the setup, the drawing indicates that
\[
\tan \theta = \frac{y}{L} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.10 \text{ m}}{0.35 \text{ m}}\right) = 16^\circ
\]

But Equation 27.1 indicates that \(\sin \theta = m \lambda / d\). Solving this expression for \(m\) and using \(\theta = 16^\circ\), we find

\[
m = \frac{d \sin \theta}{\lambda} = \frac{(1.4 \times 10^{-5} \text{ m}) \sin 16^\circ}{625 \times 10^{-9} \text{ m}} = 6.2
\]

Thus, for version A, there are \(\boxed{6}\) bright fringes on the screen (on one side of the central bright fringe), not counting the central bright fringe (which corresponds to \(m = 0\)). For version B, we find

\[
\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.10 \text{ m}}{0.50 \text{ m}}\right) = 11^\circ
\]

\[
m = \frac{d \sin \theta}{\lambda} = \frac{(1.4 \times 10^{-5} \text{ m}) \sin 11^\circ}{625 \times 10^{-9} \text{ m}} = 4.3
\]

For this version, there are \(\boxed{4}\) bright fringes on the screen (on one side of the central bright fringe), again not counting the central bright fringe.

8. **REASONING** The drawing shows a top view of the double slit and the screen, as well as the position of the central bright fringe \((m = 0)\) and the \(m^{th}\) bright fringe. The angle \(\theta\) to the \(m^{th}\) bright fringe is given by Equation 27.1 as \(\sin \theta = m \lambda / d\), where \(\lambda\) is the wavelength of the light and \(d\) is the separation between the slits. The largest that the angle can be is \(\theta = 90.0^\circ\). This condition will tell us how many bright fringes can be formed on either side of the central bright fringe.

**SOLUTION** Substituting \(\theta = 90.0^\circ\) into Equation 27.1 and solving for \(m\) gives

\[
m = \frac{d \sin 90.0^\circ}{\lambda} = \frac{(3.76 \times 10^{-6} \text{ m}) \sin 90.0^\circ}{625 \times 10^{-9} \text{ m}} = 6.02
\]

Therefore, the greatest number of bright fringes is \(\boxed{6}\).
9. **REASONING** The drawing shows a top view of the double slit and the screen, as well as the position of the central bright fringe \((m = 0)\) and the second-order bright fringe \((m = 2)\). The vertical distance \(y\) in the drawing can be obtained from the tangent function as 
\[ y = L \tan \theta. \]
According to Equation 27.1, the angle \(\theta\) is related to the wavelength \(\lambda\) of the light and the separation \(d\) between the slits by 
\[ \sin \theta = \frac{m\lambda}{d}, \]
where \(m = 0, 1, 2, 3, \ldots\) If the angle \(\theta\) is small, then \(\tan \theta \approx \sin \theta\), so that
\[ y = L \tan \theta \approx L \sin \theta = L \left( \frac{m\lambda}{d} \right) \]  
(1)

We will use this relation to find the value of \(y\) when \(\lambda = 585\) nm.

**SOLUTION** When \(\lambda = 425\) nm, we know that \(y = 0.0180\) m, so
\[ 0.0180\ m = L \left[ \frac{m(425\ nm)}{d} \right] \]  
(2)

Dividing Equation (1) by Equation (2) and algebraically eliminating the common factors of \(L, m,\) and \(d\), we find that
\[ \frac{y}{0.0180\ m} = \left( \frac{L}{d} \right) \left( \frac{m\lambda}{d} \right) = \frac{\lambda}{425\ nm} \]

When \(\lambda = 585\) nm, the distance to the second-order bright fringe is
\[ y = (0.0180\ m) \left( \frac{585\ nm}{425\ nm} \right) = 0.0248\ m \]

10. **REASONING AND SOLUTION** The first-order orange fringes occur farther out from the center than do the first-order blue fringes. Therefore, the screen must be moved toward the slits so that the orange fringes will appear on the screen. The distance between the screen and the slits is \(L\), and the amount by which the screen must be moved toward the slits is \(L_{\text{blue}} - L_{\text{orange}}\). We know that \(L_{\text{blue}} = 0.500\ m\), and must, therefore, determine \(L_{\text{orange}}\). We begin with Equation 27.1, as it applies to first-order fringes, that is, \(\sin \theta = \lambda/d\). Furthermore, as discussed in Example 1 in the text, \(\tan \theta = y/L\), where \(y\) is the distance from the center of the screen to a fringe. Since the angle \(\theta\) locating the fringes is small, \(\tan \theta \approx \sin \theta\), and we have that
\[ \frac{\lambda}{d} \approx \frac{y}{L} \quad \text{or} \quad L = \frac{yd}{\lambda} \]
Writing this result for \(L\) for both colors and dividing the two equations gives
\[
\frac{L_{\text{blue}}}{L_{\text{orange}}} = \frac{yd / \lambda_{\text{blue}}}{yd / \lambda_{\text{orange}}} = \frac{\lambda_{\text{orange}}}{\lambda_{\text{blue}}}
\]
where we have recognized that \( y \) is one-half the screen width (the same for both colors), and that the slit separation \( d \) is the same for both colors. Using the result above, we find that

\[
L_{\text{blue}} - L_{\text{orange}} = L_{\text{blue}} - \left( \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}} \right) L_{\text{blue}} = L_{\text{blue}} \left( 1 - \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}} \right)
\]

\[
= (0.500 \text{ m}) \left[ 1 - \frac{471 \text{ nm}}{611 \text{ nm}} \right] = 0.115 \text{ m}
\]

11. **REASONING** The light that travels through the plastic has a different path length than the light that passes through the unobstructed slit. Since the center of the screen now appears dark, rather than bright, destructive interference, rather than constructive interference occurs there. This means that the difference between the number of wavelengths in the plastic sheet and that in a comparable thickness of air is \( \frac{1}{2} \).

**SOLUTION** The wavelength of the light in the plastic sheet is given by Equation 27.3 as

\[
\lambda_{\text{plastic}} = \frac{\lambda_{\text{vacuum}}}{n} = \frac{586 \times 10^{-9} \text{ m}}{1.60} = 366 \times 10^{-9} \text{ m}
\]

The number of wavelengths contained in a plastic sheet of thickness \( t \) is

\[
N_{\text{plastic}} = \frac{t}{\lambda_{\text{plastic}}} = \frac{t}{366 \times 10^{-9} \text{ m}}
\]

The number of wavelengths contained in an equal thickness of air is

\[
N_{\text{air}} = \frac{t}{\lambda_{\text{air}}} = \frac{t}{586 \times 10^{-9} \text{ m}}
\]

where we have used the fact that \( \lambda_{\text{air}} \approx \lambda_{\text{vacuum}} \). Destructive interference occurs when the difference, \( N_{\text{plastic}} - N_{\text{air}} \), in the number of wavelengths is \( \frac{1}{2} \):

\[
N_{\text{plastic}} - N_{\text{air}} = \frac{1}{2}
\]

\[
\frac{t}{366 \times 10^{-9} \text{ m}} - \frac{t}{586 \times 10^{-9} \text{ m}} = \frac{1}{2}
\]

Solving this equation for \( t \) yields \( t = 487 \times 10^{-9} \text{ m} = 487 \text{ nm} \).
12. **REASONING** Since the plastic layer looks dark, we need to obtain the condition for destructive interference in terms of the film thickness $t$ and the wavelength $\lambda_{\text{plastic}}$ of the light in the plastic layer. The light travels in air ($n_{\text{air}} = 1.00$) toward the plastic ($n_{\text{plastic}} = 1.61$), where the first reflection occurs. This light travels from a smaller toward a larger index of refraction and, therefore, experiences a phase change upon reflection that is equivalent to $1/2 \lambda_{\text{plastic}}$. The light that penetrates the plastic layer travels toward the glass ($n_{\text{glass}} = 1.52$), where the second reflection occurs. This light travels from a larger toward a smaller index of refraction and, therefore, experiences no phase change upon reflection. Therefore, the net phase change due to reflection is $1/2 \lambda_{\text{plastic}}$. This half-wavelength must be combined with the extra travel distance of $2t$ for the light in the plastic layer and set equal to an odd-integer number of half-wavelengths in the plastic, which is the condition for destructive interference. We also must remember that the wavelength of the light in the plastic is $\lambda_{\text{plastic}} = \lambda_{\text{vacuum}} / n_{\text{plastic}}$ (Equation 27.3).

**SOLUTION** Based on the discussion in the **REASONING**, we have the following equation as the condition for destructive interference

$$2t + \frac{1}{2} \lambda_{\text{plastic}} = \frac{1}{2} \lambda_{\text{plastic}} + \frac{3}{2} \lambda_{\text{plastic}} + \frac{5}{2} \lambda_{\text{plastic}} + \ldots$$

Subtracting the term $1/2 \lambda_{\text{plastic}}$ from both sides of this equation gives

$$2t = 0, \frac{2}{2} \lambda_{\text{plastic}}, \frac{4}{2} \lambda_{\text{plastic}}, \ldots = m\lambda_{\text{plastic}} \quad m = 0, 1, 2, 3, \ldots \quad (1)$$

Solving Equation (1) for the thickness $t$ and substituting $\lambda_{\text{plastic}} = \lambda_{\text{vacuum}} / n_{\text{plastic}}$ (Equation 27.3) into the result, we obtain

$$t = \frac{m\lambda_{\text{plastic}}}{2} = \frac{m\lambda_{\text{vacuum}}}{2n_{\text{plastic}}} \quad (2)$$

Since we seek the two smallest possible nonzero values for the thickness of the plastic layer, we need to apply Equation (2) with the values of $m = 1$ and $m = 2$:

$$t = \frac{m\lambda_{\text{vacuum}}}{2n_{\text{plastic}}} = \frac{(1)(589 \text{ nm})}{2(1.61)} = 183 \text{ nm} \quad \text{and} \quad t = \frac{m\lambda_{\text{vacuum}}}{2n_{\text{plastic}}} = \frac{(2)(589 \text{ nm})}{2(1.61)} = 366 \text{ nm}$$

13. **SSM REASONING** To solve this problem, we must express the condition for destructive interference in terms of the film thickness $t$ and the wavelength $\lambda_{\text{film}}$ of the light as it passes through the magnesium fluoride coating. We must also take into account any phase changes that occur upon reflection.
**SOLUTION** Since the coating is intended to be nonreflective, its thickness must be chosen so that destructive interference occurs between waves 1 and 2 in the drawing. For destructive interference, the combined phase difference between the two waves must be an odd integer number of half wavelengths. The phase change for wave 1 is equivalent to one-half of a wavelength, since this light travels from a smaller refractive index \( n_{\text{air}} = 1.00 \) toward a larger refractive index \( n_{\text{film}} = 1.38 \).

Similarly, there is a phase change when wave 2 reflects from the right surface of the film, since this light also travels from a smaller refractive index \( n_{\text{film}} = 1.38 \) toward a larger one \( n_{\text{lens}} = 1.52 \). Therefore, a phase change of one-half wavelength occurs at both boundaries, so the net phase change between waves 1 and 2 due to reflection is zero. Since wave 2 travels back and forth through the film and, since the light is assumed to be at nearly normal incidence, the extra distance traveled by wave 2 compared to wave 1 is twice the film thickness, or \( 2t \). Thus, in this case, the minimum condition for destructive interference is

\[
2t = \frac{1}{2} \lambda_{\text{film}}
\]

The wavelength of light in the coating is

\[
\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n} = \frac{565 \text{ nm}}{1.38} = 409 \text{ nm}
\]

Solving the above expression for \( t \), we find that the minimum thickness that the coating can have is

\[
t = \frac{1}{4} \lambda_{\text{film}} = \frac{1}{4} (409 \text{ nm}) = \boxed{102 \text{ nm}}
\]

14. **REASONING** The vacuum wavelength \( \lambda_{\text{vacuum}} \) of the light is related to its wavelength \( \lambda_{\text{film}} \) in the soap film by

\[
\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}
\]

(Equation 27.3), where \( n_{\text{film}} \) is the index of refraction of the soap film. Consider when monochromatic light is incident on the soap film from above. Part of this light, ray 1, reflects from the upper surface, traveling from a smaller to a larger refractive index \( (n_{\text{air}} = 1.00 \text{ to } n_{\text{film}} = 1.33) \), incurring a phase change of one-half of a wavelength. Some of the incident light, ray 2, passes into the soap film and reflects from its lower surface, traveling from a larger to a smaller refractive index \( (n_{\text{film}} = 1.33 \text{ to } n_{\text{air}} = 1.00) \). This reflection incurs no phase change. Ray 2 travels back through the film, enters the air, and combines with ray 1. The second ray travels twice through the thickness \( t \)
of the film, a total distance of $2t$ farther than the first ray. Therefore, the total phase change between the two rays is equal to the sum of $2t$ and $\frac{1}{2}\lambda_{\text{film}}$. The two rays undergo destructive interference, so that the total phase change between them is $2t + \frac{1}{2}\lambda_{\text{film}}$ and is equal to an odd number of half-wavelengths:

$$2t + \frac{1}{2}\lambda_{\text{film}} = \frac{1}{2}\lambda_{\text{film}} + \frac{3}{2}\lambda_{\text{film}} + \frac{5}{2}\lambda_{\text{film}} + \cdots$$  \hspace{1cm} (1)

We will use Equation 27.3 and Equation (1), together with the condition that the second smallest nonzero thickness for which destructive interference occurs is 296 nm, and determine the vacuum wavelength $\lambda_{\text{vacuum}}$ of the light reflecting from the film.

**SOLUTION** Subtracting $\frac{1}{2}\lambda_{\text{film}}$ from both sides of Equation (1), we find that the condition for destructive interference becomes

$$2t = 0, \frac{1}{2}\lambda_{\text{film}} + \frac{3}{2}\lambda_{\text{film}} + \cdots$$  \hspace{1cm} (2)

All of the values on the right side of Equation (2) give destructive interference. The first value gives a thickness of $t = 0$ m. The second value is the smallest nonzero thickness for destructive interference, so the next value, $\frac{3}{2}\lambda_{\text{film}}$ corresponds to the second smallest nonzero thickness:

$$2t = \frac{3}{2}\lambda_{\text{film}}$$  \hspace{1cm} (3)

Substituting $\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}$ (Equation 27.3) into Equation (3) and solving for $\lambda_{\text{vacuum}}$ yields

$$2t = \frac{3}{2}\lambda_{\text{film}} = \frac{3\lambda_{\text{vacuum}}}{2n_{\text{film}}}$$

or

$$\lambda_{\text{vacuum}} = \frac{4tn_{\text{film}}}{3} = \frac{4(296 \text{ nm})(1.33)}{3} = 525 \text{ nm}$$

15. **REASONING** When the light strikes the film from above, the wave reflected from the top surface of the film undergoes a phase shift that is equivalent to one-half of a wavelength in the film, since the light travels from a smaller refractive index ($n_{\text{air}} = 1.00$) toward a larger refractive index ($n_{\text{film}} = 1.43$). When the light reflects from the bottom surface of the film the wave undergoes another phase shift that is equivalent to one-half of a wavelength in the film, since this light also travels from a smaller refractive index ($n_{\text{film}} = 1.43$) toward a larger refractive index ($n_{\text{glass}} = 1.52$). Thus, the net phase change due to reflection from the two surfaces is equivalent to one wavelength in the film. This wavelength must be combined with the extra distance $2t$ traveled by the wave reflected from the bottom surface, where $t$ is the film thickness. Thus, the condition for destructive interference is

$$\underbrace{2t}_{\text{Extra distance traveled by wave in the film}} + \underbrace{\frac{1}{2}\lambda_{\text{film}} + \frac{1}{2}\lambda_{\text{film}}}_{\text{One wavelength net phase change due to reflection}} = \underbrace{\frac{1}{2}\lambda_{\text{film}} + \frac{3}{2}\lambda_{\text{film}} + \frac{5}{2}\lambda_{\text{film}} + \cdots}_{\text{Condition for destructive interference}}$$  \hspace{1cm} (1)
We will use this relation to find the longest possible wavelength of light that will yield destructive interference.

**SOLUTION** Note that the left-hand side of Equation (1) is greater than \( \lambda_{\text{film}} \). Thus, the right-hand side of this equation must also be greater than \( \lambda_{\text{film}} \). The smallest value that is greater than \( \lambda_{\text{film}} \) is the term \( \frac{3}{2} \lambda_{\text{film}} \). Therefore, we have that

\[
2t + \frac{1}{2} \lambda_{\text{film}} + \frac{1}{2} \lambda_{\text{film}} = \frac{3}{2} \lambda_{\text{film}} \quad \text{or} \quad \lambda_{\text{film}} = 4t = 4\left(1.07 \times 10^{-7} \text{ m}\right) = 4.28 \times 10^{-7} \text{ m}
\]

Since \( \lambda_{\text{vacuum}} = n_{\text{film}} \lambda_{\text{film}} \) (Equation 27.3), the wavelength in vacuum is

\[
\lambda_{\text{vacuum}} = n_{\text{film}} \lambda_{\text{film}} = (1.43)\left(4.28 \times 10^{-7} \text{ m}\right) = 6.12 \times 10^{-7} \text{ m}
\]

Other terms on the right-hand side of Equation (1) that are greater than \( \frac{3}{2} \lambda_{\text{film}} \) lead to smaller values of \( \lambda_{\text{vacuum}} \).

---

16. **REASONING** The wavelength in the film is \( \lambda_{\text{film}} = \lambda_{\text{vacuum}} / n_{\text{film}} \) (Equation 27.3), so that the refractive index of the film is

\[
n_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{film}}}
\]

The wavelength in a vacuum is given, and we can determine the wavelength in the film by considering the constructive interference that occurs. The drawing shows the thin film and two rays of light shining on it. At nearly perpendicular incidence, ray 2 travels a distance of \( 2t \) farther than ray 1, where \( t \) is the thickness of the film. In addition, ray 2 experiences a phase shift of \( \frac{1}{2} \lambda_{\text{film}} \) upon reflection at the bottom film surface, while ray 1 experiences the same phase shift at the upper film surface. This is because, in both cases, the light is traveling through a region where the refractive index is lower toward a region where it is higher. Therefore, there is no net phase change for the two reflected rays, and only the extra travel distance determines the type of interference that occurs. For constructive interference the extra travel distance must be an integer number of wavelengths in the film:

\[
\frac{2t}{\text{Extra distance traveled by ray 2}} + \frac{0}{\text{Zero net phase change due to reflection}} = \frac{\lambda_{\text{film}}, 2\lambda_{\text{film}}, 3\lambda_{\text{film}}...}{\text{Condition for constructive interference}}
\]

This result is equivalent to

\[
2t = m\lambda_{\text{film}} \quad m = 1, 2, 3, ...
\]
The wavelength in the film, then, is
\[ \lambda_{\text{film}} = \frac{2t}{m} \quad m = 1, 2, 3, \ldots \]  

(2)

**SOLUTION** Substituting Equation (2) into Equation (1) gives
\[ n_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{film}}} = \frac{\lambda_{\text{vacuum}}}{2t} \]

Since the given value for \( t \) is the minimum thickness for which constructive interference can occur, we know that \( m = 1 \). Thus, we find that
\[ n_{\text{film}} = \frac{m\lambda_{\text{vacuum}}}{2t} = \frac{(1)(625 \text{ nm})}{2(242 \text{ nm})} = 1.29 \]

17. **REASONING** In air the index of refraction is nearly \( n = 1 \), while in the film it is greater than one. A phase change occurs whenever light travels through a material with a smaller refractive index toward a material with a larger refractive index and reflects from the boundary between the two. The phase change is equivalent to \( \frac{1}{2} \lambda_{\text{film}} \), where \( \lambda_{\text{film}} \) is the wavelength in the film. This phase change occurs at the top surface of the film, where the light first strikes it. However, no phase change occurs when light that has penetrated the film reflects back upward from the bottom surface. This is because a phase change does not occur when light travels through a material with a larger refractive index toward a material with a smaller refractive index and reflects from the boundary between the two. Thus, to evaluate destructive interference correctly, we must consider a net phase change of \( \frac{1}{2} \lambda_{\text{film}} \) due to reflection as well as the extra distance traveled by the light within the film.

**SOLUTION** The drawing shows the soap film and the two rays of light that represent the interfering light waves. At nearly perpendicular incidence, ray 2 travels a distance of \( 2t \) further than ray 1, where \( t \) is the thickness of the film. In addition, the net phase change for the two rays is \( \frac{1}{2} \lambda_{\text{film}} \), as discussed in the reasoning section. We must combine this amount with the extra travel distance to determine the condition for destructive interference. For destructive interference, the combined total must be an odd-integer number of half-wavelengths in the film:

\[
\begin{align*}
2t + \frac{1}{2} \lambda_{\text{film}} &= \frac{1}{2} \lambda_{\text{film}} + \frac{3}{2} \lambda_{\text{film}} + \frac{5}{2} \lambda_{\text{film}} + \ldots \\
&= \frac{1}{2} \lambda_{\text{film}} \left( \lambda_{\text{film}} \right) + \frac{3}{2} \lambda_{\text{film}} \left( \lambda_{\text{film}} \right) + \frac{5}{2} \lambda_{\text{film}} \left( \lambda_{\text{film}} \right) + \ldots \\
\end{align*}
\]

Subtracting the term \( \frac{1}{2} \lambda_{\text{film}} \) from the left side of this equation and from each term on the right side, we obtain
\[ 2t = 0, \lambda_{\text{film}}, 2\lambda_{\text{film}}, \ldots \]
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The minimum nonzero thickness is \( t = \frac{\lambda_{\text{film}}}{2} \). But the wavelength in the film is related to the vacuum-wavelength according to Equation 27.3: \( \lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n} \). Thus, the minimum nonzero thickness is \( t = \frac{\lambda_{\text{vacuum}}}{(2n)} \). Applying this result to both regions of the film allows us to obtain the desired ratio:

\[
\frac{t_{\text{magenta}}}{t_{\text{yellow}}} = \frac{\lambda_{\text{vacuum, green}}}{(2n)} = \frac{\lambda_{\text{vacuum, blue}}}{(2n)} = \frac{555 \text{ nm}}{469 \text{ nm}} = 1.18
\]

18. **REASONING** When the light strikes the film of oil from above, the wave reflected from the top surface of the film undergoes a phase shift that is equivalent to one-half of a wavelength, since the light travels from the smaller refractive index of air toward the larger refractive index of oil. On the other hand, there is no phase shift when the light reflects from the bottom surface of the film, since the light travels from the larger refractive index of oil toward the smaller refractive index of water. Thus, the net phase change due to reflection from the two surfaces is equivalent to one-half of a wavelength in the film. This half-wavelength must be combined with the extra distance \( 2t \) traveled by the wave reflected from the bottom surface, where \( t \) is the film thickness. Thus, the condition for destructive interference is

\[
\frac{2t}{\text{Extra distance traveled by wave in the film}} + \frac{1}{2} \lambda_{\text{film}} = \frac{1}{2} \lambda_{\text{film}} + \frac{3}{2} \lambda_{\text{film}} + \frac{5}{2} \lambda_{\text{film}} + \cdots = \text{Condition for destructive interference}
\]

Note that the left-hand side of this equation is greater than \( \frac{1}{2} \lambda_{\text{film}} \). Thus, the right-hand side must also be greater than \( \frac{1}{2} \lambda_{\text{film}} \). The smallest value that is greater than \( \frac{1}{2} \lambda_{\text{film}} \) is the term \( \frac{3}{2} \lambda_{\text{film}} \). Therefore, we have that \( 2t + \frac{1}{2} \lambda_{\text{film}} = \frac{3}{2} \lambda_{\text{film}} \), or \( 2t = \lambda_{\text{film}} \). Since \( \lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} = (640.0 \text{ nm})/n_{\text{film}} \) (see Equation 27.3), the condition for destructive interference becomes

\[
2t = \frac{\lambda_{\text{film}}}{n_{\text{film}}} = \frac{640.0 \text{ nm}}{n_{\text{film}}}
\]

The condition for constructive interference is

\[
\frac{2t}{\text{Extra distance traveled by wave in the film}} + \frac{1}{2} \lambda'_{\text{film}} = m \lambda'_{\text{film}} = \frac{m \lambda'_{\text{film}}}{n_{\text{film}}} \quad m = 1, 2, 3, \ldots
\]

where \( \lambda'_{\text{film}} \) is the wavelength that produces constructive interference in the film. Solving this relation for \( 2t \) gives

\[
2t = \left(m - \frac{1}{2}\right) \lambda'_{\text{film}} = \left(m - \frac{1}{2}\right) \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}
\]
Setting Equations (1) and (2) equal to each other and solving for $\lambda'_{\text{vacuum}}$ yields

$$\frac{640.0 \text{ nm}}{n_{\text{film}}} = \left(m - \frac{1}{2}\right) \frac{\lambda'_{\text{vacuum}}}{n_{\text{film}}} \quad \text{or} \quad \lambda'_{\text{vacuum}} = \frac{640.0 \text{ nm}}{m - \frac{1}{2}}$$

**SOLUTION** For $m = 1$, $\lambda'_{\text{vacuum}} = 1280$ nm; for $m = 2$, $\lambda'_{\text{vacuum}} = 427$ nm; for $m = 3$, $\lambda'_{\text{vacuum}} = 256$ nm. Values of $m$ greater than 3 lead to values of $\lambda'_{\text{vacuum}}$ that are smaller than 256 nm. Thus, the only wavelength in the visible spectrum (380 to 750 nm) that will give constructive interference is $427$ nm.

19. **SSM REASONING** To solve this problem, we must express the condition for constructive interference in terms of the film thickness $t$ and the wavelength $\lambda_{\text{film}}$ of the light in the soap film. We must also take into account any phase changes that occur upon reflection.

**SOLUTION** For the reflection at the top film surface, the light travels from air, where the refractive index is smaller ($n = 1.00$), toward the film, where the refractive index is larger ($n = 1.33$). Associated with this reflection there is a phase change that is equivalent to one-half of a wavelength. For the reflection at the bottom film surface, the light travels from the film, where the refractive index is larger ($n = 1.33$), toward air, where the refractive index is smaller ($n = 1.00$). Associated with this reflection, there is no phase change. As a result of these two reflections, there is a net phase change that is equivalent to one-half of a wavelength. To obtain the condition for constructive interference, this net phase change must be added to the phase change that arises because of the film thickness $t$, which is traversed twice by the light that penetrates it. For constructive interference we find that

$$2t + \frac{1}{2} \lambda_{\text{film}} = \lambda_{\text{film}} , 2\lambda_{\text{film}}, 3\lambda_{\text{film}}, \ldots$$

or

$$2t = (m + \frac{1}{2}) \lambda_{\text{film}}, \quad \text{where} \quad m = 0, 1, 2, \ldots$$

Equation 27.3 indicates that $\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n}$. Using this expression and the fact that $m = 0$ for the minimum thickness $t$, we find that the condition for constructive interference becomes

$$2t = \left(m + \frac{1}{2}\right) \lambda_{\text{film}} = \left(0 + \frac{1}{2}\right) \left(\frac{\lambda_{\text{vacuum}}}{n}\right)$$

or

$$t = \frac{\lambda_{\text{vacuum}}}{4n} = \frac{611 \text{ nm}}{4(1.33)} = 115 \text{ nm}$$
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20. **REASONING** The number \( m \) of bright fringes in an air wedge is discussed in Example 5, where it is shown that \( m \) is related to the thickness \( t \) of the film and the wavelength \( \lambda_{\text{film}} \) of the light within the film by (see Figure 27.12)

\[
\frac{2t}{\text{Extra distance traveled by wave 2}} + \frac{1}{2} \frac{\lambda_{\text{film}}}{\text{Half-wavelength shift due to reflection of wave 1}} = m\lambda_{\text{film}} \quad \text{where} \quad m = 1, 2, 3, \ldots
\]

For a given thickness, we can solve this equation for the number of bright fringes.

**SOLUTION** Since the light is traveling in a film of air, \( \lambda_{\text{film}} = \lambda_{\text{vacuum}} = 550 \text{ nm} \). The number \( m \) of bright fringes is

\[
m = \frac{2t}{\lambda_{\text{vacuum}}} + \frac{1}{2} = \frac{2(1.37 \times 10^{-5} \text{ m})}{550 \times 10^{-9} \text{ m}} + \frac{1}{2} = 50.3
\]

Thus, there are 50 bright fringes.

21. **REASONING AND SOLUTION** The condition for destructive interference is \( 2t = m\lambda \). The thickness \( t \) of the "air wedge" at the 100th dark fringe is related to the radius \( R \) of the fringe and the radius of curvature \( r \) of the curved glass in the following way.

If \( s \) is the length of the circular arc along the curved glass from the center to the fringe, then the angle subtended by this arc is \( \theta = s/r \), as shown in Drawing A. Since \( r \) is large, the arc is almost straight and is the hypotenuse of a right triangle in the air wedge of sides \( t \) and \( R \), and angle \( (1/2)\theta \) opposite \( t \), as shown in Drawing B. In drawing B, the angle opposite \( t \) is labeled \( \alpha \). It can be seen that \( \alpha = (1/2)\theta \), because \( \alpha + \beta = 90^\circ \) and \( (1/2)\theta + \beta = 90^\circ \).

\[
(1/2)\theta \approx \tan (1/2)\theta = t/R \quad \text{so} \quad R = 2tr/s
\]
Now if \((1/2)\theta\) is very small, then \(s \approx R\), so

\[
R^2 = 2tr = (100)\lambda r = (100)(654 \times 10^{-9} \text{ m})(10.0 \text{ m})
\]

so

\[
R = 0.0256 \text{ m}
\]

22. **REASONING** We need to obtain the conditions for constructive and destructive interference in terms of the film thickness \(t\) and the wavelength \(\lambda_{\text{water}}\) of the light in the layer of water. The condition for constructive interference is that obtained in Example 12 in the text:

\[
2t = m\lambda_{\text{water}} = m\frac{\lambda_{\text{vacuum}}}{n_{\text{water}}} \quad m = 1, 2, 3, ...
\]

(1)

The condition for destructive interference is similar to Equation (1), except that instead of \(2t = m\lambda_{\text{water}}\), we have

\[
2t = \left(m + \frac{1}{2}\right)\lambda_{\text{water}}:
\]

\[
2t = \left(m + \frac{1}{2}\right)\lambda_{\text{water}} = \left(m + \frac{1}{2}\right)\frac{\lambda_{\text{vacuum}}}{n_{\text{water}}} \quad m = 0, 1, 2, 3, ...
\]

(2)

**SOLUTION**

a. Solving Equation (1) for the thickness \(t\) of the water layer, we have

For \(\lambda_{\text{vacuum}} = 432 \text{ nm}\)

\[
t = \frac{m}{2} \left(\frac{432 \text{ nm}}{1.33}\right)
\]

For \(\lambda_{\text{vacuum}} = 648 \text{ nm}\)

\[
t = \frac{m'}{2} \left(\frac{648 \text{ nm}}{1.33}\right)
\]

Equating the two expressions for \(t\), we find that \(m/m' = 1.50\). For minimum thickness, this means that \(m = 3\) and \(m' = 2\). Then

\[
t = \frac{3}{2} \left(\frac{432 \text{ nm}}{1.33}\right) = \frac{3}{2} \left(\frac{432 \text{ nm}}{1.33}\right) = 487 \text{ nm}
\]

b. Solving Equation (2) for \(\lambda_{\text{vacuum}}\), we obtain

\[
\lambda_{\text{vacuum}} = \frac{2n_{\text{water}}t}{m + \frac{1}{2}} = \frac{2\left(1.33\right)(487 \text{ nm})}{m + \frac{1}{2}}
\]

The wavelength lying within the specified range is for \(m = 2\), so that

\[
\lambda_{\text{vacuum}} = 518 \text{ nm}
\]
23. **REASONING** For both sound waves and light waves, the angle \( \theta \) that locates the first minimum or dark fringe in the diffraction pattern can be calculated from the wavelength \( \lambda \) of the waves and the width \( W \) of the doorway through which the waves pass. The width of the doorway for the sound is given, and we can determine the wavelength since it is related to the given speed and frequency of the sound. Once the angle is obtained for the sound waves, we will be able to use the relationship between \( \theta, \lambda, \) and \( W \) a second time to obtain the width of the doorway for the light, since the wavelength of the light is given.

**SOLUTION**
a. The angle \( \theta \) that locates the first minimum or dark fringe in the diffraction pattern can be obtained from Equation 27.4 (with \( m = 1 \)):

\[
\sin \theta = \frac{\lambda}{W} \quad \text{or} \quad \theta = \sin^{-1} \left( \frac{\lambda}{W} \right)
\]

According to Equation 16.1, the wavelength of the sound wave is

\[
\lambda = \frac{v}{f}
\]

where \( v \) and \( f \) are, respectively, the speed and frequency of the sound. Substituting this value for the wavelength into the expression for \( \theta \), we find that

\[
\theta = \sin^{-1} \left( \frac{\lambda}{W} \right) = \sin^{-1} \left( \frac{v}{fW} \right) = \sin^{-1} \left[ \frac{343 \text{ m/s}}{\left(2.0 \times 10^4 \text{ Hz}\right)(0.91 \text{ m})} \right] = 1.1^\circ
\]

b. For the light waves, Equation 27.4 (with \( m = 1 \)) reveals that

\[
\sin \theta = \frac{\lambda}{W} \quad \text{or} \quad W = \frac{\lambda}{\sin \theta} = \frac{580 \times 10^{-9} \text{ m}}{\sin 1.1^\circ} = 3.0 \times 10^{-5} \text{ m}
\]

Thus, in order for the diffraction of the light waves to match that of the sound waves, the light waves must pass through a very narrow “doorway.” In passing through a doorway that is 0.91 m wide, the light waves would be diffracted by an amount so small that it would be unobservable.

24. **REASONING** The angle \( \theta \) at which a dark fringe is located in the diffraction pattern of a single slit is specified by \( \sin \theta = m \frac{\lambda}{W} \) (Equation 27.4), where \( m = 1, 2, 3, \ldots \), \( \lambda \) is the wavelength of the light, and \( W \) is the width of the slit. We will apply Equation 27.4 to slit A and slit B.

**SOLUTION** Solving Equation 27.4 for the slit width \( W \), we have

\[
\sin \theta = m \frac{\lambda}{W} \quad \text{or} \quad W = \frac{m \lambda}{\sin \theta}
\]

(1)
Applying Equation (1) to each slit and forming the ratio \( \frac{W_A}{W_B} \), we obtain
\[
\frac{W_A}{W_B} = \frac{m \lambda / \sin \theta_A}{m \lambda / \sin \theta_B} = \frac{\sin \theta_B}{\sin \theta_A} = \frac{\sin 56^\circ}{\sin 34^\circ} = 1.5
\]

Note that the integer \( m \) and the wavelength \( \lambda \) are the same for each slit and are eliminated algebraically from this calculation.

25. **REASONING** This problem can be solved by using Equation 27.4 for the value of the angle \( \theta \) when \( m = 1 \) (first dark fringe).

**SOLUTION**

a. When the slit width is \( W = 1.8 \times 10^{-4} \) m and \( \lambda = 675 \) nm = \( 675 \times 10^{-9} \) m, we find, according to Equation 27.4,
\[
\theta = \sin^{-1} \left( \frac{m \lambda}{W} \right) = \sin^{-1} \left( \frac{1 \times 675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-4} \text{ m}} \right) = 0.21^\circ
\]

b. Similarly, when the slit width is \( W = 1.8 \times 10^{-6} \) m and \( \lambda = 675 \times 10^{-9} \) m, we find
\[
\theta = \sin^{-1} \left( \frac{1 \times 675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-6} \text{ m}} \right) = 22^\circ
\]

26. **REASONING** Since the width of the central bright fringe on the screen is defined by the locations of the dark fringes on either side, we consider Equation 27.4, which specifies the angle \( \theta \) that determines the location of a dark fringe. This equation is \( \sin \theta = \frac{m \lambda}{W} \), where \( \lambda \) is the wavelength, \( W \) is the slit width, and \( m = 1, 2, 3, \ldots \).

The fact that the width of the central bright fringe does not change means that the positions of the dark fringes to either side also do not change. In other words, the angle \( \theta \) remains constant. According to Equation 27.4, the condition that must be satisfied for this to happen is that the ratio \( \lambda / W \) of the wavelength to the slit width remains constant.

**SOLUTION** Since the width of the central bright fringe on the screen remains constant, the angular position \( \theta \) of the dark fringes must also remain constant. Thus, according to Equation 27.4, we have
\[
\sin \theta = \frac{m \lambda_1}{W_1} \quad \text{and} \quad \sin \theta = \frac{m \lambda_2}{W_2}
\]

Since \( \theta \) is the same for each case, it follows that
\[
\frac{m \lambda_1}{W_1} = \frac{m \lambda_2}{W_2}
\]
The term \( m \) can be eliminated algebraically from this result, so that solving for \( W_2 \) gives

\[
W_2 = \frac{W_1 \lambda_2}{\lambda_1} = \frac{(2.3 \times 10^{-6} \text{ m})(740 \text{ nm})}{510 \text{ nm}} = 3.3 \times 10^{-6} \text{ m}
\]

27. **SSM REASONING** The drawing shows a top view of the slit and screen, as well as the position of the central bright fringe and the third dark fringe. The distance \( y \) can be obtained from the tangent function as \( y = L \tan \theta \). Since \( L \) is given, we need to find the angle \( \theta \) before \( y \) can be determined. According to Equation 27.4, the angle \( \theta \) is related to the wavelength \( \lambda \) of the light and the width \( W \) of the slit by \( \sin \theta = m\lambda / W \), where \( m = 3 \) since we are interested in the angle for the third dark fringe.

**SOLUTION** We will first compute the angle between the central bright fringe and the third dark fringe using Equation 27.4 (with \( m = 3 \)):

\[
\theta = \sin^{-1} \left( \frac{m\lambda}{W} \right) = \sin^{-1} \left[ \frac{3(668 \times 10^{-9} \text{ m})}{6.73 \times 10^{-6} \text{ m}} \right] = 17.3^\circ
\]

The vertical distance is

\[
y = L \tan \theta = (1.85 \text{ m}) \tan 17.3^\circ = 0.576 \text{ m}
\]

28. **REASONING** The wavelength \( \lambda \) of the light and the width \( W \) of the single slit together determine the angle \( \theta \) of the first dark fringe, via \( \sin \theta = m \frac{\lambda}{W} \) (Equation 27.4), with \( m = 1 \):

\[
\sin \theta = \frac{\lambda}{W} \tag{1}
\]

Both the angle \( \theta \) and the position \( y \) of the dark fringe on the screen are measured from the middle of the central bright fringe. Therefore, the angle \( \theta \) may be found from \( \theta = \tan^{-1} \left( \frac{y}{L} \right) \) (Equation 1.6), where \( L \) is the distance between the slit and the screen.

**SOLUTION** Solving Equation (1) for \( \lambda \), we obtain

\[
\lambda = W \sin \theta \tag{2}
\]

Substituting \( \theta = \tan^{-1} \left( \frac{y}{L} \right) \) (Equation 1.6) into Equation (2) yields

\[
\lambda = W \sin \left[ \tan^{-1} \left( \frac{y}{L} \right) \right] = (5.6 \times 10^{-4} \text{ m}) \sin \left[ \tan^{-1} \left( \frac{3.5 \times 10^{-3} \text{ m}}{4.0 \text{ m}} \right) \right] = 4.9 \times 10^{-7} \text{ m}
\]
29. **REASONING** The distance \( y \) between the center of the diffraction pattern and a particular dark fringe depends upon the distance \( L \) between the slit and the screen and the angle \( \theta \) at which the dark fringe appears

\[
y = L \tan \theta
\]

For light with a wavelength \( \lambda \) passing through a slit of width \( W \), the angles of the dark fringes are given by \( \sin \theta = m \frac{\lambda}{W} \) (Equation 27.4), where \( m \) is an integer \((m = 1, 2, 3, \ldots)\).

The screen will only be completely dark when both diffraction patterns have a dark fringe at the same angle \( \theta \). We will use Equation 27.4 to determine the integers \((m_1\) and \(m_2)\) for which the two wavelengths \((\lambda_1 = 632 \text{ nm} \text{ and } \lambda_2 = 474 \text{ nm})\) have dark fringes at the same angle. Then, Equation (1) will determine the distance \( y \).

**SOLUTION** For an angle \( \theta \) at which both diffraction patterns have dark fringes, Equation 27.4 gives

\[
\sin \theta = m_1 \frac{\lambda_1}{W} \quad \text{and} \quad \sin \theta = m_2 \frac{\lambda_2}{W}
\]

Setting the right sides of Equations (2) equal, we see that

\[
m_1 \frac{\lambda_1}{W} = m_2 \frac{\lambda_2}{W} \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{474 \text{ nm}}{632 \text{ nm}} = 0.75
\]

The first dark fringes of the two diffraction patterns do not coincide, because setting \( m_1 = m_2 = 1 \) yields a ratio of \( m_1/m_2 = 1/1 = 1 \), which does not satisfy Equation (3). But we can see that other dark fringes do coincide, because Equation (3) is satisfied when \( m_1 = 3 \) and \( m_2 = 4 \) \((m_1/m_2 = 3/4 = 0.75)\), or when \( m_1 = 6 \) and \( m_2 = 8 \) \((m_1/m_2 = 6/8 = 0.75)\), and so forth. The first time the dark fringes overlap occurs when \( m_1 = 3 \) and \( m_2 = 4 \). Solving the first of Equations (2) for \( \theta \), and taking \( m_1 = 3 \) yields

\[
\theta = \sin^{-1} \left( m_1 \frac{\lambda_1}{W} \right) = \sin^{-1} \left[ 3 \left( \frac{632 \times 10^{-9} \text{ m}}{7.15 \times 10^{-5} \text{ m}} \right) \right] = 1.52^\circ
\]

Then, from Equation (1), we have that

\[
y = L \tan \theta = (1.20 \text{ m}) \tan 1.52^\circ = 3.18 \times 10^{-2} \text{ m} = 3.18 \text{ cm}
\]
30. **REASONING** The angle $\theta$ at which a dark fringe is located in the diffraction pattern of a single slit is specified by $\sin \theta = m \frac{\lambda}{W}$ (Equation 27.4), where $m = 1, 2, 3, \ldots$, $\lambda$ is the wavelength of the light, and $W$ is the width of the slit. The drawing at the right shows the angle $\theta$ and the positions of the slit and the screen. The width of the central bright fringe is determined by the location first dark fringe ($m=1$) on either side of midpoint of the central bright fringe. As the drawing shows, the distance between the midpoint of the central bright fringe and the first dark fringe is $y$, so the width of the central fringe is $2y$.

**SOLUTION** Using Equation 27.4 for the first-order dark fringes ($m = 1$) and referring to the drawing, we see that

$$\sin \theta = (1) \frac{\lambda}{W} = \frac{y}{\sqrt{L^2 + y^2}}$$

Since the distance $L$ between the slit and the screen equals the width $2y$ of the central bright fringe, this equation becomes

$$\frac{\lambda}{W} = \frac{y}{\sqrt{(2y)^2 + y^2}} = \frac{1}{\sqrt{5}} = 0.447$$

31. **SSM REASONING** The angle $\theta$ that specifies the location of the $m^{th}$ dark fringe is given by $\sin \theta = m\lambda / W$ (Equation 27.4), where $\lambda$ is the wavelength of the light and $W$ is the width of the slit. When $\theta$ has its maximum value of $90.0^\circ$, the number of dark fringes that can be produced is a maximum. We will use this information to obtain a value for this number.

**SOLUTION** Solving Equation 27.4 for $m$, and setting $\theta = 90.0^\circ$, we have

$$m = \frac{W \sin 90.0^\circ}{\lambda} = \frac{(5.47 \times 10^{-6} \text{ m}) \sin 90.0^\circ}{651 \times 10^{-9} \text{ m}} = 8.40$$

Therefore, the number of dark fringes is 8.
32. **REASONING AND SOLUTION** It is given that $2y = 450W$ and $L = 18000W$. We know $\frac{\lambda}{W} = \sin \theta$. Now $\sin \theta \approx \tan \theta = \frac{y}{L}$, so

\[
\frac{\lambda}{W} = \frac{y}{L} = \frac{225W}{18000W} = 0.013
\]

33. **REASONING** The drawing shows the two stars, separated by a distance $s$, that are a distance $r$ from the earth. The angle $\theta$ (in radians) is defined as the arc length divided by the radius $r$ (see Equation 8.1). Assuming that $r \gg s$, the arc length is very nearly equal to the distance $s$, so we can write $\theta = s/r$. Since $s$ is known, this relation will allow us to find the distance $r$ if we can determine the angle. If the stars can just be seen as separate objects, we know that the minimum angle $\theta_{\text{min}}$ between them is given by Equation 27.6 as $\theta_{\text{min}} \approx 1.22 \frac{\lambda}{D}$, where $\lambda$ is the wavelength of the light being observed and $D$ is the diameter of the telescope’s objective. By combining these two relations, we will be able to find the distance $r$.

**SOLUTION** Solving the relation $\theta = s/r$ for $r$, and substituting in the relation $\theta = \theta_{\text{min}} \approx 1.22 \frac{\lambda}{D}$, we obtain

\[
r = \frac{s}{\theta} = \frac{s}{1.22 \frac{\lambda}{D}} = \frac{3.7 \times 10^{11} \text{m}}{1.22 \left( \frac{550 \times 10^{-9} \text{m}}{1.02 \text{m}} \right)} = 5.6 \times 10^{17} \text{m}
\]

34. **REASONING** The drawing at the right shows the batter’s eye and the two points on opposite sides of the ball. The distance between the two points is the diameter $y$ of the ball. The distance between home plate and the pitcher’s mound is approximately $L$. The angle $\theta_{\text{min}}$ between the batter’s eye and the two points is the minimum angle required for them to be resolved by the eye. This angle is $\theta_{\text{min}} \approx 1.22 \frac{\lambda}{D}$ (Equation 27.6), where $\theta_{\text{min}}$ is in radians, $\lambda$ is the wavelength of the light, and $D$ is the diameter of the pupil of the eye.

**SOLUTION**

a. The minimum angular separation (in radians) of the points is given

\[
\theta_{\text{min}} \approx \frac{1.22 \lambda}{D} = \frac{1.22 \left( 550 \times 10^{-9} \text{m} \right)}{2.0 \times 10^{-3} \text{m}} = 3.4 \times 10^{-4} \text{ rad} \quad (27.6)
\]

Using radian measure (see Section 8.1) and referring to the drawing, we can also express $\theta_{\text{min}}$ in radians as follow and thereby obtain a value for the distance $L$: 
\[
\theta_{\text{min}} = \frac{y}{L} \quad \text{or} \quad L = \frac{y}{\theta_{\text{min}}} \approx \frac{0.0738 \text{ m}}{3.4 \times 10^{-4} \text{ rad}} = 220 \text{ m}
\]

b. Since the distance \( L = 220 \text{ m} \) at which the two points can be resolved is greater than the 18.4 m distance between the pitcher’s mound and home plate, we cannot rule out the claim. Therefore, the answer is [no].

35. **SSM REASONING** According to Rayleigh's criterion, the two taillights must be separated by a distance \( s \) sufficient to subtend an angle \( \theta_{\text{min}} \approx 1.22 \frac{\lambda}{D} \) at the pupil of the observer's eye. Recalling that this angle must be expressed in radians, we relate \( \theta_{\text{min}} \) to the distances \( s \) and \( L \).

\[
\theta_{\text{min}} \approx \frac{1.22 \lambda}{D} = 1.22 \left( \frac{660 \times 10^{-9} \text{ m}}{7.0 \times 10^{-3} \text{ m}} \right) = 1.2 \times 10^{-4} \text{ rad}
\]

According to Equation 8.1, the distance \( L \) between the observer and the taillights is

\[
L = \frac{s}{\theta_{\text{min}}} = \frac{1.2 \text{ m}}{1.2 \times 10^{-4} \text{ rad}} = 1.0 \times 10^4 \text{ m}
\]

36. **REASONING** The maximum allowable dot separation should be chosen so that neither the red, the green, nor the blue dots can be resolved separately. The Rayleigh criterion is \( \theta_{\text{min}} \approx 1.22 \frac{\lambda}{D} \) (Equation 27.6), where \( \lambda \) is the wavelength of the light and \( D \) is the diameter of the pupil in this case. In the Rayleigh criterion the angle \( \theta_{\text{min}} \) is expressed in radians. According to Equation 8.1, the angle in radians is \( \theta_{\text{min}} \approx \frac{s}{L} \), where \( s \) is the dot separation and \( L \) is the distance from the dots to the eye (the viewing distance). The maximum allowable dot separation should be chosen so that neither the red, the green, nor the blue dots can be resolved separately. This means that the distance should be determined by the blue dots. A dot separation that is slightly smaller than \( s_{\text{blue}} \) will automatically be smaller than \( s_{\text{red}} \) and \( s_{\text{green}} \). This is because the \( \theta_{\text{min}} \) values for red and green light are larger than for blue light. In other words, if the blue dots can’t be resolved separately, then neither can the red dots nor the green dots.
\textbf{SOLUTION} Using the Rayleigh criterion and Equation 8.1, we have

\[
\frac{s}{L} \approx 1.22 \frac{\lambda}{D} \quad \text{or} \quad s \approx 1.22 \frac{\lambda L}{D}
\]

The maximum allowable dot separation, then, is

\[
s_{\text{blue}} \approx \frac{1.22 \lambda L}{D} = \frac{1.22 (470 \times 10^{-9} \text{ m})(0.40 \text{ m})}{2.0 \times 10^{-3} \text{ m}} = 1.1 \times 10^{-4} \text{ m}
\]

37. \textbf{REASONING} For the hunter’s eye to resolve the squirrels as separate objects, the Rayleigh criterion must be satisfied. The Rayleigh criterion relates the angle $\theta_{\text{min}}$ (the angle subtended at the eye by the objects) to the wavelength $\lambda$ of the light and the diameter $D$ of the pupil. Thus, we can calculate the diameter if values are available for the wavelength and the angle. The wavelength is given. To obtain the angle, we will use the concept of the radian and express the angle approximately as an arc length (approximately the separation distance $s$ between the squirrels) divided by a radius (the distance $L$ between the hunter and the squirrels), as discussed in Section 8.1.

\textbf{SOLUTION}

a. The Rayleigh criterion specifies the angle $\theta_{\text{min}}$ in radians as

\[
\theta_{\text{min}} \approx 1.22 \frac{\lambda}{D}
\]  

(27.6)

Solving for $D$ gives

\[
D \approx \frac{1.22 \lambda}{\theta_{\text{min}}}
\]  

(1)

The drawing at the right shows the hunter’s eye and the squirrels. According to the discussion in Section 8.1, the angle $\theta_{\text{min}}$ can be expressed approximately in radians as

\[
\theta_{\text{min}} \approx \frac{s}{L}
\]  

(8.1)

This expression for the angle is approximate because we are assuming that the distance $s$ between the squirrels is nearly the same as the arc length on a circle of radius $L$ centered on the hunter’s eye. Substituting Equation 8.1 into Equation (1), we find that
\[ D \approx \frac{1.22 \lambda}{\theta_{\text{min}}} \approx \frac{1.22 \lambda}{s/L} = \frac{1.22 \lambda L}{s} \]

\[ D \approx \frac{1.22 \left(498 \times 10^{-9} \text{ m}\right) \left(1.6 \times 10^3 \text{ m}\right)}{0.10 \text{ m}} = 9.7 \times 10^{-3} \text{ m} \text{ or } 9.7 \text{ mm} \]

b. A diameter of 9.7 mm is beyond the normal range for the human eye. Moreover, the human eye would increase the diameter of its pupil to a maximum only under dark conditions. Hunting in the dark is difficult. **The hunter’s claim is not reasonable.**

38. **REASONING AND SOLUTION**
   a. Equation 27.6 \( \left( \theta_{\text{min}} = \frac{1.22 \lambda}{D} \right) \) gives the minimum angle \( \theta_{\text{min}} \) that two point objects can subtend at an aperture of diameter \( D \) and still be resolved. The angle must be measured in radians. For a dot separation \( s \) and a distance \( L \) between the painting and the eye, Equation 8.1 gives the angle in radians as \( \theta_{\text{min}} = \frac{s}{L} \). Therefore, we find that

\[ \frac{1.22 \lambda}{D} = \frac{s}{L} \quad \text{or} \quad L = \frac{sD}{1.22 \lambda} \]

With \( \lambda = 550 \text{ nm} \) and a pupil diameter of \( D = 2.5 \text{ mm} \), the distance \( L \) is

\[ L = \frac{sD}{1.22 \lambda} = \frac{\left(1.5 \times 10^{-3} \text{ m}\right) \left(2.5 \times 10^{-3} \text{ m}\right)}{\left(1.22\right) \left(550 \times 10^{-9} \text{ m}\right)} = 5.6 \text{ m} \]

b. The calculation here is similar to that in part a, except that \( n = 1.00 \) and \( D = 25 \text{ mm} \) for the camera. Therefore, the distance \( L \) for the camera is

\[ L = \frac{sD}{1.22 \lambda} = \frac{\left(1.5 \times 10^{-3} \text{ m}\right) \left(25 \times 10^{-3} \text{ m}\right)}{\left(1.22\right) \left(550 \times 10^{-9} \text{ m}\right)} = 56 \text{ m} \]

39. **REASONING** At the aperture the star and its planet must subtend an angle at least as large as that given by the Rayleigh criterion, which is \( \theta_{\text{min}} \approx 1.22 \lambda/D \), where \( \lambda \) is the wavelength of the light and \( D \) is the diameter of the aperture. The angle \( \theta_{\text{min}} \) is given by this criterion in radians. We can obtain the angle subtended at the telescope aperture by using the separation \( s \) between the planet and the star and the distance \( L \) of the star from the earth. According to Equation 8.1, the angle in radians is \( \theta_{\text{min}} = s/L \).

**SOLUTION** Using the Rayleigh criterion and Equation 8.1, we have

\[ \frac{s}{L} \approx 1.22 \frac{\lambda}{D} \quad \text{or} \quad D \approx \frac{1.22 \lambda L}{s} = \frac{1.22 \left(550 \times 10^{-9} \text{ m}\right) \left(4.2 \times 10^{17} \text{ m}\right)}{1.2 \times 10^{11} \text{ m}} = 2.3 \text{ m} \]
40. **REASONING** Initially, the field mice are too close together to be resolved as separate objects. But as the eagle gets closer, the angle \( \theta \) between the field mice increases. When it reaches the value \( \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \) (Equation 27.6), where \( \lambda \) is the wavelength of the light that the eagle’s eye detects and \( D \) is the diameter of the eagle’s pupil, the eagle will be able to resolve them. We can also express the angle \( \theta_{\text{min}} \) in terms of the distance \( s \) between the field mice and the distance \( r \) between the eagle and the mice via \( \theta_{\text{min}} = \frac{s}{r} \) (Equation 8.1). Let \( r_0 \) denote the distance between the eagle and the mice at the instant when the eagle begins to dive. Then the time \( t \) it takes the eagle to travel a distance \( r_0 - r \) toward the mice at a constant speed \( v \) is given by Equation 2.1:

\[
t = \frac{r_0 - r}{v}
\]

(2.1)

**SOLUTION** Setting the right sides of \( \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \) (Equation 27.6) and \( \theta_{\text{min}} = \frac{s}{r} \) (Equation 8.1) equal and solving for \( r \) yields

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = \frac{s}{r} \quad \text{or} \quad 1.22 \lambda = sD \quad \text{or} \quad r = \frac{sD}{1.22 \lambda}
\]

(1)

Substituting Equation (1) into Equation 2.1, we obtain

\[
t = \frac{r_0 - r}{v} = \frac{r_0 - \frac{sD}{1.22 \lambda}}{v} = \frac{176 \text{ m} - (0.010 \text{ m})(6.0 \times 10^{-3} \text{ m})}{1.22 \left(550 \times 10^{-9} \text{ m}\right)} = 5.1 \text{ s}
\]

41. **SSM REASONING** Assuming that the angle \( \theta_{\text{min}} \) is small, the distance \( y \) between the blood cells is given by

\[
y = f \theta_{\text{min}}
\]

(8.1)

where \( f \) is the distance between the microscope objective and the cells (which is given as the focal length of the objective). However, the minimum angular separation \( \theta_{\text{min}} \) of the cells is given by the Rayleigh criterion as \( \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \) (Equation 27.6), where \( \lambda \) is the wavelength of the light and \( D \) is the diameter of the objective. These two relations can be used to find an expression for \( y \) in terms of \( \lambda \).
**SOLUTION**

a. Substituting Equation 27.6 into Equation 8.1 yields

\[ y = f \theta_{\text{min}} = f \left( \frac{1.22 \lambda}{D} \right) \]

Since it is given that \( f = D \), we see that \( y = 1.22 \lambda \).

b. Because \( y \) is proportional to \( \lambda \), the wavelength must be shorter to resolve cells that are closer together.

---

**42. REASONING AND SOLUTION**

a. Equation 27.6 \( (\theta_{\text{min}} = 1.22 \lambda / D) \) gives the minimum angle \( \theta_{\text{min}} \) that two point objects can subtend at an aperture of diameter \( D \) and still be resolved. The angle must be measured in radians. For a separation \( s \) between the two circles and a distance \( L \) between the concentric arrangement and the camera, Equation 8.1 gives the angle in radians as \( \theta_{\text{min}} = s / L \). Therefore, we find that

\[ \frac{1.22 \lambda}{D} = \frac{s}{L} \quad \text{or} \quad L = \frac{sD}{1.22 \lambda} \]

Since \( s = 0.040 \text{ m} – 0.010 \text{ m} = 0.030 \text{ m} \), we calculate that

\[ L = \frac{sD}{1.22 \lambda} = \frac{(0.030 \text{ m})(12.5 \times 10^{-3} \text{ m})}{(1.22)(555 \times 10^{-9} \text{ m})} = 550 \text{ m} \]

b. The calculation here is similar to that in part a, except that the separation \( s \) is between one side of a diameter of the small circle and the other side, or \( s = 0.020 \text{ m} \):

\[ L = \frac{sD}{1.22 \lambda} = \frac{(0.020 \text{ m})(12.5 \times 10^{-3} \text{ m})}{(1.22)(555 \times 10^{-9} \text{ m})} = 370 \text{ m} \]

---

**43. SSM REASONING** The angle that specifies the third-order maximum of a diffraction grating is \( \sin \theta = m \lambda / d \) (Equation 27.7), where \( m = 3 \), \( \lambda \) is the wavelength of the light, and \( d \) is the separation between the slits of the grating. The separation is equal to the width of the grating \( (1.50 \text{ cm}) \) divided by the number of lines \( (2400) \).

**SOLUTION** Solving Equation 27.7 for the wavelength, we obtain

\[ \lambda = \frac{d \sin \theta}{m} = \frac{\left(1.50 \times 10^{-2} \text{ m} \right)}{2400 \times 3} \sin 18.0^\circ = 6.44 \times 10^{-7} \text{ m} = 644 \text{ nm} \]
44. **REASONING** The number of lines per centimeter that a grating has is the reciprocal of the spacing between the slits of the grating. We can determine the slit spacing by considering the angle $\theta$ that defines the position of a principal bright fringe. This angle is related to the order $m$ of the fringe, the wavelength $\lambda$ of the light, and the spacing $d$ between the slits. Thus, we can use the values given for $\theta$, $m$, and $\lambda$ to determine $d$.

**SOLUTION** The number of lines per centimeter that the grating has is $N$ and is the reciprocal of the spacing $d$ between the slits:

$$N = \frac{1}{d} \quad (1)$$

where $d$ must be expressed in centimeters. The relationship that determines the angle defining the position of a principal bright fringe is

$$\sin \theta = m \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \ldots \quad (27.7)$$

Solving this equation for $d$ and applying the result for a second-order fringe ($m = 2$) gives

$$d = \frac{m \lambda}{\sin \theta} = \frac{2 \left(495 \times 10^{-9} \text{ m}\right)}{\sin 9.34^\circ} = 6.10 \times 10^{-6} \text{ m} \quad \text{or} \quad 6.10 \times 10^{-4} \text{ cm}$$

Substituting this result into Equation (1), we find that

$$N = \frac{1}{d} = \frac{1}{6.10 \times 10^{-4} \text{ cm}} = \boxed{1640 \text{ lines/cm}}$$

45. **REASONING** For a diffraction grating the angle $\theta$ that locates a bright fringe can be found using Equation 27.7, $\sin \theta = m \lambda / d$, where $\lambda$ is the wavelength, $d$ is the separation between the grating slits, and the order $m$ is $m = 0, 1, 2, 3, \ldots$. By applying this relation to both cases, we will be able to determine the unknown wavelength, because the order and the slit separation are the same for both.

**SOLUTION** We will use $\lambda_1$ and $\lambda_2$ to denote the known and unknown wavelengths, respectively. The corresponding angles are $\theta_1$ and $\theta_2$. Applying Equation 27.7 to the two cases, we have

$$\sin \theta_1 = \frac{m \lambda_1}{d} \quad \text{and} \quad \sin \theta_2 = \frac{m \lambda_2}{d}$$

Dividing the expression for case 2 by the expression for case 1 gives

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{m \lambda_2 / d}{m \lambda_1 / d} = \frac{\lambda_2}{\lambda_1}$$

Solving for $\lambda_2$, we find

$$\lambda_2 = \lambda_1 \frac{\sin \theta_2}{\sin \theta_1} = (420 \text{ nm}) \frac{\sin 41^\circ}{\sin 26^\circ} = \boxed{630 \text{ nm}}$$
46. **REASONING** The drawing shows the angle $\theta$ that locates a principal maximum on the screen, along with the separation $L$ between the grating and the screen and the distance $y$ from the midpoint of the screen. It follows from the drawing that $y = L \tan \theta$, which becomes $y = L \sin \theta$, since we are dealing with small angles ($\tan \theta \approx \sin \theta$).

For a diffraction grating, the angle $\theta$ that locates a principal maximum can be found using $\sin \theta = m\lambda / d$ (Equation 27.7), where $\lambda$ is the wavelength, $d$ is the separation between the grating slits, and the order $m$ is $m = 0, 1, 2, 3, \ldots$.

We will use the above relations to obtain an expression for the separation between adjacent principal maxima.

**SOLUTION** The distance $y$ is $y = L \sin \theta$. But $\sin \theta = m\lambda / d$, according to Equation 27.7, so that our expression for $y$ becomes

$$y = L \sin \theta = \frac{Lm\lambda}{d}$$

The separation between adjacent principal maxima is

$$\Delta y = \frac{L(m+1)\lambda}{d} - \frac{Lm\lambda}{d} = \frac{L\lambda}{d}$$

The number of lines per meter $N$ is the reciprocal of the slit separation $d$, so $N = 1/d$. Substituting this result into the equation above gives $\Delta y = L\lambda N$. Applying this expression to gratings A and B, we have

$$\Delta y_A = L\lambda N_A \quad \text{and} \quad \Delta y_B = L\lambda N_B$$

Forming the ratio of these two expressions, we have

$$\frac{\Delta y_B}{\Delta y_A} = \frac{L\lambda N_B}{L\lambda N_A} = \frac{N_B}{N_A}$$

$$N_B = \frac{N_A \Delta y_B}{\Delta y_A} = \frac{(2000 \text{ m}^{-1})(3.2 \text{ cm})}{2.7 \text{ cm}} = 2400 \text{ m}^{-1}$$
47. **REASONING AND SOLUTION** The geometry of the situation is shown below.

From the geometry, we have

\[ \tan \theta = \frac{y}{L} = \frac{0.60 \text{ mm}}{3.0 \text{ mm}} = 0.20 \quad \text{or} \quad \theta = 11.3^\circ \]

Then, solving Equation 27.7 with \( m = 1 \) for the separation \( d \) between the slits, we have

\[ d = \frac{m \lambda}{\sin \theta} = \frac{(1) (780 \times 10^{-9} \text{ m})}{\sin 11.3^\circ} = 4.0 \times 10^{-6} \text{ m} \]

48. **REASONING** For a diffraction grating, a principal maximum is located at an angle \( \theta \) that is specified by \( \sin \theta = m \frac{\lambda}{d} \) (Equation 27.7), where \( m = 0, 1, 2, 3, \ldots \), \( \lambda \) is the wavelength of the light, and \( d \) is the separation between the slits. We are not given values for \( \lambda \) or \( d \). However, we can determine the ratio \( \lambda/d \) by using Equation 27.7 with the fact that the first-order principal maximum \((m = 1)\) is located at an angle of 18.0°. Then, we can use the value of \( \lambda/d \) in Equation 27.7 to calculate the angle at which the third-order maximum \((m = 3)\) is located.

**SOLUTION** Using Equation 27.7 for the first-order maximum \((m = 1)\) and solving it for \( \lambda/d \), we have that

\[ \sin \theta = m \frac{\lambda}{d} \quad \text{or} \quad \frac{\lambda}{d} = \frac{\sin \theta}{m} = \frac{\sin 18.0^\circ}{1} = 0.309 \]

Using Equation 27.7 for the third-order maximum \((m = 3)\) with this value for \( \lambda/d \), we can obtain the corresponding value of \( \theta \):

\[ \sin \theta = m \frac{\lambda}{d} \quad \text{or} \quad \theta = \sin^{-1} \left( m \frac{\lambda}{d} \right) = \sin^{-1} \left[ (3)(0.309) \right] = 68.0^\circ \]
49. **REASONING** The angle that specifies the $m$th principal maximum of a diffraction grating is $\sin \theta = \frac{m \lambda}{d}$ (Equation 27.7), where $\lambda$ is the wavelength of the light, and $d$ is the separation between the slits of the grating. The angle $\theta$ is known, and the separation $d$ (in cm) between the slits is equal to the reciprocal of the number of lines per centimeter (2604 cm$^{-1}$).

**SOLUTION** Solving Equation 27.7 for the wavelength, we obtain

$$\lambda = \frac{d \sin \theta}{m} = \frac{\left( \frac{1}{2604 \, \text{cm}^{-1}} \right) \sin 30.0^\circ}{m} = \frac{1.92 \times 10^{-4} \, \text{cm}}{m} = 1920 \, \text{nm}$$

For $m = 1$, $\lambda = 1920 \, \text{nm}$; for $m = 2$, $\lambda = 960 \, \text{nm}$; For $m = 3$, $\lambda = 640 \, \text{nm}$; For $m = 4$, $\lambda = 480 \, \text{nm}$; for $m = 5$, $\lambda = 384 \, \text{nm}$. For $m > 5$, the values for the wavelength are smaller than 384 nm. Thus, the two wavelengths that fall within the range of 410 to 660 nm are $640 \, \text{nm}$ and $480 \, \text{nm}$.

50. **REASONING**

a. From the following drawing we see that $\tan \theta = y/L$, so that $y = L \tan \theta$. The angle between the central bright fringe and the 2nd order bright fringe is given by $\sin \theta = \frac{m \lambda}{d}$ (Equation 27.7), where $m = 2$ in this case, $\lambda$ is the wavelength of the light, and $d$ is the separation between the slits.

Since $y = L \tan \theta$ and $\tan \theta \approx \sin \theta$, we have that $y = L \sin \theta$. But $\sin \theta = \frac{m \lambda}{d}$, so $y = L \frac{m \lambda}{d}$.

b. The wavelength $\lambda_{\text{water}}$ of light in water is related to the wavelength $\lambda_{\text{vacuum}}$ in vacuum by $\lambda_{\text{water}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{water}}}$ (Equation 27.3), where $n_{\text{water}}$ is the index of refraction of water. Since $n_{\text{water}} = 1.33$, the wavelength in water is less than that in a vacuum (or in air). Therefore, the distance $y$, which is proportional to the wavelength (see part a above), is smaller when the apparatus is submerged in water.

**SOLUTION**

a. From the **REASONING**, the distance from the central bright fringe to the 2nd order bright fringe is

$$y = \frac{Lm \lambda}{d} = \frac{(0.15 \, \text{m})(2)(410 \times 10^{-9} \, \text{m})}{1.2 \times 10^{-5} \, \text{m}} = 0.010 \, \text{m}$$

b. When the apparatus is immersed in water, the wavelength of the light becomes smaller:
\[ \lambda_{\text{water}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{water}}} = \frac{410 \times 10^{-9} \text{ m}}{1.33} = 308 \times 10^{-9} \text{ m} \]

In this case, the distance \( y \) becomes
\[ y_{\text{water}} = \frac{L m \lambda_{\text{water}}}{d} = \frac{(0.15 \text{ m})(2)(308 \times 10^{-9} \text{ m})}{1.2 \times 10^{-5} \text{ m}} = 0.0077 \text{ m} \]

51. **SSM REASONING** The angle \( \theta \) that locates the first-order maximum produced by a grating with 3300 lines/cm is given by Equation 27.7, \( \sin \theta = \frac{m \lambda}{d} \), with the order of the fringes given by \( m = 0, 1, 2, 3, \ldots \) Any two of the diffraction patterns will overlap when their angular positions are the same.

**SOLUTION** Since the grating has 3300 lines/cm, we have
\[ d = \frac{1}{3300 \text{ lines/cm}} = 3.0 \times 10^{-4} \text{ cm} = 3.0 \times 10^{-6} \text{ m} \]

a. In first order, \( m = 1 \); therefore, for violet light,
\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) = \sin^{-1} \left( \frac{1 \left(410 \times 10^{-9} \text{ m} \right)}{3.0 \times 10^{-6} \text{ m}} \right) = 7.9^\circ \]

Similarly for red light,
\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) = \sin^{-1} \left( \frac{1 \left(660 \times 10^{-9} \text{ m} \right)}{3.0 \times 10^{-6} \text{ m}} \right) = 13^\circ \]

b. Repeating the calculation for the second order maximum \( (m = 2) \), we find that
\[
\begin{array}{c|c|c}
(m = 2) & \text{for violet} & \theta = 16^\circ \\
& \text{for red} & \theta = 26^\circ \\
\end{array}
\]

c. Repeating the calculation for the third order maximum \( (m = 3) \), we find that
\[
\begin{array}{c|c|c}
(m = 3) & \text{for violet} & \theta = 24^\circ \\
& \text{for red} & \theta = 41^\circ \\
\end{array}
\]

d. Comparisons of the values for \( \theta \) calculated in parts (a), (b) and (c) show that the second and third orders overlap.
52. **REASONING** The angle $\theta$ of a principal maximum formed by light passing through a diffraction grating with slit separation $d$ is given by $\sin \theta = m \frac{\lambda}{d}$ (Equation 27.7), where $m$ is an integer ($m = 0, 1, 2, 3, \ldots$) and $\lambda$ is the wavelength of the light. We seek the change $\Delta \theta$ in the angle of the seventh-order principal maximum, so we will take $m = 7$ in Equation 27.7. As the diffraction grating undergoes the temperature change $\Delta T = +100.0 \, ^\circ C$, its expansion increases the slit separation from $d_1$ to $d_2$. From Equation 27.7, then we have

$$\sin \theta_1 = \frac{7\lambda}{d_1} \quad \text{and} \quad \sin \theta_2 = \frac{7\lambda}{d_2}$$  \hspace{1cm} (1)

where $\theta_1$ and $\theta_2$ are the angles of the seventh-order principal maximum before and after the temperature changes. Because the grating expands, $d_2$ is greater than $d_1$, and we see from Equations (1) that $\theta_2$ is less than $\theta_1$. This means that the seventh-order bright fringe moves closer to the central maximum as the grating heats up. Therefore, we expect the algebraic sign of the angle change $\Delta \theta = \theta_2 - \theta_1$ to be negative.

To determine the new slit separation $d_2$, we will make use of $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), where $\alpha = 1.30 \times 10^{-4} \, (\text{C}^\circ)^{-1}$ is the coefficient of thermal expansion of the diffraction grating, $L_0 = d_1$ is the initial slit separation, $\Delta T$ is the temperature change, and $\Delta L = d_2 - d_1$ is the amount by which the slit separation increases. Making these substitutions into Equation 12.2 yields

$$d_2 - d_1 = \alpha d_1 \Delta T$$  \hspace{1cm} (2)

**SOLUTION** Solving Equation (2) for $d_2$, we obtain

$$d_2 = d_1 + \alpha d_1 \Delta T = d_1 \left(1 + \alpha \Delta T\right)$$  \hspace{1cm} (3)

Substituting Equation (3) into the second of Equations (1) yields

$$\sin \theta_2 = \frac{7\lambda}{d_2} = \frac{7\lambda}{d_1(1 + \alpha \Delta T)}$$  \hspace{1cm} (4)

Taking the inverse sine of Equation (4) and the first of Equations (1), then, we have

$$\Delta \theta = \theta_2 - \theta_1 = \sin^{-1} \left[ \frac{7\lambda}{d_1(1 + \alpha \Delta T)} \right] - \sin^{-1} \left( \frac{7\lambda}{d_1} \right)$$

$$= \sin^{-1} \left\{ \frac{7 \left( 656.0 \times 10^{-9} \, \text{m} \right)}{1.250 \times 10^{-5} \, \text{m} \left[ 1 + \left( \frac{1.30 \times 10^{-4}}{\text{C}^\circ} \right) \left( 100.0 \, \text{C}^\circ \right) \right]} \right\} - \sin^{-1} \left[ \frac{7 \left( 656.0 \times 10^{-9} \, \text{m} \right)}{1.250 \times 10^{-5} \, \text{m}} \right]$$

$$= -0.29^\circ$$
53. **REASONING AND SOLUTION**

a. The angular positions of the specified orders are equal, so \( \frac{\lambda}{d_A} = 2 \frac{\lambda}{d_B} \), or

\[
\frac{d_B}{d_A} = \frac{2}{1}
\]

b. Similarly, we have for the \( m_A \) order of grating A and the \( m_B \) order of grating B that

\[
m_A \frac{\lambda}{d_A} = m_B \frac{\lambda}{d_B}, \text{ so } m_A = \frac{m_B}{2}.
\]

The next highest orders which overlap are

\[
m_B = 4, \ m_A = 2 \quad \text{and} \quad m_B = 6, \ m_A = 3
\]

54. **REASONING** In air the index of refraction is nearly \( n = 1 \), while in the film it is \( n = 1.33 \). A phase change occurs whenever light travels through a material with a smaller refractive index toward a material with a larger refractive index and reflects from the boundary between the two. The phase change is equivalent to \( \frac{1}{2} \frac{\lambda}{\text{film}} \), where \( \lambda_{\text{film}} \) is the wavelength in the film.

In the film the index of refraction is \( n = 1.33 \), while in the glass it is \( n = 1.52 \). This situation is like that discussed above and, once again, the phase change is equivalent to \( \frac{1}{2} \frac{\lambda}{\text{film}} \), where \( \lambda_{\text{film}} \) is the wavelength in the film.

Both the light reflected from the air-film interface and from the film-glass interface experience phase changes, each of which is equivalent to \( \frac{1}{2} \frac{\lambda}{\text{film}} \). In other words, there is no net phase change between the waves reflected from the two interfaces, and only the extra travel distance of the light within the film leads to the destructive interference.

**SOLUTION** As mentioned in the **REASONING**, only the extra travel distance of the light within the film leads to the destructive interference. The extra distance is \( 2t \), where \( t \) is the film thickness. The condition for destructive interference in this case is

\[
\text{Extra travel distance in film} \quad \frac{2t}{\text{Condition for destructive interference}} = \frac{1}{2} \lambda_{\text{film}}, \frac{1}{2} \lambda_{\text{film}}, \frac{3}{2} \lambda_{\text{film}}, \ldots = (m + \frac{1}{2}) \lambda_{\text{film}} \quad \text{where } m = 0, 1, 2, 3, \ldots
\]

Solving for the wavelength gives

\[
\lambda_{\text{film}} = \frac{2t}{m + \frac{1}{2}} \quad m = 0, 1, 2, 3, \ldots
\]

According to Equation 27.3, we have \( \lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n} \). Using this substitution in our result for \( \lambda_{\text{film}} \), we obtain

\[
\lambda_{\text{vacuum}} = \frac{2nt}{m + \frac{1}{2}} \quad m = 0, 1, 2, 3, \ldots
\]
For the first four values of $m$ and the given values for $n$ and $t$, we find

\[ m = 0 \quad \lambda_{\text{vacuum}} = \frac{2nt}{m + \frac{1}{2}} = \frac{2(1.33)(465 \text{ nm})}{0 + \frac{1}{2}} = 2470 \text{ nm} \]

\[ m = 1 \quad \lambda_{\text{vacuum}} = 825 \text{ nm} \]

\[ m = 2 \quad \lambda_{\text{vacuum}} = 495 \text{ nm} \]

\[ m = 3 \quad \lambda_{\text{vacuum}} = 353 \text{ nm} \]

The range of visible wavelengths (in vacuum) extends from 380 to 750 nm. Therefore, the only visible wavelength in which the film appears dark due to destructive interference is 495 nm.

55. SSM REASONING In order for the two rays to interfere constructively and thereby form a bright interference fringe, the difference $\Delta \ell$ between their path lengths must be an integral multiple $m$ of the wavelength $\lambda$ of the light:

\[ \Delta \ell = m \lambda \]  

In Equation (1), $m$ can take on any integral value ($m = 0, 1, 2, 3, \ldots$). In this case, the rays meet at the eight-order bright fringe, so we have that $m = 8$.

SOLUTION Solving Equation (1) for $\lambda$, and substituting $m = 8$, we obtain

\[ \lambda = \frac{\Delta \ell}{m} = \frac{4.57 \times 10^{-6} \text{ m}}{8} = 5.71 \times 10^{-7} \text{ m} \]

Using the equivalence 1 nm = $10^{-9}$ m, we convert this result to nanometers:

\[ \lambda = \left(5.71 \times 10^{-7} \text{ m}\right) \left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 571 \text{ nm} \]

56. REASONING The loudspeakers are in-phase sources of identical sound waves. The waves from one speaker travel a distance $\ell_1$ in reaching point A and the waves from the second speaker travel a distance $\ell_2$. The condition that leads to constructive interference is $\ell_2 - \ell_1 = m \lambda$, where $\lambda$ is the wavelength of the waves and $m = 0, 1, 2, 3 \ldots$. In other words, the two distances are the same or differ by an integer number of wavelengths. Point A is the midpoint of a side of the square, so that the distances $\ell_1$ and $\ell_2$ are the same, and constructive interference occurs.

As you walk toward the corner, the waves from one of the sources travels a greater distance in reaching you than does the other wave. This difference in the distances increases until it reaches one half of a wavelength, at which spot destructive interference occurs and you hear no sound. As you walk on, the difference in distances continues to increase, and you
gradually hear a louder and louder sound. Ultimately, at the corner, the difference in distances becomes one wavelength, constructive interference occurs, and you hear a maximally loud sound.

The general condition that leads to constructive interference is \( \ell_2 - \ell_1 = m\lambda \), where \( m = 0, 1, 2, 3 \ldots \) There are many possible values for \( m \), and we wish to determine what the specific value is. At point A the value is \( m = 0 \). As you walk from point A toward either empty corner, the maximal loudness that indicates constructive interference does not occur again until you arrive at the corner. Thus, the next possibility for \( m \) applies at the corner; in other words, \( m = 1 \).

**SOLUTION** Consider the constructive interference that occurs at either empty corner. Using \( L \) to denote the length of a side of the square and taking advantage of the Pythagorean theorem, we have

\[
\ell_1 = L \quad \text{and} \quad \ell_2 = \sqrt{L^2 + L^2} = \sqrt{2} L
\]

The specific condition for the constructive interference at the empty corner is (using \( m = 1 \))

\[
\ell_2 - \ell_1 = \sqrt{2} L - L = (1) \lambda
\]

Solving for the wavelength of the waves gives

\[
\lambda = L\left(\sqrt{2} - 1\right) = (4.6 \text{ m})\left(\sqrt{2} - 1\right) = 1.9 \text{ m}
\]

57. **SSM REASONING** The slit separation \( d \) is given by Equation 27.1 with \( m = 1 \); namely \( d = \lambda / \sin \theta \). As shown in Example 1 in the text, the angle \( \theta \) is given by \( \theta = \tan^{-1}\left(\frac{y}{L}\right) \).

**SOLUTION** The angle \( \theta \) is

\[
\theta = \tan^{-1}\left(\frac{0.037 \text{ m}}{4.5 \text{ m}}\right) = 0.47^\circ
\]

Therefore, the slit separation \( d \) is

\[
d = \frac{\lambda}{\sin \theta} = \frac{490 \times 10^{-9} \text{ m}}{\sin 0.47^\circ} = 6.0 \times 10^{-5} \text{ m}
\]

58. **REASONING** The width \( W \) of the slit and the wavelength \( \lambda \) of the light are related to the angle \( \theta \) defining the location of a dark fringe in the single-slit diffraction pattern, so we can determine the width from values for the wavelength and the angle. The wavelength is given. We can obtain the angle from the width given for the central bright fringe on the screen and the distance between the screen and the slit. To do this, we will use trigonometry and the fact that the width of the central bright fringe is defined by the first dark fringe on either side of the central bright fringe.
SOLUTION The angle that defines the location of a dark fringe in the diffraction pattern can be determined according to

\[ \sin \theta = m \frac{\lambda}{W} \quad m = 1, 2, 3, \ldots \]  

(27.4)

Recognizing that we need the case for \( m = 1 \) (the first dark fringe on either side of the central bright fringe determines the width of the central bright fringe) and solving for \( W \) give

\[ W = \frac{\lambda}{\sin \theta} \]  

(1)

Referring to Figure 27.23 in the text, we see that the width of the central bright fringe is \( 2y \), where trigonometry shows that

\[ 2y = 2L \tan \theta \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{2y}{2L} \right) = \tan^{-1} \left[ \frac{0.050 \text{ m}}{2(0.60 \text{ m})} \right] = 2.4^\circ \]

Substituting this value for \( \theta \) into Equation (1), we find that

\[ W = \frac{\lambda}{\sin \theta} = \frac{510 \times 10^{-9} \text{ m}}{\sin 2.4^\circ} = \left[ 1.2 \times 10^{-5} \text{ m} \right] \]

59. REASONING The diameter \( D \) of the spectator’s pupils and the wavelength \( \lambda \) of the light being viewed determine the minimum angular separation \( \theta_{\text{min}} \) (in radians) at which adjacent cards can be separately resolved, according to \( \theta_{\text{min}} \approx 1.22 \frac{\lambda}{D} \) (Equation 27.6). If the spectator’s pupils are narrower than the diameter \( D \) given by Equation 27.6, the cards will not appear separated, and their colors will blur together. Thus \( D \) is the upper limit on the diameter of the spectator’s pupils. The angular separation (in radians) of the cards is found from Equation 8.1: \( \theta_{\text{min}} \approx \frac{s}{L} \), where \( s \) is the distance between adjacent cards, and \( L \) is the distance between the cards and the spectator.

SOLUTION Setting the right sides of \( \theta_{\text{min}} \approx 1.22 \frac{\lambda}{D} \) (Equation 27.6) and \( \theta_{\text{min}} \approx \frac{s}{L} \) (Equation 8.1) equal, we obtain

\[ \theta_{\text{min}} \approx \frac{s}{L} \approx 1.22 \frac{\lambda}{D} \]  

(1)

Solving Equation (1) for \( D \) yields

\[ D \approx 1.22 \frac{\lambda L}{s} = 1.22 \left( \frac{480 \times 10^{-9} \text{ m}}{5.0 \times 10^{-2} \text{ m}} \right)(160 \text{ m}) = 1.9 \times 10^{-3} \text{ m} = \left[ 1.9 \text{ mm} \right] \]
60. **REASONING AND SOLUTION** The angular position of a third order fringe is given by
\[ \theta = \sin^{-1} \frac{3 \lambda}{d}, \]
and the position of the fringe on the screen is
\[ y = L \tan \theta. \]

For the third-order red fringe,
\[ \theta = \sin^{-1} \left( \frac{3 \times 665 \times 10^{-9} \text{ m}}{0.158 \times 10^{-3} \text{ m}} \right) = 0.724^\circ \]
and
\[ y = (2.24 \text{ m}) \tan 0.724^\circ = 2.83 \times 10^{-2} \text{ m}. \]

For the third-order green fringe, \( \theta = 0.615^\circ \) and \( y = 2.40 \times 10^{-2} \text{ m}. \)

The distance between the fringes is
\[ 2.83 \times 10^{-2} \text{ m} - 2.40 \times 10^{-2} \text{ m} = 4.3 \times 10^{-3} \text{ m}. \]

61. **SSM REASONING** For a diffraction grating, the angular position \( \theta \) of a principal maximum on the screen is given by Equation 27.7 as \( \sin \theta = m \lambda / d \) with \( m = 0, 1, 2, 3, \ldots \)

**SOLUTION** When the fourth-order principal maximum of light \( A \) exactly overlaps the third-order principal maximum of light \( B \), we have
\[ \sin \theta_A = \sin \theta_B \]
\[ \frac{4 \lambda_A}{d} = \frac{3 \lambda_B}{d} \quad \text{or} \quad \frac{\lambda_A}{\lambda_B} = \frac{3}{4}. \]

62. **REASONING** The beam diffracts outward when it leaves the spotlight, so that the circular spot on the moon has a diameter \( d \) that is greater than the diameter \( D \) of the spotlight. The diffraction angle \( \theta \), measured from the middle of the beam to the first circular dark fringe, which is the edge of the central bright spot on the moon, is expected to be small. Therefore, the diameter \( d \) of the central bright spot on the moon is approximately equal to the arc length \( s = r \theta_{\text{spot}} \) (Equation 8.1), where \( r \) is the distance between the Earth and the moon and \( \theta_{\text{spot}} \) is the angle subtended by the central bright spot (see the drawing):
\[ d = r \theta_{\text{spot}} \] (1)

Note that \( \theta \) is measured from the middle of the central bright spot, so that the angle \( \theta_{\text{spot}} \) subtended by the entire bright spot is twice as large:
\[ \theta_{\text{spot}} = 2\theta \]  

(2)

The first circular dark fringe is located by the angle \( \theta = \sin^{-1}\left(\frac{1.22 \frac{\lambda}{D}}{\sin \theta} \right) \) (Equation 27.5), where \( D \) is the diameter of the spotlight and \( \lambda \) is the wavelength of the light.

**SOLUTION** Substituting Equation (2) into Equation (1) yields

\[ d = r\theta_{\text{spot}} = 2r\theta \]  

(3)

Substituting \( \theta = \sin^{-1}\left(\frac{1.22 \frac{\lambda}{D}}{\sin \theta} \right) \) (Equation 27.5) into Equation (3), we obtain

\[ d = 2r\theta = 2r \sin^{-1}\left(\frac{1.22 \frac{\lambda}{D}}{\sin \theta} \right) = 2(3.77 \times 10^8 \text{ m}) \sin^{-1}\left(\frac{1.22 \frac{694.3 \times 10^{-9} \text{ m}}{0.20 \text{ m}}}{} \right) = 3.2 \times 10^3 \text{ m} \]

63. **REASONING** The width of the central bright fringe is defined by the location of the first dark fringe on either side of it. According to Equation 27.4, the angle \( \theta \) locating the first dark fringe can be obtained from \( \sin \theta = \frac{\lambda}{W} \), where \( \lambda \) is the wavelength of the light and \( W \) is the width of the slit. According to the drawing, \( \tan \theta = \frac{y}{L} \), where \( y \) is half the width of the central bright fringe and \( L \) is the distance between the slit and the screen.

**SOLUTION** Since the angle \( \theta \) is small, we can use the fact that \( \sin \theta \approx \tan \theta \). Since \( \sin \theta = \frac{\lambda}{W} \) and \( \tan \theta = \frac{y}{L} \), we have

\[ \frac{\lambda}{W} = \frac{y}{L} \quad \text{or} \quad W = \frac{\lambda}{y} \]

Applying this result to both slits gives

\[ \frac{W_2}{W_1} = \frac{\lambda L/y_2}{\lambda L/y_1} = \frac{y_1}{y_2} \]

\[ W_2 = W_1 \frac{y_1}{y_2} = \left(3.2 \times 10^{-5} \text{ m}\right) \frac{1}{2} \left(1.2 \text{ cm}\right) = 2.0 \times 10^{-5} \text{ m} \]
64. **REASONING** When the light in the glass strikes the wedge of air, the wave reflected from the bottom surface of the glass does not experience a phase shift. This is because the light is traveling from a larger refractive index \( n_{\text{glass}} = 1.5 \) toward a smaller refractive index \( n_{\text{air}} = 1.0 \). When the light reflects from the top surface of the plastic, the wave undergoes a phase shift that is equivalent to one-half of a wavelength in the air film, since the light is traveling from a smaller refractive index \( n_{\text{air}} = 1.0 \) toward a larger refractive index \( n_{\text{plastic}} = 1.2 \).

Thus, the net phase change due to reflection from the two surfaces is equivalent to one-half of a wavelength. At point A the air wedge has no thickness, so the light reflected from the top surface of the plastic is one-half of a wavelength out of phase with the light reflected from the bottom surface of the glass. This half-wavelength difference means destructive interference occurs, resulting in the dark fringe at A.

At the second dark fringe (the first is at A), the down-and-back distance traveled by the light in the air wedge is one wavelength in the air film. At the third dark fringe, the down-and-back distance traveled by the light in the air wedge is two wavelengths. Therefore, at the seventh dark fringe at B, the down-and-back distance traveled by the light in the air wedge is six wavelengths. Since the down-and-back distance is six wavelengths, the thickness of the air wedge at B is one-half of this distance, or three wavelengths.

**SOLUTION** If the thickness of the air wedge is \( t \), the condition for destructive interference is

\[
2t + \frac{1}{2} \lambda_{\text{air}} = \frac{1}{2} \lambda_{\text{air}}, \frac{3}{2} \lambda_{\text{air}}, \frac{5}{2} \lambda_{\text{air}}, \ldots
\]

Condition for destructive interference

\[
= \left(m + \frac{1}{2}\right) \lambda_{\text{air}} \quad m = 0, 1, 2, 3, \ldots
\]

The case \( m = 0 \) corresponds to the first dark fringe at A. Therefore, the dark fringe at B corresponds to \( m = 6 \). Solving this equation for the thickness of the air wedge at B gives

\[
2t = \left(m + \frac{1}{2}\right) \lambda_{\text{air}} - \frac{1}{2} \lambda_{\text{air}} = m\lambda_{\text{air}}
\]

\[
t = \frac{1}{2} m \lambda_{\text{air}} = \frac{1}{2} (6)(520 \text{ nm}) = 1560 \text{ nm}
\]

65. **REASONING AND SOLUTION** The last maximum formed by the grating corresponds to \( \theta = 90.0^\circ \), so that

\[
m = \frac{d}{\lambda} \sin 90.0^\circ = \frac{d}{\lambda} = \frac{1.78 \times 10^{-6} \text{ m}}{471 \times 10^{-9} \text{ m}} = 3.78
\]

\[
m = \frac{d}{\lambda} = \frac{1.78 \times 10^{-6} \text{ m}}{471 \times 10^{-9} \text{ m}} = 3.78
\]
where we have used \( d = \frac{1}{5620 \text{ lines/cm}} = 1.78 \times 10^{-4} \text{ cm} = 1.78 \times 10^{-6} \text{ m} \).

Thus, the last maximum formed by the grating is \( m = 3 \). This maximum lies at

\[
\theta = \sin^{-1} \left( \frac{\lambda}{d} \right) = \sin^{-1} \left[ 3 \left( \frac{1}{3.78} \right) \right] = 52.5^\circ
\]

The distance from the center of the screen to the \( m = 3 \) maximum is

\[
y = (0.750 \text{ m}) \tan 52.5^\circ = 0.977 \text{ m}
\]

The screen must have a width of \( 2y = 1.95 \text{ m} \)

66. **Reasoning** The bright rings occur wherever the thickness of the drop causes constructive interference between a ray of light reflected from the top surface of the drop and a ray reflected from the bottom surface of the drop. The edge of the drop, where the thickness is zero and phase change can only occur due to reflection, is bright. Constructive interference at zero thickness shows that the reflections of the two rays produce no net phase change. At the other bright rings (and the bright spot at the center), then, where the thickness \( t \) is not zero, the phase change must be entirely due to the path difference \( 2t \) between the two rays. For constructive interference to occur, this phase change must be equal to an integer number of wavelengths \( \lambda_{\text{oil}} \) of light in the oil, so we have that

\[
2t = 0, \lambda_{\text{oil}}, 2\lambda_{\text{oil}}, 3\lambda_{\text{oil}}, \ldots
\]  

(1)

The first of the 56 bright rings appears at the edge of the drop, where the thickness is zero. This corresponds to the first value (0 wavelengths) on the right side of Equation (1). For blue light, there are 55 other bright rings, plus the bright spot at the center, where the thickness \( t_{\text{center}} \) of the drop is greatest. Therefore, we can say that the path difference \( 2t_{\text{center}} \) at the center of the drop is equal to \( 55 + 1 = 56 \) wavelengths of blue light in the oil:

\[
2t_{\text{center}} = 56\lambda_{\text{oil, blue}}
\]  

(2)

When red light shines on the drop, there is a bright spot at the center, so the path difference \( 2t_{\text{center}} \) is equal to an integer number \( m \) of wavelengths of red light in the oil:

\[
2t_{\text{center}} = m\lambda_{\text{oil, red}}
\]  

(3)

By comparison with Equation (2), we see that there will be \( m \) bright rings in red light.
The wavelength of light in the oil is given by $\lambda_{\text{oil}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{oil}}}$ (Equation 27.3), where $\lambda_{\text{vacuum}}$ is the vacuum wavelength of the light reflecting from the drop, and $n_{\text{oil}}$ is the refractive index of the oil. Applying Equation 27.3 to the two colors of light, and noting that the index of refraction is the same for both the blue and red wavelengths, we obtain

$$\lambda_{\text{oil, blue}} = \frac{\lambda_{\text{vacuum, blue}}}{n_{\text{oil}}} \quad \text{and} \quad \lambda_{\text{oil, red}} = \frac{\lambda_{\text{vacuum, red}}}{n_{\text{oil}}}$$  \hspace{1cm} (4)

**SOLUTION** Setting the right sides of Equations (2) and (3) equal, since the thickness $t_{\text{center}}$ at the center of the drop is the same for both wavelengths of light, we see that

$$2t_{\text{center}} = m\lambda_{\text{oil, red}} = 56\lambda_{\text{oil, blue}}$$  \hspace{1cm} (5)

Substituting Equations (4) into Equation (5), we obtain

$$m \left( \frac{\lambda_{\text{vacuum, red}}}{n_{\text{oil}}} \right) = 56 \left( \frac{\lambda_{\text{vacuum, blue}}}{n_{\text{oil}}} \right) \quad \text{or} \quad m = \frac{56\lambda_{\text{vacuum, blue}}}{\lambda_{\text{vacuum, red}}} = \frac{56(455 \text{ nm})}{637 \text{ nm}} = 40$$

Therefore, there are **40 bright rings** when the light is red.
1. (e) In each of the situations in answers a-d, the person and the frame of reference is subject to an acceleration. In an accelerated reference frame Newton’s law of inertia is not valid, so the reference frame is not an inertial reference frame.

2. (a) The worker measures the proper time, because he is at rest with respect to the light and views the flashes as occurring at the same place.

3. 1.89 s

4. (d) To see the proper length of an object, an observer must be at rest with respect to the two points defining that length. Observers in either spacecraft see the other spacecraft as moving. Therefore, neither the observers in spacecraft A nor those in spacecraft B see the proper length of the other spacecraft.

5. 8.19 light-years

6. (b) The runner sees home plate move away from his feet and first base arrive at his feet. Thus, the runner sees both events occurring at the same place and measures the proper time. The catcher is the one at rest with respect to home plate and first base. Therefore, he measures the proper length between the two points.

7. (c) According to the theory of special relativity, the equations apply when both observers have constant velocities with respect to an inertial reference frame.

8. (c) The observers will always disagree about the time interval and the length, as indicated by the time-dilation and length-contraction equations. However, each will measure the same relative speed for the other’s motion.

9. 22.7 m

10. (c) The expression \( p = \frac{mv}{\sqrt{1 - v^2/c^2}} \) for the magnitude of the relativistic momentum applies at any speed \( v \). When it is used, the conservation of linear momentum is valid for an isolated system no matter what the speeds of the various parts of the system are.

11. (d) Both of the expressions can be used provided that \( v \ll c \). Expression B differs from expression A only by a negligible amount in this limit of small speeds.

12. 0.315 kg⋅m/s
13. (b) The mass $m$ of an object is proportional to the object’s rest energy $E_0$, according to 

$$E_0 = mc^2.$$ 

The rest energy includes all forms of energy except kinetic energy, which plays no role here, because the glass is not moving. To freeze half the liquid into ice, energy in the form of heat must be removed from the liquid, so the water in possibility B has more mass than the water in possibility A. To freeze the remaining liquid into ice, more heat must be removed, so the water in possibility A has more mass than the water in possibility C. Thus, the ranking in descending order (largest first) is B, A, C.

14. $1.88 \times 10^{-5} \text{ kg}$

15. (b) The total energy is the sum of the rest energy and the kinetic energy. The rest energy includes all forms of energy (including potential energy) except kinetic energy.

16. $2.60 \times 10^8 \text{ m/s}$

17. (a) According to Equation 28.7, the magnitude of the momentum is

$$p = \frac{\sqrt{E^2 - m^2 c^4}}{c},$$

where $E$ is the total energy. The total energy is $E = E_0 + KE$. Since the kinetic energy is equal to the rest energy, the total energy is $E = 2E_0$. Substituting this result into the expression for $p$ and using the fact that $mc^2 = E_0$ give

$$p = \frac{\sqrt{4E_0^2 - E_0^2}}{c} = \frac{\sqrt{3mc^2}}{c}.$$
1. **REASONING**  
   a. The two events in this problem are the creation of the pion and its subsequent decay (or breaking apart). Imagine a reference frame attached to the pion, so the pion is stationary relative to this reference frame. To a hypothetical person who is at rest with respect to this reference frame, these two events occur at the same place, namely, at the place where the pion is located. Thus, this hypothetical person measures the proper time interval \( \Delta t_0 \) for the decay of the pion. On the other hand, the person standing in the laboratory sees the two events occurring at different locations, since the pion is moving relative to that person. The laboratory person, therefore, measures a dilated time interval \( \Delta t \). The relation between these two time intervals is given by  
   \[
   \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \quad \text{(Equation 28.1)}
   \]

   b. According to the hypothetical person who is at rest in the reference frame attached to the moving pion, the distance \( x \) that the laboratory travels before the pion breaks apart is equal to the speed \( v \) of the laboratory relative to the pion times the proper time interval \( \Delta t_0 \), or  
   \[ x = v \Delta t_0 \]  
   The speed of the laboratory relative to the pion is the same as the speed of the pion relative to the laboratory, namely, 0.990c.

   **SOLUTION**  
   a. The proper time interval is  
   \[
   \Delta t_0 = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = (3.5 \times 10^{-8} \text{ s}) \sqrt{1 - \left(\frac{0.990c}{c}\right)^2} = 4.9 \times 10^{-9} \text{ s}
   \]  
   \[ (28.1) \]

   b. The distance \( x \) that the laboratory travels before the pion breaks apart, as measured by the hypothetical person, is  
   \[
   x = v \Delta t_0 = \frac{(0.990)(3.00 \times 10^8 \text{ m/s})(4.9 \times 10^{-9} \text{ s})}{0.990c} = 1.5 \text{ m}
   \]

2. **REASONING**  
   The time interval \( \Delta t_0 = 25 \text{ s} \) measured on earth is the proper time interval. This is because an observer on earth makes his measurement by noting the time it takes for a spot on the antenna to move around a complete circle. Hence, such an observer is at rest with respect to this spot as it starts around the circle and finishes its rotation in the same place. The time interval \( \Delta t = 42 \text{ s} \) measured by instruments on the moving spacecraft is the dilated time interval. This is because the instruments “observe” the earth to be moving relative to the spaceship and a spot on the antenna to start and end its rotational path at different places. The two time intervals are related by the time-dilation equation.
\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \] (Equation 28.1), where \( v \) is the speed of the spaceship with respect to the earth and \( c \) is the speed of light in a vacuum.

**SOLUTION** Solving the time dilation equation for \( v/c \), we find

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \quad \text{or} \quad \sqrt{1 - \left(\frac{v^2}{c^2}\right)} = \frac{\Delta t_0}{\Delta t} \]

or \[ \left[\sqrt{1 - \left(\frac{v^2}{c^2}\right)}\right]^2 = \left(\frac{\Delta t_0}{\Delta t}\right)^2 \quad \text{or} \quad 1 - \left(\frac{v^2}{c^2}\right) = \left(\frac{\Delta t_0}{\Delta t}\right)^2 \]

or \[ \frac{v^2}{c^2} = 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \quad \text{or} \quad \frac{v}{c} = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1 - \left(\frac{25}{42}\right)^2} = 0.80 \]

3. **SSM REASONING** The total time for the trip is one year. This time is the proper time interval \( \Delta t_0 \), because it is measured by an observer (the astronaut) who is at rest relative to the beginning and ending events (the times when the trip started and ended) and who sees them at the same location in spacecraft. On the other hand, the astronaut measures the clocks on earth to run at the dilated time interval \( \Delta t \), which is the time interval of one hundred years. The relation between the two time intervals is given by Equation 28.1, which can be used to find the speed of the spacecraft.

**SOLUTION** The dilated time interval \( \Delta t \) is related to the proper time interval \( \Delta t_0 \) by

\[ \Delta t = \Delta t_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)} \] . Solving this equation for the speed \( v \) of the spacecraft yields

\[ v = c \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = c \sqrt{1 - \left(\frac{1 \text{ yr}}{100 \text{ yr}}\right)^2} = 0.999 \, 95c \] (28.1)

4. **REASONING** When you measure your breathing rate, you count \( N = 8.0 \) breaths during a proper time interval of \( \Delta t_0 = 1.0 \) minutes, and in so doing you determine a rate of \( R_0 = N/\Delta t_0 = (8.0 \text{ breaths})/(1.0 \text{ minute}) = 8.0 \text{ breaths/minute} \). When measured by monitors on the earth, the \( N = 8.0 \) breaths occur in a dilated time interval \( \Delta t \) that is related to the proper time interval by \[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \] (Equation 28.1). The breathing rate \( R \) measured by a monitor on the earth, then, is given by
\[ R = \frac{N}{\Delta t} \]  

\textit{SOLUTION} Substituting \( \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \) (Equation 28.1) into Equation (1), we obtain

\[ R = \frac{N}{\Delta t} = \frac{N}{\Delta t_0} = \frac{N \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{(0.975c)^2}{c^2}} = 1.8 \text{ breaths/minute} \]

5. \textit{REASONING} The observer is moving with respect to the oscillating object. Therefore, to the observer, the oscillating object is moving with a speed of \( v = 1.90 \times 10^8 \text{ m/s} \), and the observer measures a dilated time interval for the period of oscillation. To determine this dilated time interval \( \Delta t = T_{dilated} \), we must use the time-dilation equation:

\[ \Delta t = T_{dilated} = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

where \( \Delta t_0 \) is the proper time interval, as measured in the reference frame to which the fixed end of the spring is attached. The proper time interval is the period \( T \) of the oscillation as given by Equations 10.4 and 10.11:

\[ \Delta t_0 = T = 2\pi \sqrt{\frac{m}{k}} \]

where \( m \) is the mass of the object and \( k \) is the spring constant.

\textit{SOLUTION} Substituting Equation (1) into the time-dilation equation gives

\[ T_{dilated} = \frac{2\pi \sqrt{\frac{m}{k}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2\pi \sqrt{\frac{6.00 \text{ kg}}{76.0 \text{ N/m}}}}{\sqrt{1 - \frac{(1.90 \times 10^8 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}}} = 2.28 \text{ s} \]

6. \textit{REASONING} The distance \( d \) traveled by the ship, according to an observer on earth, is equal to the product of the speed \( v \) of the ship relative to earth and the elapsed time \( \Delta t \) measured by the earthbound observer, according to Equation 2.1:

\[ d = v\Delta t \]
The time interval $\Delta t$ is the dilated time interval and is related to the proper time interval $\Delta t_0$ for the journey (as measured by an observer on the ship) via Equation 28.1:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (28.1)$$

In this equation, $v$ is the speed of the ship relative to earth, and $c$ is the speed of light in a vacuum. We will use Equation 28.1 to determine the speed $v$ of the ship, and then Equation 2.1 to find the distance the ship travels, according to the earthbound observer.

**SOLUTION** Squaring both sides of Equation 28.1 and solving for the ratio $v^2/c^2$, we obtain

$$(\Delta t)^2 = \frac{(\Delta t_0)^2}{1 - \frac{v^2}{c^2}} \quad \text{or} \quad 1 - \frac{v^2}{c^2} = \frac{(\Delta t_0)^2}{(\Delta t)^2} \quad \text{or} \quad 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 = \frac{v^2}{c^2} \quad (1)$$

Solving Equation (1) for $v$ yields

$$v^2 = c^2 \left[1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2\right] \quad \text{or} \quad v = c \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} \quad (2)$$

Substituting Equation (2) into Equation 2.1, we find that

$$d = v\Delta t = c \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} \Delta t$$

$$= (3.0 \times 10^8 \text{ m/s}) \sqrt{1 - \left(\frac{9.2 \text{ yr}}{12 \text{ yr}}\right)^2} (12 \text{ yr}) \left(\frac{3.16 \times 10^7 \text{ s}}{1 \text{ yr}}\right) = 7.3 \times 10^{16} \text{ m}$$

7. **REASONING AND SOLUTION** The proper time is the time it takes for the bacteria to double its number, i.e., $\Delta t_0 = 24.0$ hours. For the earth based sample to grow to 256 bacteria, it would take 8 days ($2^n = 256$ or $n = 8$). The "doubling time" for the space culture would be

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{24.0 \text{ h}}{\sqrt{1 - \left(\frac{0.866 c}{c}\right)^2}} = 48.0 \text{ h} \quad \text{or} \quad 2 \text{ days}$$

In eight earth days, the space bacteria would undergo $n \approx \left(\frac{1}{2}\right)^8 = 4$ "doublings". The number of space bacteria is

Number of Space Bacteria = $2^n = 2^4 = 16$
8. **REASONING** The distance \( L_0 = 4.1 \times 10^6 \) m is the proper distance, because it is the distance between the two cities as measured by an observer on the earth who is at rest with respect to the cities. The distance \( L \) measured by the voyagers aboard the UFO is the contracted distance, because the voyagers are moving relative to the cities. The two distances are related by the length-contraction equation \( L = L_0 \sqrt{1 - \left( \frac{v}{c} \right)^2} \) (Equation 28.2), where \( v = 0.70c \) is the speed of the UFO with respect to the earth and \( c \) is the speed of light in a vacuum.

**SOLUTION** Using Equation 28.2, we find that the contracted distance measured by the UFO-voyagers is

\[
L = L_0 \sqrt{1 - \left( \frac{v}{c} \right)^2} = \left( 4.1 \times 10^6 \text{ m} \right) \sqrt{1 - \left( \frac{0.70c}{c} \right)^2} = 2.9 \times 10^6 \text{ m}
\]

9. **SSM REASONING** All standard meter sticks at rest have a length of 1.00 m for observers who are at rest with respect to them. Thus, 1.00 m is the proper length \( L_0 \) of the meter stick. When the meter stick moves with speed \( v \) relative to an earth-observer, its length \( L = 0.500 \) m will be a contracted length. Since both \( L_0 \) and \( L \) are known, \( v \) can be found directly from Equation 28.2, \( L = L_0 \sqrt{1 - \left( \frac{v^2}{c^2} \right)} \).

**SOLUTION** Solving Equation 28.2 for \( v \), we find that

\[
v = c \sqrt{1 - \left( \frac{L}{L_0} \right)^2} = (3.00 \times 10^8 \text{ m/s}) \sqrt{1 - \left( \frac{0.500 \text{ m}}{1.00 \text{ m}} \right)^2} = 2.60 \times 10^8 \text{ m/s}
\]

10. **REASONING** The distance between earth and the center of the galaxy is the proper length \( L_0 \), because it is the distance measured by an observer who is at rest relative to the earth and the center of the galaxy. A person on board the spaceship is moving with respect to them and measures a contracted length \( L \) that is related to the proper length by Equation 28.2 as \( L = L_0 \sqrt{1 - \left( \frac{v^2}{c^2} \right)} \). The contracted distance is also equal to the product of the spaceship’s speed \( v \) the time interval measured by a person on board the spaceship. This time interval is the proper time interval \( \Delta t_0 \) because the person on board the spaceship measures the beginning and ending events (the times when the trip starts and ends) at the same location relative to a coordinate system fixed to the spaceship. Thus, the contracted distance is also \( L = v \Delta t_0 \). By setting the two expressions for \( L \) equal to each other, we can find the how long the trip will take according to a clock on board the spaceship.
**SOLUTION** Setting $L = L_0 \sqrt{1 - (v^2/c^2)}$ equal to $L = v \Delta t_0$ and solving for the proper time interval $\Delta t_0$ gives

$$\Delta t_0 = \frac{L_0}{v} \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

$$= \frac{(23\,000\,\text{ly}) \left(\frac{9.47 \times 10^{15}\,\text{m}}{1\,\text{ly}}\right)}{0.9990 \left(3.00 \times 10^8\,\text{m/s}\right)} \sqrt{1 - \left[\frac{(0.9990c)^2}{c^2}\right]} \left(\frac{1\,\text{yr}}{3.16 \times 10^7\,\text{s}}\right) = 1.0 \times 10^3\,\text{yr}$$

11. **REASONING** The tourist is moving at a speed of $v = 1.3\,\text{m/s}$ with respect to the path and, therefore, measures a contracted length $L$ instead of the proper length of $L_0 = 9.0\,\text{km}$. The contracted length is given by the length-contraction equation, Equation 28.2.

**SOLUTION** According to the length-contraction equation, the tourist measures a length that is

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (9.0\,\text{km}) \sqrt{1 - \frac{(1.3\,\text{m/s})^2}{(3.0\,\text{m/s})^2}} = 8.1\,\text{km}$$

12. **REASONING** The Martian measures the proper time interval $\Delta t_0$, because the Martian measures the beginning and ending events (the times when the trip starts and ends) at the same location relative to a coordinate system fixed to the spaceship.

The given distance between Mars and Venus is the distance as measured by a person on earth. That person is at rest relative to the two planets and, hence, measures the proper length. The Martian, who is moving relative to the planets, does not measure the proper length, but measures a contracted length.

According to the Martian, the time of the trip $\Delta t_0$ is equal to the contracted length that he measures divided by the speed $v$ of the spaceship.

**SOLUTION**

a. The contracted length $L$ measured by the Martian is related to the proper length $L_0$ by Equation 28.2 as

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (1.20 \times 10^{11}\,\text{m}) \sqrt{1 - \frac{(0.80c)^2}{c^2}} = 7.2 \times 10^{10}\,\text{m}$$
13. **REASONING AND SOLUTION**

The diameter $D$ of the planet, as measured by a moving spacecraft, is given in terms of the proper diameter $D_0$ by Equation 28.2. Taking the ratio of the diameter $D_A$ of the planet measured by spaceship A to the diameter $D_B$ measured by spaceship B, we find

$$
\frac{D_A}{D_B} = \frac{D_0\sqrt{1 - \frac{v_A^2}{c^2}}}{D_0\sqrt{1 - \frac{v_B^2}{c^2}}} = \sqrt{\frac{1 - (0.60c)^2}{1 - (0.80c)^2}} = 1.3
$$

14. **REASONING**

a. The two events are the creation of the particle and its subsequent disintegration. Relative to a stationary reference frame fixed to the laboratory, these two events occur at different locations, because the particle is moving relative to this reference frame. The proper distance $L_0$ is the distance $(1.05 \times 10^{-3} \text{ m})$ given in the statement of the problem, because this distance is measured by an observer in the laboratory who is at rest with respect to these locations.

b. The distance measured by a hypothetical person traveling with the particle is a contracted distance, because it is measured by a person who is moving relative to the two locations. The contracted distance $L$ is related to the proper distance $L_0$ by the length-contraction formula, $L = L_0\sqrt{1 - v^2/c^2}$ (Equation 28.2).

c. The proper lifetime $\Delta t_0$ of the particle is the lifetime as registered in a reference frame attached to the particle. In this reference frame the two events occur at the same location. The proper lifetime is equal to the contracted distance $L$, which is measured in this reference frame, divided by the speed $v$ of the particle, or $\Delta t_0 = L/v$.

d. The particle’s contracted lifetime $\Delta t$ is related to its proper lifetime $\Delta t_0$ by the time-dilation formula, $\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2}$ (Equation 28.1).

**SOLUTION**

a. The proper distance is $L_0 = 1.05 \times 10^{-3} \text{ m}$.

b. The distance measured by a hypothetical person traveling with the particle is
\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (1.05 \times 10^{-3} \text{ m}) \sqrt{1 - \left(\frac{0.990c}{c}\right)^2} = 1.48 \times 10^{-4} \text{ m} \] (28.2)

c. The proper lifetime \( \Delta t_0 \) is equal to the contracted distance \( L \) divided by the speed \( v \) of the particle:

\[ \Delta t_0 = L \left( \frac{1.48 \times 10^{-4} \text{ m}}{(0.990)(3.00 \times 10^8 \text{ m/s})} \right) = \frac{4.98 \times 10^{-13} \text{ s}}{0.990c} \]

\[ \Delta t = \Delta t_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{\left(4.98 \times 10^{-13} \text{ s}\right)}{\sqrt{1 - \left(\frac{0.990c}{c}\right)^2}} = 3.53 \times 10^{-12} \text{ s} \] (28.1)

15. **REASONING** Length contraction occurs only along the direction of the motion. Those dimensions that are perpendicular to the motion are not contracted. In this problem, then, we expect side \( x_0 \) to be contracted to a value of \( x \), whereas side \( y_0 \), being perpendicular, will be unaffected by the motion. It is because of the contraction of side \( x_0 \) that the person aboard the rocket will measure an angle \( \theta \) that is different than 30.0º. To determine \( \theta \), we will use the fact that the tangent of an angle in a right triangle is the opposite side divided by the adjacent side (see Equation 1.3) of the triangle. To take into account the length contraction, we will use the length-contraction equation \( x = x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) (Equation 28.2), where \( v = 0.70c \) is the speed of the rocket with respect to the space station and \( c \) is the speed of light in a vacuum.

**SOLUTION** Since side \( x_0 \) is contracted to a value of \( x \) while side \( y_0 \) is unaffected by the motion and since the tangent of an angle in a right triangle is the opposite side divided by the adjacent side (see Equation 1.3), we know that

\[ \tan \theta = \frac{y_0}{x} \] (1)

In Equation (1) the contracted length \( x \) is related to the length \( x_0 \) by the length-contraction equation \( x = x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) (Equation 28.2). Substituting this expression for \( x \) into Equation (1) gives

\[ \tan \theta = \frac{y_0}{x} = \frac{y_0}{x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}} \] (2)

Note that \( \tan 30.0^\circ = \frac{y_0}{x_0} \), so that Equation (2) becomes
\[
\tan \theta = \frac{y_0}{x_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\tan 30.0^\circ}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\tan 30.0^\circ}{\sqrt{1 - \left(\frac{0.730c}{c}\right)^2}} = 0.845
\]

Thus, we find that
\[\theta = \tan^{-1} 0.845 = 40.2^\circ\]

16. **REASONING** To the observer at rest relative to the cube, its length, width, and height are all equal to the proper length \(L_0 = 0.11\) m of one of the cube’s sides. Suppose that the moving observer is moving parallel to the width of the cube and, therefore, measures a contracted length \(L\) for the width. Note, however, that this observer still measures the proper length \(L_0\) for the other two dimensions of the cube, since they are perpendicular to the direction of motion. The shortened width of the cube is given by \(L = L_0 \sqrt{1 - \frac{v^2}{c^2}}\) (Equation 28.2), where \(v\) is the speed of the observer relative to the cube. Accordingly, the volume \(V = L_0 \times L \times L_0 = L_0^2 L\) of the cube is smaller for the moving observer than it is for the observer at rest relative to the cube. Given the mass \(m\) of the cube, then, the moving observer calculates a density of \(\rho = \frac{m}{V} = \frac{m}{L_0^2 L}\) (Equation 11.1) for the cube that is greater than the density of glass.

**SOLUTION** Substituting \(L = L_0 \sqrt{1 - \frac{v^2}{c^2}}\) (Equation 28.2) into \(\rho = \frac{m}{L_0^2 L}\) (Equation 11.1), we obtain
\[
\rho = \frac{m}{L_0^2 L} = \frac{m}{(L_0^2) L_0 \sqrt{1 - \frac{v^2}{c^2}}} = \frac{m}{L_0^3 \sqrt{1 - \frac{v^2}{c^2}}}
\]

Rearranging Equation (1) and solving for the quantity \(\frac{v^2}{c^2}\), we find that
\[
\frac{m}{\rho L_0^3} = \sqrt{1 - \frac{v^2}{c^2}} \quad \text{or} \quad \left(\frac{m}{\rho L_0^3}\right)^2 = 1 - \frac{v^2}{c^2} \quad \text{or} \quad \frac{v^2}{c^2} = 1 - \left(\frac{m}{\rho L_0^3}\right)^2
\]

Taking the square root of both sides of Equation (2) and solving for \(v\) yields
\[
\frac{v}{c} = \sqrt{1 - \left(\frac{m}{\rho L_0^3}\right)^2} \quad \text{or} \quad v = c \sqrt{1 - \left(\frac{m}{\rho L_0^3}\right)^2} = c \sqrt{1 - \left[\frac{3.2 \text{ kg}}{(7800 \text{ kg/m}^3)(0.11 \text{ m})^3}\right]^2} = 0.951c
\]
17. **REASONING** Only the sides of the rectangle that lie in the direction of motion will experience length contraction. In order to make the rectangle look like a square, each side must have a length of $L = 2.0$ m. Thus, we move along the long side, taking the proper length to be $L_0 = 3.0$ m. We can solve for the speed using Equation 28.2. Then, with this speed, we can use the relation for length contraction to find $L$ for the short side as we move along it.

**SOLUTION** From Equation 28.2, \( L = L_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)} \), we find that

\[
v = c \sqrt{1 - \left(\frac{L}{L_0}\right)^2} = c \sqrt{1 - \left(\frac{2.0 \text{ m}}{3.0 \text{ m}}\right)^2} = 0.75 \, c
\]

Moving at this speed along the short side, we take $L_0 = 2.0$ m and find $L$:

\[
L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = (2.0 \text{ m}) \sqrt{1 - \left(\frac{0.75 \, c}{c}\right)^2} = 1.3 \, \text{m}
\]

The observed dimensions of the rectangle are, therefore, \(3.0 \, \text{m} \times 1.3 \, \text{m}\), since the long side is not contracted due to motion along the short side.

18. **REASONING** The magnitude $p$ of the relativistic momentum of a particle (mass $m$ and speed $v$) is given by Equation 28.3:

\[
p = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \frac{p_0}{\sqrt{1 - v^2 / c^2}} \quad (1)
\]

where we have used the fact that the magnitude $p_0$ of the nonrelativistic momentum of a particle is $p_0 = mv$ (Equation 7.2).

**SOLUTION** When the magnitude of the relativistic momentum of a particle is three times the magnitude of its nonrelativistic momentum, we have $p = 3p_0$, so that Equation (1) becomes

\[
3p_0 = \frac{p_0}{\sqrt{1 - v^2 / c^2}} \quad \text{or} \quad \sqrt{1 - v^2 / c^2} = \frac{1}{3}
\]

Squaring and rearranging the last expression gives

\[
1 - \frac{v^2}{c^2} = \frac{1}{9} \quad \text{or} \quad \frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}
\]

Taking the square root, we find

\[
v = \frac{\sqrt{8}}{\sqrt{9}} \, c = 0.943c = 2.83 \times 10^8 \, \text{m/s}
\]
19. **REASONING** The magnitude of the relativistic momentum $p$ of the proton is related to its relativistic total energy $E$ by $E^2 = p^2 c^2 + m^2 c^4$ (Equation 28.7), where $m = 1.67 \times 10^{-27}$ kg is the mass of a proton (see the inside front cover of the text) and $c$ is the speed of light in a vacuum.

**SOLUTION** Solving Equation 28.7 for $p$, we obtain

$$p^2 c^2 = E^2 - m^2 c^4 \quad \text{or} \quad p^2 = \frac{E^2}{c^2} - m^2 c^2 \quad \text{or} \quad p = \sqrt{\frac{E^2}{c^2} - m^2 c^2}$$

Therefore, the magnitude of the relativistic momentum of the proton is

$$p = \sqrt{\left(\frac{2.7 \times 10^{-10}}{3.00 \times 10^8} \text{ m/s}\right)^2 - \left(1.67 \times 10^{-27} \text{ kg}\right)^2 \left(3.00 \times 10^8 \text{ m/s}\right)^2} = 7.5 \times 10^{-19} \text{ kg} \cdot \text{m/s}$$

20. **REASONING** The magnitude $p_{rel}$ of the relativistic momentum of the spacecraft is given by $p_{rel} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$ (Equation 28.3), where $m$ is the mass of the spacecraft, $v$ is its speed, and $c$ is the speed of light in a vacuum. The numerator of Equation 28.3 is the magnitude of the nonrelativistic momentum $p = mv$ (Equation 7.2), so we have that

$$p_{rel} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{p}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We are told that the pilot measures the proper time interval $\Delta t_0$ between two events to be half the dilated time interval $\Delta t$: $\Delta t_0 = \frac{1}{2} \Delta t$. Thus, the dilated time interval is

$$\Delta t = 2\Delta t_0$$

In general, the proper time interval $\Delta t_0$ and the dilated time interval $\Delta t$ are related by

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**SOLUTION** Substituting Equation (2) into Equation 28.1, and solving for the expression $\sqrt{1 - \frac{v^2}{c^2}}$ that also appears in Equation (1), we obtain
**SPECIAL RELATIVITY**

\[ \Delta t = 2 \Delta t_0 = \frac{\Delta x_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad 2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} \quad (3) \]

Substituting Equation (3) into Equation (1), we find that

\[ p_{\text{rel}} = \frac{p}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{p}{(\frac{1}{2})} = 2p = 2 \left(1.3 \times 10^{13} \text{ kg} \cdot \text{m/s}\right) = 2.6 \times 10^{13} \text{ kg} \cdot \text{m/s} \]

21. **SSM REASONING** The height of the woman as measured by the observer is given by Equation 28.2 as \( h = h_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \), where \( h_0 \) is her proper height. In order to use this equation, we must determine the speed \( v \) of the woman relative to the observer. We are given the magnitude of her relativistic momentum, so we can determine \( v \) from \( p \).

**SOLUTION** According to Equation 28.3 \( p = mv/\sqrt{1 - v^2/c^2} \), so \( mv = p\sqrt{1 - v^2/c^2} \)

Squaring both sides, we have

\[ m^2v^2 = p^2(1 - v^2/c^2) \quad \text{or} \quad m^2v^2 + p^2 \frac{v^2}{c^2} = p^2 \]

\[ v^2 \left( m^2 + \frac{p^2}{c^2} \right) = p^2 \quad \text{or} \quad v^2 = \frac{p^2}{m^2 + \frac{p^2}{c^2}} \]

Solving for \( v \) and substituting values, we have

\[ v = \frac{p}{\sqrt{m^2 + \frac{p^2}{c^2}}} = \frac{2.0 \times 10^{10} \text{ kg} \cdot \text{m/s}}{(55 \text{ kg})^2 + \left(\frac{2.0 \times 10^{10} \text{ kg} \cdot \text{m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2} = 2.3 \times 10^8 \text{ m/s} \]

Then, the height that the observer measures for the woman is

\[ h = h_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = (1.6 \text{ m}) \sqrt{1 - \left(\frac{2.3 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right)^2} = 1.0 \text{ m} \]

22. **REASONING** In special relativity the momentum of a particle is given by Equation 28.3 as \( p = mv/\sqrt{1 - \left(\frac{v^2}{c^2}\right)} \). Because of the \( \sqrt{1 - \left(\frac{v^2}{c^2}\right)} \) term in the denominator, doubling the particle’s speed more than doubles its momentum.
An examination of Equation 28.3 shows that the relativistic momentum is directly proportional to the mass \( m \). Thus, halving the particle’s mass also halves its momentum.

The relation \( p = \frac{mv}{\sqrt{1-v^2/c^2}} \) indicates that the magnitudes of the relativistic momenta for particles a, b, and c are:

\[
p_a = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{1}{2}m(2v) = \frac{mv}{\sqrt{1-(2v)^2/c^2}} = \frac{1}{4}m(4v) = \frac{mv}{\sqrt{1-(4v)^2/c^2}} = \frac{mv}{\sqrt{1-16v^2/c^2}}.
\]

These results show that particle c has the greatest momentum magnitude, followed by particle b and then by particle a.

**SOLUTION** The momenta of the three particles are:

\[
\text{Particle } a \quad p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{1.20 \times 10^{-8} \text{ kg}}{0.200 \text{ m/s}} \left( \frac{3.00 \times 10^8 \text{ m/s}}{c} \right) = 0.735 \text{ kg} \cdot \text{m/s}
\]

\[
\text{Particle } b \quad p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{\frac{1}{2} \times 1.20 \times 10^{-8} \text{ kg}}{0.200 \text{ m/s}} \left( \frac{3.00 \times 10^8 \text{ m/s}}{c} \right) = 0.786 \text{ kg} \cdot \text{m/s}
\]

\[
\text{Particle } c \quad p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{\frac{1}{4} \times 1.20 \times 10^{-8} \text{ kg}}{0.200 \text{ m/s}} \left( \frac{3.00 \times 10^8 \text{ m/s}}{c} \right) = 1.20 \text{ kg} \cdot \text{m/s}
\]

As expected, the ranking of the momenta (largest first) is c, b, a.

23. **SSM REASONING** The magnitude \( p \) of the relativistic momentum of an object is given by \( p = \frac{mv}{\sqrt{1-v^2/c^2}} \) (Equation 28.3), where \( m \) is the object’s mass, \( v \) is the object’s speed, and \( c \) is the speed of light in a vacuum. The principle of conservation of linear momentum (see Section 7.2) states that the total momentum of a system is conserved when no net external force acts on the system. This principle applies at speeds approaching the speed of light in a vacuum, provided that Equation 28.3 is used for the individual momenta of the objects that comprise the system.
SOLUTION The total momentum of the man/woman system is conserved, since friction is negligible, so that no net external force acts on the system. Therefore, the final total momentum \( p_m + p_w \) must equal the initial total momentum, which is zero. As a result, \( p_m = -p_w \) where Equation 28.3 must be used for the momenta \( p_m \) and \( p_w \). Thus, we find

\[
\frac{m_m v_m}{\sqrt{1 - \left(\frac{v_m}{c}\right)^2}} = -\frac{m_w v_w}{\sqrt{1 - \left(\frac{v_w}{c}\right)^2}}
\]  

(1)

We know that \( m_m = 88 \text{ kg}, m_w = 54 \text{ kg}, \) and \( v_w = +2.5 \text{ m/s} \). Remember that \( c \) has the hypothetical value of \( 3.00 \text{ m/s} \). Solving Equation (1) for \( v_m \) reveals that \( v_m = \pm 2.0 \text{ m/s} \). We choose the negative value, since the man and woman recoil from one another and it is stated that the woman moves away in the positive direction. Therefore, we find that \( v_m = -2.0 \text{ m/s} \).

24. **REASONING AND SOLUTION** The mass equivalent is given by \( E_0 = KE = mc^2 \) or

\[
m = \frac{KE}{c^2} = \frac{7.8 \times 10^{-13} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 8.7 \times 10^{-30} \text{ kg}
\]

(28.5)

25. **SSM REASONING** According to the work-energy theorem, Equation 6.3, the work that must be done on the electron to accelerate it from rest to a speed of 0.990c is equal to the kinetic energy of the electron when it is moving at 0.990c.

**SOLUTION** Using Equation 28.6, we find that

\[
KE = mc^2 \left( \frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right) - 1}} \right)
\]

\[
= (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left( \frac{1}{\sqrt{1 - (0.990c)^2 / c^2} - 1} \right) = 5.0 \times 10^{-13} \text{ J}
\]

26. **REASONING** Compressing the spring stores elastic potential energy, which increases the total energy of the spring. Because the spring is not in motion, it is the spring’s rest energy \( E_0 \) that increases, as well as its mass \( m \), according to \( E_0 = mc^2 \) (Equation 28.5). We will use the hypothetical value of \( c = 3.00 \times 10^2 \text{ m/s} \) for the speed of light in a vacuum in Equation...
28.5. The increase of $\Delta m = 0.010 \text{ g} = 0.010 \times 10^{-3} \text{ kg}$ in the mass of the spring, therefore, corresponds to an increase $\Delta E_0$ in the rest energy of the spring, where

$$\Delta E_0 = (\Delta m)c^2$$

(1)

The increase of the rest energy is equal to the spring’s elastic potential energy $PE_{\text{elastic}} = \frac{1}{2}kx^2$ (Equation 10.13), where $k$ is the spring constant of the spring, and $x$ is the distance by which the spring is compressed from its equilibrium length. Therefore, we have that

$$\Delta E_0 = PE_{\text{elastic}} = \frac{1}{2}kx^2$$

(2)

**SOLUTION** Setting the right sides of Equations (1) and (2) equal to one another and solving for $x^2$ yields

$$\Delta E_0 = \frac{1}{2}kx^2 = (\Delta m)c^2 \quad \text{or} \quad x^2 = \frac{2(\Delta m)c^2}{k}$$

(3)

Taking the square root of Equation (3), we obtain

$$x = \sqrt{\frac{2(\Delta m)c^2}{k}} = c\sqrt{\frac{2(\Delta m)}{k}} \left( 3.00 \times 10^2 \text{ m/s} \right) \sqrt{\frac{2(0.010 \times 10^{-3} \text{ kg})}{850 \text{ N/m}}} = 0.046 \text{ m}$$

27. **REASONING** The mass $m$ of the aspirin is related to its rest energy $E_0$ by Equation 28.5, $E_0 = mc^2$. Since it requires $1.1 \times 10^8 \text{ J}$ to operate the car for twenty miles, we can calculate the number of miles that the car can go on the energy that is equivalent to the mass of one tablet.

**SOLUTION** We begin by converting the mass $m$ from milligrams (mg) to kilograms (kg):

$$\left( 325 \text{ mg} \right) \left( \frac{1 \text{ g}}{1000 \text{ mg}} \right) \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) = 325 \times 10^{-6} \text{ kg}$$

The number $N$ of miles the car can go on one aspirin tablet is

$$N = \frac{E_0}{\left(1.1 \times 10^8 \text{ J}\right)/\left(20.0 \text{ mi}\right)} = \frac{mc^2}{\left(1.1 \times 10^8 \text{ J}\right)/\left(20.0 \text{ mi}\right)}$$

$$= \frac{(325 \times 10^{-6} \text{ kg}) \left(3.0 \times 10^8 \text{ m/s}\right)^2}{\left(1.1 \times 10^8 \text{ J}\right)/\left(20.0 \text{ mi}\right)} = 5.3 \times 10^6 \text{ mi}$$

28. **REASONING** In order to change a certain mass of ice at 0 °C into liquid water at 0 °C heat must be added, and heat is a form of energy. Therefore, the energy of the liquid water is
greater than that of the ice. According to special relativity, energy and mass are equivalent. Since the liquid water has the greater energy, it also has the greater mass.

Heat must also be added to boil water into steam. Following the same type of reasoning as in the case of melting, we conclude that the steam has the greater mass.

The amount of heat $Q$ that must be supplied to change the phase of $m$ kilograms of a substance is given by Equation 12.5 as $Q = mL$, where $L$ is the latent heat of the substance. Since the latent heat of vaporization $L_v$ for water is greater than the latent heat of fusion $L_f$ (see Table 12.3), the change in mass is greater when liquid water turns into steam at 100 °C than when ice turns into liquid water at 0 °C.

**SOLUTION** The change in mass $\Delta m$ associated with a change in rest energy $\Delta E_0$ is given by Equation 28.5 as $\Delta m = \Delta E_0/c^2$. The change in rest energy is the heat $Q$ that must be added to change the phase of the water, so that $Q = mL$. Thus, the change in mass is $\Delta m = Q/c^2 = mL/c^2$.

a. According to Table 12.3, the latent heat of fusion for water is $L_f = 3.35 \times 10^5$ J/kg. The change in mass associated with the ice to liquid-water phase change at 0 °C is, then,

$$\Delta m = \frac{mL_f}{c^2} = \frac{(2.00 \text{ kg})(3.35 \times 10^5 \text{ J/kg})}{(3.00 \times 10^8 \text{ m/s})^2} = 7.44 \times 10^{-12} \text{ kg}$$

b. According to Table 12.3, the latent heat of vaporization for water is $L_v = 2.26 \times 10^6$ J/kg. The change in mass associated with the liquid-water to steam phase change at 100 °C is

$$\Delta m = \frac{mL_v}{c^2} = \frac{(2.00 \text{ kg})(2.26 \times 10^6 \text{ J/kg})}{(3.00 \times 10^8 \text{ m/s})^2} = 5.02 \times 10^{-11} \text{ kg}$$

As expected, the change in mass for the liquid-to-steam phase change is greater than that for the ice-to-liquid phase change.

29. **SSM REASONING AND SOLUTION**

a. In Section 28.6 it is shown that when the speed of a particle is $0.01c$ (or less), the relativistic kinetic energy becomes nearly equal to the nonrelativistic kinetic energy. Since the speed of the particle here is $0.001c$, the ratio of the relativistic kinetic energy to the nonrelativistic kinetic energy is $\frac{1}{10}$.

b. Taking the ratio of the relativistic kinetic energy, Equation 28.6, to the nonrelativistic kinetic energy, $\frac{1}{2} mv^2$, we find that
30. **REASONING AND SOLUTION** The energy $E_0$ produced in one year is the product of the power $P$ generated and the time $t$, $E_0 = P \cdot t$. This energy is equivalent to an amount of mass $m$ given by Equation 28.5 as $E_0 = mc^2$. Thus, we have that
\[
m c^2 = P t \quad \text{or} \quad m = \frac{P t}{c^2}
\]
The mass of nuclear fuel consumed in one year ($3.15 \times 10^7$ s) is
\[
m = \frac{P t}{c^2} = \frac{(3.0 \times 10^9 \text{ W})(3.15 \times 10^7 \text{ s})}{(3.00 \times 10^8 \text{ m/s})^2} = 1.1 \text{ kg}
\]

31. **REASONING** The rate $R$ that the quasar is losing mass is equal to the mass $m$ it loses divided by the time $t$ during which the loss occurs, or $R = m/t$. The mass that the quasar loses is equivalent to a certain amount of energy $E_0$; this equivalency is expressed by $m = E_0/c^2$ (Equation 28.5), where $c$ is the speed of light. According to Equation 6.10b, the energy radiated is equal to the average power $\bar{P}$ times the time, $E_0 = \bar{P}t$. The average power is the rate at which the quasar radiates energy.

**SOLUTION** Combining $R = m/t$ and $m = E_0/c^2$, we find that
\[
R = \frac{m}{t} = \frac{E_0}{c^2} \cdot \frac{1}{t} = \frac{E_0}{c^2 t}
\]
Since the energy radiated by the quasar is $E_0 = \bar{P}t$, the rate at which the quasar loses mass can be written as
\[
R = \frac{1}{c^2} \left( \frac{E_0}{t} \right) = \frac{1}{c^2} \left( \frac{\bar{P}t}{t} \right) = \frac{\bar{P}}{c^2} = \frac{1.0 \times 10^{41} \text{ W}}{(3.00 \times 10^8 \text{ m/s})^2} = 1.1 \times 10^{24} \text{ kg/s}
\]
REASONING According to the work-energy theorem (see Section 6.2), the change in the kinetic energy equals the work \( W \) done to accelerate the electron. Since the electron starts from rest, its initial kinetic energy is zero, so its final kinetic energy \( KE \) equals the work done. This conclusion is valid for either nonrelativistic or relativistic speeds. The given potential difference has a magnitude of \( |\Delta V| = 2.40 \times 10^7 \) V and is related to the work done according to \( |\Delta V| = W / |q_0| \) (Equation 19.4 without the minus sign), where \( |q_0| = 1.60 \times 10^{-19} \) C is the magnitude of the charge of the electron. The relativistic kinetic energy of the electron is \( KE = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \) (Equation 28.6), where \( m = 9.11 \times 10^{-31} \) kg is the electron’s mass, \( v \) is the electron’s speed, and \( c \) is the speed of light in a vacuum.

SOLUTION

a. Recognizing that the electron’s final kinetic energy \( KE \) equals the work \( W \) done to accelerate the electron, we have

\[
KE = W
\]

Using Equation 19.4 to obtain the work, we have

\[
|\Delta V| = \frac{W}{|q_0|} \quad \text{or} \quad W = |\Delta V||q_0|
\]

Substituting Equation (2) into Equation (1), we find that the relativistic kinetic energy of the electron is

\[
KE = W = |\Delta V||q_0| = \left(2.40 \times 10^7 \right)\left(1.60 \times 10^{-19} \right) = 3.84 \times 10^{-12} \text{ J}
\]

b. Solving Equation 28.6 for the speed \( v \), we proceed as follows:

\[
KE = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \quad \text{or} \quad \frac{1}{\sqrt{1-v^2/c^2}} = \frac{KE}{mc^2} + 1
\]

or

\[
1 - v^2/c^2 = \left[\frac{1}{(KE)/(mc^2) + 1}\right]^2 \quad \text{or} \quad v = c\sqrt{1 - \left[\frac{1}{(KE)/(mc^2) + 1}\right]^2}
\]

or

\[
v = c\sqrt{1 - \left[\frac{1}{3.84 \times 10^{-12} \left(9.11 \times 10^{-31} \right)\left(3.00 \times 10^8 \right)^2 + 1}\right]^2} = 0.999781 c
\]
33. **REASONING** The total energy $E$ and the magnitude $p$ of the relativistic momentum are related according to Equation 28.7:

$$E^2 = p^2c^2 + m^2c^4 \quad \text{or} \quad p^2 = \frac{E^2 - m^2c^4}{c^2} \quad (28.7)$$

We are given a value for the total energy, but do not have a value for the mass $m$. However, we recognize that the rest energy is $E_0 = mc^2$ (Equation 28.5). With this substitution, Equation 28.7 becomes

$$p^2 = \frac{E^2 - m^2c^4}{c^2} = \frac{E^2 - E_0^2}{c^2}$$

We can obtain a value for the rest energy, because the total energy is the sum of the kinetic energy and the rest energy or $E = KE + E_0$. In other words, the rest energy is $E_0 = E - KE$, and we have values for both $E$ and $KE$. Using this substitution for the rest energy, our expression for $p^2$ becomes

$$p^2 = \frac{E^2 - E_0^2}{c^2} = \frac{E^2 - (E - KE)^2}{c^2} \quad (1)$$

**SOLUTION** Using Equation (1), we find that

$$p = \sqrt{\frac{E^2 - (E - KE)^2}{c^2}} = \sqrt{\left(\frac{5.0 \times 10^{15} \text{ J}}{3.0 \times 10^8 \text{ m/s}}\right)^2 - \left[\left(5.0 \times 10^{15} \text{ J}\right) - \left(2.0 \times 10^{15} \text{ J}\right)\right]^2} = 1.3 \times 10^7 \text{ kg} \cdot \text{m/s}$$

34. **REASONING** If the speeds of your car and the truck are much less than the speed of light, the relative speed at which the truck approaches you is the same in parts (a) and (b). Let’s suppose that you are traveling due east, which is taken to be the positive direction, and the truck is traveling due west, which is the negative direction. The relative velocities are:

- $v_{TC} = \text{velocity of the Truck relative to the Car}$
- $v_{TG} = \text{velocity of the Truck relative to the Ground}$
- $v_{CG} = \text{velocity of the Car relative to the Ground}$ (Note that the velocity $v_{GC}$ of the Ground relative to the Car is $v_{GC} = -v_{CG}$.)

The velocity $v_{TC}$ of the truck with respect to the car is equal to the velocity $v_{TG}$ of the truck with respect to the ground plus the velocity $v_{GC}$ of the ground with respect to the car: $v_{TC} = v_{TG} + v_{GC}$, as discussed in Section 3.4.

When $v_{TG} = -35 \text{ m/s}$ and $v_{GC} = -25 \text{ m/s}$, the velocity of the truck relative to the car is $v_{TC} = -60 \text{ m/s}$, where the minus sign indicates that the relative velocity is westward.
When \( v_{TG} = -55 \text{ m/s} \) and \( v_{CG} = +5.0 \text{ m/s} \), the relative velocity of the truck with respect to the car is still \( v_{TC} = v_{TG} + v_{GC} = -55 \text{ m/s } - 5 \text{ m/s} = -60 \text{ m/s} \). In either case, the speed is the magnitude of \( v_{TC} \), or 60 m/s.

However, the relative velocities and, hence, the relative speeds would not be the same in parts (a) and (b) if the speeds were comparable to the speed of light. According to special relativity, the correct relation is the velocity-addition formula, Equation 28.8:

\[
v_{TC} = \frac{v_{TG} + v_{GC}}{1 + \frac{v_{TG}v_{GC}}{c^2}}
\]

Because of the presence of the term \( v_{TG}v_{GC}/c^2 \) in the denominator, different results are obtained when \( v_{TG} = -35 \text{ m/s} \) and \( v_{GC} = -25 \text{ m/s} \) than when \( v_{TG} = -55 \text{ m/s} \) and \( v_{GC} = -5.0 \text{ m/s} \).

**SOLUTION**

a. When \( v_{TG} = -35 \text{ m/s} \) and \( v_{GC} = -25 \text{ m/s} \), the velocity of the truck relative to the car is

\[
v_{TC} = \frac{v_{TG} + v_{GC}}{1 + \frac{v_{TG}v_{GC}}{c^2}} = \frac{-35 \text{ m/s } - 25 \text{ m/s}}{1 + \frac{(35 \text{ m/s})(25 \text{ m/s})}{(65 \text{ m/s})^2}} = -49.7 \text{ m/s}
\]

The speed of the truck relative to the car is the magnitude of this result, or \( 49.7 \text{ m/s} \).

b. When \( v_{TG} = -55 \text{ m/s} \) and \( v_{GC} = -5.0 \text{ m/s} \), the velocity of the truck relative to the car is

\[
v_{TC} = \frac{v_{TG} + v_{GC}}{1 + \frac{v_{TG}v_{GC}}{c^2}} = \frac{-55 \text{ m/s } - 5.0 \text{ m/s}}{1 + \frac{(55 \text{ m/s})(5.0 \text{ m/s})}{(65 \text{ m/s})^2}} = -56.3 \text{ m/s}
\]

The speed of the truck relative to the car is the magnitude of this result, or \( 56.3 \text{ m/s} \).

35. **REASONING** Let’s define the following relative velocities, assuming that the spaceship and exploration vehicle are moving in the positive direction.

- \( v_{ES} \) = velocity of Exploration vehicle relative to the Spaceship.
- \( v_{EO} \) = velocity of Exploration vehicle relative to an Observer on earth = +0.70c
- \( v_{SO} \) = velocity of Spaceship relative to an Observer on earth = +0.50c

The velocity \( v_{ES} \) can be determined from the velocity-addition formula, Equation 28.8:
The velocity \( v_{OS} \) of the observer on earth relative to the spaceship is not given. However, we know that \( v_{OS} \) is the negative of \( v_{SO} \), so \( v_{OS} = -v_{SO} = -(+0.50c) = -0.50c \).

**SOLUTION** The velocity of the exploration vehicle relative to the spaceship is

\[
v_{ES} = \frac{v_{EO} + v_{OS}}{1 + \frac{v_{EO}v_{OS}}{c^2}}
\]

The speed of the exploration vehicle relative to the spaceship is the magnitude of this result or \( 0.31c \).

36. **REASONING** We assume that the direction away from the earth is the positive direction. With this assumption, we have the following relative velocities:

- \( v_{IE} \): the velocity of ions relative to the ship
- \( v_{IS} \): the velocity of ions relative to the spaceship = \(-0.80c\).
- \( v_{SE} \): the velocity of spaceship relative to earth = \(+0.70c\)

Note that the velocity \( v_{IE} \) of the ions relative to the spaceship is negative because the spaceship is moving away from the earth (in the positive direction), and the ions are emitted from the engine in the opposite or negative direction. It is the velocity \( v_{IE} \) that we seek.

**SOLUTION** These velocities defined in the **REASONING** are related by the velocity-addition formula, Equation 28.7, according to which we have

\[
v_{IE} = \frac{v_{IS} + v_{SE}}{1 + \frac{v_{IS}v_{SE}}{c^2}} = \frac{-0.80c + 0.70c}{1 + \frac{(-0.80c)(+0.70c)}{c^2}} = -0.23c
\]

37. **REASONING** The velocity of the *Enterprise 2*, as measured by an earth-based observer, is given by

\[
v_{2e} = \frac{v_{21} + v_{1e}}{1 + \frac{v_{21}v_{1e}}{c^2}} \quad (28.8)
\]

where

- \( v_{2e} \): velocity of *Enterprise 2* relative to earth
- \( v_{21} \): velocity of *Enterprise 2* relative to *Enterprise 1*
- \( v_{1e} \): velocity of *Enterprise 1* relative to earth
All of these variables are known, so \( v_{2e} \) can be determined.

**SOLUTION** The velocity of *Enterprise 2* relative to the earth is

\[
v_{2e} = \frac{v_{2l} + v_{1e}}{1 + \frac{v_{2l}v_{1e}}{c^2}} = \frac{(+0.31c) + (+0.65c)}{1 + \frac{(0.31c)(0.65c)}{c^2}} = +0.80c
\]

38. **REASONING** We define the following relative velocities, assuming that the rocket approaching the earth from the right is traveling in the positive direction:

- \( v_{RL} \) = velocity of the Right rocket relative to the Left rocket
- \( v_{RE} \) = velocity of the Right rocket relative to the person on Earth = +0.75c
- \( v_{LE} \) = velocity of the Left rocket relative to the person on Earth = –0.65c

The velocity \( v_{RL} \) can be found from the velocity-addition formula, Equation 28.8:

\[
v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE}v_{EL}}{c^2}}
\]

The velocity \( v_{RE} \) is given, but \( v_{EL} \), the velocity of the earth relative to the left rocket, is not. However, we know that \( v_{EL} \) is the negative of \( v_{LE} \), so \( v_{EL} = –v_{LE} = –(–0.65c) = +0.65c \).

**SOLUTION** The velocity of the right rocket relative to the left rocket is

\[
v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE}v_{EL}}{c^2}} = \frac{+0.75c + 0.65c}{1 + \frac{(0.75c)(0.65c)}{c^2}} = +0.94c
\]

The relative speed between the two rockets is the magnitude of this result, or \[0.94c\].

39. **SSM REASONING AND SOLUTION**

In all parts of this problem, the direction of the intergalactic cruiser, the ions, and the laser light is taken to be the positive direction.

a. According to the second postulate of special relativity, all observers measure the speed of light to be \( c \), regardless of their velocities relative to each other. Therefore, the aliens aboard the hostile spacecraft see the photons of the laser approach \[ at the speed of light, c \].

b. To find the velocity of the ions relative to the aliens, we define the relative velocities as follows:
\( v_{IS} = \text{velocity of the Ions relative to the alien Spacecraft} \)
\( v_{IC} = \text{velocity of the Ions relative to the intergalactic Cruiser} = +0.950c \)
\( v_{CS} = \text{velocity of the intergalactic Cruiser relative to the alien Spacecraft} = +0.800c \)

These velocities are related by the velocity-addition formula, Equation 28.8. The velocity of the ions relative to the alien spacecraft is:

\[
\begin{align*}
v_{IS} &= \frac{v_{IC} + v_{CS}}{1 + \frac{v_{IC}v_{CS}}{c^2}} \\
&= \frac{+0.950c + 0.800c}{1 + \frac{(+0.950c)(+0.800c)}{c^2}} = +0.994c
\end{align*}
\]

c. The aliens see the laser light (photons) moving with respect to the cruiser at a velocity

\[
U = +1.000c - 0.800c = +0.200c
\]

d. The aliens see the ions moving away from the cruiser at a velocity

\[
U' = +0.994c - 0.800c = +0.194c
\]

40. **REASONING** The passengers would measure a length for the spaceships that does not match the constructed length if the two ships have a nonzero relative speed. This would mean that the ships are moving with respect to one another. If so, the phenomenon of length contraction would occur, and the passengers in either ship would measure a contracted length.

To calculate the contracted length \( L \), the length contraction formula must be used, as given in Equation 28.2:

\[
L = L_0 \sqrt{1 - \frac{v_{AB}^2}{c^2}} \tag{28.2}
\]

where \( L_0 \) is the proper length of the spaceships, that is, the length measured by an observer at rest with respect to them. In other words, the proper length is the constructed length. The velocity \( v_{AB} \) in Equation 28.2 is the velocity of spaceship A with respect to spaceship B. It can be obtained from the velocities of each ship with respect to the earth by using the velocity addition equation, as given in Equation 28.8:

\[
v_{AB} = \frac{v_{AE} + v_{EB}}{1 + \frac{v_{AE}v_{EB}}{c^2}} \tag{28.8}
\]

where \( v_{AE} \) is the velocity of spaceship A with respect to the earth and \( v_{EB} \) is the velocity of the earth with respect to spaceship B. We note that \( v_{EB} = -v_{BE} \), where \( v_{BE} \) is the velocity of spaceship B with respect to the earth. Substituting \( v_{EB} = -v_{BE} \) into Equation 28.8 gives
\[ v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} \quad (1) \]

**SOLUTION** Using Equation (1) to calculate \( v_{AB} \) for use in Equation 28.2, we find

\[ v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} = \frac{0.850c - 0.500c}{1 - (0.850c)(0.500c)} = 0.609c \]

Here we have taken the direction in which the spaceships are traveling to be the positive direction. Substituting this result into Equation 28.2 reveals that

\[ L = L_0 \sqrt{1 - \frac{v_{AB}^2}{c^2}} = (1.50 \text{ km}) \sqrt{1 - \frac{(0.609c)^2}{c^2}} = 1.19 \text{ km} \]

41. **REASONING**

The following relative velocities are pertinent to this problem. It is assumed that particle 1 is moving in the positive direction, and, therefore, particle 2 is moving in the negative direction.

- \( v_{P_{12}} \) = velocity of particle 1 (\( P_1 \)) relative to particle 2 (\( P_2 \))
- \( v_{P_{1L}} \) = velocity of particle 1 (\( P_1 \)) relative to an observer in the Laboratory
  \[ = +2.10 \times 10^8 \text{ m/s} \]
- \( v_{P_{2L}} \) = velocity of particle 2 (\( P_2 \)) relative to an observer in the Laboratory
  \[ = -2.10 \times 10^8 \text{ m/s} \]

The velocity \( v_{P_{12}} \) can be obtained from the velocity-addition formula, Equation 28.8:

\[ v_{P_{12}} = \frac{v_{P_{1L}} + v_{P_{2L}}}{1 + \frac{v_{P_{1L}}v_{P_{2L}}}{c^2}} \]

The velocity \( v_{P_{1L}} \) is given, but \( v_{P_{2L}} \), the velocity of the laboratory observer relative to particle 2, is not. However, we know that \( v_{P_{2L}} \) is the negative of \( v_{P_{2L}} \), so \( v_{P_{2L}} = -v_{P_{2L}} = -(-2.10 \times 10^8 \text{ m/s}) = +2.10 \times 10^8 \text{ m/s} \).

**SOLUTION**

a. According to the velocity-addition formula, the velocity of particle 1 relative to particle 2 is

\[ v_{P_{12}} = \frac{v_{P_{1L}} + v_{P_{2L}}}{1 + \frac{v_{P_{1L}}v_{P_{2L}}}{c^2}} = \frac{+2.10 \times 10^8 \text{ m/s} + 2.10 \times 10^8 \text{ m/s}}{1 + \frac{(+2.10 \times 10^8 \text{ m/s})(+2.10 \times 10^8 \text{ m/s})}{c^2}} = +2.82 \times 10^8 \text{ m/s} \]
The speed of one particle as seen by the other particle is the magnitude of this result, or $2.82 \times 10^8 \text{ m/s}$.

b. The relativistic momentum is given by Equation 28.3, where the speed is that determined in part a. Therefore,

$$p = \frac{mv_{p_2}}{\sqrt{1 - \left(\frac{v_{p_2}}{c}\right)^2}} = \frac{\left(2.16 \times 10^{-25} \text{ kg}\right) \left(+2.82 \times 10^8 \text{ m/s}\right)}{\sqrt{1 - \left(\frac{2.82 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} = 1.8 \times 10^{-16} \text{ kg} \cdot \text{m/s}$$

42. **REASONING** The total energy $E$ for each particle is $E = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}$ (Equation 28.4), where $m = 9.11 \times 10^{-31} \text{ kg}$ is the mass of each particle, $v = 0.20c$ is the speed of each particle, and $c$ is the speed of light in a vacuum. The total energy of the electromagnetic radiation that appears after the collision is twice the value given by Equation 28.4. Note that we will not use $E_0 = mc^2$ (Equation 28.5), which gives the rest energy $E_0$, because the particles are moving and are not at rest.

**SOLUTION** Applying Equation 28.4 for the total energy of each particle, we find that the energy of the electromagnetic radiation that appears after the collision is

$$E_{\text{electromagnetic radiation}} = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{2mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{2 \left(9.11 \times 10^{-31} \text{ kg}\right) \left(3.00 \times 10^8 \text{ m/s}\right)^2}{\sqrt{1 - \left(\frac{0.20c}{c}\right)^2}} = 1.7 \times 10^{-13} \text{ J}$$

43. **SSM REASONING** Assume that traveler A moves at a speed of $v_A = 0.70c$ and traveler B moves at a speed of $v_B = 0.90c$, both speeds being with respect to the earth. Each traveler is moving with respect to the earth and the distant star, so each measures a contracted length $L_A$ or $L_B$ for the distance traveled. However, an observer on earth is at rest with respect to the earth and the distant star (which is assumed to be stationary with respect to the earth), so he or she would measure the proper length $L_0$. For each traveler the contracted length is given by the length-contraction equation as stated in Equation 28.2:
\[ L_A = L_0 \sqrt{1 - \frac{v_A^2}{c^2}} \quad \text{and} \quad L_B = L_0 \sqrt{1 - \frac{v_B^2}{c^2}} \]

It is important to note that the proper length \( L_0 \) is the same in each application of the length-contraction equation. Thus, we can combine the two equations and eliminate it. Then, since we are given values for \( L_A, v_A, \) and \( v_B, \) we will be able to determine \( L_B. \)

**SOLUTION** Dividing the expression for \( L_B \) by the expression for \( L_A \) and eliminating \( L_0, \) we obtain

\[
\frac{L_B}{L_A} = \frac{L_0 \sqrt{1 - \frac{v_B^2}{c^2}}}{L_0 \sqrt{1 - \frac{v_A^2}{c^2}}} = \frac{\sqrt{1 - \frac{v_B^2}{c^2}}}{\sqrt{1 - \frac{v_A^2}{c^2}}}
\]

\[
L_B = L_A \sqrt{\frac{1 - \frac{v_B^2}{c^2}}{1 - \frac{v_A^2}{c^2}}} = 6.5 \text{ light-years}
\]

\[
L_B = L_A \sqrt{\frac{1 - \frac{(0.90c)^2}{c^2}}{1 - \frac{(0.70c)^2}{c^2}}} = 4.0 \text{ light-years}
\]

44. **REASONING** The relativistic momentum \( p \) of an object of mass \( m \) is given by

\[ p = \frac{mv}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \] (Equation 28.3), where \( v \) is the speed of the object and \( c \) is the speed of light in a vacuum. For part (a), we will solve Equation 28.3 to determine the mass of the ion. In part (b), we will use the mass determined in part (a) in Equation 28.3 to calculate the relativistic momentum of the ion at its final speed.

**SOLUTION**

a. Solving Equation 28.3 for \( m \) and using the initial speed of \( v = 0.460c, \) we obtain

\[
m = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{v} \left( \frac{5.08 \times 10^{-17} \text{ kg} \cdot \text{m/s}}{0.460 (3.00 \times 10^8 \text{ m/s})} \right) = 3.27 \times 10^{-25} \text{ kg}
\]

b. Substituting the value for \( m \) found in part (a) and the final speed of \( v = 0.920c \) into Equation 28.3 yields
45. **REASONING** The expression for time dilation is, according to Equation 28.1,

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

For a given event, it relates the proper time interval \( \Delta t_0 \) to the time interval \( \Delta t \) that would be measured by an observer moving at a speed \( v \) relative to the frame of reference in which the event takes place.

We must consider two situations; in the first situation, the Klingon spacecraft has a speed of 0.75\( c \) with respect to the earth. In the second situation, the craft has a speed of 0.94\( c \) relative to the earth. We will refer to these two situations as A and B, respectively.

Since the proper time interval always has the same value, \((\Delta t_0)_A = (\Delta t_0)_B\). We can express both sides of this expression using Equation 28.1. The result can be solved for \( \Delta t_B \).

**SOLUTION** Use of Equation 28.1 gives

\[ \Delta t_A \sqrt{1 - \frac{v_A^2}{c^2}} = \Delta t_B \sqrt{1 - \frac{v_B^2}{c^2}} \]

\[ \Delta t_B = \Delta t_A \frac{\sqrt{1 - \frac{v_A^2}{c^2}}}{\sqrt{1 - \frac{v_B^2}{c^2}}} = \Delta t_A \frac{1 - \frac{(v_A / c)^2}{1 - \frac{(0.75c / c)^2}{1 - \frac{(0.94c / c)^2}}}{1 - \frac{(0.94c / c)^2}}}{1 - \frac{(0.75c / c)^2}} = 72 \text{ h} \]

46. **REASONING** The total linear momentum of the system is conserved, since no net external force acts on the system. Therefore, the final total momentum \( p_1 + p_2 \) of the two fragments must equal the initial total momentum, which is zero since the particle is initially at rest. As a result, \( p_1 = -p_2 \), where Equation 28.3 must be used for the magnitudes of the momenta \( p_1 \) and \( p_2 \). Thus, we find

\[ \frac{m_1 v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{-m_2 v_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \]

**SOLUTION** Letting fragment 2 be the more-massive fragment, we have that \( v_1 = +0.800c \), \( m_1 = 1.67 \times 10^{-27} \text{ kg} \), and \( m_2 = 5.01 \times 10^{-27} \text{ kg} \). Squaring both sides of the above equation, rearranging terms, substituting the known values for \( v_1 \), \( m_1 \), and \( m_2 \), we find that
\[
\frac{v_2^2}{1 - (v_2^2/c^2)} = \frac{m_1^2 v_1^2}{m_2^2 [1 - (v_1^2/c^2)]} = \frac{(1.67 \times 10^{-27} \text{ kg})^2 (+0.800c)^2}{(5.01 \times 10^{-27} \text{ kg})^2 [1 - (+0.800c/c)^2]} = 0.1975c^2
\]

Therefore,
\[
v_2^2 = 0.1975c^2 [1 - (v_2^2/c^2)] = 0.1975c^2 - 0.1975v_2^2
\]
Solving for \( v_2 \) gives
\[
v_2 = \pm \sqrt{\frac{0.1975c^2}{1.1975}} = \pm 0.406c
\]
We reject the positive root, since then both fragments would be moving in the same direction after the break-up and the system would have a non-zero momentum. According to the principle of conservation of momentum, the total momentum after the break-up must be zero, just as it was before the break-up. The momentum of the system will be zero only if the velocity \( v_2 \) is opposite to the velocity \( v_1 \). Hence, we chose the negative root and
\[
v_2 = -0.406c
\]

47. SSM REASONING Since the crew is initially at rest relative to the escape pod, the length of 45 m is the proper length \( L_0 \) of the pod. The length of the escape pod as determined by an observer on earth can be obtained from the relation for length contraction given by Equation 28.2, \( L = L_0 \sqrt{1 - \left(v_{PE}^2/c^2\right)} \). The quantity \( v_{PE} \) is the speed of the escape pod relative to the earth, which can be found from the velocity-addition formula, Equation 28.8. The following are the relative velocities, assuming that the direction away from the earth is the positive direction:

\[
\begin{align*}
v_{PE} &= \text{velocity of the escape Pod relative to Earth.} \\
v_{PR} &= \text{velocity of escape Pod relative to the Rocket} = -0.55c. \text{ This velocity is negative because the rocket is moving away from the earth (in the positive direction), and the escape pod is moving in an opposite direction (the negative direction) relative to the rocket.} \\
v_{RE} &= \text{velocity of Rocket relative to Earth} = +0.75c
\end{align*}
\]
These velocities are related by the velocity-addition formula, Equation 28.8.

SOLUTION The relative velocity of the escape pod relative to the earth is
\[
v_{PE} = \frac{v_{PR} + v_{RE}}{1 + \frac{v_{PR} v_{RE}}{c^2}} = \frac{-0.55c + 0.75c}{1 + \frac{(-0.55c)(0.75c)}{c^2}} = +0.34c
\]
The speed of the pod relative to the earth is the magnitude of this result, or 0.34c. The length of the pod as determined by an observer on earth is

\[ L = L_0 \sqrt{1 - \frac{v_{PE}^2}{c^2}} = (45 \text{ m}) \sqrt{1 - \left(\frac{0.34c}{c}\right)^2} = 42 \text{ m} \]

48. **REASONING** The total relativistic energy \( E \) is related to the rest energy \( E_0 = mc^2 \) and the speed \( v \) according to Equation 28.4:

\[ E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(28.4)

A greater total energy \( E \) does not necessarily mean that an object has a greater speed. It is not the total energy alone that matters, but the ratio \( E/E_0 \) of the total energy to the rest energy. According to Equation 28.4 this ratio is

\[ \frac{E}{E_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(1)

Larger values for this ratio mean that the speed \( v \) is greater. When the speed is greater, square root term in the denominator on the right-hand side of Equation (1) is smaller, so the reciprocal of the square root term is larger.

By considering the ratio given in Equation (1), we can rank the speeds of the objects. This ratio is listed in the following table:

<table>
<thead>
<tr>
<th>Object</th>
<th>Total Energy ((E))</th>
<th>Rest Energy ((E_0))</th>
<th>(E/E_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.00(\varepsilon)</td>
<td>(\varepsilon)</td>
<td>2.00</td>
</tr>
<tr>
<td>B</td>
<td>3.00(\varepsilon)</td>
<td>(\varepsilon)</td>
<td>3.00</td>
</tr>
<tr>
<td>C</td>
<td>3.00(\varepsilon)</td>
<td>2.00(\varepsilon)</td>
<td>1.50</td>
</tr>
</tbody>
</table>

The ratio \(E/E_0\) is greatest for object B and smallest for object C. Thus, the ranking of the speeds is

B (largest), A, C

**SOLUTION** Solving Equation (1) for the speed \( v \), we obtain
\[ \frac{E}{E_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \left( \frac{E}{E_0} \right)^2 = \frac{1}{1 - \frac{v^2}{c^2}} \quad \text{or} \quad \frac{1}{(E/E_0)^2} = 1 - \frac{v^2}{c^2} \quad \text{or} \quad v = c \sqrt{1 - \frac{1}{(E/E_0)^2}} \]

Applying this result to each object, we find

\[ v_A = c \sqrt{1 - \frac{1}{(E/E_0)^2}}_A = c \sqrt{1 - \frac{1}{2.00^2}} = 0.866c \]

\[ v_B = c \sqrt{1 - \frac{1}{(E/E_0)^2}}_B = c \sqrt{1 - \frac{1}{3.00^2}} = 0.943c \]

\[ v_C = c \sqrt{1 - \frac{1}{(E/E_0)^2}}_C = c \sqrt{1 - \frac{1}{1.50^2}} = 0.745c \]

These results are consistent with our expected ranking. In particular, note that the largest total energy for object C does not imply that its speed is the largest.

49. **REASONING** The first twin, traveling at the higher speed \( v_1 = 0.900c \), arrives at the distant planet first. Thereafter, the first twin is at rest relative to the earth, and ages at the same rate as people back on the earth. As measured by an observer on the earth, the dilated time intervals \( \Delta t_1, \Delta t_2 \) for the journeys of the two twins are related to the proper time intervals \( \Delta t_{01}, \Delta t_{02} \) by \( \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \) (Equation 28.1), so we have that

\[ \Delta t_{01} = \Delta t_1 \sqrt{1 - \frac{v_1^2}{c^2}} \quad \text{and} \quad \Delta t_{02} = \Delta t_2 \sqrt{1 - \frac{v_2^2}{c^2}} \quad (1) \]

The proper time intervals \( \Delta t_{01} \) and \( \Delta t_{02} \) in Equations (1) give the aging experienced by each twin during the journey. The additional aging that the first twin undergoes between the time of his arrival at the distant planet and the arrival of the second twin is the difference between the time of the second twin’s journey and the time of the first twin’s journey, both as measured by an observer on the earth:

\[ \text{Additional aging} = \Delta t_2 - \Delta t_1 \quad (2) \]

Lastly, we will use \( v = \frac{d}{\Delta t} \) (Equation 2.1) to determine the time intervals for each twin’s journey as measured from the earth, where \( d = 12.0 \) light-years is the distance between the earth and the distant planet. The distance \( d \) and the time intervals \( \Delta t_1, \Delta t_2 \) are all measured...
by an observer on the earth, so we do not need to apply the special theory of relativity. Therefore, we have that

$$\Delta t_1 = \frac{d}{v_1} \quad \text{and} \quad \Delta t_2 = \frac{d}{v_2}$$

(3)

**SOLUTION**

a. When the second twin arrives at the distant planet, the final age of the first twin is given by

$$A_1 = A + \Delta t_{01} + \Delta t_2 - \Delta t_1,$$

where $A = 19.0 \text{ years}$ is the initial age of both twins and we have used Equation (2) to take into account the additional aging. The second twin’s final age is the initial age $A$ of the twin plus the time $\Delta t_{02}$ that elapses during the journey, as measured by that twin: $A_2 = A + \Delta t_{02}$. Therefore, the net difference between their ages will be

$$A_2 - A_1 = A + \Delta t_{02} - (A + \Delta t_{01} + \Delta t_2 - \Delta t_1) = \Delta t_{02} - \Delta t_{01} - \Delta t_2 + \Delta t_1$$

(4)

Substituting Equations (1) into Equation (4) yields

$$A_2 - A_1 = \Delta t_{02} - \Delta t_{01} - \Delta t_2 + \Delta t_1 = \Delta t_2 \left(1 - \frac{v_1^2}{c^2}\right) - \Delta t_1 \left(1 - \frac{v_2^2}{c^2}\right)$$

(5)

Substituting Equations (3) into Equation (5), we obtain

$$A_2 - A_1 = \Delta t_2 \left(1 - \frac{v_2^2}{c^2}\right) - \Delta t_1 \left(1 - \frac{v_1^2}{c^2}\right) = \frac{d}{v_2} \left(1 - \frac{v_2^2}{c^2}\right) - \frac{d}{v_1} \left(1 - \frac{v_1^2}{c^2}\right)$$

(6)

In applying Equation (6), we make use of the fact that the speed $c$ of light in a vacuum is equal to 1 light-year per year: $c = 1 \text{ light-year/year}$:

$$A_2 - A_1 = \frac{12.0 \text{ light-years}}{0.500 \text{ light-years/year}} \left(1 - \frac{(0.500c)^2}{c^2}\right) - \frac{12.0 \text{ light-years}}{0.900 \text{ light-years/year}} \left(1 - \frac{(0.900c)^2}{c^2}\right) = 4.3 \text{ years}$$

b. Since $A_2 - A_1$ is positive, $A_2$ is greater than $A_1$. Thus, when the twins meet again at the earliest possible time, the second twin (traveling at 0.500 $c$) is older.
1. (a) At the higher temperature, the intensity per unit wavelength is greater, and the maximum occurs at a shorter wavelength (see Section 29.2).

2. (b) An X-ray photon has a much greater frequency than does a microwave photon (see Section 24.2). The X-ray photon also has a greater energy \( E \), because \( E = hf \) (Equation 29.2), where \( h \) is Planck’s constant and \( f \) is the frequency. The wavelength \( \lambda \) and frequency of a photon are related by \( \lambda = c/f \) (Equation 16.1), where \( c \) is the speed of light in a vacuum. Since the microwave photon has the smaller frequency, it has the greater wavelength.

3. (c) A green photon has a greater frequency than does a red photon (see Section 24.2). Therefore, the green photon possesses a greater energy \( E \), because \( E = hf \) (Equation 29.2), where \( h \) is Planck’s constant and \( f \) is the frequency.

4. (e) As the wavelength of a photon becomes smaller, its frequency and its energy become larger (see Section 29.3). Since the energy of an incident photon is equal to the maximum kinetic energy of the photoelectron plus the work function of the metal, increasing the incident energy increases the maximum kinetic energy of the photoelectron. The work function is a characteristic of the metal, and does not depend on the photon.

5. (d) The maximum kinetic energy of the ejected photoelectrons depends on the energy of the incident photons. Doubling the number of photons per second that strikes the surface does not change the energy of the incident photons, which depends only on the frequency of the photons. Thus, the maximum kinetic energy of the ejected photoelectrons does not change. However, doubling the number of photons per second that strikes the surface means that twice as many photoelectrons per second are ejected from the surface.

6. (b) Whether or not electrons are ejected from the surface of the metal depends on the energy of the incident photons and the work function of the metal. The work function depends on the type of metal (e.g., aluminum or copper) from which the plate is made. (See Section 29.3).

7. \( W_0 = 3.6 \times 10^{-19} \text{ J} \)

8. \( KE_{\text{max}} = 2.7 \times 10^{-20} \text{ J} \)

9. (d) According to the discussion in Section 29.4, a photon has a momentum whose magnitude \( p \) is related to its wavelength \( \lambda \) by \( p = h/\lambda \).
10. (b) In the Compton effect, an X-ray photon strikes an electron and, like two billiard balls (particles) colliding on a pool table, the X-ray photon scatters in one direction and the electron recoils in another direction after the collision.

11. (a) In the Compton effect, some of the energy of the incident photon is given to the recoil electron. Therefore, the energy of the scattered photon is less than that of the incident photon. Since the energy of a photon depends inversely on its wavelength (see Section 29.3), the wavelength of the scattered photon is greater than that of the incident photon.

12. \( \lambda' = 0.35 \text{ nm} \)

13. (c) The de Broglie wavelength \( \lambda \) depends inversely on the magnitude \( p \) of the momentum; \( \lambda = h/p \) (Equation 29.8), where \( h \) is Planck’s constant. Particle A, having the smaller charge, has the smaller electric potential energy (see Section 19.2). Consequently, after accelerating through the potential difference, particle A has the smaller kinetic energy, and hence, the smaller momentum. Thus, particle A has the longer de Broglie wavelength.

14. (e) The de Broglie wavelength \( \lambda \) depends inversely on the magnitude \( p \) of the momentum; \( \lambda = h/p \) (Equation 29.8), where \( h \) is Planck’s constant. Therefore, as the momentum decreases, the wavelength increases, and vice versa. In A, the proton is moving opposite to the direction of the electric field, so the proton is slowing down, and its momentum is decreasing. In B, the proton is accelerating, and its momentum is increasing. In C, the proton moves parallel to the magnetic field. According to the discussion in Section 21.2, the proton does not experience a force, so its momentum remains constant. In D the proton is moving perpendicular to the direction of the magnetic field. According to the discussion in Section 21.3, such a situation does not change the magnitude of the proton’s momentum.

15. \( \lambda = 2.1 \times 10^{-14} \text{ m} \)

16. (b) According to the Heisenberg uncertainty principle, the uncertainty \( \Delta y \) in a particle’s position is related to the uncertainty \( \Delta p_y \) in its momentum by (see Equation 29.10)\[ \Delta y \geq \frac{h}{4\pi \Delta p_y} \]. If \( \Delta p_y = 0 \text{ kg\cdotm/s} \), then \( \Delta y \) becomes infinitely large.
CHAPTER 29 | PARTICLES AND WAVES

PROBLEMS

1. **REASONING** The energy of a photon of frequency \( f \) is, according to Equation 29.2, \( E = hf \), where \( h \) is Planck’s constant. Since the frequency and wavelength are related by \( f = c / \lambda \) (see Equation 16.1), the energy of a photon can be written in terms of the wavelength as \( E = hc / \lambda \). These expressions can be solved for both the wavelength and the frequency.

**SOLUTION**

a. The wavelength of the photon is

\[
\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{1.22 \times 10^{-18} \text{ J}} = 1.63 \times 10^{-7} \text{ m}
\]

b. Using the answer from part (a), we find that the frequency of the photon is

\[
f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1.63 \times 10^{-7} \text{ m}} = 1.84 \times 10^{15} \text{ Hz}
\]

Alternatively, we could use Equation 29.2 directly to obtain the frequency:

\[
f = \frac{E}{h} = \frac{1.22 \times 10^{-18} \text{ J}}{6.63 \times 10^{-34} \text{ J s}} = 1.84 \times 10^{15} \text{ Hz}
\]

c. The wavelength and frequency values shown in Figure 24.9 indicate that this photon corresponds to electromagnetic radiation in the ultraviolet region of the electromagnetic spectrum.

2. **REASONING** All electromagnetic waves are comprised of photons. The energy \( E \) of a photon is \( E = hf \) (Equation 29.2), where \( h = 6.63 \times 10^{-34} \text{ J s} \) is Planck’s constant and \( f \) is the frequency of the wave. Since the AM wave has the smaller frequency, the energy \( E_{AM} \) of the AM photon is smaller than the energy \( E_{FM} \) of the FM photon.

**SOLUTION** Knowing that the total energy of \( N_{AM} \) AM photons equals the energy of one FM photon and knowing that Equation 29.2 applies to each type of photon, we have

\[
(N_{AM})E_{AM} = (1)E_{FM} \quad \text{or} \quad (N_{AM})hf_{AM} = hf_{FM}
\]

Solving Equation (1) for \( N_{AM} \), we find that

\[
N_{AM} = \frac{hf_{FM}}{hf_{AM}} = \frac{91.9 \times 10^6 \text{ Hz}}{665 \times 10^3 \text{ Hz}} = 138
\]
3. **REASONING** According to Equation 29.3, the work function $W_0$ is related to the photon energy $hf$ and the maximum kinetic energy $KE_{\text{max}}$ by $W_0 = hf - KE_{\text{max}}$. This expression can be used to find the work function of the metal.

**SOLUTION** $KE_{\text{max}}$ is 6.1 eV. The photon energy (in eV) is, according to Equation 29.2,

$$hf = \left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^{15} \text{ Hz}\right) \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 12.4 \text{ eV}$$

The work function is, therefore,

$$W_0 = hf - KE_{\text{max}} = 12.4 \text{ eV} - 6.1 \text{ eV} = 6.3 \text{ eV}$$

4. **REASONING** The intensity $S = 680 \text{ W/m}^2$ of the photons is equal to the total amount of energy delivered by the photons per second per square meter of surface area. Therefore, the intensity of the photons is equal to the energy $E$ of one photon multiplied by the number $N$ of photons that reach the surface of the earth per second per square meter:

$$S = NE \quad (1)$$

The energy $E$ of each photon is given by $E = hf$ (Equation 29.2), where $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ is Planck’s constant and $f$ is the frequency of the photon. We will use $f = \frac{c}{\lambda}$ (Equation 16.1) to determine the frequency $f$ of the photons from their wavelength $\lambda$ and the speed $c$ of light in a vacuum.

**SOLUTION** Solving Equation (1) for $N$, we obtain $N = S/E$. Substituting $E = hf$ (Equation 29.2) into this result yields

$$N = \frac{S}{E} = \frac{S}{hf} \quad (2)$$

Substituting $f = \frac{c}{\lambda}$ (Equation 16.1) into Equation (2), we find that

$$N = \frac{S}{hf} = \frac{S}{h \left(\frac{c}{\lambda}\right)} = \frac{S \lambda}{hc} = \frac{\left(680 \text{ W/m}^2\right) \left(730 \times 10^{-9} \text{ m}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \text{ m/s}\right)}$$

$$= 2.5 \times 10^{21} \text{ photons/\left(s \cdot \text{m}^2\right)}$$
5. **REASONING** The energy of the photon is related to its frequency by Equation 29.2, \( E = hf \). Equation 16.1, \( v = f \lambda \), relates the frequency and the wavelength for any wave.

**SOLUTION** Combining Equations 29.2 and 16.1, and noting that the speed of a photon is \( c \), the speed of light in a vacuum, we have

\[
\lambda = \frac{c}{f} = \frac{c}{(E/h)} = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{6.4 \times 10^{-19} \text{ J}} = 3.1 \times 10^{-7} \text{ m} = 310 \text{ nm}
\]

6. **REASONING** The photons of this wave must carry at least enough energy to equal the work function. Then the electrons are ejected with zero kinetic energy. Since the energy of a photon is \( E = hf \) according to Equation 29.2, where \( f \) is the frequency of the wave, we have that \( W_0 = hf \). Equation 16.1 relates the frequency to the wavelength \( \lambda \) according to \( f = c/\lambda \), where \( c \) is the speed of light. Thus, it follows that \( W_0 = hc/\lambda \).

**SOLUTION** Using Equations 29.2 and 16.1, we find that

\[
W_0 = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{485 \times 10^{-9} \text{ m}} = 4.10 \times 10^{-19} \text{ J}
\]

Since 1 eV = 1.60 × 10^{-19} J, it follows that

\[
W_0 = \left(4.10 \times 10^{-19} \text{ J}\right)\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 2.56 \text{ eV}
\]

7. **REASONING AND SOLUTION** In the first case, the energy of the incident photon is given by Equation 29.3 as

\[
hf = KE_{\text{max}} + W_0 = 0.68 \text{ eV} + 2.75 \text{ eV} = 3.43 \text{ eV}
\]

In the second case, a rearrangement of Equation 29.3 yields

\[
KE_{\text{max}} = hf - W_0 = 3.43 \text{ eV} - 2.17 \text{ eV} = 1.26 \text{ eV}
\]

8. **REASONING** To identify the metal, we need to determine its work function \( W_0 \). The conservation of energy, as applied to the photoelectric effect, indicates that

\[
hf = \frac{1}{2}mv^2_{\text{max}} + W_0 \quad (29.3)
\]

where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant, \( f \) is the frequency of the radiation, \( m = 9.11 \times 10^{-31} \text{ kg} \) is the mass of an electron, and \( v_{\text{max}} = 3.75 \times 10^5 \text{ m/s} \) is the maximum speed of the ejected electrons. We can solve this equation for \( W_0 \).
**SOLUTION** Solving Equation 29.3 for the work function, we obtain

\[ W_0 = hf - \frac{1}{2}mv_{\text{max}}^2 \]  

(1)

In this equation we have no value for the frequency \( f \). However, we know that the frequency and the wavelength \( \lambda \) are related according to \( f \lambda = c \) (Equation 16.1), so that \( \lambda = c / f \), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum. Substituting this result for \( \lambda \) into Equation (1), we find that

\[ W_0 = hf - KE_{\text{max}} = \frac{hc}{\lambda} - \frac{1}{2}mv_{\text{max}}^2 \]

\[ = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{238 \times 10^{-9} \text{ m}} - \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.75 \times 10^5 \text{ m/s})^2 = 7.716 \times 10^{-19} \text{ J} \]

\[ = (7.716 \times 10^{-19} \text{ J})\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 4.82 \text{ eV} \]

The metal with this work function is **gold**.

9. **SSM REASONING AND SOLUTION** The number of photons per second, \( N \), entering the owl's eye is \( N = \frac{SA}{E} \), where \( S \) is the intensity of the beam, \( A \) is the area of the owl's pupil, and \( E \) is the energy of a single photon. Assuming that the owl's pupil is circular, \( A = \pi r^2 = \pi \left(\frac{1}{2}d\right)^2 \), where \( d \) is the diameter of the owl's pupil. Combining Equations 29.2 and 16.1, we have \( E = hf = hc / \lambda \). Therefore,

\[ N = \frac{SA\lambda}{hc} = \frac{(5.0 \times 10^{-13} \text{ W/m}^2)\pi \left(\frac{1}{2}(8.5 \times 10^{-3} \text{ m})\right)^2 (510 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})} = \boxed{73 \text{ photons/s}} \]

10. **REASONING** We will first calculate the potential energy of the system at each of the two separations, and then find the energy difference for the two configurations. Since the electric potential energy lost by the system is carried off by a photon that is emitted during the process, the energy difference must be equal to the energy of the photon. The wavelength of the photon can then be found using Equation 29.2 with Equation 16.1: \( E = hf = hc / \lambda \).

**SOLUTION** The initial potential energy of the system is (see Equations 19.3 and 19.6)

\[ \text{EPE}_i = eV_i = e\left(\frac{kq}{r_i}\right) \]

\[ = (1.6 \times 10^{-19} \text{ C})\left[\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(8.30 \times 10^{-6} \text{ C})}{0.420 \text{ m}}\right] = 2.84 \times 10^{-14} \text{ J} \]
The final potential energy is
\[ E_{PE_2} = eV_2 = (1.6 \times 10^{-19} \ C) \left[ \frac{(8.99 \times 10^9 \ N \cdot m^2 / C^2)(8.30 \times 10^{-6} \ C)}{1.58 \ m} \right] = 7.56 \times 10^{-15} \ J \]

The energy difference, and therefore the energy of the emitted photon, is
\[ \Delta E = E_{PE_1} - E_{PE_2} = 2.84 \times 10^{-14} \ J - 7.56 \times 10^{-15} \ J = 2.08 \times 10^{-14} \ J \]

The wavelength of this photon is
\[ \lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \ J \cdot s)(3.00 \times 10^8 \ m / s)}{2.08 \times 10^{-14} \ J} = 9.56 \times 10^{-12} \ m \]

11. **REASONING** The wavelength \( \lambda \) of the light shining on the surface is related to the maximum kinetic energy \( KE_{max} \) of the electrons ejected from the surface by
\[ hf = KE_{max} + W_0 \] (Equation 29.3), where \( h \) is Planck’s constant, \( f = \frac{c}{\lambda} \) (Equation 16.1) is the frequency of the light, and \( W_0 \) is the work function of the surface. Substituting Equation 16.1 into Equation 29.3, we see that
\[ hf = \frac{hc}{\lambda} = KE_{max} + W_0 \]  
(1)

The work function \( W_0 \) is a property of the metal surface itself, so it remains the same for any wavelength of incident light. When the wavelength of the incident light is \( \lambda_1 = 221 \ nm \), the maximum kinetic energy of the ejected electrons is \( KE_{max,1} = 3.28 \times 10^{-19} \ J \), and when the wavelength is \( \lambda_2 \), the maximum kinetic energy is twice as great: \( KE_{max,2} = 2KE_{max,1} \). We will use this information, with Equation (1), to determine the unknown wavelength \( \lambda_2 \).

**SOLUTION** Solving Equation (1) for the work function \( W_0 \) yields
\[ W_0 = -KE_{max} + \frac{hc}{\lambda} \]  
(2)

Because the work function \( W_0 \) isn’t affected by changing the wavelength of the incident light from \( \lambda_1 \) to \( \lambda_2 \), we have that
\[ W_0 = -KE_{max,2} + \frac{hc}{\lambda_2} = -KE_{max,1} + \frac{hc}{\lambda_1} \]

or
\[ KE_{max,2} - KE_{max,1} + \frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} \]  
(3)

Solving Equation (3) for \( \lambda_2 \) and substituting \( KE_{max,2} = 2 KE_{max,1} \), we obtain
\[ \lambda_2 = \frac{hc}{KE_{max,2} - KE_{max,1} + \frac{hc}{\lambda_1}} = \frac{hc}{2KE_{max,1} - KE_{max,1} + \frac{hc}{\lambda_1}} = \frac{1}{\frac{KE_{max,1} + \frac{1}{\lambda_1}}{hc}} \]  
(4)
In the last step of Equation (4), we have divided both the numerator and the denominator by the product \(hc\). Substituting the given values into Equation (4), we find that

\[ \lambda_2 = \frac{1}{\left(\frac{3.28 \times 10^{-19}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}\right) \left(3.00 \times 10^8 \text{ m/s}\right)^2 + \left(1.62 \times 10^{-7}\right)} = 1.62 \times 10^{-7} \text{ m} = 162 \text{ nm} \]

12. **REASONING** The total energy \(Q\) delivered by \(N\) photons is \(NE\), where \(E\) is the energy carried by one photon, so that \(N = Q/E\). Equation 29.2 indicates that the photon energy is \(E = hf\), where \(h\) is Planck’s constant and \(f\) is the frequency. Thus, the number \(N\) of photons can be written as

\[ N = \frac{Q}{E} = \frac{Q}{hf} \quad \text{(1)} \]

Equation 16.1 relates the frequency \(f\) of a photon to its wavelength \(\lambda\) according to \(f = c/\lambda\), where \(c\) is the speed of light in a vacuum. Therefore, Equation (1) can be expressed as

\[ N = \frac{Q}{hf} = \frac{\lambda Q}{hc} \quad \text{(2)} \]

According to Equation 12.4, the heat \(Q\) required to raise the temperature of a substance by an amount \(\Delta T\) is \(Q = c_{\text{specific heat}}m\Delta T\), where \(c_{\text{specific heat}}\) is the specific heat capacity of the substance and \(m\) is its mass.

**SOLUTION** Combining Equation 12.4 with Equation (2), the number of photons required to raise the temperature by an amount \(\Delta T\) is

\[ N = \frac{\lambda Q}{hc} \quad \text{or} \quad N = \frac{\lambda c_{\text{specific heat}}m\Delta T}{hc} \]

Applying this result to each type of photon, we obtain

\[ N_{\text{infrared}} = \frac{(6.0 \times 10^{-5} \text{ m}) \left[\frac{840 \text{ J}/(\text{kg} \cdot \text{C}^\circ)}{2.0 \text{ C}^\circ}\right] (0.50 \text{ kg})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \text{ m/s}\right)} = 2.5 \times 10^{23} \]

\[ N_{\text{blue}} = \frac{(4.7 \times 10^{-7} \text{ m}) \left[\frac{840 \text{ J}/(\text{kg} \cdot \text{C}^\circ)}{2.0 \text{ C}^\circ}\right] (0.50 \text{ kg})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \text{ m/s}\right)} = 2.0 \times 10^{21} \]

13. **SSM REASONING AND SOLUTION**

a. According to Equation 24.5b, the electric field can be found from \(E = \sqrt{S/(\varepsilon_0c)}\). The intensity \(S\) of the beam is
\[ S = \frac{\text{Energy per unit time}}{A} = \frac{Nhf}{A} = \frac{Nh(c/\lambda)}{A} \]

\[ = \frac{(1.30 \times 10^{18} \text{ photons/s})(6.63 \times 10^{-34} \text{ J s})}{\pi (1.00 \times 10^{-3} \text{ m})^2} \left( \frac{3.00 \times 10^8 \text{ m/s}}{514.5 \times 10^{-9} \text{ m}} \right) \]

\[ = 1.60 \times 10^5 \text{ W/m}^2 \]

where \( N \) is the number of photons per second emitted. Then,

\[ E = \sqrt{S/(\varepsilon_0 c)} = \frac{7760 \text{ N/C}}{mL_{f}c} \]

b. According to Equation 24.3, the average magnetic field is

\[ B = E/c = 2.59 \times 10^{-5} \text{ T} \]

14. **REASONING** The heat required to melt the ice is given by \( Q = mL_{f} \), where \( m \) is the mass of the ice and \( L_{f} \) is the latent heat of fusion for water (see Section 12.8). Since, according to Equation 29.2, each photon carries an energy of \( E = hf \), the energy content of \( N \) photons is \( E_{\text{Total}} = Nhf \). According to Equation 16.1, \( f = c/\lambda \), so we have

\[ E_{\text{Total}} = Nh \frac{c}{\lambda} \]

If we assume that all of the photon energy is used to melt the ice, then, \( E_{\text{Total}} = Q \), so that

\[ \frac{Nh}{\lambda E_{\text{Total}}} = \frac{mL_{f}}{Q} \]

This expression may be solved for \( N \) to determine the required number of photons.

**SOLUTION**

a. We find that

\[ N = \frac{mL_{f}}{hc} = \frac{(2.0 \text{ kg})(33.5 \times 10^4 \text{ J/kg})(620 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})} = 2.1 \times 10^{24} \text{ photons} \]

b. The number \( N' \) of molecules in 2.0-kg of water is

\[ N' = (2.0 \text{ kg}) \left( \frac{1 \text{ mol}}{18 \times 10^{-3} \text{ kg}} \right) \left( \frac{6.022 \times 10^{23} \text{ molecules}}{1 \text{ mol}} \right) = 6.7 \times 10^{25} \text{ molecules} \]
Therefore, on average, the number of water molecules that one photon converts from the ice phase to the liquid phase is

\[
\frac{N'}{N} = \frac{6.7 \times 10^{25} \text{ molecules}}{2.1 \times 10^{24} \text{ photons}} = 32 \text{ molecules/ photon}
\]

15. **REASONING** The frequency \( f \) of a photon is related to its energy \( E \) by \( f = E / h \) (Equation 29.2), where \( h \) is Planck’s constant. As discussed in Section 29.4, the energy \( E \) is related to the magnitude \( p \) of the photon’s momentum by \( E = pc \), where \( c \) is the speed of light in a vacuum. By combining these two relations, we see that the frequency can be expressed in terms of \( p \) as \( f = pc/h \).

**SOLUTION**

a. Substituting values for \( p \), \( c \), and \( h \), into the relation \( f = pc/h \) gives

\[
f = \frac{pc}{h} = \frac{(2.3 \times 10^{-29} \text{ kg} \cdot \text{m/s}) (3.00 \times 10^8 \text{ m/s})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 1.0 \times 10^{13} \text{ Hz}
\]

b. An inspection of Figure 24.9 shows that this frequency lies in the infrared region of the electromagnetic spectrum.

16. **REASONING** The momentum of the photon is related to its wavelength \( \lambda \) and Planck’s constant \( h \). The momentum (nonrelativistic) of the ball depends on its mass \( m \) and speed \( v \). We can set the two momenta equal and solve directly for the speed.

**SOLUTION** The momentum \( p_{\text{photon}} \) of the photon and the momentum \( p_{\text{ball}} \) of the ball are

\[
p_{\text{photon}} = \frac{h}{\lambda} \quad (29.6) \quad \text{and} \quad p_{\text{ball}} = mv \quad (7.2)
\]

Since \( p_{\text{photon}} = p_{\text{ball}} \), we have

\[
\frac{h}{\lambda} = mv \quad \text{or} \quad v = \frac{h}{\lambda m} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(720 \times 10^{-9} \text{ m})(2.2 \times 10^{-3} \text{ kg})} = 4.2 \times 10^{-25} \text{ m/s}
\]

17. **SSM REASONING** The angle \( \theta \) through which the X-rays are scattered is related to the difference between the wavelength \( \lambda' \) of the scattered X-rays and the wavelength \( \lambda \) of the incident X-rays by Equation 29.7 as

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)
\]
where $h$ is Planck’s constant, $m$ is the mass of the electron, and $c$ is the speed of light in a vacuum. We can use this relation directly to find the angle, since all the other variables are known.

**SOLUTION** Solving Equation 29.7 for the angle $\theta$, we obtain

$$\cos \theta = 1 - \frac{mc}{h} (\lambda' - \lambda)$$

$$= 1 - \left( \frac{9.11 \times 10^{-31} \text{ kg}}{6.63 \times 10^{-34} \text{ J s}} \right) \left( \frac{3.00 \times 10^{8} \text{ m/s}}{0.2703 \times 10^{-9} \text{ m} - 0.2685 \times 10^{-9} \text{ m}} \right) = 0.26$$

$$\theta = \cos^{-1} (0.26) = 75^\circ$$

18. **REASONING** The wavelength $\lambda$ of the incident X-rays is related to the wavelength $\lambda'$ of the scattered X-rays by $\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$ (Equation 29.7), where $h = 6.626 \times 10^{-34}$ J s is Planck’s constant, $c = 2.998 \times 10^{8}$ m/s is the speed of light in a vacuum, $m = 9.109 \times 10^{-31}$ kg is the mass of an electron, and $\theta = 122.0^\circ$ is the angle at which the X-rays are scattered. The wavelength $\lambda'$ of the scattered photon is found from the magnitude $p'$ of its momentum via $p' = \frac{h}{\lambda'}$ (Equation 29.6).

**SOLUTION** Solving $\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$ (Equation 29.7) for $\lambda$, we obtain

$$\lambda = \lambda' - \frac{h}{mc} (1 - \cos \theta)$$

Solving $p' = \frac{h}{\lambda'}$ (Equation 29.6) for $\lambda'$ yields $\lambda' = \frac{h}{p'}$, which, on substitution into Equation (1), gives

$$\lambda = \lambda' - \frac{h}{mc} (1 - \cos \theta) = \frac{h}{p'} - \frac{h}{mc} (1 - \cos \theta)$$

$$= \frac{6.626 \times 10^{-34} \text{ J s}}{1.856 \times 10^{-24} \text{ kg m/s}} - \left( \frac{6.626 \times 10^{-34} \text{ J s}}{9.109 \times 10^{-31} \text{ kg}} \right) \left( 1 - \cos 122.0^\circ \right)$$

$$= 3.533 \times 10^{-10} \text{ m} = 0.3533 \text{ nm}$$

19. **REASONING** There are no external forces that act on the system, so the conservation of linear momentum applies. Since the photon is scattered at $\theta = 180^\circ$, the collision is "head-on," and all motion occurs along the horizontal direction, which we take as the $x$ axis.
The incident photon is assumed to be moving along the +x axis. For an initially stationary electron, the conservation of linear momentum states that:

\[
\begin{align*}
\frac{p}{-p'} + \frac{P_{\text{electron}}}{P_{\text{incident photon}}} &= \frac{P_{\text{scattered photon}}}{-P_{\text{recoil photon}}} \\
\end{align*}
\]

where the momentum of the scattered photon is negative since it moves along the \(-x\) direction (the scattering angle is 180°). Using the relation \(p = \frac{h}{\lambda}\) (Equation 29.6), where \(h\) is Planck’s constant and \(\lambda\) is the wavelength of the photon, we can write the expression for the momentum of the electron as

\[
P_{\text{electron}} = p + p' = \frac{h}{\lambda} + \frac{h}{\lambda'} = h\left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right)
\]

**SOLUTION** Substituting numerical values into the equation above, we have

\[
P_{\text{electron}} = (6.626 \times 10^{-34} \text{ J} \cdot \text{s})\left(\frac{1}{0.2750 \times 10^{-9} \text{ m}} + \frac{1}{0.2825 \times 10^{-9} \text{ m}}\right) = 4.755 \times 10^{-24} \text{ kg} \cdot \text{m/s}
\]

20. **REASONING** Before the scattering, the electron is at rest and has no momentum. Thus, the total initial momentum consists only of the photon’s momentum, which points along the +x axis. The total initial momentum has no y component. Since the total momentum is conserved, the total momentum after the scattering must be the same as it was before and, therefore, has no y component.

The total momentum after the scattering is the sum of the momentum of the scattered photon and that of the scattered electron, and it only has an x component. But the scattered photon is moving along the \(-y\) axis, so its momentum has no x component. Therefore, the momentum of the electron must have an x component.

The total momentum after the scattering is the sum of the momentum of the scattered photon and that of the scattered electron, and it has no y component. But the scattered photon is moving along the \(-y\) axis, so its momentum points along the \(-y\) axis. Therefore, this contribution to the total final momentum must be cancelled by part of the momentum of the scattered electron, which must have a component along the +y axis.

**SOLUTION** Since the total momentum is conserved and since the scattered photon has no momentum in the x direction, the momentum of the scattered electron must have an x component that equals the momentum of the incident photon. According to Equation 29.6, the magnitude \(p\) of the momentum of the incident photon is \(p = \frac{h}{\lambda}\), where \(h\) is Planck’s constant and \(\lambda\) is the wavelength. Therefore, the momentum of the scattered electron has a component in the +x direction that is

\[
p_x = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.00 \times 10^{-12} \text{ m}} = 7.37 \times 10^{-23} \text{ kg} \cdot \text{m/s}
\]
The momentum of the scattered electron has a component along the +y direction. This component cancels the momentum of the scattered photon that points along the −y direction. To find the momentum of the scattered photon, we first need to determine its wavelength $\lambda'$, which we can do using Equation 29.7:

$$\lambda' = \lambda + \frac{h}{mc} \left(1 - \cos 90.0^\circ\right)$$

$$= 9.00 \times 10^{-12} \text{ m} + \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.11 \times 10^{-31} \text{ kg} \left(3.00 \times 10^8 \text{ m/s}\right)} \approx 1.14 \times 10^{-11} \text{ m}$$

Again using Equation 29.6, we find that the momentum of the scattered electron has a component in the +y direction that is

$$p_y = \frac{h}{\lambda'} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.14 \times 10^{-11} \text{ m}} \approx 5.82 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

21. **SSM REASONING** The change in wavelength that occurs during Compton scattering is given by Equation 29.7:

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$

or

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos 180^\circ) = \frac{2h}{mc}$$

$$(\lambda' - \lambda)_{\text{max}}$$ is the maximum change in the wavelength, and to calculate it we need a value for the mass $m$ of a nitrogen molecule. This value can be obtained from the mass per mole $M$ of nitrogen ($N_2$) and Avogadro's number $N_A$, according to $m = M / N_A$ (see Section 14.1).

**SOLUTION** Using a value of $M = 0.0280 \text{ kg/mol}$, we obtain the following result for the maximum change in the wavelength:

$$\left(\lambda' - \lambda\right)_{\text{max}} = \frac{2h}{mc} = \frac{2h}{\left(\frac{M}{N_A}\right)c} = \frac{2}{\left(\frac{0.0280 \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}^{-1}}\right)} \left(3.00 \times 10^8 \text{ m/s}\right)$$

$$= 9.50 \times 10^{-17} \text{ m}$$

22. **REASONING** The change in the wavelength in Compton’s experiment is

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$ (Equation 29.7), where $\lambda'$ is the wavelength of the scattered photon, $\lambda$ is the wavelength of the incident photon, $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ is Planck’s constant,
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9.11 10 kg

m = 9.11 × 10⁻³¹ kg is the mass of an electron, \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum, and \( \theta \) is the scattering angle. We do not know the value of the incident wavelength \( \lambda \). However, we do know that it is the same for the two given values of the wavelength \( \lambda' \) of the scattered photons. Thus, we can apply Equation 29.7 for each of the given values of \( \lambda' \) and eliminate the incident wavelength from our calculation.

**SOLUTION** Applying Equation 29.7 for each of the given wavelength values, we have

\[
\lambda'_1 - \lambda = \frac{h}{mc}(1 - \cos \theta_1) \quad \text{and} \quad \lambda'_2 - \lambda = \frac{h}{mc}(1 - \cos \theta_2)
\]

Subtracting the first equation from the second gives

\[
\left( \lambda'_2 - \lambda' \right) - \left( \lambda'_1 - \lambda \right) = \frac{h}{mc}(1 - \cos \theta_2) - \frac{h}{mc}(1 - \cos \theta_1)
\]

\[
\lambda'_2 - \lambda'_1 = \frac{h}{mc}(-\cos \theta_2 + \cos \theta_1)
\]

\[
= \frac{\left( 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \right)}{\left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 3.00 \times 10^8 \text{ m/s} \right)}(-\cos 70.0^\circ + \cos 30.0^\circ) = 1.27 \times 10^{-12} \text{ m}
\]

23. **REASONING** Energy is conserved during the collision. This means that the energy \( E \) of the incident photon must equal the kinetic energy \( KE \) of the recoil electron plus the energy \( E' \) the scattered photon:

\[
E = KE + E'
\] (1)

The energy \( E \) of a photon is related to its frequency \( f \) by \( E = hf \) (Equation 29.2), where \( h \) is Planck’s constant. The frequency, in turn, is related to the wavelength \( \lambda \) by \( f = c/\lambda \) (Equation 16.1), where \( c \) is the speed of light in a vacuum. Substituting \( f = c/\lambda \) into \( E = hf \) gives

\[
E = hc/\lambda
\] (2)

The wavelength \( \lambda' \) of the scattered photon depends on the wavelength \( \lambda \) of the incident photon according to Equation 29.7, so that we have

\[
\lambda' = \lambda + \frac{h}{mc}(1 - \cos \theta)
\]

Since the photon is scattered straight backward, \( \theta = 180^\circ \), and

\[
\lambda' = \lambda + \frac{h}{mc}(1 - \cos 180^\circ) = \lambda - \frac{2h}{mc}
\] (3)

**SOLUTION** The kinetic energy of the recoil electron is given by \( KE = \frac{1}{2}mv^2 \) (Equation 6.2), where \( m \) is its mass and \( v \) is its speed. Substituting this expression into Equation (1), we have \( E = \frac{1}{2}mv^2 + E' \). Solving for the speed \( v \) of the electron gives
\( v = \sqrt{\frac{2(E - E')}{m}} \) \hspace{1cm} (4)

From Equation (2), we also know that \( E = \frac{hc}{\lambda} \) and \( E' = \frac{hc}{\lambda'} \). Substituting these expressions into Equation (4), we find that the speed of the electron can be written as

\[
v = \sqrt{\frac{2hc}{m} \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right)}
\]

Since \( \lambda' = \lambda + \frac{2h}{mc} \) [Equation (3)], the speed of the electron is

\[
v = \sqrt{\frac{2hc}{m} \left( \frac{1}{\lambda} - \frac{1}{\lambda + \frac{2h}{mc}} \right)}
\]

\[
= \sqrt{\frac{2(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (3.00 \times 10^8 \text{ m/s})}{9.11 \times 10^{-31} \text{ kg}}}
\times \sqrt{\frac{1}{0.45000 \times 10^{-9} \text{ m}} - \frac{1}{0.45000 \times 10^{-9} \text{ m} + \frac{2(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{9.11 \times 10^{-31} \text{ kg} (3.00 \times 10^8 \text{ m/s})}}}
\]

\[
= 3.22 \times 10^6 \text{ m/s}
\]

24. **REASONING** Since the net external force acting on the system (the photon and the electron) is zero, the conservation of linear momentum applies. In addition, there are no nonconservative forces, so the conservation of total energy applies as well. Since the photon scatters at an angle of \( \theta = 180.0^\circ \) in Figure 29.10, the collision is "head-on." Thus, the motion takes place entirely along the horizontal direction, which we will take as the \( x \) axis, with the right as being the positive direction.

The conservation of linear momentum gives rise to Equation 29.7, which relates the difference \( \lambda' - \lambda \) between the scattered and incident X-ray photon wavelengths to the scattering angle \( \theta \) of the electron as

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta) = \frac{h}{mc} (1 - \cos 180^\circ) = \frac{2h}{mc}
\]

\[
(1)
\]

The conservation of total energy is written as

\[
\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2}mv^2
\]

\[
(2)
\]
Equations (1) and (2) will permit us to find the wavelength $\lambda$ of the incident X-ray photon.

**SOLUTION** Solving Equation (1) for $\lambda'$ and substituting the result into Equation (2) gives

$$\frac{hc}{\lambda} = \frac{hc}{2h/mc} + \frac{1}{2}mv^2$$

Algebraically rearranging this result, we obtain a quadratic equation for $\lambda$:

$$\lambda^2 + \left(\frac{2h}{mc}\right)\lambda - \frac{2h^2}{m\left(\frac{1}{2}mv^2\right)} = 0$$

where we have used $h = 6.63 \times 10^{-34}$ J-s, $m = 9.11 \times 10^{-31}$ kg, $c = 3.00 \times 10^8$ m/s, and $v = 4.67 \times 10^6$ m/s. Solving this quadratic equation for $\lambda$, we obtain

$$\lambda = 3.09 \times 10^{-10} \text{ m}$$

25. **REASONING** The de Broglie wavelength $\lambda$ is related to Planck’s constant $h$ and the magnitude $p$ of the particle’s momentum. The magnitude of the momentum is related to the mass $m$ and the speed $v$ at which the bacterium is moving. Since the mass and the speed are given, we can calculate the wavelength directly.

**SOLUTION** The de Broglie wavelength is

$$\lambda = \frac{h}{p} \quad (29.8)$$

The magnitude of the momentum is $p = mv$ (Equation 7.2), which we can substitute into Equation 29.8 to show that the de Broglie wavelength of the bacterium is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(2 \times 10^{-15} \text{ kg})(0.33 \text{ m/s})} = 1 \times 10^{-18} \text{ m}$$

26. **REASONING AND SOLUTION**

a. We know $E = hc/\lambda$ for a photon. The energy of the photon is

$$E = 5.0 \text{ eV} \left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 8.0 \times 10^{-19} \text{ J}$$

The wavelength is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{8.0 \times 10^{-19} \text{ J}} = 2.5 \times 10^{-7} \text{ m}$$
b. The speed of the 5.0-eV electron is

\[
v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(8.0 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 1.3 \times 10^6 \text{ m/s}
\]

The de Broglie wavelength is

\[
\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.3 \times 10^6 \text{ m/s})} = 5.6 \times 10^{-10} \text{ m}
\]

27. [SSM] REASONING In order for the person to diffract to the same extent as the sound wave, the de Broglie wavelength of the person must be equal to the wavelength of the sound wave.

SOLUTION

a. Since the wavelengths are equal, we have that

\[
\lambda_{\text{sound}} = \lambda_{\text{person}}
\]

\[
\lambda_{\text{sound}} = \frac{h}{m_{\text{person}} v_{\text{person}}}
\]

Solving for \( v_{\text{person}} \), and using the relation \( \lambda_{\text{sound}} = v_{\text{sound}} / f_{\text{sound}} \) (Equation 16.1), we have

\[
v_{\text{person}} = \frac{h}{m_{\text{person}} (v_{\text{sound}} / f_{\text{sound}})} = \frac{hf_{\text{sound}}}{m_{\text{person}} v_{\text{sound}}}
\]

\[
= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(128 \text{ Hz})}{(55.0 \text{ kg})(343 \text{ m/s})} = 4.50 \times 10^{-36} \text{ m/s}
\]

b. At the speed calculated in part (a), the time required for the person to move a distance of one meter is

\[
t = \frac{x}{v} = \frac{1.0 \text{ m}}{4.50 \times 10^{-36} \text{ m/s}} \left( \frac{1.0 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ day}}{24.0 \text{ h}} \right) \left( \frac{1 \text{ year}}{365.25 \text{ days}} \right) = 7.05 \times 10^{27} \text{ years}
\]

Factors to convert seconds to years

28. REASONING The de Broglie wavelength \( \lambda \) of a particle is \( \lambda = h / p \) (Equation 29.8), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant and \( p \) is the magnitude of the particle’s momentum. Since we are ignoring relativistic effects, the magnitude of the particle’s momentum is \( p = mv \) (Equation 7.2), where \( m \) is the mass of the particle and \( v \) is the speed.
SOLUTION Using Equation 7.2 for the magnitude of the particle’s momentum, we can write Equation 29.8 for the de Broglie wavelength as follows:

\[ \lambda = \frac{h}{p} = \frac{h}{mv} \]  

(1)

Applying Equation (1) to the electron and the proton, we find that

\[
\frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}} = \frac{m_{\text{proton}}}{m_{\text{electron}}} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = 1830
\]

We have taken the masses of the electron and proton from the table on the inside of the front cover of the text.

29. REASONING AND SOLUTION The average kinetic energy of a helium atom is

\[ KE = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 6.07 \times 10^{-21} \text{ J} \]

The speed of the atom corresponding to the average kinetic energy is

\[ v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2(6.07 \times 10^{-21} \text{ J})}{6.65 \times 10^{-27} \text{ kg}}} = 1.35 \times 10^{3} \text{ m/s} \]

The de Broglie wavelength is

\[ \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(6.65 \times 10^{-27} \text{ kg})(1.35 \times 10^{3} \text{ m/s})} = 7.38 \times 10^{-11} \text{ m} \]

30. REASONING According to Equation 27.1, the angle \( \theta \) that locates the first-order bright fringes \((m = 1)\) is specified by \( \sin \theta = \lambda/d \), where \( \lambda \) is the wavelength and \( d \) is the separation between the slits. The wavelength of the electron is the de Broglie wavelength, which is given by \( \lambda = h/p \) (Equation 29.8), where \( h \) is Planck’s constant and \( p \) is the magnitude of the momentum of the electron.

SOLUTION Combining Equations 27.1 and 29.8, we find that the angle locating the first-order bright fringes is specified by

\[ \sin \theta = \frac{\lambda}{d} = \frac{h}{pd} \]

Dividing this result for case A by that for case B, we find
PARTICLES AND WAVES

\[
\frac{\sin \theta_A}{\sin \theta_B} = \frac{h / (p_A d)}{h / (p_B d)} = \frac{p_B}{p_A} \quad \text{or} \quad p_B = \frac{p_A \sin \theta_A}{\sin \theta_B}
\]

\[
p_B = \left(1.2 \times 10^{-22} \text{ kg} \cdot \text{m/s}\right) \sin \left(1.6 \times 10^{-4} \text{ degrees}\right) \over \sin \left(4.0 \times 10^{-4} \text{ degrees}\right) = 4.8 \times 10^{-23} \text{ kg} \cdot \text{m/s}
\]

31. **SSM REASONING** The de Broglie wavelength \( \lambda \) is related to Planck’s constant \( h \) and the magnitude \( p \) of the particle’s momentum. The magnitude of the momentum can be related to the particle’s kinetic energy. Thus, using the given wavelength and the fact that the kinetic energy doubles, we will be able to obtain the new wavelength.

**SOLUTION** The de Broglie wavelength is

\[
\lambda = \frac{h}{p}
\]

The kinetic energy and the magnitude of the momentum are

\[
KE = \frac{1}{2} mv^2 \quad \text{(6.2)} \quad p = mv \quad \text{(7.2)}
\]

where \( m \) and \( v \) are the mass and speed of the particle. Substituting Equation 7.2 into Equation 6.2, we can relate the kinetic energy and momentum as follows:

\[
KE = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2m(KE)}
\]

Substituting this result for \( p \) into Equation 29.8 gives

\[
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(KE)}}
\]

Applying this expression for the final and initial wavelengths \( \lambda_f \) and \( \lambda_i \), we obtain

\[
\lambda_f = \frac{h}{\sqrt{2m(KE)_f}} \quad \text{and} \quad \lambda_i = \frac{h}{\sqrt{2m(KE)_i}}
\]

Dividing the two equations and rearranging reveals that

\[
\frac{\lambda_f}{\lambda_i} = \frac{\sqrt{2m(KE)_f}}{h} \cdot \frac{h}{\sqrt{2m(KE)_i}} = \sqrt{\frac{(KE)_i}{(KE)_f}} \quad \text{or} \quad \lambda_f = \lambda_i \sqrt{\frac{(KE)_i}{(KE)_f}}
\]

Using the given value for \( \lambda_i \) and the fact that \( KE_f = 2(KE_i) \), we find
\[ \lambda_f = \sqrt[2]{\frac{K_{E_f}}{K_{E_i}}} = \sqrt[2]{\frac{2.7 \times 10^{-10} \text{ m}}{1.9 \times 10^{-10} \text{ m}}} = 1.9 \times 10^{-10} \text{ m} \]

32. **REASONING** The de Broglie wavelength \( \lambda \) of the woman is found from \( p = \frac{h}{\lambda} \) (Equation 29.8), where \( p \) is the magnitude of her momentum and \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck's constant. We will use \( p = m v \) (Equation 7.2) to determine the magnitude \( p \) of the woman’s momentum from her mass \( m \) and her speed \( v \) at the instant she strikes the water.

Once the woman jumps from the cliff, she is in free fall with an initial speed of \( v_0 = 0 \text{ m/s} \), and an acceleration \( a = -9.8 \text{ m/s}^2 \). Since we have taken upward to be the positive direction, her displacement during the fall is \( H = -9.5 \text{ m} \). Her final speed \( v \), then, is given by

\[ v^2 = v_0^2 + 2aH \quad (2.9) \]

**SOLUTION** Solving \( p = \frac{h}{\lambda} \) (Equation 29.8) for \( \lambda \) yields \( \lambda = \frac{h}{p} \). Substituting \( p = m v \) (Equation 7.2) into this result, we obtain

\[ \lambda = \frac{h}{mv} \quad (1) \]

Substituting \( v_0 = 0 \text{ m/s} \) into Equation (2.9) and taking the square root of both sides, we find that

\[ v^2 = (0 \text{ m/s})^2 + 2aH \quad \text{or} \quad v^2 = 2aH \quad \text{or} \quad v = \sqrt{2aH} \quad (2) \]

Substituting Equation (2) into Equation (1), we find the de Broglie wavelength of the woman at the instant she strikes the water to be:

\[ \lambda = \frac{h}{mv} = \frac{h}{m \sqrt{2aH}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(41 \text{ kg}) \sqrt{2(-9.8 \text{ m/s}^2)(-9.5 \text{ m})}} = 1.2 \times 10^{-36} \text{ m} \]

33. **REASONING** The width of the central bright fringe in the diffraction patterns will be identical when the electrons have the same de Broglie wavelength as the wavelength of the photons in the red light. The de Broglie wavelength of one electron in the beam is given by Equation 29.8, \( \lambda_{\text{electron}} = \frac{h}{p} \), where \( p = m v \).

**SOLUTION** Following the reasoning described above, we find

\[ \lambda_{\text{red light}} = \lambda_{\text{electron}} \]

\[ \lambda_{\text{red light}} = \frac{h}{m_{\text{electron}} v_{\text{electron}}} \]
Solving for the speed of the electron, we have

\[
v_{\text{electron}} = \frac{h}{m_{\text{electron}} \lambda_{\text{red light}}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(661 \times 10^{-9} \text{ m})} = 1.10 \times 10^3 \text{ m/s}
\]

34. **REASONING** The linear momentum \( p \) of a particle is given by \( p = mv \) (Equation 7.2), where \( m \) and \( v \) are its mass and velocity. Since particle A is initially at rest, its momentum is zero. The initial momentum of particle B is \( p_{0B} = m_B v_{0B} \). This is also the total initial linear momentum of the two-particle system.

After the collision the combined mass of the two particles is \( m_A + m_B \), and the common velocity is \( v_f \). Thus, the total linear momentum of the system after the collision is \( p_f = (m_A + m_B) v_f \). From Section 7.2, we know that the total linear momentum of an isolated system is conserved. An isolated system is one in which the vector sum of the external forces acting on the system is zero. Since there are no external forces acting on the particles, the two-particle system is an isolated system. Thus, the total linear momentum of the system after the collision equals the total linear momentum before the collision.

The de Broglie wavelength \( \lambda \) is inversely related to the magnitude \( p \) of a particle’s momentum by \( \lambda = h/p \) (Equation 29.8), where \( h \) is Planck’s constant.

**SOLUTION** The de Broglie wavelength \( \lambda_f \) of the object that moves off after the collision is given by \( \lambda_f = h/p_f \) (Equation 29.8). Since momentum is mass times velocity, the magnitude of the momentum that the object has after the collision is \( p_f = (m_A + m_B) v_f \) where \( v_f \) is the common speed of the two particles. We can evaluate this momentum by using the law of conservation of momentum, which indicates that the total momentum after the collision is the same as it is before the collision. Before the collision only particle B is moving, so that the magnitude of the total momentum at that time has a value of \( m_B v_{0B} \), where \( v_{0B} \) is the initial speed of particle B. Assuming the particles travel along the \( +x \) axis, we write the conservation of linear momentum as follows:

\[
\underbrace{+(m_A + m_B) v_f}_{\text{Total momentum after collision}} = \underbrace{+m_B v_{0B}}_{\text{Total momentum before collision}}
\]

Using this result, we find that the desired de Broglie wavelength is

\[
\lambda_f = \frac{h}{p_f} = \frac{h}{(m_A + m_B) v_f} = \frac{h}{m_B v_{0B}}
\]

But the term on the far right is just the given de Broglie wavelength of the incident particle B. Therefore, we conclude that \[ \lambda_f = 2.0 \times 10^{-34} \text{ m} \].
35. **REASONING** The de Broglie wavelength $\lambda$ of the electron is related to the magnitude $p$ of its momentum by $\lambda = h/p$ (Equation 29.8), where $h$ is Planck’s constant. If the speed of the electron is much less than the speed of light, the magnitude of the electron’s momentum is given by $p = mv$ (Equation 7.2). Thus, the de Broglie wavelength can be written as $\lambda = h/(mv)$.

When the electron is at rest, it has electric potential energy, but no kinetic energy. The electric potential energy $\text{EPE}$ is given by $EPE = eV$ (Equation 19.3), where $e$ is the magnitude of the charge on the electron and $V$ is the potential difference. When the electron reaches its maximum speed, it has no potential energy, but its kinetic energy is $\frac{1}{2}mv^2$. The conservation of energy states that the final total energy of the electron equals the initial total energy:

$$\frac{1}{2}mv^2 = eV$$

Solving this equation for the final speed gives $v = \sqrt{\frac{2eV}{m}}$. Substituting this expression for $v$ into $\lambda = h/(mv)$ gives $\lambda = \frac{h}{\sqrt{2meV}}$.

**SOLUTION** After accelerating through the potential difference, the electron has a de Broglie wavelength of

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})(418 \text{ V})}} = 6.01 \times 10^{-11} \text{ m}$$

36. **REASONING AND SOLUTION** The energy of the photon is $E = hf = hc/\lambda_{\text{photon}}$, while the kinetic energy of the particle is $\text{KE} = (1/2)mv^2 = h^2/(2m\lambda^2)$. Equating the two energies and rearranging the result gives $\lambda_{\text{photon}}/\lambda = (2mc/h)\lambda$. Now the speed of the particle is $v = 0.050c$, so $\lambda = h/(0.050mc)$, and

$$\lambda_{\text{photon}}/\lambda = 2/0.050 = 4.0 \times 10^1$$

37. **REASONING** Suppose the object is moving along the $+y$ axis. The uncertainty in the object’s position is $\Delta y = 2.5 \text{ m}$. The minimum uncertainty $\Delta p_y$ in the object’s momentum is specified by the Heisenberg uncertainty principle (Equation 29.10) in the form $(\Delta p_y)(\Delta y) = h/(4\pi)$. Since momentum is mass $m$ times velocity $v$, the uncertainty in the velocity $\Delta v$ is related to the uncertainty in the momentum by $\Delta v = (\Delta p_y)/m$.

**SOLUTION**

a. Using the uncertainty principle, we find the minimum uncertainty in the momentum as follows:
\[
\left( \Delta p_y \right) \left( \Delta y \right) = \frac{h}{4\pi} \\
\Delta p_y = \frac{h}{4\pi \Delta y} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi (2.5 \text{ m})} = 2.1 \times 10^{-35} \text{ kg} \cdot \text{m/s}
\]

b. For a golf ball this uncertainty in momentum corresponds to an uncertainty in velocity that is given by
\[
\Delta v_y = \frac{\Delta p_y}{m} = \frac{2.1 \times 10^{-35} \text{ kg} \cdot \text{m/s}}{0.045 \text{ kg}} = 4.7 \times 10^{-34} \text{ m/s}
\]

c. For an electron this uncertainty in momentum corresponds to an uncertainty in velocity that is given by
\[
\Delta v_y = \frac{\Delta p_y}{m} = \frac{2.1 \times 10^{-35} \text{ kg} \cdot \text{m/s}}{9.11 \times 10^{-31} \text{ kg}} = 2.3 \times 10^{-5} \text{ m/s}
\]

38. **REASONING** The Heisenberg uncertainty principle specifies the relationship between the uncertainty \( \Delta y \) in a particle’s position and the uncertainty \( \Delta p_y \) in the particle’s linear momentum. This principle is stated as follows: \((\Delta p_y)(\Delta y) \geq h/(4\pi)\) (Equation 29.10), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant. Since we seek the minimum uncertainty in the proton’s momentum, we will use the equals sign in Equation 29.10 and omit the greater-than symbol (>). For our purposes in this problem, then, the uncertainty principle becomes
\[
(\Delta p_y)(\Delta y) = \frac{h}{4\pi} \quad (1)
\]

**SOLUTION** Solving Equation (1) for \( \Delta p_y \), we find that
\[
\Delta p_y = \frac{h}{4\pi \Delta y} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi (5.5 \times 10^{-13} \text{ m})} = 9.6 \times 10^{-21} \text{ kg} \cdot \text{m/s}
\]

39. **[SSM] REASONING** The Heisenberg uncertainty principle specifies the relationship between the uncertainty \( \Delta y \) in a particle’s position and the uncertainty \( \Delta p_y \) in the particle’s linear momentum. This principle is stated as follows: \((\Delta p_y)(\Delta y) \geq h/(4\pi)\) (Equation 29.10), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant. Since we seek the minimum uncertainty in the speed (which is proportional to the momentum) of the oxygen molecule, we will use the equals sign in Equation 29.10 and omit the greater-than symbol (>). For our purposes in this problem, then, the uncertainty principle becomes
\[
(\Delta p_y)(\Delta y) = \frac{h}{4\pi} \quad (1)
\]
We can use Equation (1) to calculate the minimum uncertainty in the speed, because the magnitude \( p_y \) of the momentum is related to the speed \( v_y \) according to \( p_y = mv_y \) (Equation 7.2), where \( m \) is the mass of the oxygen molecule.

**SOLUTION** Substituting \( p_y = mv_y \) (Equation 7.2) into Equation (1), we obtain

\[
(\Delta p_y)(\Delta y) = \frac{h}{4\pi} \quad \text{or} \quad (m\Delta v_y)(\Delta y) = \frac{h}{4\pi} \tag{2}
\]

Solving Equation (2) for the uncertainty \( \Delta v_y \) in the speed, we find that

\[
\Delta v_y = \frac{h}{4\pi m\Delta y} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi \left( 5.3 \times 10^{-26} \text{ kg} \right) \left( 0.12 \times 10^{-3} \text{ m} \right)} = 8.3 \times 10^{-6} \text{ m/s}
\]

40. **REASONING** When particles pass through the slit the great majority fall on the screen between the first dark fringes on either side of the central bright fringe. The first dark fringes are located by the angles \(-\theta\) (below the midpoint) and \(+\theta\) (above the midpoint), so this is the minimum range of angles over which the particles spread out. As Figure 29.14 shows, the angle \( \theta \) is found from \( \theta = \tan^{-1} \left( \frac{\Delta p_y}{p_x} \right) \) (Equation 1.4), where \( \Delta p_y \) is the uncertainty in the \( y \) component \( p_y \) of a particle’s momentum, and \( p_x \) is the \( x \) component of a particle’s momentum. The given de Broglie wavelength \( \lambda = 0.200 \text{ mm} \) of the particles is the wavelength they possess before passing through the slit, when their momentum has only the \( x \) component \( p_x \). Therefore, the \( x \) component of each particle’s momentum is given by

\[
p_x = \frac{h}{\lambda} \quad \text{(Equation 29.8), where } h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s is Planck’s constant.}
\]

We are given that the uncertainty in the position of each particle along the \( y \) direction is equal to one-half the width of the slit: \( \Delta y = \frac{1}{2} W \). The minimum uncertainty in the \( y \) component \( \Delta p_y \) of a particle’s momentum, then, is found from the Heisenberg uncertainty principle:

\[
(\Delta p_y)(\Delta y) = \frac{h}{4\pi} \tag{29.10}
\]

**SOLUTION** Solving Equation 29.10 for \( \Delta p_y \) and substituting \( \Delta y = \frac{1}{2} W \), we obtain

\[
\Delta p_y = \frac{h}{4\pi (\Delta y)} = \frac{h}{4\pi \left( \frac{1}{2} W \right)} = \frac{h}{2\pi W} \tag{1}
\]

Substituting Equation (1) and \( p_x = \frac{h}{\lambda} \) (Equation 29.8) into \( \theta = \tan^{-1} \left( \frac{\Delta p_y}{p_x} \right) \) (Equation 1.6), we find that
\[ \theta = \tan^{-1} \left( \frac{\Delta p_y}{p_x} \right) = \tan^{-1} \left( \frac{j^'}{2\pi W} \right) = \tan^{-1} \left( \frac{\lambda}{2\pi W} \right) = \tan^{-1} \left( \frac{633 \times 10^{-9} \text{ m}}{2\pi (0.200 \times 10^{-3} \text{ m})} \right) \]

\[ = 0.0289^\circ \]

Therefore, the particles spread out over the range \(-0.0289^\circ\) to \(+0.0289^\circ\).

41. **SOLUTION**  The minimum uncertainty \(\Delta y\) in the position of the particle is related to the minimum uncertainty \(\Delta p_y\) in the momentum via the Heisenberg uncertainty principle. To cast this relationship into a form that gives us the desired percentage for the minimum uncertainty in the speed, we note that the minimum uncertainty in the position is specified as the de Broglie wavelength \(\lambda\). We can then express the de Broglie wavelength in terms of Planck’s constant \(h\) and the magnitude \(p_y\) of the particle’s momentum. The magnitude of the momentum is related to the mass \(m\) and the speed \(v_y\) of the particle.

**SOLUTION**  The percentage minimum uncertainty in the speed is

\[ \text{Percentage} = \frac{\Delta v_y}{v_y} \times 100\% \quad (1) \]

According to the Heisenberg uncertainty principle, the minimum uncertainty \(\Delta p_y\) in the momentum and the minimum uncertainty \(\Delta y\) in the position of the particle are related according to

\[ (\Delta p_y)(\Delta y) = \frac{h}{4\pi} \quad (29.10) \]

We know that \(\Delta y\) is equal to the de Broglie wavelength \(\lambda = h/ p_y\) (Equation 29.8), where the magnitude of the momentum is \(p_y = mv_y\) (Equation 7.2). Thus, we have

\[ \Delta y = \lambda = \frac{h}{p_y} = \frac{h}{mv_y} \]

Substituting this result for \(\Delta y\) into Equation 29.10, we obtain

\[ (\Delta p_y)(\Delta y) = (\Delta p_y) \left( \frac{h}{mv_y} \right) = \frac{h}{4\pi} \quad (2) \]

The last step in our transformation of the uncertainty principle is to realize that \(\Delta p_y = \Delta (mv_y) = m\Delta v_y\), since the mass is constant. Substituting this expression for \(\Delta p_y\) into Equation (2) shows that
\[(\Delta p_y) \left( \frac{h}{mv_y} \right) = (m\Delta v_y) \left( \frac{h}{mv_y} \right) = \frac{h}{4\pi} \quad \text{or} \quad \frac{\Delta v_y}{v_y} = \frac{1}{4\pi} \]

Using this result in Equation (1), we find that

\[
\text{Percentage} = \frac{\Delta v_y}{v_y} \times 100% = \frac{1}{4\pi} \times 100% = \left[ 8.0\% \right]
\]

42. **REASONING** The mass \( m \) of the particle is related to its rest energy \( E_0 \) by \( E_0 = mc^2 \) (Equation 28.5). Therefore, if there is a minimum uncertainty \( \Delta E_0 \) in measuring the rest energy of the particle, there will be a corresponding uncertainty \( \Delta m \) in measuring its mass:

\[
\Delta E_0 = (\Delta m)c^2 \quad \text{or} \quad \Delta m = \frac{\Delta E_0}{c^2}
\]  

(1)

The minimum uncertainty \( \Delta E_0 \) in the particle’s rest energy is related to the length of time \( \Delta t \) the particle exists in a state by the Heisenberg uncertainty principle: \((\Delta E_0)(\Delta t) = \frac{h}{4\pi}\)

(Equation 29.11), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant.

**SOLUTION** Solving Equation 29.11 for the minimum uncertainty \( \Delta E_0 \), we obtain

\[
\Delta E_0 = \frac{h}{4\pi(\Delta t)}
\]  

(2)

Substituting Equation (2) into Equation (1), we find that

\[
\Delta m = \frac{\Delta E_0}{c^2} = \frac{h}{4\pi(\Delta t)c^2} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi \left( 7.4 \times 10^{-20} \text{ s} \right) \left( 3.00 \times 10^8 \text{ m/s} \right)^2} = \left[ 7.9 \times 10^{-33} \text{ kg} \right]
\]

43. **REASONING AND SOLUTION** The de Broglie wavelength \( \lambda \) is given by Equation 29.8 as \( \lambda = h/p \), where \( p \) is the magnitude of the momentum of the particle. The magnitude of the momentum is \( p = mv \), where \( m \) is the mass and \( v \) is the speed of the particle. Using this expression in Equation 29.8, we find that

\[
\frac{\lambda}{m} = \frac{h}{mv} \quad \text{or} \quad \frac{v}{m\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.67 \times 10^{-27} \text{ kg})(0.282 \times 10^{-9} \text{ m})} = \left[ 1.41 \times 10^3 \text{ m/s} \right]
\]

44. **REASONING** The energy of a photon is related to its frequency and Planck’s constant. The frequency, in turn, is related to the speed and wavelength of the light. Thus, we can relate the energy to the wavelength. The given relationship between the wavelengths will then allow us to determine the unknown energy.
**SOLUTION** The energy $E$ of a photon with frequency $f$ is

$$E = hf$$  \hspace{1cm} (29.2)$$

where $h$ is Planck’s constant. The frequency is related to the speed $c$ and wavelength $\lambda$ of the light according to

$$f = \frac{c}{\lambda}$$  \hspace{1cm} (16.1)$$

Substituting this expression for $f$ into Equation 29.2 gives

$$E = hf = h \frac{c}{\lambda}$$

Applying this result to both sources, we have

$$E_B = h \frac{c}{\lambda_B} \quad \text{and} \quad E_A = h \frac{c}{\lambda_A}$$

Dividing the two expressions gives

$$\frac{E_B}{E_A} = \frac{h \frac{c}{\lambda_B}}{h \frac{c}{\lambda_A}} = \frac{\lambda_A}{\lambda_B}$$

Using the given value for $E_A$ and the fact that $\lambda_B = 3\lambda_A$ in this result shows that

$$E_B = E_A \frac{\lambda_A}{\lambda_B} = \left(2.1 \times 10^{-18} \text{ J}\right) \left(\frac{\lambda_A}{3\lambda_A}\right) = 7.0 \times 10^{-19} \text{ J}$$

**45. SSM REASONING AND SOLUTION** The de Broglie wavelength $\lambda$ is given by Equation 29.8 as $\lambda = h/p$, where $p$ is the magnitude of the momentum of the particle. The magnitude of the momentum is $p = mv$, where $m$ is the mass and $v$ is the speed of the particle. Using this expression in Equation 29.8, we find that $\lambda = h/(mv)$, or

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.67 \times 10^{-27} \text{ kg})(1.30 \times 10^{-14} \text{ m})} = 3.05 \times 10^7 \text{ m/s}$$

The kinetic energy of the proton is
\[ \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg})(3.05 \times 10^7 \text{ m/s})^2 = 7.77 \times 10^{-13} \text{ J} \]

46. **REASONING** The de Broglie wavelength \( \lambda \) of a particle is inversely proportional to the magnitude \( p \) of its momentum, as we see from \( \lambda = \frac{h}{p} \) (Equation 29.8), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant. The electron is moving at a speed \( v = 0.88c \), which is close to the speed of light in a vacuum \( (c = 3.00 \times 10^8 \text{ m/s}) \). Therefore, we will use \( p = \sqrt{1 - \frac{v^2}{c^2}} \) (Equation 28.3) to determine the magnitude of the electron’s relativistic momentum, where \( m = 9.11 \times 10^{-31} \text{ kg} \) is the electron’s mass.

**SOLUTION** Substituting Equation 28.3 into Equation 9.8, we find that

\[
\lambda = \frac{h}{p} = \frac{h}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{h}{mv} \sqrt{1 - \frac{v^2}{c^2}}
\]

\[
= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(0.88)(3.00 \times 10^8 \text{ m/s})} \sqrt{1 - \left(\frac{0.88c}{c}\right)^2} = 1.3 \times 10^{-12} \text{ m}
\]

47. **REASONING** According to Equation 29.3, the relation between the photon energy, the maximum kinetic energy of an ejected electron, and the work function of a metal surface is

\[
hf = \text{KE}_{\text{max}} + W_0
\]

Equation 16.1 relates the frequency \( f \) of a photon to its wavelength \( \lambda \) via \( f = \frac{c}{\lambda} \), where \( c \) is the speed of light in a vacuum. The maximum kinetic energy \( \text{KE}_{\text{max}} \) is related to the mass \( m \) and maximum speed \( v_{\text{max}} \) of the ejected electron by \( \text{KE}_{\text{max}} = \frac{1}{2}mv^2 \) (Equation 6.2). With these substitutions, Equation 29.3 becomes

\[
hf = \text{KE}_{\text{max}} + W_0 \quad \text{or} \quad \frac{hc}{\lambda} = \frac{1}{2}mv_{\text{max}}^2 + W_0
\]
**SOLUTION** Solving Equation (1) for the wavelength gives

\[ \lambda = \frac{hc}{\frac{1}{2} m v_{\text{max}}^2 + W_0} \]

\[
\lambda_A = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{\frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(7.30 \times 10^5 \text{ m/s})^2 + 4.80 \times 10^{-19} \text{ J}} = 2.75 \times 10^{-7} \text{ m}
\]

\[
\lambda_B = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{\frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^5 \text{ m/s})^2 + 4.80 \times 10^{-19} \text{ J}} = 3.35 \times 10^{-7} \text{ m}
\]

48. **REASONING** The speed \( v \) of a particle is related to the magnitude \( p \) of its momentum by \( v = p/m \) (Equation 7.2). The magnitude of the momentum is related to the particle’s de Broglie wavelength \( \lambda \) by \( p = h/\lambda \) (Equation 29.8), where \( h \) is Planck’s constant. Thus, the speed of a particle can be expressed as \( v = h/(m \lambda) \). We will use this relation to find the speed of the proton.

**SOLUTION** The speeds of the proton and electron are

\[
v_{\text{proton}} = \frac{h}{m_{\text{proton}} \lambda_{\text{proton}}} \quad \text{and} \quad v_{\text{electron}} = \frac{h}{m_{\text{electron}} \lambda_{\text{electron}}}
\]

Dividing the first equation by the second equation, and noting that \( \lambda_{\text{electron}} = \lambda_{\text{proton}} \), we obtain

\[
\frac{v_{\text{proton}}}{v_{\text{electron}}} = \frac{m_{\text{electron}} \lambda_{\text{electron}}}{m_{\text{proton}} \lambda_{\text{proton}}} = \frac{m_{\text{electron}}}{m_{\text{proton}}}
\]

Using values for \( m_{\text{electron}} \) and \( m_{\text{proton}} \) taken from the inside of the front cover, we find that the speed of the proton is

\[
v_{\text{proton}} = v_{\text{electron}} \left( \frac{m_{\text{electron}}}{m_{\text{proton}}} \right) = (4.50 \times 10^6 \text{ m/s}) \left( \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) = 2.45 \times 10^3 \text{ m/s}
\]

49. **REASONING** When the electron is at rest, it has electric potential energy, but no kinetic energy. The electric potential energy EPE is given by \( EPE = eV \) (Equation 19.3), where \( e \) is the magnitude of the charge on the electron and \( V \) is the potential difference. When the
electron reaches its maximum speed, it has no potential energy, but its kinetic energy is \( \frac{1}{2}mv^2 \). The conservation of energy states that the final total energy of the electron equals the initial total energy:

\[
\frac{1}{2}mv^2 = eV
\]

Solving this equation for the potential difference gives \( V = \frac{mv^2}{2e} \).

The speed of the electron can be expressed in terms of the magnitude \( p \) of its momentum by \( v = \frac{p}{m} \) (Equation 7.2). The magnitude of the electron’s momentum is related to its de Broglie wavelength \( \lambda \) by \( p = \frac{h}{\lambda} \) (Equation 29.8), where \( h \) is Planck’s constant. Thus, the speed can be written as \( v = \frac{h}{m\lambda} \). Substituting this expression for \( v \) into \( V = \frac{mv^2}{2e} \) gives \( V = \frac{h^2}{2me\lambda^2} \).

**SOLUTION** The potential difference that accelerates the electron is

\[ V = \frac{h^2}{2me\lambda^2} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})(0.900 \times 10^{-11} \text{ m})^2} = 1.86 \times 10^4 \text{ V} \]

50. **REASONING** The wavelength \( \lambda \) of the photon is related to its frequency \( f \) by \( \lambda = \frac{c}{f} \) (Equation 16.1), where \( c \) is the speed of light. The frequency of the photon is proportional to its energy \( E \) via \( f = \frac{E}{h} \) (Equation 29.2), where \( h \) is Planck’s constant. Thus, \( \lambda = \frac{ch}{E} \). The photon energy is equal to the sum of the maximum kinetic energy \( KE_{\text{max}} \) of the ejected electron and the work function \( W_0 \) of the metal; \( E = KE_{\text{max}} + W_0 \) (Equation 29.3). Substituting this expression for \( E \) into \( \lambda = \frac{ch}{KE_{\text{max}} + W_0} \) gives

\[ \lambda = \frac{c h}{KE_{\text{max}} + W_0} \quad (1) \]

The maximum kinetic energy is related to the maximum speed \( v_{\text{max}} \) by \( KE_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 \) (Equation 6.2), where \( m \) is the mass of the electron.

**SOLUTION** Substituting \( KE_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 \) into Equation (1), and converting the work function from electron-volts to joules, gives
\[
\lambda = \frac{ch}{\frac{1}{2}mv_{\text{max}}^2 + W_0}
\]
\[
= \frac{(3.00 \times 10^8 \text{ m/s}) (6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\frac{1}{2} (9.11 \times 10^{-31} \text{ kg}) (1.2 \times 10^6 \text{ m/s})^2 + (2.3 \text{ eV}) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right)} = 1.9 \times 10^{-7} \text{ m}
\]

51. **REASONING**

a. Consider one square meter of the sail’s surface. Each of the \( N \) photons that strike this square meter in a one-second interval \( (\Delta t = 1.0 \text{ s}) \) has an initial momentum \( p \) that is determined by its wavelength \( \lambda \), according to \( p = \frac{h}{\lambda} \) (Equation 29.6), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant. Each photon is fully reflected, so the final momentum of a photon is equal to \( -p \). The magnitude of the change \( \Delta p \) in a photon’s momentum, then, is equal to \( \Delta p = |p - (-p)| = 2p \), and the magnitude \( \Delta P \) of the net momentum change undergone by all \( N \) photons is given by \( \Delta P = N\Delta p = 2Np \). In order to cause this momentum change, the sail exerts an impulse of magnitude \( F\Delta t = \Delta P \) (Equation 7.4) on the photons, where \( F \) is the magnitude of the force exerted on the photons per square meter of the sail. By Newton’s Third Law, that force magnitude is equal to the magnitude of the force exerted by the \( N \) photons on one square meter of the sail. From Equation 7.4, then, we have that

\[
F\Delta t = \Delta P = 2Np
\]

Lastly, the magnitude \( \Sigma F = F \) of the net force necessary for the sail to attain the desired acceleration of \( a = 9.8 \times 10^{-6} \text{ m/s}^2 \) is given by Newton’s Second Law, \( F = ma \) (Equation 4.1), where \( m = 3.0 \times 10^{-3} \text{ kg} \) is the mass of one square meter of the sail. As instructed, we have ignored all other forces acting on the sail.

b. The intensity \( S \) of the laser beam depends on the total energy delivered to the sail by

\[
S = \frac{\text{Total energy}}{\Delta t \cdot A}
\]

(Equation 24.4), where \( A \) is the area of the sail and \( \Delta t \) is the time interval.

The total energy is equal to the number \( N \) of photons that strike the area in one second times the energy \( E \) of a single photon. Therefore, the intensity of the laser beam is

\[
S = \frac{\text{Total energy}}{\Delta t \cdot A} = \frac{NE}{\Delta t \cdot A}
\]

The energy \( E \) of a single photon is given by \( E = hf \) (Equation 29.2), where \( f = \frac{c}{\lambda} \) (Equation 16.1) is the photon’s frequency and \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum.
\textbf{SOLUTION}

a. Solving Equation (1) for \( N \), we obtain

\[
N = \frac{F \Delta t}{2p}
\]  
(3)

Substituting \( p = \frac{h}{\lambda} \) (Equation 29.6) and \( F = ma \) (Equation 4.1) into Equation (3) yields

\[
N = \frac{F \Delta t}{2p} = \frac{ma \Delta t}{2 \left( \frac{h}{\lambda} \right)} = \frac{ma \Delta t \lambda}{2h}
\]

\[
= \left( 3.0 \times 10^{-3} \text{ kg} \right) \left( 9.8 \times 10^{-6} \text{ m/s}^2 \right) \left( 1.00 \text{ s} \right) \left( 225 \times 10^{-9} \text{ m} \right)
\]

\[
= \frac{5.0 \times 10^{18}}{2 \left( 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \right)}
\]

b. Substituting \( f = \frac{c}{\lambda} \) (Equation 16.1) into \( E = hf \) (Equation 29.2) gives \( E = hf = \frac{hc}{\lambda} \).

Substituting this result into Equation (2), we obtain

\[
S = \frac{NE}{\Delta t A} = \frac{Nh c}{\Delta t A \lambda}
\]  
(4)

Equation (4) applies to the intensity reaching an area \( A = 1.0 \text{ m}^2 \) of the sail in a time \( \Delta t = 1.0 \text{ s} \). Therefore, in order for the sail to accelerate at the desired rate, the intensity of the laser must be

\[
S = \frac{Nh c}{\Delta t A \lambda} = \frac{\left( 5.0 \times 10^{18} \right) \left( 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \right) \left( 3.00 \times 10^8 \text{ m/s} \right)}{(1.00 \text{ s}) (1.00 \text{ m}^2) \left( 225 \times 10^{-9} \text{ m} \right)} = 4.4 \text{ W/m}^2
\]
1. (d) The Bohr model deals with a single negatively charged electron in orbit about a positively charged nucleus. Since only one electron is assumed to be present, the model does not take into account the electrostatic repulsion between electrons that would exist in a multiple-electron atom, such as helium (two electrons).

2. (c) According to Equation 30.10, the radius \( r_n \) of an orbit is proportional to the square of the quantum number \( n \). An increase in the radius by a factor of four, therefore, means that \( n \) has doubled. According to Equation 30.8, the magnitude \( L_n \) of the orbital angular momentum is proportional to \( n \). This means that \( L_n \) doubles when \( r_n \) doubles.

3. (b) The shortest wavelength \( \lambda_{\text{shortest}} \) in a given series can be calculated from Equation 30.14 (with \( \frac{1}{n_i^2} = 0 \)) as \( \frac{1}{\lambda_{\text{shortest}}} = \frac{RZ^2}{n_i^2} \), where \( R \) is the Rydberg constant and \( Z \) is the number of protons in the nucleus.

4. 1282 nm

5. (e) To remove an electron completely from an atom and place it at rest infinitely far from the nucleus of the atom, energy must be supplied in the amount of the ionization energy.

6. (e) In the quantum mechanical picture, the principal quantum number \( n \) can have the values 1, 2, 3, \ldots. The orbital quantum number \( \ell \) can have the values \( \ell = 0, 1, 2, \ldots, (n-1) \). Thus, for \( n = 3 \), the possible values for \( \ell \) are 0, 1, and 2.

7. (a) According to Equation 30.15, the magnitude \( L \) of the orbital angular momentum is \( L = \sqrt{\ell (\ell + 1)} \frac{\hbar}{2\pi} \). This expression can be solved for \( \ell \), the orbital angular momentum quantum number, with the result that \( \ell = 4 \). Since \( \ell \) can be at most \( n - 1 \), the principal quantum number \( n \) must be 5 or larger. Of the options given in this case, 5 is the only feasible answer.

8. (c) The spin angular momentum of the electron plays no role in the Bohr model, but it does play a role in the quantum mechanical picture.

9. \( 6.33 \times 10^{-34} \text{ J} \cdot \text{s} \)

10. (e) The Pauli exclusion principle states that no two electrons in an atom can have the same set of the four quantum numbers \( n, \ell, m_\ell, \text{ and } m_s \).
11. (a) A subshell can contain up to a maximum of $2(2\ell + 1)$ electrons, where $\ell$ is the orbital quantum number. For the 5f subshell, $\ell = 3$, so the maximum number of electrons is $2(2\ell + 1) = 14$. For the 6h subshell, $\ell = 5$, so the maximum number of electrons is $2(2\ell + 1) = 22$. Therefore, 19 electrons can fit into the 6h subshell but not the 5f subshell.

12. (c) The maximum number is obtained by adding the maximum number for each subshell within the $n = 5$ shell. The value of the orbital quantum number $\ell$ defines a subshell, and for $n = 5$, the possible values for $\ell$ are 0, 1, 2, 3, and 4. Each subshell can contain up to a maximum of $2(2\ell + 1)$ electrons. Thus, the $\ell = 0$ subshell can hold 2 electrons, the $\ell = 1$ subshell can hold 6 electrons, the $\ell = 2$ subshell can hold 10 electrons, the $\ell = 3$ subshell can hold 14 electrons, and the $\ell = 4$ subshell can hold 18 electrons, for a total of 50.

13. (d) For the s, p, and d subshells the values for $\ell$ are 0, 1, and 2, respectively. Each subshell can contain up to a maximum of $2(2\ell + 1)$ electrons. Thus, the s subshell can hold 2 electrons, the p subshell can hold 6 electrons, and the d subshell can hold 10 electrons. In addition, the order in which the subshells fill is important. The subshells fill in order of increasing principal quantum number $n$ and, for a given $n$, in order of increasing $\ell$. There are exceptions, however. For example, the 4s subshell ($\ell = 0$) fills before the 3d subshell ($\ell = 2$). This answer shows the reverse, namely, that the 3d subshell fills before the 4s subshell.

14. (c) More energy is required to knock a K-shell electron completely out of an atom, when the number $Z$ of protons in the nucleus is greater. This is because the electrostatic force of attraction that the nucleus exerts on an electron is greater when the positive nuclear charge is greater.

15. $2.3 \times 10^{-11}$ m

16. (b) According to Equation 30.17, the cutoff wavelength is inversely proportional to the voltage applied across the tube. Therefore, when the voltage increases by a factor of two, the cutoff wavelength decreases by a factor of two.

17. (d) This statement correctly describes a population inversion (see Section 30.8).

18. (d) The population inversions used in a lasers involve a higher (not lower) energy state that is metastable (see Section 30.8).
CHAPTER 30 | THE NATURE OF THE ATOM

PROBLEMS

1. **SSM REASONING** Assuming that the hydrogen atom is a sphere of radius $r_{\text{atom}}$, its volume $V_{\text{atom}}$ is given by $\frac{4}{3}\pi r_{\text{atom}}^3$. Similarly, if the radius of the nucleus is $r_{\text{nucleus}}$, the volume $V_{\text{nucleus}}$ is given by $\frac{4}{3}\pi r_{\text{nucleus}}^3$.

**SOLUTION**

a. According to the given data, the nuclear dimensions are much smaller than the orbital radius of the electron; therefore, we can treat the nucleus as a point about which the electron orbits. The electron is normally at a distance of about $5.3 \times 10^{-11}$ m from the nucleus, so we can treat the atom as a sphere of radius $r_{\text{atom}} = 5.3 \times 10^{-11}$ m. The volume of the atom is

$$V_{\text{atom}} = \frac{4}{3}\pi r_{\text{atom}}^3 = \frac{4}{3}\pi\left(5.3 \times 10^{-11}\ \text{m}\right)^3 = 6.2 \times 10^{-31}\ \text{m}^3$$

b. Similarly, since the nucleus has a radius of approximately $r_{\text{nucleus}} = 1 \times 10^{-15}$ m, its volume is

$$V_{\text{nucleus}} = \frac{4}{3}\pi r_{\text{nucleus}}^3 = \frac{4}{3}\pi\left(1 \times 10^{-15}\ \text{m}\right)^3 = 4 \times 10^{-45}\ \text{m}^3$$

c. The percentage of the atomic volume occupied by the nucleus is

$$\frac{V_{\text{nucleus}}}{V_{\text{atom}}} \times 100\% = \frac{\frac{4}{3}\pi r_{\text{nucleus}}^3}{\frac{4}{3}\pi r_{\text{atom}}^3} \times 100\% = \left(\frac{1 \times 10^{-15}\ \text{m}}{5.3 \times 10^{-11}\ \text{m}}\right)^3 \times 100\% = 7 \times 10^{-13}\ %$$

2. **REASONING** The density $\rho$ is the mass $m$ per unit volume $V$ or $\rho = m/V$ (Equation 11.1). The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where $r$ is the radius. Substituting this expression for the volume into Equation 11.1, we see that the density can be written as

$$\rho = \frac{m}{\frac{4}{3}\pi r^3} = \frac{3m}{4\pi r^3} \quad (1)$$

We will apply Equation (1) to the nucleus and to the complete atom in order to determine the desired density ratio.

**SOLUTION** Applying Equation (1) to the nucleus and to the complete atom and noting that the mass of the complete atom is the sum of the masses of the nuclear proton and the orbiting electron, we find that the desired density ratio is
\[
\frac{\rho_{\text{nucleus}}}{\rho_{\text{atom}}} = \frac{3m_{\text{proton}}/(4\pi r_{\text{proton}}^3)}{3(m_{\text{proton}} + m_{\text{electron}})/(4\pi r_{\text{atom}}^3)} = \frac{m_{\text{proton}} r_{\text{proton}}^3}{(m_{\text{proton}} + m_{\text{electron}}) r_{\text{atom}}^3}
\]

\[
= \frac{(1.67 \times 10^{-27} \text{ kg})(5.3 \times 10^{-11} \text{ m})^3}{(1.67 \times 10^{-27} \text{ kg} + 9.11 \times 10^{-31} \text{ kg})(1.0 \times 10^{-15} \text{ m})^3} = 1.5 \times 10^{14}
\]

We have used the values for the masses of the proton and the electron given on the inside of the front cover of the text.

3. **REASONING** According to the discussion in Conceptual Example 1, the radius of the electron orbit is about \(1 \times 10^5\) times as great as the radius of the nucleus. In the scale model, the radius of the electron’s orbit must also be \(1 \times 10^5\) times as great as the radius of the ball.

**SOLUTION** Since the ball that represents the nucleus has a radius of 3.2 cm, the distance between the nucleus and the nearest electron in the model must be

\[
(3.2 \text{ cm})(1 \times 10^5) = (3.2 \times 10^5 \text{ cm}) \left(\frac{1 \text{ mi}}{1.61 \times 10^5 \text{ cm}}\right) = 2 \text{ mi}
\]

4. **REASONING** In the absence of relativistic effects, the kinetic energy is \(KE = \frac{1}{2}mv^2\), where \(m\) is the mass of the \(\alpha\) particle and \(v\) is its speed. We can relate the kinetic energy to the de Broglie wavelength \(\lambda\) by recognizing two things. The first is that \(\lambda = h/p\), according to Equation 29.8, where \(h\) is Planck’s constant and \(p\) is the magnitude of the \(\alpha\) particle’s momentum. The second is that the magnitude of the momentum is \(p = mv\). With this substitution for \(p\), the de Broglie wavelength becomes

\[
\lambda = h = \frac{h}{mv} \quad \text{or} \quad mv = \frac{h}{\lambda}
\]

Substituting the expression for \(mv\) into the kinetic-energy expression gives

\[
KE = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{h^2}{2m\lambda^2}
\]

**SOLUTION** Using Equation (1), we find that

\[
KE = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2 \left(6.64 \times 10^{-27} \text{ kg}\right) \left(1.4 \times 10^{-14} \text{ m}\right)^2} = 1.7 \times 10^{-13} \text{ J}
\]
5. **REASONING** The distance of closest approach can be obtained by setting the kinetic energy KE of the $\alpha$ particle equal to the electric potential energy EPE of the $\alpha$ particle. According to Equation 19.3, we have that $EPE = (2e)V$, where $2e$ is the charge on the $\alpha$ particle and $V$ is the electric potential created by a gold nucleus. According to Equation 19.6, the electric potential of the gold nucleus ($\text{charge} = Ze = 79e$) is $V = k(79e)/r$, where $r$ is the distance between the $\alpha$ particle and the gold nucleus. Therefore, we have that

$$EPE = (2e)V = (2e)\frac{k(79e)}{r} \tag{1}$$

In this expression, we note that $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ and $e = 1.602 \times 10^{-19} \text{ C}$.

**SOLUTION** Solving Equation (1) for the distance $r$ we obtain

$$r = \frac{(2e)k(79e)}{EPE} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)2(79)(1.602 \times 10^{-19} \text{ C})^2}{5.0 \times 10^{-13} \text{ J}} = 7.3 \times 10^{-14} \text{ m}$$

6. **REASONING** The work $W_{AB}$ done on the proton by the electric force in moving the electron from infinity (position $A$) to the “surface” of the nucleus (position $B$) is given by $W_{AB} = -q_0(V_B - V_A)$ (Equation 19.4), where $q_0 = e = 1.6 \times 10^{-19} \text{ C}$ is the charge of a proton, $V_B$ is the electric potential at position $B$, and $V_A$ is the electric potential at position $A$. The electric potentials $V_A$, $V_B$ are due to the net charge $q = +Ze$ of the $Z = 29$ protons in the copper nucleus. The electric potential $V$ a distance $r$ from a point charge $q = +Ze$ is given by $V = \frac{kq}{r} = \frac{kZe}{r}$ (Equation 19.6), where $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. Therefore, when the proton is infinitely far from the copper nucleus (position $A$), the electric potential is $V_A = 0 \text{ V}$, and when the proton is at the “surface” of the copper nucleus (position $B$), the electric potential is

$$V_B = \frac{kZe}{r_B} \tag{1}$$

where $r_B = 4.8 \times 10^{-15} \text{ m}$ is the radius of the copper nucleus. We note that the work $W_{AB}$ given by Equation 19.4 will be in joules (J), which we will convert to electron volts (eV) via the equivalence $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

**SOLUTION** Substituting $V_A = 0 \text{ V}$ and $q_0 = -e$ into $W_{AB} = -q_0(V_B - V_A)$ (Equation 19.4) yields

$$W_{AB} = -q_0(V_B - V_A) = -e(V_B - 0 \text{ V}) = -eV_B \tag{2}$$

Substituting Equation (1) into Equation (2), we obtain
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\[ W_{AB} = -eV_B = -\frac{kZe^2}{r_B} = -\frac{29(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{4.8 \times 10^{-15} \text{ m}} \]

Converting this result to electron volts with the equivalence 1 eV = 1.6\times10^{-19} \text{ J}, we find that

\[ W_{AB} = \left[ -\frac{29(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{4.8 \times 10^{-15} \text{ m}} \right] \left( \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = -8.7 \times 10^6 \text{ eV} \]

7. SSM REASONING According to the Bohr model, the energy \( E_n \) (in eV) of the electron in an orbit is given by Equation 30.13: \( E_n = -13.6 \left( \frac{Z^2}{n^2} \right) \). In order to find the principal quantum number of the state in which the electron in a doubly ionized lithium atom \( \text{Li}^{2+} \) has the same total energy as a ground state electron in a hydrogen atom, we equate the right hand sides of Equation 30.13 for the hydrogen atom and the lithium ion. This gives

\[ -(13.6) \left( \frac{Z^2}{n^2} \right)_H = -(13.6) \left( \frac{Z^2}{n^2} \right)_{\text{Li}} \quad \text{or} \quad n_{\text{Li}}^2 = \left( \frac{n^2}{Z^2} \right)_H Z_{\text{Li}}^2 \]

This expression can be evaluated to find the desired principal quantum number.

SOLUTION For hydrogen, \( Z = 1 \), and \( n = 1 \) for the ground state. For lithium \( \text{Li}^{2+} \), \( Z = 3 \). Therefore,

\[ n_{\text{Li}}^2 = \left( \frac{n^2}{Z^2} \right)_H Z_{\text{Li}}^2 = \left( \frac{1^2}{3^2} \right) \left( 3^2 \right) \quad \text{or} \quad n_{\text{Li}} = 3 \]

8. REASONING The atomic number for helium is \( Z = 2 \). The ground state is the \( n = 1 \) state, the first excited state is the \( n = 2 \) state, and the second excited state is the \( n = 3 \) state. With \( Z = 2 \) and \( n = 3 \), we can use Equation 30.10 to find the radius of the ion.

SOLUTION The radius of the second excited state is

\[ r_3 = (5.29 \times 10^{-11} \text{ m}) \frac{n^2}{Z} = (5.29 \times 10^{-11} \text{ m}) \frac{3^2}{2} = 2.38 \times 10^{-10} \text{ m} \] (30.10)

9. REASONING According to the Bohr model, the energy (in joules) of the \( n \)th orbit of an atom containing a single electron is

\[ E_n = -\left( 2.18 \times 10^{-18} \text{ J} \right) \frac{Z^2}{n^2} \] (30.12)

where \( Z \) is the atomic number of the atom. The ratio of the energies of the two atoms can be obtained directly by using this relation.
**SOLUTION** Taking the ratio of the energy $E_{n,\text{Be}^+}$ of the $n$th orbit of a beryllium atom ($Z_{\text{Be}^+} = 4$) to the energy $E_{n,\text{H}}$ of the $n$th orbit of a hydrogen ($Z_\text{H} = 1$) atom gives

$$\frac{E_{n,\text{Be}^+}}{E_{n,\text{H}}} = \frac{-(2.18 \times 10^{-18} \text{ J}) Z_{\text{Be}^+}^2}{n^2} = \frac{Z_{\text{Be}^+}^2}{Z_\text{H}^2} = \frac{(4)^2}{(1)^2} = 16$$

10. **REASONING** The energy levels in a hydrogen atom are given by $E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}$ (Equation 30.13), where $Z = 1$ is the number of protons in the hydrogen nucleus and $n = 1, 2, 3, ...$. For the first excited state, the value of $n$ is $n = 2$. We will use Equation 30.13 to calculate the corresponding quantum number $n$ for the energy level into which the electron moves after gaining an additional 2.86 eV of energy.

**SOLUTION** Using Equation 30.13, we have

$$E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \quad \text{or} \quad n = \sqrt{-(13.6 \text{ eV}) \frac{Z^2}{E_n}} \quad (1)$$

The energy $E_n$ of the final state is the energy $E_2$ of the first excited state plus the additional energy of 2.86 eV, so that we know that

$$E_n = E_2 + 2.86 \text{ eV} \quad (2)$$

The energy of the first excited state can be determined from Equation 30.13 as follows:

$$E_2 = -(13.6 \text{ eV}) \frac{Z^2}{n^2} = -(13.6 \text{ eV}) \frac{1^2}{2^2} = -3.40 \text{ eV}$$

With this value for $E_2$, Equation (2) becomes

$$E_n = E_2 + 2.86 \text{ eV} = -3.40 \text{ eV} + 2.86 \text{ eV} = -0.54 \text{ eV}$$

We can now use Equation (1) to determine $n$:

$$n = \sqrt{-(13.6 \text{ eV}) \frac{Z^2}{E_n}} = \sqrt{-(13.6 \text{ eV}) \frac{1^2}{(-0.54 \text{ eV})}} = 5$$

11. **SSM REASONING** According to Equation 30.14, the wavelength $\lambda$ emitted by the hydrogen atom when it makes a transition from the level with $n_i$ to the level with $n_f$ is given by
\[ \frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4}{\hbar^3 c} (Z^2) \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \] with \( n_i, n_f = 1, 2, 3, \ldots \) and \( n_i > n_f \)

where \( 2\pi^2 m k^2 e^4 / (\hbar^3 c) = 1.097 \times 10^7 \text{ m}^{-1} \) and \( Z = 1 \) for hydrogen. Once the wavelength for the particular transition in question is determined, Equation 29.2 \( (E = hf = hc / \lambda ) \) can be used to find the energy of the emitted photon.

**SOLUTION** In the Paschen series, \( n_f = 3 \). Using the above expression with \( Z = 1, n_i = 7 \) and \( n_f = 3 \), we find that

\[ \frac{1}{\lambda} = \left( 1.097 \times 10^7 \text{ m}^{-1} \right) \left( 1^2 \right) \left( \frac{1}{3^2} - \frac{1}{7^2} \right) \quad \text{or} \quad \lambda = 1.005 \times 10^{-6} \text{ m} \]

The photon energy is

\[
E = \frac{hc}{\lambda} = \frac{\left( 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \right) \left( 3.00 \times 10^8 \text{ m/s} \right)}{1.005 \times 10^{-6} \text{ m}} = 1.98 \times 10^{-19} \text{ J}
\]

12. **REASONING** The ionization energy for a given state is the energy needed to remove the electron completely from the atom. The removed electron has no kinetic energy and no electric potential energy, so its total energy is zero.

The ionization energy for a given excited state is less than the ionization energy for the ground state. In the excited state the electron already has part of the energy necessary to achieve ionization, so less energy is required to ionize the atom from the excited state than from the ground state.

**SOLUTION**

a. The energy of the \( n^{\text{th}} \) state in the hydrogen atom is given by Equation 30.13 as

\[ E_n = -\left( 13.6 \text{ eV} \right) \frac{Z^2}{n^2} \]. When \( n = \infty \), \( E_\infty = 0 \text{ J} \), and when \( n = 4 \),

\[ E_4 = -\left( 13.6 \text{ eV} \right) \frac{1^2}{4^2} = -0.850 \text{ eV} \]. The difference in energies between these two states is the ionization energy:

\[ \text{Ionization energy} = E_\infty - E_4 = 0.850 \text{ eV} \]

b. In the same manner, it can be shown that the ionization energy for the \( n = 1 \) state is 13.6 eV. The ratio of the ionization energies is

\[
\frac{0.850 \text{ eV}}{13.6 \text{ eV}} = 0.0625
\]
13. **REASONING** Since the atom emits two photons as it returns to the ground state, one is emitted when the electron falls from \( n = 3 \) to \( n = 2 \), and the other is emitted when it subsequently drops from \( n = 2 \) to \( n = 1 \). The wavelengths of the photons emitted during these transitions are given by Equation 30.14 with the appropriate values for the initial and final numbers, \( n_i \) and \( n_f \).

**SOLUTION** The wavelengths of the photons are

\[
\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right) \left(1 \right)^2 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.524 \times 10^6 \text{ m}^{-1} \quad (30.14)
\]

\[\lambda = 6.56 \times 10^{-7} \text{ m}\]

\[
\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right) \left(1 \right)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 8.228 \times 10^6 \text{ m}^{-1} \quad (30.14)
\]

\[\lambda = 1.22 \times 10^{-7} \text{ m}\]

14. **REASONING** The energy levels in a hydrogen atom are given by \( E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \) (Equation 30.13), where \( Z = 1 \) is the number of protons in the hydrogen nucleus and \( n = 1, 2, 3, \ldots \). To use Equation 30.13, we need a value for the quantum number \( n \). We can obtain this value from the radius \( r_n \), since \( r_n = \left(5.29 \times 10^{-11} \text{ m}\right) \frac{n^2}{Z} \) (Equation 30.10).

**SOLUTION** Solving Equation 30.10 for \( n^2 \), we have

\[r_n = \left(5.29 \times 10^{-11} \text{ m}\right) \frac{n^2}{Z} \quad \text{or} \quad n^2 = \frac{r_n Z}{5.29 \times 10^{-11} \text{ m}} \quad (1)\]

Substituting Equation (1) into Equation 30.13, we find

\[
E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} = -(13.6 \text{ eV}) \frac{Z^2}{r_n Z / \left(5.29 \times 10^{-11} \text{ m}\right)}
\]

\[= -(13.6 \text{ eV}) \frac{Z \left(5.29 \times 10^{-11} \text{ m}\right)}{r_n} = -(13.6 \text{ eV}) \left(\frac{1}{4.761 \times 10^{-10} \text{ m}}\right) = -1.51 \text{ eV}\]

15. **REASONING** The Bohr expression as it applies to any one-electron species of atomic number \( Z \), is given by Equation 30.13: \( E_n = -(13.6 \text{ eV})(Z^2 / n^2) \). For certain values of the quantum number \( n \), this expression predicts equal electron energies for singly ionized
helium \( \text{He}^+ \) \((Z = 2)\) and doubly ionized lithium \( \text{Li}^{2+} \) \((Z = 3)\). As stated in the problem, the quantum number \( n \) is different for the equal energy states for each species.

**SOLUTION** For equal energies, we can write

\[
(E_n)_{\text{He}} = (E_n)_{\text{Li}} \quad \text{or} \quad -(13.6 \text{ eV}) \frac{Z_{\text{He}}^2}{n_{\text{He}}^2} = -(13.6 \text{ eV}) \frac{Z_{\text{Li}}^2}{n_{\text{Li}}^2}
\]

Simplifying, this becomes

\[
\frac{Z_{\text{He}}^2}{n_{\text{He}}^2} = \frac{Z_{\text{Li}}^2}{n_{\text{Li}}^2} \quad \text{or} \quad \frac{4}{n_{\text{He}}^2} = \frac{9}{n_{\text{Li}}^2}
\]

Thus,

\[
n_{\text{He}} = n_{\text{Li}} \sqrt{\frac{4}{9}} = n_{\text{Li}} \left(\frac{2}{3}\right)
\]

Therefore, the value of the helium energy level is equal to the lithium energy level for any value of \( n_{\text{He}} \) that is two-thirds of \( n_{\text{Li}} \). For quantum numbers less than or equal to 9, an equality in energy levels will occur for \( n_{\text{He}} = 2, 4, 6 \) corresponding to \( n_{\text{Li}} = 3, 6, 9 \). The results are summarized in the following table.

<table>
<thead>
<tr>
<th>( n_{\text{He}} )</th>
<th>( n_{\text{Li}} )</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>-13.6 eV</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>-3.40 eV</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>-1.51 eV</td>
</tr>
</tbody>
</table>

16. **REASONING** We expect \( Z_{\text{effective}} \) to be greater than one. Since the orbiting “inner” electrons spend part of the time on the side of the nucleus opposite to the outermost electron, they are not always between the nucleus and the outermost electron. Therefore, they are sometimes farther away from the outermost electron than the nuclear protons are. As a result, the repulsive force that they exert on the outermost electron sometimes has a magnitude less than the attractive force exerted by the nuclear protons. The net effect of this is that the 10 “inner” electrons only partially shield the outermost electron from the attractive force of 10 nuclear protons, and it follows that \( Z_{\text{effective}} \) is greater than one.

In the Bohr model the orbital radius \( r_n \) is

\[
r_n = (5.29 \times 10^{-11} \text{ m}) \frac{n^2}{Z} \quad (30.10)
\]

where \( n \) is the principal quantum number. The radius is inversely proportional to \( Z \). Therefore, we expect that the radius calculated with \( Z = 11 \) will be less than the radius.
calculated with \( Z = Z_{\text{effective}} \), where \( Z_{\text{effective}} > 1 \), but less than 11. This makes sense, because 11 protons would pull the outermost electron in more tightly than 1 or 2 protons would.

**SOLUTION**

a. According to the Bohr model, the total energy \( E \) of the electron is

\[
E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \tag{30.13}
\]

To remove an electron with this energy from an atom, it is necessary to give the electron positive energy of a magnitude equal to that in Equation 30.13. This energy is the ionization energy \( E_{\text{ionization}} \). Then, the electron will be raised from the \( n \)th level to the \( n = \infty \) level, where the total energy of the electron is zero and the electron is separated from the atom. Thus, with \( Z = Z_{\text{effective}} \) and \( n = 3 \) for the outermost electron in a sodium atom (\( 1s^2 \ 2s^2 \ 2p^6 \ 3s^1 \)) we have

\[
E_{\text{ionization}} = (13.6 \text{ eV}) \frac{Z_{\text{effective}}^2}{3^2} \quad \text{or} \quad Z_{\text{effective}} = \sqrt{\frac{3^2 E_{\text{ionization}}}{13.6 \text{ eV}}} = \sqrt{\frac{3^2 (5.1 \text{ eV})}{13.6 \text{ eV}}} = 1.8
\]

As expected, \( Z_{\text{effective}} \) is less than 11 and greater than 1.

b. Using Equation 30.10 for the radius with \( Z = 11 \) and \( Z = Z_{\text{effective}} = 1.8 \), we obtain

\[
[Z = 11] \quad r_3 = \left( 5.29 \times 10^{-11} \text{ m} \right) \frac{3^2}{11} = \left[ 4.3 \times 10^{-11} \text{ m} \right]
\]

\[
[Z = 1.8] \quad r_3 = \left( 5.29 \times 10^{-11} \text{ m} \right) \frac{3^2}{1.8} = \left[ 2.6 \times 10^{-10} \text{ m} \right]
\]

As expected, the radius is smaller when \( Z = 11 \).

17. **SSM REASONING** A wavelength of 410.2 nm is emitted by the hydrogen atoms in a high-voltage discharge tube. This transition lies in the visible region (380–750 nm) of the hydrogen spectrum. Thus, we can conclude that the transition is in the Balmer series and, therefore, that \( n_f = 2 \). The value of \( n_i \) can be found using Equation 30.14, according to which the Balmer series transitions are given by

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n_i^2} \right) \quad n = 3, 4, 5, \ldots
\]

This expression may be solved for \( n_i \) for the energy transition that produces the given wavelength.
**SOLUTION** Solving for \( n_i \), we find that

\[
n_i = \sqrt{\frac{1}{2^2 - \frac{1}{R \lambda}}} = \sqrt{\frac{1}{\frac{1}{1.097 \times 10^7 \text{ m}^{-1}} (410.2 \times 10^{-9} \text{ m})}} = 6
\]

Therefore, the initial and final states are identified by \( n_i = 6 \) and \( n_f = 2 \).

18. **REASONING** To obtain the quantum number of the higher level from which the electron falls, we will use Equation 30.14 for the reciprocal of the wavelength \( \lambda \) of the photon:

\[
\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)
\]

where \( R \) is the Rydberg constant and \( n_f \) and \( n_i \), respectively, are the quantum numbers of the final and initial levels. Although we are not directly given the wavelength, we do have a value for the magnitude \( p \) of the photon’s momentum, and the momentum and the wavelength are related according to Equation 29.6:

\[
p = \frac{h}{\lambda}
\]

where \( h \) is Planck’s constant. Using Equation 30.14 to substitute for \( \frac{1}{\lambda} \), we obtain

\[
p = \frac{h}{\lambda} = hR \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)
\]

**SOLUTION** Rearranging Equation (1) gives

\[
\frac{1}{n_f^2} - \frac{1}{n_i^2} = \frac{p}{hR} \quad \text{or} \quad \frac{1}{n_i^2} - \frac{1}{n_f^2} = \frac{p}{hR}
\]

Thus, we find

\[
\frac{1}{n_i^2} = \frac{1}{n_f^2} - \frac{p}{hR} = \frac{1}{1^2} - \frac{5.452 \times 10^{-27} \text{ kg} \cdot \text{m/s}}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s}) (1.097 \times 10^7 \text{ m}^{-1})} = 0.2499 \quad \text{or} \quad n_i = 2
\]

19. **SSM REASONING** For either series of lines, the wavelengths \( \lambda \) can be obtained from

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) (Z)^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad \text{(Equation 30.14), where} \ Z = 1 \ \text{is the number of protons in the hydrogen nucleus,} \ n_i, n_f = 1, 2, 3, \ldots, \text{and} \ n_i > n_f.
\]
**SOLUTION** For the Paschen series, $n_f = 3$. The range of wavelengths occurs for values of $n_i = 4$ to $n_i = \infty$. Using Equation 30.14, we find that the shortest wavelength occurs for $n_i = \infty$ and is

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(1)^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{3^2} \right) \quad \text{or} \quad \lambda = 8.204 \times 10^{-7} \text{ m}$$

The longest wavelength in the Paschen series occurs for $n_i = 4$ and is

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{3^2} - \frac{1}{4^2} \right) \quad \text{or} \quad \lambda = 1.875 \times 10^{-6} \text{ m}$$

For the Brackett series, $n_f = 4$. The range of wavelengths occurs for values of $n_i = 5$ to $n_i = \infty$. Using Equation 30.14, we find that the shortest wavelength occurs for $n_i = \infty$ and is

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(1)^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{4^2} \right) \quad \text{or} \quad \lambda = 1.459 \times 10^{-6} \text{ m}$$

The longest wavelength in the Brackett series occurs for $n_i = 5$ and is

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{4^2} - \frac{1}{5^2} \right) \quad \text{or} \quad \lambda = 4.051 \times 10^{-6} \text{ m}$$

Since the longest wavelength in the Paschen series falls within the Brackett series, the wavelengths of the two series overlap.

---

20. **REASONING** For each species, the wavelengths appearing in the line spectra can be calculated using Equation 30.14:

$$\frac{1}{\lambda} = RZ^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The shortest wavelength for a given series of lines occurs when an electron is in an initial energy level with a principal quantum number of $n_i = \infty$. Therefore, $1/n_i^2 = 1/\infty = 0$, and the shortest wavelength is given by

$$\frac{1}{\lambda} = \frac{RZ^2}{n_f^2}$$
In this expression, the value of \( n_f \) is the same for the series for Li\(^{2+}\) and for Be\(^{3+}\). The term \( R \) is the Rydberg constant and is also the same for both ionic species. Thus, rearranging this equation gives

\[
\lambda Z^2 = \frac{n_f^2}{R} = \text{constant}
\]

which is the basis for our solution.

**SOLUTION** Since \( \lambda Z^2 \) is the same for each species, we have

\[
\left( \lambda Z^2 \right)_{\text{Li}} = \left( \lambda Z^2 \right)_{\text{Be}} \quad \text{or} \quad \lambda_{\text{Be}} \frac{\lambda Z^2}{Z^2_{\text{Be}}} = \frac{(40.5 \text{ nm})(3)^2}{(4)^2} = 22.8 \text{ nm}
\]

21. **REASONING AND SOLUTION**
   a. To find an expression for \( v_n \), we use Equation 30.8, \( L_n = m v_n r_n = n h/(2 \pi) \), and substitute for \( r_n \) from Equation 30.9:

\[
m v_n \left( \frac{h^2 n^2}{4 \pi^2 m k e^2 Z} \right) = \frac{nh}{2 \pi} \quad \text{or} \quad v_n = \frac{2\pi k e^2 Z}{nh}
\]

b. For the hydrogen atom (\( Z = 1 \)) in the \( n = 1 \) orbit,

\[
v_n = \frac{2\pi}{1} \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2 (1)}{(1)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})} = 2.19 \times 10^6 \text{ m/s}
\]

c. For the \( n = 2 \) orbit of hydrogen,

\[
v_n = \frac{2\pi}{2} \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2 (1)}{(2)(6.626 \times 10^{-34} \text{ J} \cdot \text{s})} = 1.09 \times 10^6 \text{ m/s}
\]

d. The speeds found in parts (b) and (c) are well below the speed of light, \( 3.0 \times 10^8 \text{ m/s} \), and are consistent with ignoring relativistic effects.

22. **REASONING** In either the ground state (\( n = 1 \)) or the first excited state (\( n = 2 \)), the electron’s angular speed \( \omega_n \) is found from

\[
v_n = r_n \omega_n \quad \text{(8.9)}
\]

where \( v_n \) is the electron’s orbital speed, \( r_n \) is the orbital radius, and \( \omega_n \) is measured in radians per second. The speed \( v_n \) of the electron depends upon the principal quantum number \( n \) via

\[
L_n = m v_n r_n = n \frac{h}{2\pi} \quad n = 1, 2, 3, \ldots
\]
where $m$ is the mass of the electron. The radius $r_n$ of the electron’s orbit is given by

$$r_n = \left(5.29 \times 10^{-11} \text{ m}\right) \frac{n^2}{Z} = \left(5.29 \times 10^{-11} \text{ m}\right)n^2 \quad n = 1, 2, 3, \ldots$$  \hspace{1cm} (30.10)$$

where we have used $Z = 1$, the atomic number of hydrogen. After using Equations 8.9, 30.8, and 30.10 to determine the angular speed $\omega_n$ of the electron in radians per second, we will convert the result to revolutions per second with the equivalence 1 revolution = $2\pi$ radians.

**SOLUTION**

a. Solving Equation 8.9 for $\omega_n$ yields

$$\omega_n = \frac{v_n}{r_n} \quad (1)$$

Solving Equation 30.8 for $v_n$, we obtain

$$v_n = \frac{nh}{2\pi m r_n} \quad (2)$$

Substituting Equation (2) into Equation (1), we find that

$$\omega_n = \frac{v_n}{r_n} = \frac{nh}{2\pi m r_n} = \frac{nh}{r_n^2} \quad (3)$$

Substituting Equation 30.10 into Equation (3) yields

$$\omega_n = \frac{nh}{2\pi m r_n^2} = \frac{nh}{2\pi m \left(5.29 \times 10^{-11} \text{ m}\right)^2 \left(n^2\right)^2} = \frac{h}{2\pi m \left(5.29 \times 10^{-11} \text{ m}\right)^2 n^3} \quad (4)$$

To find the ground-state angular speed of the electron in revolutions per second, we substitute $n = 1$ into Equation (4) and multiply by the appropriate conversion factor:

$$\omega_1 = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)}{2\pi \left(9.11 \times 10^{-31} \text{ kg}\right) \left(5.29 \times 10^{-11} \text{ m}\right)^2 (1)^3} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = \left(6.59 \times 10^{15} \text{ rev/s}\right)$$

b. When the electron is in the first excited state, the value of the principal quantum number is $n = 2$, so by Equation (4) we have that

$$\omega_2 = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)}{2\pi \left(9.11 \times 10^{-31} \text{ kg}\right) \left(5.29 \times 10^{-11} \text{ m}\right)^2 (2)^3} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = \left(8.23 \times 10^{14} \text{ rev/s}\right)$$
23. **REASONING**
   a. The ground state is the \( n = 1 \) state, the first excited state is the \( n = 2 \) state, and the second excited state is the \( n = 3 \) state. The total energy (in eV) of a hydrogen atom in the \( n = 3 \) state is given by Equation 30.13.

   b. According to quantum mechanics, the magnitude \( L \) of the angular momentum is given by Equation 30.15 as \( L = \sqrt{\ell (\ell + 1)} \left( \frac{\hbar}{2\pi} \right) \), where \( \ell \) is the orbital quantum number. The discussion in Section 30.5 indicates that the maximum value that \( \ell \) can have is one less than the principal quantum number, so that \( \ell_{\text{max}} = n - 1 \).

   c. Equation 30.16 gives the \( z \)-component \( L_z \) of the angular momentum as \( L_z = m_\ell \left( \frac{\hbar}{2\pi} \right) \), where \( m_\ell \) is the magnetic quantum number. According to the discussion in Section 30.5, the maximum value that \( m_\ell \) can attain is when it is equal to the orbital quantum number, which is \( \ell_{\text{max}} \).

**SOLUTION**

a. The total energy of the hydrogen atom is given by Equation 30.13. Using \( n = 3 \), we have

\[
E_3 = -\left( \frac{13.6 \text{ eV}}{3^2} \right) = -1.51 \text{ eV}
\]

b. The maximum orbital quantum number is \( \ell_{\text{max}} = n - 1 = 3 - 1 = 2 \). The maximum angular momentum \( L_{\text{max}} \) has a magnitude given by Equation 30.15:

\[
L_{\text{max}} = \sqrt{\ell_{\text{max}} (\ell_{\text{max}} + 1)} \left( \frac{\hbar}{2\pi} \right) = \sqrt{2(2+1)} \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} = 2.58 \times 10^{-34} \text{ J} \cdot \text{s}
\]

c. The maximum value for the \( z \)-component \( L_z \) of the angular momentum is

\[
L_z = m_\ell \left( \frac{\hbar}{2\pi} \right) = (2) \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} = 2.11 \times 10^{-34} \text{ J} \cdot \text{s}
\]

24. **REASONING** The orbital quantum number \( \ell \) can have any integer value from 0 up to \( n - 1 \), so that it must be less than the principle quantum number \( n \) and it cannot be a negative number. If for example, \( n = 4 \), \( \ell \) can have only the values 0, 1, 2, and 3.

The magnetic quantum number \( m_\ell \) can only have integer values, including 0, from \( -\ell \) to \( +\ell \). Thus, it can have a negative value. For instance, if \( \ell = 2 \), \( m_\ell \) can have the values \(-2, -1, 0, +1, \) and \(+2\).
SOLUTION Of the five states listed in the table, three are not possible. The ones that are not possible, and the reasons they are not possible, are:

<table>
<thead>
<tr>
<th>State</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>The quantum number $\ell$ must be less than $n$</td>
</tr>
<tr>
<td>(c)</td>
<td>The quantum number $m_\ell$ must be less than or equal to $\ell$.</td>
</tr>
<tr>
<td>(d)</td>
<td>The quantum number $\ell$ cannot be negative.</td>
</tr>
</tbody>
</table>

25. REASONING The orbital quantum number $\ell$ has values of 0, 1, 2, ..., $(n - 1)$, according to the discussion in Section 30.5. Since $\ell = 5$, we can conclude, therefore, that $n \geq 6$. This knowledge about the principal quantum number $n$ can be used with Equation 30.13,

$$E_n = -\left(13.6 \text{ eV}\right)\frac{Z^2}{n^2},$$

to determine the smallest value for the total energy $E_n$.

SOLUTION The smallest value of $E_n$ (i.e., the most negative) occurs when $n = 6$. Thus, using $Z = 1$ for hydrogen, we find

$$E_n = -\left(13.6 \text{ eV}\right)\frac{1^2}{6^2} = -0.378 \text{ eV}$$

26. REASONING We can determine the orbital quantum number $\ell$ from the maximum allowed magnitude of the magnetic quantum number $m_\ell$ (see Section 30.5). For a given value of $\ell$ the allowed values for $m_\ell$ are

$$-\ell, \ldots, -2, -1, 0, +1, +2, \ldots, +\ell$$

Once the value for $\ell$ is identified, we can determine what values for the principal quantum number $n$ are possible (see Section 30.5). For a given value of $n$, the permissible values for $\ell$ are

$$\ell = 0, 1, 2, 3, \ldots, (n - 1)$$

SOLUTION Since the maximum magnitude for the magnetic quantum number is $|m_\ell| = 4$, we conclude that $\boxed{\ell = 4}$. Furthermore, we know that the value for $\ell$ must be less than or equal to $n - 1$. Therefore, the smallest possible value of the principal quantum number is $\boxed{n = 5}$.

27. SSM REASONING The maximum value for the magnetic quantum number is $m_\ell = \ell$; thus, in state A, $\ell = 2$, while in state B, $\ell = 1$. According to the quantum mechanical theory
of angular momentum, the magnitude of the orbital angular momentum for a state of given \( \ell \) is \( L = \sqrt{\ell(\ell+1)(\hbar/2\pi)} \) (Equation 30.15). This expression can be used to form the ratio \( L_A / L_B \) of the magnitudes of the orbital angular momenta for the two states.

**SOLUTION** Using Equation 30.15, we find that

\[
\frac{L_A}{L_B} = \frac{\sqrt{2(2+1)}\frac{\hbar}{2\pi}}{\sqrt{1(1+1)}\frac{\hbar}{2\pi}} = \frac{6}{\sqrt{2}} = \sqrt{3} = 1.732
\]

28. **REASONING** The greater the magnitude of the magnetic quantum number \( m_\ell \), the greater the magnitude of the \( z \) component \( L_z \) of the electron’s angular momentum, as we see from

\[
L_z = m_\ell \frac{\hbar}{2\pi} \quad (\text{Equation 30.16}), \quad \text{where} \quad \hbar = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \quad \text{is Planck’s constant.}
\]

The maximum value of the magnetic quantum number \( m_\ell \) can have is the value of the orbital quantum number \( \ell \), so we know that \( m_\ell = \ell \). The angular momentum \( L \) of the electron can be found from the orbital quantum number \( \ell \) via \( L = \ell(\ell+1)\frac{\hbar}{2\pi} \) (Equation 30.15).

**SOLUTION** Squaring both sides of Equation 30.15 and solving for the quantity \( \ell(\ell+1) \), we obtain

\[
L^2 = \ell(\ell+1)\left(\frac{\hbar}{2\pi}\right)^2 \quad \text{or} \quad \ell(\ell+1) = \left(\frac{2\pi L}{\hbar}\right)^2 \quad (1)
\]

The orbital quantum number \( \ell \) can only take on nonnegative integral values: \( \ell = 0, 1, 2, 3, \ldots \). The quantity \( \ell(\ell+1) \) in Equation (1), therefore, is a product of two successive integers, and must itself be an integer. Substituting \( L = 8.948 \times 10^{-34} \text{ J} \cdot \text{s} \) into Equation (1) yields

\[
\ell(\ell+1) = \left[ \frac{2\pi \left(8.948 \times 10^{-34} \text{ J} \cdot \text{s}\right)}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} \right]^2 = 72 \quad (2)
\]

The only positive integer \( \ell \) that satisfies Equation (2) is 8: \( \ell(\ell+1) = 8(8+1) = 8(9) = 72 \).

Substituting \( m_\ell = \ell = 8 \) into \( L_z = m_\ell \frac{\hbar}{2\pi} \) (Equation 30.16), we obtain the maximum possible magnitude for the \( z \) component of the electron’s angular momentum:

\[
L_z = 8 \left( \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} \right) = \frac{8.436 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi}
\]
29. **REASONING** The total energy \( E_n \) for a hydrogen atom in the quantum mechanical picture is the same as in the Bohr model and is given by Equation 30.13:

\[
E_n = -(13.6 \text{ eV}) \frac{1}{n^2}
\]  

(30.13)

Thus, we need to determine values for the principal quantum number \( n \) if we are to calculate the three smallest possible values for \( E \). Since the maximum value of the orbital quantum number \( \ell \) is \( n - 1 \), we can obtain a minimum value for \( n \) as \( n_{\text{min}} = \ell + 1 \). But how to obtain \( \ell \)? It can be obtained, because the problem statement gives the maximum value of \( L_z \), the \( z \) component of the angular momentum. According to Equation 30.16, \( L_z \) is

\[
L_z = m_\ell \frac{h}{2\pi}
\]  

(30.16)

where \( m_\ell \) is the magnetic quantum number and \( h \) is Planck’s constant. For a given value of \( \ell \) the allowed values for \( m_\ell \) are as follows: \(-\ell, \ldots, -2, -1, 0, +1, +2, \ldots, +\ell\). Thus, the maximum value of \( m_\ell \) is \( \ell \), and we can use Equation 30.16 to calculate the maximum value of \( m_\ell \) from the maximum value given for \( L_z \).

**SOLUTION** Solving Equation 30.16 for \( m_\ell \) gives

\[
m_\ell = \frac{2\pi L_z}{h} = \frac{2\pi \left(4.22 \times 10^{-34} \text{ J} \cdot \text{s}\right)}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 4
\]

As explained in the **REASONING**, this maximum value for \( m_\ell \) indicates that \( \ell = 4 \). Therefore, a minimum value for \( n \) is

\[
n_{\text{min}} = \ell + 1 = 4 + 1 = 5 \quad \text{or} \quad n \geq 5
\]

This means that the three energies we seek correspond to \( n = 5 \), \( n = 6 \), and \( n = 7 \). Using Equation 30.13, we find them to be

\[
\begin{align*}
[n = 5] & \quad E_5 = -(13.6 \text{ eV}) \frac{1}{5^2} = -0.544 \text{ eV} \\
[n = 6] & \quad E_6 = -(13.6 \text{ eV}) \frac{1}{6^2} = -0.378 \text{ eV} \\
[n = 7] & \quad E_7 = -(13.6 \text{ eV}) \frac{1}{7^2} = -0.278 \text{ eV}
\end{align*}
\]

30. **REASONING** From the drawing, we see that the angle \( \theta \) is related to the magnitude \( L \) of the orbital angular momentum and the magnitude \( L_z \) of the \( z \) component of the orbital angular momentum by
\[ \theta = \cos^{-1} \left( \frac{L_z}{L} \right) \]  

(1.5)

The magnitude \( L \) of the orbital angular momentum is given by \( L = \sqrt{\ell (\ell + 1)} \frac{h}{2\pi} \) (Equation 30.15), where \( \ell \) is the orbital quantum number and \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant. The orbital quantum number can have any integer value from \( \ell = 0 \) to \( \ell = n-1 \), where \( n = 5 \) is the principal quantum number. The magnitude \( L_z \) of the \( z \) component of the orbital angular momentum is given by \( L_z = m_\ell \frac{h}{2\pi} \) (Equation 30.16), where \( m_\ell \) is the magnetic quantum number, an integer ranging in value from \(-\ell \) to \( \ell \). For a given magnitude \( L \) of the orbital angular momentum (and, hence, a given value of the orbital quantum number \( \ell \)), the smaller the angle \( \theta \) becomes, the greater the magnitude \( L_z \) of the \( z \) component of the orbital angular momentum. Therefore, we will determine the smallest value of \( \theta \) by choosing the greatest possible value of the magnetic quantum number: \( m_\ell = \ell \).

**SOLUTION** Substituting \( m_\ell = \ell \) into \( L_z = m_\ell h \frac{1}{2\pi} \) (Equation 30.16), we obtain

\[ L_z = \ell \frac{h}{2\pi} \]  

(1)

Substituting \( L = \sqrt{\ell (\ell + 1)} \frac{h}{2\pi} \) and Equation (1) into Equation (1.5) yields

\[ \theta = \cos^{-1} \left( \frac{L_z}{L} \right) = \cos^{-1} \left( \frac{\ell \frac{h}{2\pi}}{\sqrt{\ell (\ell + 1)} \frac{h}{2\pi}} \right) = \cos^{-1} \left( \ell \sqrt{\ell (\ell + 1)} \right) = \cos^{-1} \left( \sqrt{\ell + 1} \right) \]  

(2)

We must choose the value of the orbital quantum number \( \ell \) such that the angle \( \theta \) in Equation (2) is as small as possible. This means making the quantity inside the square root as large as possible. Given that \( \ell \) is an integer, we see that the quantity in the square root of Equation (2) will be of the form \( \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \). Therefore, to obtain the smallest value for \( \theta \), we must choose the greatest possible value for the orbital quantum number \( \ell \), which is \( \ell = n-1 = 5-1 = 4 \). Substituting this value into Equation (2), we obtain

\[ \theta = \cos^{-1} \left( \sqrt{\frac{4}{5}} \right) = \cos^{-1} \left( \frac{4}{5} \right) = 26.6^\circ \]

31. **REASONING**
a. The maximum number of electrons that the \( n = 1, \ell = 0 \) subshell can contain is 2 (see Figure 30.16). If the atom is in its ground state, the third electron must go into the \( n = 2, \ell = 0 \) subshell. For this subshell, the magnetic quantum number can only be zero (\( m_\ell = 0 \)), and the electron can have a spin quantum number of either \( m_s = +\frac{1}{2} \) or \( -\frac{1}{2} \).

b. If the atom is in its first excited state, the third electron must go into the \( n = 2, \ell = 1 \) subshell. For \( \ell = 1 \), there are three possibilities for \( m_\ell \): \( m_\ell = +1, 0, \text{ and } -1 \). For each value of \( m_\ell \), the electron can have a spin quantum number of either \( m_s = +\frac{1}{2} \) or \( -\frac{1}{2} \).

**SOLUTION**

a. According to the discussion in the REASONING section, the third electron can have one of two possibilities for the four quantum numbers: \( n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2} \) and \( n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2} \).

b. When the third electron is in the first excited state, there are six possibilities for the four quantum numbers:

\[
\begin{align*}
&n = 2, \ell = 1, m_\ell = +1, m_s = +\frac{1}{2}, & n = 2, \ell = 1, m_\ell = +1, m_s = -\frac{1}{2}, \\
&n = 2, \ell = 1, m_\ell = 0, m_s = +\frac{1}{2}, & n = 2, \ell = 1, m_\ell = 0, m_s = -\frac{1}{2}, \\
&n = 2, \ell = 1, m_\ell = -1, m_s = +\frac{1}{2}, & n = 2, \ell = 1, m_\ell = -1, m_s = -\frac{1}{2}.
\end{align*}
\]

32. **REASONING** For use here, we recall that the \( \ell = 0 \) subshell is denoted by s, the \( \ell = 1 \) subshell is denoted by p, and the \( \ell = 2 \) subshell is denoted by d. In Figure 30.16 we see that the 4s subshell fills before the 3d subshell and that the 5s subshell fills before the 4d subshell. We also see in Figure 30.16 that s subshells can contain up to 2 electrons, p subshells can contain up to 6 electrons, and d subshells can contain up to 10 electrons.

**SOLUTION** Following the style in Table 30.3, we find that the ground state electronic configuration for the 39 electrons in yttrium is

\[1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^1\]

33. **REASONING** According to the discussion in Section 30.6, the \( \ell \)th subshell can hold at most \( 2(2\ell + 1) \) electrons. For \( n = 5 \), we can have the following values for \( \ell \): \( \ell = 0, 1, 2, 3, \text{ and } 4 \).

**SOLUTION** The maximum number \( N_\ell \) of electrons that can be put into each subshell is:
\[ \ell = 0 \text{ subshell: } N_0 = 2[2(0)+1] = 2 \]
\[ \ell = 1 \text{ subshell: } N_1 = 2[2(1)+1] = 6 \]
\[ \ell = 2 \text{ subshell: } N_2 = 2[2(2)+1] = 10 \]
\[ \ell = 3 \text{ subshell: } N_3 = 2[2(3)+1] = 14 \]
\[ \ell = 4 \text{ subshell: } N_4 = 2[2(4)+1] = 18 \]

The maximum number \( N \) of electrons that can be put into the \( n = 5 \) shell is:

\[ N = 2 + 6 + 10 + 14 + 18 = 50 \]

34. **ReASONING** The orbital quantum number \( \ell \) can have any integer value from 0 up to \( n - 1 \). If, for example, \( n = 4 \), \( \ell \) can have the values 0, 1, 2, and 3.

For a given value of the orbital quantum number \( \ell \), the magnetic quantum number \( m_\ell \) can have any integer value, including 0, from \(-\ell\) to \(+\ell\). For instance, if \( \ell = 2 \), \( m_\ell \) can have the values \(-2, -1, 0, +1, \) and \(+2\).

**SOLUTION** Of the five subshell configurations, three are not allowed. The ones that are not allowed, and the reasons they are not allowed, are:

(b) The principal quantum number is \( n = 2 \) and the orbital quantum number is \( \ell = 2 \) (the d subshell). Since \( \ell \) must be less than \( n \), this subshell configuration is not permitted.

(c) This subshell has 4 electrons. However, according to the Pauli exclusion principle, only two electrons can be in the s subshell \( (\ell = 0) \). Therefore, this subshell configuration is not allowed.

(d) This subshell has 8 electrons. However, according to the Pauli exclusion principle, only six electrons can be in the p subshell \( (\ell = 1) \). Therefore, this subshell configuration is not allowed.

35. **SSM REASONING** In the theory of quantum mechanics, there is a selection rule that restricts the initial and final values of the orbital quantum number \( \ell \). The selection rule states that when an electron makes a transition between energy levels, the value of \( \ell \) may not remain the same or increase or decrease by more than one. In other words, the rule requires that \( \Delta \ell = \pm 1 \).

**SOLUTION**

a. For the transition \( 2s \rightarrow 1s \), the electron makes a transition from the 2s state \((n = 2, \ell = 0)\) to the 1s state \((n = 1, \ell = 0)\). Since the value of \( \ell \) is the same in both states, \( \Delta \ell = 0 \), and we can conclude that this energy level transition is [not allowed].
b. For the transition $2p \rightarrow 1s$, the electron makes a transition from the $2p$ state ($n=2$, $\ell=1$) to the $1s$ state ($n=1$, $\ell=0$). The value of $\ell$ changes so that $\Delta \ell = 0 - 1 = -1$, and we can conclude that this energy level transition is **allowed**.

c. For the transition $4p \rightarrow 2p$, the electron makes a transition from the $4p$ state ($n=4$, $\ell=1$) to the $2p$ state ($n=2$, $\ell=1$). Since the value of $\ell$ is the same in both states, $\Delta \ell = 0$, and we can conclude that this energy level transition is **not allowed**.

d. For the transition $4s \rightarrow 2p$, the electron makes a transition from the $4s$ state ($n=4$, $\ell=0$) to the $2p$ state ($n=2$, $\ell=1$). The value of $\ell$ changes so that $\Delta \ell = 1 - 0 = +1$, and we can conclude that this energy level transition is **allowed**.

e. For the transition $3d \rightarrow 3s$, the electron makes a transition from the $3d$ state ($n=3$, $\ell=2$) to the $3s$ state ($n=3$, $\ell=0$). The value of $\ell$ changes so that $\Delta \ell = 0 - 2 = -2$, and we can conclude that this energy level transition is **not allowed**.

36. **REASONING** The first “noble gas” is the element with an atomic number $Z$ that is equal to the number of electrons that can fit into the first energy level ($n=1$), and the atomic number $Z$ of the second “noble gas” is equal to the total number of electrons that can fit into the first and second ($n=2$) energy levels. The first energy level has only an s subshell, while the second energy level has both an s subshell and a p subshell. The number of electrons that will fit into either kind of subshell is equal to the number of unique configurations of the quantum numbers $\ell$, $m_\ell$, and $m_s$. Using the “new” rule for the spin quantum number $m_s$ and the “old” rules for $\ell$ and $m_\ell$, we will count up all of the possible electron configurations for both subshells.

**SOLUTION**

a. In an s subshell, both the orbital quantum number $\ell$ and the magnetic quantum number $m_\ell$ must be zero: $\ell=0$, $m_\ell=0$. This means that there is only one (1) combination of these two quantum numbers to be matched with the three (3) values of the spin quantum number $m_s$. Therefore, there are three ($1 \times 3 = 3$) unique configurations of the quantum numbers in the s subshell. We conclude that the first “noble gas” under these quantum rules would be the element with atomic number $Z=3$, which is **lithium (Li)**.

b. In a p subshell, the orbital quantum number is $\ell=1$, and the magnetic quantum number can have three values: $m_\ell=-1,0,1$. This makes for three (3) combinations of $\ell$ and $m_\ell$, each of which may be combined with one of the three (3) unique values of $m_s$, so there are a total of nine ($3 \times 3 = 9$) unique configurations in the p subshell. For a ground-state configuration to fill the $n=1$, $\ell=0$ subshell (3 electrons), the $n=2$, $\ell=0$ subshell (3 electrons), and the $n=2$, $\ell=1$ subshell (9 electrons), the element must have $3 + 3 + 9 = 15$
electrons. The element phosphorus (P) has an atomic number \( Z = 15 \) and, therefore, would be the second “noble gas” under these quantum rules.

37. **REASONING** According to the Bohr model, wavelength \( \lambda \) is specified by Equation 30.14. In this expression, the number \( Z \) of nuclear protons appears. It is this number that we seek, because it will identify the element that it is producing the X-rays. Strictly speaking, the Bohr model applies only to one-electron atoms, because it neglects the repulsive force between electrons in a multiple-electron atom. The production of the K\( _\alpha \) X-ray involves the K-shell, in which each of the two electrons exerts on the other a repulsive force that balances (approximately) the attractive force of one nuclear proton. In effect, one of the K-shell electrons shields the other from the force of that proton. Therefore, in our calculation of the wavelength we will use replace \( Z \) in Equation 30.14 by \( Z - 1 \) as follows:

\[
\frac{1}{\lambda} = R (Z - 1)^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)
\]

where \( n_i = 2 \) and \( n_f = 1 \) for the K\( _\alpha \) X-ray and \( R = 1.097 \times 10^7 \text{ m}^{-1} \).

**SOLUTION** Solving Equation (1) for \( Z \), we obtain

\[
Z = \sqrt{\frac{1/(\lambda R)}{1/n_f^2 - 1/n_i^2}} + 1 = \left( \frac{1/(4.5 \times 10^{-9} \text{ m})(1.097 \times 10^7 \text{ m}^{-1})}{1/1^2 - 1/2^2} \right)^{1/2} + 1 = 6.2
\]

Therefore, referring to the periodic table on the inside of the back cover of the text, we see that the element is likely to be carbon \( (Z = 6) \).

38. **REASONING** One electron volt is the kinetic energy acquired when an electron accelerates from rest through a potential difference of one volt. Thus, we can determine the desired potential difference if we can find the energy (in electron volts) that the electron has when it strikes the copper target. According to the Bohr model, the energy of a K-shell electron is given by \( E_n = -(13.6 \text{ eV}) Z^2 / n^2 \) (Equation 30.13), with \( n = 1 \). The incoming electron must have at least enough energy to raise the K-shell electron from this low energy level up to the 0-eV level that corresponds to a very large distance from the nucleus. Only then will the incoming electron knock the K-shell electron entirely out of a copper atom in the target.

**SOLUTION** We will use Equation 30.13 to estimate the minimum energy that an incoming electron must have. However, strictly speaking, this equation applies only to one-electron atoms, because it neglects the repulsive force between electrons in a multiple-electron atom. In the copper K-shell, each electron exerts on the other electron a repulsive force that balances (approximately) the attractive force of one nuclear proton. In effect, one of the K-shell electrons shields the other from the force of that proton. Therefore, in our calculation we replace \( Z \) in Equation 30.13 by \( Z - 1 \) and find that
\[ E_i = -(13.6 \text{ eV}) \left( \frac{(Z-1)^2}{n_i^2} \right) = -(13.6 \text{ eV}) \left( \frac{29-1}{12} \right) = -1.07 \times 10^4 \text{ eV} \]

Thus, to raise the K-shell electron up to the 0-eV level, the minimum energy for an incoming electron is \( 1.07 \times 10^4 \text{ eV} \). Since a one volt potential difference is required to give the incoming electron an energy of 1.00 eV, a potential difference of \( 1.07 \times 10^4 \text{ V} \) is required to give the incoming electron an energy of \( 1.07 \times 10^4 \text{ eV} \).

39. **REASONING** The wavelength \( \lambda \) of a photon is \( \lambda = \frac{c}{f} \), according to Equation 16.1, where \( f \) is the frequency and \( c \) is the speed of light in a vacuum. The frequency is given by Equation 29.2 as \( f = \frac{E}{h} \), where \( E \) is the energy of the Bremsstrahlung photon and \( h \) is Planck’s constant. Substituting this expression for \( f \) into the expression for the wavelength gives

\[ \lambda = \frac{c}{\frac{E}{h}} = \frac{hc}{E} \]

The energy of the photon is 35.0% of the kinetic energy \( KE \) of the electron that collides with the metal target. According to our discussions in Section 19.2, the electron acquires its kinetic energy by accelerating from rest through a potential difference \( V \), the kinetic energy being \( KE = eV \), where \( e \) is the magnitude of the charge on the electron. Thus, we have that

\[ E = 0.350 \text{ KE} = 0.350 \text{ eV} \]

Substituting this result into the wavelength expression shows that

\[ \lambda = \frac{hc}{E} = \frac{hc}{0.350 \text{ eV}} \]

**SOLUTION** Using Equation (1), we find that

\[ \lambda = \frac{hc}{0.350 \text{ eV}} = \left( 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \right) \left( 3.00 \times 10^8 \text{ m/s} \right) \left\{ \frac{1.60 \times 10^{-19} \text{ C}}{52.0 \times 10^3 \text{ V}} \right\} = 6.83 \times 10^{-11} \text{ m} \]

40. **REASONING** If the electron undergoing the \( K_a \) transition to the \( n_f = 1 \) shell from the \( n_i = 2 \) shell were the only electron in the niobium atom, the Bohr model would predict the wavelength \( \lambda_{\text{predicted}} \) of the emitted X-ray via

\[ \frac{1}{\lambda_{\text{predicted}}} = \left( 1.097 \times 10^7 \text{ m}^{-1} \right) \left(Z^2 \right) \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

(Equation 30.14), where \( Z = 41 \) is the atomic number of niobium. But in actuality, the charge \( +Ze \) of the niobium nucleus is partially screened by the single electron (charge = \( -e \)) remaining in the \( n = 1 \) shell “below” the electron in the \( n_i = 2 \) shell. Therefore, we will use \( Z - 1 \) in Equation 30.14 to account for this partial screening of the nuclear charge.

**SOLUTION**

a. Taking the reciprocal of Equation 30.14 and replacing \( Z \) with \( Z - 1 \) yields
Substituting \( n_f = 1, n_i = 2, \) and \( Z = 41 \) into Equation (1), we find that

\[
\lambda_{\text{predicted}} = \frac{1}{(1.097 \times 10^7 \text{ m}^{-1})(41-1)^2 \left[ \left( \frac{1}{n_f^2} \right) - \left( \frac{1}{n_i^2} \right) \right]} = 7.596 \times 10^{-11} \text{ m}
\]

The difference \( \Delta \lambda \) between the predicted wavelength and the observed wavelength is

\[
\Delta \lambda = \lambda_{\text{predicted}} - \lambda_{\text{observed}} = 7.596 \times 10^{-11} \text{ m} - 7.462 \times 10^{-11} \text{ m} = 1.34 \times 10^{-12} \text{ m}
\]

b. Expressed as a percentage of the observed wavelength, the difference is

\[
\frac{\Delta \lambda}{\lambda_{\text{observed}}} \times 100\% = \frac{1.34 \times 10^{-12} \text{ m}}{7.462 \times 10^{-11} \text{ m}} \times 100\% = 1.80\%
\]

41. **SSM REASONING** As discussed in text Example 11, the wavelengths of the \( K_\alpha \) electrons depend upon the quantity \( Z - 1 \), where \( Z \) is the atomic number of the target. The Bohr model predicts that the energies of the electrons are given by

\[
E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}
\]

(Equation 30.13), where \( n \) can take on any integer value greater than zero. However, the \( K_\alpha \) photons correspond to a transition to the \( n = 1 \) shell from the \( n = 2 \) shell, and during this transition the nuclear charge is partly screened by the electron that remains in the \( n = 1 \) shell, as discussed in text Example 10. We will replace \( Z \) with \( Z - 1 \) in Equation 30.13 to account for this screening. Therefore, the energy of the electron’s initial and final states are calculated from

\[
E_n = -(13.6 \text{ eV}) \frac{(Z-1)^2}{n^2}
\]

The energy \( E \) of the emitted X-ray is equal to the higher initial energy \( E_2 \) of the electron minus the lower final energy \( E_1 \):

\[
E = E_2 - E_1
\]

**SOLUTION** Substituting Equation (1) into Equation (2), we obtain

\[
E = -(13.6 \text{ eV}) \frac{(Z-1)^2}{2^2} - \left[-(13.6 \text{ eV}) \frac{(Z-1)^2}{1^2}\right] = (13.6 \text{ eV}) (Z-1)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right)
\]

\[
E = \frac{3}{4} (13.6 \text{ eV}) (Z-1)^2
\]
Solving Equation (3) for \((Z - 1)^2\) and taking the square root of both sides yields

\[
(Z - 1)^2 = \frac{4E}{3(13.6 \text{ eV})} \quad \text{or} \quad Z = \sqrt[3]{\frac{4E}{3(13.6 \text{ eV})}} + 1
\]

The prediction of the Bohr model is, then,

\[
Z = \sqrt[3]{\frac{4(9890 \text{ eV})}{3(13.6 \text{ eV})}} + 1 = 32.1
\]

The closest integer to this result is \(Z = 32\); the atomic number of germanium (Ge).

42. **REASONING** Using Equation 30.13 for the total energy \(E_1\) of an electron in the K-shell \((n = 1)\) and using \(Z - 1\) instead of \(Z\) to account for shielding (see Example 10), we have

\[
E_1 = -(13.6 \text{ eV}) \left(\frac{(Z-1)^2}{l^2}\right)
\]

When striking the metal target, an incoming electron must have at least enough energy to raise the K-shell electron from this low energy level up to the 0-eV level that corresponds to a very large distance from the nucleus. Only then will the incoming electron knock the K-shell electron entirely out of a target atom. Thus, the value for the minimum energy \(E_{\text{min}}\) that an incoming electron must have is

\[
E_{\text{min}} = (13.6 \text{ eV}) (Z - 1)^2
\]

We can use Equation 30.14 to estimate the \(K_{\alpha}\) wavelength \(\lambda\) by recognizing that an electron in the L shell \((n_l = 2)\) falls into the K shell \((n_f = 1)\) when the \(K_{\alpha}\) photon is emitted. In addition, we use \(Z - 1\) instead of \(Z\) to account for shielding (see Example 11). Thus, we have

\[
\frac{1}{\lambda} = R (Z - 1)^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R (Z - 1)^2 \left(\frac{1}{l^2} - \frac{1}{2^2}\right)
\]

According to Equation (2), the wavelength \(\lambda\) is inversely proportional to \((Z - 1)^2\). Therefore, a larger value of the wavelength implies a smaller value of \(Z - 1\). If \(Z - 1\) is smaller, then \(Z\) must also be smaller. Thus, since \(\lambda_A\) is greater than \(\lambda_B\), it follows that \(Z_A\) is less than \(Z_B\). According to Equation (1), \(E_{\text{min}}\) is directly proportional to \((Z - 1)^2\). This means that when \(Z\) is larger, \(E_{\text{min}}\) is also larger. However, we have just seen that \(Z_A\) is less than \(Z_B\), so we conclude that \(E_{\text{min}, A}\) is less than \(E_{\text{min}, B}\).
**SOLUTION** Applying Equation (1) to both metals A and B, we have

\[
\frac{E_{\text{min, A}}}{E_{\text{min, B}}} = \frac{(13.6 \text{ eV})(Z_A - 1)^2}{(13.6 \text{ eV})(Z_B - 1)^2} = \frac{(Z_A - 1)^2}{(Z_B - 1)^2}
\]

(3)

According to Equation (2), the wavelength \( \lambda_\alpha \) and \((Z - 1)^2\) are inversely proportional. Thus, Equation (3) becomes

\[
\frac{E_{\text{min, A}}}{E_{\text{min, B}}} = \frac{1/\lambda_A}{1/\lambda_B} = \frac{\lambda_B}{\lambda_A}
\]

(4)

Since \( \lambda_A = 2\lambda_B \), we have

\[
\frac{E_{\text{min, A}}}{E_{\text{min, B}}} = \frac{\lambda_B}{\lambda_A} = \frac{\lambda_B}{2\lambda_B} = 0.50
\]

As expected \( E_{\text{min, A}} \) is less than \( E_{\text{min, B}} \).

---

43. **SSM REASONING** In the spectrum of X-rays produced by the tube, the cutoff wavelength \( \lambda_0 \) and the voltage \( V \) of the tube are related according to Equation 30.17, \( V = \frac{hc}{(e\lambda_0)} \). Since the voltage is increased from zero until the \( K_\alpha \) X-ray just appears in the spectrum, it follows that \( \lambda_0 = \lambda_\alpha \) and \( V = \frac{hc}{(e\lambda_\alpha)} \). Using Equation 30.14 for \( 1/\lambda_\alpha \), we find that

\[
V = \frac{hc}{e\lambda_\alpha} = \frac{hcR(Z - 1)^2}{e} \left( \frac{1}{1^2} - \frac{1}{2^2} \right)
\]

In this expression we have replaced \( Z \) with \( Z-1 \), in order to account for shielding, as explained in Example 11 in the text.

**SOLUTION** The desired voltage is, then,

\[
V = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \text{ m/s}\right) \left(1.097 \times 10^7 \text{ m}^{-1}\right) \left(47-1\right)^2}{\left(1.60 \times 10^{-19} \text{ C}\right)} \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = 21600 \text{ V}
\]

---

44. **REASONING** We know from our study of waves (see Section 16.2) that the wavelength of a wave is inversely proportional to its frequency. As discussed in Section 29.3, the frequency of a wave, such as an X–ray photon, is directly proportional to its energy. The energy needed to create the wave comes from the kinetic energy of the electron impinging on the metal target. We can determine the electron’s kinetic energy from its speed, since the
two are related. Thus, we will be able to evaluate the wavelength of the X-ray photon from a knowledge of the electron’s speed.

SOLUTION  As discussed in Section 16.2, the wavelength \( \lambda \) of a wave is related to its frequency \( f \) and speed \( v \) by \( \lambda = \frac{v}{f} \) (Equation 16.1). In a vacuum, an electromagnetic wave travels at the speed \( c \) of light. Substituting \( v = c \) into \( \lambda = \frac{v}{f} \) gives \( \lambda = \frac{c}{f} \). An X-ray photon is an electromagnetic wave that is a discrete packet of energy. The photon’s frequency \( f \) is related to its energy \( E \) by \( f = \frac{E}{h} \) (Equation 29.2), where \( h \) is Planck’s constant. Substituting this expression for \( f \) into \( \lambda = \frac{c}{f} \) gives

\[
\lambda = \frac{c}{f} = \frac{c}{\left( \frac{E}{h} \right)} = \frac{ch}{E}
\]

The energy needed to produce an X-ray photon comes from the kinetic energy of an electron striking the target. If the speed of the electron is much less than the speed of light in a vacuum, its kinetic energy \( KE \) given by Equation 6.2 as

\[
KE = \frac{1}{2}mv^2
\]

where \( m \) and \( v \) are the mass and speed of the electron. We are given that the electron decelerates to one-quarter of its original speed, so that its loss of kinetic energy is

\[
\text{Loss of kinetic energy} = \frac{1}{2}mv^2 - \frac{1}{2}m\left( \frac{1}{4}v \right)^2 = \frac{15}{32}mv^2
\]

The kinetic energy lost by the decelerating electron goes into creating the X-ray photon, so that \( E = \frac{15}{32}mv^2 \). Substituting this expression for \( E \) into Equation (1) gives

\[
\lambda = \frac{ch}{E} = \frac{ch}{\frac{15}{32}mv^2} = \frac{(3.00 \times 10^8 \text{ m/s})(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{\frac{15}{32} \times (9.11 \times 10^{-31} \text{ kg})(6.00 \times 10^7 \text{ m/s})^2} = 1.29 \times 10^{-10} \text{ m}
\]

45. SSM REASONING The number of photons emitted by the laser will be equal to the total energy carried in the beam divided by the energy per photon.

SOLUTION  The total energy carried in the beam is, from the definition of power,

\[
E_{\text{total}} = Pt = (1.5 \text{ W})(0.050 \text{ s}) = 0.075 \text{ J}
\]

The energy of a single photon is given by Equations 29.2 and 16.1 as

\[
E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{514 \times 10^{-9} \text{ m}} = 3.87 \times 10^{-19} \text{ J}
\]

where we have used the fact that 514 nm = 514 × 10⁻⁹ m. Therefore, the number of photons emitted by the laser is
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\[
\frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{0.075 \text{ J}}{3.87 \times 10^{-19} \text{ J/photons}} = 1.9 \times 10^{17} \text{ photons}
\]

46. **REASONING**  The energy \( E \) delivered a single laser photon is \( E = hf \) (Equation 29.2), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant and \( f \) is the frequency. The frequency is related to the wavelength \( \lambda \) according to \( f\lambda = c \) (Equation 16.1), so that \( f = c / \lambda \), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum. Substituting this expression for the frequency into Equation 29.2 gives the following equation for the energy of a single photon:

\[
E = hf = \frac{hc}{\lambda}
\]  

(1)

The total energy of \( N \) photons is \( NE \).

**SOLUTION**  Using Equation (1), we set the total energy delivered by \( N \) photons from the carbon dioxide laser equal to the energy delivered by a single photon from the excimer laser:

\[
N \left( \frac{hc}{\lambda_{\text{carbon dioxide}}} \right) = \frac{hc}{\lambda_{\text{excimer}}}
\]

Solving for \( N \), we find that

\[
N = \frac{\lambda_{\text{carbon dioxide}}}{\lambda_{\text{excimer}}} = \frac{1.06 \times 10^{-5} \text{ m}}{193 \times 10^{-9} \text{ m}} = 54.9
\]

Since only a whole number of photons is possible, the minimum number of photons required of the carbon dioxide laser is \( \boxed{55} \).

47. **REASONING**  The total energy \( E_{\text{tot}} \) of a single pulse is equal to the number \( N \) of photons in the pulse multiplied by the energy \( E \) of each photon:

\[
E_{\text{tot}} = NE
\]  

(1)

The energy \( E \) of each photon is given by \( E = hf \) (Equation 29.2), where \( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \) is Planck’s constant and \( f \) is the frequency of the photon. We will use \( f = \frac{c}{\lambda} \) (Equation 16.1) to determine the frequency \( f \) of the photons from their wavelength \( \lambda \) and the speed \( c \) of light in a vacuum. To find the total energy \( E_{\text{tot}} \) of each pulse, we will make use of the fact that the average power \( P_{\text{avg}} \) of the laser is equal to the total energy of a single pulse divided by the duration \( \Delta t \) of the pulse:
\[ P_{av} = \frac{E_{tot}}{\Delta t} \quad (6.10b) \]

**SOLUTION** Solving Equation (1) for \( N \), we obtain

\[ N = \frac{E_{tot}}{E} \quad (2) \]

Solving Equation (6.10b) for \( E_{tot} \) yields \( E_{tot} = P_{av} \Delta t \). Substituting this result and \( E = hf \) (Equation 29.2) into Equation (2), we find that

\[ N = \frac{E_{tot}}{E} = \frac{P_{av} \Delta t}{hf} \quad (3) \]

Substituting \( f = \frac{c}{\lambda} \) (Equation 16.1) into Equation (3), we find that the number of photons in each pulse is:

\[ N = \frac{P_{av} \Delta t}{hf} = \frac{P_{av} \Delta t}{h \left( \frac{c}{\lambda} \right)} = \frac{P_{av} \Delta t \lambda}{hc} = \frac{(5.00 \times 10^{-3} \text{ W})(25.0 \times 10^{-3} \text{ s})(633 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^{8} \text{ m/s})} \]

\[ = 3.98 \times 10^{14} \]

48. **REASONING** The external source of energy must “pump” the electrons from the ground state \( E_0 \) to the metastable state \( E_2 \). The population inversion occurs between the metastable state \( E_2 \) and the one below it, \( E_1 \). The lasing action occurs between the two states that have the population inversion, the \( E_2 \) and \( E_1 \) states.

**SOLUTION**

a. From the drawing, we see that the energy required to raise an electron from the \( E_0 \) state to the \( E_2 \) state is \( 0.289 \text{ eV} \).

b. The lasing action occurs between the \( E_2 \) and \( E_1 \) states, and so the energy \( E \) of the emitted photon is the difference between them; \( E = E_2 - E_1 \). According to Equations 29.2 and 16.1, the wavelength \( \lambda \) of the photon is related to its energy via \( \lambda = hc/E \), so that

\[ \lambda = \frac{hc}{E_2 - E_1} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^{8} \text{ m/s})}{(0.289 \text{ eV} - 0.165 \text{ eV})(1.60 \times 10^{-19} \text{ J/1 eV})} = 1.00 \times 10^{-5} \text{ m} \]
c. An examination of Figure 24.9 shows that the photon lies in the infrared region of the electromagnetic spectrum.

49. **REASONING** The number of photons is the energy delivered to the iris divided by the energy per photon. The energy \( E \) of a photon is given by Equation 29.2 as 
\[
E = hf
\]
where \( h \) is Planck’s constant and \( f \) is the photon frequency. The frequency is related to the wavelength \( \lambda \) by 
\[
f = \frac{c}{\lambda}
\]
according to Equation 16.1, where \( c \) is the speed of light in a vacuum. Substituting this expression for \( f \) into the expression for \( E \) gives
\[
E = hf = \frac{hc}{\lambda}
\]

**SOLUTION** Using Equation (1), we find that the number of photons is
\[
\text{Number of photons} = \frac{\frac{4.1\times10^{-3} \text{ J}}{E}}{\frac{hc}{\lambda}} = \frac{(4.1\times10^{-3} \text{ J})\lambda}{hc} = \frac{(4.1\times10^{-3} \text{ J})(1064\times10^{-9} \text{ m})}{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})} = 2.2\times10^{16}
\]

50. **REASONING AND SOLUTION** For either laser, the number of photons is given by the energy \( E \) produced divided by the energy per photon. Since power \( P \) is energy per unit time, the energy produced in a time \( t \) is \( E = Pt \). For a photon frequency \( f \) the energy per photon is \( hf \). But the photon frequency \( f \) and wavelength \( \lambda \) are related to the speed of light \( c \) by \( f = c/\lambda \). Therefore, the energy per photon is \( hc/\lambda \). The number of photons is, then,
\[
\text{Number of photons} = \frac{\text{Energy produced}}{\text{Energy per photon}} = \frac{Pt}{hc/\lambda} = \frac{Pt\lambda}{hc}
\]
Since each laser produces the same number of photons, it follows that
\[
\left(\frac{Pt\lambda}{hc}\right)_{\text{He/Ne}} = \left(\frac{Pt\lambda}{hc}\right)_{SS}
\]
or
\[
\lambda_{\text{He/Ne}} = \frac{(Pt\lambda)_{SS}}{(P\lambda)_{\text{He/Ne}}}
\]
where the "He/Ne" and "SS" refer to the helium/neon and solid state lasers, respectively.
The time for the helium/neon laser is, then,
\[
\lambda_{\text{He/Ne}} = \frac{(Pt\lambda)_{SS}}{(P\lambda)_{\text{He/Ne}}} = \frac{(1.0\times10^{14} \text{ W})(1.1\times10^{-11} \text{ s})(1060 \text{ nm})}{(1.0\times10^{-3} \text{ W})(633 \text{ nm})} = 1.8\times10^6 \text{ s}
\]
This is a time of \( (1.8\times10^6 \text{ s})\left[(1 \text{ day})/(8.64\times10^4 \text{ s})\right] = 21 \text{ days} \).
51. **REASONING**  For use here, we recall that the \( \ell = 0 \) subshell is denoted by s, the \( \ell = 1 \) subshell is denoted by p, and the \( \ell = 2 \) subshell is denoted by d. In Figure 30.16 we see that the 4s subshell fills before the 3d subshell and the 5s subshell fills before the 4d subshell. We also see in Figure 30.16 that s subshells can contain up to 2 electrons, p subshells can contain up to 6 electrons, and d subshells can contain up to 10 electrons.

**SOLUTION**  Following the style in Table 30.3, we find that the ground state electronic configuration for the 48 electrons in cadmium is

\[
1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10}
\]

52. **REASONING**

a. The total energy \( E_n \) for a single electron in the \( n^{th} \) state is given by

\[
E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2}
\]

where \( Z = 2 \) for helium. The minimum amount of energy required to remove the electron from the ground state \( (n = 1) \) is that needed to move the electron into the state for which \( n = 2 \). This amount equals the difference between the two energy levels.

b. The ionization energy defined as the minimum amount of energy required to remove the electron from the \( n = 1 \) orbit to the highest possible excited state \( (n = \infty) \).

**SOLUTION**

a. The minimum amount of energy required to remove the electron from the ground state \( (n = 1) \) and move it into the state for which \( n = 2 \) is

\[
\text{Minimum energy} = E_2 - E_1 = \frac{-(13.6 \text{ eV})(2)^2}{2^2} - \left[ \frac{-(13.6 \text{ eV})(2)^2}{1^2} \right] = 40.8 \text{ eV}
\]

b. The ionization energy is the difference between the ground-state energy \( (n = 1) \) and the energy in the highest possible excited state \( (n = \infty) \). Thus,

\[
\text{Ionization energy} = E_{\infty} - E_1 = \frac{-(13.6 \text{ eV})(2)^2}{(\infty)^2} - \left[ \frac{-(13.6 \text{ eV})(2)^2}{1^2} \right] = 54.4 \text{ eV}
\]

53. **REASONING AND SOLUTION**  The table below lists the possible sets of the four quantum numbers that correspond to the electrons in a completely filled 4f subshell:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \ell )</th>
<th>( m_\ell )</th>
<th>( m_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1/2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>-1/2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1/2</td>
</tr>
</tbody>
</table>
54. **REASONING** This problem is similar to Example 11 in the text. We use Equation 30.14 with the initial value of \( n \) being \( n_i = 2 \), and the final value being \( n_f = 1 \). As in Example 11, we use a value of \( Z \) that is one less than the atomic number of the atom in question (in this case, a value of \( Z = 41 \) rather than 42); this accounts approximately for the shielding effect of the single K-shell electron in canceling out the attraction of one nuclear proton.

**SOLUTION** Using Equation 30.14, we obtain

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(41)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right) \quad \text{or} \quad \lambda = 7.230 \times 10^{-11} \text{ m}
\]

55. **SSM REASONING AND SOLUTION**

a. The longest wavelength in the Pfund series occurs for the transition \( n = 6 \) to \( n = 5 \), so that according to Equation 30.14 with \( Z = 1 \), we have

\[
\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right) = \left( 1.097 \times 10^7 \text{ m}^{-1} \right) \left( \frac{1}{5^2} - \frac{1}{6^2} \right) \quad \text{or} \quad \lambda = 7458 \text{ nm}
\]

b. The shortest wavelength occurs when \( 1/n^2 = 0 \), so that

\[
\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right) = \left( 1.097 \times 10^7 \text{ m}^{-1} \right) \left( \frac{1}{5^2} \right) \quad \text{or} \quad \lambda = 2279 \text{ nm}
\]

c. The lines in the Pfund series occur in the **infrared region**.

56. **REASONING** The cutoff wavelength \( \lambda_0 \) depends only on the voltage \( V \) across the X-ray tube, according to Equation 30.17; \( \lambda_0 = hc/(eV) \). Since the voltage does not change, the cutoff wavelength remains the same, independent of the target material.

The wavelength of the \( K_{\alpha} \) photon is given by Equation 30.14, with \( n_i = 2 \) and \( n_f = 1 \). The wavelength \( \lambda \) is proportional to \( 1/(Z - 1)^2 \), so the wavelength decreases for larger values of
Z. Since silver has a larger value of $Z$ than molybdenum, the wavelength of the $K_{\alpha}$ photon decreases when silver replaces molybdenum.

SOLUTION

a. The cutoff wavelength $\lambda_0$, which is the same for either target material, is

$$\lambda_0 = \frac{hc}{eV} \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.60 \times 10^{-19} \text{ C}} \right) \left( \frac{3.00 \times 10^8 \text{ m/s}}{35.0 \times 10^3 \text{ V}} \right) = 3.55 \times 10^{-11} \text{ m} \quad (30.17)$$

b. The $K_{\alpha}$ photon is emitted when an electron drops from the $n_i = 2$ to the $n_f = 1$ level. The wavelengths emitted by molybdenum and silver are:

**Molybdenum**

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(42 - 1)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = 1.38 \times 10^{10} \text{ m}^{-1} \quad (30.14)$$

$$\lambda = 7.23 \times 10^{-11} \text{ m}$$

**Silver**

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(47 - 1)^2 \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = 1.74 \times 10^{10} \text{ m}^{-1} \quad (30.14)$$

$$\lambda = 5.74 \times 10^{-11} \text{ m}$$

57. **REASONING AND SOLUTION**

a. For the angular momentum, Bohr's value is given by Equation 30.8, with $n = 1$,

$$L_n = \frac{nh}{2\pi} = \frac{h}{2\pi}$$

According to quantum theory, the angular momentum is given by Equation 30.15. For $n = 1$, $\ell = 0$

$$L = \sqrt{\ell(\ell+1)} \left( \frac{h}{2\pi} \right) = \sqrt{0(0+1)} \left( \frac{h}{2\pi} \right) = 0 \text{ J} \cdot \text{s}$$

b. For $n = 3$; Bohr theory gives

$$L_n = \frac{nh}{2\pi} = \frac{3h}{2\pi}$$

while quantum mechanics gives

$$[n = 3, \ell = 0] \quad L = \sqrt{\ell(\ell+1)} \left( \frac{h}{2\pi} \right) = \sqrt{0(0+1)} \left( \frac{h}{2\pi} \right) = 0 \text{ J} \cdot \text{s}$$
\[ n = 3, \ell = 1 \]
\[ L = \sqrt{\ell (\ell + 1)} \left( \frac{h}{2\pi} \right) = \sqrt{1(1+1)} \left( \frac{h}{2\pi} \right) = \frac{\sqrt{2} h}{2\pi} \]

\[ n = 3, \ell = 2 \]
\[ L = \sqrt{\ell (\ell + 1)} \left( \frac{h}{2\pi} \right) = \sqrt{2(2+1)} \left( \frac{h}{2\pi} \right) = \frac{\sqrt{6} h}{2\pi} \]

58. **REASONING** The energy levels and radii of a hydrogenic species of atomic number \( Z \) are given by Equations 30.13 and 30.10, respectively: \( E_n = -(13.6 \text{ eV})(Z^2 / n^2) \) and \( r_n = (5.29 \times 10^{-11} \text{ m})(n^2 / Z) \). We can use Equation 30.13 to find the value of \( Z \) for the unidentified ionized atom and then calculate the radius of the \( n = 5 \) orbit using Equation 30.10.

**SOLUTION** Solving Equation 30.13 for atomic number \( Z \) of the unknown species, we have

\[ Z = \sqrt{\frac{E_n n^2}{-13.6 \text{ eV}}} = \sqrt{\frac{-(30.6 \text{ eV})(2)^2}{-13.6 \text{ eV}}} = 3 \]

Therefore, the radius of the \( n = 5 \) orbit is

\[ r_5 = (5.29 \times 10^{-11} \text{ m}) \left( \frac{5^2}{3} \right) = 4.41 \times 10^{-10} \text{ m} \]

59. **SSM REASONING** Singly ionized helium, \( \text{He}^+ \), is a hydrogen-like species with \( Z = 2 \). The wavelengths of the series of lines produced when the electron makes a transition from higher energy levels into the \( n_f = 4 \) level are given by Equation 30.14 with \( Z = 2 \) and \( n_f = 4 \):

\[ \frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(2^2) \left( \frac{1}{4^2} - \frac{1}{n_i^2} \right) \]

**SOLUTION** Solving this expression for \( n_i \) gives

\[ n_i = \left[ \frac{1}{4^2} - \frac{1}{4\lambda(1.097 \times 10^7 \text{ m}^{-1})} \right]^{-1/2} \]

Evaluating this expression at the limits of the range for \( \lambda \), we find that \( n_i = 19.88 \) for \( \lambda = 380 \text{ nm} \), and \( n_i = 5.58 \) for \( \lambda = 750 \text{ nm} \). Therefore, the values of \( n_i \) for energy levels from which the electron makes the transitions that yield wavelengths in the range between 380 \text{ nm} \) and 750 \text{ nm} \) are \( 6 \leq n_i \leq 19 \).
60. **REASONING AND SOLUTION** From the diagram given with the problem statement we see that an integral number of half wavelengths fit into the "box". That is, \( n\lambda/2 = L \), where \( n = 1, 2, 3, \ldots \) Using the de Broglie equation yields \( \lambda = \frac{h}{mv} \). Combining these two expressions yields, \( nh/(2mv) = L \), and rearranging gives the velocity as \( v = nh/(2mL) \).

Finally, the kinetic energy can be written

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{nh}{2mL}\right)^2
\]

\[
KE = \frac{n^2h^2}{8mL^2} \quad \text{where } n=1,2,3,\ldots
\]

61. **SSM REASONING AND SOLUTION** The shortest wavelength, \( \lambda_s \), occurs when \( n_i = \infty \), so that \( 1/n_i = 0 \). In that case Equation 30.14 becomes

\[
\frac{1}{\lambda_s} = \frac{RZ^2}{n_f^2} \quad \text{or} \quad RZ^2 = \frac{n_f^2}{\lambda_s}
\]

The longest wavelength in the series occurs when \( n_i = n_f + 1 \).

\[
\frac{1}{\lambda_i} = RZ^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \left( \frac{n_f^2}{\lambda_s} \right) \left( \frac{1}{n_f^2} - \frac{1}{(n_f + 1)^2} \right)
\]

A little algebra gives \( n_f \) as follows:

\[
\frac{\lambda_i}{\lambda_s} = \frac{41.02 \times 10^{-9} \text{ m}}{22.79 \times 10^{-9} \text{ m}} = \frac{(n_f + 1)^2}{2n_f + 1}
\]

Rearranging this equation gives

\[
n_f^2 - 1.600n_f - 0.800 = 0
\]

Solving the quadratic equation yields only one positive root, which is \( n_f = 2 \). Therefore, \( 1/\lambda_s = 1/(22.79 \times 10^{-9} \text{ m}) = RZ^2/n_f^2 = RZ^2/4 \), which gives \( Z = 4 \). As a result, the next-to-the-longest wavelength is:

\[
\frac{1}{\lambda} = R(4)^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right) \quad \text{or} \quad \lambda = 30.39 \times 10^{-9} \text{ m} = \boxed{30.39 \text{ nm}}
\]
1. (e) In the notation $^{A}_{Z}$In, the symbol $Z$ represents the number of protons, and the symbol $A$ represents the number of protons plus the number of neutrons.

2. (c) Because the atomic number of the nucleus is 37, there are 37 protons in the nucleus. Thus, there must be 37 electrons in orbit about the electrically neutral atom.

3. $r = 4.8 \times 10^{-15}$ m

4. (e) According to the discussion in Section 31.1, the nuclear density of all nuclei is approximately the same.

5. (c) The graph in Section 31.2 displays the nucleon number versus the proton number for the elements, and it shows that the number of neutrons in the nucleus is about equal to the number of protons for elements whose proton number is less than or equal to $Z = 8$.

6. (d) The mass defect depends is equal to the mass of the separated nucleons (63 u) minus the mass (62.5 u) of the stable nucleus (see Section 31.3).

7. (a) Both $\alpha$ and $\beta^-$ decays produce a daughter nucleus that has a different atomic number than the parent nucleus (see Section 31.4). Thus, each decay results in a new element.

8. (b) When a nucleus decays, electric charge is conserved. The number of protons in the $^{240}_{92}$U daughter nucleus and the $\alpha$ particle (92 + 2) is the same as the number (94) of protons in the $^{244}_{94}$Pu parent nucleus. Also, when a nucleus decays, the nucleon number is conserved. The number of nucleons in the $^{240}_{92}$U daughter nucleus and the $\alpha$ particle (240 + 4) is the same as the number (244) of protons in the $^{244}_{94}$Pu parent nucleus.

9. (d) The radius of a nucleus depends on the number of nucleons it contains (see Section 31.1). In $\alpha$ decay the number of nucleons in the daughter nucleus is four less than that in the parent nucleus, while in $\beta^-$ and $\gamma$ decay the number of nucleons is the same as that in the parent nucleus (see Section 31.4).

10. (b) When a nucleus decays, electric charge is conserved. The combined charge (80 − 1) of the $^{198}_{80}$Hg daughter nucleus and the $\beta^-$ particle is the same as the charge (79) in the $^{198}_{79}$Au parent nucleus. Also, when a nucleus decays, the nucleon number is conserved. The number (198 + 0)
of nucleons in the $^{198}_{80}$Hg daughter nucleus and the $\beta^-$ particle is the same as the number (198) of nucleons in the $^{198}_{79}$Au parent nucleus.

11. (c) The activity of a sample is directly proportional to the number of radioactive nuclei (see Equation 31.4). Therefore, if the mass of a substance increases, the number of nuclei increases. The decay constant $\lambda$ depends on the specific type of radioactive nuclei in the sample, e.g., $^{14}_6$C, but not on the mass of the radioactive material (See Equations 31.4 and 31.5). Since the half-life is inversely proportional to the decay constant (see Equation 31.6), the half-life also remains constant when the mass of the substance increases.

12. $N/N_0 = 0.63$

13. (b) The activity of an isotope depends on the number of nuclei present and the decay constant of the isotope (see Section 31.6). The decay constant, on the other hand, is inversely proportional to the half-life (see Equation 31.6). Thus, it is possible for the two samples to have the same activity, provided the ratio of the number of nuclei to the half-life is the same for both.

14. (e) Since the half-life is 1 day, one-half of the sample remains after 1 day, and one-half of that amount (or one-quarter of the original) remains after 2 days.

15. (a) The activity of a radioactive sample is equal to the number $N$ of nuclei present times the decay constant $\lambda$ of the isotope (see Section 31.6). The decay constant is inversely proportional to the half-life $T_{1/2}$ (see Equation 31.6). Thus, the activity is proportional to the ratio $N/T_{1/2}$, and this ratio is greatest for Sample 1.

16. (d) The half-life is the time required for one-half of the original nuclei to disintegrate (see Section 31.6).

17. (b) The $^{14}_6$C dating technique assumes that the isotope $^{14}_6$C was ingested by a living organism (see Section 31.7). At one time, all of these items were part of a living animal.

18. (e) The $^{14}_6$C activity when the animal died was 0.23 Bq per gram of carbon (see Section 31.7). The present activity is 0.10 Bq per gram of carbon, which is less than half of the original activity. Since the half-life of $^{14}_6$C is 5730 yr, the bones must be more than 5000 years old.

19. Age = $6.9 \times 10^3$ years
CHAPTER 31 \textbf{NUCLEAR PHYSICS AND RADIOACTIVITY}

PROBLEMS

1. \textbf{SSM REASONING} For an element whose chemical symbol is X, the symbol for the nucleus is $^{A}_Z X$, where $A$ represents the total number of protons and neutrons (the nucleon number) and $Z$ represents the number of protons in the nucleus (the atomic number). The number of neutrons $N$ is related to $A$ and $Z$ by Equation 31.1: $A = Z + N$.

\textbf{SOLUTION} For the nucleus $^{208}_{82}$Pb, we have $Z = 82$ and $A = 208$.

a. The net electrical charge of the nucleus is equal to the total number of protons multiplied by the charge on a single proton. Since the $^{208}_{82}$Pb nucleus contains 82 protons, the net electrical charge of the $^{208}_{82}$Pb nucleus is

$$q_{\text{net}} = (82)(+1.60 \times 10^{-19} \text{ C}) = +1.31 \times 10^{-17} \text{ C}$$

b. The number of neutrons is $N = A - Z = 208 - 82 = 126$.

c. By inspection, the number of nucleons is $A = 208$.

d. The approximate radius of the nucleus can be found from Equation 31.2, namely

$$r = (1.2 \times 10^{-15} \text{ m})(A/3) = (1.2 \times 10^{-15} \text{ m})(208/3) = 7.1 \times 10^{-15} \text{ m}$$

e. The nuclear density is the mass per unit volume of the nucleus. The total mass of the nucleus can be found by multiplying the mass $m_{\text{nucleon}}$ of a single nucleon by the total number $A$ of nucleons in the nucleus. Treating the nucleus as a sphere of radius $r$, the nuclear density is

$$\rho = \frac{m_{\text{total}}}{V} = \frac{m_{\text{nucleon}} A}{\frac{4}{3} \pi r^3} = \frac{m_{\text{nucleon}} A}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m}) A^{1/3}} = \frac{m_{\text{nucleon}}}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3}$$

Therefore,

$$\rho = \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg/m}^3$$
2. **REASONING** The radius \( r \) of a nucleus is given approximately in meters as
\[
r \approx \left( 1.2 \times 10^{-15} \text{ m} \right)^{1/3} \quad \text{(Equation 31.2)}
\]
where \( A \) is the nucleon number. The nucleon number is given by \( A = Z + N \) (Equation 31.1), where \( Z \) is the number of protons and \( N \) is the number of neutrons in the nucleus.

**SOLUTION** Substituting Equation 31.1 for \( A \) into Equation 31.2, we find that the radius is
\[
r \approx \left( 1.2 \times 10^{-15} \text{ m} \right)^{1/3} \left( Z + N \right)^{1/3}
\]
\[
= \left( 1.2 \times 10^{-15} \text{ m} \right) (18 + 22)^{1/3} = \left[ 4.1 \times 10^{-15} \text{ m} \right]
\]

3. **REASONING** For an element whose chemical symbol is \( X \), the symbol for the nucleus is \( ^A_Z X \) where \( A \) represents the number of protons and neutrons (the nucleon number) and \( Z \) represents the number of protons (the atomic number) in the nucleus.

**SOLUTION**

a. The symbol \( ^{195}_{78} X \) indicates that the nucleus in question contains \( Z = 78 \) protons, and \( N = A - Z = 195 - 78 = \boxed{117 \text{ neutrons}} \). From the periodic table, we see that \( Z = 78 \) corresponds to \( \text{platinum, Pt} \).

b. Similar reasoning indicates that the nucleus in question is \( ^{32}_{16} S \), and the nucleus contains \( N = A - Z = 32 - 16 = \boxed{16 \text{ neutrons}} \).

c. Similar reasoning indicates that the nucleus in question is \( ^{63}_{29} \text{Cu} \), and the nucleus contains \( N = A - Z = 63 - 29 = \boxed{34 \text{ neutrons}} \).

d. Similar reasoning indicates that the nucleus in question is \( ^{11}_{5} \text{B} \), and the nucleus contains \( N = A - Z = 11 - 5 = \boxed{6 \text{ neutrons}} \).

e. Similar reasoning indicates that the nucleus in question is \( ^{239}_{94} \text{Pu} \), and the nucleus contains \( N = A - Z = 239 - 94 = \boxed{145 \text{ neutrons}} \).

4. **REASONING AND SOLUTION** Solving Equation 31.2 for \( A \) gives
\[
A = r^3 / \left( 1.2 \times 10^{-15} \text{ m} \right)^3
\]

If \( r \) is doubled, then \( A \) will increase by a factor of \( 2^3 = \boxed{8} \).
5. **REASONING**

a. The number of protons in a given nucleus $^A_ZX$ is specified by its atomic number $Z$.

b. The number $N$ of neutrons in a given nucleus $^A_ZX$ is equal to the nucleon number $A$ (the number of protons and neutrons) minus the atomic number $Z$ (the number of protons): $N = A - Z$ (Equation 31.1).

c. In an electrically neutral niobium atom, the number of electrons in orbit about the nucleus is equal to the number of protons in the nucleus.

**SOLUTION**

a. The number of protons in the uranium $^{238}_{92}U$ nucleus is $Z = 92$.

b. The number $N$ of neutrons in the $^{202}_{80}Hg$ nucleus is $N = A - Z = 202 - 80 = 122$.

c. The number of electrons that orbit the $^{93}_{41}Nb$ nucleus in the neutral niobium atom is equal to the number of protons in the nucleus, or $41$.

6. **REASONING** Assuming that the nuclei are spherical, we will use $Area = 4\pi r^2$ to find the surface area of a nucleus from its radius $r$. Under the same assumption, the radius $r$ of a nucleus is given by $r = (1.2 \times 10^{-15} \text{ m})A^{1/3}$ (Equation 31.2), where $A$ is the number of nucleons in the nucleus.

**SOLUTION** The ratio of the surface area $Area_{209}$ of the largest nucleus ($A_{209} = 209$) to the surface area $Area_1$ of the smallest nucleus ($A_1 = 1$) is

$$\frac{Area_{209}}{Area_1} = \frac{4\pi r_{209}^2}{4\pi r_1^2} = \frac{r_{209}^2}{r_1^2} = \left(\frac{r_{209}}{r_1}\right)^2$$  

(Equation 1)

Substituting $r = (1.2 \times 10^{-15} \text{ m})A^{1/3}$ (Equation 31.2) into Equation (1), we obtain

$$\frac{Area_{209}}{Area_1} = \left(\frac{r_{209}}{r_1}\right)^2 = \left[\frac{(1.2 \times 10^{-15} \text{ m})A_{209}^{1/3}}{(1.2 \times 10^{-15} \text{ m})A_1^{1/3}}\right]^2 = \left(\frac{A_{209}}{A_1}\right)^{2/3} = \left(\frac{A_{209}}{A_1}\right)^{2/3} = \left(\frac{209}{1}\right)^{2/3} = 35.2$$

7. **REASONING** To identify the unknown nucleus in the form $^A_ZX$, we need to determine the atomic mass number $A$ (which is the total number of nucleons in the nucleus) and the atomic number $Z$ (which is the number of protons in the nucleus). Then we can use the periodic table to identify the nucleus X. To determine $A$, we will use the given ratio of the nuclear
radii (the radius of a nucleus is proportional to $A^{1/3}$) and the fact that $A = 3$ for the tritium nucleus. Since $A$ is the total number of nucleons in the nucleus, the value determined for the unknown nucleus is the number $N$ of neutrons plus the number $Z$ of protons. We know that $N$ is the same for both species and can determine it from the information in the symbol $^3_1T$ for the tritium nucleus. Thus, we will be able to determine the value of $Z$ for the unknown nucleus.

**SOLUTION** The radius $r$ of a nucleus is

$$ r = \left(1.2 \times 10^{-15} \text{ m} \right) A^{1/3} \tag{31.2} $$

Applying this expression to both nuclei gives

$$ \frac{r_X}{r_T} = \left( \frac{1.2 \times 10^{-15} \text{ m}}{A_X} \right) A^{1/3}_X \left( \frac{A^{1/3}_X}{A^{1/3}_T} \right) \quad \text{or} \quad \left( \frac{r_X}{r_T} \right)^3 = \frac{A_X}{A_T} $$

Solving for $A_X$, we find that

$$ A_X = A_T \left( \frac{r_X}{r_T} \right)^3 = 3 \cdot (1.10)^3 = 4 $$

Since $A_X$ is the number $N$ of neutrons plus the number $Z$ of protons in the unknown nucleus, it follows that

$$ 4 = N + Z $$

The value of $N$ is the same as it is for the tritium nucleus $^3_1T$, for which $3 = N + 1$, so that $N = 2$. Thus, for the unknown nucleus, we have

$$ 4 = N + Z = 2 + Z \quad \text{or} \quad Z = 4 - 2 = 2 $$

From the periodic table, we identify the nucleus for which $Z = 2$ as the helium (He) nucleus:

$$ ^4_2\text{He} $$

8. **REASONING** The nucleus is roughly spherical, so its volume is $V = \frac{4}{3} \pi r^3$. The radius $r$ is given by Equation 31.2 as $r \approx (1.2 \times 10^{-15} \text{ m}) A^{1/3}$, where $A$ is the atomic mass number or nucleon number. Therefore, the volume is given by $V = \frac{4}{3} \pi \left(1.2 \times 10^{-15} \text{ m} \right)^3 A$ and is proportional to $A$. We can apply this expression to the unknown nucleus and to the nickel nucleus, knowing that the ratio of the two volumes is 2:1. This ratio provides the solution we seek.

**SOLUTION** Applying the expression for the volume to each nucleus gives
The nucleon number is equal to the number of neutrons \( N \) plus the number of protons or the atomic number \( Z \), so \( A = N + Z \). Therefore, the atomic number for the unknown nucleus is

\[
Z = A - N = 120 - 70 = 50
\]

The unknown nucleus, then, is \( ^{120}_{50}X \). Reference to the periodic table reveals that the element that has an atomic number of \( Z = 50 \) is tin (Sn). Thus, \( ^{A}_{Z}X=^{120}_{50}Sn \).

\[ V = \frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3 \]

\[
\frac{V_{Ni}}{A} = \frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3 (60) = \frac{A}{60} = 2 \quad \text{or} \quad A = 120
\]

(Solution)

According to Equation 31.2, the radius of a nucleus in meters is

\[
r = \left(1.2 \times 10^{-15} \text{ m}\right) A^{1/3}
\]

where \( A \) is the nucleon number. If we treat the neutron star as a uniform sphere, its density (Equation 11.1) can be written as

\[
\rho = \frac{M}{V} = \frac{M}{\frac{4}{3} \pi r^3}
\]

Solving for the radius \( r \), we obtain,

\[
r = \frac{3}{4} \frac{M}{\pi \rho}
\]

This expression can be used to find the radius of a neutron star of mass \( M \) and density \( \rho \).

**Solution**  As discussed in Conceptual Example 1, nuclear densities have the same approximate value in all atoms. If we consider a uniform spherical nucleus, then the density of nuclear matter is approximately given by

\[
\rho = \frac{M}{V} \approx \frac{A \times \text{(mass of a nucleon)}}{\frac{4}{3} \pi r^3} = \frac{A \times \text{(mass of a nucleon)}}{\frac{4}{3} \pi \left(1.2 \times 10^{-15} \text{ m}\right) A^{1/3}}
\]

\[
= \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi (1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg/m}^3
\]

The mass of the sun is \( 1.99 \times 10^{30} \text{ kg} \) (see inside of the front cover of the text). Substituting values into the expression for \( r \) determined above, we find

\[
r = \frac{3}{4} \frac{(0.40)(1.99 \times 10^{30}) \text{ kg}}{\pi (2.3 \times 10^{17} \text{ kg/m}^3)} = 9.4 \times 10^{-3} \text{ m}
\]

10. **Reasoning**

a. The neutrons packed into the tennis ball are like a giant spherical nucleus containing \( N \) neutrons. Because no protons are present, the atomic number \( Z \) of the sphere is zero, and the atomic mass number \( A \) is equal to \( N \), as we see from \( A = Z + N = 0 + N = N \) (Equation 31.1).
Because the neutrons are packed into a spherical shape as densely as the nucleons in a nucleus, the mass number $A$ of the sphere is related to its radius by

$$r = (1.2 \times 10^{-15} \text{ m}) A^{1/3}$$

(Equation 31.2). Substituting $N = A$ into Equation 31.2 gives

$$r = (1.2 \times 10^{-15} \text{ m}) N^{1/3}$$

(1)

b. According to Newton’s 2nd law, the magnitude $a$ of the object’s acceleration is equal to the magnitude $F$ of the gravitational force exerted on it divided by its mass $m$: $a = F/m$. The magnitude of that gravitational force is given by

$$F = \frac{GMm}{R^2}$$

(Equation 4.3), where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ is the universal gravitational constant, $R = 2.0 \text{ m}$ is the distance between the object and the center of the tennis ball, and $M$ is the mass of the tennis ball. The mass of the tennis ball is the mass $m_n = 1.67 \times 10^{-27} \text{ kg}$ of a single neutron multiplied by the number $N$ of neutrons found in part (a), so we have that

$$M = N m_n$$

(2)

### SOLUTION

a. Cubing both sides of Equation (1) and solving for $N$, we obtain

$$r^3 = (1.2 \times 10^{-15} \text{ m})^3 N \quad \text{or} \quad N = \left(\frac{r}{1.2 \times 10^{-15} \text{ m}}\right)^3 = \left(\frac{0.032 \text{ m}}{1.2 \times 10^{-15} \text{ m}}\right)^3 = 1.9 \times 10^{40}$$

b. Substituting $F = \frac{GMm}{R^2}$ into $a = \frac{F}{m}$, we find that

$$a = \frac{F}{m} = \frac{GM m_n}{R^2} = \frac{GM}{R^2}$$

(3)

Substituting Equation (2) into Equation (3) yields

$$a = \frac{GM}{R^2} = \frac{G N m_n}{R^2} = \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)(1.9 \times 10^{40})(1.67 \times 10^{-27} \text{ kg}) = 530 \text{ m/s}^2$$

11. **SSM REASONING** To obtain the binding energy, we will calculate the mass defect and then use the fact that 1 u is equivalent to 931.5 MeV. The atomic mass given for $^7_3$Li includes the 3 electrons in the neutral atom. Therefore, when computing the mass defect, we must account for these electrons. We do so by using the atomic mass of 1.007 825 u for the hydrogen atom $^1_1$H, which also includes the single electron, instead of the atomic mass of a proton.

### SOLUTION

Noting that the number of neutrons is $7 - 3 = 4$, we obtain the mass defect $\Delta m$ as follows:
\[
\Delta m = 3(1.007\,825\,\text{u}) + 4(1.008\,665\,\text{u}) - 7.016\,003\,\text{u} = 4.2132 \times 10^{-2}\,\text{u}
\]

Since 1 u is equivalent to 931.5 MeV, the binding energy is

\[
\text{Binding energy} = \left(4.2132 \times 10^{-2}\,\text{u}\right)\left(\frac{931.5\,\text{MeV}}{1\,\text{u}}\right) = 39.25\,\text{MeV}
\]

12. **REASONING** The mass defect of the nucleus in atomic mass units (u) and the binding energy of the nucleus in MeV are related. The relationship between the two is specified by the fact that 1 u = 931.5 MeV. With the aid of this fact, we can convert the binding energy given in MeV into a mass defect in atomic mass units.

**SOLUTION** Following the procedure that we use for any conversion of units, we obtain the mass defect \(\Delta m\) in atomic mass units (u) by multiplying the binding energy \(E_{\text{binding}}\) in MeV by a conversion factor in the form \((1\,\text{u})/(931.5\,\text{MeV})\):

\[
\Delta m_{\text{in atomic mass units}} = E_{\text{binding}} \left(\frac{1\,\text{u}}{931.5\,\text{MeV}}\right) = \left(225.0\,\text{MeV}\right)\left(\frac{1\,\text{u}}{931.5\,\text{MeV}}\right) = 0.2415\,\text{u}
\]

13. **REASONING** The mass defect is the total mass of the stationary separated nucleons (protons and neutrons) minus the mass of the intact nucleus. The given atomic masses are for the electrical neutral atoms and, therefore, include the mass of the electrons. This will cause no problem, provided that we use the atomic mass of the hydrogen atom (including its electron) when determining the mass of the separated nucleons. Referring to Table 31.1 we find that the mass of a neutron is 1.008 665 u and the mass of a hydrogen atom is 1.007 825 u.

**SOLUTION**

a. The helium \(_2^3\text{He}\) nucleus contains 2 protons and \(3 - 2 = 1\) neutron. Thus, the mass defect \(\Delta m\) is

\[
\Delta m = 2(1.007\,825\,\text{u}) + 1(1.008\,665\,\text{u}) - 3.016\,030\,\text{u} = 0.008\,285\,\text{u}
\]

b. The tritium \(_1^3\text{T}\) nucleus contains 1 proton and \(3 - 1 = 2\) neutrons. Thus, the mass defect \(\Delta m\) is

\[
\Delta m = 1(1.007\,825\,\text{u}) + 2(1.008\,665\,\text{u}) - 3.016\,050\,\text{u} = 0.009\,105\,\text{u}
\]
c. The mass defect for tritium $^3_1T$ is greater than that for helium $^3_2He$. The mass defect is related to the binding energy as follows:

$$\text{Binding energy} = (\Delta m)c^2$$

(31.3)

The binding energy is the energy that must be supplied to the intact nucleus in order to separate it into its constituent nucleons. Thus,

more energy must be supplied to tritium $^3_1T$ than to helium $^3_2He$.

14. **REASONING**

a. The gravitational potential energy of the earth-boulder system is given by $\text{PE} = mgh$ (Equation 6.5), where $m$ is the boulder’s mass, $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity, and $h$ is the height of the shaft. Equation 6.5 gives the amount of gravitational potential energy the system loses as the boulder falls to the bottom, which is the binding energy when the boulder is at the bottom of the shaft.

b. The earth-boulder system loses energy as it falls, so it also loses mass. The binding energy is proportional to the change $\Delta m$ in the system’s mass, according to Equation 31.3:

$$\text{Binding energy} = (\Delta m)c^2$$

Here, $c = 3.00 \times 10^8 \text{ m/s}$ is the speed of light in a vacuum.

**SOLUTION**

a. The binding energy is equal to the potential energy the earth-boulder system loses when the boulder falls down the mineshaft. Therefore, we have that

$$\text{Binding energy} = \text{PE} = mgh = (245 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \times 10^3 \text{ m}) = 7.2 \times 10^6 \text{ J}$$

b. Solving the expression $\text{Binding energy} = (\Delta m)c^2$ (Equation 31.3) for $\Delta m$, we obtain

$$\Delta m = \frac{\text{Binding energy}}{c^2} = \frac{7.2 \times 10^6 \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = 8.0 \times 10^{-11} \text{ kg}$$

15. **REASONING** The atomic mass given for $^{206}_{82}\text{Pb}$ includes the 82 electrons in the neutral atom. Therefore, when computing the mass defect, we must account for these electrons. We do so by using the atomic mass of 1.007 825 u for the hydrogen atom $^1_1\text{H}$, which also includes the single electron, instead of the atomic mass of a proton. To obtain the binding energy in MeV, we will use the fact that 1 u is equivalent to 931.5 MeV.

**SOLUTION**

a. Noting that the number of neutrons is $206 - 82 = 124$, we can obtain the mass defect $\Delta m$ as follows:
\[ \Delta m = 82(1.007\,825\, u) + 124(1.008\,665\, u) - 205.974\,440\, u = 1.741\,670\, u \]

b. Since 1 u is equivalent to 931.5 MeV, the binding energy is

\[
\text{Binding energy} = (1.741\,670\, u) \left( \frac{931.5\, \text{MeV}}{1\, u} \right) = 1622\, \text{MeV}
\]

c. The binding energy per nucleon is

\[
\text{Binding energy per nucleon} = \frac{\text{Binding energy}}{\text{Number of nucleons}} = \frac{1622\, \text{MeV}}{206} = 7.87\, \text{MeV/nucleon}
\]

**16. REASONING AND SOLUTION**

a. The mass defect is

\[ \Delta m = 13.005\,738\, u + 1.008\,665\, u - 14.003\,074\, u = 0.011\,329\, u \]

The binding energy of the neutron is then

\[
(0.011\,329\, u) \left( \frac{931.5\, \text{MeV}}{1\, u} \right) = 10.55\, \text{MeV}
\]

b. The mass defect is

\[ \Delta m = 13.003\,355\, u + 1.007\,825\, u - 14.003\,074\, u = 0.008\,106\, u \]

The binding energy of the proton is then

\[
(0.008\,106\, u) \left( \frac{931.5\, \text{MeV}}{1\, u} \right) = 7.55\, \text{MeV}
\]

c. The neutron is more tightly bound, since it has the larger binding energy.

**17. SSM REASONING** Since we know the difference in binding energies for the two isotopes, we can determine the corresponding mass defect. Also knowing that the isotope with the larger binding energy contains one more neutron than the other isotope gives us enough information to calculate the atomic mass difference between the two isotopes.

**SOLUTION** The mass defect corresponding to a binding energy difference of 5.03 MeV is

\[
(5.03\, \text{MeV}) \left( \frac{1\, u}{931.5\, \text{MeV}} \right) = 0.005\,40\, u
\]
Since the isotope with the larger binding energy has one more neutron \((m = 1.008\, 665\, \text{u})\) than the other isotope, the difference in atomic mass between the two isotopes is

\[
1.008\, 665\, \text{u} - 0.005\, 40\, \text{u} = 1.003\, 27\, \text{u}
\]

18. **REASONING** The binding energy of a nucleus is the energy required to separate the nucleus into its constituent protons and neutrons. The binding energy of the nucleus is equivalent to a certain amount \(\Delta m\) of mass, called the mass defect of the nucleus. According to Equation 31.3 the relation between the two is

\[
\text{Binding energy} = (\Delta m)c^2
\]

where \(c\) is the speed of light in a vacuum. The mass defect of a nucleus is the total mass of the separated protons and neutrons minus the mass of the intact nucleus. As discussed in the text this expression is equivalent to \(1\, \text{u} = 931.5\, \text{MeV}\).

The energy required to break all the nuclei into their constituent protons and neutrons is equal to the number of atoms (and, hence, nuclei) in the metal times the binding energy of a single nucleus.

As discussed in Section 14.1, the number \(N\) of nuclei (or atoms) in a material is equal to the number \(n\) of moles times Avogadro’s number \(N_A\) (the number of nuclei per mole). Thus, \(N = nN_A\). However, the number of moles is equal to the mass \(m\) of the metal divided by its mass per mole. Recall that the mass per mole (in grams per mole) has the same numerical value as the atomic mass of the substance.

**SOLUTION** The binding energy of a nucleus is the mass defect for the \(^{63}_{29}\text{Cu}\) nucleus, expressed in energy units of MeV \((1\, \text{u} = 931.5\, \text{MeV})\). Since the copper nucleus has 29 protons and 34 neutrons, its mass defect is

\[
\Delta m = 29(1.007\, 825\, \text{u}) + 34(1.008\, 665\, \text{u}) - 62.939\, 598\, \text{u} = 0.581\, 937\, \text{u}
\]

The binding energy (in MeV) is

\[
\text{Binding energy} = (0.581\, 937\, \text{u})\left(\frac{931.5\, \text{MeV}}{1\, \text{u}}\right) = 542\, \text{MeV}
\]

The number \(N\) of nuclei in the penny is equal to the number \(n\) of moles times Avogadro’s number \(N_A\), so \(N = nN_A\). The number of moles is equal to the mass \(m\) (3.00 g) of the penny divided by its mass per mole. Since the mass (in grams) per mole has the same numerical value as the atomic mass of the substance, the mass per mole is 62.939 598 g/mol. Thus, the number of nuclei is
\[ N = n N_A \left( \frac{m}{\text{Mass per mole}} \right) N_A \]
\[ = \left( \frac{3.00 \text{ g}}{62.939 \text{ 598 g/mol}} \right) (6.02 \times 10^{23} \text{ nuclei/mol}) = 2.87 \times 10^{22} \text{ nuclei} \]

The energy required to break all the copper nuclei into their constituent protons and neutrons is
\[ (542 \text{ MeV})(2.87 \times 10^{22} \text{ nuclei}) = 1.56 \times 10^{25} \text{ MeV} \]

19. **SSM REASONING** As discussed in Section 31.4, \( \beta^+ \) decay occurs when the nucleus emits a positron, which has the same mass as an electron but carries a charge of \(+e\) instead of \(-e\). The general form for \( \beta^+ \) decay is

\[
\begin{array}{c}
\text{Parent nucleus} \\
^{A Z}_{-1} P \rightarrow \quad \quad ^{A Z-1}_{+1} D + \quad \quad ^{0}_{+1} e \\
\text{Daughter nucleus} \\
\beta^+ \text{ particle} \\
\text{(positron)}
\end{array}
\]

**SOLUTION**

a. Therefore, the \( \beta^+ \) decay process for \( ^{18}_{9} F \) is \( ^{18}_{9} F \rightarrow ^{18}_{8} O + ^{0}_{+1} e \).

b. Similarly, the \( \beta^+ \) decay process for \( ^{15}_{8} O \) is \( ^{15}_{8} O \rightarrow ^{15}_{7} N + ^{0}_{+1} e \).

20. **REASONING** As discussed in Section 31.4, \( \beta^- \) decay occurs when the nucleus emits an electron, which carries a charge of \(-e\). The general form for \( \beta^- \) decay is

\[
\begin{array}{c}
\text{Parent nucleus} \\
^{A Z}_{-1} P \rightarrow \quad \quad ^{A Z+1}_{-1} D + \quad \quad ^{0}_{-1} e \\
\text{Daughter nucleus} \\
\beta^- \text{ particle} \\
\text{(electron)}
\end{array}
\]

**SOLUTION** Therefore, the \( \beta^- \) process for \( ^{14}_{6} C \) is \( ^{14}_{6} C \rightarrow ^{14}_{7} N + ^{0}_{-1} e \).

21. **SSM REASONING** The reaction and the atomic masses are:

\[
\text{^{191}_{76}Os} \rightarrow ^{191}_{77}Ir + ^{0}_{-1} e
\]

\[
\text{190.960 \text{ 920 u}} \rightarrow \quad \quad \quad \quad \quad \text{190.960 \text{ 584 u}}
\]

When the \( ^{191}_{76}Os \) nucleus is converted into an iridium \( ^{191}_{77}Ir \) nucleus, the number of orbital electrons remains the same, so the resulting iridium atom is missing one orbital electron.
However, the given mass includes all 77 electrons of a neutral iridium atom. In effect, then, the value of 190.960 584 u for \(^{191}_{77}\text{Ir}\) already includes the mass of the \(\beta^-\) particle. Since energy is released during the decay, the combined mass of the iridium \(^{191}_{77}\text{Ir}\) daughter nucleus and the \(\beta^-\) particle is less than the mass of the osmium \(^{191}_{76}\text{Os}\) parent nucleus. The difference in mass is equivalent to the energy released. To obtain the energy released in MeV, we will use the fact that 1 u is equivalent to 931.5 MeV.

**SOLUTION** The mass decrease that accompanies the \(\beta^-\) decay of osmium \(^{191}_{76}\text{Os}\) is 190.960 920 u – 190.960 584 u = 3.36 \(\times\) \(10^{-4}\) u. Since 1 u is equivalent to 931.5 MeV, the energy released is

\[
\text{Energy released} = \left(3.36 \times 10^{-4}\ \text{u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 0.313 \text{ MeV}
\]

22. **REASONING** The reaction and the atomic masses are:

\[
\overset{211}{82}\text{Pb} \rightarrow \overset{211}{83}\text{Bi} + \overset{0}{-1}\text{e}
\]

When a \(^{211}_{82}\text{Pb}\) nucleus decays into a bismuth \(^{211}_{83}\text{Bi}\) nucleus, the number of orbital electrons in the bismuth atom is the same (82) as that in the parent lead atom; thus, the bismuth atom is missing one orbital electron. However, the atomic mass for bismuth \(^{211}_{83}\text{Bi}\) (210.987 255 u) includes all 83 electrons for the neutral atom. We note that the combination \(^{211}_{83}\text{Bi} + \overset{0}{-1}\text{e}\) contains 82 + 1 = 83 electrons, so it is this combination that has an atomic mass of 210.987 255 u. Energy is released during \(\beta^-\) decay, so the combined mass of the bismuth \(^{211}_{83}\text{Bi}\) daughter nucleus and the \(\beta^-\) particle is less than the mass of the lead \(^{211}_{82}\text{Pb}\) parent nucleus. The difference in mass is equivalent to the energy released. To obtain the energy in MeV, we will use the fact that 1 u is equivalent to 931.5 MeV.

**SOLUTION** The mass difference that accompanies the \(\beta^-\) decay is given by 210.988 735 u – 210.987 255 u = 1.48 \(\times\) \(10^{-3}\) u. Since 1 u is equivalent to 931.5 MeV, the energy released is

\[
\text{Energy released} = \left(1.48 \times 10^{-3}\ \text{u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 1.38 \text{ MeV}
\]

23. **REASONING AND SOLUTION** The mass of the products is

\[
m = 222.017 \ 57\ \text{u} + 4.002 \ 60\ \text{u} = 226.020\ 17\ \text{u}
\]

The mass defect for the decay is \(\Delta m = 226.025\ 40\ \text{u} – 226.020\ 17\ \text{u} = 0.005\ 23\ \text{u}\), which corresponds to an energy of
(0.00523 u) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 4.87 \text{ MeV}

24. **REASONING**
   a. An \( \alpha \) decay process always takes the form

   \[
   \frac{A}{Z} P \rightarrow \frac{A-4}{Z-2} D + \frac{4}{2} \text{He}
   \]

   Working back from the identity of the daughter nucleus \( \left( ^{207}_{82}\text{Pb} \right) \), we will determine the identity of the parent nucleus.

   a. The \( \beta^- \) decay process looks rather different from that of an \( \alpha \) decay:

   \[
   \frac{A}{Z} P \rightarrow \frac{A+1}{Z+1} D + \frac{0}{-1} \text{e}
   \]

   Knowing the identity of the daughter nucleus \( \left( ^{207}_{82}\text{Pb} \right) \) will allow us to identify the parent nucleus.

   **SOLUTION**
   a. An \( \alpha \) decay reduces the atomic mass number \( A \) of the parent species by 4. Because the atomic mass number of the daughter nucleus, lead \( ^{207}_{82}\text{Pb} \), is 207, we have that

   \[
   A - 4 = 207 \quad \text{or} \quad A = 207 + 4 = 211
   \]

   Similarly, given that the atomic number of the daughter nucleus is 82, the atomic number \( Z \) of the parent nucleus must be \( Z = 82 + 2 = 84 \). This atomic number identifies the parent nucleus as polonium (Po). Therefore, the decay process is \( ^{211}_{84}\text{Po} \rightarrow ^{207}_{82}\text{Pb} + ^{4}_{2}\text{He} \).

   b. In a \( \beta^- \) decay, the atomic mass number \( A \) is unaffected, but the atomic number \( Z \) increases by 1. Therefore, the parent nucleus in this decay process has an atomic number that is 1 less than that of the daughter nucleus: \( Z = 82 - 1 = 81 \). Thallium (Tl) is the element with the atomic number 81, so we have that \( ^{207}_{81}\text{Tl} \rightarrow ^{207}_{82}\text{Pb} + ^{0}_{-1}\text{e} \).

25. **REASONING AND SOLUTION**
   a. The decay reaction is: \( ^{242}_{94}\text{Pu} \rightarrow ^{A}_{Z}\text{X} + ^{4}_{2}\text{He} \). Therefore, \( 242 = A + 4 \), so that \( A = 238 \). In addition, \( 94 = Z + 2 \), so that \( Z = 92 \). Thus, the daughter nucleus is \( ^{238}_{92}\text{U} \).
b. The decay reaction is \(^{24}_{11}\)Na\(\rightarrow^{A}_{z}\)X\(+^0_1\)e. Therefore, \(24 = A\). In addition, \(11 = Z - 1\), so that \(Z = 12\). Thus the daughter nucleus is \(^{24}_{12}\)Mg.

c. The decay reaction is \(^{13}_{7}\)N\(\rightarrow^{A}_{z}\)X\(+^0_1\)e. Therefore, \(13 = A\). In addition, \(7 = Z + 1\), so that \(Z = 6\). Thus, the daughter nucleus is \(^{13}_{6}\)C.

26. **REASONING** The \(\gamma\)-ray photon is emitted when the nucleus changes from one energy state to a lower energy state. The energy of the photon is the difference \(\Delta E\) between the two nuclear energy levels, in a way similar to that discussed in Section 30.3 for the energy levels of the electron in the hydrogen atom. In that section, we saw that \(\Delta E\) is related to the frequency \(f\) and Planck’s constant \(h\). Thus, we can determine the energy from a value for the frequency. However, this value is not given. Instead, the wavelength \(\lambda\) is given. The frequency can be obtained from the wavelength, since the two quantities are related to the speed \(c\) of light.

**SOLUTION** Section 30.3 discusses the fact that the photon emitted when the electron in a hydrogen atom changes from a higher to a lower energy level has an energy \(\Delta E\), which is the difference between the energy levels. A similar situation exists here when the nucleus changes from a higher to a lower energy level. The \(\gamma\)-ray photon that is emitted has an energy \(\Delta E\) given by

\[
\Delta E = hf
\]  

The frequency is related to the wavelength according to

\[
f = \frac{c}{\lambda}
\]  

Substituting this expression into Equation 30.4 shows that

\[
\Delta E = hf = h\left(\frac{c}{\lambda}\right) = \left(6.63 \times 10^{-34} \, \text{J} \cdot \text{s}\right)\left(\frac{3.00 \times 10^8 \, \text{m/s}}{1.14 \times 10^{11} \, \text{m}}\right) = 1.74 \times 10^{-14} \, \text{J}
\]

Finally, we convert this result from joules (J) to MeV:

\[
\Delta E = \left(1.74 \times 10^{-14} \, \text{J}\right)\left(\frac{1 \, \text{eV}}{1.60 \times 10^{-19} \, \text{J}}\right)\left(\frac{1 \, \text{MeV}}{1 \times 10^6 \, \text{eV}}\right) = 0.109 \, \text{MeV}
\]

27. **REASONING** Since energy is released during the decay, the combined mass of the lead \(^{206}_{82}\)Pb daughter nucleus and the \(\alpha\) particle is less than the mass of the polonium \(^{210}_{84}\)Po parent nucleus. The difference in mass is equivalent to the energy released. Since the recoil of the lead nucleus is being ignored and we are assuming that all the released energy goes
into the kinetic energy $KE$ of the $\alpha$ particle, it follows that released energy $= KE = \frac{1}{2}mv^2$ (Equation 6.2). Thus, the speed of the $\alpha$ particle is $v = \sqrt{\frac{2(KE)}{m}}$.

**SOLUTION** The decay reaction is

$$\frac{^{210}_{84}\text{Po}}{\text{209.982 848 u}} \rightarrow \frac{^{206}_{82}\text{Pb}}{\text{205.974 440 u}} + \frac{^4_2\text{He}}{\text{4.002 603 u}}$$

The difference in the masses is

$$209.982 \text{ 848 u} - 205.974 \text{ 440 u} - 4.002 \text{ 603 u} = 5.805 \times 10^{-3} \text{ u}$$

This mass difference corresponds to an energy of

$$\left(5.805 \times 10^{-3} \text{ u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 5.407 \text{ MeV}$$

This energy is the kinetic energy of the $\alpha$ particle (mass $m = 6.6447 \times 10^{-27} \text{ kg}$, see Example 2), so the speed of the $\alpha$ particle is

$$v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2\left(5.407 \times 10^6 \text{ eV}\right)\left(1.60 \times 10^{-19} \text{ J}\right)}{1 \text{ eV} \times 6.6447 \times 10^{-27} \text{ kg}}} = 1.61 \times 10^7 \text{ m/s}$$

28. **REASONING** When a nucleus undergoes an $\alpha$ decay, it emits one $\alpha$ particle that contains two protons and two neutrons. Therefore, each $\alpha$ decay decreases the atomic mass number $A$ by four. A $\beta^-$ decay does not affect the atomic mass number $A$ at all, so the difference $A_d - A_p$ between the atomic mass number $A_d = 208$ of the daughter nucleus $\left(\frac{^{208}_{82}\text{Pb}}{}\right)$ and the atomic mass number $A_p = 230$ of the parent nucleus $\left(\frac{^{230}_{86}\text{Rn}}{}\right)$ after the emission of $N_\alpha$ alpha particles is

$$A_d - A_p = -4N_\alpha \quad (1)$$

The negative sign in Equation (1) indicates a reduction in the atomic mass number in going from the parent to the daughter. In order to determine the difference $Z_d - Z_p$ between the final atomic number $Z_d = 82$ of the daughter nucleus and the atomic number $Z_p = 86$ of the parent nucleus, we must take into account both the loss of two protons for each $\alpha$ particle emitted, and the gain of one proton for each $\beta^-$ particle emitted. Therefore, we have that

$$Z_d - Z_p = -2N_\alpha + N_\beta \quad (2)$$

where $N_\beta$ is the number of $\beta^-$ particles emitted in the decay series.

**SOLUTION** Solving Equation (1) for $N_\alpha$, we obtain
\[ N_\alpha = \frac{A_d - A_p}{-4} = \frac{208 - 220}{-4} = 3 \]

Solving Equation (2) for \( N_\beta \) yields
\[ N_\beta = 2N_\alpha + Z_d - Z_p = 2(3) + 82 - 86 = 2 \]

Therefore, in this decay series, \( 3 \alpha \) particles and \( 2 \beta^- \) particles are emitted.

29. **REASONING AND SOLUTION**  The conservation of linear momentum applied to the reaction gives
\[ m_\alpha v_\alpha + m_\beta v_\beta = 0 \]  (1)

The energy released in the decay is assumed to be kinetic, so
\[ (1/2)m_\alpha v_\alpha^2 + (1/2)m_\beta v_\beta^2 = 4.3 \text{ MeV} \]  (2)

Solving Equation (1) for \( v_\alpha \), substituting into Equation (2) and rearranging, gives the kinetic energy of the thorium atom.
\[ \frac{1}{2} m_\alpha v_\alpha^2 = \frac{4.3 \text{ MeV}}{1 + \frac{m_\alpha}{m_T}} = \frac{4.3 \text{ MeV}}{234.0436 \text{ u}} = 0.072 \text{ MeV} \]

The kinetic energy of the \( \alpha \) particle is, then, \( 4.3 \text{ MeV} - 0.072 \text{ MeV} = 4.2 \text{ MeV} \).

30. **REASONING** As Section 7.2 discusses, the principle of conservation of linear momentum indicates that the total momentum of an isolated system is conserved. Assuming that no external net force acts on the beryllium nucleus, it is isolated and the emission of the gamma ray satisfies this principle. The magnitude of the momentum of the gamma ray photon is \( h/\lambda \), according to Equation 29.6, where \( h \) is Planck’s constant and \( \lambda \) is the wavelength. We take the direction of the photon as the positive direction. Then, using Equation 7.2, we write the momentum of the recoiling beryllium nucleus as \(-mv_{\text{Be}}\), where \( m \) is the mass and \( v_{\text{Be}} \) is the speed. The conservation principle indicates that the total momentum after the emission must be equal to the total momentum before the emission, so we have
\[ -mv_{\text{Be}} + \frac{h}{\lambda} = 0 \]

This expression can be solved for the wavelength.

**SOLUTION** Solving the conservation-of-momentum expression, we obtain
\[ \lambda = \frac{h}{mv_{\text{Be}}} \]

Before we can use this equation, the mass of the beryllium must be converted from atomic mass units to kilograms:

\[ m_{\text{Be}} = (7.017 \text{ u}) \left( \frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) = 1.165 \times 10^{-26} \text{ kg} \]

The wavelength, then, is

\[ \lambda = \frac{h}{mv_{\text{Be}}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.165 \times 10^{-26} \text{ kg})(2.19 \times 10^4 \text{ m/s})} = 2.60 \times 10^{-12} \text{ m} \]

31. **REASONING** Energy is released during the \( \beta \) decay. To find the energy released, we determine how much the mass has decreased because of the decay and then calculate the equivalent energy. The reaction and masses are shown below:

\[
\begin{array}{ccc}
^{22}_{11}\text{Na} & \rightarrow & ^{22}_{10}\text{Ne} + ^{0}_{1}\text{e} \\
21.994 \text{ 434 u} & & 21.991 \text{ 383 u} + 5.485 \text{ 799 u} \times 10^{-4}
\end{array}
\]

**SOLUTION** The decrease in mass is

\[ 21.994 \text{ 434 u} - (21.991 \text{ 383 u} + 5.485 \text{ 799 u} \times 10^{-4} \text{ u} + 5.485 \text{ 799 u} \times 10^{-4}) = 0.001 \text{ 954 u} \]

where the extra electron mass takes into account the fact that the atomic mass for sodium includes the mass of 11 electrons, whereas the atomic mass for neon includes the mass of only 10 electrons.

Since 1 u is equivalent to 931.5 MeV, the released energy is

\[ (0.001 \text{ 954 u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 1.82 \text{ MeV} \]

32. **REASONING** The number \( N \) of radioactive nuclei that remains in a sample after a time \( t \) is given by \( N = N_0 e^{-\lambda t} \) (Equation 31.5), where \( N_0 \) is the number present initially and \( \lambda \) is the decay constant. The decay constant is related to the half-life by \( \frac{T_{1/2}}{0.693} = \frac{1}{\lambda} \) (Equation 31.6); therefore, \( \lambda = \frac{0.693}{T_{1/2}} \) and we can write Equation 31.5 as follows:

\[ \frac{N}{N_0} = e^{- \left( \frac{0.693}{T_{1/2}} \right) t} \quad (1) \]

This equation can be solved for the half-life. Another way to obtain the half-life is to remember that the half-life is the time required for one-half of the original number of radioactive nuclei to disintegrate. Thus, two half-lives are required for the number of
radioactive nuclei to decrease to one-fourth the initial number, and three half-lives are required for the number of radioactive nuclei to decrease to one-eighth the initial number. We see, then, that \(3T_{1/2} = 9.0\) days and expect that Equation (1) will reveal a half-life of 3.0 days.

**SOLUTION** Solving Equation (1) for the half-life, we find that
\[
\frac{N}{N_0} = e^{-\left(\frac{0.693}{T_{1/2}}\right)t} \quad \text{or} \quad \ln \left(\frac{N}{N_0}\right) = -\left(\frac{0.693}{T_{1/2}}\right)t
\]

or
\[
T_{1/2} = -\left[\frac{0.693}{\ln(N/N_0)}\right]t = -\left[\frac{0.693}{\ln(1/8)}\right](9.0\ \text{days}) = 3.0\ \text{days}
\]

This answer is what we expected.

33. **SSM REASONING** The basis of our solution is the fact that only one-half of the radioactive nuclei present initially remain after a time equal to one half-life. After a time period that equals two half-lives, the number of nuclei remaining is \(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\) of the initial number. After three half-lives, the number remaining is \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}\) of the initial number, and so on. Thus, to determine the fraction of nuclei remaining after a given time period, we need only to know how many half-lives that period contains.

**SOLUTION** For sample A, the number of nuclei remaining is \(\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}\) of the initial number. We can see, then, that the time period involved is equal to two half lives of radioactive isotope A. But we know that \(T_{1/2,B} = \frac{1}{2}T_{1/2,A}\), so that this time period must be equal to four half-lives of radioactive isotope B. The fraction \(f\) of the B nuclei that remain, therefore, is
\[
f = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}
\]

34. **REASONING** The decay constant \(\lambda\) is related to the half-life by \(T_{1/2} = \frac{0.693}{\lambda}\) (Equation 31.6). We can solve this equation for \(\lambda\). However, the half-life is given as 14.28 days. We will need to convert this value into seconds (s), so that the answer for \(\lambda\) will be in the desired units of \(s^{-1}\). For this conversion, we will use the fact that 1 day = \(8.64 \times 10^4\) s (see the page facing the inside of the front cover of the text).

**SOLUTION** Solving Equation 31.6 for \(\lambda\), we find
\[
T_{1/2} = \frac{0.693}{\lambda} \quad \text{or} \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(14.28\ \text{days})} \left(\frac{1\ \text{day}}{8.64 \times 10^4\ \text{s}}\right) = \frac{5.62 \times 10^{-7}}{\text{s}^{-1}}
\]
35. **REASONING AND SOLUTION** The amount remaining is 0.0100% = 0.000 100. We know
\[ N / N_0 = e^{-0.693 t / T_{1/2}} \]. Therefore, we find
\[ t = -\frac{T_{1/2}}{0.693} \ln \left( \frac{N}{N_0} \right) = -\frac{29.1 \text{ yr}}{0.693} \ln(0.000100) = 387 \text{ yr} \]

36. **REASONING** The number \( N \) of radioactive nuclei remaining after a time \( t \) is given by
\[ N = N_0 e^{-\lambda t} \] (31.5)
where \( N_0 \) is the number of radioactive nuclei present at \( t = 0 \) s, and \( \lambda \) is the decay constant. The decay constant is related to the half-life \( T_{1/2} \) of the nuclei by \( \lambda = 0.693 / T_{1/2} \) (Equation 31.6). Substituting this expression for \( \lambda \) into Equation 31.5 gives
\[ N = N_0 e^{-0.693 t / T_{1/2}} \] (1)

**SOLUTION** Applying Equation (1) to each type of nucleus, we obtain
\[ \frac{N_{\text{Sr}}} {N_{\text{Cs}}} = \frac{N_{0, \text{Sr}} e^{-0.693 t / T_{1/2, \text{Sr}}}} {N_{0, \text{Cs}} e^{-0.693 t / T_{1/2, \text{Cs}}}} = \left(7.80 \times 10^{-3}\right) \frac{e^{-0.693 (15.0 \text{ yr})} / (29.1 \text{ yr})} {e^{-0.693 (15.0 \text{ yr})} / (2.06 \text{ yr})} = 0.848 \]

37. **SSM REASONING** We can find the decay constant from Equation 31.5, \( N = N_0 e^{-\lambda t} \). If we multiply both sides by the decay constant \( \lambda \), we have
\[ \lambda N = \lambda N_0 e^{-\lambda t} \quad \text{or} \quad A = A_0 e^{-\lambda t} \]
where \( A_0 \) is the initial activity and \( A \) is the activity after a time \( t \). Once the decay constant is known, we can use the same expression to determine the activity after a total of six days.

**SOLUTION** Solving the expression above for the decay constant \( \lambda \), we have
\[ \lambda = -\frac{1}{t} \ln \left( \frac{A}{A_0} \right) = -\frac{1}{2 \text{ days}} \ln \left( \frac{285 \text{ disintegrations/min}} {398 \text{ disintegrations/min}} \right) = 0.167 \text{ days}^{-1} \]

Then the activity four days after the second day is
\[ A = (285 \text{ disintegrations/min}) e^{-0.167 \text{ days}^{-1}(4.00 \text{ days})} = 146 \text{ disintegrations/min} \]
38. **REASONING AND SOLUTION** According to Equation 31.5, the fraction of an initial sample remaining after a time \( t \) is \( \frac{N}{N_0} = e^{-\lambda t} \), where \( \lambda \) is the decay constant. The decay constant is related to the half-life \( T_{1/2} \). According to Equation 31.6, the decay constant is \( \lambda = \frac{0.693}{T_{1/2}} \). Therefore, the fraction remaining is

\[
\frac{N}{N_0} = e^{-0.693 \frac{t}{T_{1/2}}} = e^{-0.693 \frac{(30.0 \text{ days})}{(8.04 \text{ days})}} = 0.0753
\]

This fraction corresponds to a percentage of \( 7.53\% \).

39. **SSM REASONING AND SOLUTION** According to Equation 31.5, \( N = N_0 e^{-\lambda t} \), the decay constant is

\[
\lambda = -\frac{1}{t} \ln \left( \frac{N}{N_0} \right) = -\frac{1}{20 \text{ days}} \ln \left( \frac{8.14 \times 10^{14}}{4.60 \times 10^{15}} \right) = 0.0866 \text{ days}^{-1}
\]

The half-life is, from Equation 31.6,

\[
T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.0866 \text{ days}^{-1}} = 8.00 \text{ days}
\]

40. **REASONING** The “activity” \( \left| \frac{\Delta N}{\Delta t} \right| \) of the lottery on the second day is the number of contestants per day eliminated on the second day. To determine the activity, we will find the number \( N \) of contestants that remains after the first day, and multiply that number by \( 10\% \) per day \( = 0.10 \text{ d}^{-1} \), because \( 10\% \) of the existing contestants are eliminated per day:

\[
\left| \frac{\Delta N}{\Delta t} \right| = (0.10 \text{ d}^{-1}) N \tag{1}
\]

In nuclear decay, the decay constant \( \lambda \) is related to the activity \( \left| \frac{\Delta N}{\Delta t} \right| \) via \( \left| \frac{\Delta N}{\Delta t} \right| = \lambda N \) (Equation 31.4) where \( N \) is the number of nuclei present in a given sample. Comparison of Equations 31.4 and (1) will allow us to determine the decay constant \( \lambda \) of the lottery. Once we have determined the decay constant, we will use \( T_{1/2} = \frac{0.693}{\lambda} \) (Equation 31.6) to calculate the half-life of the lottery.

**SOLUTION**

a. On the first day, \( 10\% \) of the original 5800 contestants are eliminated, and \( 90\% \) advance to the next round. Therefore, the number \( N \) of contestants at the beginning of the second day is \( N = 0.90(5800) = 5220 \). Using Equation (1), the activity on the second day is
b. Comparing Equation (1) to \( \frac{\Delta N}{\Delta t} = \lambda N \) (Equation 31.4), we see that the factor \( 0.10 \, \text{d}^{-1} \), which is each contestant’s chance of being eliminated from the lottery per day, plays the role of the decay constant. Therefore, we have that \( \lambda = 0.10 \, \text{d}^{-1} \).

c. From Equation 31.6, the half-life of the lottery is
\[
T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.10 \, \text{d}^{-1}} = 6.9 \, \text{d}
\]

41. **REASONING AND SOLUTION**  The activity is \( A = \lambda N \). The decay constant is
\[
\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5.27 \, \text{yr}) \left( \frac{3.156 \times 10^7 \, \text{s}}{1 \, \text{yr}} \right)} = 4.17 \times 10^{-9} \, \text{s}^{-1}
\]

As discussed in Section 14.1, the number \( N \) of nuclei is the number of moles of nuclei times Avogadro’s number (which is the number of nuclei per mole). Thus,
\[
N = \left( \frac{0.50 \, \text{g}}{59.9 \, \text{g/mol}} \right) \left( \frac{6.02 \times 10^{23} \, \text{mol}^{-1}}{\text{Avogadro’s number}} \right) = 5.0 \times 10^{21}
\]

Therefore, \( A = \lambda N = (4.17 \times 10^{-9} \, \text{s}^{-1})(5.0 \times 10^{21}) = 2.1 \times 10^{13} \, \text{Bq} \).

42. **REASONING**  The heat \( Q \) needed to melt a mass \( m \) of ice is given by \( Q = mL_f \) (Equation 12.5), where \( L_f = 33.5 \times 10^4 \, \text{J/kg} \) is the latent heat of fusion for water (see Table 12.3).

The energy \( E \) released when a nucleus disintegrates is related to the mass decrease \( \Delta m \) by \( E = (\Delta m)c^2 \) (Equation 28.5). A mass decrease of one atomic mass unit \( (\Delta m = 1 \, \text{u}) \) corresponds to a released energy of 931.5 MeV. The total energy \( E_{\text{Total}} \) released is just the number of disintegrations times the energy for each one, or \( E_{\text{Total}} = nE \).

In a time period equal to one half-life, the number of radioactive nuclei that decay is \( \frac{1}{2} N_0 \), where \( N_0 \) is the number present initially.

**SOLUTION**  According to Equation 12.5, the mass of ice melted is \( m = Q/L_f \). The heat \( Q \) is provided by the total energy from the decay or \( Q = E_{\text{Total}} = nE \), as we discussed previously in
The REASONING. The mass of water melted, then, becomes \( m = nE/L_t \). To use this expression, we need the energy \( E \) that is released by one disintegration.

The disintegration process is

\[
\begin{align*}
^{224}_{88}\text{Ra} & \rightarrow ^{220}_{86}\text{Rn} + ^4_2\text{He} \\
224.020 \text{ 186 u} & \rightarrow 220.011 \text{ 368 u} + 4.002 \text{ 603 u} \\
& \rightarrow 224.013 \text{ 971 u}
\end{align*}
\]

As usual, the masses are atomic masses and include the mass of the orbital electrons. This causes no error here, however, because the same total number of electrons is included on the left and right sides of the arrow in the process above. The decrease in mass, then, is

\[
224.020 \text{ 186 u} - 224.013 \text{ 971 u} = 0.006215 \text{ u}
\]

Since 931.5 MeV of energy corresponds to 1 u and since 1 eV = \( 1.60 \times 10^{-19} \) J, the energy release by one disintegration is

\[
E = (0.006215 \text{ u}) \left( \frac{931.5 \times 10^6 \text{ eV}}{1 \text{ u}} \right) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 9.26 \times 10^{-13} \text{ J}
\]

Since the time period being considered is equal to one half-life, the number \( n \) of disintegrations that occurs is one half the number of radioactive nuclei initially present. We can now calculate the mass of ice melted as follows:

\[
m = \frac{nE}{L_t} = \frac{1}{2} \left( 2.69 \times 10^{21} \right) \left( 9.26 \times 10^{-13} \text{ J} \right) = \frac{33.5 \times 10^4 \text{ J/kg}}{1 \text{ kg}} = 3720 \text{ kg}
\]

43. REASONING The mass \( m \) of a substance is equal to the number \( n \) of moles times the mass per mole (see Section 14.1). The mass per mole (in grams per mole) has the same numerical value as the atomic mass of the substance. Since the atomic mass of gold is 197.968 u, its mass per mole is 197.968 g/mol. Thus, \( m = n \) (197.968 g/mol). Recall also that the number \( n \) of moles is equal to the number \( N \) of gold atoms divided by Avogadro’s number \( N_A \); \( n = N/N_A \). Therefore, the mass of gold can be written as

\[
m = n(197.968 \text{ g/mol}) = \left( \frac{N}{N_A} \right) (197.968 \text{ g/mol}) \tag{1}
\]

The activity \( |\Delta N/\Delta t| \) of a radioactive sample is the number of disintegrations per second that occurs. The activity is related to the decay constant \( \lambda \) and the number \( N \) of atoms by

\[
\text{Activity} = \left| \frac{\Delta N}{\Delta t} \right| = \lambda N \tag{31.4}
\]

The minus sign has been removed from Equation 31.4, since we have taken the absolute value of both sides of this equation. Solving Equation 31.4 for \( N \) and substituting the result into Equation (1), we find
Finally, we recognize that the decay constant $\lambda$ is related to the half-life $T_{1/2}$ by $\lambda = 0.693/T_{1/2}$ (Equation 31.6). Substituting this expression for $\lambda$ into Equation (2) gives

$$m = \frac{|\Delta N|}{\Delta t} \frac{T_{1/2}}{N_A} (197.968 \text{ g/mol})$$

**SOLUTION** Expressing the activity of radioactive gold in becquerels (1 Ci = $3.70 \times 10^{10}$ Bq) and the half-life in seconds (1 day = $8.64 \times 10^4$ s), we find that the mass of gold required to produce an activity of 315 Ci is

$$m = \frac{|\Delta N|}{\Delta t} \frac{T_{1/2}}{N_A} (197.968 \text{ g/mol})$$

$$= \frac{(315 \text{ Ci}) \left( \frac{3.70 \times 10^{10} \text{ Bq}}{1 \text{ Ci}} \right) \left( 2.69 \text{ days} \right) \left( \frac{8.64 \times 10^4 \text{ s}}{1 \text{ day}} \right) \left( 197.968 \text{ g/mol} \right)}{(0.693) \left( 6.02 \times 10^{23} \text{ mol}^{-1} \right)}$$

$$= 1.29 \times 10^{-3} \text{ g}$$

44. **REASONING** The average distance $x$ traveled by the neutrons before 75.0% of them decay is $x = vt$, where $v$ is their speed and $t$ is the time for 75.0% of them to decay. The speed of each neutron is related to its kinetic energy $KE$ ($= 5.00$ eV) by $KE = \frac{1}{2}mv^2$ (Equation 6.2). Solving this equation for the speed gives

$$v = \sqrt{\frac{2(KE)}{m}}$$

Thus, the distance traveled can be written as

$$x = vt = \sqrt{\frac{2(KE)}{m}} t$$

The time can be obtained by noting that the number $N$ of neutrons remaining after a time $t$ is given by $N = N_0 e^{-\lambda t}$ (Equation 31.5), where $N_0$ is the original number of neutrons and $\lambda$ is the decay constant. The decay constant is related to the half-life $T_{1/2}$ of a neutron by $\lambda = 0.693/T_{1/2}$ (Equation 31.6), so Equation 31.5 can be written as
Dividing both sides of this equation by \( N_0 \), taking the natural logarithm, and solving for \( t \) gives

\[
\frac{N}{N_0} = e^{-\left(\frac{0.693}{T_{1/2}}\right) t} \\
\ln\left(\frac{N}{N_0}\right) = -\left(\frac{0.693}{T_{1/2}}\right) t \\
t = -\frac{T_{1/2}}{0.693} \ln\left(\frac{N}{N_0}\right)
\]

**SOLUTION** Substituting the result for \( t \) in Equations (2) into Equation (1) and noting that \( N/N_0 = 0.750 \) (since the number of neutrons decreases to 75.0% of its initial value), we obtain

\[
x = \sqrt{\frac{2(KE)}{m}} t = \sqrt{\frac{2(KE)}{m}} \left(\frac{T_{1/2}}{0.693}\right) \ln\left(\frac{N}{N_0}\right)
\]

\[
= \sqrt{\frac{2(5.00 \text{ eV})(1.60 \times 10^{-19} \text{ J})}{1 \text{ eV}}} \left[\frac{-1.60 \times 10^{-19} \text{ J}}{1.675 \times 10^{-27} \text{ kg}}\right] \ln(0.750) = 8.01 \times 10^6 \text{ m}
\]

### 45. **SSM REASONING**

According to Equation 31.5, the number of nuclei remaining after a time \( t \) is \( N = N_0 e^{-\lambda t} \). Using this expression, we find the ratio \( N_A / N_B \) as follows:

\[
\frac{N_A}{N_B} = \frac{N_0A e^{-\lambda_A t}}{N_0B e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t}
\]

where we have used the fact that initially the numbers of the two types of nuclei are equal \((N_{0A} = N_{0B})\). Taking the natural logarithm of both sides of the equation above shows that

\[
\ln\left(\frac{N_A}{N_B}\right) = -(\lambda_A - \lambda_B)t \
\lambda_A - \lambda_B = -\frac{\ln\left(\frac{N_A}{N_B}\right)}{t}
\]

**SOLUTION** Since \( N_A / N_B = 3.00 \) when \( t = 3.00 \text{ days} \), it follows that

\[
\lambda_A - \lambda_B = \frac{-\ln(3.00)}{3.00 \text{ days}} = -0.366 \text{ days}^{-1}
\]

But we need to find the half-life of species B, so we use Equation 31.6, which indicates that \( \lambda = 0.693/T_{1/2} \). With this expression for \( \lambda \), the result for \( \lambda_A - \lambda_B \) becomes

\[
0.693 \left(\frac{1}{T_{1/2}^A} - \frac{1}{T_{1/2}^B}\right) = -0.366 \text{ days}^{-1}
\]

Since \( T_{1/2}^B = 1.50 \text{ days} \), the result above can be solved to show that \( T_{1/2}^A = 7.23 \text{ days} \).
46. **REASONING** The number \( N \) of radioactive nuclei remaining after a time \( t \) is given by Equations 31.5 and 31.6 as

\[
N = N_0 e^{-0.693 t / T_{1/2}}
\]  

(1)

where \( N_0 \) is the number of radioactive nuclei present at \( t = 0 \) s and \( T_{1/2} = 5730 \text{ yr} \) is the half-life for the decay of \(^{14}\text{C}\). The activity \( A \) is proportional to the number of radioactive nuclei that a sample contains, so it follows from Equation (1) that

\[
A = A_0 e^{-0.693 t / T_{1/2}}
\]  

(2)

The age of the sample can be obtained by solving Equation (2) for \( t \).

**SOLUTION**

a. Solving Equation (2) for \( t \) and using \( A_0 = 0.23 \text{ Bq} \), we find that

\[
\frac{A}{A_0} = e^{-(0.693/T_{1/2})t} \quad \text{or} \quad \ln \left( \frac{A}{A_0} \right) = - \left( \frac{0.693}{T_{1/2}} \right) t
\]

or

\[
t = - \frac{T_{1/2} \ln \left( \frac{A}{A_0} \right)}{0.693} = - \left( \frac{5730 \text{ yr}}{0.693} \right) \ln \left[ \frac{(0.0061 \text{ Bq}) / (0.23 \text{ Bq})}{0.693} \right] = 3.0 \times 10^4 \text{ yr}
\]

b. If the value of 0.23 Bq were 40% larger, we would have \( A_0 = 1.4(0.23 \text{ Bq}) \), and would find that

\[
t = - \frac{T_{1/2} \ln \left( \frac{A}{A_0} \right)}{0.693} = - \left( \frac{5730 \text{ yr}}{0.693} \right) \ln \left[ \left( \frac{0.0061 \text{ Bq}}{1.4 \times 0.23 \text{ Bq}} \right) / 0.693 \right] = 3.3 \times 10^3 \text{ yr}
\]

47. **SSM REASONING** According to Equation 31.5, the number of nuclei remaining after a time \( t \) is \( N = N_0 e^{-\lambda t} \). If we multiply both sides of this equation by the decay constant \( \lambda \), we have \( \lambda N = \lambda N_0 e^{-\lambda t} \). Recognizing that \( \lambda N \) is the activity \( A \), we have \( A = A_0 e^{-\lambda t} \), where \( A_0 \) is the activity at time \( t = 0 \). \( A_0 \) can be determined from the fact that we know the mass of the specimen, and that the activity of one gram of carbon in a living organism is 0.23 Bq. The decay constant \( \lambda \) can be determined from the value of 5730 yr for the half-life of \(^{14}\text{C}\) using Equation 31.6. With known values for \( A_0 \) and \( \lambda \), the given activity of 1.6 Bq can be used to determine the age \( t \) of the specimen.

**SOLUTION** For \(^{14}\text{C}\), the decay constant is

\[
\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}
\]

The activity at time \( t = 0 \) is \( A_0 = (9.2 \text{ g})(0.23 \text{ Bq/g}) = 2.1 \text{ Bq} \). Since \( A = 1.6 \text{ Bq} \) and \( A_0 = 2.1 \text{ Bq} \), the age of the specimen can be determined from
\[ A = 1.6 \text{ Bq} = (2.1 \text{ Bq}) e^{-(1.21 \times 10^{-4} \text{ yr}^{-1}) t} \]

Taking the natural logarithm of both sides leads to

\[ \ln \left( \frac{1.6 \text{ Bq}}{2.1 \text{ Bq}} \right) = -(1.21 \times 10^{-4} \text{ yr}^{-1}) t \]

Therefore, the age of the specimen is

\[ t = \frac{\ln \left( \frac{1.6 \text{ Bq}}{2.1 \text{ Bq}} \right)}{-1.21 \times 10^{-4} \text{ yr}^{-1}} = 2.2 \times 10^3 \text{ yr} \]

48. **REASONING** Initially, the rock contains a number \( N_0 \) of uranium \( ^{238}_{92} \text{U} \) atoms. After a time \( t \) passes, only 60.0% of these atoms are still present. Therefore the final number \( N \) of uranium \( ^{238}_{92} \text{U} \) atoms in the rock is given by

\[ N = 0.600 N_0 \quad (1) \]

In general, the relationship between the initial and final numbers \( N_0, N \) of atoms in a radioactive sample after a time \( t \) is

\[ N = N_0 e^{-\lambda t} \quad (31.5) \]

where \( \lambda = \frac{\ln 2}{T_{1/2}} \) (Equation 31.6) is the decay constant and \( T_{1/2} \) is the half-life of the radioactive nuclei.

**SOLUTION** Solving Equation 31.5 for the term \( e^{-\lambda t} \) and then taking the natural logarithm of both sides, we are able to solve for \( t \):

\[ e^{-\lambda t} = \frac{N}{N_0} \quad \text{or} \quad -\lambda t = \ln \left( \frac{N}{N_0} \right) \quad \text{or} \quad t = -\frac{\ln \left( \frac{N}{N_0} \right)}{\lambda} \quad (2) \]

Substituting \( \lambda = \frac{\ln 2}{T_{1/2}} \) (Equation 31.6) and Equation (1) into Equation (2) yields

\[ t = -\frac{\ln \left( \frac{0.600 N_0}{N_0} \right)}{\lambda} = -\frac{\ln \left( \frac{0.600}{1} \right)}{\ln 2} = T_{1/2} \ln (0.600) = \frac{4.47 \times 10^9 \text{ yr} \ln (0.600)}{\ln 2} = 3.29 \times 10^9 \text{ yr} \]
49. **REASONING AND SOLUTION**
   a. According to Equations 31.5 and 31.6, the ratio \( \frac{N}{N_0} \) is given by
   \[
   \frac{N}{N_0} = e^{-0.693 \frac{t}{T_{1/2}}} = e^{-0.693 \frac{(5.00 \text{ yr})}{(5730 \text{ yr})}} = 0.999
   \]
   b. Similarly, we find
   \[
   \frac{N}{N_0} = e^{-0.693 \frac{t}{T_{1/2}}} = e^{-0.693 \frac{(3600 \text{ s})}{(122.2 \text{ s})}} = 1.36 \times 10^{-9}
   \]
   c. Similarly, we find
   \[
   \frac{N}{N_0} = e^{-0.693 \frac{t}{T_{1/2}}} = e^{-0.693 \frac{(5.00 \text{ yr})}{(12.33 \text{ yr})}} = 0.755
   \]

50. **REASONING** In the radiocarbon method, the number of radioactive nuclei remaining at a given instant is related to the number present initially, the time that has passed since the individual died, and the decay constant for \(^{14}\text{C} \). Thus, to determine how long ago the individual died, we will need information about the number of nuclei present in the material discovered with the mummy and the number present initially, which can be related to the activity of the material found with the body and the initial activity. We will also need the decay constant, which can be obtained from the half-life of \(^{14}\text{C} \).

**SOLUTION** The number \( N \) of radioactive nuclei present at a time \( t \) is
   \[
   N = N_0 e^{-\lambda t} \tag{31.5}
   \]
where \( N_0 \) is the number present initially at \( t = 0 \) s and \( \lambda \) is the decay constant for \(^{14}\text{C} \).

Solving this equation for \( t \), we find that
   \[
   \frac{N}{N_0} = e^{-\lambda t} \quad \text{or} \quad \ln \left( \frac{N}{N_0} \right) = -\lambda t \quad \text{or} \quad t = -\left( \frac{1}{\lambda} \right) \ln \left( \frac{N}{N_0} \right)
   \]

Since the activity \( A \) is proportional to the number \( N \) of radioactive nuclei, this expression for \( t \) becomes
   \[
   t = -\left( \frac{1}{\lambda} \right) \ln \left( \frac{N}{N_0} \right) = -\left( \frac{1}{\lambda} \right) \ln \left( \frac{A}{A_0} \right) \tag{1}
   \]

The decay constant is related to the half-life \( T_{1/2} \) according to
   \[
   \lambda = \frac{0.693}{T_{1/2}} \tag{31.6}
   \]

Substituting this expression into Equation (1) reveals that
\[
t = -\left( \frac{1}{\lambda} \right) \ln \left( \frac{A}{A_0} \right) = -\left( \frac{1}{0.693/T_{1/2}} \right) \ln \left( \frac{A}{A_0} \right)
\]

Noting that \( A/A_0 = 0.785 \) and that \( T_{1/2} = 5730 \text{ yr} \), we find that

\[
t = -\left( \frac{T_{1/2}}{0.693} \right) \ln \left( \frac{A}{A_0} \right) = -\left( \frac{5730 \text{ yr}}{0.693} \right) \ln (0.785) = 2.00 \times 10^3 \text{ yr}
\]

51. **SSM REASONING** According to Equation 31.5, \( N = N_0 e^{-\lambda t} \). If we multiply both sides by the decay constant \( \lambda \), we have

\[
\lambda N = \lambda N_0 e^{-\lambda t} \quad \text{or} \quad A = A_0 e^{-\lambda t}
\]

where \( A_0 \) is the initial activity and \( A \) is the activity after a time \( t \). The decay constant \( \lambda \) is related to the half-life through Equation 31.6: \( \lambda = 0.693 / T_{1/2} \). We can find the age of the fossils by solving for the time \( t \). The maximum error can be found by evaluating the limits of the accuracy as given in the problem statement.

**SOLUTION** The age of the fossils is

\[
t = -\frac{T_{1/2}}{0.693} \ln \left( \frac{A}{A_0} \right) = -\frac{5730 \text{ yr}}{0.693} \ln \left( \frac{0.100 \text{ Bq}}{0.23 \text{ Bq}} \right) = \boxed{6900 \text{ yr}}
\]

The maximum error can be found as follows. When there is an error of +10.0 %, \( A = 0.100 \text{ Bq} + 0.0100 \text{ Bq} = 0.110 \text{ Bq} \), and we have

\[
t = -\frac{5730 \text{ yr}}{0.693} \ln \left( \frac{0.110 \text{ Bq}}{0.23 \text{ Bq}} \right) = 6100 \text{ yr}
\]

Similarly, when there is an error of -10.0 %, \( A = 0.100 \text{ Bq} - 0.0100 \text{ Bq} = 0.090 \text{ Bq} \), and we have

\[
t = -\frac{5730 \text{ yr}}{0.693} \ln \left( \frac{0.090 \text{ Bq}}{0.23 \text{ Bq}} \right) = 7800 \text{ yr}
\]

The maximum error in the age of the fossils is 7800 yr – 6900 yr = \boxed{900 \text{ yr}}.

52. **REASONING** The number of radioactive nuclei remaining after a time \( t \) is given by Equations 31.5 and 31.6 as

\[
N = N_0 e^{-0.693t/T_{1/2}}
\]

where \( N_0 \) is the number of radioactive nuclei present at \( t = 0 \text{ s} \) and \( T_{1/2} \) is the half life for the decay. The activity \( A \) is proportional to the number of radioactive nuclei that a sample contains, so it follows that
\[ A = A_0 e^{-0.693 t/T_{1/2}} \]  

We know that the activity for the ancient carbon in the sample is 0.011 Bq per gram of carbon, whereas for the fresh carbon it is 0.23 Bq per gram of carbon. Knowing the percentage composition of the contaminated sample, we can determine its activity \( A \) by using the given percentages with the known activities:

\[ A_{\text{Contaminated}} = 0.980A_{\text{Ancient}} + 0.020A_{\text{Fresh}} \]  

The true age and the apparent age can be obtained by applying Equation (1) to the corresponding activities.

**SOLUTION**

a. Using Equation (1), we find that the true age is

\[ A_{\text{Ancient}} = A_0 e^{-0.693 t/T_{1/2}} \quad \text{or} \quad 0.011 \text{ Bq} = (0.23 \text{ Bq}) e^{-0.693 t/(5730 \text{ yr})} \]

Taking the natural logarithm of both sides of this result gives

\[ \ln \left( \frac{0.011 \text{ Bq}}{0.23 \text{ Bq}} \right) = -\frac{0.693 t}{5730 \text{ yr}} \]

\[ \frac{- (5730 \text{ yr}) \ln \left( \frac{0.011 \text{ Bq}}{0.23 \text{ Bq}} \right)}{0.693} = 25000 \text{ yr} \]

b. Using Equation (2) to find the activity of the contaminated sample, we obtain

\[ A_{\text{Contaminated}} = 0.980A_{\text{Ancient}} + 0.020A_{\text{Fresh}} \]

\[ = 0.980(0.011 \text{ Bq}) + 0.020(0.23 \text{ Bq}) = 0.0154 \text{ Bq} \]

Using this activity in Equation (1), we find that the apparent age of the sample is

\[ \frac{- (5730 \text{ yr}) \ln \left( \frac{0.0154 \text{ Bq}}{0.23 \text{ Bq}} \right)}{0.693} = 22000 \text{ yr} \]

53. **REASONING** The mass defect \( \Delta m \) is related to the total binding energy and the square of the speed \( c \) of light. Thus, to determine \( \Delta m \) we will use Figure 31.5 to obtain the binding energy per nucleon for the oxygen \( ^{16}O \) nucleus and multiply this value by the total number of nucleons (16) to find the total binding energy. Note that the data in Figure 31.5 is given in MeV per nucleon and that an MeV is not an SI unit compatible with kilograms. Thus, we will need to convert MeV to joules (J).

**SOLUTION** The mass defect \( \Delta m \) is given by
\[ \Delta m = \frac{\text{Binding energy}}{c^2} \]  

(31.3)

In Figure 31.5 we can see that the binding energy per nucleon for oxygen \(^{16}\text{O}\) is 8.00 MeV per nucleon. Therefore, the total binding energy for the 16 nucleons in the nucleus is

\[ \text{Binding energy} = (8.00 \text{ MeV/nucleon})(16 \text{ nucleons}) = 128 \text{ MeV} \]

Using this value in Equation 31.3 and converting the energy units of MeV into joules (J), we find that

\[ \Delta m = \frac{128 \text{ MeV}}{c^2} = \frac{1 \times 10^6 \text{ eV}}{1 \text{ MeV}} \times \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \times \left(3.00 \times 10^8 \text{ m/s}\right)^2 = 2.28 \times 10^{-28} \text{ kg} \]

54. **REASONING** According to Coulomb’s law, the magnitude of the electrostatic force between two protons is \( F = ke^2/r^2 \) (Equation 18.1), where \( e \) is the magnitude of the charge on each proton and \( r \) is the distance between them. The electrostatic force given by Coulomb’s law has the least possible magnitude when the two charges are as widely separated as possible, so that the distance \( r \) between them is as great as possible. In the nucleus, this means that the two protons in question must be located at opposite ends of the diameter of the nucleus.

**SOLUTION** To find the magnitude of the least possible electrostatic force that either of two protons can exert on the other, we need to know the diameter of the gold nucleus. The diameter is twice the radius, so that Equation 31.2 indicates that the diameter of the gold nucleus is \( d = 2(1.2 \times 10^{-15} \text{ m})A^{1/3} \), where \( A \) is the nucleon number. For gold \(^{197}\text{Au}\) it follows that \( A = 197 \). Using Coulomb’s law with a separation between the protons that equals the diameter \( d \) of the nucleus, we find

\[ F_{\text{Least possible}} = \frac{ke^2}{d^2} = \frac{ke^2}{2(1.2 \times 10^{-15} \text{ m})A^{1/3}} = \frac{8.99 \times 10^9 \text{ N \cdot m}^2/\text{C}^2}{2(1.2 \times 10^{-15} \text{ m})(197)^{1/3}} = 1.2 \text{ N} \]

55. **REASONING** The half-life \( T_{1/2} \) of a given isotope is directly related to the decay constant \( \lambda \) by \( T_{1/2} = \frac{0.693}{\lambda} \) (Equation 31.6). The decay constant is found from \( N = N_0e^{-\lambda t} \) (Equation 31.5), where \( N \) and \( N_0 \) are, respectively, the final and initial numbers of nuclei in the sample, and \( t \) is the time interval. We do not know either the initial or the final number of nuclei in
the sample, but we are told that the final number $N$ is 93.8% of the initial number $N_0$. Therefore, we have that $N = 0.938 N_0$.

**SOLUTION** Solving $N = N_0 e^{-\lambda t}$ (Equation 31.5) for the decay constant $\lambda$, we obtain

$$\frac{N}{N_0} = e^{-\lambda t} \quad \text{or} \quad \ln \left( \frac{N}{N_0} \right) = -\lambda t \quad \text{or} \quad \lambda = -\frac{\ln \left( \frac{N}{N_0} \right)}{t} \quad (1)$$

Substituting Equation (1) into $T_{1/2} = \frac{0.693}{\lambda}$ (Equation 31.6), we find that

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{-\frac{\ln \left( \frac{N}{N_0} \right)}{t}} = \frac{0.693 t}{\ln \left( \frac{N}{N_0} \right)} \quad (2)$$

Substituting the time $t = 4.51 \times 10^9 \text{ yr}$ and $N = 0.938 N_0$ into Equation (2) yields

$$T_{1/2} = \frac{0.693}{\ln \left( \frac{0.938 N_0}{N_0} \right)} = \frac{0.693 \left( 4.51 \times 10^9 \text{ yr} \right)}{\ln (0.938)} = 4.88 \times 10^{10} \text{ yr}$$

56. **REASONING** Since the released energy $E$ is shared among the three particles, we know that

$$E = E_{\text{Daughter}} + E_{\text{Beta particle}} + E_{\text{Antineutrino}} \quad \text{or} \quad E_{\text{Antineutrino}} = E - E_{\text{Daughter}} - E_{\text{Beta particle}} \quad (1)$$

The value for $E_{\text{Beta particle}}$ is known. For a given value of $E$, $E_{\text{Antineutrino}}$ will have its maximum possible value when $E_{\text{Daughter}}$ is zero.

The energy $E$ released during the decay is related to the mass decrease $\Delta m$ by $E = (\Delta m) c^2$ (Equation 28.5), where $c$ is the speed of light in a vacuum. A mass decrease of one atomic mass unit ($\Delta m = 1 \text{ u}$) corresponds to a released energy of 931.5 MeV.

**SOLUTION** We begin by calculating the decrease in mass for the decay process, which is shown as follows, along with the given atomic mass values:

$$\frac{32}{15} \text{P} \rightarrow \frac{32}{16} \text{S} + \frac{0}{1} \text{e}^- + \bar{\nu} \quad \text{Antineutrino}$$

When the $\frac{32}{15} \text{P}$ nucleus of a phosphorus atom is converted into a $\frac{32}{16} \text{S}$ nucleus, the number of orbital electrons remains the same, so the resulting sulfur atom is missing one orbital electron. However, the given atomic mass for sulfur includes all 16 electrons of a neutral atom. In effect, then, the value of 31.972 070 u already includes the mass of the $\beta^-$.
particle. The mass decrease for the decay is

\[ 31.973\,907\,u - 31.972\,070\,u = 0.001\,837\,u \]

The released energy \( E \) is

\[ E = \left( 0.001\,837\,u \right) \left( \frac{931.5\,\text{MeV}}{1\,u} \right) = 1.711\,\text{MeV} \]

Using Equation (1), we can now find the maximum possible energy carried away by the antineutrino

\[ E_{\text{Antineutrino}} = E - E_{\text{Daughter}} - E_{\text{Beta particle}} \]

\[ = (1.711\,\text{MeV}) - (0\,\text{MeV}) - (0.90\,\text{MeV}) = 0.81\,\text{MeV} \]

57. **REASONING** We can determine the identity of \( X \) in each of the decay processes by noting that for each process, the sum of \( A \) and \( Z \) for the decay products must equal the values of \( A \) and \( Z \) for the parent nuclei.

**SOLUTION**

a. \( ^{211}_{82}\text{Pb} \rightarrow ^{211}_{83}\text{Bi} + X \)

Using the reasoning discussed above, \( X \) must have \( A = 0 \) and \( Z = -1 \). Therefore \( X \) must be an electron, \( _{-1}^0 \text{e} \); \( X \) represents a \( \beta^- \) particle (electron).

b. \( ^{11}_{6}\text{C} \rightarrow ^{11}_{5}\text{B} + X \)

Similar reasoning suggests that \( X \) must have \( A = 0 \) and \( Z = +1 \). Therefore \( X \) must be a positron, \( _{+1}^0 \text{e} \); \( X \) represents a \( \beta^+ \) particle (positron).

c. \( ^{231}_{90}\text{Th}^* \rightarrow ^{231}_{90}\text{Th} + X \)

Similar reasoning suggests that \( X \) must have \( A = 0 \) and \( Z = 0 \). Therefore \( X \) must be a gamma ray; \( X \) represents a \( \gamma \) ray.

d. \( ^{210}_{84}\text{Po} \rightarrow ^{206}_{82}\text{Pb} + X \)

Using the reasoning discussed above, \( X \) must have \( A = 4 \) and \( Z = 2 \). Therefore \( X \) must be a helium nucleus, \( _2^4 \text{He} \); \( X \) represents an \( \alpha \) particle (helium nucleus).

58. **REASONING** The activity \( |\Delta N/\Delta t| \) of a radioactive sample is the number of disintegrations per second that occurs. The activity is related to the decay constant \( \lambda \) by

\[ \text{Activity} = \frac{\Delta N}{\Delta t} = \lambda N \]  

(31.4)

where \( N \) is the number of radioactive nuclei present. The minus sign has been removed from Equation 31.4, since we have taken the absolute value of both sides of this equation. The
decay constant is related to the half-life $T_{1/2}$ by \( \lambda = \frac{0.693}{T_{1/2}} \) (Equation 31.6). As discussed in Section 14.1, the number $N$ of radium nuclei is the number $n$ of moles of nuclei times Avogadro’s number $N_A$ (which is the number of nuclei per mole). Thus, the activity can be expressed as

\[
\text{Activity} = \lambda N = \frac{0.693}{T_{1/2}} (nN_A)
\]  \hspace{1cm} (1)

However, the number of moles is equal to the mass of radium (one gram) divided by its mass per mole. Recall that the mass (in grams) per mole has the same numerical value as the atomic mass of the substance. Since the atomic mass of radium is 226 u, its mass per mole is 226 g/mol.

**SOLUTION** Using Equation (1), we find that the activity of one gram of radium is

\[
\text{Activity} = \frac{0.693}{T_{1/2}} (nN_A)
\]

\[
= \frac{0.693}{(1.6 \times 10^3 \text{ yr})} \left( \frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \left( \frac{1.00 \text{ g}}{226 \text{ g/mol}} \right) \left( \frac{6.02 \times 10^{23} \text{ mol}^{-1}}{\text{Avogadro’s number}} \right)
\]

\[
= 3.7 \times 10^{10} \text{ disintegrations/s}
\]

59. **SSM REASONING AND SOLUTION** As shown in Figure 31.17, if the first dynode produces 3 electrons, the second produces 9 electrons (3\(^2\)), the third produces 27 electrons (3\(^3\)), so the $N^{th}$ produces $3^N$ electrons. The number of electrons that leaves the 14\(^th\) dynode and strikes the 15\(^th\) dynode is

\[
3^{14} = 4,782,969 \text{ electrons}
\]

60. **REASONING** Recall from Section 14.1 that the mass $m$ of a substance is equal to the number $n$ of moles times the mass per mole. The mass (in grams) per mole has the same numerical value as the atomic mass of the substance, so the mass per mole for strontium is 89.908 g/mol. Thus, $m = n$ (89.908 g/mol). Also, the number $n$ of moles is equal to the number $N$ of strontium atoms divided by Avogadro’s number $N_A$; $n = N / N_A$. The mass of strontium can, therefore, be written as

\[
m = n(89.908 \text{ g/mol}) = \left( \frac{N}{N_A} \right)(89.908 \text{ g/mol})
\]  \hspace{1cm} (1)

The activity $|\Delta N / \Delta t|$ of a radioactive sample is the number of disintegrations per second that occurs. The activity is related to the decay constant $\lambda$ and the number $N$ of atoms by
Activity = \( \frac{\Delta N}{\Delta t} \) = \( \lambda N \)  

(31.4)

The minus sign has been removed from Equation 31.4, since we have taken the absolute value of both sides of this equation. Solving Equation 31.4 for \( N \) and substituting the result into Equation (1), we find

\[
m = \left( \frac{\Delta N}{\Delta t} \right) (89.908 \text{ g/mol}) \]

(2)

Finally, we recognize that the decay constant \( \lambda \) is related to the half-life \( T_{1/2} \) by \( \lambda = 0.693/ T_{1/2} \) (Equation 31.6). Substituting this expression for \( \lambda \) into Equation (2) gives

\[
m = \frac{\Delta N}{\Delta t} \frac{T_{1/2} (89.908 \text{ g/mol})}{(0.693) N_A} \]

(3)

**SOLUTION** Using Equation (3), we find that

\[
m = \frac{\Delta N}{\Delta t} \frac{T_{1/2} (89.908 \text{ g/mol})}{(0.693) N_A} \]

\[
= \frac{(6.0 \times 10^5 \text{ Bq}) \left[ (29.1 \text{ yr}) \left( \frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \right] (89.908 \text{ g/mol})}{(0.693) (6.02 \times 10^{23} \text{ mol}^{-1})} = 1.2 \times 10^{-7} \text{ g} \]

61. **REASONING AND SOLUTION** The \( \beta^- \) decay reaction is

\[
{}^{208}_{81}\text{TI} \rightarrow {}^{208}_{82}\text{Pb} + {}^6_{-1}\text{e} \]

so

\[
{}^Z_X \rightarrow {}^{208}_{82}\text{Pb} + {}^4_{2}\text{He} \]

gives

\[
{}^Z_X = {}^{212}_{84}\text{Po} \]

62. **REASONING**

a. Each \( \alpha \) decay process takes the general form

\[
{}^Z_X \rightarrow {}^{A-4}_{Z-2}\text{D} + {}^4_{2}\text{He} \]

Parent nucleus \quad \text{Daughter nucleus} \quad \text{Particle}
Therefore, each \( \alpha \) decay reduces the atomic mass number \( A \) by 4 and the atomic number \( Z \) by 2. Writing out the overall decay process for four \( \alpha \) decays will allow us to determine the atomic mass number and atomic number of the “great-great-granddaughter” of the thorium \( ^{228}_{90}\text{Th} \) nucleus.

b. The total amount of energy \( E \) released by the four alpha decays is found from the difference in mass \( \Delta m \) between the parent nucleus, thorium \( ^{228}_{90}\text{Th} \), and the products of the decays, which are four alpha particles and the “great-great-granddaughter” nucleus. After calculating the mass difference in atomic mass units (u), we will find the equivalent amount of released energy, given that 1 u of mass is equivalent to 931.5 MeV of energy.

**SOLUTION**

a. Four successive \( \alpha \) decays reduce the atomic mass number \( A \) of the parent nucleus by \( 4 \times 4 = 16 \), and the atomic number \( Z \) by \( 4 \times 2 = 8 \). Therefore, the overall reaction process may be summarized as

\[
^{A\ Z}_{Z-4}\text{P} \rightarrow ^{A-16\ Z-8}_{Z-8}\text{D} + 4\left(^4\text{He}\right)
\]

Substituting \( A = 228 \), \( Z = 90 \) and the appropriate chemical symbols into this reaction process, we obtain

\[
^{228\ 90}_{90}\text{Th} \rightarrow ^{212\ 82}_{82}\text{Pb} + 4\left(^4\text{He}\right)
\]

Therefore, the product of the four \( \alpha \) decays is lead \( ^{212}_{82}\text{Pb} \).

b. The mass of thorium \( ^{228}_{90}\text{Th} \) is 228.028 715 u, the mass of lead \( ^{212}_{82}\text{Pb} \) is 211.991 871 u (see Appendix F at the back of the book), and the mass of a single \( \alpha \) particle is 4.002 603 u. Following the reaction process above, the mass difference \( \Delta m \) is

\[
\Delta m = \frac{228.028\ 715\ u}{\text{Mass of parent}} - \frac{211.991\ 871\ u}{\text{Mass of great-great-granddaughter}} - 4\left(4.002\ 603\ u\right) = 0.026\ 432\ u
\]

The energy equivalent of this mass difference, then, is

\[
E = \left(0.026\ 432\ u\right)\left(\frac{931.5\ \text{MeV}}{1\ u}\right) = 24.62\ \text{MeV}
\]
1. (c) The biologically equivalent dose (in rems) is the absorbed dose (in rads) times the relative biological effectiveness. (See Equation 32.4.)

2. (d) Equation 32.3 defines the relative biological effectiveness as the dose of 200-keV X-rays that produces a certain biological effect divided by the dose of radiation that produces the same biological effect. Thus, since $\text{RBE}_A = 2 \times \text{RBE}_B$, it takes a smaller dose of A to produce the same biological effect as B, smaller by a factor of 2.

3. $9.0 \times 10^{-3}$ rem

4. (b) Since electric charge must be conserved, we know that the number of protons must be the same before and after the reaction takes place. Therefore, $Z + 7 = 6 + 1$, so $Z = 0$. We also know that the total number of nucleons must be conserved, so the total number before and after the reaction takes place must be the same. Therefore, $A + 14 = 14 + 1$, so $A = 1$. With a single uncharged nucleon in the nucleus, the unknown species must be a neutron $^1_0\text{n}$.

5. (a) The symbol for the α particle is $^4_2\text{He}$, and the symbol for the proton $p$ is $^1_1\text{H}$. Therefore, there are $2 + 13 = 15$ protons present before the reaction takes place and $15 + 1 = 16$ protons present after the reaction takes place, which violates the conservation of electric charge. There are $27 + 4 = 31$ nucleons present before the reaction takes place and $1 + 31 = 32$ nucleons present after the reaction takes place, which violates the conservation of nucleon number.

6. (e) The compound nucleus is $^{236}_{92}\text{U}$ for any X and Y. Since the total number of nucleons is conserved, it follows that $1 + 235 = A_X + A_Y + \eta$, where $\eta$ is the number of neutrons $^1_0\text{n}$ produced by the reaction. Therefore, greater values of $\eta$ lead to smaller values for $A_X$ and $A_Y$. Since electric charge also is conserved, it follows that $0 + 92 = Z_X + Z_Y + \eta(0)$. Therefore, $Z_X + Z_Y = 92$ for any X and Y.
7. (b) In a fission reactor each fission event, on average, must produce at least one neutron. Otherwise there would be no neutrons to cause additional fission events, and it would not be possible to establish a controlled chain reaction. If the fission products of the reaction each have a binding energy per nucleon that is less than the binding energy per nucleon of the starting nucleus, the reaction does not produce energy. It only produces energy when the fission products have, on average, a binding energy per nucleon that is greater than the binding energy per nucleon of the starting nucleus.

8. (a) In order for a fusion reaction to be potentially energy-producing, the binding energy per nucleon of the starting nuclei must be smaller than the binding energy per nucleon of the product nucleus. Since the maximum binding energy per nucleon occurs at a nucleon number of about 60, the starting nuclei with nucleon numbers of 30 in reaction I have the smaller binding energy per nucleon, as required.

9. (c) The energy produced by a fusion reaction is the mass defect $\Delta m$ (in u) for the reaction times 931.5 MeV/u, since an energy of 931.5 MeV is equivalent to 1 u. The mass defect is the total mass of the initial nuclei minus the total mass of the product nuclei. Thus, to obtain the ranking, we need only calculate the mass defect for each reaction from the given masses and rank the defects in descending order. The results are: reaction I ($\Delta m = 0.0035$ u), reaction II ($\Delta m = 0.0138$ u), reaction III ($\Delta m = 0.0053$ u).

10. (d) The quark theory explains the electric charge that each hadron carries. An antiproton carries a charge of $-e$, which is opposite the charge of $+e$ that a proton carries. Only possibility D shows a charge of $-e\left(\frac{1}{3}e - \frac{2}{3}e - \frac{2}{3}e\right)$.

11. (e) Hubble’s law indicates that a galaxy located at a distance $d$ from the earth is moving away from the earth at a speed that is directly (not inversely) proportional to $d$. 
1. **REASONING** The biologically equivalent dose (in rems) is the product of the absorbed dose (in rads) and the relative biological effectiveness (RBE), according to Equation 32.4. We can apply this equation to each type of radiation. Since the biologically equivalent doses of the neutrons and $\alpha$ particles are equal, we can solve for the unknown RBE.

**SOLUTION** Applying Equation 32.4 to each type of particle and using the fact that the biological equivalent doses are equal, we find that

$$\text{Biologically equivalent dose of } \alpha \text{ particles} \times \text{RBE}_{\alpha} = \text{Biologically equivalent dose of neutrons} \times \text{RBE}_{\text{neutrons}}$$

Solving for $\text{RBE}_{\alpha}$ and noting that $(\text{Absorbed dose})_{\text{neutrons}} = 6(\text{Absorbed dose})_{\alpha}$, we have

$$\text{RBE}_{\alpha} = \frac{(\text{Absorbed dose})_{\text{neutrons}}}{(\text{Absorbed dose})_{\alpha}} \times \text{RBE}_{\text{neutrons}} = \frac{6(\text{Absorbed dose})_{\alpha}}{(\text{Absorbed dose})_{\alpha}} \times (2.0) = [12]$$

2. **REASONING** The $\alpha$ particles and the protons cause the same biological damage. Therefore, they have same biologically equivalent dose (BED). However, they do not have the same absorbed dose (AD), because they do not have the same relative biological effectiveness (RBE). The BED and the AD are related according to $\text{BED} = \text{AD} \times \text{RBE}$ (Equation 32.4).

**SOLUTION** Using Equation 32.4 and the fact that the two types of radiation have the same biologically equivalent dose, we have

$$\text{BED}_{\alpha \text{ particles}} = \text{BED}_{\text{protons}} \quad \text{or} \quad \text{AD}_{\alpha \text{ particles}} \times \text{RBE}_{\alpha \text{ particles}} = \text{AD}_{\text{protons}} \times \text{RBE}_{\text{protons}}$$

Solving this result for $\text{AD}_{\alpha \text{ particles}}$, we find that

$$\text{AD}_{\alpha \text{ particles}} = \frac{\text{AD}_{\text{protons}} \times \text{RBE}_{\text{protons}}}{\text{RBE}_{\alpha \text{ particles}}} = \frac{(60 \text{ rad})(10)}{15} = 40 \text{ rad}$$
3. **REASONING** The absorbed dose (AD) in grays (Gy) is the energy absorbed divided by the tumor mass: \( AD = \frac{\text{Energy absorbed}}{\text{Mass}} \) (Equation 32.2). Because both tumors receive the same absorbed dose, the absorbed dose found in (a) will allow us to determine the energy absorbed by the second tumor, again via Equation 32.2.

**SOLUTION**

a. Substituting the given values into Equation 32.2, we obtain

\[
AD = \frac{\text{Energy absorbed}}{\text{Mass}} = \frac{1.7 \text{ J}}{0.12 \text{ kg}} = 14 \text{ Gy}
\]

b. For the second tumor, the absorbed dose is still \( AD = 14 \text{ Gy} \), but the mass is now 0.15 kg. Solving Equation 32.2 for the energy absorbed, we obtain

\[
\text{Energy absorbed} = (AD)\text{Mass} = (14 \text{ Gy})(0.15 \text{ kg}) = 2.1 \text{ J}
\]

4. **REASONING**

a. The absorbed dose (AD) and the biologically equivalent dose (BED) are related by Equation 32.4:

\[
BED = AD \times RBE \tag{32.4}
\]

where \( AD \) is the absorbed dose measured in rads, and \( RBE \) is the relative biological effectiveness of the cosmic rays. The cosmic rays are protons, for which the \( RBE \) is 10. We note that the biologically equivalent dose is given in millirems, where 1 mrem = 10^{-3} rem.

b. The absorbed dose is related to the energy absorbed and the mass of the person by Equation 32.2:

\[
AD = \frac{\text{Energy absorbed}}{\text{Mass}} \tag{32.2}
\]

If the energy is measured in joules, and the mass in kilograms, the absorbed dose is given in grays (Gy), where 1 Gy = 1 J/kg. To convert the absorbed dose found in (a) from rads to grays, we will use the equivalence 1 rad = 0.01 Gy = 0.01 J/kg.

**SOLUTION**

a. Solving Equation (32.4) for the absorbed dose (AD), we find that

\[
AD = \frac{\text{BED}}{RBE} = \frac{24 \times 10^{-3} \text{ rem}}{10} = 2.4 \times 10^{-3} \text{ rad}
\]

b. Solving Equation (32.2) for the mass and using 1 rad = 0.01 Gy = 0.01 J/kg, we find that

\[
\text{Mass} = \frac{\text{Energy absorbed}}{AD} = \frac{1.9 \times 10^{-3} \text{ J}}{(2.4 \times 10^{-3} \text{ rad})} = 79 \text{ kg}
\]
5. **SSM REASONING AND SOLUTION** According to Equation 32.2, the absorbed dose (AD) is equal to the energy absorbed by the tumor divided by its mass:

\[
AD = \frac{\text{Energy absorbed}}{\text{Mass}} = \frac{(25 \text{ s})(1.6 \times 10^{10} \text{ s}^{-1})(4.0 \times 10^6 \text{ eV})}{0.015 \text{ kg}} \times \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}
\]

\[
= 1.7 \times 10^4 \text{ Gy} = 1.7 \times 10^3 \text{ rad}
\]

The biologically equivalent dose (BED) is equal to the product of the absorbed dose (AD) and the RBE (see Equation 32.4):

\[
\text{BED} = AD \times \text{RBE} = (1.7 \times 10^3 \text{ rad})(14) = 2.4 \times 10^4 \text{ rem}
\]  

(32.4)

6. **REASONING** The energy \( E \) that is absorbed is related to the absorbed dose (AD) in grays and the mass \( M = 75 \text{ kg} \) of the person, according to \( AD = \frac{E}{M} \) (Equation 32.2). This equation can be solved for \( E \). We are not given a value for the absorbed dose, but it can be obtained from the given biologically equivalent dose (BED) of \( \text{BED} = 45 \text{ mrem} \). This is possible because the BED (in rems) and the AD (in rads) are related according to \( \text{BED} = AD \times \text{RBE} \) (Equation 32.4).

**SOLUTION** Solving Equation 32.2 for the energy \( E \) that is absorbed, we have

\[
AD = \frac{E}{M} \quad \text{or} \quad E = (AD)M
\]

(1)

Solving Equation 32.4 for absorbed dose, we obtain

\[
\text{BED} = AD \times \text{RBE} \quad \text{or} \quad AD = \frac{\text{BED}}{\text{RBE}}
\]

(2)

Substituting Equation (2) into Equation (1), we find that

\[
E = (AD)M = \left( \frac{\text{BED}}{\text{RBE}} \right)M = \left( \frac{45 \times 10^{-3}}{12} \text{ rad} \right)(75 \text{ kg}) = 0.28 \text{ rad} \cdot \text{kg}
\]

In this result, the unit for the mass is the SI unit called the kilogram (kg), whereas the unit for the absorbed dose is the non-SI unit called the rad. However, we know that the rad is related to the SI unit called the gray, according to \( 1 \text{ rad} = 0.01 \text{ gray} = 0.01 \text{ J/kg} \). Using this conversion factor, we find that the energy \( E \) in joules (J) is

\[
E = (0.28 \text{ rad} \cdot \text{kg}) \left( \frac{0.01 \text{ J/kg}}{1 \text{ rad}} \right) = 2.8 \times 10^{-3} \text{ J}
\]
7. **REASONING** When the cancerous growth absorbs energy from the radiation, the growth heats up. According to the discussion in Section 12.7, the rise in temperature depends on the heat absorbed, as well as on the mass and specific heat capacity of the growth. As we have seen in Section 32.1, the energy absorbed from the radiation is equal to the product of the absorbed dose and the mass of the growth.

**SOLUTION** When a substance, such as the cancerous growth, has a mass $m$ and absorbs an amount of energy $Q$, the change $\Delta T$ in its temperature is given by

$$\Delta T = \frac{Q}{cm}$$  \hspace{1cm} (12.4)

where $c$ is the specific heat capacity of the substance. The energy $Q$ absorbed is equal to the absorbed dose of the radiation times the mass $m$:

$$Q = (\text{Absorbed dose})m$$  \hspace{1cm} (32.2)

Substituting Equation 32.2 into Equation 12.4 gives

$$\Delta T = \frac{Q}{cm} = \frac{(\text{Absorbed dose})\mu r}{c\mu r} = \frac{\text{Absorbed dose}}{c}$$

$$= \frac{2.1 \text{ Gy}}{4200 \text{ J/(kg} \cdot \text{C})} = 5.0 \times 10^{-4} \text{ C}$$

8. **REASONING** The relation between the rad and gray units is presented in Section 32.1 as 1 rad = 0.01 gray. If, for instance, we wanted to convert an absorbed dose of 2.5 grays into rads, we would use the conversion procedure:

$$(2.5 \text{ Gy})\left(\frac{1 \text{ rad}}{0.01 \text{ Gy}}\right) = 250 \text{ rad}$$

In general, the conversion relation is

$$\text{Absorbed dose (in rad)} = \left[\text{Absorbed dose (in Gy)}\right]\left(\frac{1 \text{ rad}}{0.01 \text{ Gy}}\right)$$  \hspace{1cm} (1)

**SOLUTION**

According to Equation 32.4, the relative biological effectiveness (RBE) is given by

$$\text{RBE} = \frac{\text{Biologically equivalent dose}}{\text{Absorbed dose (in rad)}}$$

The absorbed dose (in rad) is related to the absorbed dose (in Gy) by Equation (1), so the RBE can be expressed as
Biologically equivalent dose

\[ RBE = \frac{\text{Biologically equivalent dose}}{[\text{Absorbed dose (in Gy)}] \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right)} \]

The absorbed dose (in Gy) is equal to the energy \( E \) absorbed by the tissue divided by its mass \( m \) (Equation 32.2), so the RBE can be written as

\[ RBE = \frac{2.5 \times 10^{-2} \text{ rem}}{\left( \frac{E}{m} \right) \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right)} = \frac{2.5 \times 10^{-2} \text{ rem}}{6.2 \times 10^{-3} \text{ J} / 21 \text{ kg} \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right)} = 0.85 \]

9. **REASONING AND SOLUTION** According to Equation 32.2, the energy absorbed is equal to the product of the absorbed dose (AD) and the mass of the tumor:

\[ \text{Energy} = \text{AD} \times \text{Mass} = (12 \text{ Gy})(2.0 \text{ kg}) = 24 \text{ J} \]

This energy is carried by \( \Delta N \) particles in time \( \Delta t \), so that

\[ \text{Energy} = \left( \frac{\Delta N}{\Delta t} \right)(850 \text{ s})(0.40 \times 10^6 \text{ eV}) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) \]

Therefore,

\[ \frac{\Delta N}{\Delta t} = 4.4 \times 10^{11} \text{ s}^{-1} \]

10. **REASONING** According to Equation 32.2 the absorbed dose is given by

\[ \text{Absorbed dose} = \frac{\text{Energy absorbed}}{\text{Mass of absorbing material}} \]

According to Equation 12.5, the amount of energy \( Q \) needed to boil a mass \( m \) of liquid water is

\[ Q = mL_v \]

where \( L_v = 22.6 \times 10^5 \text{ J/kg} \) is the latent heat of vaporization (see Table 12.3). Substituting this expression for \( Q \) into the expression for the absorbed dose, we find that

\[ \text{Absorbed dose} = \frac{\text{Energy absorbed}}{\text{Mass of absorbing material}} = \frac{mL_v}{m} = L_v \quad (1) \]

Note that a value for the mass \( m \) of the water is not needed, since it is eliminated algebraically from Equation (1). Furthermore, we see that the absorbed dose has the same units as does the latent heat, namely, 1 J/kg = 1 Gy. To obtain the absorbed dose in rads, we will use the fact that 1 rad = 0.01 Gy.
**SOLUTION** Using Equation (1) and converting units, we obtain

\[
\text{Absorbed dose (in rads)} = L_v \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right) = \left( 22.6 \times 10^5 \text{ J/kg} \right) \left( \frac{1 \text{ rad}}{0.01 \text{ Gy}} \right) = 2.26 \times 10^8 \text{ rad}
\]

11. **SSM REASONING** The number of nuclei in the beam is equal to the energy absorbed by the tumor divided by the energy per nucleus (130 MeV). According to Equation 32.2, the energy (in joules) absorbed by the tumor is equal to the absorbed dose (expressed in grays) times the mass of the tumor. The absorbed dose (expressed in rads) is equal to the biologically equivalent dose divided by the RBE of the radiation (see Equation 32.4). We can use these concepts to determine the number of nuclei in the beam.

**SOLUTION** The number \( N \) of nuclei in the beam is equal to the energy \( E \) absorbed by the tumor divided by the energy per nucleus. Since the energy absorbed is equal to the absorbed dose (in Gy) times the mass \( m \) (see Equation 32.2) we have

\[
N = \frac{E}{\text{Energy per nucleus}} = \left[ \text{Absorbed dose (in Gy)} \right] \frac{m}{\text{Energy per nucleus}}
\]

We can express the absorbed dose in terms of rad units, rather than Gy units, by noting that 1 rad = 0.01 Gy. Therefore,

\[
\text{Absorbed dose (in Gy)} = \text{Absorbed dose (in rad)} \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right)
\]

The number of nuclei can now be written as

\[
N = \frac{E}{\text{Energy per nucleus}} = \left[ \text{Absorbed dose (in rad)} \right] \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) \frac{m}{\text{Energy per nucleus}}
\]

We know from Equation 32.4 that the Absorbed dose (in rad) is equal to the biologically equivalent dose divided by the RBE, so that

\[
N = \frac{E}{\text{Energy per nucleus}} = \left[ \frac{\text{Biologically equivalent dose}}{\text{RBE}} \right] \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) \frac{m}{\text{Energy per nucleus}}
\]

\[
= \left[ \frac{180 \text{ rem}}{16} \right] \left( \frac{0.01 \text{ Gy}}{1 \text{ rad}} \right) \left( 0.17 \text{ kg} \right) \left( \frac{130 \times 10^6 \text{ eV}}{1 \text{ eV}} \right) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 9.2 \times 10^8
\]
12. **REASONING** The reaction \( ^{10}_5\text{B}(\alpha, p)^Z\text{X} \), where “\( \alpha \)” is an \( \alpha \) particle \( \left(^4_2\text{He}\right) \) and “\( p \)” is a proton \( \left(^1_1\text{H}\right) \), can be written as

\[
^{10}_5\text{B} + ^4_2\text{He} \rightarrow ^1_1\text{H} + ^Z_2\text{X}
\]

The total number of nucleons (protons plus neutrons) is conserved during any nuclear reaction. This conservation law will allow us to determine the atomic mass number \( A \).

The total electric charge (number of protons) is also conserved during any nuclear reaction. This conservation law will allow us to determine the atomic number \( Z \). From a knowledge of \( Z \), we can use the periodic table to identify the element \( \text{X} \).

**SOLUTION** The conservation of the total number of nucleons states that the total number of nucleons before the reaction \((10 + 4)\) is equal to the total number after the reaction \((1 + A)\):

\[
10 + 4 = 1 + A \quad \text{or} \quad A = 13
\]

The conservation of total electric charge states that the number of protons before the reaction \((5 + 2)\) is equal to the total number after the reaction \((1 + Z)\):

\[
5 + 2 = 1 + Z \quad \text{or} \quad Z = 6
\]

By consulting a periodic table (see the inside of the back cover), we find that the element whose atomic number is 6 is carbon:

\[
\text{X} = \text{C (or carbon)}
\]

13. **SSM REASONING** The reaction given in the problem statement is written in the shorthand form: \( ^{17}_8\text{O} (\gamma, \alpha n)^{12}_6\text{C} \). The first and last symbols represent the initial and final nuclei, respectively. The symbols inside the parentheses denote the incident particles or rays (left side of the comma) and the emitted particles or rays (right side of the comma).

**SOLUTION** In the shorthand form of the reaction, we note that the designation \( \alpha n \) refers to an \( \alpha \) particle (which is a helium nucleus \( ^4_2\text{He} \)) and a neutron \( ^1_0\text{n} \). Thus, the reaction is

\[
\gamma + ^{17}_8\text{O} \rightarrow ^{12}_6\text{C} + ^4_2\text{He} + ^1_0\text{n}
\]

14. **REASONING** Protons and neutrons are nucleons, and the total number of nucleons before the reaction is equal to the total number of nucleons after the reaction.

Only the proton and the electron have electrical charge. The neutron and the \( \gamma \)-ray photon are electrically neutral. The total electric charge of the particles before the reaction is equal to the total electric charge of particles after the reaction.
**SOLUTION**

a. The total number of nucleons before the reaction is \( A + 14 \). The total number of nucleons after the reaction is \( 1 + 17 \). Setting these two numbers equal to each other yields \( A = 4 \). The net electric charge before the reaction is \( Z + 7 \). The net electric charge after the reaction is \( 1 + 8 \). Setting these two numbers equal to each other yields \( Z = 2 \). The unknown particle is \( ^4_2X = ^{14}_2He \).

b. The total number of nucleons before the reaction is \( 15 + A \). The total number of nucleons after the reaction is \( 12 + 4 \). Setting these two numbers equal to each other yields \( A = 1 \). The net electric charge before the reaction is \( 7 + Z \). The net electric charge after the reaction is \( 6 + 2 \). Setting these two numbers equal to each other yields \( Z = 1 \). The unknown particle is \( ^{1}_1X = ^{1}_1H \).

c. The total number of nucleons before the reaction is \( 1 + 27 \). The total number of nucleons after the reaction is \( A + 1 \). Setting these two numbers equal to each other yields \( A = 27 \). The net electric charge before the reaction is \( 1 + 13 \). The net electric charge after the reaction is \( Z + 0 \). Setting these two numbers equal to each other yields \( Z = 14 \). The unknown particle is \( ^{27}_{14}X = ^{27}_{14}Si \).

d. The total number of nucleons before the reaction is \( 7 + 1 \). The total number of nucleons after the reaction is \( 4 + A \). Setting these two numbers equal to each other yields \( A = 4 \). The net electric charge before the reaction is \( 3 + 1 \). The net electric charge after the reaction is \( 2 + Z \). Setting these two numbers equal to each other yields \( Z = 2 \). The unknown particle is \( ^{4}_{2}X = ^{4}_{2}He \).

15. **REASONING** The conservation of nucleon number states that the total number of nucleons (protons plus neutrons) before the reaction occurs must be equal to the total number of nucleons after the reaction. This conservation law will allow us to find the atomic mass number \( A \) of the unknown nucleus. The conservation of electric charge states that the net electric charge of the particles before the reaction must be equal to the net charge after the reaction. This conservation law will allow us to find the atomic number \( Z \) of the unknown nucleus. With a knowledge of the atomic number, we can use the periodic table to identify the element.

**SOLUTION**

a. The total number of nucleons before the reaction is \( 1 + 232 = 233 \). The total number of nucleons after the reaction is \( A \). Setting these two numbers equal to each other yields \( A = 233 \). The net electric charge before the reaction is \( 0 + 90 = 90 \). The net electric charge after the reaction is \( Z \). Setting these two numbers equal to each other yields \( Z = 90 \). A check of the periodic table shows that this element is \( ^{233}_{90}Thorium \).
b. The $^{233}_{90}\text{Th}$ nucleus subsequently undergoes $\beta^-$ decay $\left(0^+_1\text{e}\right)$, as does its daughter. The first reaction is $^{233}_{90}\text{Th} \rightarrow A^Z\text{X} + 0^+_1\text{e}$. By employing an analysis similar to that used in part (a), the unknown nucleus is found to be $^{233}_{91}\text{Pa}$. This daughter nucleus also undergoes $\beta^-$ decay according to the reaction $^{233}_{91}\text{Pa} \rightarrow A^Z\text{X} + 0^+_1\text{e}$. Using the analysis of part (a) again, we see that the final unknown nucleus is $^{233}_{92}\text{U}$.

16. **REASONING** In the shorthand form discussed in the text, the following symbolism is used:
- $^4_2\text{He}$ is an $\alpha$ particle and is denoted by $\alpha$.
- $^1_0\text{n}$ is a neutron and is denoted by $n$.
- $^1_1\text{H}$ is a proton and is denoted by $p$.
- $\gamma$ is a gamma ray photon and is denoted by $\gamma$.

**SOLUTION**

a. The reaction $^4_2\text{He} + ^{27}_{13}\text{Al} \rightarrow ^{30}_{15}\text{P} + ^1_0\text{n}$ can be written as $^{27}_{13}\text{Al}(\alpha, n)^{30}_{15}\text{P}$.

b. The reaction $^1_1\text{H} + ^9_4\text{Be} \rightarrow ^6_3\text{Li} + ^4_2\text{He}$ can be written as $^9_4\text{Be}(p, \alpha)^6_3\text{Li}$.

c. The reaction $^1_0\text{n} + ^{55}_{25}\text{Mn} \rightarrow ^{56}_{25}\text{Mn} + \gamma$ can be written as $^{55}_{25}\text{Mn}(n, \gamma)^{56}_{25}\text{Mn}$.

17. **REASONING** During each reaction, both the total electric charge of the nucleons and the total number of nucleons are conserved. For each type of nucleus that participates in a reaction, the atomic mass number $A$ specifies the number of nucleons, and the atomic number $Z$ specifies the number of protons. We will use these conserved quantities to calculate the atomic mass number and atomic number for each of the unknown entities, thereby identifying them.

**SOLUTION**

a. In the reaction described by $^{34}_{18}\text{Ar}(n, \alpha)$, a neutron $\left(^1_0\text{n}\right)$ induces a reaction in which the argon $^{34}_{18}\text{Ar}$ nucleus is broken into an $\alpha$ particle $\left(^4_2\text{He}\right)$ and an unknown entity, which we designate as $^A_Z\text{X}$. The reaction process can be written as $^{34}_{18}\text{Ar} + ^1_0\text{n} \rightarrow ^4_2\text{He} + ^A_Z\text{X}$.
Conservation of the total number of nucleons gives \(34 + 1 = 4 + A\), or \(A = 34 + 1 - 4 = 31\). Conservation of the total electric charge yields \(18 + 0 = 2 + Z\), or \(Z = 18 - 2 = 16\). The atomic number of sulfur is \(Z = 16\), so the unknown entity must be sulfur \(\boxed{^{31}_{16}S}\).

b. The reaction \(^{82}_{34}\text{Se}(?,n)^{82}_{35}\text{Br}\) can be written as

\[
^{82}_{34}\text{Se} + \frac{A}{Z}\text{X} \rightarrow \frac{1}{0}\text{n} + ^{82}_{35}\text{Br}
\]

Since the total number of nucleons is conserved, we see that \(82 + A = 1 + 82\), or \(A = 1\). Conservation of the total electric charge indicates that \(34 + Z = 0 + 35\), or \(Z = 35 - 34 = 1\). The unknown entity has \(A = Z = 1\), so it must be a proton \(\boxed{^1_1\text{H}}\).

c. The reaction \(^{58}_{28}\text{Ni}(\frac{40}{18}\text{Ar},?)^{57}_{27}\text{Co}\) can be written as

\[
^{58}_{28}\text{Ni} + \frac{40}{18}\text{Ar} \rightarrow \frac{A}{Z}\text{X} + ^{57}_{27}\text{Co}
\]

Conservation of the total number of nucleons reveals that the atomic mass number of the unknown entity is given by \(58 + 40 = A + 57\), or \(A = 98 - 57 = 41\). Conservation of the total electric charge reveals that the atomic number is given by \(28 + 18 = Z + 27\), or \(Z = 46 - 27 = 19\). Potassium (K) has an atomic number \(Z = 19\), so the unknown entity is the potassium nucleus \(\boxed{^{41}_{19}\text{K}}\).

d. From the notation \(?(\gamma,\alpha)^{16}_{8}\text{O}\), we see that the reaction is induced by a \(\gamma\)-ray photon, which has \(A = Z = 0\). Therefore, the reaction process is

\[
\frac{A}{Z}\text{X} + \frac{0}{0}\gamma \rightarrow \frac{4}{2}\text{He} + ^{16}_{8}\text{O}
\]

The unknown entity, then, has \(A = 4 + 16 = 20\), and \(Z = 2 + 8 = 10\). The atomic number \(Z = 10\) corresponds to neon, so the unknown entity is neon \(\boxed{^{20}_{10}\text{Ne}}\).

18. **REASONING AND SOLUTION** The reaction can be written as

\[
\frac{A}{Z}\text{X} + ^{63}_{29}\text{Cu} \rightarrow ^{62}_{29}\text{Cu} + ^{1}_{1}\text{H} + ^{0}_{0}\text{n}
\]

Therefore, \(A + 63 = 62 + 1 + 1\), so that we find \(A = 1\). In addition, \(Z + 29 = 29 + 1\), so that \(Z = 1\). Thus, the unknown particle \(\frac{A}{Z}\text{X}\) is a proton \(\frac{1}{1}\text{H}\). Therefore, the nucleus formed temporarily has

\[
Z = 29 + 1 = 30 \quad \text{and} \quad A = 63 + 1 = 64
\]

This nucleus is \(\boxed{\text{zinc, } ^{64}_{30}\text{Zn}}\).
19. **REASONING** Energy is released from this reaction. Consequently, the combined mass of the daughter nucleus $^{12}_6$C and the $\alpha$ particle $^4_2$He is less than the combined mass of the parent nucleus $^{14}_7$N and $^2_1$H. The mass defect is equivalent to the energy released. We proceed by determining the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV (see Section 31.3).

**SOLUTION** The reaction and the atomic masses are as follows:

$$\frac{2}{1}H + \frac{14}{7}N \rightarrow \frac{12}{6}C + \frac{4}{2}He$$

The mass defect $\Delta m$ for this reaction is

$$\Delta m = 2.014102\text{ u} + 14.003074\text{ u} - 12.000000\text{ u} - 4.002603\text{ u} = 0.014573\text{ u}$$

Since 1 u = 931.5 MeV, the energy released is

$$(0.014573\text{ u}) \left(\frac{931.5\text{ MeV}}{1\text{ u}}\right) = 13.6\text{ MeV}$$

20. **REASONING** During the fission reaction, the total electric charge of the nucleons is conserved, so that we can set the total number of protons before the reaction equal to the total number after the reaction. The total number of nucleons is also conserved, so that we can set the total number before the reaction equal to the total number after the reaction. These two conserved quantities will allow us to identify the nucleus $^A_ZX$.

**SOLUTION** The conservation of total charge and total number of nucleons leads to the equations listed below:

<table>
<thead>
<tr>
<th>Conserved Quantity</th>
<th>Before reaction = After Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total electric charge (number of protons)</td>
<td>$92 = 51 + Z$</td>
</tr>
<tr>
<td>Total number of nucleons</td>
<td>$1 + 235 = 133 + A + 4$</td>
</tr>
</tbody>
</table>

Solving the equation for total electric charge reveals that $Z = 92 - 51 = 41$. Solving the equation for total number of nucleons reveals that $A = 1 + 235 - 133 - 4 = 99$. The periodic table indicates that the element with $Z = 41$ is niobium (Nb). Thus, we find that

$$^{41}_{99}X = ^{99}_{41}\text{Nb}$$
21. **REASONING** The rest energy of the uranium nucleus can be found by taking the atomic mass of the $^{235}_{92}$U atom, subtracting the mass of the 92 electrons, and then using the fact that 1 u is equivalent to 931.5 MeV (see Section 31.3). According to Table 31.1, the mass of an electron is $5.485799 \times 10^{-4}$ u. Once the rest energy of the uranium nucleus is found, the desired ratio can be calculated.

**SOLUTION** The mass of $^{235}_{92}$U is 235.043924 u. Therefore, subtracting the mass of the 92 electrons, we have

$$\text{Mass of } ^{235}_{92}\text{U nucleus} = 235.043924 \text{ u} - 92(5.485799 \times 10^{-4} \text{ u}) = 234.993455 \text{ u}$$

The energy equivalent of this mass is

$$\left(234.993455 \text{ u}\right) \left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 2.189 \times 10^5 \text{ MeV}$$

Therefore, the ratio is

$$\frac{200 \text{ MeV}}{2.189 \times 10^5 \text{ MeV}} = 9.0 \times 10^{-4}$$

22. **REASONING** The fission reaction is

$$^0_1n + ^{235}_{92}\text{U} \rightarrow ^{132}_{50}\text{Sn} + ^{101}_{42}\text{Mo} + \eta^0_1n$$

where $\eta$ is the number of neutrons produced. This reaction must satisfy the conservation of nucleon number. Using this conservation law, we will be able to determine $\eta$.

**SOLUTION** The conservation of nucleon number states that the total number of nucleons present before and after the reaction takes place are the same. Therefore, we have

$$\frac{1+235}{\text{Before}} = \frac{132+101+\eta(1)}{\text{After}} \quad \text{or} \quad \eta = 1+235-132-101 = 3$$

23. **REASONING AND SOLUTION** The energy of the neutron after the first collision is $(1.5 \times 10^6 \text{ eV})(0.65)$. After the second collision it is $(1.5 \times 10^6 \text{ eV})(0.65)(0.65)$. Thus, after the $n^{th}$ collision it is

$$(1.5 \times 10^6 \text{ eV})(0.65)^n = 0.040 \text{ eV}$$

Solving for $n$ gives
\[ n \log(0.65) = \log\left( \frac{0.040 \text{ eV}}{1.5 \times 10^6 \text{ eV}} \right) = -7.57 \quad \text{or} \quad n = \frac{-7.57}{\log(0.65)} = 40.4 \]

Therefore, 40 collisions will reduce the energy to something slightly greater than 0.040 eV, and to reduce the energy to at least 0.040 eV, 41 collisions are needed.

24. **REASONING**  According to Equation 6.10b, the power \( P \) is the energy \( E \) being produced divided by the time \( t \):

\[ P = \frac{E}{t} \]

The energy is the number \( N \) of fissions times the energy per fission \( E_{\text{PF}} \), or \( E = NE_{\text{PF}} \). Substituting this result into the expression for power gives

\[ P = \frac{E}{t} = \frac{NE_{\text{PF}}}{t} \tag{1} \]

**SOLUTION** Using Equation (1), we obtain

\[ P = \frac{NE_{\text{PF}}}{t} = \frac{(2.0 \times 10^9)(2.0 \times 10^2 \text{ MeV})}{1.0 \text{ s}} \]

Since we are asked for the power in watts (1 W = 1 J/s), it is necessary to convert the energy units of MeV into joules. For this purpose we note that 1 MeV = 1 \times 10^6 eV and 1 eV = 1.60 \times 10^{-19} J. We find, then, that

\[ P = \frac{(2.0 \times 10^9)(2.0 \times 10^2 \text{ MeV})}{1.0 \text{ s}} \left( \frac{1 \times 10^6 \text{ eV}}{1 \text{ MeV}} \right) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 6.4 \times 10^8 \text{ W} \]

25. **REASONING**  The mass of the two fragments is related to the 225.0 MeV of energy released when the fission reaction occurs. This is because the released energy is determined by the amount by which the mass of the initial species exceeds the mass of the final species:

\[ \frac{1}{0}n + \frac{235}{92}U \rightarrow \text{(2 fragments)} + \frac{3}{0}n \]

We are given the masses of the neutron \( \frac{1}{0}n \) and the uranium \( \frac{235}{92}U \) in atomic mass units (u), and we know that 1 u = 931.5 MeV.

**SOLUTION**  The amount \( \Delta m \) by which the mass of the initial species exceeds the mass of the final species is
\[ \Delta m = m_{\text{neutron}} + m_{\text{uranium}} - m_{\text{fragments}} - 3m_{\text{neutron}} \]  

(1)

where \( m_{\text{neutron}} \) is the mass of a single neutron. Solving Equation (1) for \( m_{\text{fragments}} \) gives

\[ m_{\text{fragments}} = m_{\text{uranium}} - 2m_{\text{neutron}} - \Delta m \]  

(2)

In Equation (2) the term \( \Delta m \) can be determined from the 225.0 MeV of energy released when the fission reaction occurs by using the fact that \( 1 \text{ u} = 931.5 \text{ MeV} \):

\[ \Delta m = (225.0 \text{ MeV}) \left( \frac{1 \text{ u}}{931.5 \text{ MeV}} \right) = 0.2415 \text{ u} \]  

(3)

Substituting Equation (3) and the masses of \( ^1_0 \text{n} \) and \( ^{235}_{92} \text{U} \) into Equation (2), we find that

\[ m_{\text{fragments}} = m_{\text{uranium}} - 2m_{\text{neutron}} - \Delta m \]

\[ = (235.043 924 \text{ u}) - 2(1.008 665 \text{ u}) - (0.2415 \text{ u}) = 232.7851 \text{ u} \]

26. **REASONING** We first determine the energy released by 1.0 kg of \( ^{235}_{92} \text{U} \). Using the data given in the problem statement, we can then determine the number of kilograms of coal that must be burned to produce the same energy.

**SOLUTION** The energy equivalent of one atomic mass unit is given in the text (see Section 31.3) as

\[ 1 \text{ u} = 1.4924 \times 10^{-10} \text{ J} = 931.5 \text{ MeV} \]

Therefore, the energy released in the fission of 1.0 kg of \( ^{235}_{92} \text{U} \) is

\[
(1.0 \text{ kg of } ^{235}_{92} \text{U}) \left( \frac{1.0 \times 10^3 \text{ g/kg}}{235 \text{ g/mol}} \right) \left( \frac{6.022 \times 10^{23} \text{ nuclei}}{1.0 \text{ mol}} \right) \times \left( \frac{2.0 \times 10^2 \text{ MeV}}{\text{nuclei}} \right) \left( \frac{1.4924 \times 10^{-10} \text{ J}}{931.5 \text{ MeV}} \right) = 8.2 \times 10^{13} \text{ J}
\]

When 1.0 kg of coal is burned, about \( 3.0 \times 10^7 \text{ J} \) is released; therefore the number of kilograms of coal that must be burned to produce an energy of \( 8.2 \times 10^{13} \text{ J} \) is

\[ m_{\text{coal}} = \left( 8.2 \times 10^{13} \text{ J} \right) \left( \frac{1.0 \text{ kg}}{3.0 \times 10^7 \text{ J}} \right) = 2.7 \times 10^6 \text{ kg} \]
27. **SSM REASONING AND SOLUTION** The binding energy per nucleon for a nucleus with $A = 239$ is about 7.6 MeV per nucleon, according to Figure 32.9. The nucleus fragments into two pieces of mass ratio 0.32 : 0.68. These fragments thus have nucleon numbers:

$$A_1 = (0.32)(A) = 76 \quad \text{and} \quad A_2 = (0.68)(A) = 163$$

Using Figure 32.9 we can estimate the binding energy per nucleon for $A_1$ and $A_2$. We find that the binding energy per nucleon for $A_1$ is about 8.8 MeV, representing an increase of $8.8 \text{ MeV} - 7.6 \text{ MeV} = 1.2 \text{ MeV per nucleon}$. Since there are 76 nucleons present, the energy released for $A_1$ is

$$76 \times 1.2 \text{ MeV} = 91 \text{ MeV}$$

Similarly, for $A_2$, we see that the binding energy increases to 8.0 MeV per nucleon, the difference being $8.0 \text{ MeV} - 7.6 \text{ MeV} = 0.4 \text{ MeV per nucleon}$. Since there are 163 nucleons present, the energy released for $A_2$ is

$$163 \times 0.4 \text{ MeV} = 70 \text{ MeV}$$

The energy released per fission is $E = 91 \text{ MeV} + 70 \text{ MeV} = 160 \text{ MeV}$. 

28. **REASONING** A mass $m$ of water that passes through the core absorbs an amount of heat $Q$. According to Equation 12.4, $Q = cm\Delta T$, where $c = 4420 \text{ J/(kg} \cdot \text{C}^\circ\text{)}$ is the specific heat capacity of the water and $\Delta T = T - T_0$ is the difference between the final temperature $T = 287 \text{ °C}$ and initial temperature $T_0 = 216 \text{ °C}$ of the water. We will use $P = \frac{Q}{t}$ (Equation 6.10b) to determine the amount $Q$ of heat that the water must absorb from the core in a time $t$ in order to keep the core from heating up, where $P = 5.6 \times 10^9 \text{ W}$ is the thermal output of the core. These relations will allow us to determine the mass of water that passes through the core each second.

**SOLUTION** Solving $Q = cm\Delta T$ (Equation 12.4) for $m$ yields

$$m = \frac{Q}{c\Delta T} \quad \text{(1)}$$

Solving $P = \frac{Q}{t}$ (Equation 6.10b) for $Q$ yields $Q = Pt$. Substituting this result into Equation (1), we find that

$$m = \frac{Q}{c\Delta T} = \frac{Pt}{c\Delta T} \quad \text{(2)}$$

With $t = 1.0 \text{ s}$, we obtain the mass of water that passes through the core each second:
29. **REASONING**  We first determine the total power generated (used and wasted) by the plant. Energy is power times the time, according to Equation 6.10, and given the energy, we can determine how many kilograms of $^{235}_{92}U$ are fissioned to produce this energy.

**SOLUTION** Since the power plant produces energy at a rate of $8.0 \times 10^8$ W when operating at 25% efficiency, the total power produced by the power plant is

$$\left(8.0 \times 10^8 \text{ W}\right)4 = 3.2 \times 10^9 \text{ W}$$

The energy equivalent of one atomic mass unit is given in the text (see Section 31.3) as

$$1 \text{ u} = 1.4924 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$$

Since each fission produces $2.0 \times 10^2$ MeV of energy, the total mass of $^{235}_{92}U$ required to generate $3.2 \times 10^9$ W for a year ($3.156 \times 10^7$ s) is

$$\frac{3.2 \times 10^9 \text{ J/s}}{3.156 \times 10^7 \text{ s}} \times \frac{931.5 \text{ MeV}}{1.4924 \times 10^{-10} \text{ J}} \times \frac{1.0}{2.0 \times 10^2 \text{ MeV}} \times \frac{0.235 \text{ kg}}{6.022 \times 10^{23} \text{ $^{235}_{92}U$ nuclei}} = 1200 \text{ kg}$$

30. **REASONING** Each generation of fissions lasts for an average time of $t_{avg} = 1.2 \times 10^{-8}$ s, and each generation has a higher fission rate and power output than the previous generation. The total time $T$ it takes for the power output to reach the stated level is equal to the number $N$ of generations times the average time of a generation:

$$T = N t_{avg}$$  \hfill (1)$$

The power output $P$ from a single generation of fissions is directly proportional to the number of fissions in that generation. Thus, the power output in the critical state is $25 \times 10^3 \text{ W} = P_0 = K(1.00)$, where $K$ is the proportionality constant. The power output of the first generation of fissions in the supercritical state is $P_1 = K(1.00)(1.01) = 1.01 P_0$. The
power output of the second generation of fissions in the supercritical state is
\[ P_2 = K(1.00)(1.01)(1.01) = (1.01)^2 P_0. \]
We can see, then, that the power output of the \( N \)th generation of fissions in the supercritical state is
\[ P_N = (1.01)^N P_0 \]  
(2)
where \( P_N = 3300 \text{ MW} = 3300\times10^6 \text{ W}. \) Equations (1) and (2) will allow us to determine the total elapsed time \( T. \)

**SOLUTION** Solving for the term in Equation (2) containing \( N, \) we obtain
\[ (1.01)^N = \frac{P_N}{P_0} \]  
(3)
Taking the natural logarithm of both sides of Equation (3) and solving for \( N \) yields
\[ N \ln (1.01) = \ln \left( \frac{P_N}{P_0} \right) \quad \text{or} \quad N = \frac{\ln \left( \frac{P_N}{P_0} \right)}{\ln (1.01)} \]  
(4)
Substituting Equation (4) into Equation (1), we find that
\[ T = N t_{\text{avg}} = \frac{\ln \left( \frac{P_N}{P_0} \right) t_{\text{avg}}}{\ln (1.01)} = \frac{\ln \left( \frac{3300\times10^6 \text{ W}}{25\times10^3 \text{ W}} \right)(1.2\times10^{-8} \text{ s})}{\ln (1.01)} = 1.4\times10^{-5} \text{ s} \]

31. **REASONING** The energy released can be found from the mass defect of the reaction. According to the discussion in Sections 31.3 and 31.4, the mass defect is equal to the sum of the individual masses before the reaction minus the sum of the masses after the reaction. Since the mass of each atom is given in atomic mass units (u), we can find the energy released (in MeV) from the mass defect by using the relation \( 1 \text{ u} = 931.5 \text{ MeV}. \)

**SOLUTION** The reaction is
\[ _1^2 \text{H} + _1^2 \text{H} \rightarrow _1^3 \text{H} + _1^1 \text{H} \]
\[ 2.014\text{102 u} \quad 2.014\text{102 u} \quad 3.016\text{050 u} \quad 1.007\text{825 u} \]
The sum of the masses before the reaction is \( 2.014\text{102 u} + 2.014\text{102 u} = 4.028\text{204 u}. \) The sum of the masses after the reaction is \( 3.016\text{050 u} + 1.007\text{825 u} = 4.023\text{875 u}. \) The mass defect is
\[ \Delta m = 4.028\text{204 u} - 4.023\text{875 u} = 0.004\text{329 u} \]
The energy released (in MeV) is
\[ \text{Energy} = (0.004\text{329 u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 4.03 \text{ MeV} \]
32. **REASONING** If energy is released during a reaction, the total rest energy of the particles after the reaction must be less than the total rest energy before the reaction. Since energy and mass are equivalent, the total mass of the particles after the reaction is less than the total mass of the particles before the reaction.

According to Einstein’s relation between mass and energy, Equation 28.5, the difference $\Delta m$ in total masses is related to the energy $\Delta E_0$ released by the reaction by $\Delta m = \Delta E_0/c^2$, where $c$ is the speed of light in a vacuum.

**SOLUTION** The difference between the total mass before the reaction and the total mass after the reaction is

$$\Delta m = \frac{1.0078 \text{ u} + 1.0087 \text{ u}}{\text{Total mass before reaction}} - \frac{2.0141 \text{ u}}{\text{Total mass after reaction}} = 0.0024 \text{ u}$$

Note that the $\gamma$-ray photon has no rest mass, so that we can ignore it in our calculations. Since $1 \text{ u} = 931.5 \text{ MeV}$, the energy released is

$$E = (0.0024 \text{ u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 2.2 \text{ MeV}$$

33. **REASONING** The reaction is

$$\begin{align*}
\text{\textsuperscript{2}}\text{H} + \text{\textsuperscript{2}}\text{H} &\rightarrow \text{\textsuperscript{3}}\text{He} + \text{\textsuperscript{1}}\text{n} \\
2.0141 \text{ u} + 2.0141 \text{ u} &\rightarrow 3.0160 \text{ u} + 1.0087 \text{ u}
\end{align*}$$

The energy produced by a fusion reaction is the mass defect $\Delta m$ (in u) for the reaction times $931.5 \text{ MeV/u}$, since $931.5 \text{ MeV}$ is the energy equivalent of 1 u. The mass defect is the total mass of the initial nuclei minus the total mass of the product nuclei.

**SOLUTION** Using the given masses, we obtain

$$\text{Energy} = (\Delta m) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right)$$

$$= (2.0141 \text{ u} + 2.0141 \text{ u} - 3.0160 \text{ u} - 1.0087 \text{ u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 3.3 \text{ MeV}$$

34. **REASONING** The conservation of nucleon number states that the total number of nucleons (protons plus neutrons) before the reaction occurs must be equal to the total number of nucleons after the reaction. This conservation law will allow us to find the atomic mass number $A$ of the unknown particle Y. The conservation of electric charge states that the net electric charge of the particles before the reaction must be equal to the net charge after the reaction. This conservation law will allow us to find the atomic number $Z$ of the unknown particle X.
**SOLUTION**

a. The total number of nucleons before the reaction is \(1 + A\). The total number of nucleons after the reaction is 3. Setting these two numbers equal to each other yields \(A = 2\). The net electric charge before the reaction is \(Z + 1\). The net electric charge after the reaction is 1. Setting these two numbers equal to each other yields \(Z = 0\). The nucleon \(\frac{1}{2}X = \frac{1}{0}n\) is a neutron. The nucleon \(\frac{4}{1}Y = \frac{2}{1}H\) is a deuterium nucleus.

b. The sum of the atomic masses before the reaction is \(1.0087 \text{ u} + 2.0141 \text{ u} = 3.0228 \text{ u}\). The sum of the atomic mass after the reaction is 3.0161 \text{ u}. The difference between the sums is \(3.0228 \text{ u} – 3.0161 \text{ u} = 0.0067 \text{ u}\). This mass difference is equivalent to an energy of

\[
(0.0067 \text{ u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 6.2 \text{ MeV}
\]

35. **SSM REASONING** To find the energy released per reaction, we follow the usual procedure of determining how much the mass has decreased because of the fusion process. Once the energy released per reaction is determined, we can determine the mass of lithium \(\frac{6}{3}\text{Li}\) needed to produce \(3.8 \times 10^{10} \text{ J}\).

**SOLUTION** The reaction and the masses are shown below:

\[
\frac{2}{1}H + \frac{6}{3}\text{Li} \rightarrow \frac{2}{4}\text{He} + \frac{4}{2}\text{He}
\]

The mass defect is, therefore, \(2.014 \text{ u} + 6.015 \text{ u} – 2(4.003 \text{ u}) = 0.023 \text{ u}\). Since 1 u is equivalent to 931.5 MeV, the released energy is 21 MeV, or since the energy equivalent of one atomic mass unit is given in Section 31.3 as 1 u = 1.4924 \times 10^{-10} \text{ J} = 931.5 \text{ MeV},

\[
(21 \text{ MeV}) \left( \frac{1.4924 \times 10^{-10} \text{ J}}{931.5 \text{ MeV}} \right) = 3.4 \times 10^{-12} \text{ J}
\]

In 1.0 kg of lithium \(\frac{6}{3}\text{Li}\), there are

\[
(1.0 \text{ kg of } \frac{6}{3}\text{Li}) \left( \frac{1.0 \times 10^3 \text{ g}}{1.0 \text{ kg}} \right) \left( \frac{6.022 \times 10^{23} \text{ nuclei/mol}}{6.015 \text{ g/mol}} \right) = 1.0 \times 10^{26} \text{ nuclei}
\]

Therefore, 1.0 kg of lithium \(\frac{6}{3}\text{Li}\) would produce an energy of
(3.4 \times 10^{-12} \text{ J/nuclei})(1.0 \times 10^{26} \text{ nuclei}) = 3.4 \times 10^{14} \text{ J}

If the energy needs of one household for a year is estimated to be $3.8 \times 10^{10} \text{ J}$, then the amount of lithium required is

$$\frac{3.8 \times 10^{10} \text{ J}}{3.4 \times 10^{14} \text{ J/kg}} = 1.1 \times 10^{-4} \text{ kg}$$

36. **REASONING** The conservation of linear momentum states that the total momentum of an isolated system remains constant. Here, we have the following fusion reaction:

$$^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0\text{n}$$

in which the two initial nuclei are assumed to be at rest. Therefore, the initial total momentum before the fusion occurs is zero, since the momentum vector is the mass times the velocity vector. As a result of momentum conservation, we can conclude that the final total momentum of the neutron $^1_0n$ and the $^4_2\text{He}$ particle is also zero. This means that the momentum vector of the neutron points opposite to the momentum vector of the $^4_2\text{He}$ particle and each has the same magnitude. Using $p$ to denote the magnitude of the momentum, we have that

$$p_n = p_\alpha \quad (1)$$

The magnitude $p$ of the momentum is the mass $m$ times the speed $v$, since we are ignoring relativistic effects. Thus, according to Equation (1), we have

$$m_n v_n = m_\alpha v_\alpha \quad \text{or} \quad v_n = v \frac{m_\alpha}{m_n}$$

From the masses given along with the reaction, we see that $m_\alpha > m_n$. Therefore, the speed of the neutron is greater than the speed of the $^4_2\text{He}$ particle.

Using the facts that $p = mv$ and $\text{KE} = \frac{1}{2}mv^2$, we have

$$\text{KE} = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \quad (2)$$

The total energy released in the reaction is $E$, and it is all in the form of kinetic energy of the two product nuclei. Therefore, it follows that

$$E = \text{KE}_\alpha + \text{KE}_n \quad \text{or} \quad \text{KE}_n = E - \text{KE}_\alpha \quad (3)$$
**SOLUTION** Using Equation (2) to substitute into Equation (3), we have

\[
KE_n = E - KE_\alpha = E - \frac{p_\alpha^2}{2m_\alpha}
\]  

(4)

But \(p_n = p_\alpha\) according to Equation (1), so that Equation (4) becomes

\[
KE_n = E - \frac{p_n^2}{2m_\alpha}
\]

(5)

According to Equation (2), \(p_n^2 = 2m_n (KE_n)\), so that Equation (5) can be rewritten as

\[
KE_n = E - \frac{p_n^2}{2m_\alpha} = E - \frac{2m_n (KE_n)}{2m_\alpha} = E - \frac{m_n (KE_n)}{m_\alpha}
\]

Solving for \(KE_n\) reveals that

\[
KE_n = \frac{E}{1 + m_n / m_\alpha} = \frac{17.6 \text{ MeV}}{1 + (1.0087 \text{ u})/(4.0026 \text{ u})} = 14.1 \text{ MeV}
\]

Using Equation (3), we find that

\[
KE_\alpha = E - KE_n = 17.6 \text{ MeV} - 14.1 \text{ MeV} = 3.5 \text{ MeV}
\]

The particle with the greater kinetic energy has the greater speed. Thus, the neutron has the greater speed, as expected.

---

37. **REASONING AND SOLUTION**

a. The number \(n\) of atoms of hydrogen and its isotopes in 1 kg of water (molecular mass = 18 u) is

\[
n = \frac{2(6.02 \times 10^{23}/\text{mol})(1.00 \times 10^3 \text{ g})}{18.015 \text{ g/mol}} = 6.68\times10^{25}
\]

If deuterium makes up 0.015% of hydrogen in this number of atoms, the number \(N\) of deuterium atoms in 1 kg of water is

\[
N = (1.5 \times 10^{-4})(6.68 \times 10^{25}) = \boxed{1.0\times10^{22}}
\]

b. Each deuterium nucleus provides 7.2 MeV of energy, so the energy from 1 kg of water is

\[
E = (1.0\times10^{22})(7.2\times10^6 \text{ eV})\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) = 1.15\times10^{10} \text{ J}
\]
To supply $1.1 \times 10^{20}$ J of energy, we would need

$$m = \frac{1.1 \times 10^{20} \text{ J}}{1.15 \times 10^{10} \text{ J/kg}} = 9.6 \times 10^9 \text{ kg}$$

38. **REASONING AND SOLUTION** We will use the atomic mass of hydrogen $^1\text{H}$ (1.007 825 u), deuterium $^2\text{H}$ (2.014 102 u), helium $^3\text{He}$ (3.016 030 u), helium $^4\text{He}$ (4.002 603 u), a positron $^0\text{e}$ (0.000 549 u), and an electron (0.000 549 u). The mass defect $\Delta m$ for two reactions of type (1) is

$$\Delta m = 2(1.007 \, 825 \, \text{u} + 1.007 \, 825 \, \text{u} - 2.014 \, 102 \, \text{u} - 0.000 \, 549 \, \text{u} - 0.000 \, 549 \, \text{u}) = 0.000 \, 900 \, \text{u}$$

In this result, one of the two values of 0.000 549 u accounts for the positron. The other is present because the two hydrogen atoms contain a total of two electrons, whereas a single deuterium atom contains only one electron. The mass defect for two reactions of type (2) is

$$\Delta m = 2(1.007 \, 825 \, \text{u} + 2.014 \, 102 \, \text{u} - 3.016 \, 030 \, \text{u}) = 0.011 \, 794 \, \text{u}$$

The mass defect for one reaction of type (3) is

$$\Delta m = 3.016 \, 030 \, \text{u} + 3.016 \, 030 \, \text{u} - 4.002 \, 603 \, \text{u} - 1.007 \, 825 \, \text{u} - 1.007 \, 825 \, \text{u} = 0.013 \, 807 \, \text{u}$$

The total mass defect for the proton-proton cycle is the sum of three values just calculated:

$$\Delta m_{\text{total}} = 0.000 \, 900 \, \text{u} + 0.011 \, 794 \, \text{u} + 0.013 \, 807 \, \text{u} = 0.026 \, 501 \, \text{u}$$

Since 1 u = 931.5 MeV, the energy released is

$$(0.026 \, 501 \, \text{u}) \left(\frac{931.5 \, \text{MeV}}{1 \, \text{u}}\right) = 24.7 \, \text{MeV}$$

39. **REASONING** The energy released in the decay is the difference between the rest energy of the $\pi^-$ and the sum of the rest energies of the $\mu^-$ and $\bar{\nu}_\mu$. The rest energies of $\pi^-$ and $\mu^-$ can be found in Table 32.3, and the rest energy of $\bar{\nu}_\mu$ is approximately zero.

**SOLUTION** The rest energies of the particles are $\pi^- (139.6 \, \text{MeV})$, $\mu^- (105.7 \, \text{MeV})$ and $\bar{\nu}_\mu (\approx 0 \, \text{MeV})$. The energy released is $139.6 \, \text{MeV} - 105.7 \, \text{MeV} = 33.9 \, \text{MeV}$. 
40. **REASONING** The energies \( E_1, E_2 \) of the \( \gamma \)-ray photons come from the total relativistic energy \( E \) of the neutral pion, which loses both its rest energy and its kinetic energy when it disintegrates. Therefore, we have that

\[
E_1 + E_2 = E
\]  

(1)

The total relativistic energy of the neutral pion is found from \( E = mc^2 \sqrt{1 - \frac{v^2}{c^2}} \) (Equation 28.4), where \( c = 3.00 \times 10^8 \) m/s is the speed of light in a vacuum, \( v = 0.780c \) is the speed of the neutral pion, and \( mc^2 = E_0 = 135.0 \) MeV (Equation 28.5) is the rest energy of the neutral pion.

**SOLUTION** Solving Equation (1) for \( E_2 \), we obtain

\[
E_2 = E - E_1
\]  

(2)

Substituting \( E = mc^2 \sqrt{1 - \frac{v^2}{c^2}} \) (Equation 28.4) into Equation (2) yields

\[
E_2 = E - E_1 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - E_1 = \frac{135.0 \text{ MeV}}{\sqrt{1 - (0.780c)^2}} - 192 \text{ MeV} = 24 \text{ MeV}
\]

41. **REASONING** The \( K^- \) particle has a charge of \( -e \) and contains one quark and one antiquark. Therefore, the charge on the quark and the charge on the antiquark must add to give a total of \( -e \). Any quarks or antiquarks that cannot possibly lead to this charge are the ones we seek.

**SOLUTION**

a. Any quark that has a charge of \( \frac{2}{3}e \) (\( u \), \( c \), or \( t \)) cannot be present in the \( K^- \) particle, because if it were, then the antiquark that is present would need to have a charge of \( -\frac{5}{3}e \) to give a total charge of \( -e \). Since there are no antiquarks with a charge of \( -\frac{5}{3}e \), we conclude that

the \( K^- \) particle does not contain \( u \), \( c \), or \( t \) quarks

b. Any antiquark that has a charge of \( \frac{1}{3}e \) (\( \bar{u} \), \( \bar{c} \), or \( \bar{t} \)) cannot be present in the \( K^- \) particle, because if it were, then the quark that is present would need to have a charge of
$-\frac{4}{3}e$ to give a total charge of $-e$. Since there are no quarks with a charge of $-\frac{4}{3}e$, we conclude that

the $K^-$ particle does not contain $\bar{d}$, $\bar{s}$, or $\bar{b}$ antiquarks.

Note: the $K^-$ particle contains the $s$ quark and the $\bar{u}$ antiquark.

42. **REASONING** The given reaction is an example of the conversion of energy into matter, and when this occurs, energy is conserved. Therefore, we will approach this problem by setting the total energy of the two protons equal to the total energy of the resulting neutron, proton, and pion. As a guide, we show here the reaction and the energy of each participating particle:

\[
p' + p \rightarrow n + p + \pi^+
\]

**SOLUTION** Applying the principle of conservation of energy to the given reaction, we find

\[
(938.3 \text{ MeV}) + \text{KE} + (938.3 \text{ MeV}) = (939.6 \text{ MeV}) + (938.3 \text{ MeV}) + (139.6 \text{ MeV})
\]

Solving this result for KE, we obtain

\[
\text{KE} = (939.6 \text{ MeV}) + (938.3 \text{ MeV}) + (139.6 \text{ MeV})
\]

\[
- (938.3 \text{ MeV}) - (938.3 \text{ MeV}) = 140.9 \text{ MeV}
\]

43. **SSM REASONING** The momentum of a photon is given in the text as $p = E/c$ (see the discussion leading to Equation 29.6). This expression applies to any massless particle that travels at the speed of light. In particular, assuming that the neutrino has no mass and travels at the speed of light, it applies to the neutrino. Once the momentum of the neutrino is determined, the de Broglie wavelength can be calculated from Equation 29.6 ($p = h/\lambda$).

**SOLUTION**

a. The momentum of the neutrino is, therefore,

\[
p = \frac{E}{c} = \left( \frac{35 \text{ MeV}}{3.00 \times 10^8 \text{ m/s}} \right) \left( \frac{1.4924 \times 10^{-10} \text{ J}}{931.5 \text{ MeV}} \right) = 1.9 \times 10^{-20} \text{ kg} \cdot \text{m/s}
\]

where we have used the fact that $1.4924 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$ (see Section 31.3).
b. According to Equation 29.6, the de Broglie wavelength of the neutrino is

\[ \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.9 \times 10^{-20} \text{ kg} \cdot \text{m/s}} = 3.5 \times 10^{-14} \text{ m} \]

44. **REASONING** The kinetic energies of the electron and its antiparticle are negligible, so we may assume that they are at rest before their annihilation. Therefore, the initial total linear momentum of the system of the two particles is zero. Assuming that the system is isolated, the principle of conservation of linear momentum holds and indicates that the final total momentum of the system (that of the two \( \gamma \)-ray photons) is also zero. Since photons cannot be at rest, the only way for the total momentum of the two photons to be zero is for the photons to have momenta of equal magnitudes and opposite directions. We conclude, then, that each photon has a momentum of magnitude \( p = \frac{E}{c} \) (Equation 29.6), where \( E \) is the energy of either photon and \( c \) is the speed of light in a vacuum.

Before annihilation, each of the two particles has a negligible amount of kinetic energy, so the only energy each possesses is its rest energy \( E_0 = mc^2 \) (Equation 28.5), where \( m \) is the mass of an electron (and also the mass of its antiparticle). From the energy conservation principle, we know that the total rest energy \( 2E_0 \) of the electron and its antiparticle is equal to the total energy \( 2E \) of the two \( \gamma \)-ray photons. Therefore, we have that \( 2E_0 = 2E \), or

\[ E_0 = E = mc^2 \quad (1) \]

**SOLUTION** Substituting Equation (1) into \( p = \frac{E}{c} \) (Equation 29.6), we obtain

\[ p = \frac{E}{c} = \frac{mc^2}{c} = mc = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s}) = 2.73 \times 10^{-22} \text{ kg} \cdot \text{m/s} \]

45. **REASONING AND SOLUTION** In order for the proton to come within a distance \( r \) of the second proton, it must overcome the Coulomb repulsive force. Therefore, the kinetic energy of the incoming proton must equal the Coulombic potential energy of the system.

\[ \text{KE} = \text{EPE} = \frac{ke^2}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{8.0 \times 10^{-15} \text{ m}} = 2.9 \times 10^{-14} \text{ J} \]

\[ \text{KE} = (2.9 \times 10^{-14} \text{ J}) \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = 1.8 \times 10^5 \text{ eV} = 0.18 \text{ MeV} \]
46. **REASONING** According to Equation 32.4, the biologically equivalent dose is equal to the product of the absorbed dose and the RBE (relative biological effectiveness).

If the absorbed doses are the same, then the radiation with the larger RBE produces the greater biological effect.

If the two types of radiation have the same RBE, then the radiation that produces the greater absorbed dose produces the greater biological effect.

**SOLUTION** Using Equation 32.4, the biologically equivalent dose can be determined for each type of radiation. The rankings are given in the last column of the table.

<table>
<thead>
<tr>
<th>Radiation</th>
<th>Biologically Equivalent Dose = (Absorbed dose)(RBE)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ rays</td>
<td>((20 \times 10^{-3} \text{ rad})(1) = 20 \times 10^{-3} \text{ rem})</td>
<td>3</td>
</tr>
<tr>
<td>Electrons</td>
<td>((30 \times 10^{-3} \text{ rad})(1) = 30 \times 10^{-3} \text{ rem})</td>
<td>2</td>
</tr>
<tr>
<td>Protons</td>
<td>((5 \times 10^{-3} \text{ rad})(10) = 50 \times 10^{-3} \text{ rem})</td>
<td>1</td>
</tr>
<tr>
<td>Slow neutrons</td>
<td>((5 \times 10^{-3} \text{ rad})(2) = 10 \times 10^{-3} \text{ rem})</td>
<td>4</td>
</tr>
</tbody>
</table>

47. **SSM REASONING** The lambda particle contains three different quarks, one of which is the up quark \(u\), and contains no antiquarks. Therefore, the remaining two quarks must be selected from the down quark \(d\), the strange quark \(s\), the charmed quark \(c\), the top quark \(t\), and the bottom quark \(b\). Choosing among these possibilities must be done consistent with the fact that the lambda particle has an electric charge of zero.

**SOLUTION** Since the lambda particle has an electric charge of zero and since \(u\) has a charge of \(+2e/3\), the charges of the remaining two quarks must add up to a total charge of \(-2e/3\). This eliminates the quarks \(c\) and \(t\) as choices, because they each have a charge of \(+2e/3\). We are left, then, with \(d\), \(s\), and \(b\) as choices for the remaining two quarks in the lambda particle. The three possibilities are as follows:

\[
(1) \ u,d,s \quad (2) \ u,d,b \quad (3) \ u,s,b
\]

48. **REASONING** The reaction specified is

\[
\frac{2}{1} \text{H} + \frac{22}{11} \text{Na} \rightarrow \frac{4}{2} X + \frac{4}{2} \text{He}
\]

This reaction must satisfy the conservation of nucleon number and the conservation of electric charge. Using these laws, we will be able to identify the unknown species \(\frac{4}{2} X\).
**SOLUTION** The conservation of nucleon number states that the total number of nucleons present before and after the reaction takes place are the same. Therefore, we have

\[
2 + 22 = A + 4 \quad \text{or} \quad A = 2 + 22 - 4 = 20
\]

The conservation of electric charge states that the total number of protons present before and after the reaction takes place are the same

\[
1 + 11 = Z + 2 \quad \text{or} \quad Z = 1 + 11 - 2 = 10
\]

Therefore, with \(A = 20\) and \(Z = 10\), we consult the periodic table on the inside of the back cover and find that the unknown species \(^{20}_{10}X\) is the nucleus of neon \(^{20}_{10}\text{Ne}\).

49. **REASONING** The energy released can be found from the mass defect of the reaction. According to the discussion in Sections 31.3 and 31.4, the mass defect is equal to the sum of the individual masses before the reaction minus the sum of the masses after the reaction. Since the mass of each nucleus is given in atomic mass units (u), we can find the energy released (in MeV) from the mass defect by using the relation \(1 \text{ u} = 931.5 \text{ MeV}\).

**SOLUTION** The reaction is

\[
_{6}^{1}\text{n} + _{92}^{235}\text{U} \rightarrow _{54}^{140}\text{Xe} + _{38}^{94}\text{Sr} + 2_{0}^{1}\text{n}
\]

The sum of the masses before the reaction is 1.009 u + 235.044 u = 236.053 u. The sum of the masses after the reaction is 139.922 u + 93.915 u + 2(1.009 u) = 235.855 u. The mass defect is

\[
\Delta m = 236.053 \text{ u} - 235.855 \text{ u} = 0.198 \text{ u}
\]

The energy released (in MeV) is

\[
\text{Energy} = (0.198 \text{ u}) \left( \frac{931.5 \text{ MeV}}{1 \text{ u}} \right) = 184 \text{ MeV}
\]

50. **REASONING** According to Equation 32.2, the absorbed dose is the energy absorbed divided by the mass of absorbing material:

\[
\text{Absorbed dose} = \frac{\text{Energy absorbed}}{\text{Mass of absorbing material}}
\]

The energy absorbed in this case is the sum of three terms: (1) the heat needed to melt a mass \(m\) of ice at 0.0 °C into liquid water at 0.0 °C, which is \(mL_{f}\), according to the definition of the latent heat of fusion \(L_{f}\) (see Section 12.8); (2) the heat needed to raise the temperature
of liquid water by an amount $\Delta T$, which is $cm\Delta T$, where $c$ is the specific heat capacity and $\Delta T$ is the change in temperature from 0.0 to 100.0 °C, according to Equation 12.4; (3) the heat needed to vaporize liquid water at 100.0 °C into steam at 100.0 °C, which is $mL_v$, according to the definition of the latent heat of vaporization $L_v$ (see Section 12.8). Once the energy absorbed is determined, the absorbed dose can be determined using Equation 32.2.

SOLUTION Using the value of $c = 4186$ J/(kg·C°) for liquid water from Table 12.2 and the values of $L_f = 33.5 \times 10^4$ J/kg and $L_v = 22.6 \times 10^5$ J/kg from Table 12.3, we find that

$$\text{Absorbed dose in grays} = \frac{\text{Energy}}{\text{mass}} = \frac{mL_f + cm\Delta T + mL_v}{m} = L_f + c\Delta T + L_v$$

$$= 33.5 \times 10^4 \text{ J/kg} + \left[\frac{4186 \text{ J/(kg·C°)}}{(100.0 \text{ C°})}\right](100.0 \text{ C°}) + 22.6 \times 10^5 \text{ J/kg}$$

$$= 3.01 \times 10^6 \text{ J/kg}$$

Using the fact that 0.01 Gy = 1 rad, we find that

$$\text{Absorbed dose} = \left(3.01 \times 10^6 \text{ Gy}\right)\left(\frac{1 \text{ rad}}{0.01 \text{ Gy}}\right) = 3.01 \times 10^8 \text{ rad}$$

51. [SSM] REASONING To find the energy released per reaction, we follow the usual procedure of determining how much the mass has decreased because of the fusion process. Once the energy released per reaction is determined, we can determine the amount of gasoline that must be burned to produce the same amount of energy.

SOLUTION The reaction and the masses are shown below:

$$\begin{align*}
\frac{3}{2} \text{H} & \rightarrow \frac{4}{2} \text{He} + \frac{1}{1} \text{H} + \frac{1}{0} \text{n}
\end{align*}$$

The mass defect is, therefore,

$$3(2.0141 \text{ u}) - 4.0026 \text{ u} - 1.0078 \text{ u} - 1.0087 \text{ u} = 0.0232 \text{ u}$$

Since 1 u is equivalent to 931.5 MeV, the released energy is 21.6 MeV, or since it is shown in Section 31.3 that $931.5 \text{ MeV} = 1.4924 \times 10^{-10} \text{ J}$, the energy released per reaction is

$$\left(21.6 \text{ MeV}\right)\left(\frac{1.4924 \times 10^{-10} \text{ J}}{931.5 \text{ MeV}}\right) = 3.46 \times 10^{-12} \text{ J}$$

To find the total energy released by all the deuterium fuel, we need to know the number of deuterium nuclei present. Noting that $6.1 \times 10^{-6} \text{ kg} = 6.1 \times 10^{-3} \text{ g}$, we find that the number of deuterium nuclei is
(6.1 \times 10^{-3} \text{ g}) \left( \frac{6.022 \times 10^{23} \text{ nuclei/mol}}{2.0141 \text{ g/mol}} \right) = 1.8 \times 10^{21} \text{ nuclei}

Since each reaction consumes three deuterium nuclei, the total energy released by the deuterium fuel is

\frac{1}{3} (3.46 \times 10^{-12} \text{ J/nuclei})(1.8 \times 10^{21} \text{ nuclei}) = 2.1 \times 10^9 \text{ J}

If one gallon of gasoline produces 2.1 \times 10^9 \text{ J} of energy, then the number of gallons of gasoline that would have to be burned to equal the energy released by all the deuterium fuel is

\left( 2.1 \times 10^9 \text{ J} \right) \left( \frac{1.0 \text{ gal}}{2.1 \times 10^9 \text{ J}} \right) = 1.0 \text{ gal}

52. **REASONING**  The mass of a single uranium $^{235}_{92}$U atom is $m = 235 \text{ u}$ (see Appendix F), where $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$. The total mass $M$ of uranium $^{235}_{92}$U consumed is the mass $m$ multiplied by the number $N$ of uranium atoms:

$$M = Nm \quad (1)$$

To determine the number $N$ of uranium $^{235}_{92}$U atoms needed to generate the total energy output $E_{tot}$ of the United States in one year, we will divide $E_{tot}$ by the energy $E = 2.0 \times 10^2 \text{ MeV}$ released by a single fission:

$$N = \frac{E_{tot}}{E} \quad (2)$$

Because $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, we have that

$$1 \text{ MeV} = \left(1.0 \times 10^6 \text{ eV} \right) \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} = 1.6 \times 10^{-13} \text{ J}$$

**SOLUTION** Substituting Equation (2) into Equation (1) yields

$$M = Nm = \frac{E_{tot} m}{E} \quad (3)$$

Substituting the given values and the conversion factors into Equation (3), we obtain the mass of uranium $^{235}_{92}$U that would be needed to supply the energy output of the United States for one year:

$$M = \frac{E_{tot} m}{E} = \left(1.1 \times 10^{20} \right) \left(235 \text{ u} \right) \left(1.66 \times 10^{-27} \text{ kg} \right) \left( \frac{1 \text{ MeV}}{2.0 \times 10^2 \text{ MeV}} \right) \left( \frac{1 \text{ MeV}}{1.6 \times 10^{-13} \text{ J}} \right) = 1.3 \times 10^6 \text{ kg}$$
53. **REASONING AND SOLUTION**
   a. The number of nuclei in one gram of U-235 can be obtained as follows:

   \[
   1 \text{ gram of U-235} = \left( \frac{1}{235} \text{ mol} \right) \left( 6.02 \times 10^{23} \text{ nuclei/mol} \right) = 2.56 \times 10^{21} \text{ nuclei}
   \]

   Each nucleus yields \(2.0 \times 10^2\) MeV of energy, so we have

   \[
   E = \left(2.0 \times 10^8 \text{ eV} \right) \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) \left(2.56 \times 10^{21} \text{ nuclei} \right) = 8.2 \times 10^{10} \text{ J}
   \]

   b. If 30.0 kWh of energy are used per day, the total energy use per year is

   \[
   E_{\text{total}} = (30.0 \text{ kWh/d}) \left( \frac{3.60 \times 10^6 \text{ J}}{1 \text{ kWh}} \right) \left( \frac{365 \text{ d}}{1 \text{ yr}} \right) = 3.94 \times 10^{10} \text{ J/yr}
   \]

   The amount of U-235 needed in a year, then, is

   \[
   m = \frac{3.94 \times 10^{10} \text{ J}}{8.2 \times 10^{10} \text{ J/g}} = 0.48 \text{ g}
   \]

54. **REASONING AND SOLUTION** Using Equation 32.1,

   \[
   \text{Exposure} = \frac{q}{\left(2.58 \times 10^{-4}\right)m}
   \]

   we can get the amount of charge produced in 1 kilogram of dry air when exposed to 1.0 R of X-rays as follows:

   \[
   q = (2.58 \times 10^{-4})(1.0 \text{ R})(1 \text{ kg}) = 2.58 \times 10^{-4} \text{ C}
   \]

   Alternatively, we know that 1.0 R deposits \(8.3 \times 10^{-3}\) J of energy in 1 kg of dry air. Thus, \(8.3 \times 10^{-3}\) J of energy produces \(2.58 \times 10^{-4}\) C. To find how much energy \(E\) is required to produce \(1.60 \times 10^{-19}\) C, we set up a ratio as follows:

   \[
   \frac{E}{8.3 \times 10^{-3} \text{ J}} = \frac{1.60 \times 10^{-19} \text{ C}}{2.58 \times 10^{-4} \text{ C}} \quad \text{or} \quad E = 5.1 \times 10^{-18} \text{ J}
   \]

   Converting to eV, we find

   \[
   E = (5.1 \times 10^{-18} \text{ J})(1 \text{ eV})/(1.6 \times 10^{-19} \text{ J}) = 32 \text{ eV}
   \]